Module E3

Problem 1

A two-unit system is given by the following data

$$C_1(P_{g1}) = 0.015 \cdot (P_{g1})^2 + 2 \cdot (P_{g1}) + 6$$

$$C_2(P_{g2}) = 0.025 \cdot (P_{g2})^2 + 7 \cdot (P_{g2}) + 3$$

The total system demand is 500MW. The lower and upper limits for each generator unit are 20 and 300MW, respectively.

- (a) Determine the optimal dispatch ignoring inequality constraints
- (b) And identify whether it is a feasible dispatch or not (support your answer)

Solution to problem 1

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$$F(P_{g1}, P_{g2}, \lambda) = C_1(P_{g1}) + C_2(P_{g2}) - \lambda(P_{g1} + P_{g2}) - 500$$
$$\frac{\partial F}{\partial P_{g1}} = 0 \Rightarrow \frac{\partial C_1}{\partial P_{g1}} - \lambda = 0 \qquad \frac{\partial F}{\partial P_{g2}} = 0 \Rightarrow \frac{\partial C_2}{\partial P_{g2}} - \lambda = 0$$
$$\frac{\partial F}{\partial \lambda} = 0 \Rightarrow P_{g1} + P_{g2} - 500 = 0$$

The above equation result in

(a)

0.03	0	-1]	$\left[P_{g1} \right]$		-2]	P_{g1}		375	
0	0.05	-1	P_{g2}	=	-7	\Rightarrow	P_{g2}	=	125	
1	1	0	Lλ		500		lλ		13.25	

(b) And we note that it is not feasible because $P_{g1} > P_{g1MAX} = 300MW$

Problem 2

Generator cost rate functions, in \$/hr, for a three unit system are given as

$$C_1(P_1) = 0.004P_1^2 + 5.3P_1 + 500$$

$$C_2(P_2) = 0.006P_2^2 + 5.5P_2 + 400$$

$$C_3(P_3) = 0.009P_3^2 + 5.8P_3 + 200$$

Limits on the generation levels are $200 \le P_1 \le 450$, $150 \le P_2 \le 350$, $100 \le P_3 \le 225$. These three generators must supply a total demand of 975 MW.

- (a) Form the linear matrix equation necessary to solve the unconstrained optimization problem.
- (b) The solution to the unconstrained optimization problem is $P_1 = 482.9MW$, $P_2 = 305.3MW$,
 - $P_3 = 186.5 MW$. For this solution (i.e., ignoring limits)
 - (i) Compute lambda
 - (ii) Determine the total cost rate

- (iii) How much would the total cost rate change if the total load increased from 975 to 976 MW? (Indicate whether the total cost rate increases or decreases).
- (c) Form the linear matrix equation necessary to solve the next iteration of getting the solution to this problem.

$$\frac{\partial C_1}{\partial P_1} = 0.008P_1 + 5.3 \Rightarrow 0.008P_1 - \lambda = -5.3$$
$$\frac{\partial C_2}{\partial P_2} = 0.012P_2 + 5.5 \Rightarrow 0.012P_2 - \lambda = -5.5$$
$$\frac{\partial C_3}{\partial P_3} = 0.018P_3 + 5.8 \Rightarrow 0.018P_3 - \lambda = -5.8$$

(a)

0.008	0	0	-1]	$\left\lceil P_1 \right\rceil$		-5.3
0	0.012	0	-1	P_2	_	-5.5
0	0	0.018	-1	P_3	-	-5.8
1	1	1	0	P_4		975

(c) (i) We can use any of the
$$\frac{\partial C_i}{\partial P_i}$$

 $\lambda = 0.008 \cdot (482.9) + 5.3 = 0.012 \cdot (305.3) + 5.5 = 0.018 \cdot (186.8) + 5.8$
 $= 9.163 \$ / MW - hr$

(ii)

$$C_{Total} = C_1(P_1) + C_2(P_2) + C_3(P_3)$$

= 0.004(482.9)² + 5.3(482.9) + 500
+ 0.006(305.3)² + 5.5(305.3) + 400
+ 0.009(186.8)² + 5.8(186.8) + 200
= 3992.1 + 2638.4 + 1597.5
= 8228\$ / hr

(iii) A 1 KW change in load would <u>increase</u> the total cost by $\lambda = \$9.163 / hr$

(c) Now since P_1 exceeds its limit, we need to bring in the corresponding constraint with its Lagrange Multiplier.

$$F = C_1 + C_2 + C_3 - \lambda (P_1 + P_2 + P_3 - 975) - \mu (P_1 - 450)$$

And when we apply KKT, we will get

-0.008	0	0	-1	-1	$\left\lceil P_1 \right\rceil$		-5.3
0	0.012	0	-1	0	P_2		-5.5
0	0	0.018	-1	0	P_3	=	-5.8
1	1	1	0	0	λ		975
1	0	0	0	0_	μ		450

Problem 3

A three-unit system is given by the following data. The total system demand is 1100MW. Generator constraints are $0 < P_{g1} < 550$, $0 < P_{g2} < 300$, $0 < P_{g3} < 300$

$$C_1(P_{g_1}) = 0.010 \cdot (P_{g_1})^2 + 0.3 \cdot (P_{g_1}) + 1$$

$$C_2(P_{g_2}) = 0.030 \cdot (P_{g_2})^2 + 0.2 \cdot (P_{g_2}) + 3$$

$$C_3(P_{g_3}) = 0.020 \cdot (P_{g_3})^2 + 0.9 \cdot (P_{g_3}) + 5$$

- (a) Identify the objective function for this optimization problem.
- (b) Identify the Lagrangian function assuming no constraints are binding.
- (c) Identify the KKT conditions assuming no constraints are binding.
- (d) Find the solution to the problem assuming no constraints are binding.
- (e) Find the solution to the problem accounting for any binding constraints.
- (f) Find the total cost of supplying the 1100MW using the solution found in part (e)
- (g) Approximately the total cost of supplying the 1100MW change if the upper limit on generator 1 was increased from 550MW to 560MW.

Solution to problem 3

(a)
$$f(P_g) = \overline{C_1(P_{g1})} + C_2(P_{g2}) + C_3(P_{g3})$$

(b)
$$L[\lambda, P_g] = f(P_g) - \lambda [P_{g1} + P_{g2} + P_{g3} - 1100]$$

(c)

$$\frac{\partial L}{\partial P_{g1}} = 0.02P_1 + 0.3 - \lambda = 0$$

$$\frac{\partial L}{\partial P_{g2}} = 0.06P_2 + 0.2 - \lambda = 0$$

$$\frac{\partial L}{\partial P_{g3}} = 0.04P_3 + 0.9 - \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = P_1 + P_2 + P_3 - 1100 = 0$$

(d)

0.02	0	0	-1	P_{g1}		-0.3	
0	0.06	0	-1	P_{g2}	_	-0.2	
0	0	0.04	-1	P_{g3}	=	-0.9	
1	1	1	0	L λ		1100	

$$\Rightarrow P_{g1} = 607.3MW$$

$$\Rightarrow \overline{P_{g2}} = 204.1MW$$

$$\Rightarrow \overline{P_{g3}} = 288.6MW$$

$$\Rightarrow \underline{\lambda} = 12.45\$ / MW - hr$$

(d) We note that P_{g_1} violates its upper limit. Therefore we add in the equation $P_{g_1}=550$ and also argument the KKT condition for the first equation to be $0.02P_{g_1} + 0.3 - \lambda - \mu_1 = 0$. So our equation become

$$\begin{bmatrix} 0.02 & 0 & 0 & -1 & -1 \\ 0 & 0.06 & 0 & -1 & 0 \\ 0 & 0 & 0.04 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_{g1} \\ P_{g2} \\ P_{g3} \\ \lambda \\ \mu_1 \end{bmatrix} = \begin{bmatrix} -0.3 \\ -0.2 \\ P_{g3} \\ \delta \\ \mu_1 \end{bmatrix}$$
$$\Rightarrow P_{g1} = 550MW$$
$$\Rightarrow P_{g2} = 227MW$$
$$\Rightarrow P_{g2} = 227MW$$
$$\Rightarrow \lambda = 13.82\$/MW - hr$$
$$\Rightarrow \mu_1 = 2.52\$/MW$$

But $\,P_{\rm g3}\,$ violates its upper limit so we must reformulate again:

$$\begin{bmatrix} 0.02 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0.06 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0.04 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_{g1} \\ P_{g2} \\ P_{g3} \\ \lambda \\ \mu_1 \\ \mu_3 \end{bmatrix} = \begin{bmatrix} -0.3 \\ -0.2 \\ -0.9 \\ 550 \\ 300 \\ 1100 \\ \Rightarrow P_{g1} = 550 \\ F_{g2} = 250 \\ F_{g3} = 300 \\ W \\ \Rightarrow \overline{P_{g3} = 300 \\ W} \\ \Rightarrow \overline{P_{g3} = 300 \\ W} \\ \Rightarrow \overline{P_{g3} = 300 \\ W} \\ \Rightarrow \overline{\mu_1 = -3.9 \\ MW} \\ \Rightarrow \overline{\mu_3 = -2.3 \\ MW} \\ \end{bmatrix}$$

(f) $C_1(550) + C_2(250) + C_3(300) = 3191 + 1928 + 2075 = \underline{7194}$

(g)
$$7194 - (3.9) \cdot (10) = \underline{7155}$$

Problem 4

A three-unit system is given by the following data. The total system demand is 1100MW. Generator constraints are $0 < P_{g1} < 700$, $0 < P_{g2} < 200$, $0 < P_{g3} < 252.3$.

$$C_{1}(P_{g1}) = 0.008 \cdot (P_{g1})^{2} + 0.5 \cdot (P_{g1}) + 5$$

$$C_{2}(P_{g2}) = 0.030 \cdot (P_{g2})^{2} + 0.2 \cdot (P_{g2}) + 3$$

$$C_{3}(P_{g3}) = 0.020 \cdot (P_{g3})^{2} + (P_{g3}) + 5$$

- (a) Set up the linear matrix equation to solve the economic dispatch problem, assuming all constraints are satisfied (i.e., ignore constraints. DO NOT solve the equation.
- (b) The solution to the problem in (a) is $P_{g1} = 664.5MW$, $P_{g2} = 182.2MW$, and $P_{g3} = 253.3MW$. Reformulate this linear matrix equation to solve the economic dispatch problem for this system, accounting for any violated constraints. Again, you DO NOT need to actually solve the equation, just set it up.
- (c) Using only the cost function for generator 1, $C_1(P_{g1})$, together with information given in the part b problem statement, determine the system lambda for the solution to the unconstrained problem.

Solution to problem 4

(a)	0.016 0 0 1	0 0.06 0 1	0 0 0.04 1	$ \begin{array}{c} -1 \\ -1 \\ -1 \\ 0 \end{array} $	$\begin{bmatrix} \mathbf{P}_{g1} \\ \mathbf{P}_{g2} \\ \mathbf{P}_{g3} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{g3} \\ \mathbf{I}_{g3} \end{bmatrix}$	-0.5 -0.2 1 1100	5 2 0			
(b)	0.016 0 1 0	0 0.06 0 1 0	0 0 0.04 1 1	$ \begin{array}{ccc} -1 & 0 \\ -1 & 0 \\ -1 & -1 \\ 0 & 0 \\ 0 & 0 \end{array} $	$\begin{bmatrix} P_{g1} \\ P_{g2} \\ P_{g3} \\ \lambda \\ \mu_{1} \end{bmatrix}$		$\begin{bmatrix} -0.5 \\ -0.2 \\ -1 \\ 1100 \\ 252.3 \end{bmatrix}$			
(c)	$\lambda = \frac{\partial c_1}{\partial P_g}$	-=0.0	$016P_{g1}$	+0.5 = 0).016∙(6	564.:	5)+0.5	= 11.1	32\$/1	<u>MW – hr</u>

Problem 5

Recall that the "system lambda" is the cost to the system owner of producing the next MW over the next hour; it is equal to the incremental cost of an individual unit when the system is economically dispatched for minimum cost and the unit is not at an upper or lower generation limit. A two-unit system is given by the following data.

$$C_1(P_{g1}) = 0.015 \cdot (P_{g1})^2 + 2 \cdot (P_{g1}) + 6$$

$$C_2(P_{g2}) = 0.020 \cdot (P_{g2})^2 + 6 \cdot (P_{g1}) + 4$$

The demand is 300MW

- 1. Write the KKT conditions that must be satisfied at the optimal solution to this problem, assuming that both units are operating between their respective upper and lower limits.
- 2. Set up the linear matrix equation to solve the economic dispatch problem for this system, assuming that both units are operating between their respective upper and lower limits. Do NOT solve the system of equations.
- 3. The solution to the problem in (2) is $P_{g1} = 228.57 MW$, $P_{g2} = 71.43 MW$. Assuming that each unit has a minimum generation capability of 80 MW.
 - (a) Indicate why the given solution is not feasible.
 - (b) Identify the optimal feasible solution
 - (c) Identify the incremental costs of each unit at the optimal feasible solution
 - (d) Identify the system lambda at the optimal feasible solution
 - (e) Would the total cost of supplying the 300MW increase or decrease (relative to the total cost corresponding to the optimal feasible solution) if the minimum generation capabilities on both units were changed to 79MW ?

1.
$$L[P_{g1}, P_{g2}, \lambda] = C_1(P_{g1}) + C_2(P_{g2}) - \lambda[P_{g1} + P_{g2} - 300]$$

$$\frac{\partial L}{\partial P_{g1}} = 0.03P_1 + 2 - \lambda = 0$$

$$\frac{\partial L}{\partial P_{g2}} = 0.04P_2 + 6 - \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = P_1 + P_2 - 300 = 0$$

2.
$$\begin{bmatrix} 0.03 & 0 & -1 \\ 0 & 0.04 & -1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_{g1} \\ P_{g2} \\ \lambda \end{bmatrix} = \begin{bmatrix} -2 \\ -6 \\ 300 \end{bmatrix} \text{ or } \begin{bmatrix} 0.03 & 0 & 1 \\ 0 & 0.04 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_{g1} \\ P_{g2} \\ -\lambda \end{bmatrix} = \begin{bmatrix} -2 \\ -6 \\ 300 \end{bmatrix}$$

3. (a) Because Generator 2 is below its minimum capability

(b)
$$P_{g2} = 80MW, P_{g1} = 300 - 80 \Longrightarrow 220MW - P_{g1}$$

(c)
$$\frac{\partial C_1}{\partial P_{g_1}} = 0.03P_{g_1} + 2 = 0.03 \cdot (220) + 2 = \frac{8.6\$}{MW - hr} = IC_1$$

 $\frac{\partial C_2}{\partial P_{g_2}} = 0.04P_{g_2} + 6 = 0.04 \cdot (80) + 6 = \frac{9.2\$}{MW - hr} = IC_2$

- (d) lambda = 8.6%/MW-hr (Since unit 1 would supply the next MW-hr)
- (e) Total cost would decrease

Problem 6

The 'system λ ' is the cost to the system owner of producing the next MW over the next hour. It is equal to the incremental cost of an individual unit when the system is economically dispatched for minimum cost and the unit is not at an upper or lower generation limit. A three-unit system is given by the following data. Total system demand is 1000 MW.

$$C_1(P_{g1}) = 0.008 \cdot (P_{g1})^2 + 0.5 \cdot (P_{g1}) + 5$$

$$C_2(P_{g2}) = 0.015 \cdot (P_{g2})^2 + 2 \cdot (P_{g2}) + 6$$

$$C_3(P_{g3}) = 0.020 \cdot (P_{g3})^2 + P_{g3} + 5$$

- a) Set up the linear matrix equation to solve the economic dispatch problem for this system. DO NOT solve the equation.
- b) The solution to the problem in (a) is $P_{g1} = 549.6$ MW, $P_{g2} = 243.1$ MW, and $P_{g3} = 207.3$ MW. Assume that each unit has a maximum generation capability of 350 MW. Reformulate the linear matrix equation to solve the economic dispatch problem for this system. Again, DO NOT solve the system.
- c) What is the incremental cost for unit 1 under the condition specified in part (b)? Do you think the system λ is greater than or less than this value?

a)

$$\frac{\partial C_1}{\partial P_{g_1}} = 0.016 \cdot (P_{g_1}) + 0.5 \qquad \Rightarrow \qquad 0.01(P_{g_1}) - \lambda = -0.5$$
$$\frac{\partial C_2}{\partial P_{g_2}} = 0.03 \cdot (P_{g_2}) + 2 \qquad \Rightarrow \qquad 0.03(P_{g_2}) - \lambda = -2$$
$$\frac{\partial C_3}{\partial P_{g_3}} = 0.04 \cdot (P_{g_3}) + 1 \qquad \Rightarrow \qquad 0.04(P_{g_3}) - \lambda = -1$$

$$P_{g1} + P_{g2} + P_{g3} = 1000$$

0.016	0	0	1]		P_{g1}		-0.5	
0	0.03	0	1		P_{g2}	_	-2	
0	0	0.04	1	×	P_{g3}	_	-1	
1	1	1	0		$-\lambda$		1000	

b) Our Lagrangian becomes

 $L(Pg1, Pg2, Pg3, \lambda, \mu) = C_1(P_{g1}) + C_2(P_{g2}) + C_3(P_{g3}) - \lambda(P_{g1} + P_{g2} + P_{g3} - 1000) - \mu(P_{g1} - P_{g1} \max)$

and application of KKT results in

0.016	0	0	1	1		$\left\lceil Pg1 \right\rceil$		- 0.5
0	0.03	0	1	0		Pg2		-2
0	0	0.04	1	0	×	Pg3	=	-1
1	1	1	0	0		$-\lambda$		1000
1	0	0	0	0		_ – μ		350

c)

IC1 = (0.016)(350) + 0.5 = \$6.10 / MWhr

The system lambda must be greater than 6.1 \$/MWhr because ???????

Problem 7

Generator 1 has an incremental cost curve of:

$$IC_1(P_{g1}) = 0.05(P_{g1}) + 2.0$$

and limits of:

$$10 MW \le P_{g1} \le 100 MW.$$

The generator operates in an economically dispatched system. In this system, it is found that supplying an additional 5 MW costs an additional 50/hr. Determine P_{g1} .

Solution to problem 7

$$\lambda = \frac{\Delta \$ / hr}{\Delta P} = \frac{50}{5} = 10 \$ / MWhr$$
$$IC(P_{g1}) = 0.05(P_{g1}) + 2.0 = 10 \implies P_{g1} = 160 MW$$

But this result is outside of the power generation bounds. Therefore, $P_{g1} = 100$.

Problem 8

A system consists of two generators supplying a load. Generators 1 and 2 have incremental cost curves as indicated below:

$$IC_1(P_{g1}) = 0.04(P_{g1}) + 2.0$$
$$IC_2(P_{g2}) = 0.06(P_{g2}) + 1.0.$$

and limits of:

$$10 MW \le P_{g1} \le 100 MW$$
$$30 MW \le P_{g2} \le 100 MW$$

- a) In this system, when the load is 140 MW, what is the dispatch of these two units?
- b) In this system, when the load is 190 MW, what is the dispatch of these two units?
- c) In this system, under a certain economically dispatched scenario (a scenario different than in part (a) and (b)), it is found that supplying an additional 1 MW costs an additional 5.68/hr. Determine P_{g1} and P_{g2} .

Solution to problem 8

a)

$$\begin{bmatrix} 0.04 & 0 & 1 \\ 0 & 0.06 & 1 \\ 1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} Pg1 \\ Pg2 \\ -\lambda \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 140 \end{bmatrix}$$

$$P_{g1} = 74.0 \quad P_{g2} = 66.0 \quad \lambda = 4.96$$

b)

$$\begin{bmatrix} 0.04 & 0 & 1 \\ 0 & 0.06 & 1 \\ 1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} Pg1 \\ Pg2 \\ -\lambda \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 190 \end{bmatrix} \qquad P_{g1} = 104.0 \quad P_{g2} = 86.0 \quad \lambda = 6.16$$

But in this case Pg1 is above the acceptable limits. We must re-optimize with the

Generator 1 limit taken into consideration:

0.04	0	1	1	P_{g1}		-2			
0	0.06	1	0	P_{g2}	_	-1		$P_{g1} = 100.0$	$P_{g2} = 90.0$
1	1	0	0	$ -\lambda $	-	190		$\lambda = 6.4$	$\mu = -0.4$
L 1	0	0	0	$\left\lfloor -\mu \right\rfloor$		100			
$IC_1(P_{g1})$ =	= 0.04(<i>F</i>	• _{g1})⊣	+ 2.0 =	5.68	⇒	P_{g1}	=92 <i>MW</i>		
$IC_2(P_{g2})$	= 0.06	P_{g2}	+1.0=	= 5.68	=	$\Rightarrow P_{g2}$	= 78 <i>MW</i>		

Problem 9

c)

A two unit system has incremental cost curves (the derivatives of the cost curves) of $IC_1=0.01P_1+5$, and $IC_2=0.02P_2+4$, where P_1 and P_2 are given in MW. The demand is 300 MW. Ignoring limits on the generators, determine the values of P_1 and P_2 that minimize the cost of supplying the 300 MW.

Solution to problem 9

0.01	0	-1]	$\begin{bmatrix} P_1 \end{bmatrix}$		-5		$\begin{bmatrix} P_1 \end{bmatrix}$		166.67	
0	0.02	-1 >	$\langle P_2$	=	-4	\Rightarrow	P_2	=	133.33	
1	1	0	λ		300		_λ_		6.67	

Problem 10

A two-generator system is operating on economic dispatch and supplying 420 Mw of load. The total cost of supply is computed from the final EDC solution (i.e., all constraints are satisfied) and found to be \$3000/hr. From this same final solution, the LaGrange multipliers are found to be:

Equality constraint	$\lambda = 15/Mw-hr$
$Pg_1 \ge 20 \ Mw$	$\mu_{1,L} = 0$
$Pg_1 \leq 300 Mw$	µ _{1,H} =0
$Pg_2 \ge 10Mw$	µ _{2,L} =0
$Pg_2 \le 200 \ Mw$	$\mu_{2,H}$ = -\$4.00/Mw-hr

Here the subscripts "L" and "H" indicate "Low limit" and "High limit," respectively, and refer to the corresponding inequality constraint. For each question below, you must provide some basis or reasoning for your response.

- (a) What would be the (approximate) total cost of supply if the total demand was increased to 421 Mw?
- (b) What would be the total cost of supply if the lower limit for generator 2 was increased from 10 Mw to 11 Mw?
- (c) What would be the total cost of supply if the upper limit for generator 2 was increased from 200 Mw to 201 Mw?
- (d) What are the generation levels in Mw of generators 1 and 2?
- (e) What is the incremental cost for generator 1?

- (a) 3000+15=3015
- (b) 3000+0=3000
- (c) 3000-4=<u>2996</u>
- (d) Pg2=200 MW, Pg1=420-200=220 MW
- (e) IC(Pg1)= λ =\$15/Mw-hr

Problem 11

A two generator system has cost curves ($\frac{h}{r}$) of C₁(P₁)=0.006P₁²+5P₁+3, and C₂(P₂)=0.01P₂²+4P₂+2, where P_1 and P_2 are given in MW. The total demand is P_T =500 MW. The limits on these generators are $0 \le P_1 \le 300$ and $0 \le P_2 \le 300$.

- a. Determine the unconstrained values of P_1 and P_2 that minimize the cost of supplying the 500 MW, and indicate whether this solution is feasible or not.
- For the solution found in (a), how much would the total cost of supply change if the total demand b. increased to 501 MW for one hour ?
- Use the complementary condition (the third condition in the KKT conditions), to identify the c. values of each Lagrange multiplier associated with the inequality constraints.
- Repeat (a) if the total demand is 550 MW, and d.
 - i. Determine how much would the total cost of supply change if the total demand increased to 551MW.
 - ii. Determine λ and the incremental cost of both units.
 - iii. Identify the values of each Lagrange multiplier associated with the inequality constraints.
 - iv. What is the total system cost rate at this solution?
 - v. What is the total system cost rate if the lower limit of unit 3 is increased to 101 MW?
 - vi. What is the total system cost rate if the upper limit of unit 3 is increased to 201 MW?
 - vii. What is the total system cost rate if both limits of unit 2 are increased to 101 MW and 201 MW, respectively?

Solution to problem 11

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1

a. The uncontrained solution is found from applying the first and second KKT conditions. This results in:

$$F = 0.006P_1^2 + 5P_1 + 3 + 0.01P_2^2 + 4P_2 + 2 - \lambda(P_1 + P_2 - 500)$$

$$\partial F / \partial P_1 = 0.012P_1 + 5 - \lambda = 0$$

$$\partial F / \partial P_2 = 0.02P_2 + 4 - \lambda = 0$$

$$P_1 + P_2 - 500 = 0$$

In matrix form, this becomes:

$$\begin{bmatrix} 0.012 & 0 & -1 \\ 0 & 0.02 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} -5 \\ -4 \\ 500 \end{bmatrix} \implies \begin{bmatrix} P_1 \\ P_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 281.25 \\ 218.75 \\ 8.375 \end{bmatrix}$$

Both generation values are within their constraints; therefore the solution is feasible.

The LaGrange multiplier λ provides the change in the objective function (which is the total cost of b. supply) for a 1 MW increase in demand over the next hour, which is exactly what this question is asking, therefore the answer is \$8.375.

- c. Because the unconstrained solution is feasible, there are no inequality constraints that are binding. Therefore, by the complementary condition, it must be the case that all Lagrange multipliers on the inequality constraints must be zero, i.e., $\mu_i = 0 \forall i$.
- d. If demand is 550MW, then the unconstrained solution is found as

0.012	0	-1]	$\left\lceil P_1 \right\rceil$		-5		P_1		312.5
0	0.02	-1	P_2	=	-4	\Rightarrow	P_2	=	237.5
1	1	0	Lλ		550		λ		8.75

Observe that the 300MW upper bound on P_1 is violated, so the solution is not feasible. Therefore we need to bring in the inquality upper bound constrain on P₁ as an equality. This means our Lagrangian and KKT conditions become:

$$F = 0.006P_1^2 + 5P_1 + 3 + 0.01P_2^2 + 4P_2 + 2 - \lambda(P_1 + P_2 - 500) - \mu_1(P_1 - 300)$$

$$\frac{\partial F}{\partial P_1} = 0.012P_1 + 5 - \lambda - \frac{=}{\mu_1} = 0$$

$$\frac{\partial F}{\partial P_2} = 0.02P_2 + 4 - \lambda = 0$$

$$P_1 + P_2 - 550 = 0$$

$$P_1 = 300$$

In matrix form, this becomes:

Iı natrix form, this

$$\begin{bmatrix} 0.012 & 0 & -1 & -1 \\ 0 & 0.02 & -1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \lambda \\ = \\ \mu_1 \end{bmatrix} = \begin{bmatrix} -5 \\ -4 \\ 550 \\ 300 \end{bmatrix} \implies \begin{bmatrix} P_1 \\ P_2 \\ \lambda \\ = \\ \mu_1 \end{bmatrix} = \begin{bmatrix} 300 \\ 250 \\ 9 \\ -0.4 \end{bmatrix}$$

i. Determine how much would the total cost rate change if the total demand increased to 551MW.

\$9/hr.

ii. Identify λ and the incremental cost of both units.

λ=9.

iii. Identify the values of each Lagrange multiplier associated with the inequality constraints.

 $\mu_1 = -0.4$ but all other Lagrange multipliers associated with the inequality constraints are zero, i.e.,

$$\mu_{1} = 0$$

$$= 0$$

$$\mu_{2} = 0$$

$$\mu_{2} = 0$$

iv. What is the total system cost rate at this solution? $f=C_1(P_1)+C_2(P_2)=$

$$f = 0.006P_1^2 + 5P_1 + 3 + 0.01P_2^2 + 4P_2 + 2$$

= 0.006(300)² + 5(300) + 3 + 0.01(250)² + 4(250) + 2
= 3670

v. What is the total system cost rate if the upper limit of unit 1 is increased to 301 MW? 3670-0.4=3669.6

vi. What is the total system cost rate if both low limits of units 1 and 2 are increased to 1 MW?

3670