Module B3

3.1 Sinusoidal steady-state analysis (single-phase), a review

3.2 Three-phase analysis

Kirtley

Chapter 2: AC Voltage, Current and Power

2.1 Sources and Power

2.2 Resistors, Inductors, and Capacitors

Chapter 4: Polyphase systems

4.1 Three-phase systems

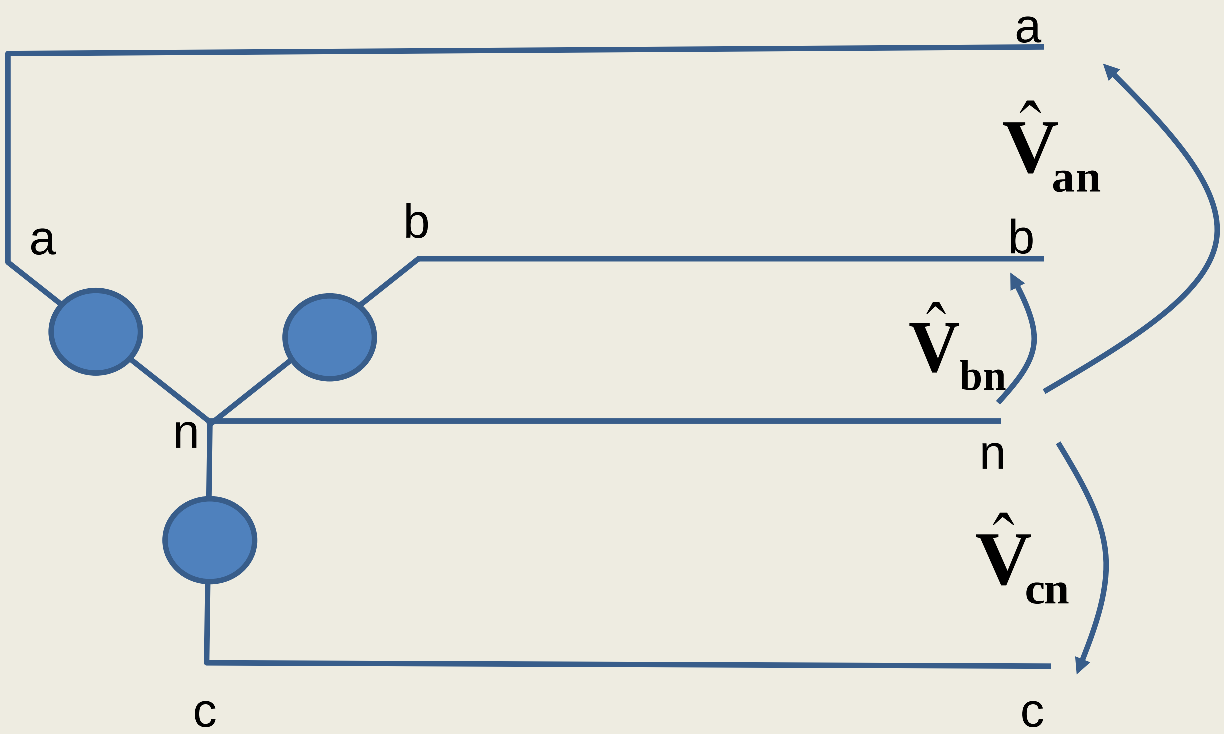
4.2 Line-Line Voltages

Three-phase power

All of what we have done in the previous slides is for "single phase" circuits. However, almost all transmission systems in the US are 3-phase AC systems (the only exceptions are a few DC transmission lines). Three-phase AC is preferred over single-phase AC because a 3-phase system provides constant power (not pulsating as we saw before) and because the cost/MW of transmission capacity is more attractive.

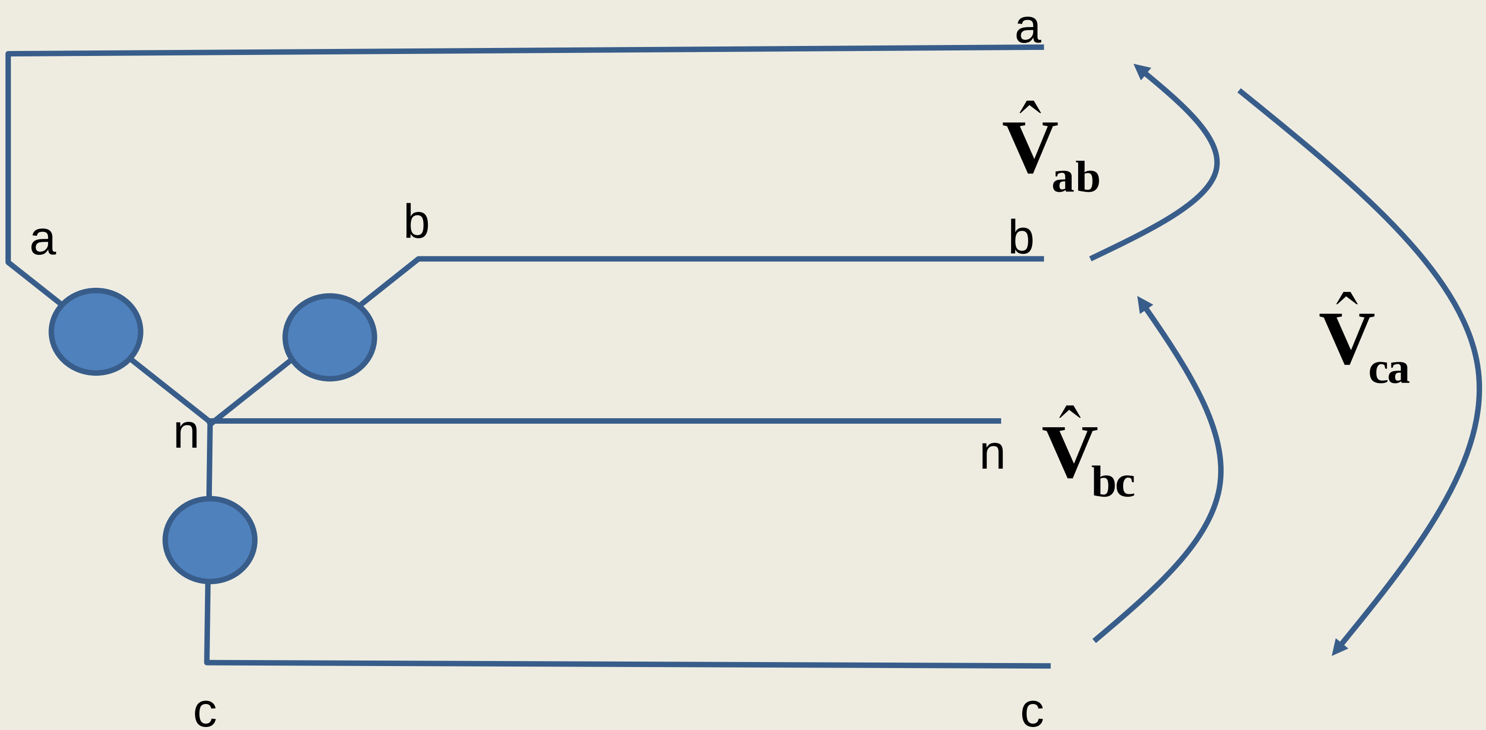
A wind generator also supplies 3-phase power. A circuit diagram for the stator of a typical 3-phase wind generator is provided in the next two slides.

Line-to-neutral voltages



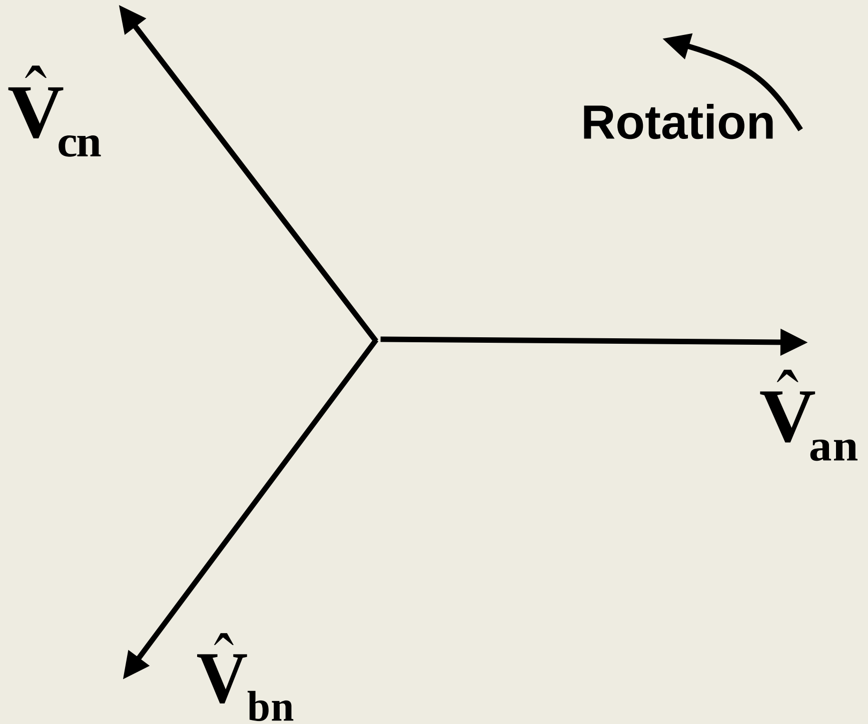
The identified voltages are referred to as "line-to-neutral voltages," or "phase voltages."

Line-to-line voltages



The identified voltages are referred to as “line-to-line voltages,” or just “line voltages.”

Phasor diagram for line-neutral voltages



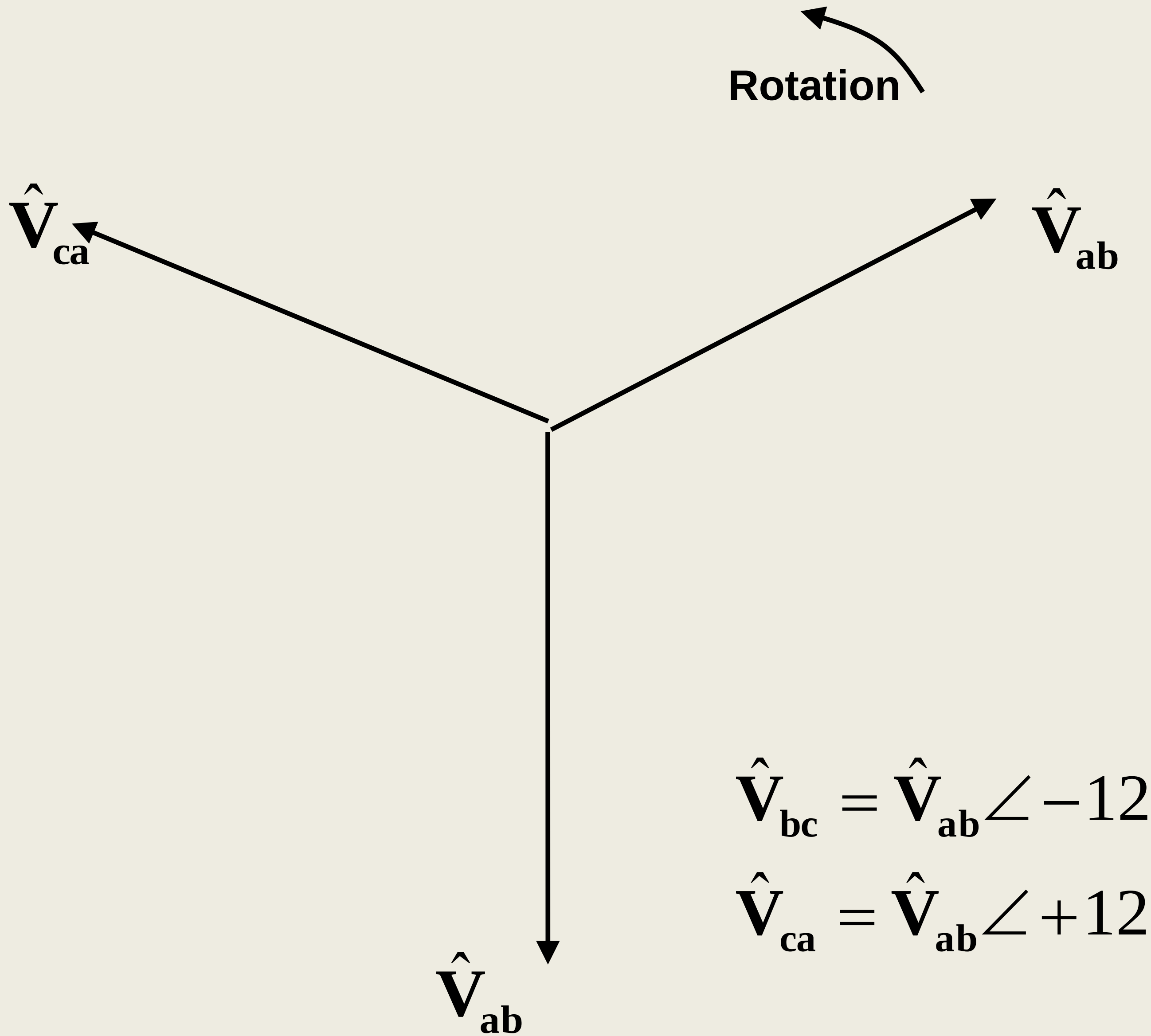
What is rotating?
➔The peak value of the sinusoid, which is projected onto one of the axes to obtain the instantaneous value of the quantity.

$$\hat{\mathbf{V}}_{\mathbf{bn}} = \hat{\mathbf{V}}_{\mathbf{an}} \angle -120°$$

$$\hat{\mathbf{V}}_{\mathbf{cn}} = \hat{\mathbf{V}}_{\mathbf{an}} \angle +120°$$

www.animations.physics.unsw.edu.au/jw/phasor-addition.html

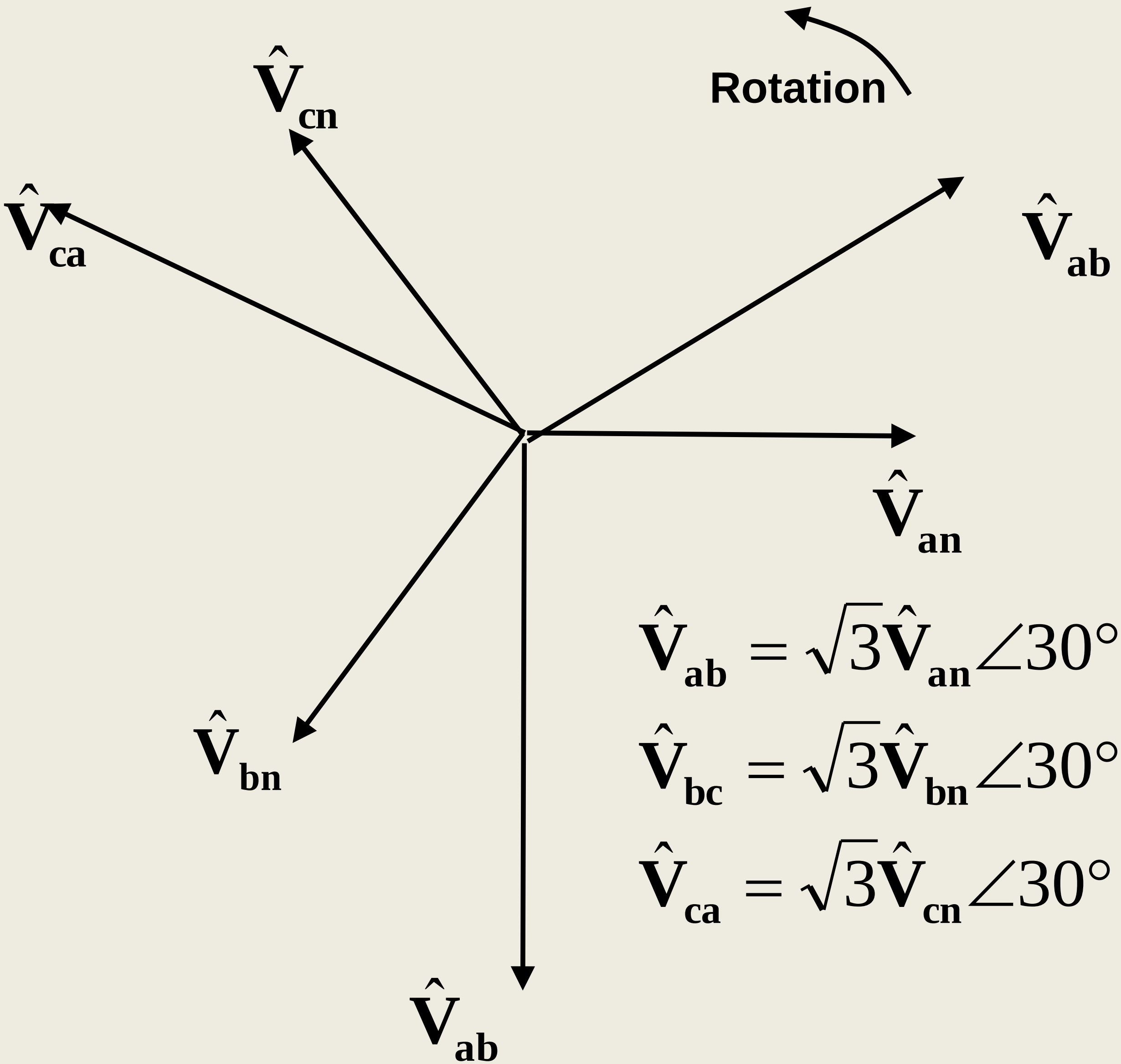
Phasor diagram for line-line voltages



$$\hat{V}_{bc} = \hat{V}_{ab}\angle -120^\circ$$

$$\hat{V}_{ca} = \hat{V}_{ab}\angle +120^\circ$$

Relating phase and line voltages



$$\hat{\mathbf{V}}_{ab} = \sqrt{3}\hat{\mathbf{V}}_{an}\angle 30^\circ$$

$$\hat{\mathbf{V}}_{bc} = \sqrt{3}\hat{\mathbf{V}}_{bn}\angle 30^\circ$$

$$\hat{\mathbf{V}}_{ca} = \sqrt{3}\hat{\mathbf{V}}_{cn}\angle 30^\circ$$

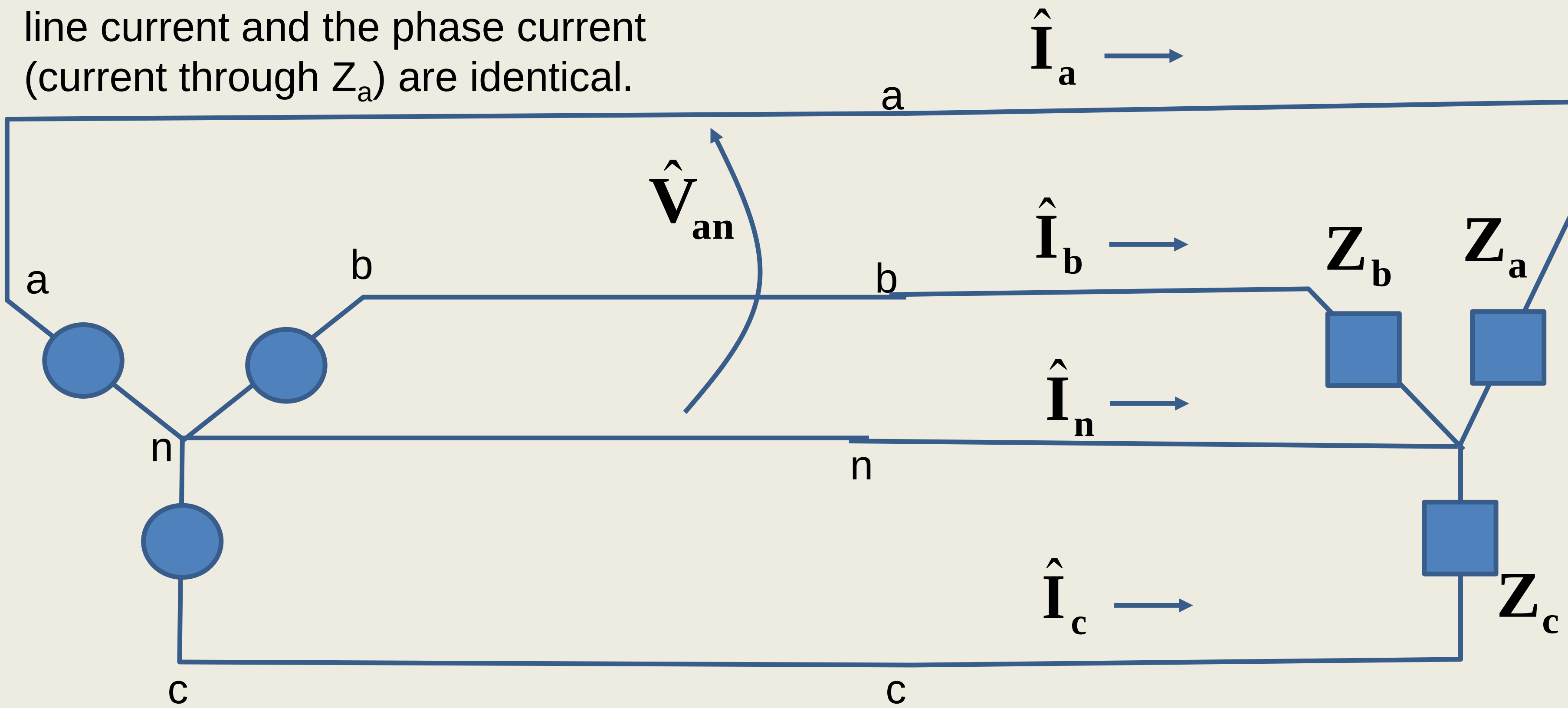
Balanced conditions

Balanced 3-phase conditions have:

- Line and phase voltages related as in previous slides.
- $Z_a = Z_b = Z_c$

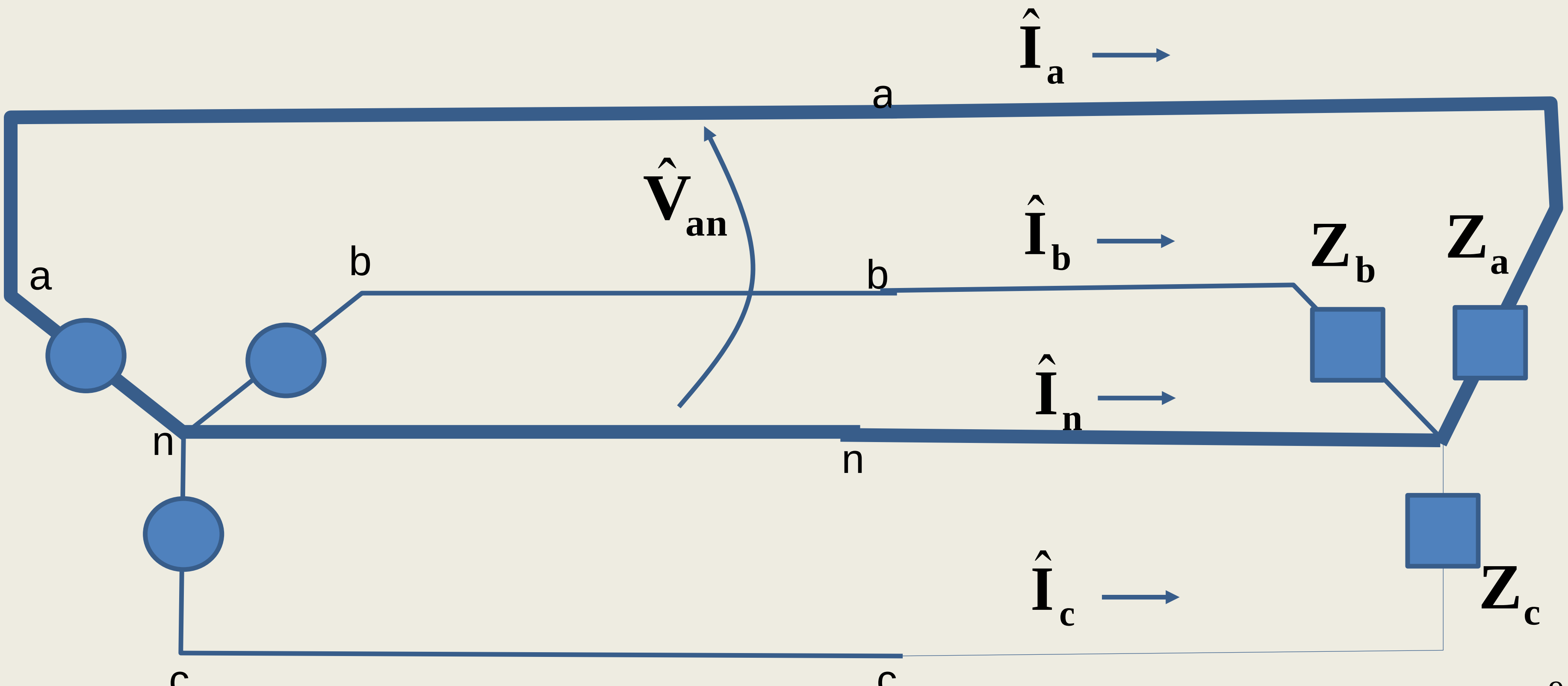
This results in: $\hat{\mathbf{I}}_b = \hat{\mathbf{I}}_a \angle -120°, \quad \hat{\mathbf{I}}_c = \hat{\mathbf{I}}_a \angle +120°, \quad \hat{\mathbf{I}}_n = 0$

Note: In Wye-connected loads, the line current and the phase current (current through Z_a) are identical.

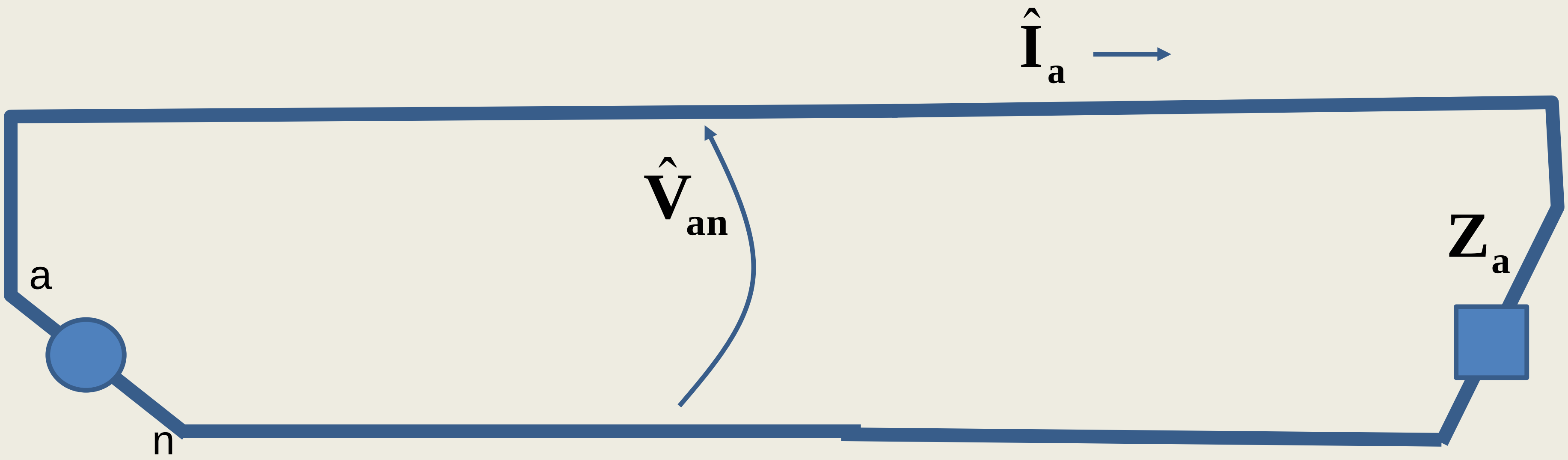


Per-phase analysis

Under balanced conditions, we may perform single-phase analysis on a "lifted-out" a-phase and neutral circuit, as shown below.



Per-phase analysis

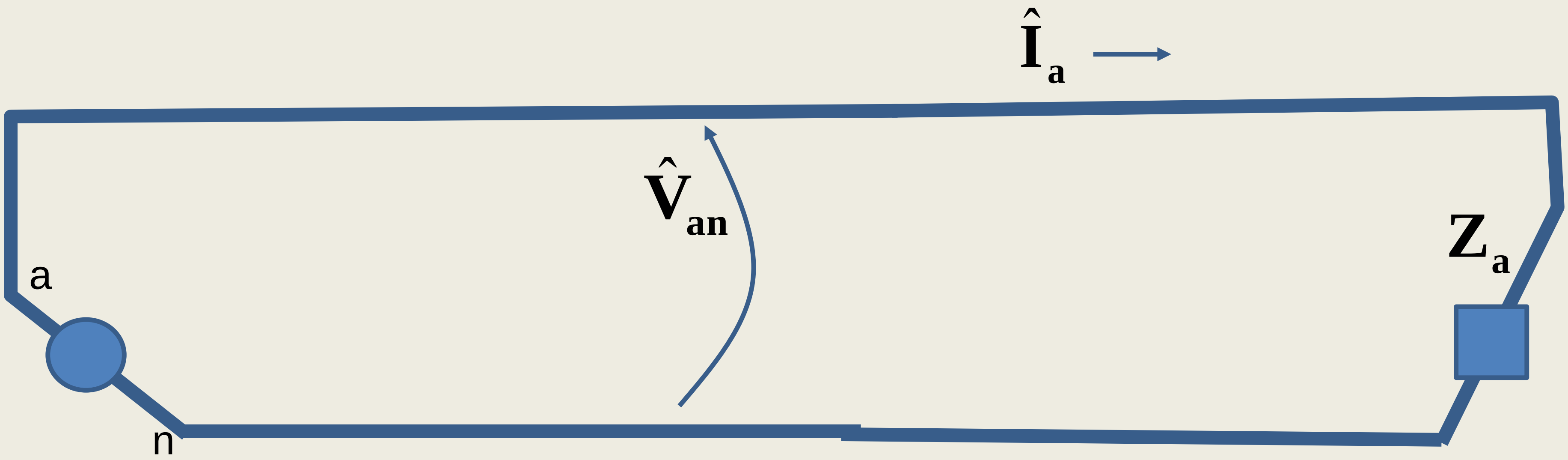


Now it is clear that:

$$\hat{\mathbf{I}}_\mathbf{a} = \frac{\hat{\mathbf{V}}_\mathbf{an}}{Z_a} \qquad \mathbf{S}_{1\varphi} = \hat{\mathbf{V}}_\mathbf{an}\hat{\mathbf{I}}_\mathbf{a}^* = P_{1\phi} + jQ_{1\phi}$$

Also, we still have: $P_{1\phi} = V_{an} I_a \cos\theta,$ $\qquad Q_{1\phi} = V_{an} I_a \sin\theta$

Per-phase analysis



Following the single-phase analysis, one may then compute the 3-phase quantities according to:

$$\mathbf{S}_{3\varphi} = 3\mathbf{S}_{1\varphi} \Rightarrow P_{3\phi} = 3P_{1\phi}, \qquad Q_{3\phi} = 3Q_{1\phi}$$

Three phase power relations

The previous power relations utilize line-to-neutral voltages and line currents. Power may also be computed using line voltages, as developed in what follows:

$$P_{1\phi} = V_{an} I_a \cos\theta$$

$$\hat{\mathbf{V}}_{\mathbf{ab}} = \sqrt{3}\hat{\mathbf{V}}_{\mathbf{an}}\angle 30^\circ \Rightarrow V_{ab} = \sqrt{3}V_{an} \Rightarrow V_{an} = \frac{V_{ab}}{\sqrt{3}}$$

$$P_{1\phi} = \frac{V_{ab}}{\sqrt{3}} I_a \cos\theta = \frac{V_{ab}}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} I_a \cos\theta = \frac{V_{ab}\sqrt{3}}{3} I_a \cos\theta$$

$$P_{3\phi} = 3P_{1\phi} = 3\frac{V_{ab}\sqrt{3}}{3} I_a \cos\theta = \sqrt{3}V_{ab} I_a \cos\theta$$

Likewise, we may develop that

$$Q_{3\phi} = \sqrt{3}V_{ab} I_a \sin\theta$$

Three phase power relations

In summary:

$$\mathbf{S}_{3\varphi} = 3\mathbf{S}_{1\varphi} \Rightarrow P_{3\phi} = 3P_{1\phi}, \qquad Q_{3\phi} = 3Q_{1\phi}$$

$$P_{1\phi} = V_{an} I_a \cos\theta \qquad Q_{1\phi} = V_{an} I_a \sin\theta$$

$$P_{3\phi} = \sqrt{3} V_{ab} I_a \cos\theta \qquad Q_{3\phi} = \sqrt{3} V_{ab} I_a \sin\theta$$

Note 1: In Wye-connections, the power factor angle θ is the angle by which the line-to-neutral voltage $\hat{\mathbf{V}}_{\mathbf{an}}$ leads the phase current $\hat{\mathbf{I}}_{\mathbf{a}}$. It is not the angle by which the line-to-line voltage $\hat{\mathbf{V}}_{\mathbf{ab}}$ leads the phase current. More generally, the power factor angle at two terminals is the angle by which the voltage across those terminals leads the current into the positive terminal.

Note 2: The text uses notation V_{LL} for $V_{ab.}$