

Name (2 pts): \_\_\_\_\_

**Quiz 2, EE 303, September 14, 2017, Dr. McCalley.** Closed book, closed notes, calculator allowed, communication devices not allowed; Answer on this sheet of paper.

1. A three-phase line has an impedance of  $2+j4$  ohms/phase, and the line feeds two balanced three-phase loads that are connected in parallel. The first load is Y-connected and has an impedance of  $30+j40$  ohms/phase. The second load is delta-connected and has an impedance of  $60-j45$  ohms/phase. The line is energized at the sending end from a three-phase balanced supply of line voltage 207.85 volts. Taking the phase voltage  $V_{an}$  at the supply as reference, determine:
  - a. (30 pts) The current, real power, and reactive power drawn from the supply.
  - b. (25 pts) The line voltage at the combined loads.
  - c. (25 pts) The current per phase in each load.

**Solution:**

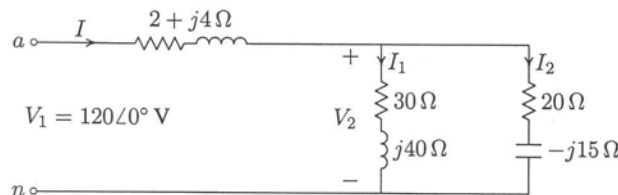
(a) The  $\Delta$ -connected load is transformed into an equivalent Y. The impedance per phase of the equivalent Y is

$$Z_2 = \frac{60 - j45}{3} = 20 - j15 \Omega$$

The phase voltage is

$$V_1 = \frac{207.85}{\sqrt{3}} = 120 \text{ V}$$

The single-phase equivalent circuit is shown in the following figure.



The total impedance is

$$Z = 2 + j4 + \frac{(30 + j40)(20 - j15)}{(30 + j40) + (20 - j15)} = 2 + j4 + 22 - j4 = 24 \Omega$$

with the phase voltage  $V_{an}$  as reference, the current in phase  $a$  is

$$I = \frac{V_1}{Z} = \frac{120\angle 0^\circ}{24} = 5 \text{ A}$$

The three-phase power supplied is

$$S = 3V_1 I^* = 3(120\angle 0^\circ)(5\angle 0^\circ) = 1800 \text{ W}$$

(b) The phase voltage at the load terminal is

$$V_2 = 120\angle 0^\circ - (2 + j4)(5\angle 0^\circ) = 110 - j20 = 111.8\angle -10.3^\circ \text{ V}$$

The line voltage at the load terminal is

$$V_{2ab} = \sqrt{3}\angle 30^\circ V_2 = \sqrt{3}(111.8)\angle 19.7^\circ = 193.64\angle 19.7^\circ \text{ V}$$

(c) The current per phase in the Y-connected load and in the equivalent Y of the  $\Delta$  load is

$$I_1 = \frac{V_2}{Z_1} = \frac{110 - j20}{30 + j40} = 1 - j2 = 2.236\angle -63.4^\circ \text{ A}; \quad I_2 = \frac{V_2}{Z_2} = \frac{110 - j20}{20 - j15} = 4 + j2 = 4.472\angle 26.56^\circ \text{ A}$$

The phase current in the original  $\Delta$ -connected load, i.e.,  $I_{ab}$  is given by

$$I_{ab} = \frac{I_2}{\sqrt{3}\angle -30^\circ} = \frac{4.472\angle 26.56^\circ}{\sqrt{3}\angle -30^\circ} = 2.582\angle 56.56^\circ \text{ A}$$

2. (18 pts) Consider the relation  $S = (\sqrt{3})|V_{LL}||I_L|[\cos\theta + j\sin\theta]$ , where  $V_{LL}$  is the line-to-line voltage phasor and  $I_L$  is the line current phasor, and  $|\bullet|$  indicates magnitude. Answer true/false for the below statements:

F \_\_\_\_ a)  $S$  is the per-phase complex power.

T \_\_\_\_ b) This relation can be used for both Y- and  $\Delta$ -connected loads.

F \_\_\_\_ c) The angle  $\theta$  is the angle by which  $V_{LL}$  leads  $I_L$ .