Module B3

More on
Three Phase Analysis

B3.2

Balanced Three Phase Circuits

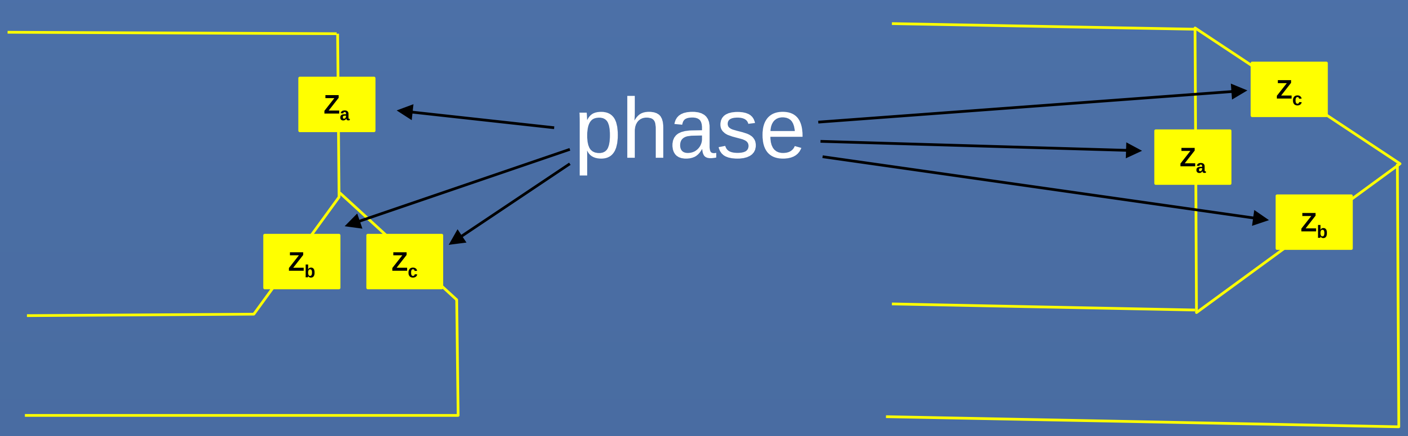
Terminology:

- phase: the impedance or voltage source element in the three-phase connection.
- phase current: current through the phase.
- phase voltage: voltage across the phase.

Notation:

I_ϕ: phase current magnitude;
V_ϕ: phase voltage magnitude;

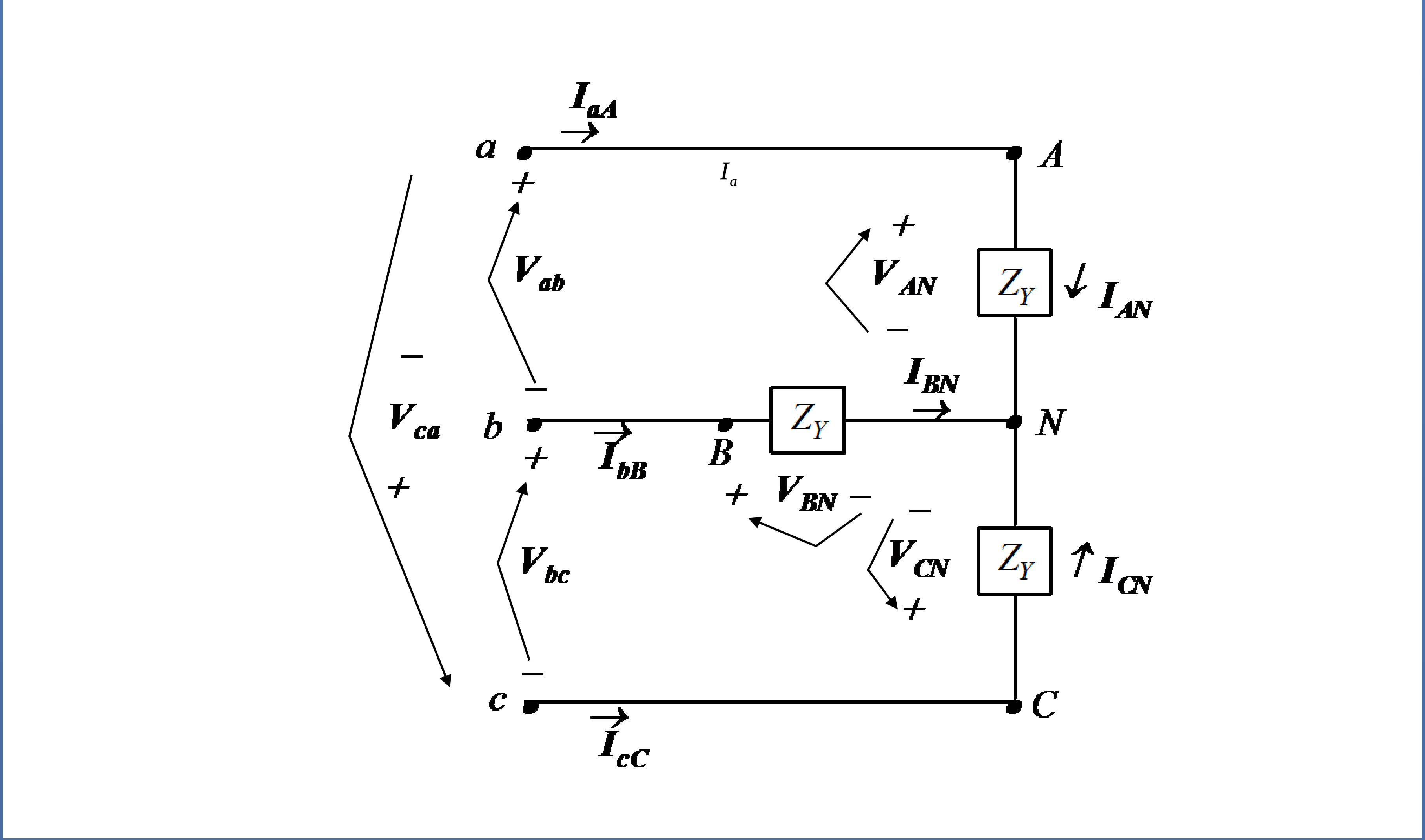
This notation will be used in the following slides to express phase current phasors.



Wye connected load **Delta connected load**

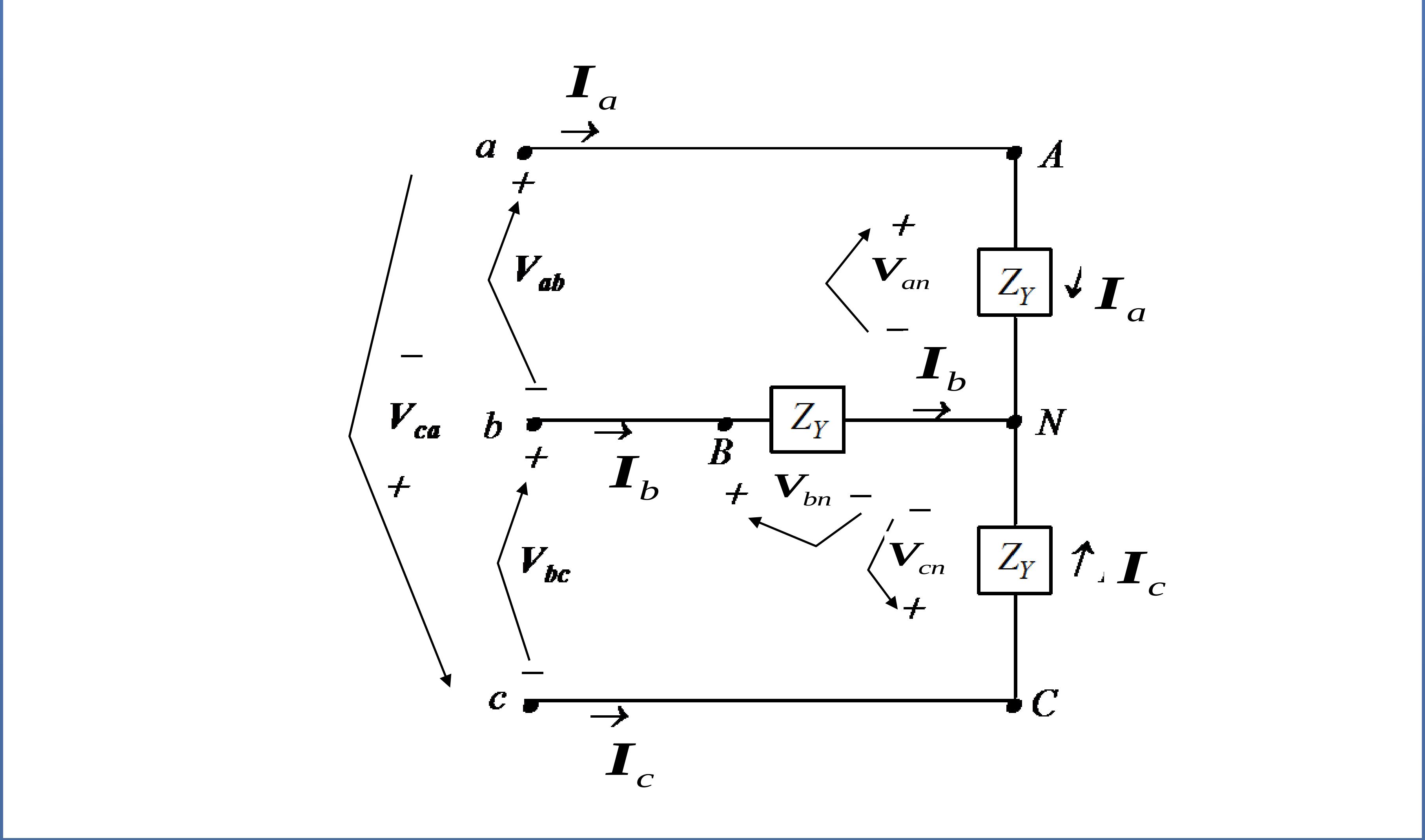
WYE Connection

Lower and upper case subscripts correspond to the particular identified nodes. They are used in this figure to be very explicit regarding each quantity. However, it is more common to use only one or the other, as indicated in the next slide.



WYE Connection

Same figure, but with more common nomenclature.



$$I_{AN} = I_{BN} = I_{CN} = I_{\phi}\,, \qquad V_{AN} = V_{BN} = V_{CN} = V_{\phi}$$

$$\boldsymbol{V_{AN}} = V_{\phi}\angle 0^{\circ}$$

$$\boldsymbol{V_{BN}} = V_{\phi}\angle -120^{\circ}$$

$$\boldsymbol{V_{CN}} = V_{\phi}\angle +120^{\circ}$$

$$\boldsymbol{V_{AB}} = \boldsymbol{V_{AN}} - \boldsymbol{V_{BN}} = V_{\phi} - V_{\phi}\angle -120^{\circ} = \sqrt{3}V_{\phi}\angle 30^{\circ}$$

$$\boldsymbol{V_{BC}} = \boldsymbol{V_{BN}} - \boldsymbol{V_{CN}} = V_{\phi}\angle -120^{\circ} - V_{\phi}\angle 120^{\circ} = \sqrt{3}V_{\phi}\angle -90^{\circ}$$

$$\boldsymbol{V_{CA}} = \boldsymbol{V_{CN}} - \boldsymbol{V_{AN}} = V_{\phi}\angle 120^{\circ} - V_{\phi}\angle 0^{\circ} = \sqrt{3}V_{\phi}\angle 150^{\circ}$$

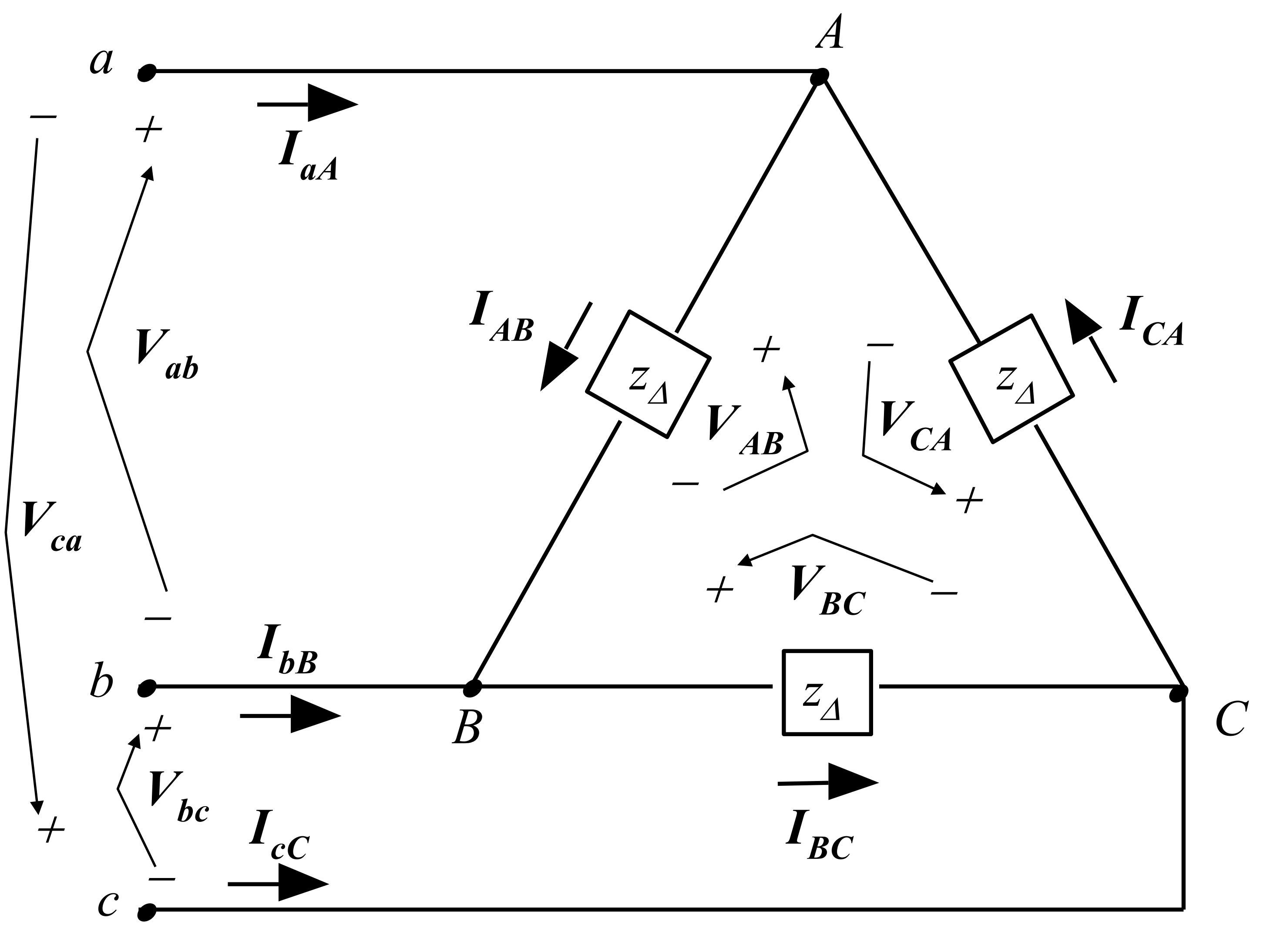
Wye connection:

Line-Line voltages are sqrt(3) times phase voltages in magnitude and lead them by 30 degrees in angle.

Y: Line-Line Voltages Lead: Y-LV-Lead

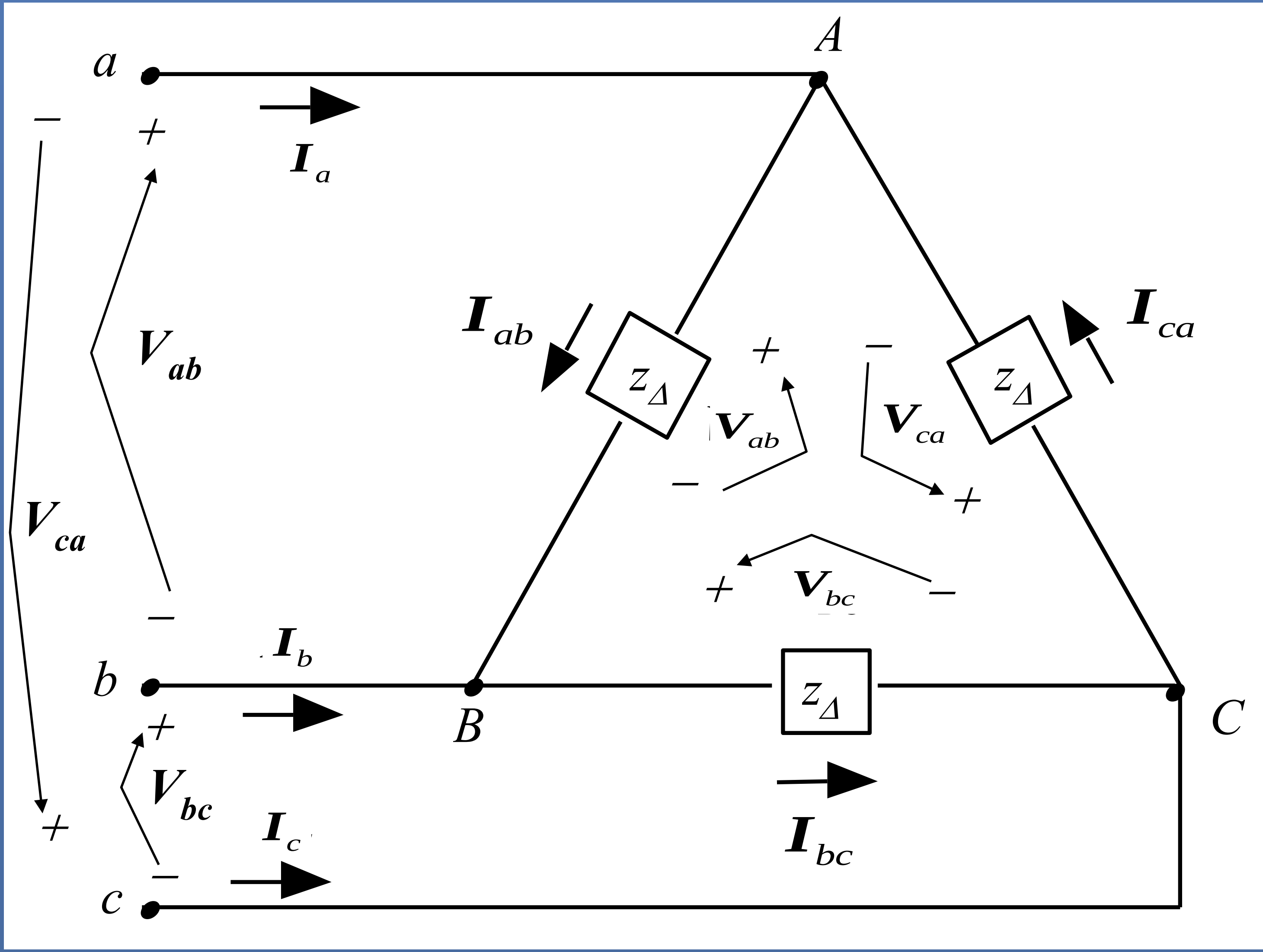
Line currents equal phase currents

Delta Connection



Delta Connection

Same figure, but with more common nomenclature.



$$I_{AB} = I_{BC} = I_{CA} = I_{\phi}, \qquad V_{AB} = V_{BC} = V_{CA} = V_{\phi}$$

$$\boldsymbol{I_{AB}} = I_{\phi}\angle 0^{\circ}$$

$$\boldsymbol{I_{BC}} = I_{\phi}\angle -120^{\circ}$$

$$\boldsymbol{I_{CA}} = I_{\phi}\angle +120^{\circ}$$

$$\mathrm{I_{aA}} = \mathrm{I_{AB}} - \mathrm{I_{CA}} = I_{\phi}\angle 0^{\circ} - I_{\phi}\angle 120^{\circ} = \sqrt{3}I_{\phi}\angle -30^{\circ}$$

$$\mathrm{I_{bB}} = \mathrm{I_{BC}} - \mathrm{I_{AB}} = I_{\phi}\angle -120^{\circ} - I_{\phi}\angle 0^{\circ} = \sqrt{3}I_{\phi}\angle -150^{\circ}$$

$$\mathrm{I_{cC}} = \mathrm{I_{CA}} - \mathrm{I_{BC}} = I_{\phi}\angle 120^{\circ} - I_{\phi}\angle -120^{\circ} = \sqrt{3}I_{\phi}\angle 90^{\circ}$$

Delta connection:

Line-line voltages equal phase voltages

Line currents are sqrt(3) times
phase currents in magnitude
and lag them by 30 degrees in angle.

Delta: Line Currents Lag: Delta-LC-LAG

Power relations for three phase circuits

$$P = 3V_{\phi} I_{\phi} \cos\theta$$

$$P = \sqrt{3} V_L I_L \cos\theta$$

$$Q = 3V_{\phi} I_{\phi} \sin\theta$$

$$Q = \sqrt{3} V_L I_L \sin\theta$$

$$S = P + jQ$$

$$|S| = \sqrt{P^2 + Q^2} = \sqrt{3} V_L I_L$$

- V_L: line-to-line voltage magnitude
- I_L: line current magnitude

- These formulat can be used for Wye or Delta
- all quantities are magnitudes
- θ is the angle for which phase voltage leads phase current
- Positive Q is for power flowing into L load
- Negative Q is for power flowing “into” C load

Per-phase analysis of 3 phase circuits

- Convert all delta connections to Y connections using

$$Z_Y = \frac{Z_\Delta}{3}$$

- “Lift out” the a-phase to neutral circuit
- Perform single phase analysis using phase quantities and per phase powers
- Multiply all powers by 3 to get solution in terms of three phase powers.

Example B3.2

Three balanced three-phase loads are connected in parallel. Load 1 is Y-connected with an impedance of $150 + j50$; load 2 is delta-connected with an impedance of $900 + j600$; and load 3 is 95.04 kVA at 0.6 pf leading. The loads are fed from a distribution line with an impedance of $3 + j24$. The magnitude of the line-to-neutral voltage at the load end of the line is 4.8 kV.

a) Calculate the total complex power at the sending end of the line.

b) What percent of the average power at the sending end of the line is delivered to the load?

Solution:

Load 1: Z1=150+j50

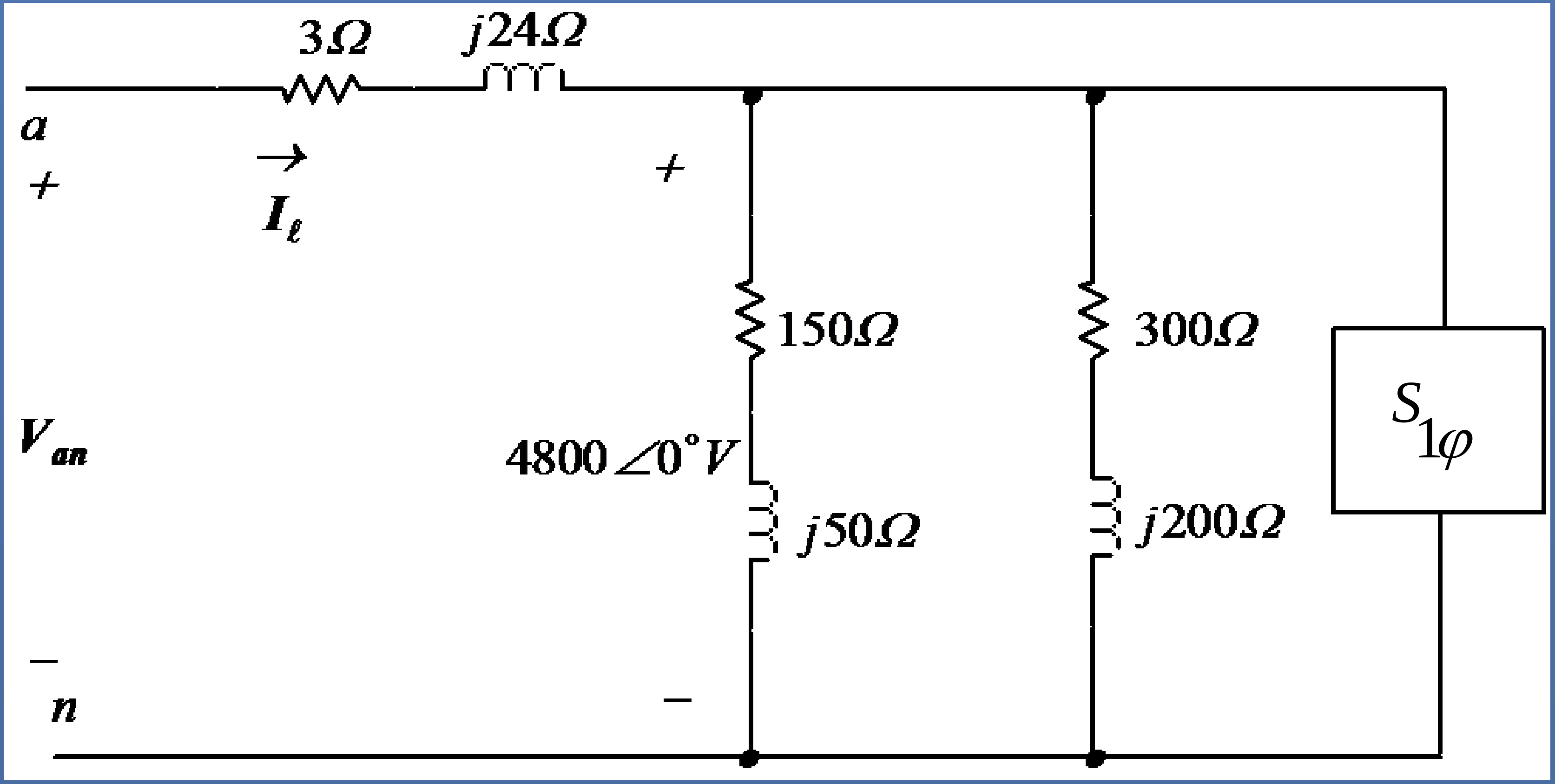
Load 2: Z2=(900+j600)/3=300+j200

Load 3:

$$S_{1\varphi} = \frac{95040}{3}(0.6 - j0.8)$$
$$= 19{,}008 - j25{,}344 \text{ VA}$$

$V_\phi = 4800$ volts

Per phase equivalent circuit



Compute current from source

$$\mathbf{I}_\ell = \frac{4800}{150 + j50} + \frac{4800}{300 + j200} + \frac{19{,}008 + j25{,}344}{4800}$$

$$= 28.8 - j9.6 + 11.0769 - j7.3846 + 3.96 + j5.28$$

$$= \quad I_1 \quad + \quad \mathrm{I}_2 \quad + \quad \mathrm{I}_3$$

$$= 43.8369 - j11.7046\,A(rms) = 45.3725\angle -14.949^\circ\,A(rms)$$

Compute losses in the transmission line

$$P_{loss} = 3\left|I_{eff}\right|^2 R = 3(45.3725)^2(3) = 18{,}528.04 \text{ W}$$

$$Q_{loss} = 3\left|I_{eff}\right|^2 X = 3(45.3725)^2(24) = 148{,}224.34 \text{ VAR}$$

Compute power consumed by load 1.

$$P_1 = 3\left|28.8 - j9.6\right|^2(150) = 414{,}720 \text{ W}$$

$$Q_1 = 3\left|28.8 - j9.6\right|^2(50) = 138{,}240 \text{ VAR}$$

Compute power consumed by load 2:

$$P_2 = 3\left|11.0769 - j7.3846\right|^2 (300) = 159{,}507.02 \text{ W}$$

$$Q_2 = 3\left|11.0769 - j7.3846\right|^2 (200) = 106{,}338.02 \text{ VAR}$$

Compute power consumed by load 3:

$$P_3 = 95{,}040(0.6) = 57{,}024 \text{ W}$$

$$Q_3 = -95{,}040(0.8) = -76{,}032 \text{ VAR}$$

Add the powers to the three loads

$$S_{totalload,3\phi} = 631{,}251 + j168{,}546 \ \text{ VA (load end)}$$

We could have also obtained this from 3VI* (see the "check" in the text)

Add in the losses to get sending end power:

$$S_{sending,3\phi} = 631{,}251 + j168{,}546 + 18{,}528.04 + j148{,}224.34 \ \text{ VA}$$
$$= 649779.04 + j316{,}770.34 \ \text{ VA}$$

Part b of the problem wants the percent of the power from the sending end that is actually delivered at the load.

$$\% P \text{ delivered} = \frac{631{,}251}{649{,}779.04} \times 100 = 97.148$$

This is a measure of efficiency.
Why is 100% of the power not delivered?