Module PE2

Induction Motor

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Prerequisite Competencies: Basic laws and theorems in Electric Circuits, typically covered in an introductory circuits course. Concept of three phase circuits, found in Module B3

Module Objectives:
1. Identify induction motors as loads on electric power systems.
2. Explain motor operation in terms of Faraday's law of induction and Lorentz's force equation.
3. Perform steady-state calculations for induction motor operation in terms of applied voltage, currents, slip, rotational speed and torque.
4. Identify basic relationship between speed-torque characteristics of an induction motor and speed-torque characteristics of typical loads.
5. Identify main criteria used in selection of a motor for industrial applications and how these criteria relate to NEMA standards.

PE2.0 Introduction

The induction motor is a three-phase AC motor and is the most widely used machine. Its characteristic features are as follows:

- It is simple and rugged
- It is low cost and requires minimum maintenance
- It has high reliability and has sufficiently high efficiency
- It needs no extra starting motor and need not be synchronized

An induction motor has basically two parts, the stator and the rotor.
The stator is the outer part of an induction motor and is stationary. It is made up of a number of stampings with slots to carry a three-phase winding. It is wound for a definite number of poles. The windings are geometrically spaced 120 degrees apart.

The rotor is the moving part in an induction motor. It rotates concentrically within the stator. A small air gap exists between a stator and a rotor. Two types of rotors are used in induction motors,
Wound rotor and Squirrel-cage rotor.

A wound rotor has a three-phase, double-layer, distributed winding. It is wound for as many poles as are on the stator. A pole refers to the magnetic poles (north and south poles) that are created when three-phase supply is given to the winding. An induction motor can have one or more pairs of poles. The speed of rotation depends on the *number of pole pairs* that an induction motor is wound for.

The three phases are wyed internally and the other ends are connected to slip-rings mounted on shaft with brushes resting on them. The brushes are connected to an external resistance that does not rotate with the rotor and can be varied to change the speed-torque characteristics. In fact an induction motor can be compared with a transformer because of the fact that just like a transformer it is a singly energized device which involves changing flux linkages with respect to a primary (stator) winding and secondary (rotor) winding. An induction motor may be considered to be a transformer with a rotating secondary.

A squirrel-cage rotor consists of thick conducting bars embedded in parallel slots. These bars are permanently short-circuited at both ends by means of short-circuiting rings. The solid bars are placed approximately parallel to the shaft and embedded in the surface of the core. The bars are normally skewed as shown. This provides more uniform torque and also reduces the humming noise and mechanical vibrations when the motor is running.

PE2.1 Production of a Rotating Magnetic Field

An induction motor operates on the principle of induction. The rotor receives power due to induction from the stator rather than direct conduction of electrical power. It is important to understand the principle of rotating magnetic field in order to understand the operation of an induction motor. When a three-phase voltage is applied to the stator winding, a rotating magnetic field of constant magnitude is produced. This rotating field is produced by the contributions of space-displaced phase windings carrying appropriate time displaced currents. These currents, which are time displaced by 120 electrical degrees, are shown in Figure PE2.1.

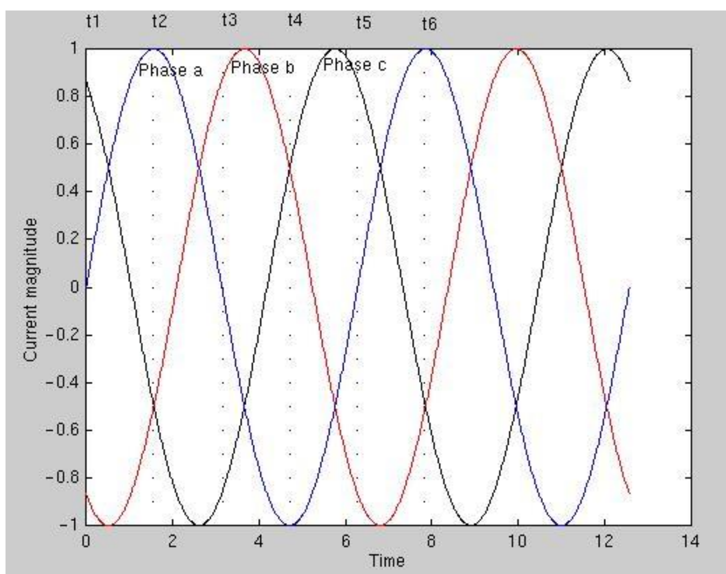

Figure PE2.1 Plot Representing the Three Phase Currents

Consider a stator structure with three-phase winding, as depicted in Figure PE2.2

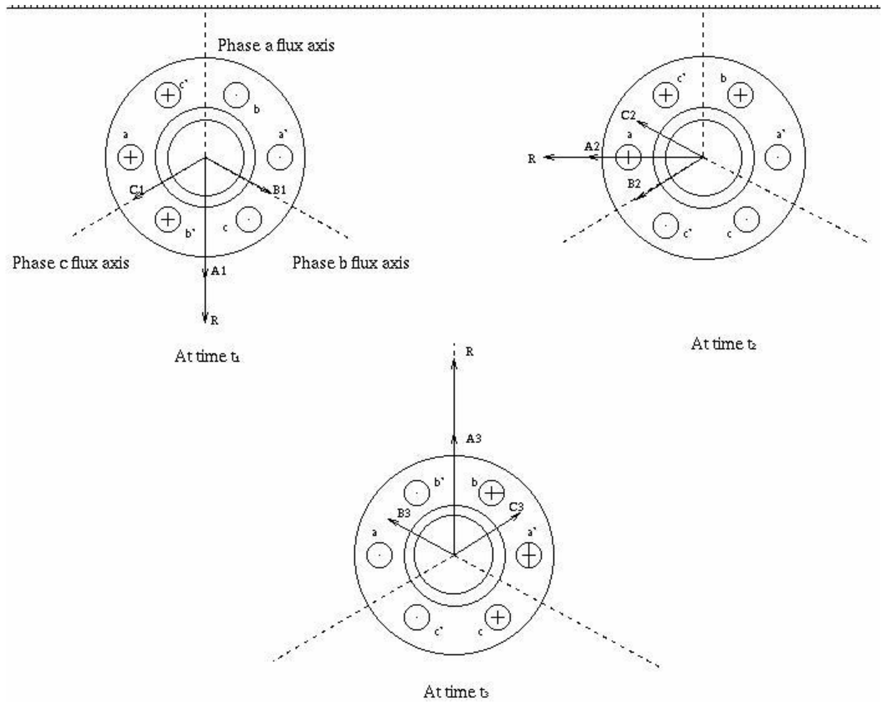

Figure PE2.2 Diagram for Resultant Flux at Three Different Time Intervals

The coil a-a' represents the entire phase windings for phase a. Similarly, b-b' and c-c' represent the windings for phases b and c. The current through each winding produces flux throughout the air gap of the motor. Note carefully;

- At any particular point in the air gap, the flux from each winding varies in time due to the variation in time of the current producing it.
- At any particular moment in time, the flux from each winding varies in space along the air gap due to the change in physical distance to the winding.

Our goal is to describe the resultant flux in the air gap, from all three windings, at any time *t*, and at any physical angle θ, where θ is illustrated in Fig PE2.2

If the currents in the three windings are balanced equal in magnitude and phased displaced in time by $120°$, and if the windings are spatially displaced along the stator circumference by $120°$, then for sinusoidal current variation at frequency ϖ (normal 377 rad/sec in North America), we may describe the time and space variation in flux from each phase as :

$$\begin{aligned} \Phi_a &= \Phi_m \cos(\omega t)\cdot\cos(\theta) \\ \Phi_b &= \Phi_m \cos(\omega t-120°)\cdot\cos(\theta-120°) \\ \Phi_c &= \Phi_m \cos(\omega t-240°)\cdot\cos(\theta-240°) \end{aligned} \qquad \text{(PE2.1)}$$

Here, Φ_m is the value flux that occurs at a location $90°$ from the winding just when the winding current peaks. It is the maximum value of flux. The trigonometric function of ωt accounts for current variation on time, and the trigonometric function of θ accounts for variation in space. Therefore, one can use these equations to visualize the pulsating-in-time flux at a particular point θ or the spatially distributed sinusoidal wave having an amplitude governed by the particular time *t*.

To get the resultant, we add:

$$\begin{aligned}\Phi_n &= \Phi_a + \Phi_b + \Phi_c \\ &= \Phi_m \cos(\omega t)\cdot\cos(\theta) \\ &\quad + \Phi_m \cos(\omega t - 120^\circ)\cdot\cos(\theta - 120^\circ) \\ &\quad + \Phi_m \cos(\omega t - 240^\circ)\cdot\cos(\theta - 240^\circ)\end{aligned}$$

Using the trigonometric identity

$$\cos(a)\cdot\cos(b) = \frac{1}{2}\cos(a-b) + \frac{1}{2}\cos(a+b)$$

we find that

$$\Phi_n = \frac{1}{2}\Phi_m \begin{bmatrix}\cos(\theta - \omega t) + \cos(\theta + \omega t) + \cos(\theta - \omega t) + \\ \cos(\theta + \omega t - 240^\circ) + \cos(\theta - \omega t)\cos(\theta + \omega t - 480^\circ)\end{bmatrix}$$

Noting that

$$\cos(\theta + \omega t) + \cos(\theta + \omega t - 240^\circ) + \cos(\theta - \omega t)\cos(\theta + \omega t - 480^\circ) = 0$$

we have

$$\Phi_n = \frac{3}{2}\Phi_m [\cos(\theta - \omega t)] \tag{PE2.2}$$

Here we see that at any particular time, the resultant flux has a sinusoidal distribution about the air gap. As time advances, this distribution retains its shape and amplitude, but it "moves" in space. The speed of this movement is ω electrical radian per second. If the machine has only *P*=2 poles, this will also be the mechanical speed of rotation. Otherwise,

$$\varpi_s = \frac{2}{p}\cdot\varpi \tag{PE2.3}$$

This speed is the mechanical rotational speed of the magnetic field. It is sometimes called the synchronous speed. We can also write the synchronous speed in revolutions per minute (rpm), N_s, according to:

$$N_s = \omega_s \frac{radian}{second} \times \frac{1 rev}{2\pi\, rad} \times \left[\frac{60\sec}{\min}\right] = \frac{60}{2\pi}\omega_s$$

and since

$$\omega_s = \frac{2}{p}\omega = \frac{2}{p}(2\pi f) = \frac{4\pi f}{p}$$

we have

$$N_s = \frac{60}{2\pi}\left(\frac{4\pi f}{p}\right) = \frac{120 f}{p}$$

PE2.2 The Concept of 'Slip'

The rotating field produced in an induction motor rotates at a mechanical speed of ω_s, the synchronous speed. The rotating field induces an emf. in the rotor-circuit. Current flows through the short-circuited rotor windings to produce a flux. The interaction between the stator flux and the rotor currents sets up a torque. This torque sets the rotor in motion. If ω_m (rad /sec) is the rotor speed, then the normalized difference between the motor speed ω_m and the magnetic field speed, ω_s is called he slip, s.

$$s = \frac{(\omega_s - \omega_m)}{\omega_s} \qquad \text{(PE2.4)}$$

$$\omega_m = (1 - s) \times \omega_s$$

In term of rpm we have: $N_m = N_s \times (1 - s)$

When the rotor is stationary, the frequency of rotor current is same as the supply frequency. But when the rotor starts revolving, the rotor current frequency becomes dependent on the relative speed between the rotor windings and the rotating magnetic field. Let f' be the rotor current frequency at any slip *s*.

f' is proportional to $(N_s - N_m)$

Hence $\frac{f'}{f} = \frac{(N_s - N_m)}{N_s} = s$

$$f' = s \times f \qquad \text{(PE2.5)}$$

We have seen that when a three-phase supply is applied to the stator windings, a rotating field is produced. This flux of constant magnitude cuts the stationary conductors of rotor. This induces an emf in the conductors, according to Faradays law of electro-magnetic induction. Since the rotor conductors form a closed circuit, a current is induced in them having a direction, per Lenz's law, to oppose its very own cause i.e., the relative speed between the rotating flux and the stationary rotor conductors. This tendency to reduce the relative speed results in a torque that rotates the rotor in the direction of the magnetic field.

Example PE 2.1

A three-phase Y-connected, 400 hp, 380 V(l-l), 50 Hz, wound rotor induction motor operating at rated conditions has a slip of 0.0159. Determine the rotor frequency.

Solution

This problem might be interpreted in two ways-frequency (i.e., speed) of rotor or frequency of rotor currents.
According to the first interpretation rotor speed is $\omega_m = (1 - s)\omega_s$

Hence $\omega_m = (1 - s)\omega_s = (1 - 0.0159) \cdot 157.1 = 154.6\, rad/\sec$

$$N_s = 120 \times \frac{f}{p} = 120 \times \frac{50}{4} = 1500\ rpm$$

$$\omega_s = \frac{2 \times \pi \times N_s}{60} = \frac{2 \times \pi \times 1500}{60} = 157.1\ rad/\sec$$

So $\omega_m = 154.6\ rad/\sec = 1476.2\ rpm$

According to the second interpretation, the frequency of rotor current f' is given by:
$f' = s \times f = (0.0159)(50) = 0.795\, Hz$

PE2.3 Equivalent Circuit of an Induction Motor

For operational analysis and to facilitate computation of performance, it is desirable to have an equivalent circuit of an induction motor. It is similar to that of a transformer except that the secondary is rotating and hence modifications have to be made to account for the mechanical power. All the parameters of the equivalent circuit are expressed on a per phase basis.

The exact equivalent circuit model of an induction motor is:

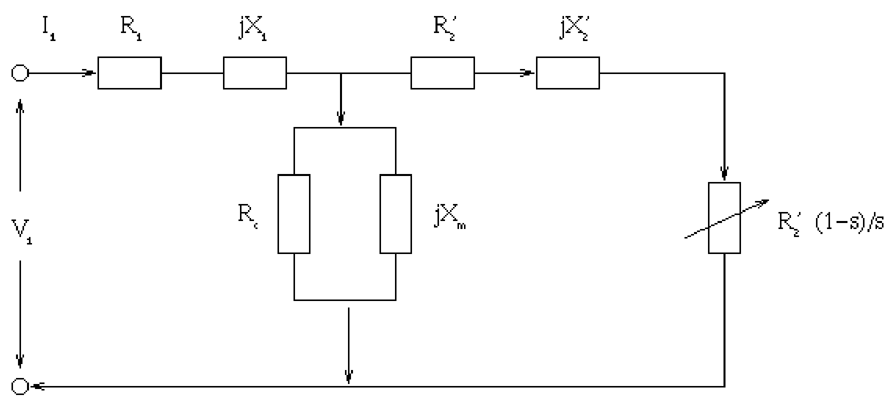

Figure PE2.3 Exact Equivalent Circuit of an Induction Motor

Where

R_1	:	is the stator phase winding resistance per phase
X_1	:	is the stator leakage reactance per phase
R_2'	:	is the equivalent rotor resistance referred to stator per phase
X_2'	:	is the equivalent rotor reactance referred to stator per phase
R_c	:	is the resistance representing core losses
X_m	:	is the magnetizing reactance per phase
V_1	:	is the per phase supply voltage to the stator
s	:	is the slip of the motor

PE2.3.1 The Stator Circuit Per Phase

From Figure PE2.3 we see that the portion of equivalent circuit that has reference to the stator side consists of the stator phase winding resistance R_1, a stator phase winding leakage reactance X_1 and a magnetizing impedance made up of the core-loss resistor R_c and the magnetizing reactance X_m. This circuit is similar to that of a transformer. However the magnitude of magnetizing current is different in the two cases. In case of a transformer it is 2 to 5 % whereas in case of an induction motor it is around 25 to 35% of the rated current necessarily because of the air gap.

PE2.3.2 The Stator-Referred Rotor Equivalent Circuit

The voltages appearing across the stator terminals and the rotor terminals are not the same. Just as in the case of a transformer, they depend on the number of the effective turns of the two windings. Thus, when a single-line equivalent circuit is to be drawn as in Figure PE2.3, all the rotor circuit quantities are referred to the stator. The prime notation in the circuit represents the stator referred rotor quantities. The relationship between actual and stator referred rotor quantities is related to the ratio of the number of turns of the rotor and the stator.

PE2.3.3 The Rotor Circuit Per Phase

The rotor current per phase can be expressed as

$$I_2 = \frac{E_2^{'}}{R_2^{'} + \frac{R_2^{'}}{s}(1-s) + jX_2^{'}} = \frac{E_2^{'}}{\frac{R_2^{'}}{s} + jX_2^{'}}$$

where $E_2^{'}$ is the voltage impressed across the rotor circuit

The corresponding interpretation is that the current $I_2^{'}$ is produced by the supply frequency voltage $E_2^{'}$ acting on a rotor circuit having an impedance of $\frac{R_2^{'}}{s} + jX_2$. This indicates that the rotor circuit is characterized by a variable resistance and constant leakage reactance. The total power associated with the rotor circuit now is

$$P = I_2^{'2}\left(\frac{R_2^{'}}{s}\right)$$

This can be written as $P = I_2^{'2}\left[R_2^{'} + \frac{R_2^{'}}{s}(1-s)\right]$

Thus, the rotor circuit now represents both the rotor winding resistance $R_2^{'}$ and the variable resistance $\frac{R_2^{'}}{s}(1-s)$ representing mechanical load instead of a single slip dependent resistance. This expression is very useful in analysis.

PE2.4 Power Flow in an Induction Motor

In an induction motor, the input is electrical power P and the output is mechanical power P_{out}. The power flow within an induction motor is best understood by means of a power flow diagram.

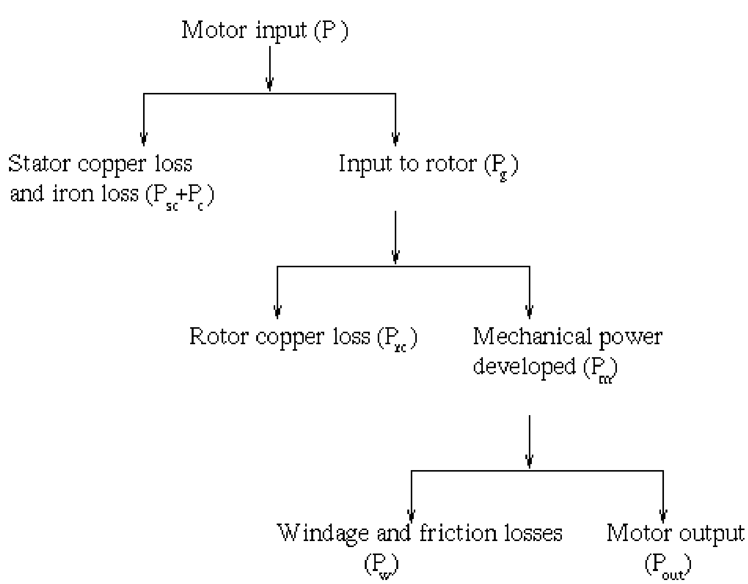

Figure PE2.4 Power Flow Diagram

A part of the total input power is lost as stator copper and iron losses P_{sc} and P_c. P_g is the power input to the rotor and from Figure PE2.3 we see that:

$$P_g = 3 \times (I_2')^2 \times \left[R_2' + R_2' \times \frac{(1-s)}{s} \right]$$

This equation can be split into two parts as follows :

$$P_g = \left[3(I_2')^2 R_2' \right] + \left[3R_2' \frac{(1-s)}{s} \right]$$

The first part represents the rotor copper losses P_{rc}

From the equivalent circuit it can be seen that the equivalent resistance $R_2 \dfrac{(1-s)}{s}$ is a variable resistance and is equivalent to the mechanical power developed. Thus, the second part corresponds to the mechanical power developed P_m to meet the load.

$$P_m = \left[3|I_2'|^2 R_2' \frac{(1-s)}{s} \right] \tag{PE2.6a}$$

This mechanical power less the windage and friction losses in the rotor gives the final mechanical power output P_{out}.

Example PE 2.2

A three-phase induction motor has 4-pole, Y-connected stator winding. It runs on a 50 Hz system with 220 V between lines. The rotor resistance per phase, referred to the stator, is 0.1Ω.
Calculate (a) rotor copper losses P_{rc} (b) mechanical power developed P_m and (c) the power across the gap, P_g.

The motor is operating at a slip of 4%, and the rotor current, referred to the stator, is $I_2' = 29.1A$.

Solution

(a) The total copper losses are $P_{rc} = 3I_2'^2 R_2' = 3(29.1)^2\, 0.1 = 255\ W$

(b) The Mechanical power developed $P_m = \dfrac{P_{rc}(1-s)}{s} = \dfrac{255(1-0.04)}{0.04} = 6120\ W$

(c) The power delivered to the rotor across the air gap is $P_g = P_{rc} + P_m = 255 + 6120 = 6375W$

PE2.5 Torque Considerations of an Induction Motor

In the previous section, the expression for the mechanical power P_m has been developed. For obtaining the expression for torque we use Equation PE2.6 and consider the Thevenin's equivalent of the actual circuit to obtain a simplified expression for I_2'.

PE2.5.1 Thevenin's Equivalent Circuit

To obtain the Thevenin's equivalent circuit of the actual equivalent circuit, it is necessary to obtain the Thevenin's equivalent impedance Z_{th} and the equivalent voltage V_{th}.

Considering Figure PE2.5, the Thevenin's equivalent has to be obtained between points A and B looking into the direction as shown by the arrow.

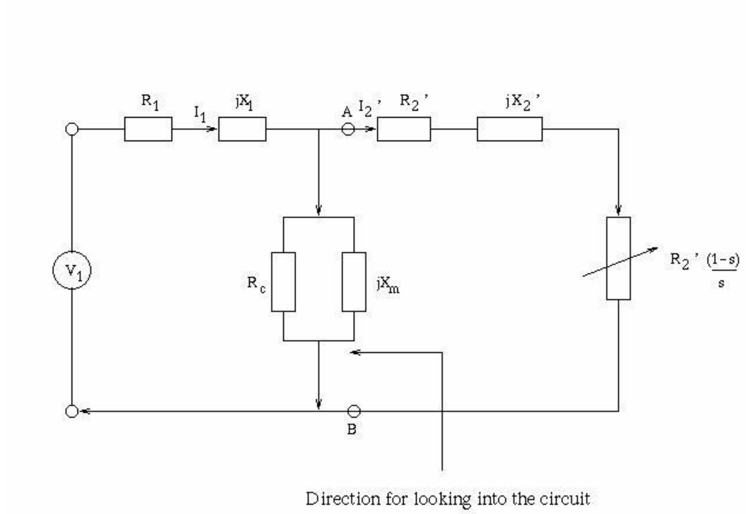

Figure PE2.5 Thevenin's Equivalent Impedance and Voltage

As per Thevenin's theorem, the voltage source V_1 is short-circuited to obtain the equivalent impedance between points A and B.

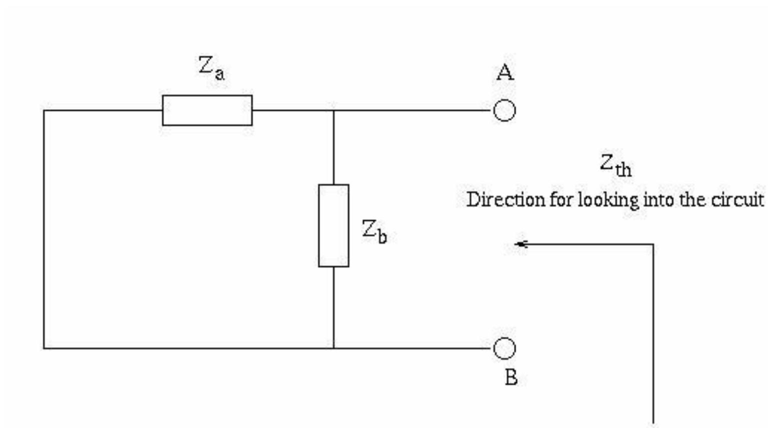

Figure PE2.6 Thevenin's Equivalent Impedance

From Figure PE2.6, we see that the equivalent impedance Z_{th} is a parallel combination of Z_a and Z_b where, $Z_a = R_1 + jX_1$ and Z_b is the parallel combination of R_c and jX_m. Thus, the Thevenin's equivalent circuit of an induction motor is seen in Figure PE2.7.

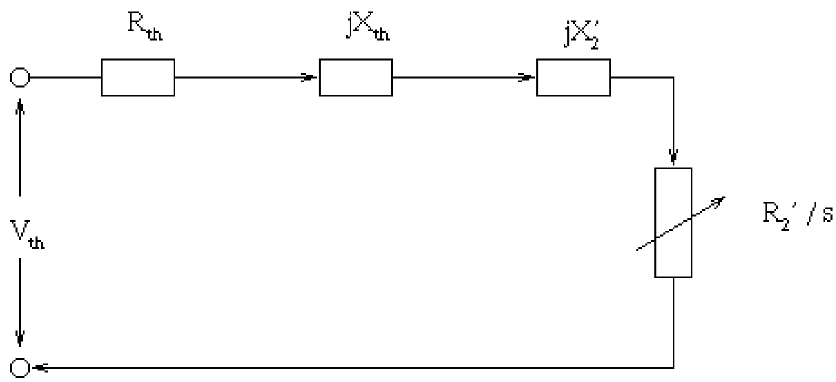

Figure PE2.7 Thevenin's Equivalent Circuit of an Induction Motor

PE2.5.2 Expression for Torque

In an induction motor, the torque developed T is given by:

$$T = \frac{P_m}{\omega_m} = \frac{3 \cdot \left|I_2'\right|^2 \cdot R_2'}{s \cdot \omega_s} \qquad \text{(PE2.6b)}$$

Where $P_m \rightarrow$ Mechanical power developed

$\omega_m \rightarrow$ Motor speed

From the Figure PE2.7, we get the expression for the current I_2' as

$$I_2' = \frac{V_{th}}{\left[\left(R_{th} + \frac{R_2'}{s}\right) + j\left(X_{th} + X_2'\right)\right]}$$

$$\left|I_2'\right|^2 = \frac{V_{th}^2}{\left[\left(R_{th} + \frac{R_2'}{s}\right)^2 + \left(X_{th} + X_2'\right)^2\right]} \qquad \text{(PE2.7)}$$

Substituting Equation PE2.7 in Equation PE2.6b,

$$T = \frac{3V_{th}^2 R_2'}{\left(s\omega_s\right)\left[\left(R_{th} + \frac{R_2'}{s}\right)^2 + \left(X_{th} + X_2'\right)^2\right]} \qquad \text{(PE2.8)}$$

PE2.5.3 Starting Torque and Maximum Torque

At the time of starting an induction motor, the speed is zero.

$\omega_m = 0$ and $s = \dfrac{\omega - \omega_m}{\omega} = 1$

Thus, to obtain the starting torque we put s=1 in the Equation PE2.8.
Hence the starting torque is

$$T_{start} = \frac{3V_{th}^{\ 2}\ R_2'}{(\omega_s)\left[(R_{th} + R_2')^2 + (X_{th} + X_2')^2\right]} \qquad \text{(PE2.9)}$$

To obtain the maximum torque produced, we initially need to find the slip at maximum torque. From basic calculus, we know that maximum torque will occur when $\dfrac{dT}{ds} = 0$. Applying this condition we get $s_{\max T}$, that is the slip at maximum torque $T_{\max}$ as:

$$s_{\max T} = \frac{R_2'}{\sqrt{\left[R_{th}^{\ 2} + (X_2' + X_{th})^2\right]}} \qquad \text{(PE2.10)}$$

Substituting Equation PE2.10 in Equation PE2.8, we get the expression for maximum torque $T_{\max}$.
Thus

$$T_{\max} = \frac{3V_{th}^{\ 2}}{(2\,\omega_s)\left[R_{th} + \sqrt{\left(R_{th}^{\ 2} + (X_2' + X_{th})^2\right)}\right]} \qquad \text{(PE2.11)}$$

Now, to simplify the equation, we neglect R_{th} and the equation for maximum torque is obtained as:

$$T_{\max} = \frac{3V_{th}^{\ 2}}{(2\,\omega_s)\left[(X_{th} + X_2')\right]} \qquad \text{(PE2.12)}$$

We now get the relationship between the starting torque T_{start} t, maximum torque $T_{\max}$ and T i.e., the torque developed at a slip say *s*.

$$\frac{T_{start}}{T_{\max}} = \frac{2\,s_{\max T}}{\left(1 + s_{\max T}^{\ 2}\right)} \qquad \text{(PE2.13)}$$

$$\frac{T_d}{T_{\max}} = \frac{2\,s_{\max T}\ s}{\left(s^2 + s_{\max T}^{\ 2}\right)} \qquad \text{(PE2.14)}$$

Example PE 2.3

A three-phase Y-connected, 400 hp, 380 V(l-l), 60 Hz, 4 pole wound rotor induction motor is operating at rated conditions. The machine parameters expressed in ohms are: R_{th}=0.00536 ohms; R_2'=0.00613 ohms; X_{th}=0.0383ohms; X_2'=0,0383 ohms. Calculate (a) the slip at which maximum torque occurs (b) the value of maximum torque (c) the starting torque. Assume V_{th} is same as V_1.

Solution

(a) Let s_{Tmax} be the slip at which maximum torque is obtained

We know that $s_{\max T} = \dfrac{R_2'}{\sqrt{R_{th}^2 + (X_{th} + X_2')^2}} = \dfrac{0.00613}{\sqrt{(0.00536)^2 + (0.083 + 0.083)^2}}$

So $s_{\max T} = 0.0798$

If we assume that $R_1 = 0$ and $R_c = \infty$, then $R_{th} = 0$ and $s_{\max T} = \dfrac{R_2'}{X_{th} + X_2'}$

Hence $s_{\max T} = \dfrac{0.00613}{0.0383 + 0.0383} = 0.08$

<u>Either of the above two values are acceptable.</u>

(b) The maximum torque can be obtained from Equation PE2.12

$$\omega_s = \frac{2}{p}\omega = \frac{2}{4}(2\pi)60 = 188.4$$

Thus

$$T_{\max} = \frac{3(219.4)^2}{2(188.4)(0.0383 + 0.0383)} = 5003.28\ N \cdot m$$

(c) From Equation PE2.13 we get the starting torque as:

$$T_{start} = \frac{2T_{\max}\, s_{\max T}}{\left(1 + s_{\max T}^{\ 2}\right)} = \frac{5003.28 \times 2 \times 0.0798}{1 + (0.0798)^2} = 793.47\, N \cdot m$$

PE2.5.4 Speed-Torque Characteristics

The expression for torque obtained from Equation PE2.8 can be rewritten as:

$$T = \frac{k_1}{\left(k_2 + sk_3 + \frac{k_4}{s}\right)}$$

Where
$$k_1 \rightarrow \frac{3 V_{th}^{\ 2}\ R_2'}{2\pi\ N_s}$$
$$k_2 \rightarrow 2\ R_{th}\ R_2'$$
$$k_3 \rightarrow R_{th}^{\ 2} + (X_{th} + X_2')^2$$
$$k_4 \rightarrow (R_2')^2$$

Initially when the motor starts, the slip is 1. So it can be assumed that k_4/s is negligible as compared to other terms (since k_4 is very small). Hence, the torque produced increases with speed N_m, since

$$T = \frac{k_1}{(k_2 + sk_3)} = \frac{k_1}{k_2 + \left(\frac{N_s - N_m}{N_s}\right)k_3}$$

However, when the motor attains stable speed, slip is negligible. Hence $k_3 \cdot s \approx 0$ and the torque decreases with speed N_m, since

$$T = \frac{k_1}{\left(k_2 + \frac{k_4}{s}\right)} = \frac{k_1}{k_2 + \left(\frac{N_s}{N_s - N_m}\right)k_4}$$

From these relationships, the general shape of speed -torque characteristics of induction motor can be obtained. This curve also results from repeated application of Equation (PE2.8)

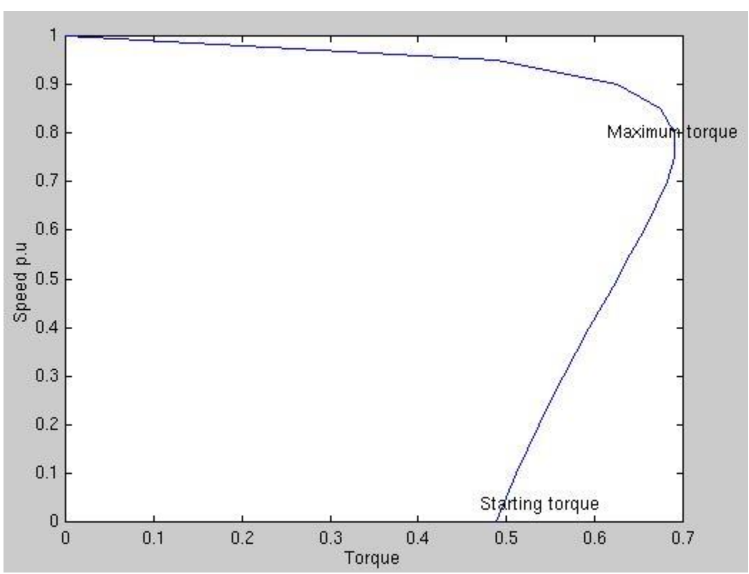

Figure PE2.8 Speed-Torque Characteristics of an Induction Motor

PE2.6 Factors Affecting the Speed-Torque Characteristics of an Induction Motor

The speed-torque characteristics are affected by various factors like applied voltage, R_2' and to some extent frequency. The main effect of change in frequency is to change the synchronous speed of the motor itself.

PE2.6.1 Applied Voltage

We know that $T \propto V^2$. Thus, not only the stationary torque but also the torque under running conditions changes with change in supply voltage.

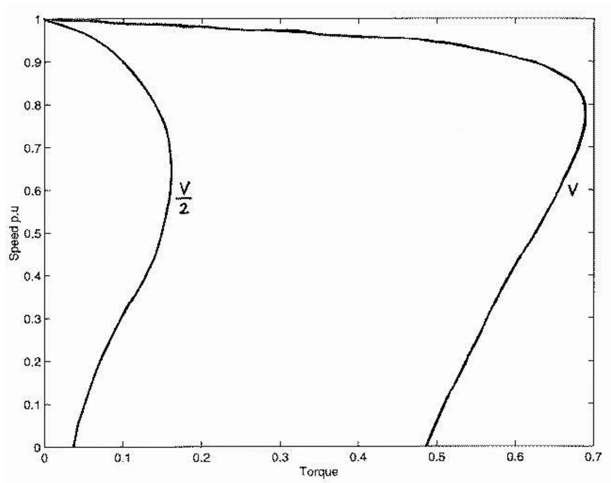

Figure PE2.9 Speed-Torque Characteristics for Different Supply Voltages

PE2.6.2 Rotor Resistance

The maximum torque produced does not depend on R_2'. However, with increase in R_2', the starting torque increases. The slip at which T_{max} is reached increases too which means that T_{max} is obtained at lower motor speeds.

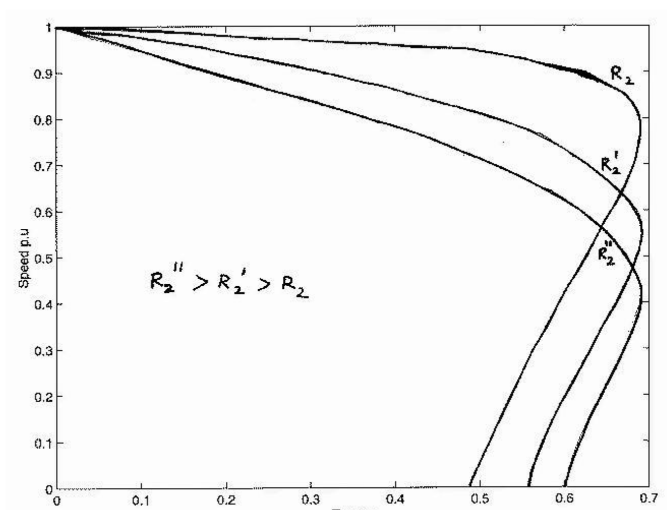

Figure PE2.10 Speed-Torque Characteristics for Different Values of Rotor Resistance

PE2.7 NEMA Specifications

The National Electrical Manufacturers Association (NEMA) standardized four basic design categories of Induction motors to match the torque-speed requirements of the most common types of mechanical loads. These basic design categories are Design A, Design B, Design C and Design D.

The Design B motor serves as the basis for comparison of motor performance with other designs. It has the broadest field of application and is used to drive centrifugal pumps, fans, blowers and machine tools. It has a relatively high efficiency, even at light loads, and a relatively high power factor at full load.

The Design A motor has essentially the same characteristics as the Design B, except for somewhat higher break-down torque. Since its starting current is higher, however, its field of application is limited.

The Design C motor has a higher locked-rotor torque, but a lower break-down torque than the Design B. The higher starting torque makes it suitable for driving plunger pumps, vibrating screens and compressors without unloading devices. The starting current and slip at rated torque are essentially the same as that for the Design B.

The Design D motor has a very high locked-rotor torque and a high slip. Its principal field of application is in high-inertia loads such as fly-wheel equipped punch presses, elevators and hoists.

The Design D rotor has relatively high-resistance, low-reactance rotor bars close to the surface. Design B and Design A have low-resistance with high reactance at the deeper bars.

Example PE 2.4

A three-phase, 2-pole, 60 Hz induction motor provides 25 hp to a load at a speed of 3420 rpm. The windage and friction losses are 250 W. Determine (a) the slip of the motor in percent (b) the developed torque (c) the shaft speed of the motor if the torque is doubled

Solution

(a) The synchronous speed, $N_s = \dfrac{120\ f}{p} = \dfrac{120 \times 60}{2} = 3600\ rpm$

Motor speed, $N_m = 3420\ rpm$

Thus, slip in percent is $s = \dfrac{N_s - N_m}{N_s} = \dfrac{3600 - 3420}{3600} \times 100 = 5\%$

(b) Mechanical power developed, $P_m = P_{out} + P_w = (25 \times 746) + 250 = 18900\ W$

The developed torque $= \dfrac{P_m}{\omega_m} = \dfrac{18900}{2} \times 2 \times \pi \times 3420 = 52.75\ N \cdot m$

(c) The input power is constant and the torque developed has been doubled. The slip also doubles and the new slip is $s = 2 \times 0.05 = 0.1$

Thus, the new speed is $N_m = N_s \cdot (1 - s) = 3600 \cdot (1 - 0.1) = 3240\ rpm$

Example PE 2.5

A three-phase, 2-pole, 50 hp, 480 V (l-l), 60 Hz, wye-connected induction motor has the following constants in ohms per phase referred to the stator: R_1 = 0.3 ohms, R_2=0.175 ohms, X_1=0.675 ohms, X_2=0.6 ohms, X_m=10.5 ohms. Calculate (a) the slip at which the maximum torque is developed (b) the maximum torque developed (c) the starting torque developed (d) the speed at which maximum torque is developed, if rotor resistance is doubled.

Solution

We first calculate the Thevenin's resistance, reactance and voltage here.

$$V_{th} = \frac{V_1 \cdot X_m}{\sqrt{R_1^{\,2} + (X_1 + X_m)^2}} = 480\big/\sqrt{3} \cdot \left[\frac{10.5}{\sqrt{0.3^2 + (0.675 + 10.5)^2}}\right] = 260.3\,V$$

$$R_{th} \approx R_1 \cdot \left[\frac{X_m}{(X_1 + X_m)}\right]^2 = 0.3\left[\frac{10.5}{0.675 + 10.5}\right]^2 = 0.265\Omega$$

$$X_{th} \approx X_1 = 0.675\Omega$$

also

$$N_s = 120 \cdot \frac{60}{2} = 3600 \ rpm$$

Note that the exact expression for R_{th} in this problem is:

$$R_{th} = R_1 \cdot \left[\frac{X_m^{\,2}}{R_1^{\,2} + (X_1 + X_m)^2}\right]$$

(a) The slip at which maximum torque is developed is
From Equation PE2.10, $s_{\max T} = 0.134$

(b) From Equation PE2.12, $T_{\max} = 3 \cdot \frac{(260.3)^2}{377 \cdot 2 \cdot (0.675 + 0.6)} = 211.44\,N \cdot m$

(c) From Equation PE2.9 $T_{start} = 51.94\ N \cdot m$

(d) Since the rotor resistance is doubled, the slip at which maximum torque occurs doubles. Thus, $s_{\max T} = 0.2688$ and the corresponding speed is $N_m = (1 - 0.2688) \times 3600 = 2632.32\ rpm$

PROBLEMS

Problem 1

A three-phase, 4-pole, 60 Hz induction motor has a line-to-line voltage applied across its terminals equal to 1214 V. The parameter values for this motor are: R1=0.3 Ω, X1=1 Ω, Rc=large, Xm=30 Ω, R'2=0.3 Ω, X'2=1 Ω. The slip corresponding to maximum torque is 0.1507. The current flowing in the rotor circuit is $|I'_2|$=279 A.

(a) What is the synchronous speed of the rotor in rad/sec?
(b) What is the difference in speed between rotor speed at maximum torque and synchronous rotor speed?
(c) Compute the Thevenin equivalent voltage and impedance "looking left" from the rotor circuit.
(d) Compute the maximum torque of this motor.

Problem 2

A three-phase, 4-pole, 60 Hz induction motor has a line-to-line voltage applied across its terminals equal to 1214 volts. The parameter values for this motor are: R_1=0.3 Ω, X_1=1 Ω, $R_c=\infty$, X_m=30 Ω, R'_2=0.3 Ω, X'_2=1 Ω. The slip corresponding to maximum torque is 0.1507.

(a) Find the Thevenin equivalent voltage and impedance seen "looking left" from the rotor circuit.
(b) Compute the speed of this motor, in mechanical rad/sec, under the maximum torque condition.
(c) Compute the maximum torque of this motor.

Problem 3

A three-phase, 4-pole, 60 Hz induction motor operates at a slip of s=0.08. The mechanical power developed at the shaft is P_D = 5 kW. (a) Find the mechanical speed of this motor. (b) Compute the power flowing across the air gap.

Problem 4

A 10 hp, 440 volt (line-to-line), three-phase induction motor has synchronous speed of 1800 and runs at 1700 rpm at the full load power output of 10 hp. The stator copper loss is 200 W and the rotational loss is 400 W. Assume the core losses are zero, i.e., P_c = 0. Determine

(a) Power developed, P_m (otherwise known as P_D)
(b) Slip
(c) Input power to the rotor, P_g
(d) Rotor copper losses, P_{rc}
(e) Total power input, P_{in}
(f) The magnitude of the current drawn by the motor, assuming the motor power factor is 0.80

Problem 5

A three-phase, 4 pole, 60 Hz induction motor operates with the parameter values: R_1=0 Ω, X_1=3 Ω, $R_c=\infty$, X_m=30 Ω, R'_2=1 Ω, X'_2=3 Ω.

(a) Compute the synchronous rotor speed of this motor, w_s, in rad/sec.
(b) Compute the slip corresponding to a rotor speed of w_m =160 rad/sec
(c) Compute the mechanical torque developed at the rotor shaft when $\left|I_2'\right| = 80$ A and the speed is w_m =160 rad/sec.

Problem 6

A three-phase, 6 pole, 60 Hz, wye-connected induction motor has a line-to-line voltage applied across its terminals of 220 volts. The parameter values for this motor are: $R_1 = 0.3$ ohms, $X_1 = 0.5$ ohms, Rc = large, Xm = 15 ohms, $R'_2 = 0.15$ ohms, $X'_2 = 0.2$ ohms. For a slip of s = 0.02, compute:

(a) The synchronous speed ω_s and the rotor speed ω_r in rad/sec.
(b) The Thevenin equivalent voltage and impedance “looking left” from the rotor circuit.
(c) The current magnitude in the rotor part of the equivalent circuit.
(d) The power delivered to the rotor across the airgap.
(e) The power and torque developed at the shaft.