EE 303, Quiz 8, Spring 2017, Dr. McCalley Name:____________________________________

Time: 15 minutes, closed book, closed notes, closed calculator

A two-unit system is given by the following data.

$$C_1\left(P_{g1}\right) = 0.015 \cdot \left(P_{g1}\right)^2 + 2 \cdot \left(P_{g1}\right) + 6$$
$$C_2\left(P_{g2}\right) = 0.020 \cdot \left(P_{g2}\right)^2 + 6 \cdot \left(P_{g2}\right) + 4$$

The demand is 300MW

1. (12) Form the Lagrangian function assuming that both units are operating between their respective upper and lower limits.
2. (12) Write the KKT conditions that must be satisfied at the optimal solution to this problem, assuming that both units are operating between their respective upper and lower limits.
3. (12) Set up the linear matrix equation to solve the economic dispatch problem for this system, assuming that both units are operating between their respective upper and lower limits. Do NOT solve the system of equations.
4. The solution to equations identified in part (2) is $P_{g1} = 228.57MW,\ P_{g2} = 71.43MW$. Assume that each unit has a minimum generation capability of 80 MW. Answer the following eight questions:
   (a) (8) Is the solution to the equations identified in part (2) feasible? If not, indicate why.
   (b) (8) Identify the optimal feasible solution in terms of values for P_{g1} and P_{g2}.
   (c) (8) Identify the incremental costs of each unit at the optimal feasible solution
   (d) (8) Identify the system lambda (also known as system incremental cost) at the optimal feasible solution.
   (e) (8) Would the total cost of supply increase, decrease, or stay the same (relative to the total cost corresponding to the optimal feasible solution of 300 MW) if the load changes to 301 MW and the system is reoptimized?
   (f) (8) Would the total cost of supply increase, decrease, or stay the same (relative to the total cost corresponding to the optimal feasible solution of 300 MW) if the load remains at 300MW and the minimum generation capability on unit 1 is changed to 79MW ?
   (g) (8) Would the total cost of supply increase, decrease, or stay the same (relative to the total cost corresponding to the optimal feasible solution of 300 MW) if the load remains at 300MW and the minimum generation capabilities on unit 2 is changed to 79MW ?
   (h) (8) The complementary conditions for units 1 and 2 minimum generation are $\mu_1(P_{g1}-80)=0$ and $\mu_2(P_{g2}-80)=0$, respectively. At the optimal feasible solution, which of the parameters μ_1 and μ_2 are zero, if any.

Solution:

1. $$\underline{\underline{L\left[P_{g1}, P_{g2}, \lambda\right] = C_1\left(P_{g1}\right) + C_2\left(P_{g2}\right) - \lambda\left[P_{g1} + P_{g2} - 300\right]}}$$

2. $$\underline{\underline{\frac{\partial L}{\partial P_{g1}} = 0.03P_1 + 2 - \lambda = 0}}$$

$$\underline{\underline{\frac{\partial L}{\partial P_{g2}} = 0.04P_2 + 6 - \lambda = 0}}$$

$$\underline{\underline{\frac{\partial L}{\partial \lambda} = P_1 + P_2 - 300 = 0}}$$

3. $$\underline{\underline{\begin{bmatrix} 0.03 & 0 & -1 \\ 0 & 0.04 & -1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_{g1} \\ P_{g2} \\ \lambda \end{bmatrix} = \begin{bmatrix} -2 \\ -6 \\ 300 \end{bmatrix}}} \text{ or } \underline{\underline{\begin{bmatrix} 0.03 & 0 & 1 \\ 0 & 0.04 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_{g1} \\ P_{g2} \\ -\lambda \end{bmatrix} = \begin{bmatrix} -2 \\ -6 \\ 300 \end{bmatrix}}}$$

4. (a) It is not feasible because Generator 2 is below its minimum capability

(b) $P_{g2} = 80MW, P_{g1} = 300 - 80 \Rightarrow \underline{\underline{220MW = P_{g1}}}$

(c) $\frac{\partial C_1}{\partial P_{g1}} = 0.03P_{g1} + 2 = 0.03 \cdot (220) + 2 = \underline{\underline{8.6\$/MW-hr}} = IC_1$

$\frac{\partial C_2}{\partial P_{g2}} = 0.04P_{g2} + 6 = 0.04 \cdot (80) + 6 = \underline{\underline{9.2\$/MW-hr}} = IC_2$

(d) lambda = <u>8.6$/MW-hr </u>(Since unit 1 is regulating)

(e) Total cost would increase.

(f) Total cost would stay the same.

(g) Total cost would decrease.

(h) μ_1 is zero.