Answers for Homework 1, Part I (Basic Linear Algebra)
HCI/ComS 575X: Computational Perception, Spring 2006

1. (20 points)

a. $-3\sqrt{6}$

b. $a^2 - 5a + 21$

c. -4

d. $-c^4 + c^3 - 16c^2 + 8c - 2$

2. (10 points)

The determinant reduces to $\sin^2\theta + \cos^2\theta$, which is equal to 1 (trigonometry identity). Therefore, it does not depend on θ.

3. (20 points)

a. $\begin{bmatrix} -7 & 4 \\ 2 & -1 \end{bmatrix}$

b. $\begin{bmatrix} -5/39 & 2/13 \\ 4/39 & 1/13 \end{bmatrix}$

c. $\begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \end{bmatrix}$

d. $\begin{bmatrix} 1/k_1 & 0 & 0 & 0 \\ 0 & 1/k_2 & 0 & 0 \\ 0 & 0 & 1/k_3 & 0 \\ 0 & 0 & 0 & 1/k_4 \end{bmatrix}$

4. (10 points)

a. $x_1 = 3, x_2 = -1$

b. $x_1 = -1, x_2 = 4, x_3 = -7$

5. (20 points)

a. 7

b. $\sqrt{14}$

(Note the second part 'b' shouldn't be there. It will be ignored as it is part of 'c' anyway.)

c. $(-14, -20, -82)$

d. $(27, 40, -42)$

6. (10 points)

a. $(0,0)$

b. $\left(-\frac{16}{13}, 0, -\frac{80}{13}\right)$

7. (10 points)

a. $\cos^{-1}\left(-\frac{11}{\sqrt{962}}\right)$ or 1.93 radians or 110.7 degrees

b. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ or 0.96 radians or 54.7 degrees

8. (10 points)

a. $2y - z + 1 = 0$

b. $x + 9y - 5z - 26 = 0$

9. (10 points)

a. $-3, 1$

b. $-2, 3, 4$

10. (10 points)

a. $1, 2, 3$

b. $-\sqrt{2}, 0, \sqrt{2}$

11. (10 points)

$$R = \begin{bmatrix} \cos 60° & -\sin 60° \\ \sin 60° & \cos 60° \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \frac{1}{2}x - \frac{\sqrt{3}}{2}y \\ \frac{\sqrt{3}}{2}x + \frac{1}{2}y \end{bmatrix}$$

12. (10 points)

(Note: these matrices are for CCW rotation.)

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

13. (10 points)

"Show that 3D rotation matrices are not commutative." One of the most straightforward ways to show this is to compute $R_x R_y$ and $R_y R_x$, showing that $R_x R_y \neq R_y R_x$.