Week	Quiz?	Lecture number	Module # &/or Text Chapter	Description	Material to download	Homework	Quiz Solutions	Other readings
1/10		1	B2, Ch 1	Introduction to energy systems	Module B2, EnergyOverview: PDF			
1/12		2	B2, Ch 1	Energy, power, and the environment		Module B3, problems 1-6, due 1/24		
1/17		3	B3, Ch 2	Three phase analysis	Module B3			
1/19		4	B3, Ch 4	Three phase analysis				
1/24	Quiz 1	5	B3, Ch 4	Three phase analysis				
1/26		6	B3, Ch 4	Three phase analysis				
1/31		7	B3, Ch 4	Three phase analysis				
2/2	Quiz 2	8	Ch 5,6	Mutual inductance, transformers				
2/7		9	Ch 5,6	Mutual inductance, transformers				
2/9		10	Ch 5,6	Mutual inductance, transformers				
2/14	Quiz 3	11	B4, Ch 7	Per unit analysis	Module B4			
2/16		12	B4, Ch 7	Per unit analysis				
2/21		13	G1, 9	Electric power generation and machine controls	Module G1			
2/23	Quiz 4	14	G1, 9	Electric power generation and machine controls				
2/28		15	T1	Electric power transmission	Module T1			
3/2		16	T1	Electric power transmission				
3/7	Quiz 5	17	T7, Ch 11	Power flow, Newton-Raphson	Module T7			
3/9		18	T7, Ch 11	Power flow, Newton-Raphson				

Module B3

3.1 Sinusoidal steady-state analysis (single-phase), a review

3.2 Three-phase analysis

Kirtley

Chapter 2: AC Voltage, Current and Power

2.1 Sources and Power

2.2 Resistors, Inductors, and Capacitors

Chapter 4: Polyphase systems

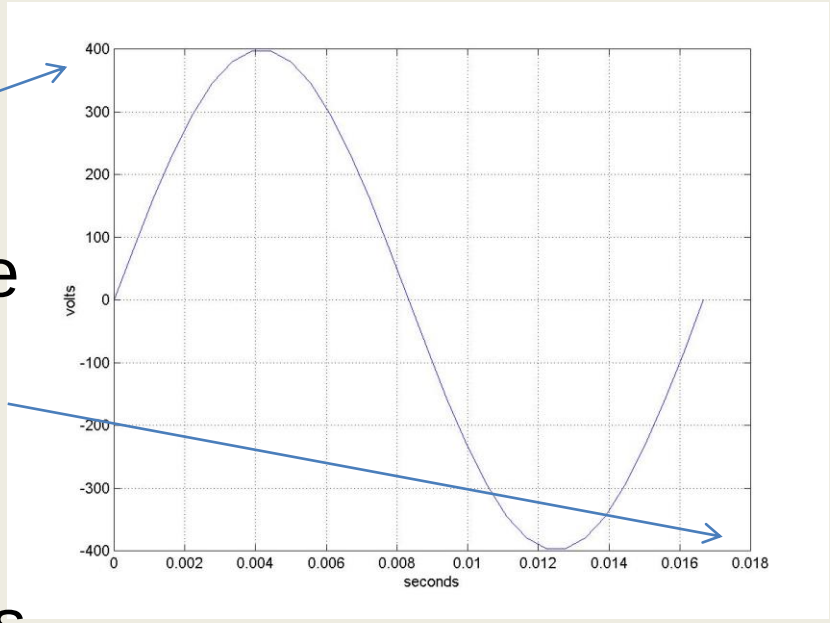
4.1 Three-phase systems

4.2 Line-Line Voltages

Basic concepts of electric power

With exception of a few High Voltage DC circuits, the entire electric grid operates to deliver “alternating current” (AC). An “AC voltage” appears below.

The maximum value of the waveform is 400 volts.

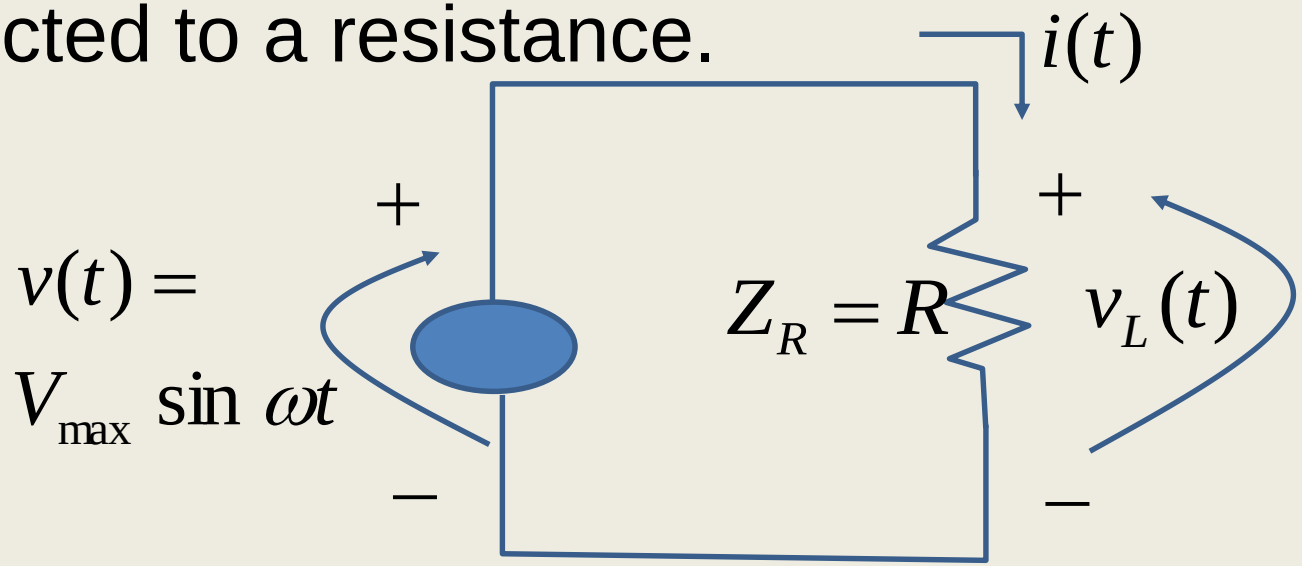


The waveform completes one cycle in 0.0167 sec. This is the “period” of the waveform and is 1/f. So, f=1/0.0167=60Hz or 60 cycles/sec.

Since 1 cycle completes 2π radians, ω=(2π rad/cyc)*60cyc/sec=377rad/sec. We can therefore express this voltage as $v(t) = V_{\max} \sin \omega t = 400 \sin(377t)$

KVL and Ohm's Law

The below is a circuit model of a voltage source connected to a resistance.



We apply Kirchoff's Voltage Law (KVL): the sum of voltages around the loop must be 0:

$$v(t) - v_L(t) = 0$$

$$\Rightarrow v_L(t) = v(t)$$

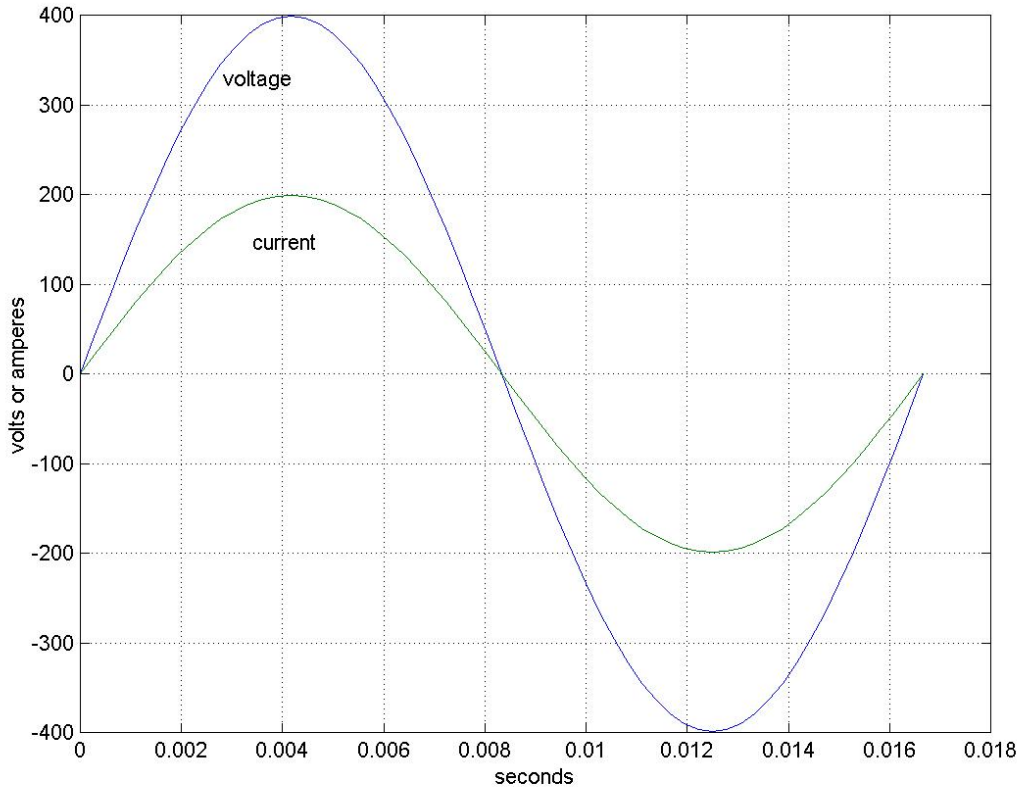
We may apply Ohm's Law to obtain the current:

$$i(t) = \frac{v_L(t)}{Z_R} = \frac{V_{\max}}{R} \sin \omega t$$

Voltage and current for resistive cct

Compare voltage & current. Assume R=2 ohms.

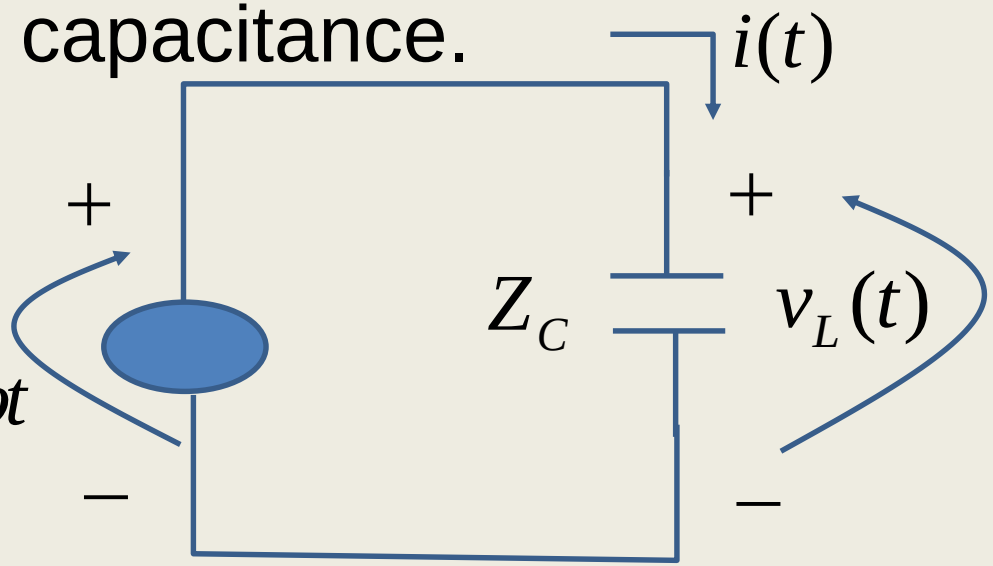
$$v_L(t) = V_{\max} \sin \omega t \qquad i(t) = \frac{V_{\max}}{R} \sin \omega t = \frac{400}{2} \sin 377t$$



Current & voltage cross 0 simultaneously

Capacitance

The below is a circuit model of a voltage source connected to a capacitance.



Again, from KVL:

$$v(t) - v_L(t) = 0$$

$$\Rightarrow v_L(t) = v(t)$$

The current is given by:

$$i(t) = C\frac{dv_L(t)}{dt} = \omega C V_{\max} \cos \omega t$$

$$= \omega C V_{\max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= \frac{V_{\max}}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

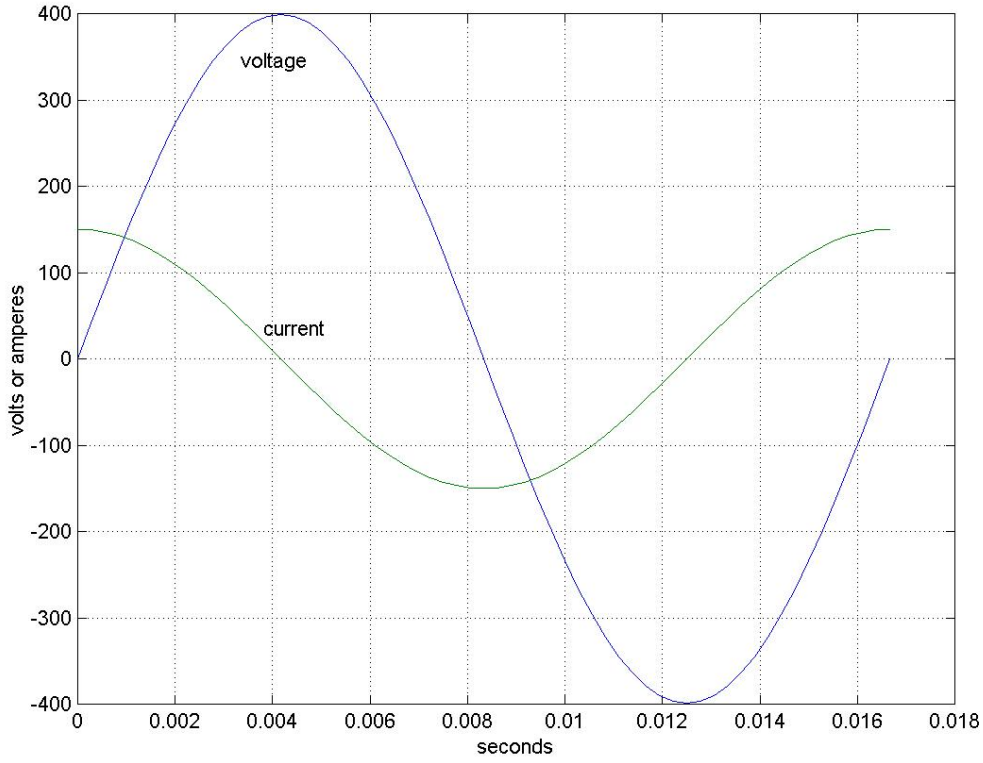
$\cos \omega t = \sin(\omega t + \pi/2)$

Voltage and current for capacitive cct

Compare voltage & current. Assume C=0.001farad.

$$v_L(t) = V_{\max} \sin \omega t \qquad i(t) = C\omega V_{\max} \sin\left(\omega t + \frac{\pi}{2}\right) =$$

$$.001 \times 377 \times 400 \sin\left(\omega t + \frac{\pi}{2}\right) = 150.8 \sin\left(\omega t + \frac{\pi}{2}\right)$$

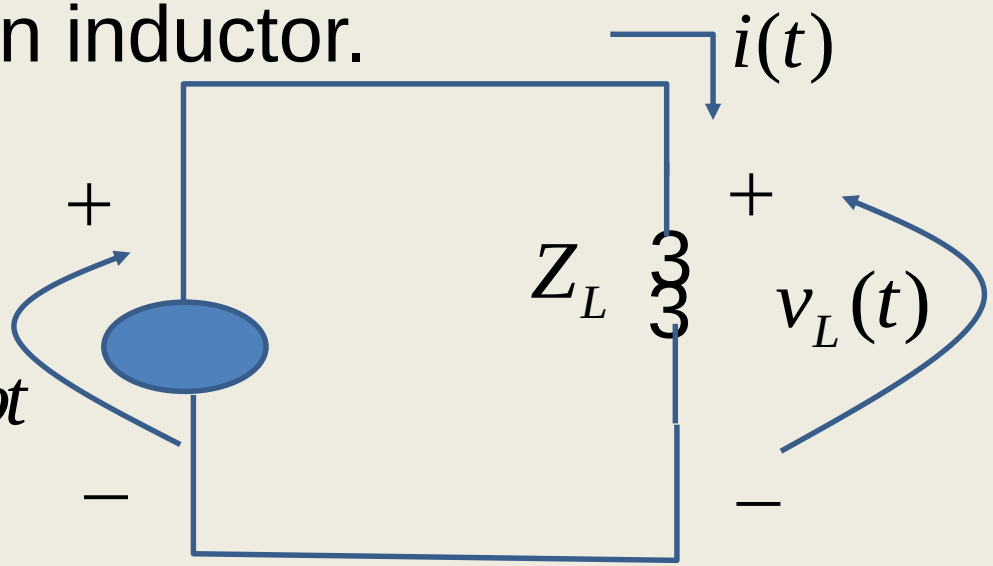


Current crosses 0 before voltage. We say "current leads voltage."

If we multiply the time axis by ω=377 rad/sec, we would see that current leads voltage by π/2 radians.

Inductance

The below is a circuit model of a voltage source connected to an inductor.



Again, from KVL:

$$v(t) - v_L(t) = 0$$

$$\Rightarrow v_L(t) = v(t)$$

The current is given by:

$$i(t) = \frac{1}{L}\int v_L(t) = \frac{-V_{\max}}{\omega L}\cos \omega t$$

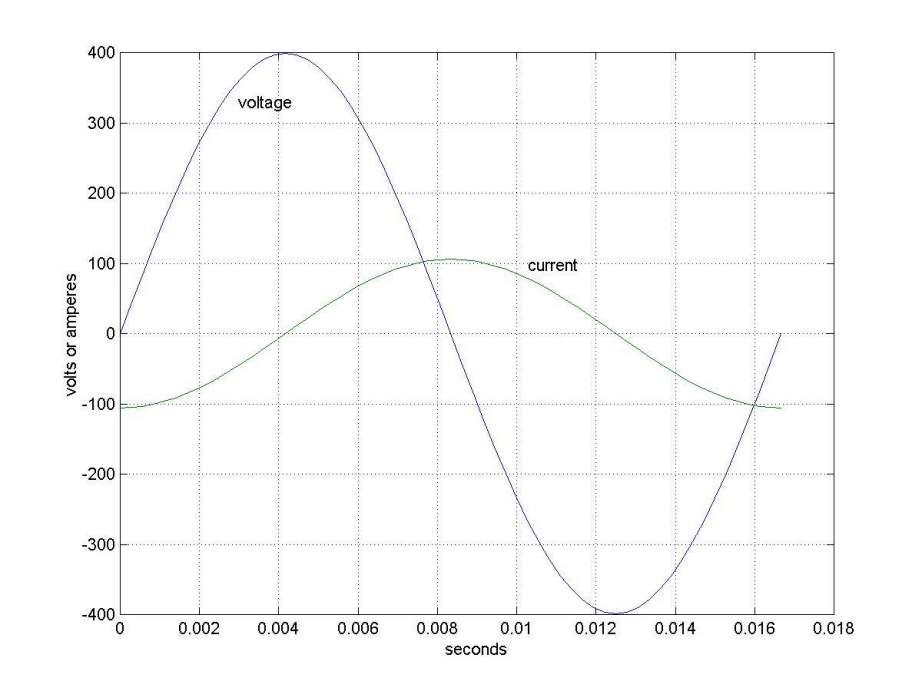
$$= \frac{-V_{\max}}{\omega L}\sin\left(\omega t + \frac{\pi}{2}\right) = \frac{V_{\max}}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

Voltage and current for inductive cct

Compare voltage & current. Assume L=.01 henry.

$$v_L(t) = V_{\max} \sin \omega t \qquad i(t) = \frac{V_{\max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= \frac{400}{377 \times 0.01} \sin\left(\omega t - \frac{\pi}{2}\right) = 106.1 \sin\left(\omega t - \frac{\pi}{2}\right)$$



Current crosses 0 after voltage. We say "current lags voltage."

If we multiply the time axis by ω=377 rad/sec, we would see that current lags voltage by π/2 radians.

Impedance

$v_L(t) = V_{\max} \sin \omega t$

RESISTIVE: $i(t) = \frac{v_L(t)}{Z_R} = \frac{V_{\max}}{R} \sin \omega t$ It is clear that Z_R=R.

CAPACITIVE: $i(t) = \frac{v_L(t)}{Z_C} = \frac{V_{\max}}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$

INDUCTIVE: $i(t) = \frac{v_L(t)}{Z_L} = \frac{V_{\max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$

Is $Z_C = 1/\omega C$? Is $Z_L = \omega L$?

No, because we also need to take care of the phase shifts.

Impedance

To do so, let's utilize the imaginary unit "j" which has the following properties:

- It is the square root of -1: $j = \sqrt{-1}$
- From the Euler relation: $e^{j\phi} = \cos\phi + j\sin\phi$
  With ϕ=π/2, we obtain: $e^{j\pi/2} = \cos\pi/2 + j\sin\pi/2 = j$
  Therefore, j has magnitude of "1" and angle π/2.

Therefore

$$j\sin\omega t = \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \frac{\sin\omega t}{j} = \sin\left(\omega t - \frac{\pi}{2}\right)$$

Then, with Z_C=1/jωC, Z_L=jωL, we get correct current expressions.

Impedance

RESISTIVE: $$i(t) = \frac{v_L(t)}{Z_R} = \frac{V_{\max}}{R} \sin \omega t$$

CAPACITIVE: $$i(t) = \frac{v_L(t)}{Z_C} = \frac{V_{\max} \sin \omega t}{1/\, j\omega C} = V_{\max}\, j\omega C \sin \omega t = V_{\max}\, \omega C \sin\left(\omega t + \frac{\pi}{2}\right)$$

INDUCTIVE: $$i(t) = \frac{v_L(t)}{Z_L} = \frac{V_{\max} \sin \omega t}{j\omega L} = \frac{V_{\max}}{\omega L} \frac{\sin \omega t}{j} = \frac{V_{\max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

Z, called impedance, is a generalization of resistance that accounts for the effects of capacitance & inductance.

Impedance

In a purely resistive circuit, $Z=Z_R=R$.
In a purely capacitive circuit, $Z=Z_C=1/j\omega C$.
In a purely inductive circuit, $Z=Z_L=j\omega L$.

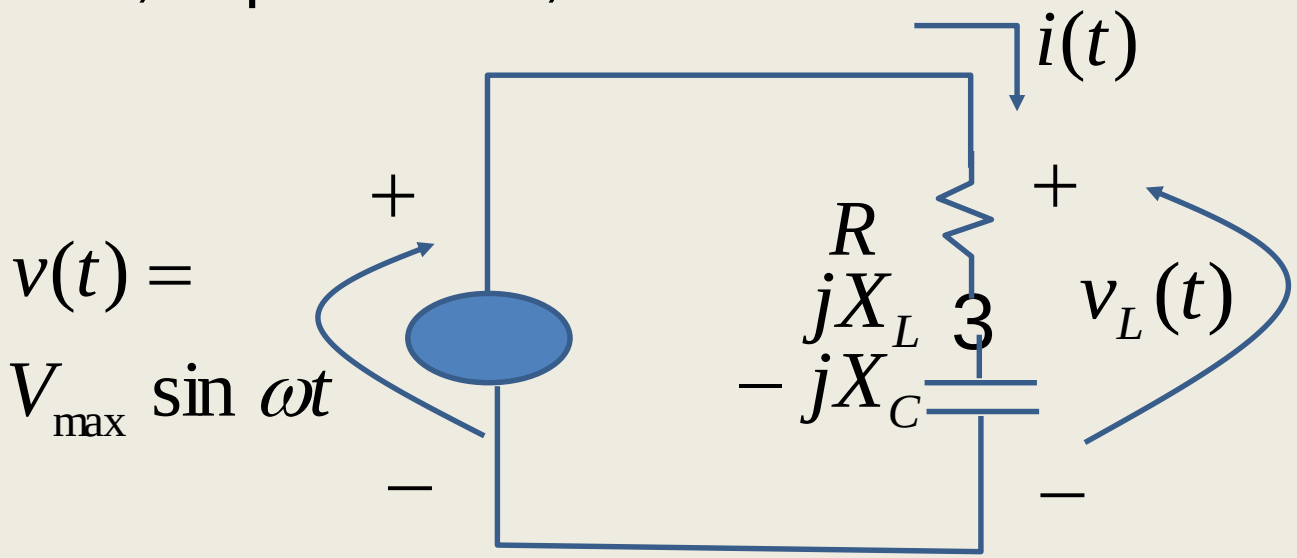
We also define:

- Capacitive reactance: $X_C=1/\omega C$
- Inductive reactance: $X_L=\omega L$

In a purely resistive circuit, $Z=Z_R=R$.
In a purely capacitive circuit, $Z=Z_C=1/j\omega C=X_C/j$.
But recall $j=\sqrt{-1}$, then $j^2=-1$. So $(X_C/j)*(j/j)=-jX_C$.
So $Z=Z_C=-jX_C$.
In a purely inductive circuit, $Z=Z_L=j\omega L=jX_L$.

Impedance

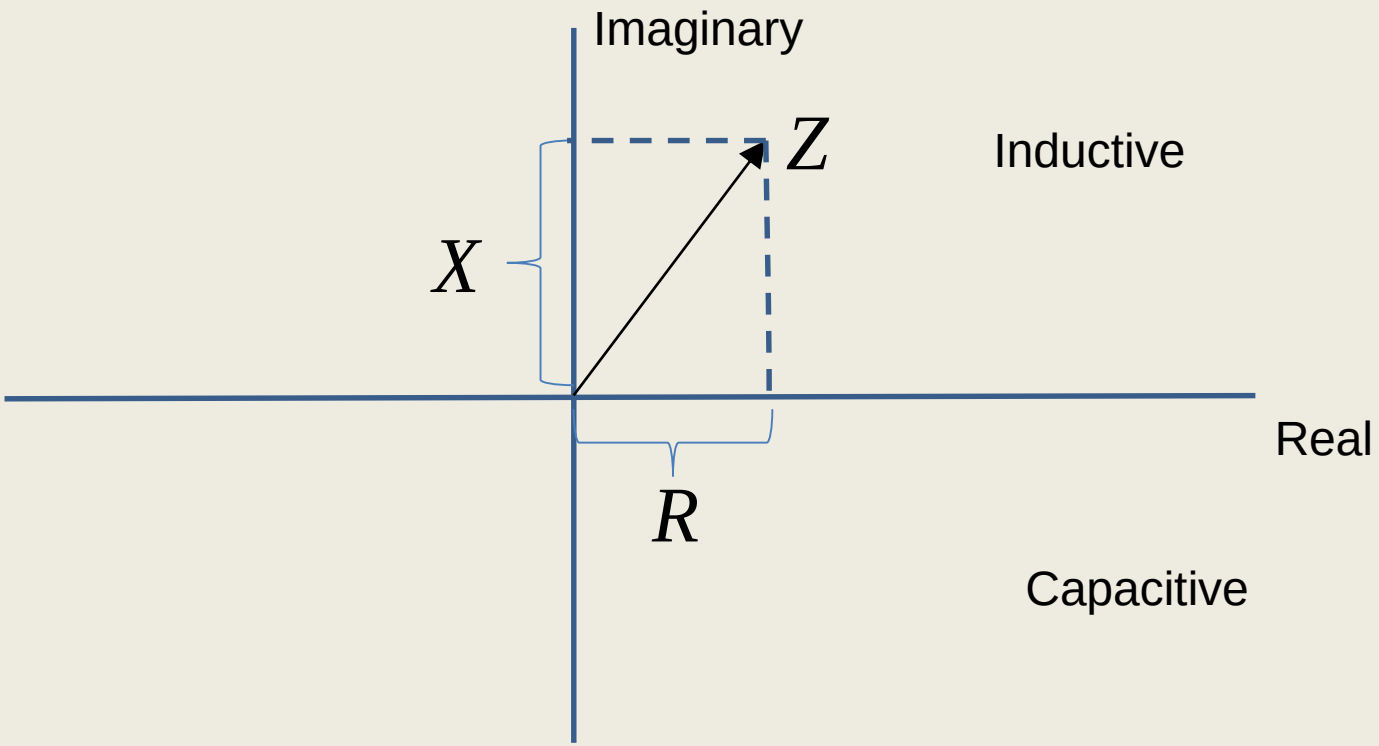
It is possible to have an impedance that is a combination of resistance, capacitance, and inductance.



Then $Z = R + jX_L - jX_C = R + j(X_L - X_C) = R + jX$

The impedance, Z, can be thought of as a vector on the complex plane.

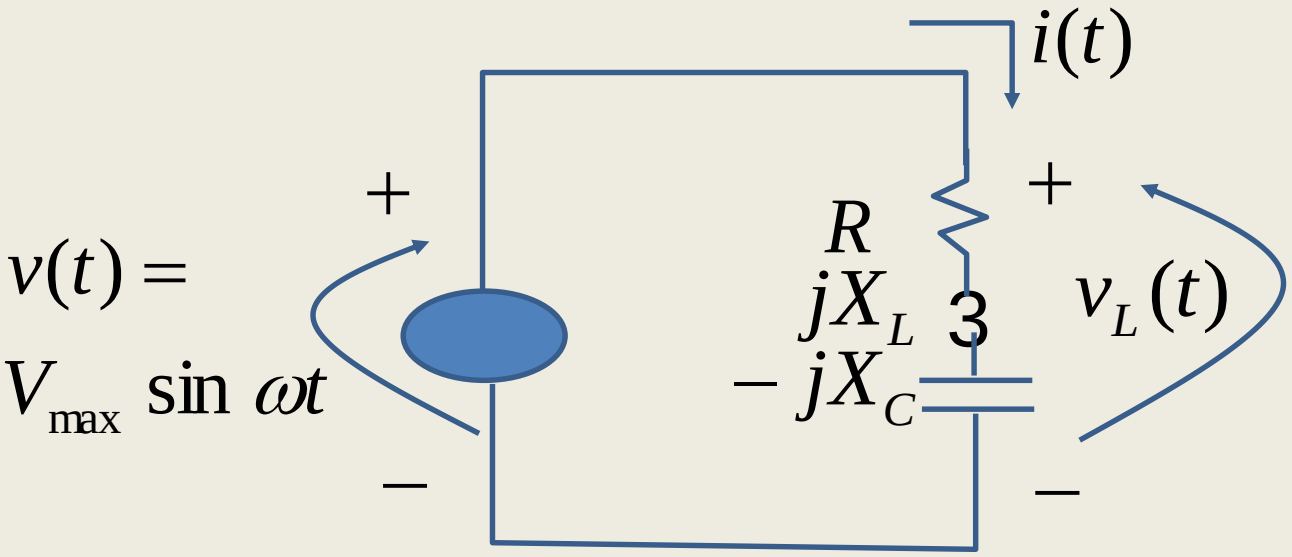
Impedance



$$Z = R + jX_L - jX_C = R + j\left(X_L - X_C\right) = R + jX$$

Example 1

Let R=2Ω, C=0.001farad, L=0.01henry, V_{max}=400.



Then Z=R+jωL-j/ωC $= 2 + j377 \times 0.01 - j/(377 \times 0.001)$

$= 2 + j(3.77 - 2.6525) = 2 + j1.1048$

The current will be

$$i(t) = \frac{v_L(t)}{Z} = \frac{V_{\max} \sin \omega t}{Z} = \frac{400 \sin \omega t}{2 + j1.1048}$$

But how to get this?

Example 1

$$i(t) = \frac{400 \sin \omega t}{2 + j1.1048}$$

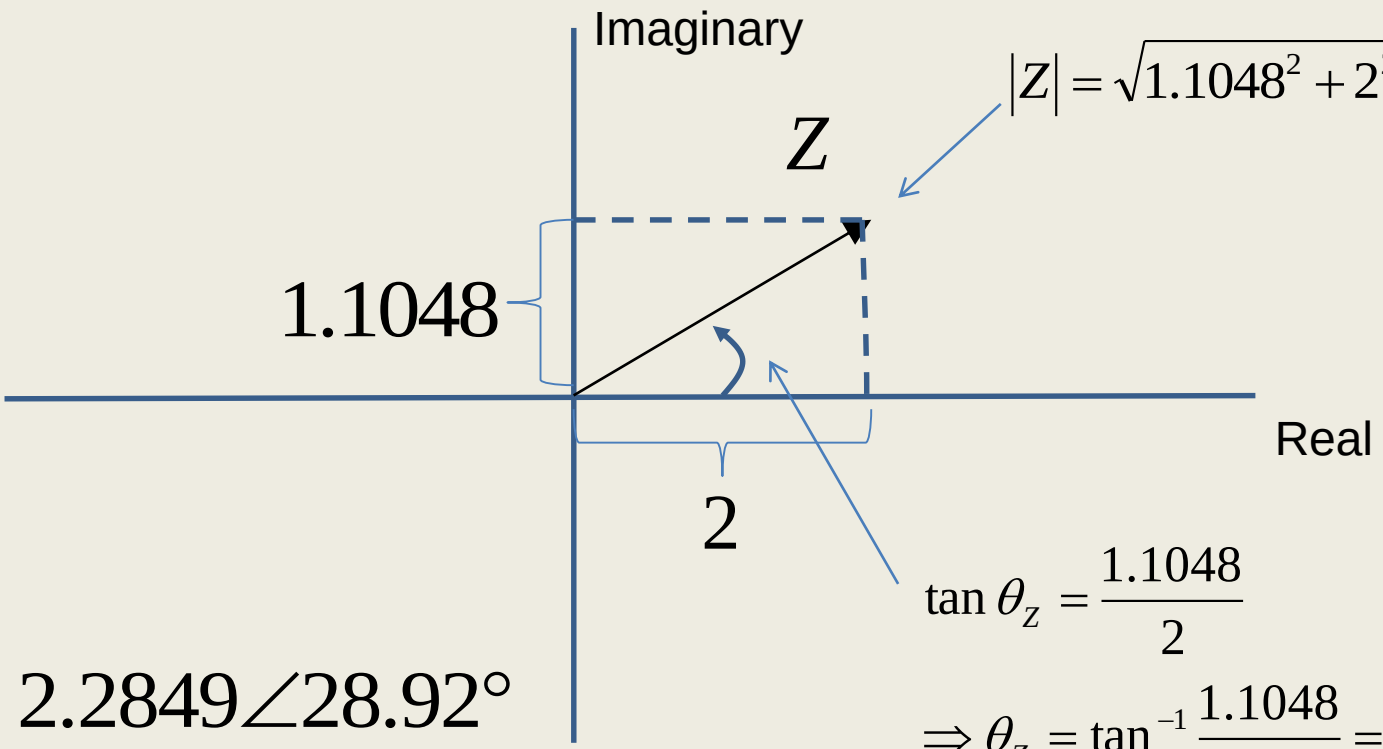
You can divide each numerical point in the sinusoidal numerator by the complex denominator, then take the magnitude and angle of the resulting complex number.

But observe that only the magnitude changes as t increases. This is because the "phasor angle" of the numerator and the vector angle of the denominator both remain constant with t. That is:

$$i(t) = \frac{400 \sin \omega t}{2 + j1.1048} \qquad = \frac{400 \angle 0 \sin \omega t}{|Z| \angle \theta_Z}$$

➔But what is $|Z| \angle \theta_Z$?

Impedance



$$|Z| = \sqrt{1.1048^2 + 2^2} = 2.2849$$

So:

$$|Z| \angle \theta_Z = 2.2849 \angle 28.92°$$

$$\tan \theta_Z = \frac{1.1048}{2}$$

$$\Rightarrow \theta_Z = \tan^{-1} \frac{1.1048}{2} =$$

$$0.5047 \text{radians} = 28.92°$$

And from the previous slide:

$$i(t) = \frac{400 \angle 0 \sin \omega t}{|Z| \angle \theta_Z} = \frac{400}{|Z|} \left(\angle (0 - \theta_Z) \right) \sin \omega t$$

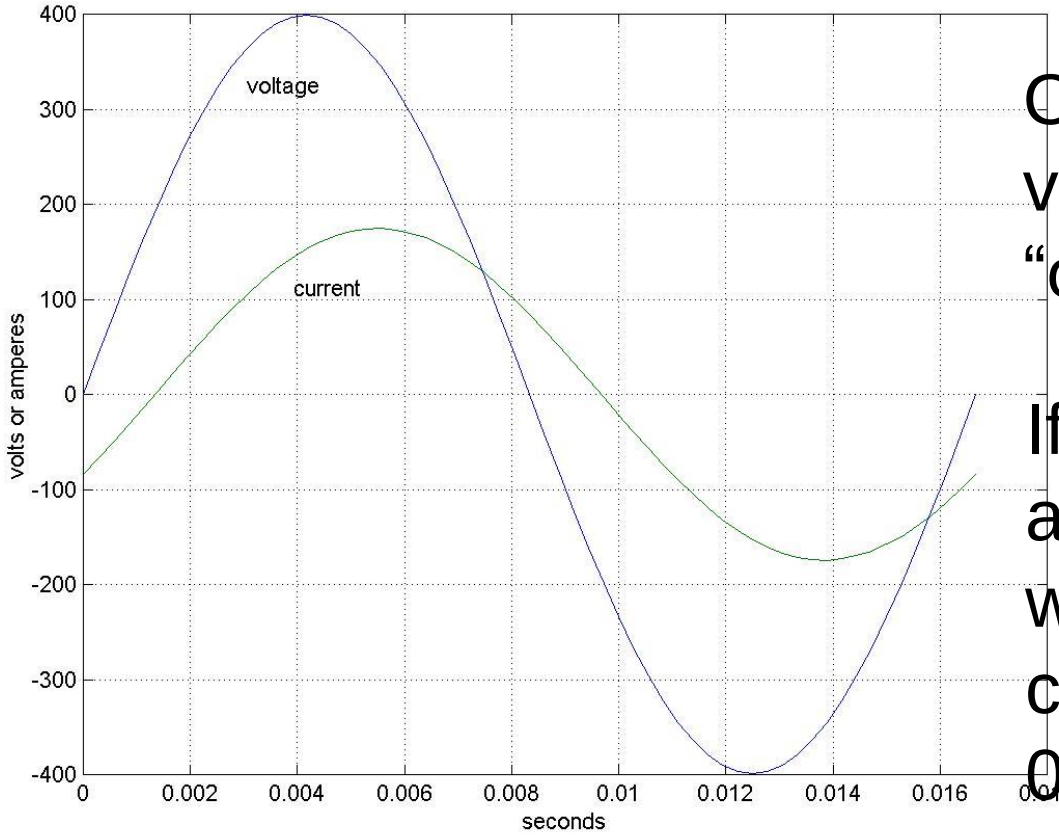
$$= \frac{400}{2.2849} \left(\angle -28.92° \right) \sin \omega t = 175.06 \angle -28.92° \sin \omega t$$

Impedance

So $i(t) = 175.06\angle -28.92° \sin \omega t$

Recalling that 28.92°=0.5047 radians, we write the above as:

$i(t) = 175.06 \sin(\omega t - 0.5047)$ ➔We can plot this:



Current crosses 0 after voltage. We say "current lags voltage."

If we multiply the time axis by ω=377 rad/sec, we would see that current lags voltage by 0.5047 radians.

Phasors

Let's review what we just did:

$$i(t) = \frac{v_L(t)}{Z} = \frac{V_{\max}\sin\omega t}{Z} = \frac{400\sin\omega t}{2 + j1.1048}$$

$$= \frac{400\angle 0\sin\omega t}{|Z|\angle\theta_Z} = \frac{400}{|Z|}\left(\angle(0-\theta_Z)\right)\sin\omega t = \frac{400}{2.2849}\left(\angle -28.92^\circ\right)\sin\omega t$$

$$= 175.06\angle -28.92^\circ \sin\omega t = 175.06\sin\left(\omega t - 0.5047\right)$$

Notice that nothing ever happened to the ωt part. Nor did the sinusoidal function change. This means that the frequency of the current is the same as the frequency of the voltage. If this is the case, then why do we need to carry the sinωt through the calculations? Why not just drop it? This is what we do when we use phasor notation, as in the next slide.

Phasors

Define:

Phasor for source voltage: $\hat{\mathbf{V}}_S = V_S \angle \phi_{V_S}$

Phasor for load voltage: $\hat{\mathbf{V}}_L = V_L \angle \phi_{V_L}$

Phasor for current: $\hat{\mathbf{I}} = I \angle \phi_I$

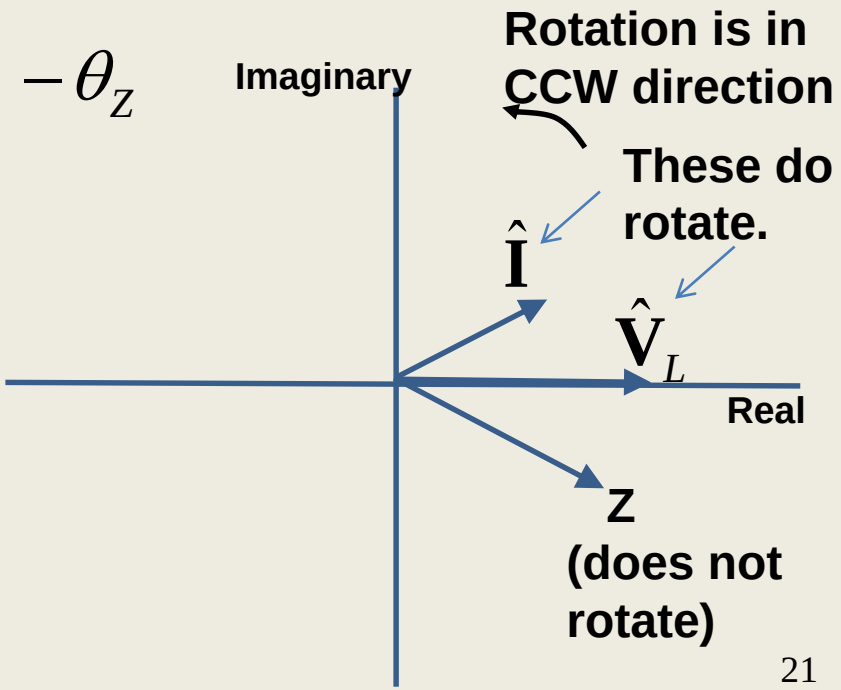
There is an implicit sinωt that we are not writing. This sinωt gives rotation to the vectors which make them phasors.

Also recall the impedance: $Z = |Z| \angle \theta_Z$

There is no implicit sinωt ➔ impedance is a vector, not a phasor!

Then…

$$\hat{\mathbf{I}} = I \angle \phi_I = \frac{\hat{\mathbf{V}}_L}{Z} = \frac{V_L \angle \phi_{V_L}}{|Z| \angle \theta_Z} = \frac{V_L}{|Z|} \angle \phi_{V_L} - \theta_Z$$



Instantaneous Power

Instantaneous power is given by $p(t) = v(t)i(t)$

Let $v(t) = V_{\max} \cos(\omega t + \theta_v)$ $\quad i(t) = I_{\max} \cos(\omega t + \theta_i)$

Then $p(t) = v(t)i(t) = V_{\max} \cos(\omega t + \theta_v) I_{\max} \cos(\omega t + \theta_i)$

$= V_{\max} I_{\max} \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$

Apply trig identity: $\cos a \cos b = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$

$$p(t) = \frac{V_{\max} I_{\max}}{2}\left[\cos(\omega t + \theta_v - \omega t - \theta_i) + \cos(\omega t + \theta_v + \omega t + \theta_i)\right]$$

$$= \frac{V_{\max} I_{\max}}{2}\left[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)\right] = \frac{V_{\max} I_{\max}}{2}\left[\cos(\theta_v - \theta_i) + \cos(2\omega t + \underbrace{2\theta_v - \theta_v}_{\theta_v} + \theta_i)\right]$$

Gathering terms in the argument of the second cos function:

$$p(t) = \frac{V_{\max} I_{\max}}{2}\left[\cos(\theta_v - \theta_i) + \cos(2(\omega t + \theta_v) - (\theta_v - \theta_i))\right]$$

Instantaneous Power

$$p(t) = \frac{V_{\max} I_{\max}}{2}\left[\cos(\theta_v - \theta_i) + \cos(2(\omega t + \theta_v) - (\theta_v - \theta_i))\right]$$

Apply trig identity to last term: $\cos(a-b) = \cos a \cos b + \sin a \sin b$

$$p(t) = \frac{V_{\max} I_{\max}}{2}\left[\cos(\theta_v - \theta_i) + \cos 2(\omega t + \theta_v)\cos(\theta_v - \theta_i) + \sin 2(\omega t + \theta_v)\sin(\theta_v - \theta_i)\right]$$

Rearrange and distribute:

$$p(t) = \frac{V_{\max} I_{\max}}{2}\cos(\theta_v - \theta_i) + \frac{V_{\max} I_{\max}}{2}\cos(\theta_v - \theta_i)\cos 2(\omega t + \theta_v) + \frac{V_{\max} I_{\max}}{2}\sin(\theta_v - \theta_i)\sin 2(\omega t + \theta_v)$$

Then: $p(t) = P + P\cos 2(\omega t + \theta_v) + Q\sin 2(\omega t + \theta_v)$

where: $P = \frac{V_{\max} I_{\max}}{2}\cos(\theta_v - \theta_i)$, $Q = \frac{V_{\max} I_{\max}}{2}\sin(\theta_v - \theta_i)$

Real power
Active power
Watts

Imaginary power
Reactive power
VARs

Example 2

Consider example 1, where V_{max}=400, θ_v=0,

$$v(t) = V_{\max} \sin(\omega t + \theta_v)$$

and we computed that I_{max}=175.06, θ_i = -.5047rad,

$$i(t) = I_{\max} \sin(\omega t + \theta_i)$$

Then:

$$P = \frac{V_{\max} I_{\max}}{2} \cos(\theta_v - \theta_i) = \frac{400 \times 175.06}{2} \cos(0 + .5047) = 30647 \quad \text{watts}$$

$$Q = \frac{V_{\max} I_{\max}}{2} \sin(\theta_v - \theta_i) = \frac{400 \times 175.06}{2} \sin(0 + .5047) = 16930 \quad \text{vars}$$

$$p(t) = P + P\cos 2(\omega t + \theta_v) + Q \sin 2(\omega t + \theta_v)$$

$$= 30647 + 30647 \cos 2(377t) + 16930 \sin 2(377t)$$

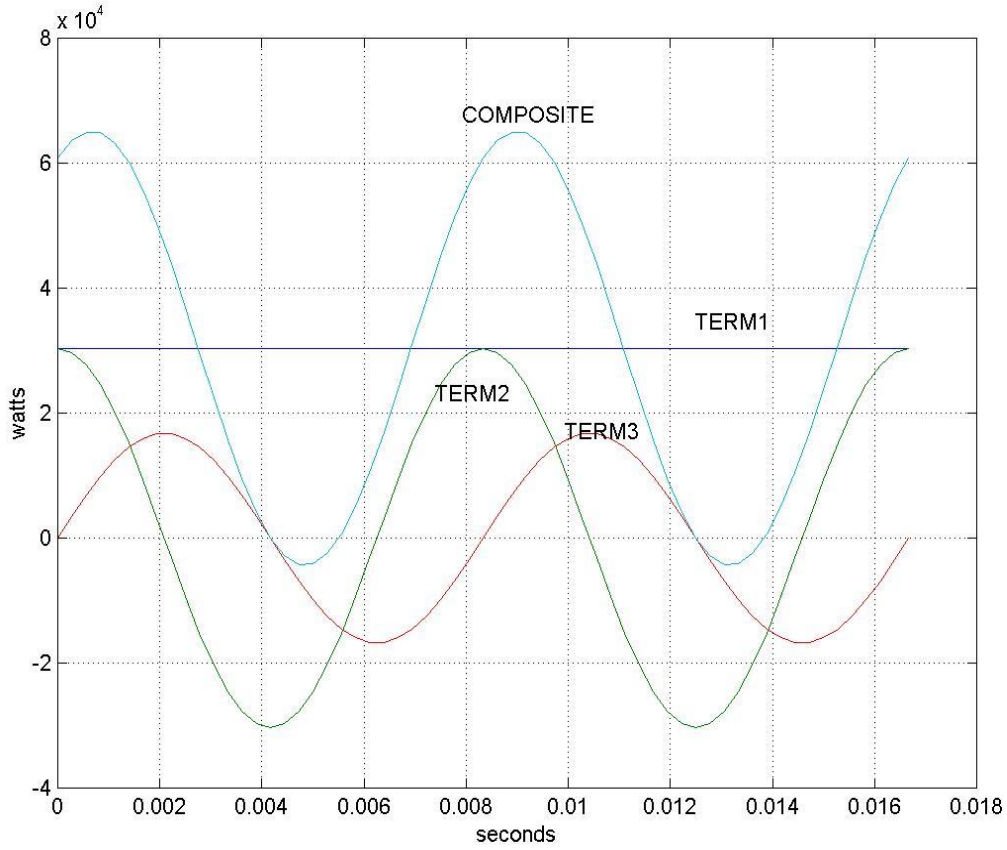
TERM1 **TERM2** **TERM3**

Example 2

$$p(t) = P + P\cos 2(\omega t + \theta_v) + Q\sin 2(\omega t + \theta_v)$$

$$= 30647 + 30647\cos 2(377t) + 16930\sin 2(377t)$$

TERM1 **TERM2** **TERM3**



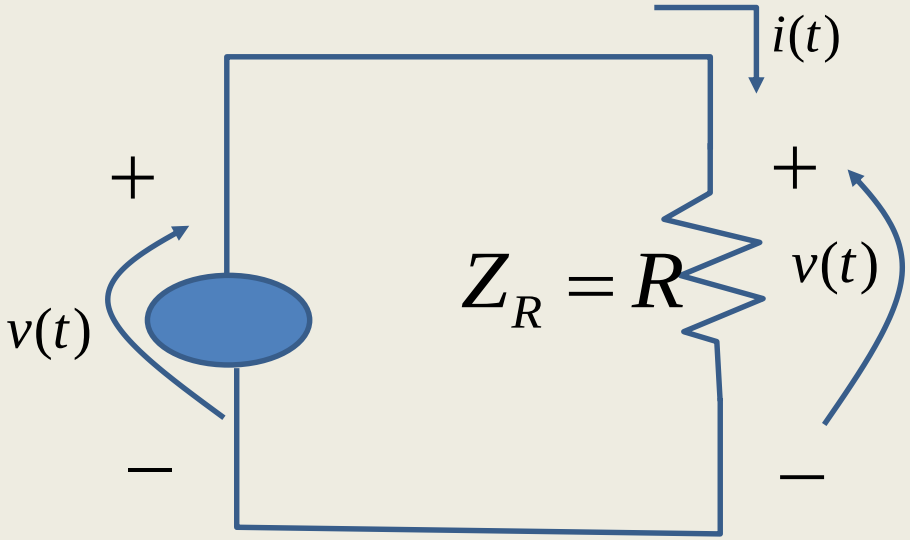
The average of a periodic signal x(t) is given by:

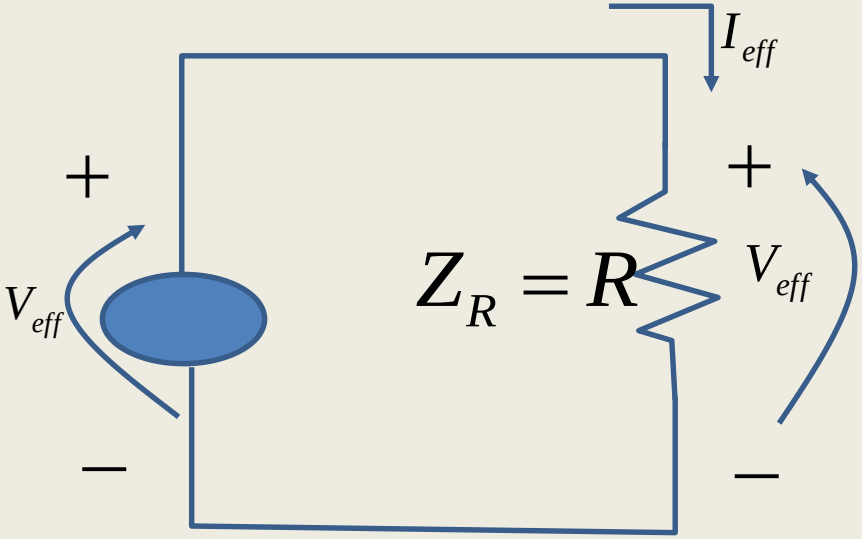
$$X_{avg} = \frac{1}{T}\int_T x(t)dt$$

- **Power is delivered only if the average power is not 0.**
- **The average value of term2 and term3 is 0. These terms are not responsible for power delivery.**
- **The average value of the composite, which is instntans power, p(t), is term1, P.**
- **Interesting side note: instntns power varies at twice the nominal frequency.**

Effective or RMS value

We want to obtain the constant (or DC) value of a **current** which delivers the same average power to a resistor.





The power delivered is:

$$P = \frac{1}{T}\int_T i^2(t)Rdt$$

The power delivered is:

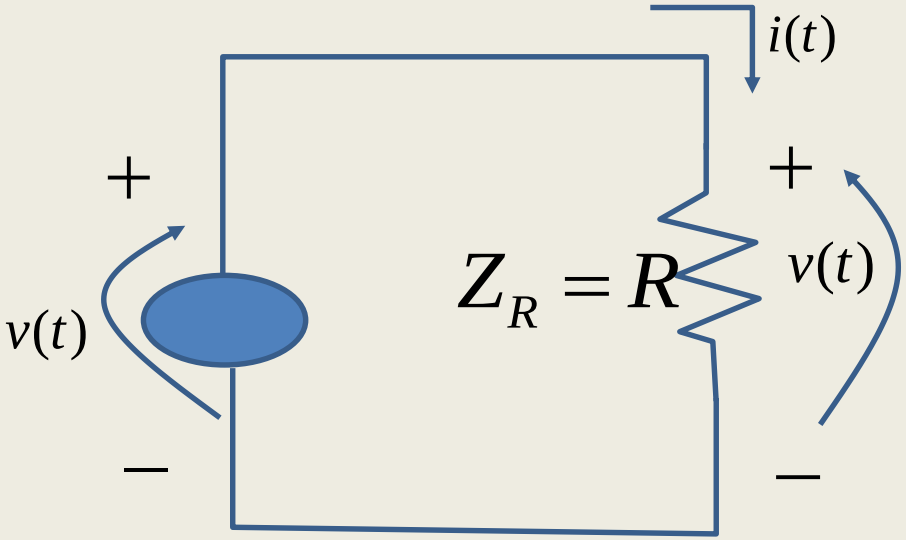
$$P = I_{eff}^2 R$$

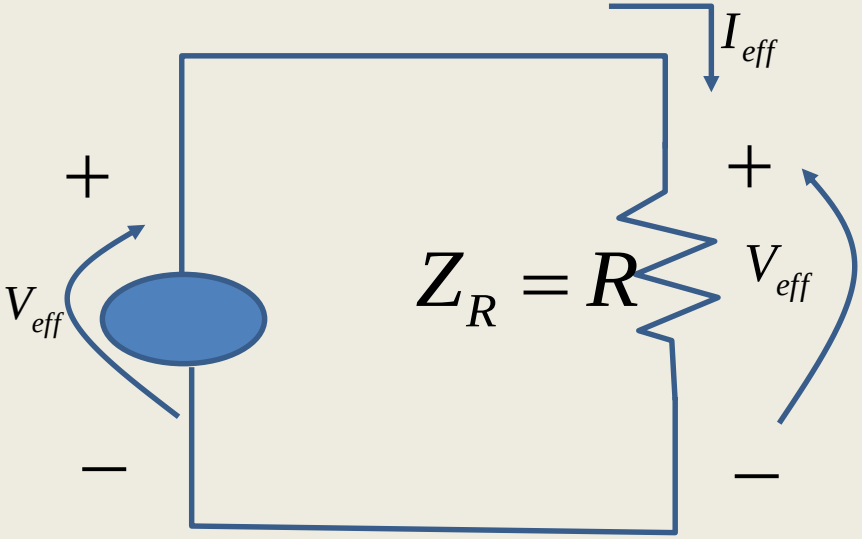
Equate

$$\frac{1}{T}\int_T i^2(t)Rdt = I_{eff}^2 R \Rightarrow I_{eff} = \sqrt{\frac{1}{T}\int_T i^2(t)dt}$$

Effective or RMS value

We want to obtain the constant (or DC) value of a **voltage** which delivers the same average power to a resistor.





The power delivered is:

$$P = \frac{1}{T}\int_T v^2(t)/R\,dt$$

The power delivered is:

$$P = V_{eff}^2/R$$

Equate

$$\frac{1}{T}\int_T v^2(t)/R\,dt = V_{eff}^2/R \Rightarrow V_{eff} = \sqrt{\frac{1}{T}\int_T v^2(t)\,dt}$$

Effective or RMS value

The root-mean-square value of any periodic signal x(t) is:

$$X_{eff} = \sqrt{\frac{1}{T}\int_T x^2(t)dt}$$

Let x(t) be v(t): $v(t) = V_{\max}\sin(\omega t + \theta_v)$

$$V_{eff} = \sqrt{\frac{1}{T}\int_T v^2(t)dt} = \sqrt{\frac{1}{T}\int_T V_{\max}^2 \sin^2(\omega t + \theta_v)dt}$$

Apply trig identity: $\sin^2 a = \frac{1}{2}\left[1 - \cos(2a)\right]$

$$V_{eff} = \sqrt{\frac{V_{\max}^2}{2T}\int_T 1 - \cos(2(\omega t + \theta_v))dt} = \sqrt{\frac{V_{\max}^2}{2T}t\Big|_0^T} = \sqrt{\frac{V_{\max}^2 T}{2T}} = \frac{V_{\max}}{\sqrt{2}}$$

Current may be similarly treated, resulting in $I_{eff} = \frac{I_{\max}}{\sqrt{2}}$

Power and Phasor notation

We previously derived power expressions for P and Q in terms of V_{max} and I_{max}. We may now express these in terms of rms (effective) values:

$$P = \frac{V_{\max} I_{\max}}{2}\cos(\theta_v - \theta_i) = \frac{\sqrt{2}V_{eff}\sqrt{2}I_{eff}}{2}\cos(\theta_v - \theta_i) = V_{eff} I_{eff}\cos(\theta_v - \theta_i)$$

$$Q = \frac{V_{\max} I_{\max}}{2}\sin(\theta_v - \theta_i) = \frac{\sqrt{2}V_{eff}\sqrt{2}I_{eff}}{2}\sin(\theta_v - \theta_i) = V_{eff} I_{eff}\sin(\theta_v - \theta_i)$$

This simplifies our power expressions. Commercial voltages and currents are always specified as rms values.

Phasor magnitudes may be rms or maximum. Unless it is otherwise specified, we assume phasor magnitudes are rms, and we will drop the subscript "eff." Thus,

$$P = VI\cos(\theta_v - \theta_i); \qquad Q = VI\sin(\theta_v - \theta_i)$$

Power factor

Power factor angle, defined θ, is the angle by which voltage across an element leads current through the element, i.e., $\theta=\theta_v-\theta_i$. It is also the angle of the load Z, i.e., $Z=|Z|\angle\theta$. One observes from the P, Q expressions that it is also the argument of the trig functions. Therefore:

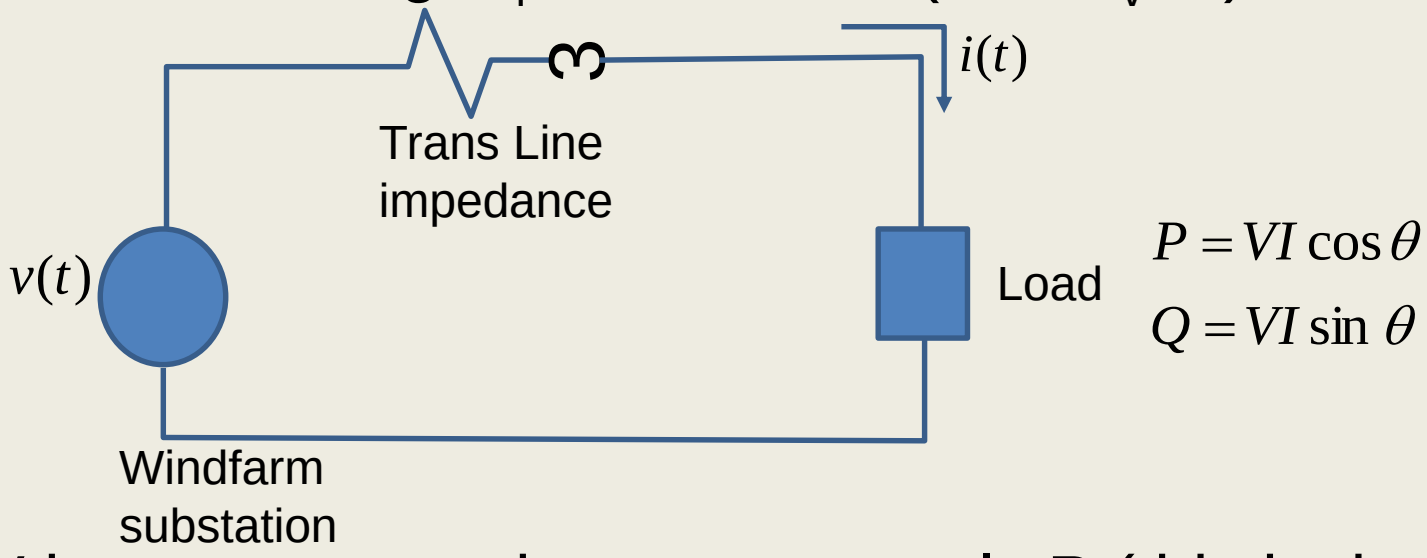
$$P = VI\cos(\theta_v-\theta_i)=VI\cos\theta; \qquad Q = VI\sin(\theta_v-\theta_i)=VI\sin\theta$$

The power factor is given as pf=cosθ. It is always positive, therefore we usually indicate pf with either the word "leading" (for capacitive load) or "lagging" (for inductive load). The following table summarizes important relations.

Load, Z	Im{Z}	θ_i (θ_v=0)	θ (θ_v=0)	pf	Q sign	Q Load
R only	0	0	0	1.0	0	0 vars
R and L	+	-	+	0 to 1, lagging	+	Absorbing vars
R and C	-	+	-	0 to 1, leading	-	Supplying vars

Power factor

Consider a windfarm at the end of a transmission line. The P and Q "load" in this case represents the utility grid. Windfarms using so-called "fixed-speed" induction gens consume vars, and so the load must be capacitive. This means current is leading, $\theta_i>0$, and $\theta<0$ (with $\theta_v=0$).



Assume V is constant, and we must supply P (this is the windfarm output). With V and P constant, from $P=VI\cos\theta$, $I\cos\theta=I\times pf$ must be constant. What do we want pf to be?

Power factor

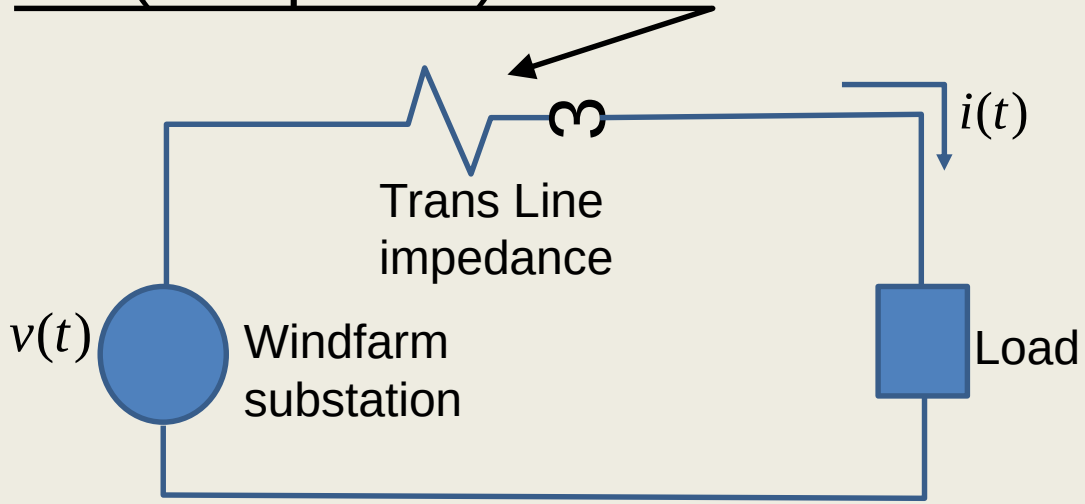
Assume P=33MW, V=39.84kV. This means that

$$P = VI\cos\theta \Rightarrow I\cos\theta = P/V = 33E6/39.84E3 = 828.3$$

Here are a few possibilities for Icosθ =I×pf.

I (amperes)	828.3	871.9	920.3	1035.4
Pf (leading)	1	0.95	0.9	0.8

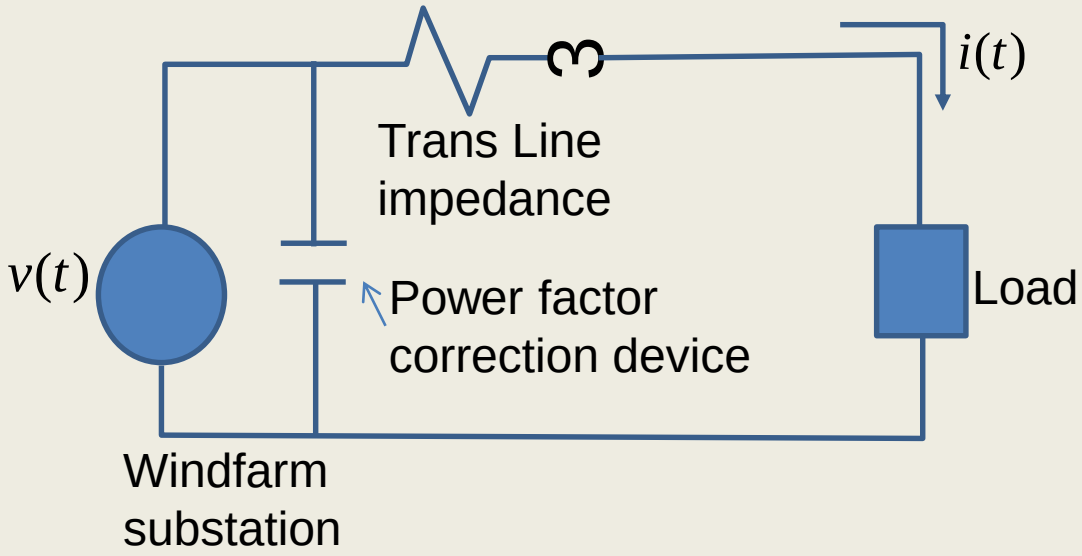
We need higher power factor to obtain lower current. Lower current is desired in order to minimize I^2R (real power) losses in the transmission line.



Power factor correction (PFC)

So what do you do if your windfarm has a power factor of 0.85 lagging but the grid code requires 0.95 lagging?

➔Install a PFC device at the windfarm's point of interconnection with the grid. This is normally the windfarm's substation.



This least expensive PFC device is a switched capacitor, or several switched capacitors. These are static devices, however. Dynamic devices are also available but are more expensive (and you get dynamic responsiveness as well).

What is reactive power?

1. It is given by: $Q = VI\sin(\theta_v - \theta_i)$
2. It occurs as the coefficient of the double frequency "sin" term in p(t):

$$p(t) = P + P\cos 2(\omega t + \theta_v) + Q\sin 2(\omega t + \theta_v)$$

3. It is the maximum value of the rate of energy flowing alternately into the reactive component of the load (away from the source) and away from the reactive component of the load (toward the source). It is only present when there is a stored energy device (C or L) in the circuit.
4. Its units are "volt-amperes-reactive" or "vars." Dimensionally, vars are the same as watts and volt-amperes.
5. Three attributes of reactive power:
   a. It does no useful work (its average value is 0).
   b. Its presence increases current magnitude for a given P and V.
   c. Derivatives of Q "flow" equations (not shown in these slides) indicate that Q is closely related to voltage magnitudes.

Complex power

Complex power is defined as:

$$S = \hat{\mathbf{V}}\hat{\mathbf{I}}^{*} = P + jQ$$

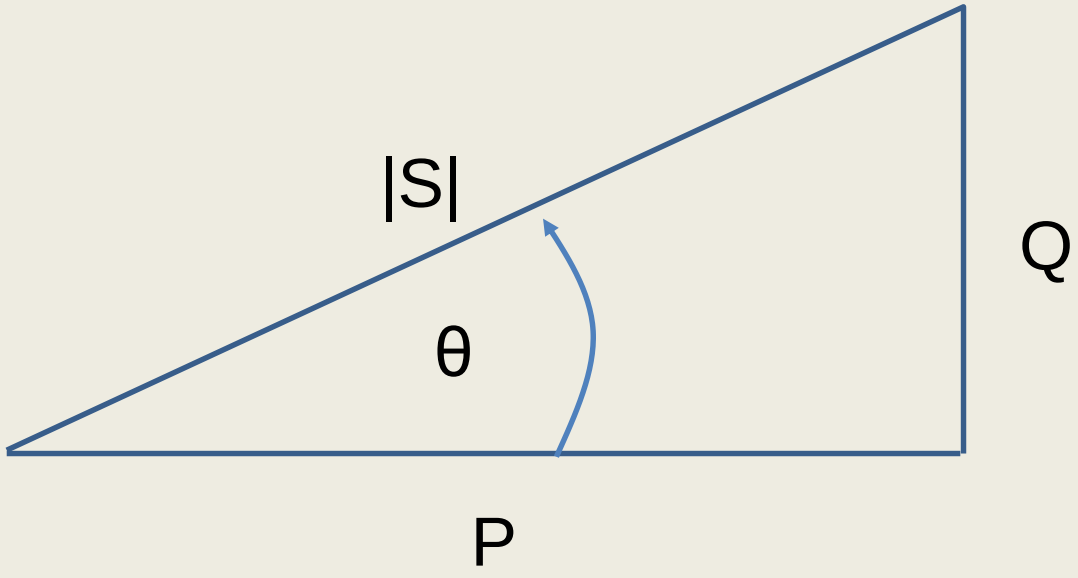
where:

$$\hat{\mathbf{V}} = \mathrm{V}\angle\theta_{\mathrm{v}} \qquad \hat{\mathbf{I}} = \mathrm{I}\angle\theta_{\mathrm{i}}$$

The asterisk next to the “I” phasor indicates conjugation:

$$\hat{\mathbf{I}}^{*} = \mathrm{I}\angle -\theta_{\mathrm{i}}$$

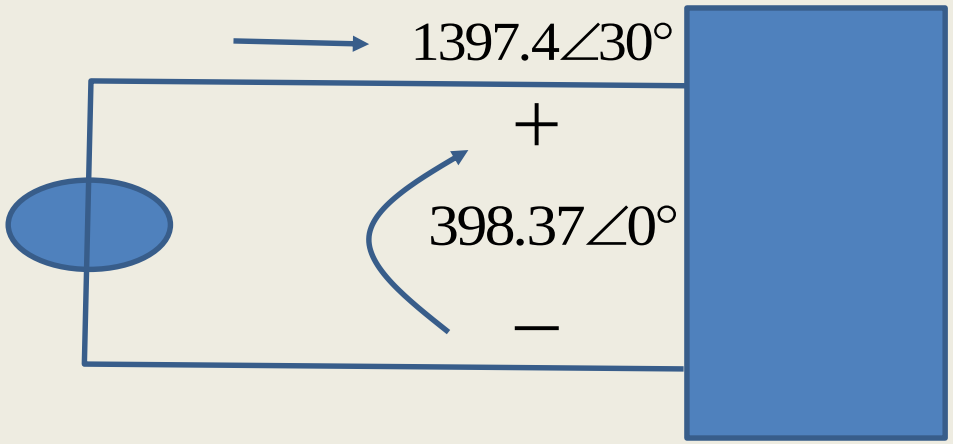
Power triangle



$$S = P + jQ \Rightarrow |S| = \sqrt{P^2 + Q^2}$$

Example 3

The below represents one of the phases of a generator supplying a load. The box to the right represents the load. The current flowing into the load is 1397.4 A at 0.866 pf leading, with 398.37 volts at its terminals. What complex power is delivered to the load?



Solution: $\theta=-\cos^{-1}(.866)=-30°$.

➔ $\theta_i=-\theta=30°$.

$$S=\hat{\mathbf{V}}\hat{\mathbf{I}}^{*}=398.37\angle 0\times(1397.4\angle 30)^{*}$$
$$=398.37\angle 0\times(1397.4\angle -30)=556680\angle -30$$
$$=556680(\cos 30-j\sin 30)=482100-j278340$$

Thus, P=0.482100MW,
Q=-0.27834MVARs

We also define the apparent power |S|, the magnitude of S:

$$S=P+jQ\Rightarrow|S|=\sqrt{P^2+Q^2}=\sqrt{.4821^2+.27834^2}=0.5567$$