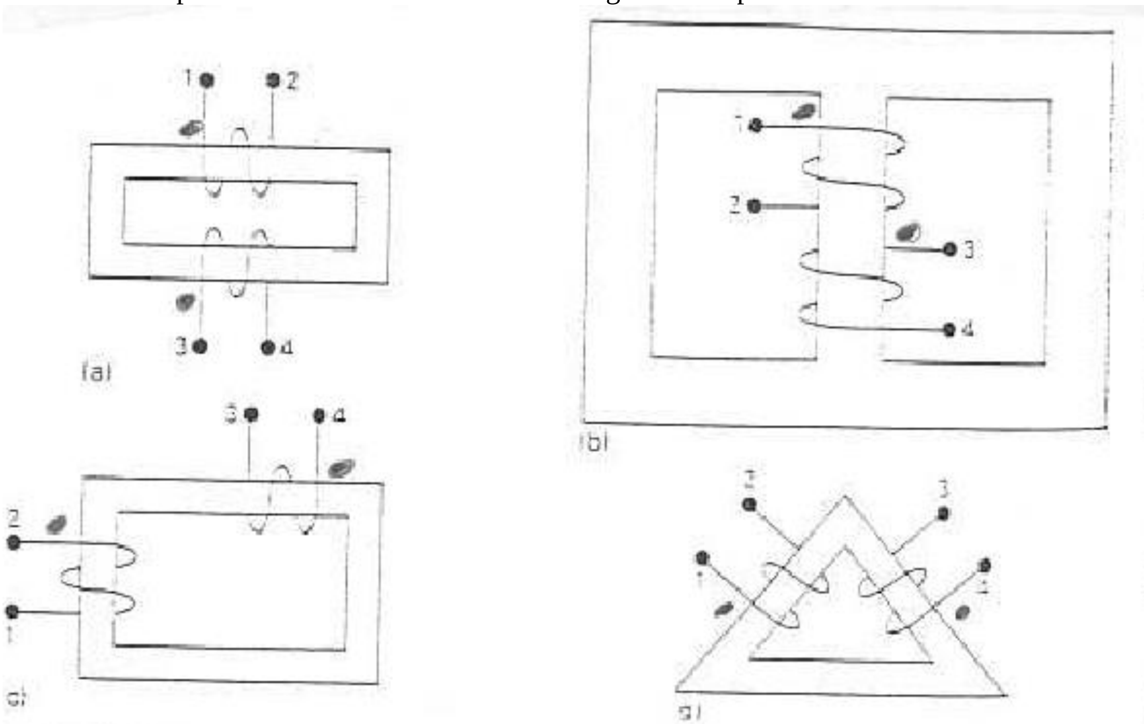
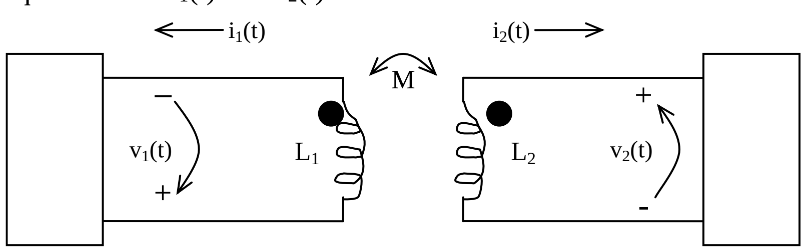
EE 303 Homework on Transformers, Dr. McCalley.

1. The physical construction of four pairs of magnetically coupled coils is shown below. Assume that the magnetic flux is confined to the core material in each structure (no leakage). Show two possible locations for the dot markings on each pair of coils.



In each case, it is possible to dot the other pair of terminals as well.

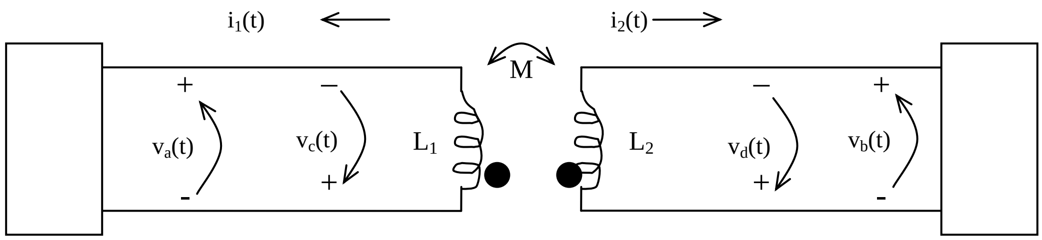
2. Write the equations for $v_1(t)$ and $v_2(t)$ for the circuit below.



$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

$$v_2(t) = -L_2 \frac{di_2(t)}{dt} - M \frac{di_1(t)}{dt}$$

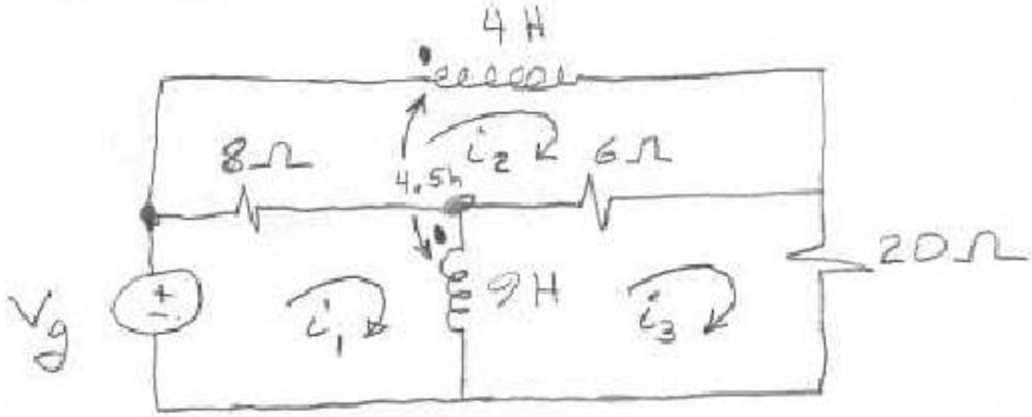
3. Write the equations for (a) $v_a(t)$ and $v_b(t)$ and (b) $v_c(t)$ and $v_d(t)$ for the circuit below.



ⓐ $V_a(t) = -L_1 \frac{di_1(t)}{dt} \;\; M \frac{di_2(t)}{dt}$, $\quad V_b(t) = -L_2 \frac{di_2(t)}{dt} - M \frac{di_1(t)}{dt}$

ⓑ $V_c(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$, $\quad V_d(t) = L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt}$

4. Write a set of mesh current equations that describe the circuit below in terms of i_1, i_2, and i_3.



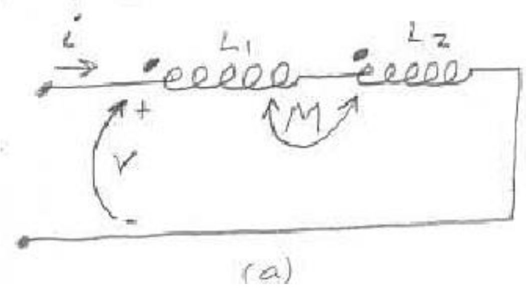
Top loop : $4\frac{di_2(t)}{dt} + 4.5\left[\frac{di_1(t)}{dt} - \frac{di_3(t)}{dt}\right] - 8\left[i_1(t) - i_2(t)\right] - 6\left[i_3(t) - i_2(t)\right] = 0$

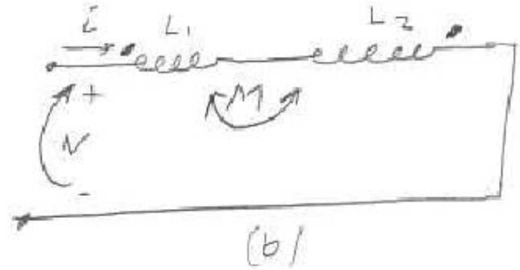
Left loop : $V_g = 9\left[\frac{di_1(t)}{dt} - \frac{di_3(t)}{dt}\right] + 4.5\frac{di_2(t)}{dt} + 8\left[i_1(t) - i_2(t)\right]$

Right loop : $9\left[\frac{di_1(t)}{dt} - \frac{di_3(t)}{dt}\right] + 4.5\frac{di_2(t)}{dt} - 6\left[i_3(t) - i_2(t)\right] - 20\left[i_3(t)\right]$

Note the last equation (right loop) should be equal to zero.

5. A pair of coupled inductors is connected in two different ways as shown below. In each case, find the differential equation relating v(t) and i(t), and then find the equivalent inductance "seen" at the terminals looking into the circuit.

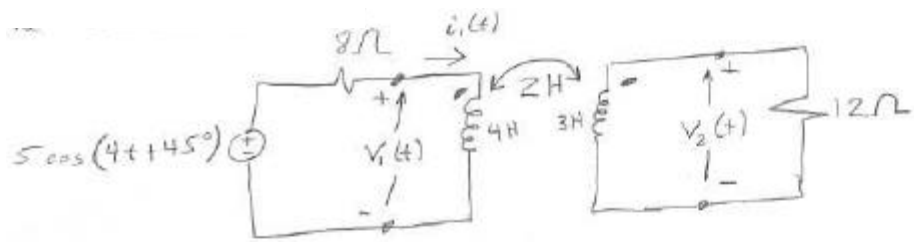




a) $v(t)=L_1di/dt+Mdi/dt+L_2di/dt+Mdi/dt$
   ➔ $v(t)=[L_1+L_2+2M]dt/dt$
   ➔ $L_{eq}=L_1+L_2+2M$

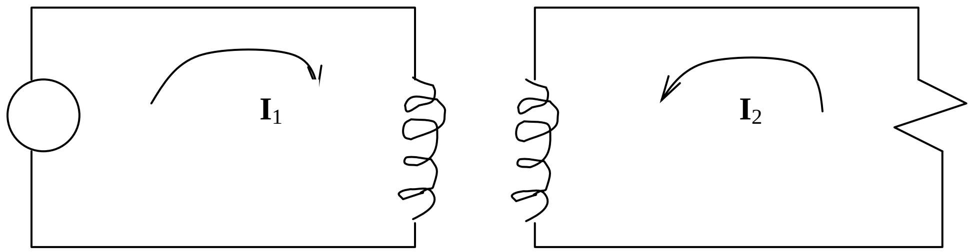
b) $v(t)=L_1di/dt-Mdi/dt+L_2di/dt-Mdi/dt$
   ➔ $v(t)=[L_1+L_2-2M]dt/dt$
   ➔ $L_{eq}=L_1+L_2-2M$

6. Use phasors to find the voltage $v_2(t)$ in the circuit below.



Since we need not do any power calculations in this problem, we may express all phasors using the maximum value as the magnitude. One important concept you must use here is that the differential equations, which characterize the relationships for *any time* under *any type of excitation*, may be converted to phasor relations for performing analysis under *steady-state conditions* when the excitation is *sinusoidal*. The transformation requires that you replace differentiation by "jω" in the phasor domain. One can see why if you just differentiate $i(t)=|I|\sin\omega t$. Then you get $di/dt=|I|\omega\cos\omega t$, which is just the original function, scaled by ω and rotated forward by 90 degrees, i.e., $di/dt=|I|\omega\sin(\omega t+90)$. So you see that the phasor for i(t) is just $I=|I|\angle 0$, and the phasor for di/dt is just $\omega|I|\angle 90=\omega|I|\angle 0\angle 90=j\omega I$.

Let's use the time domain expression first. We can use a mesh current for each side of the transformer, as shown below.



Left loop: $5\cos(4t+45^\circ) = 8i_1(t) + 4\frac{di_1(t)}{dt} + 2\frac{di_2(t)}{dt}$

Right loop: $3\frac{di_2(t)}{dt} + 2\frac{di_1(t)}{dt} + 12i_2(t) = 0$

Now let's change to phasor NOTATION:

$$5\angle 45^\circ = 8\bar{I}_1 + j\omega 4\bar{I}_1 + j\omega 2\bar{I}_2$$

$$j\omega 3\bar{I}_2 + j\omega 2\bar{I}_1 + 12\bar{I}_2 = 0$$

And with $\omega = 4$ (from the source voltage), we have

$$5\angle 45^\circ = 8\bar{I}_1 + j16\bar{I}_1 + j8\bar{I}_2 \Rightarrow 5\angle 45^\circ = (8+j16)\bar{I}_1 + j8\bar{I}_2 \quad (1)$$

$$j12\bar{I}_2 + j8\bar{I}_1 + 12\bar{I}_2 = 0 \qquad (12+j12)\bar{I}_2 + j8\bar{I}_1 = 0 \quad (2)$$

Solving the second equation for $\bar{I}_1 \Rightarrow \bar{I}_1 = \frac{-12}{j8}[1+j]\bar{I}_2 = \frac{j3}{2}[1+j]\bar{I}_2$

$\Rightarrow \bar{I}_1 = \frac{3}{2}[-1+j]\bar{I}_2$. Now substitute into (1): $(8+j16)(\frac{3}{2})[-1+j]\bar{I}_2 + j8\bar{I}_2 = 5\angle 45^\circ$

$\Rightarrow [(12+j24)[-1+j] + j8]\bar{I}_2 = 5\angle 45^\circ \Rightarrow \bar{I}_2 = 5\angle 45^\circ / [-4(9+j)] = .138\angle 141.35^\circ$

Now get $\bar{V}_2 = \bar{I}_2 R = (.138\angle 14.35^\circ)(12) = \underline{1.656\angle 141.35^\circ}$

Note the above calculation has two errors in the last two lines. These errors are:

a. next to last line: should be I_2=.138 /_ -141.35°. b. last line: should be $V_2=-I_2R$=1.656/_38.65°.

7. The ohmic values of circuit parameters of a transformer, having turns ration of $N_1/N_2=5$, are $R_1=0.5\Omega$, $R_2=0.021\Omega$, $X_1=3.2\Omega$, $X_2=0.12\Omega$, $R_c=350\Omega$, and $X_m=98\Omega$. (R_1, X_1, R_c, and X_m are given referred to the primary side; R_2 and X_2 are given referred to the secondary). Draw the approximate equivalent circuit of the transformer, with all quantities referred to (a) the primary and (b) the secondary. Show the numerical values of the circuit parameters.

The circuits are respectively shown in Fig. 2-10(*a*) and Fig. 2-10(*b*). The calculations are as follows:

(*a*)

$$R' \equiv R_1 + a^2 R_2 = 0.5 + (5)^2(0.021) = 1.025\ \Omega$$
$$X' \equiv X_1 + a^2 X_2 = 3.2 + (5)^2(0.12) = 6.2\ \Omega$$
$$R'_c = 350\ \Omega$$
$$X'_m = 98\ \Omega$$

(*b*)

$$R'' \equiv \frac{R_1}{a^2} + R_2 = \frac{0.5}{25} + 0.021 = 0.041\ \Omega$$
$$X'' \equiv \frac{X_1}{a^2} + X_2 = \frac{3.2}{25} + 0.12 = 0.248\ \Omega$$
$$R''_c = \frac{350}{25} = 14\ \Omega$$
$$X''_m = \frac{98}{25} = 3.92\ \Omega$$

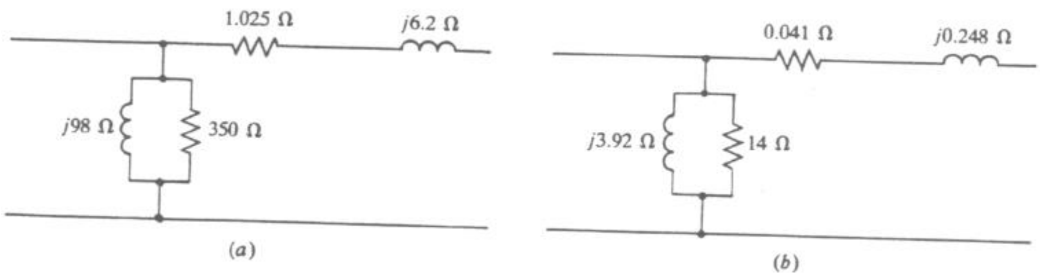


Fig. 2-10

8. The "exact" equivalent circuit parameters of a 150-kVA, 2400volt/240volt transformer are R_1=0.2 Ω, R_2=2mΩ, X_1=0.45Ω, X_2=4.5mΩ, R_c=10kΩ, and X_m=1.55kΩ. (R_1, X_1, R_c, and X_m are given referred to the primary side; R_2 and X_2 are given referred to the secondary). Using the circuit referred to the primary, determine the (a) percent voltage regulation and (b) efficiency of the transformer operating at rated load (150 kVA) with 0.8 lagging power factor. Assume that V_2=240 volts and note that percent voltage regulation is given by $(V_{no\text{-}load}-V_{load})/V_{load}$ (where these are voltage magnitudes).

See Figs. 2-3(*a*) and 2-4. Given $V_2 = 240$ V, $a = 10$, and $\theta_2 = \cos^{-1} 0.8 = -36.8°$,

$$a\mathbf{V}_2 = 2400\underline{/0°} \quad \text{V}$$

$$I_2 = \frac{150 \times 10^3}{240} = 625 \text{ A} \qquad \text{and} \qquad \frac{\mathbf{I}_2}{a} = 62.5\underline{/-36.8°} = 50 - j37.5 \quad \text{A}$$

Also, $a^2R_2 = 0.2\ \Omega$ and $a^2X_2 = 0.45\ \Omega$, so that

$$\mathbf{E}_1 = (2400 + j0) + (50 - j37.5)(0.2 + j0.45)$$
$$= 2427 + j15 = 2427\underline{/0.35°} \quad \text{V}$$
$$\mathbf{I}_m = \frac{2427\underline{/0.35°}}{1550\underline{/90°}} = 1.56\underline{/-89.65°} = 0.0095 - j1.56 \quad \text{A}$$
$$\mathbf{I}_c = \frac{2427 + j15}{10 \times 10^3} \approx 0.2427 + j0 \quad \text{A}$$

Therefore

$$\mathbf{I}_0 = \mathbf{I}_c + \mathbf{I}_m = 0.25 - j1.56 \quad \text{A}$$
$$\mathbf{I}_1 = \mathbf{I}_0 + (\mathbf{I}_2/a) = 50.25 - j39.06 = 63.65\underline{/-37.85°} \quad \text{A}$$
$$\mathbf{V}_1 = (2427 + j15) + (50.25 - j39.06)(0.2 + j0.45)$$
$$= 2455 + j30 = 2455\underline{/0.7°} \quad \text{V}$$

(*a*)

$$\text{percent regulation} \equiv \frac{V_{\text{no-load}} - V_{\text{load}}}{V_{\text{load}}} \times 100$$
$$= \frac{V_1 - aV_2}{aV_2} \times 100 = \frac{2455 - 2400}{2400} \times 100 = 2.3\%$$

(*b*)

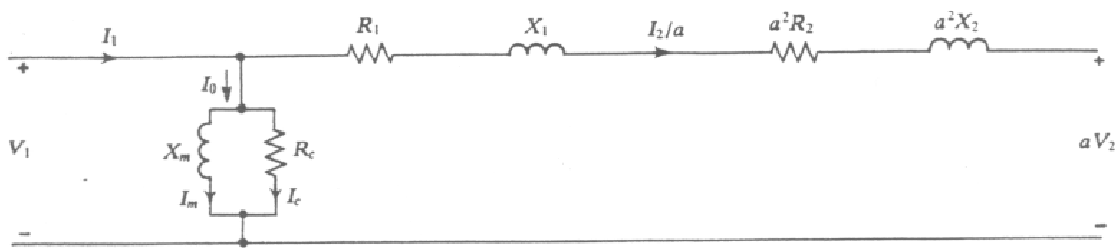
$$\text{efficiency} \equiv \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}}$$

$$\text{output} = (150 \times 10^3)(0.8) = 120 \text{ kW}$$
$$\text{losses} = I_1^2R_1 + I_c^2R_c + I_2^2R_2$$
$$= (63.65)^2(0.2) + (0.2427)^2(10 \times 10^3) + (625)^2(2 \times 10^{-3}) = 2.18 \text{ kW}$$

Hence

$$\text{efficiency} = \frac{120}{122.18} = 0.982 = 98.2\%$$

9. Using the "approximate equivalent circuit #1", shown below, repeat the calculations of problem 8 and compare the results (note that "a" is the turns ration N_1/N_2).



$$a\mathbf{V}_2 = 2400\underline{/0^\circ} \quad \text{V}$$

$$\frac{\mathbf{I}_2}{a} = 50 - j37.5 \quad \text{A}$$

$$R_1 + a^2R_2 = 0.4\ \Omega$$

$$X_1 + a^2X_2 = 0.9\ \Omega$$

Hence,

$$\mathbf{V}_1 = (2400 + j0) + (50 - j37.5)(0.4 + j0.9)$$

$$= 2453 + j30 = 2453\underline{/0.7^\circ} \quad \text{V}$$

$$\mathbf{I}_c = \frac{2453\underline{/0.7^\circ}}{10 \times 10^3} = 0.2453\underline{/0.7} \quad \text{A}$$

$$\mathbf{I}_m = \frac{2453\underline{/0.7^\circ}}{1550\underline{/90^\circ}} = 1.58\underline{/-89.3^\circ} \quad \text{A}$$

$$\mathbf{I}_0 = 0.2453 - j1.58 \quad \text{A}$$

$$\mathbf{I}_1 = 50.25 - j39.08 = 63.66\underline{/-37.9^\circ} \quad \text{A}$$

The phasor diagram is shown in Fig. 2-9.

(*a*) $$\text{percent regulation} = \frac{2453 - 2400}{2400} \times 100 = 2.2\%$$

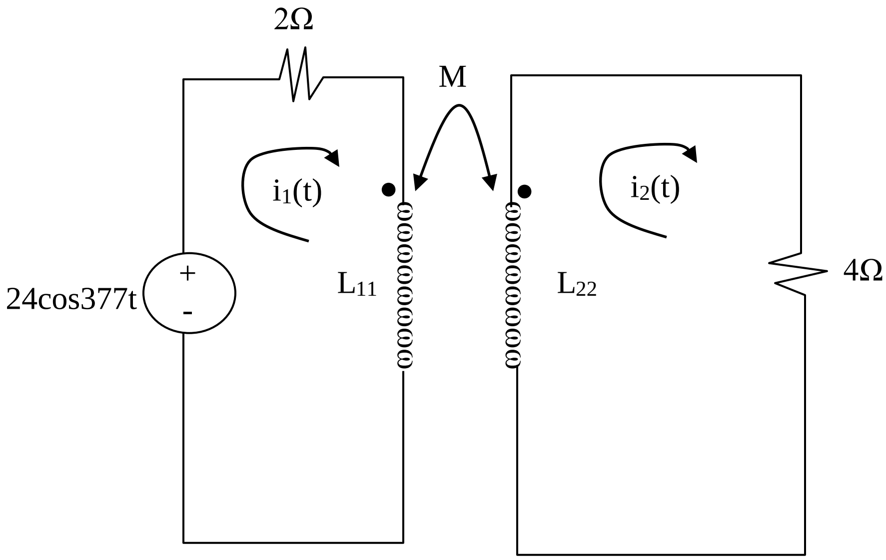
(*b*) $$\text{efficiency} = \frac{120 \times 10^3}{120 \times 10^3 + (63.66)^2(0.4) + (0.2453)^2(10 \times 10^3)} = 0.982 = 98.2\%$$

Notice that the approximate circuit yields results that are sufficiently accurate.

10. The coefficient of coupling for coupled coils is defined as

$$k = \frac{M}{\sqrt{L_{11}L_{22}}}$$

(a) Show that the idea case of no-leakage flux results in k=1.
(b) Determine for the actual case, where leakage flux exists, whether k>1 or k<1.
(c) The coupled circuit below has a coefficient of coupling of 1, Determine the energy stored in the mutually coupled inductors at time t=5 msec, where L_{11}=2.653 mH and L_{22}=10.61 mH.



Solution:

(a) We derived in class that when there is no leakage flux, $M = \sqrt{L_{11}L_{22}}$. Therefore in this case their ratio must be 1.0.

(b) From our notes, we derived that

$$L_{12} = \frac{N_1\phi_{12}}{i_2} \text{ and } L_{21} = \frac{N_2\phi_{21}}{i_1}$$

Under the no-leakage flux condition, we then used

$$\phi_{12} = \phi_{22} = \frac{\mu A}{l} N_2 i_2 \text{ and } \phi_{21} = \phi_{11} = \frac{\mu A}{l} N_1 i_1 .$$

Now we consider the case with leakage flux. Here, we assume that the percentage leakage flux from one coil is the same as the percentage leakage flux from the other coil. In this case, these relations should be written as

$$\phi_{12} = r\phi_{22} = \frac{r\mu A}{l} N_2 i_2 \text{ and } \phi_{21} = r\phi_{11} = \frac{r\mu A}{l} N_1 i_1 .$$

where r is the percentage leakage flux from each coil, 0<r<1. Then we can proceed as in the class notes. Substitution yields:

$$L_{12} = \frac{N_1 \frac{r\mu A}{l} N_2 i_2}{i_2} = rN_1N_2 \frac{\mu A}{l} = \frac{rN_1N_2}{R}$$

Likewise

$$L_{21} = \frac{N_2 \frac{r\mu A}{l} N_1 i_1}{i_1} = rN_2N_1 \frac{\mu A}{l} = \frac{rN_2N_1}{R}$$

Thus, we have that

$$L_{21} = L_{12} = M = \frac{rN_2 N_1}{R}$$

From the notes, we still have that

$$N_1 = \sqrt{L_{11} R}$$

$$N_2 = \sqrt{L_{22} R}$$

Substitution yields:

$$M = \frac{r\sqrt{L_{11}R}\sqrt{L_{22}R}}{R} = r\sqrt{L_{11}L_{22}} \Rightarrow r = \frac{M}{\sqrt{L_{11}L_{22}}}$$

And so r=k, which implies, k<1.

(c) Solution is below:

SOLUTION From the data the mutual inductance is

$$M = \sqrt{L_1 L_2} = 5.31 \text{ mH}$$

The frequency-domain equivalent circuit is shown in Fig. 12.8b, where the impedance values for X_{L_1}, X_{L_2}, and X_M are 1, 4, and 2, respectively. The mesh equations for the network are then

$$(2 + j1)\mathbf{I}_1 - 2j\mathbf{I}_2 = 24 \angle 0^\circ$$

$$-j2\mathbf{I}_1 + (4 + 4j)\mathbf{I}_2 = 0$$

Solving these equations for the two mesh currents yields

$$\mathbf{I}_1 = 9.41 \angle -11.31^\circ \text{ A} \quad \text{and} \quad \mathbf{I}_2 = 3.33 \angle +33.69^\circ \text{ A}$$

and therefore,

$$i_1(t) = 9.41 \cos(377t - 11.31^\circ) \text{ A}$$

$$i_2(t) = 3.33 \cos(377t + 33.69^\circ) \text{ A}$$

At $t = 5$ ms, $377t = 1.885$ rad or 108°, and therefore,

$$i_1(t = 5 \text{ ms}) = 9.41 \cos(108^\circ - 11.31^\circ) = -1.10 \text{ A}$$

$$i_2(t = 5 \text{ ms}) = 3.33 \cos(108^\circ + 33.69^\circ) = -2.61 \text{ A}$$

Therefore, the energy stored in the coupled inductors at $t = 5$ ms is

$$\begin{aligned} w(t)|_{t=0.005 \text{ sec}} &= \tfrac{1}{2}(2.653)(10^{-3})(-1.10)^2 + \tfrac{1}{2}(10.61)(10^{-3})(-2.61)^2 \\ &\quad - (5.31)(10^{-3})(-1.10)(-2.61) \\ &= (1.61)(10^{-3}) + (36.14)(10^{-3}) - (15.25)(10^{-3}) \\ &= 22.5 \text{ mJ} \end{aligned}$$