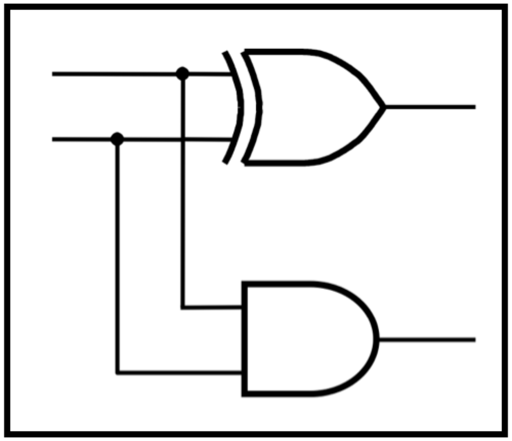


CprE 2810: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

Code Converters

CprE 2810: Digital Logic
Iowa State University, Ames, IA

Administrative Stuff

- **HW 7 is out**

- **It is due on Monday (Oct 20) @ 10pm**

- **We will start with Chapter 5 on Friday.**

Quick Review

Decoders

2-to-4 Decoder (Definition)

- Has two inputs: w_1 and w_0

- Has four outputs: y_0 , y_1 , y_2 , and y_3

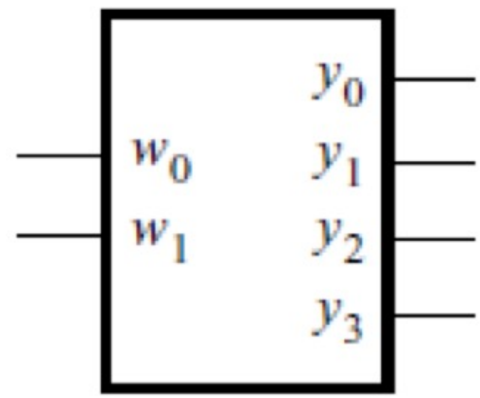
- If w_1=0 and w_0=0, then the output y_0 is set to 1
- If w_1=0 and w_0=1, then the output y_1 is set to 1
- If w_1=1 and w_0=0, then the output y_2 is set to 1
- If w_1=1 and w_0=1, then the output y_3 is set to 1

- Only one output is set to 1. All others are set to 0.

Truth Table and Graphical Symbol for a 2-to-4 Decoder

w_1	w_0	y_0	y_1	y_2	y_3
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

(a) Truth table



(b) Graphical symbol

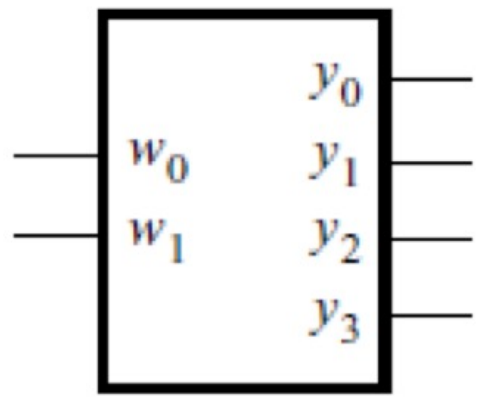
[Figure 4.13a-b from the textbook]

Truth Table and Graphical Symbol for a 2-to-4 Decoder

w_1	w_0	y_0	y_1	y_2	y_3
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

The outputs are “one-hot” encoded

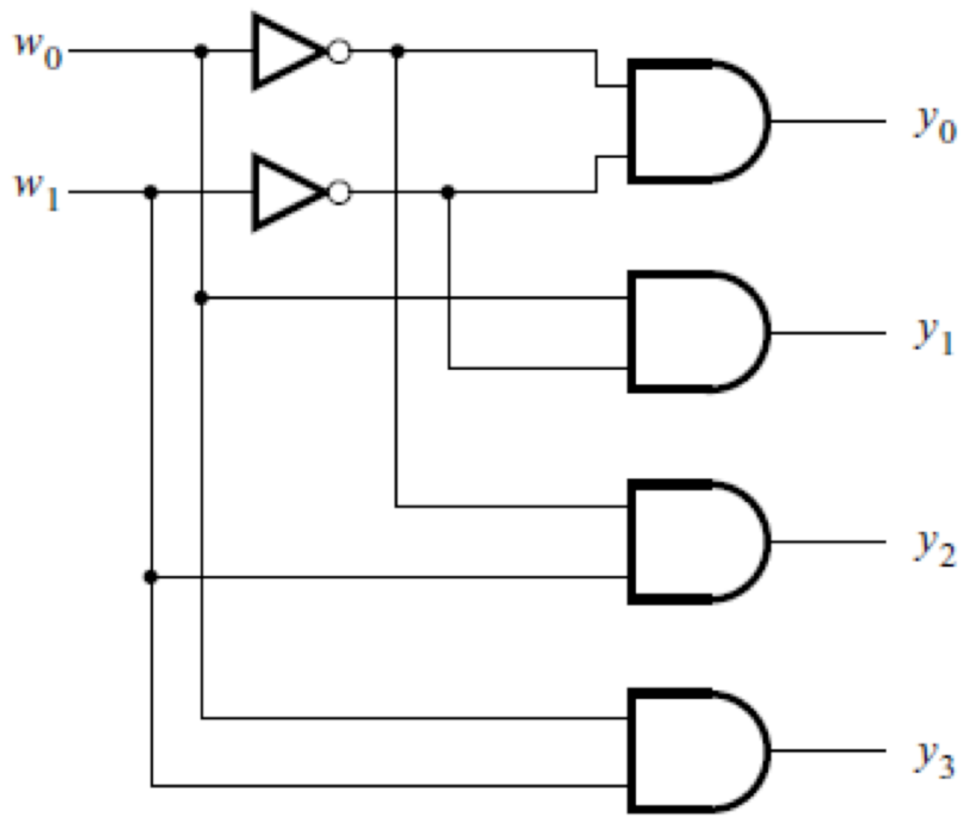
(a) Truth table



(b) Graphical symbol

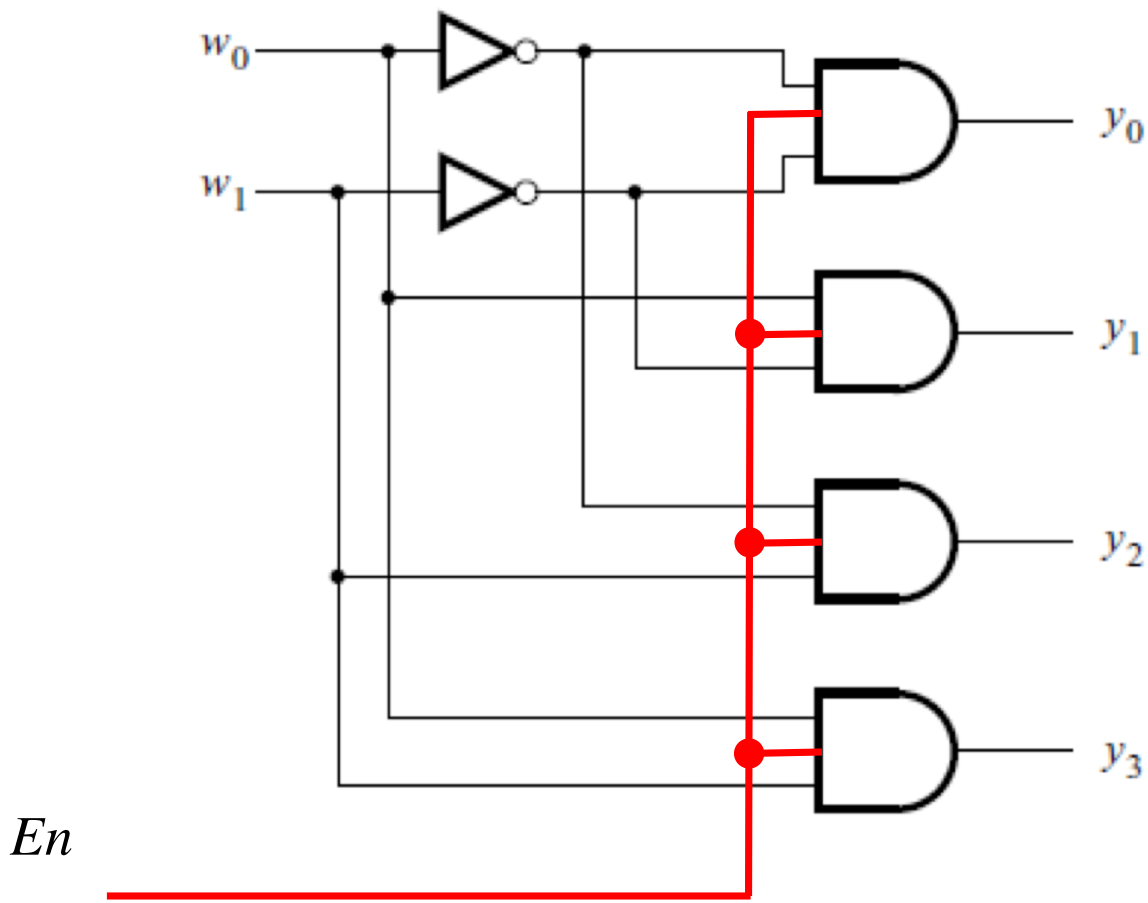
[Figure 4.13a-b from the textbook]

Truth Logic Circuit for a 2-to-4 Decoder



[Figure 4.13c from the textbook]

Adding an Enable Input



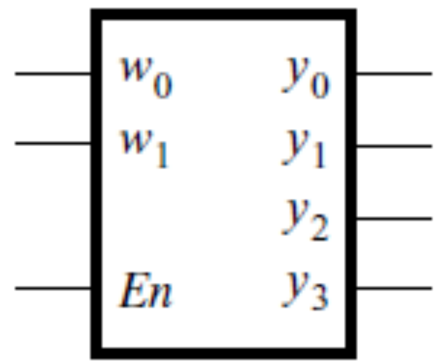
[Figure 4.13c from the textbook]

Truth Table and Graphical Symbol for a 2-to-4 Decoder with an Enable Input

En	w_1	w_0	y_0	y_1	y_2	y_3
1	0	0	1	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1
0	x	x	0	0	0	0

(a) Truth table

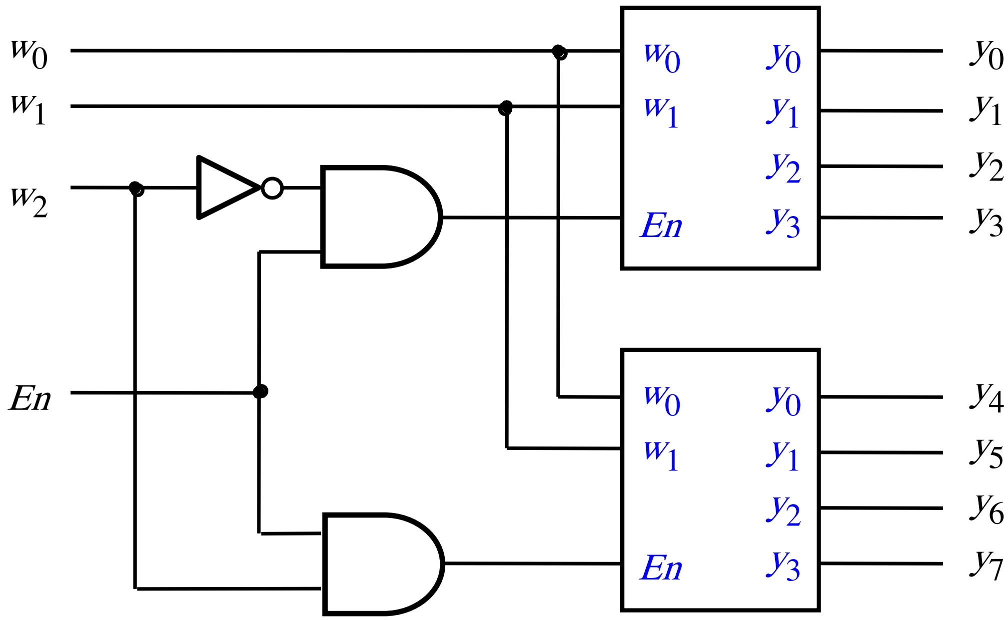
x indicates that it does not matter what the value of these variable is for this row of the truth table



(b) Graphical symbol

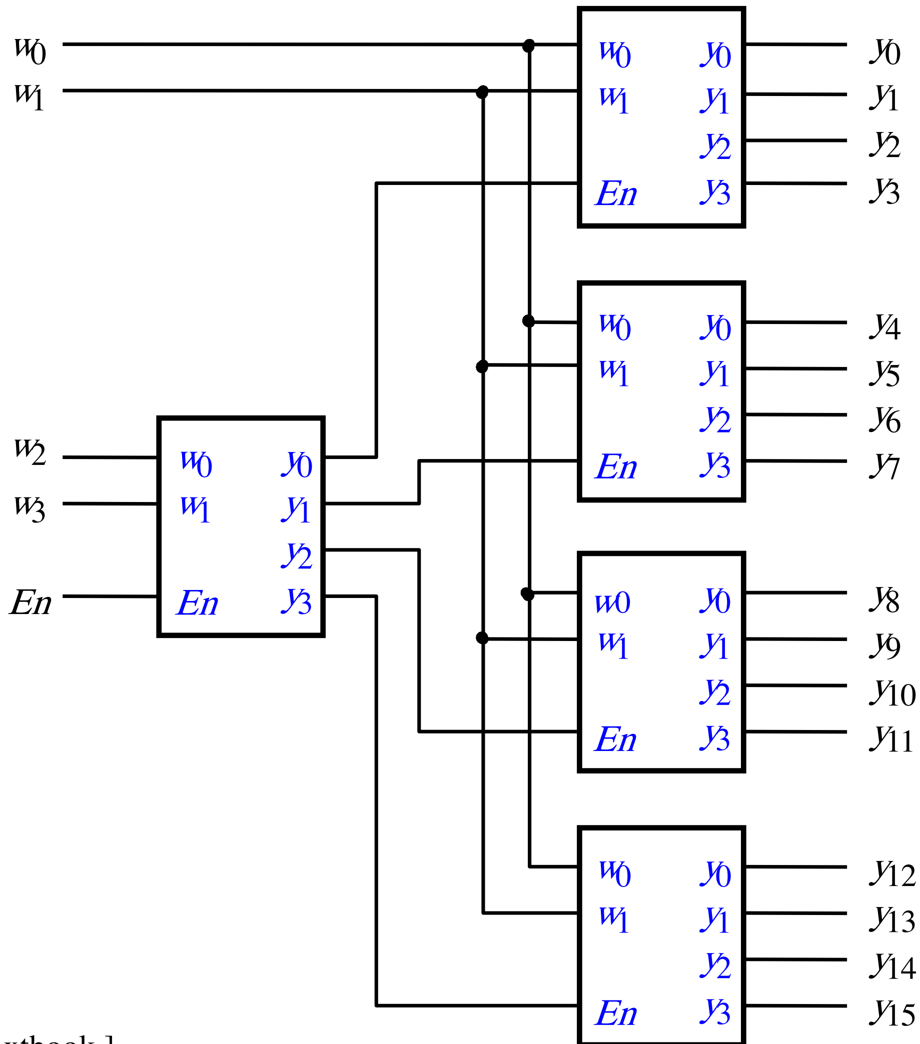
[Figure 4.14a-b from the textbook]

A 3-to-8 decoder using two 2-to-4 decoders



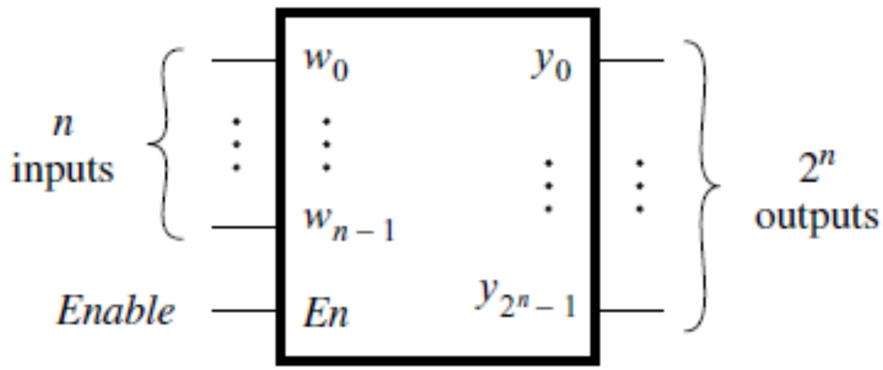
[Figure 4.15 from the textbook]

A 4-to-16 decoder built using a decoder tree



[Figure 4.16 from the textbook]

Graphical Symbol for a Binary n-to-2^n Decoder with an Enable Input



(d) An n-to-2^n decoder

A binary decoder with n inputs has 2^n outputs

The outputs of an enabled binary decoder are "one-hot" encoded, meaning that only a single bit is set to 1, i.e., it is *hot*.

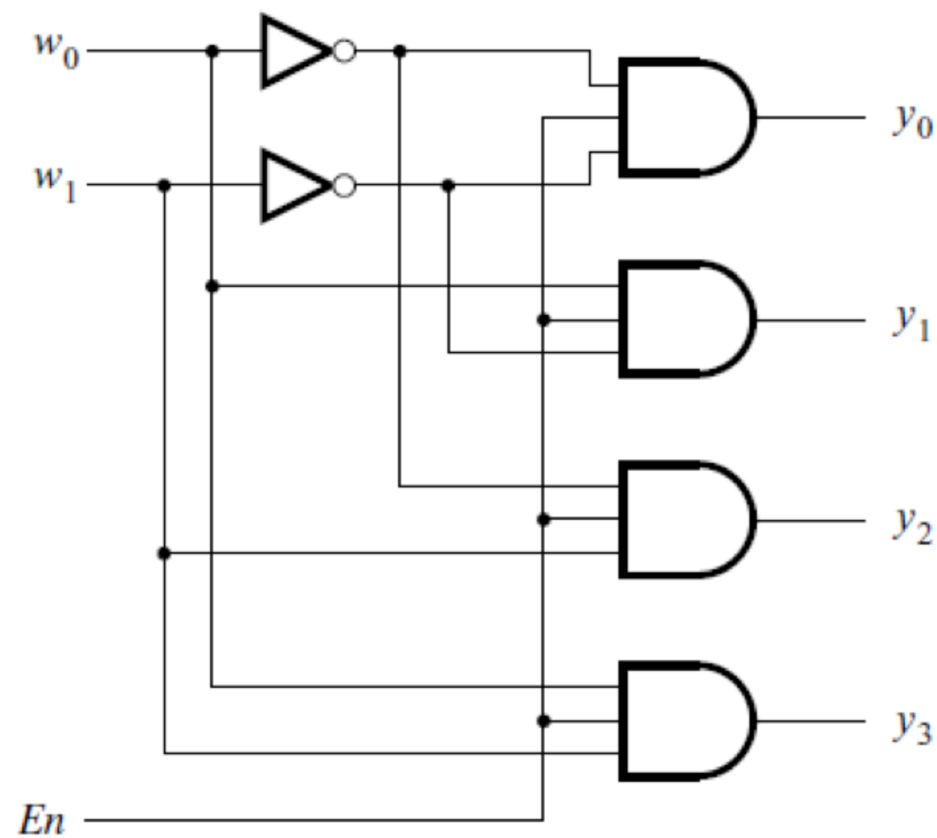
[Figure 4.14d from the textbook]

Demultiplexers

1-to-4 Demultiplexer (Definition)

- Has one data input line: D
- Has two output select lines: w_1 and w_0
- Has four outputs: y_0 , y_1 , y_2 , and y_3
- If w_1=0 and w_0=0, then the output y_0 is set to D
- If w_1=0 and w_0=1, then the output y_1 is set to D
- If w_1=1 and w_0=0, then the output y_2 is set to D
- If w_1=1 and w_0=1, then the output y_3 is set to D
- Only one output is set to D. All others are set to 0.

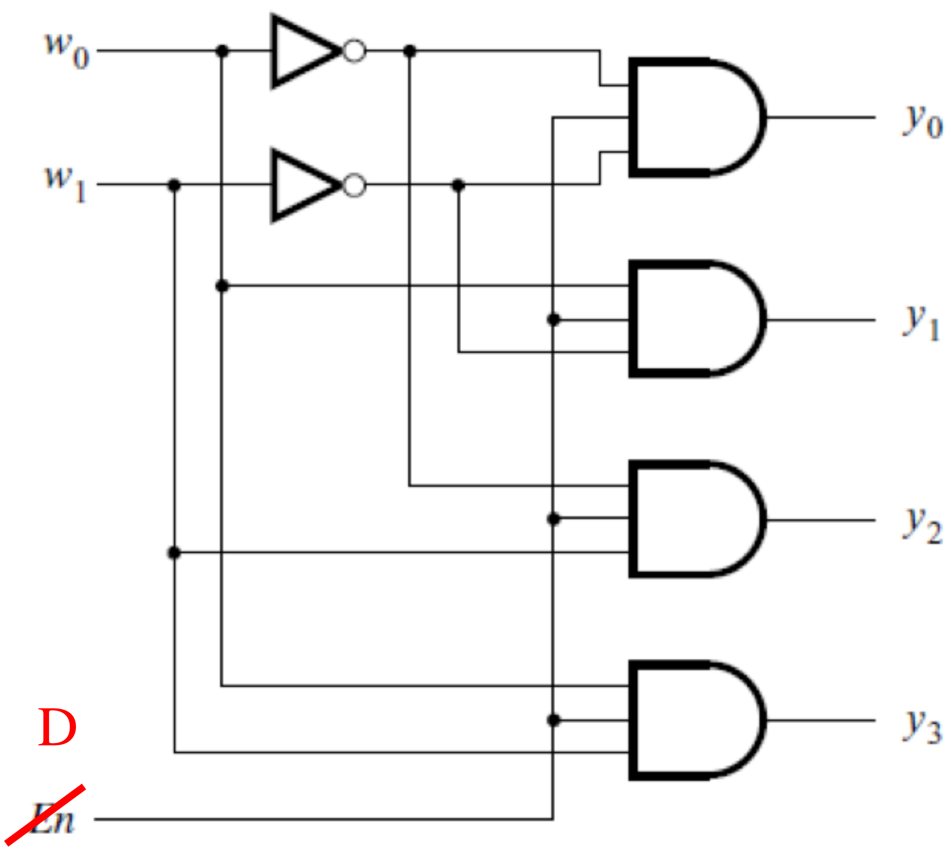
A 1-to-4 demultiplexer
built with a 2-to-4 decoder with enable



[Figure 4.14c from the textbook]

A 1-to-4 demultiplexer
built with a 2-to-4 decoder with enable

output
select
lines



the
four
output
lines

data
input
line

[Figure 4.14c from the textbook]

Encoders

Binary Encoders

4-to-2 Binary Encoder (Definition)

- **Has four inputs: w_3 , w_2 , w_1 , and w_0**
- **Has two outputs: y_1 and y_0**
- **Only one input is set to 1 ("one-hot" encoded). All others are set to 0.**
- **If w_0=1 then y_1=0 and y_0=0**
- **If w_1=1 then y_1=0 and y_0=1**
- **If w_2=1 then y_1=1 and y_0=0**
- **If w_3=1 then y_1=1 and y_0=1**

Truth table for a 4-to-2 binary encoder

w_3	w_2	w_1	w_0	y_1	y_0
0	0	0	1	0	0
0	0	1	0	0	1
0	1	0	0	1	0
1	0	0	0	1	1

The inputs are “one-hot” encoded

[Figure 4.19 from the textbook]

Expressions for 4-to-2 binary encoder

w_3	w_2	w_1	w_0	y_1	y_0
0	0	0	0		
0	0	0	1	0	0
0	0	1	0	0	1
0	0	1	1		
0	1	0	0	1	0
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0	1	1
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

w_3	w_2	w_1	w_0	y_1	y_0
0	0	0	1	0	0
0	0	1	0	0	1
0	1	0	0	1	0
1	0	0	0	1	1

Expressions for 4-to-2 binary encoder

w_3	w_2	w_1	w_0	y_1	y_0
0	0	0	0	d	d
0	0	0	1	0	0
0	0	1	0	0	1
0	0	1	1	d	d
0	1	0	0	1	0
0	1	0	1	d	d
0	1	1	0	d	d
0	1	1	1	d	d
1	0	0	0	1	1
1	0	0	1	d	d
1	0	1	0	d	d
1	0	1	1	d	d
1	1	0	0	d	d
1	1	0	1	d	d
1	1	1	0	d	d
1	1	1	1	d	d

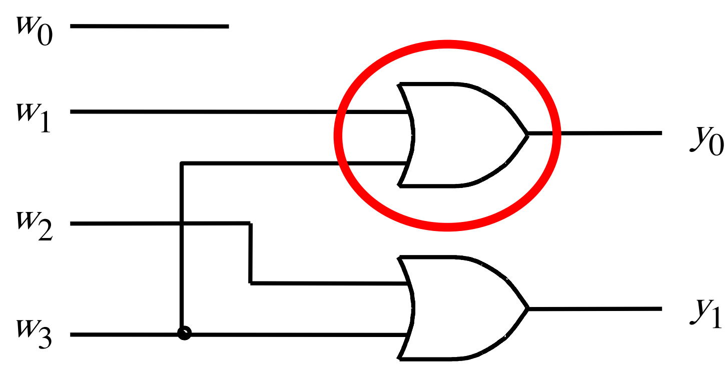
w_3	w_2	w_1	w_0	y_1	y_0
0	0	0	1	0	0
0	0	1	0	0	1
0	1	0	0	1	0
1	0	0	0	1	1

Expressions for 4-to-2 binary encoder

w_3	w_2	w_1	w_0	y_1	y_0
0	0	0	0	d	d
0	0	0	1	0	0
0	0	1	0	0	1
0	0	1	1	d	d
0	1	0	0	1	0
0	1	0	1	d	d
0	1	1	0	d	d
0	1	1	1	d	d
1	0	0	0	1	1
1	0	0	1	d	d
1	0	1	0	d	d
1	0	1	1	d	d
1	1	0	0	d	d
1	1	0	1	d	d
1	1	1	0	d	d
1	1	1	1	d	d

$w_1 w_0$ \ $w_3 w_2$	00	01	11	10
00	d	0	d	1
01	0	d	d	d
11	d	d	d	d
10	1	d	d	d

$$y_0 = (w_1 + w_3)$$

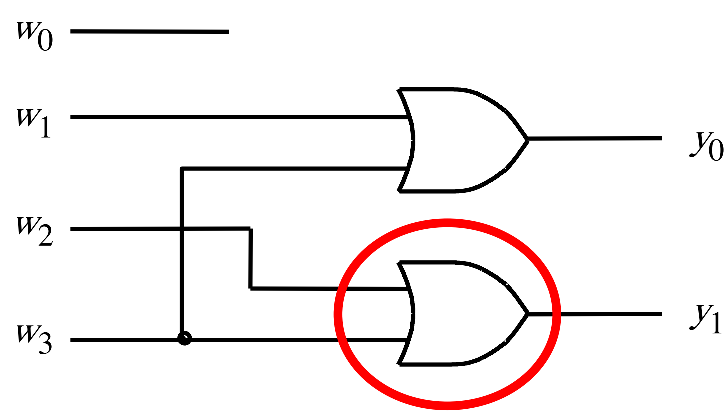


Expressions for 4-to-2 binary encoder

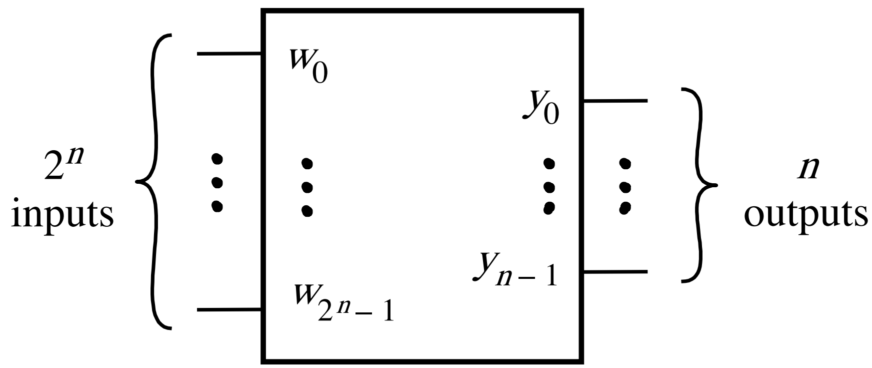
w_3	w_2	w_1	w_0	y_1	y_0
0	0	0	0	d	d
0	0	0	1	0	0
0	0	1	0	0	1
0	0	1	1	d	d
0	1	0	0	1	0
0	1	0	1	d	d
0	1	1	0	d	d
0	1	1	1	d	d
1	0	0	0	1	1
1	0	0	1	d	d
1	0	1	0	d	d
1	0	1	1	d	d
1	1	0	0	d	d
1	1	0	1	d	d
1	1	1	0	d	d
1	1	1	1	d	d

$w_1 w_0$ \ $w_3 w_2$	00	01	11	10
00	d	1	d	1
01	0	d	d	d
11	d	d	d	d
10	0	d	d	d

$$y_1 = (w_3 + w_2)$$



A 2^n-to-n binary encoder



[Figure 4.18 from the textbook]

Priority Encoders

4-to-2 Priority Encoder (Definition)

- Has four inputs: w_3 , w_2 , w_1 , and w_0
- Has two primary outputs: y_1 and y_0
- Has one other output: z

- The inputs are NOT "one-hot" encoded.
- More than one input can be set to 1 but they have priorities associated with them: w_3 – highest priority and w_0 – lowest priority.

- $y_1=0$ and $y_0=0$ (if $w_0=1$ and $w_3=w_2=w_1=0$)
- $y_1=0$ and $y_0=1$ (if $w_1=1$ and $w_3=w_2=0$)
- $y_1=1$ and $y_0=0$ (if $w_2=1$ and $w_3=0$)
- $y_1=1$ and $y_0=1$ (if $w_3=1$)

4-to-2 Priority Encoder (Definition)

- Has four inputs: w_3 , w_2 , w_1 , and w_0
- Has two primary outputs: y_1 and y_0
- Has one other output: z

- The inputs are NOT "one-hot" encoded.
- More than one input can be set to 1 but they have priorities associated with them: w_3 – highest priority and w_0 – lowest priority.

- y_1=0 and y_0=0 (if w_0=1 and w_3=w_2=w_1=0)
- y_1=0 and y_0=1 (if w_1=1 and w_3=w_2=0) w_0
- y_1=1 and y_0=0 (if w_2=1 and w_3=0) w_0 , w_1
- y_1=1 and y_0=1 (if w_3=1) w_0 , w_1 , w_2

these have lower priorities
and can be either 0 or 1.

4-to-2 Priority Encoder (Definition)

- Has four inputs: w_3 , w_2 , w_1 , and w_0
- Has two primary outputs: y_1 and y_0
- Has one other output: z

- The inputs are NOT “one-hot” encoded.
- More than one input can be set to 1 but they have priorities associated with them: w_3 – highest priority and w_0 – lowest priority.

- y_1=0 and y_0=0 (if w_0=1 and w_3=w_2=w_1=0)
- y_1=0 and y_0=1 (if w_1=1 and w_3=w_2=0)
- y_1=1 and y_0=0 (if w_2=1 and w_3=0)
- y_1=1 and y_0=1 (if w_3=1)

- z =0 if w_3=w_2=w_1=w_0=0; otherwise z=1.

Truth table for a 4-to-2 priority encoder (abbreviated version)

w_3	w_2	w_1	w_0	y_1	y_0	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	x	0	1	1
0	1	x	x	1	0	1
1	x	x	x	1	1	1

[Figure 4.20 from the textbook]

Truth table for a 4-to-2 priority encoder (abbreviated version)

w_3	w_2	w_1	w_0	y_1	y_0	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	x	0	1	1
0	1	x	x	1	0	1
1	x	x	x	1	1	1

These two are don't cares, because the y values don't matter in this case.
In other words, when z=0 there are no priority inputs that are set to 1.

[Figure 4.20 from the textbook]

Truth table for a 4-to-2 priority encoder (abbreviated version)

w_3	w_2	w_1	w_0	y_1	y_0	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	x	0	1	1
0	1	x	x	1	0	1
1	x	x	x	1	1	1

[Figure 4.20 from the textbook]

Truth table for a 4-to-2 priority encoder

	w_3	w_2	w_1	w_0	y_1	y_0	z
0 0 0 0	0	0	0	0	d	d	0
0 0 0 1	0	0	0	1	0	0	1
0 0 1 x	0	0	1	0	0	1	1
	0	0	1	1	0	1	1
0 1 x x	0	1	0	0	1	0	1
	0	1	0	1	1	0	1
	0	1	1	0	1	0	1
	0	1	1	1	1	0	1
1 x x x	1	0	0	0	1	1	1
	1	0	0	1	1	1	1
	1	0	1	0	1	1	1
	1	0	1	1	1	1	1
	1	1	0	0	1	1	1
	1	1	0	1	1	1	1
	1	1	1	0	1	1	1
	1	1	1	1	1	1	1

Expressions for 4-to-2 priority encoder

w_3	w_2	w_1	w_0	y_1	y_0	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	0	0	1	1
0	0	1	1	0	1	1
0	1	0	0	1	0	1
0	1	0	1	1	0	1
0	1	1	0	1	0	1
0	1	1	1	1	0	1
1	0	0	0	1	1	1
1	0	0	1	1	1	1
1	0	1	0	1	1	1
1	0	1	1	1	1	1
1	1	0	0	1	1	1
1	1	0	1	1	1	1
1	1	1	0	1	1	1
1	1	1	1	1	1	1

$w_1 w_0$ \ $w_3 w_2$	00	01	11	10
00	d	1	1	1
01	0	1	1	1
11	0	1	1	1
10	0	1	1	1

$$y_1 = w_3 + w_2$$

Expressions for 4-to-2 priority encoder

w_3	w_2	w_1	w_0	y_1	y_0	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	0	0	1	1
0	0	1	1	0	1	1
0	1	0	0	1	0	1
0	1	0	1	1	0	1
0	1	1	0	1	0	1
0	1	1	1	1	0	1
1	0	0	0	1	1	1
1	0	0	1	1	1	1
1	0	1	0	1	1	1
1	0	1	1	1	1	1
1	1	0	0	1	1	1
1	1	0	1	1	1	1
1	1	1	0	1	1	1
1	1	1	1	1	1	1

$w_1 w_0$ \ $w_3 w_2$	00	01	11	10
00	d	0	1	1
01	0	0	1	1
11	1	0	1	1
10	1	0	1	1

$$y_0 = w_3 + w_1 \overline{w_2}$$

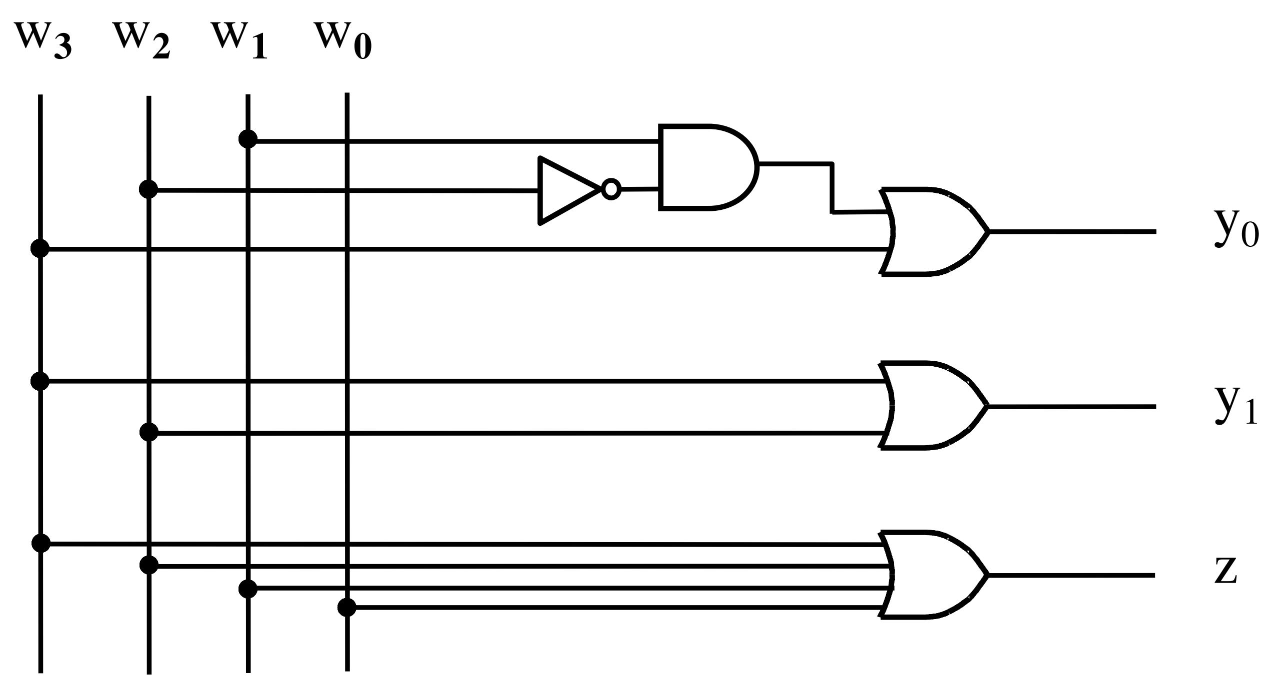
Expressions for 4-to-2 priority encoder

w_3	w_2	w_1	w_0	y_1	y_0	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	0	0	1	1
0	0	1	1	0	1	1
0	1	0	0	1	0	1
0	1	0	1	1	0	1
0	1	1	0	1	0	1
0	1	1	1	1	0	1
1	0	0	0	1	1	1
1	0	0	1	1	1	1
1	0	1	0	1	1	1
1	0	1	1	1	1	1
1	1	0	0	1	1	1
1	1	0	1	1	1	1
1	1	1	0	1	1	1
1	1	1	1	1	1	1

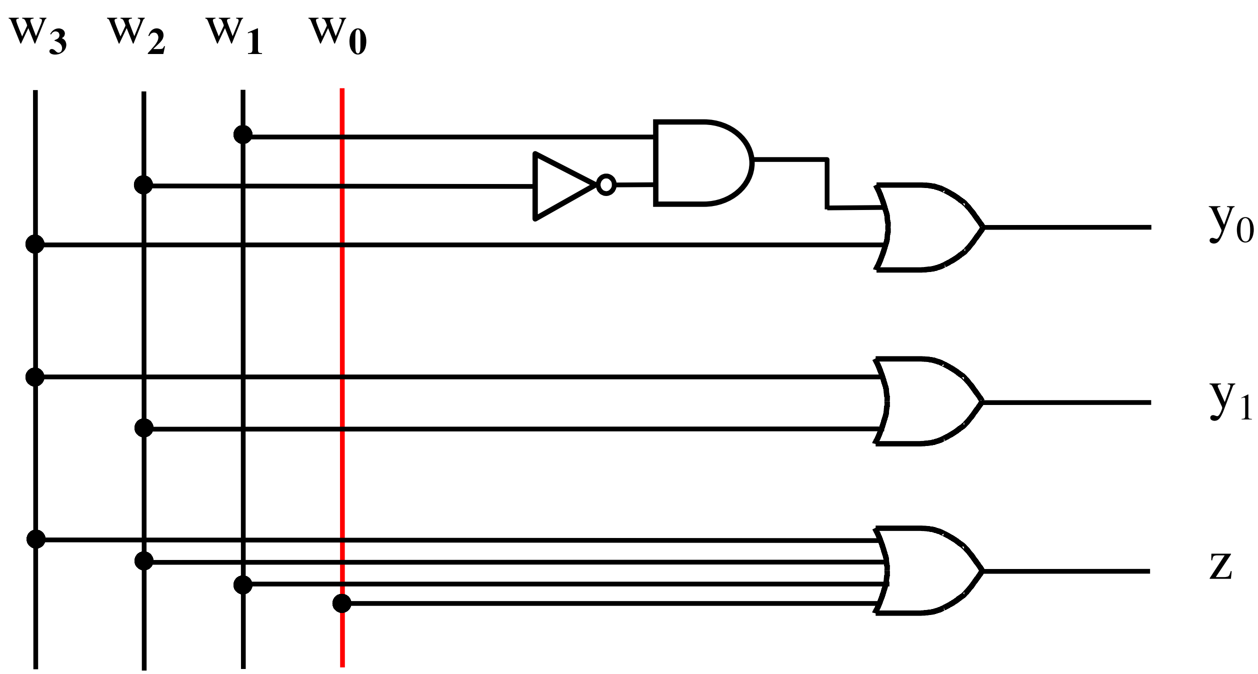
$w_1 w_0$ \ $w_3 w_2$	00	01	11	10
00	0	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$$z = w_3 + w_2 + w_1 + w_0$$

Circuit for the 4-to-2 priority encoder



Circuit for the 4-to-2 priority encoder



In this circuit w_0 is used.

The textbook derives a different circuit for the 4-to-2 priority encoder using a 4-to-2 binary encoder

w_3	w_2	w_1	w_0	y_1	y_0	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	x	0	1	1
0	1	x	x	1	0	1
1	x	x	x	1	1	1

$$i_0 = \overline{w}_3\overline{w}_2\overline{w}_1 w_0$$
$$i_1 = \overline{w}_3\overline{w}_2 w_1$$
$$i_2 = \overline{w}_3 w_2$$
$$i_3 = w_3$$

$$y_0 = i_1 + i_3$$
$$y_1 = i_2 + i_3$$

$$z = i_0 + i_1 + i_2 + i_3$$

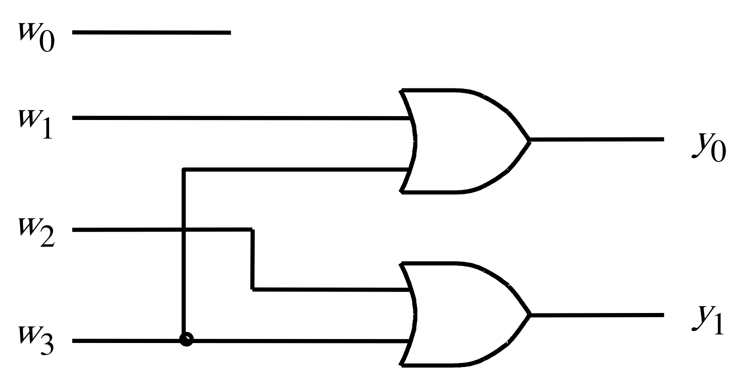
The textbook derives a different circuit for the 4-to-2 priority encoder using a 4-to-2 binary encoder

w_3	w_2	w_1	w_0	y_1	y_0	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	x	0	1	1
0	1	x	x	1	0	1
1	x	x	x	1	1	1

$$i_0 = \overline{w}_3\overline{w}_2\overline{w}_1 w_0$$
$$i_1 = \overline{w}_3\overline{w}_2 w_1$$
$$i_2 = \overline{w}_3 w_2$$
$$i_3 = w_3$$

$$y_0 = i_1 + i_3$$
$$y_1 = i_2 + i_3$$

$$z = i_0 + i_1 + i_2 + i_3$$



The textbook derives a different circuit for the 4-to-2 priority encoder using a 4-to-2 binary encoder

w_3	w_2	w_1	w_0	y_1	y_0	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	x	0	1	1
0	1	x	x	1	0	1
1	x	x	x	1	1	1

$$i_0 = \overline{w}_3\overline{w}_2\overline{w}_1 w_0$$

$$i_1 = \overline{w}_3\overline{w}_2 w_1$$

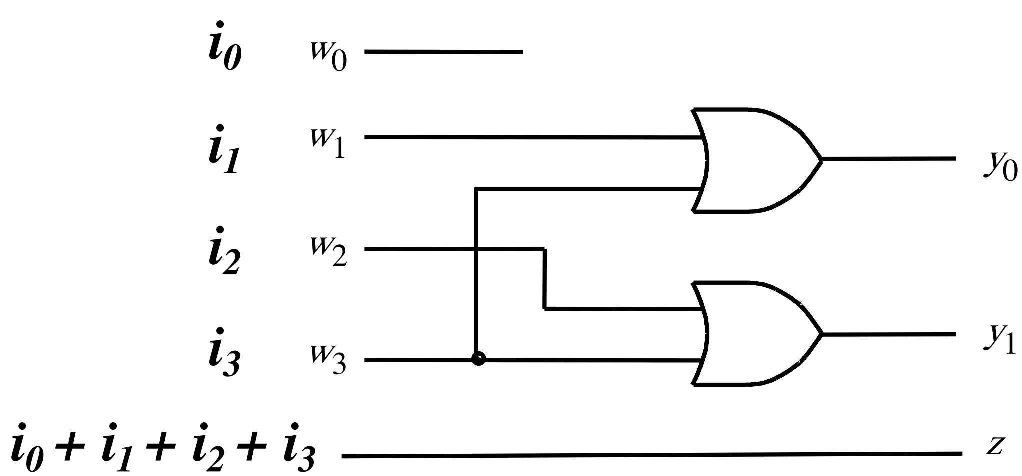
$$i_2 = \overline{w}_3 w_2$$

$$i_3 = w_3$$

$$y_0 = i_1 + i_3$$

$$y_1 = i_2 + i_3$$

$$z = i_0 + i_1 + i_2 + i_3$$



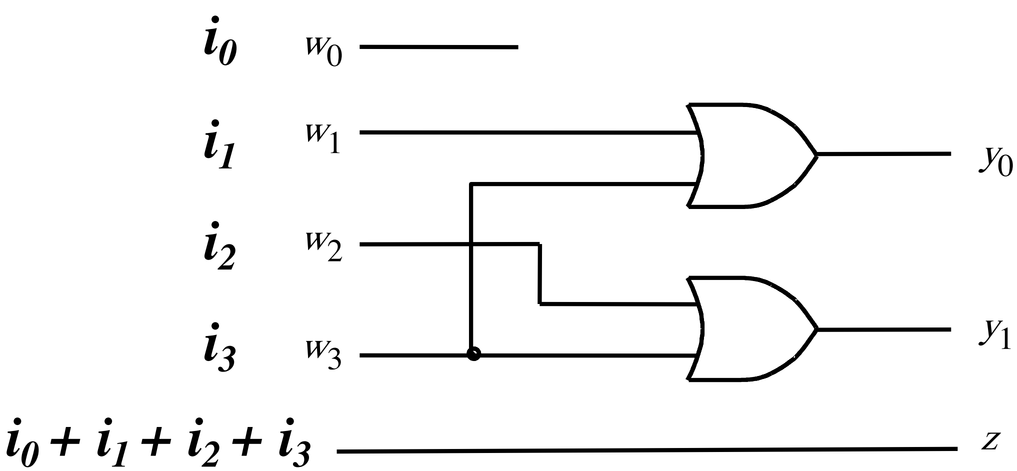
The textbook derives a different circuit for the 4-to-2 priority encoder using a 4-to-2 binary encoder

w_3	w_2	w_1	w_0	y_1	y_0	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	x	0	1	1
0	1	x	x	1	0	1
1	x	x	x	1	1	1

$$i_0 = \overline{w}_3\overline{w}_2\overline{w}_1 w_0$$
$$i_1 = \overline{w}_3\overline{w}_2 w_1$$
$$i_2 = \overline{w}_3 w_2$$
$$i_3 = w_3$$

$$y_0 = i_1 + i_3$$
$$y_1 = i_2 + i_3$$

$$z = i_0 + i_1 + i_2 + i_3$$



Try to prove that this is equivalent to the circuit that was derived above.

Let's prove this for z

w_3	w_2	w_1	w_0	y_1	y_0	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	x	0	1	1
0	1	x	x	1	0	1
1	x	x	x	1	1	1

$$i_0 = \overline{w}_3\overline{w}_2\overline{w}_1 w_0$$

$$i_1 = \overline{w}_3\overline{w}_2 w_1$$

$$i_2 = \overline{w}_3 w_2$$

$$i_3 = w_3$$

$$y_0 = i_1 + i_3$$

$$y_1 = i_2 + i_3$$

$$z = i_0 + i_1 + i_2 + i_3$$

$w_1 w_0$ \ $w_3 w_2$	00	01	11	10
00	0	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

z = ?

Let's prove this for z

w_3	w_2	w_1	w_0	y_1	y_0	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	x	0	1	1
0	1	x	x	1	0	1
1	x	x	x	1	1	1

$$i_0 = \overline{w}_3\overline{w}_2\overline{w}_1 w_0$$

$$i_1 = \overline{w}_3\overline{w}_2 w_1$$

$$i_2 = \overline{w}_3 w_2$$

$$i_3 = w_3$$

$$y_0 = i_1 + i_3$$

$$y_1 = i_2 + i_3$$

$$z = i_0 + i_1 + i_2 + i_3$$

$w_1 w_0$ \ $w_3 w_2$	00	01	11	10
00	0	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$$z = (w_0 + w_1 + w_2 + w_3)$$

Let's prove this for y_0

w_3	w_2	w_1	w_0	y_1	y_0	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	x	0	1	1
0	1	x	x	1	0	1
1	x	x	x	1	1	1

$$i_0 = \overline{w}_3\overline{w}_2\overline{w}_1 w_0$$
$$\boxed{i_1 = \overline{w}_3\overline{w}_2 w_1}$$
$$i_2 = \overline{w}_3 w_2$$
$$\boxed{i_3 = w_3}$$

$$y_0 = i_1 + i_3$$
$$y_1 = i_2 + i_3$$

$$z = i_0 + i_1 + i_2 + i_3$$

$w_1 w_0$ \ $w_3 w_2$	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	1	0	1	1
10	1	0	1	1

$y_0 = ?$

Let's prove this for y_0

w_3	w_2	w_1	w_0	y_1	y_0	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	x	0	1	1
0	1	x	x	1	0	1
1	x	x	x	1	1	1

$$i_0 = \overline{w}_3\overline{w}_2\overline{w}_1 w_0$$

$$\boxed{i_1 = \overline{w}_3\overline{w}_2 w_1}$$

$$i_2 = \overline{w}_3 w_2$$

$$\boxed{i_3 = w_3}$$

$$y_0 = i_1 + i_3$$

$$y_1 = i_2 + i_3$$

$$z = i_0 + i_1 + i_2 + i_3$$

$w_1 w_0$ \ $w_3 w_2$	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	1	0	1	1
10	1	0	1	1

$$y_0 = w_3 + w_1\overline{w_2}$$

Let's prove this for y_1

w_3	w_2	w_1	w_0	y_1	y_0	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	x	0	1	1
0	1	x	x	1	0	1
1	x	x	x	1	1	1

$$i_0 = \overline{w}_3\overline{w}_2\overline{w}_1 w_0$$
$$i_1 = \overline{w}_3\overline{w}_2 w_1$$
$$\boxed{i_2 = \overline{w}_3 w_2}$$
$$\boxed{i_3 = w_3}$$

$$y_0 = i_1 + i_3$$
$$y_1 = i_2 + i_3$$

$$z = i_0 + i_1 + i_2 + i_3$$

$w_1 w_0$ \ $w_3 w_2$	00	01	11	10
00	0	1	1	1
01	0	1	1	1
11	0	1	1	1
10	0	1	1	1

y_1 = ?

Let's prove this for y_1

w_3	w_2	w_1	w_0	y_1	y_0	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	x	0	1	1
0	1	x	x	1	0	1
1	x	x	x	1	1	1

$$i_0 = \overline{w}_3\overline{w}_2\overline{w}_1 w_0$$
$$i_1 = \overline{w}_3\overline{w}_2 w_1$$
$$i_2 = \overline{w}_3 w_2$$
$$i_3 = w_3$$

$$y_0 = i_1 + i_3$$
$$y_1 = i_2 + i_3$$

$$z = i_0 + i_1 + i_2 + i_3$$

$w_1 w_0$ \ $w_3 w_2$	00	01	11	10
00	0	1	1	1
01	0	1	1	1
11	0	1	1	1
10	0	1	1	1

$y_1 = w_3 + w_2$

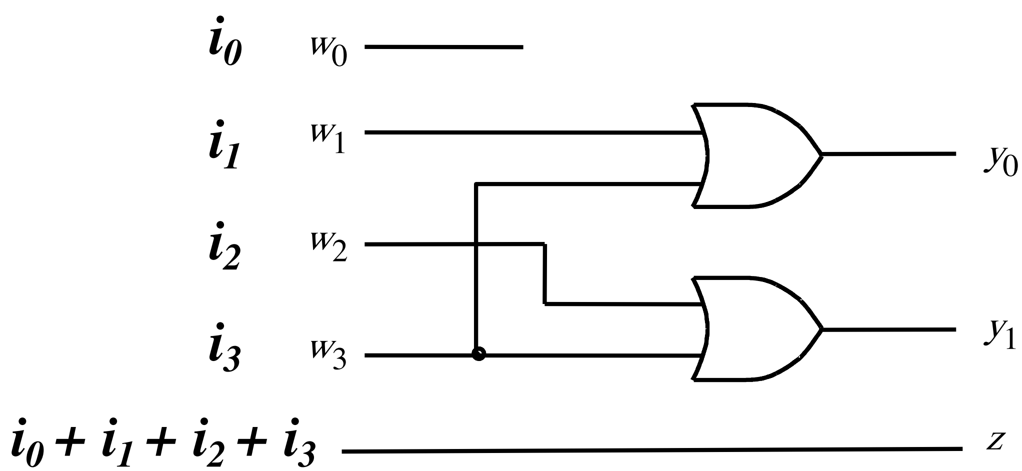
Therefore, this circuit for the 4-to-2 priority encoder is equivalent to …

w_3	w_2	w_1	w_0	y_1	y_0	z
0	0	0	0	d	d	0
0	0	0	1	0	0	1
0	0	1	x	0	1	1
0	1	x	x	1	0	1
1	x	x	x	1	1	1

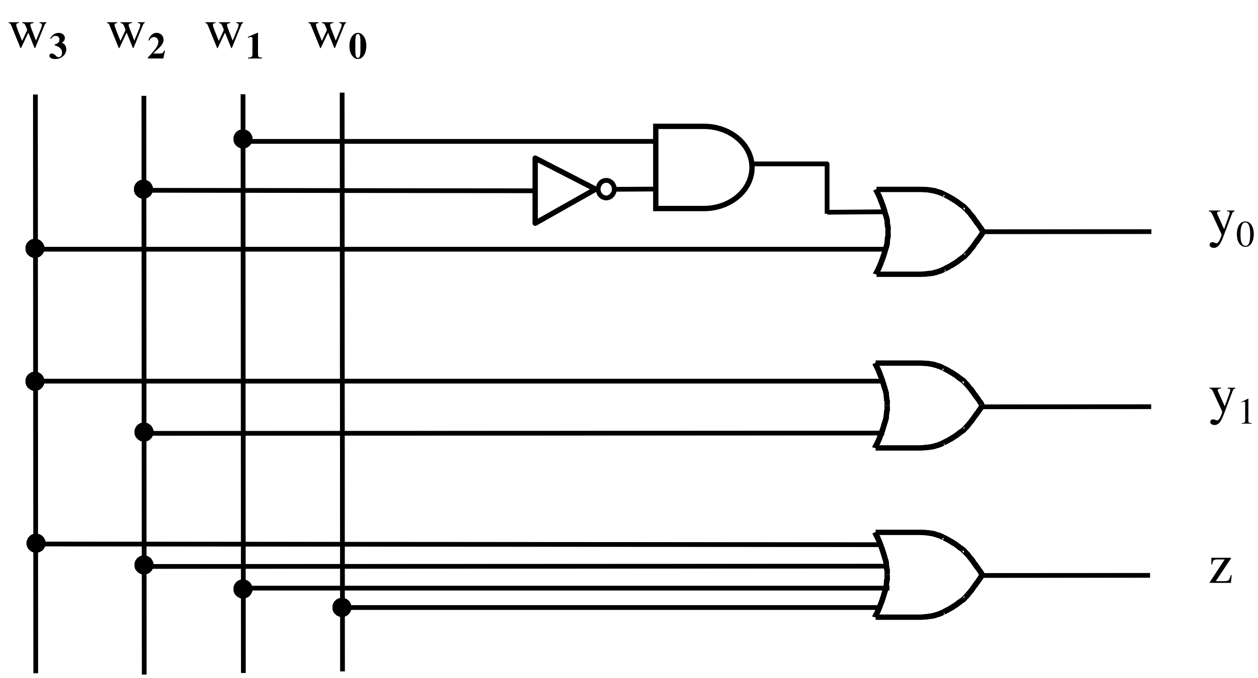
$$i_0 = \overline{w}_3\overline{w}_2\overline{w}_1 w_0$$
$$i_1 = \overline{w}_3\overline{w}_2 w_1$$
$$i_2 = \overline{w}_3 w_2$$
$$i_3 = w_3$$

$$y_0 = i_1 + i_3$$
$$y_1 = i_2 + i_3$$

$$z = i_0 + i_1 + i_2 + i_3$$



… this circuit for the 4-to-2 priority encoder



Code Converters

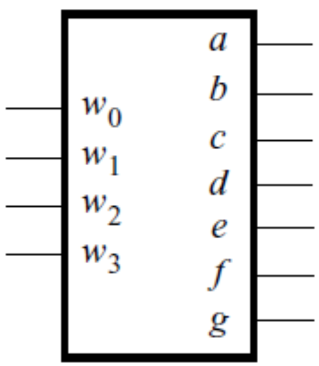
Code Converter (Definition)

- **Converts from one type of input encoding to a different type of output encoding.**

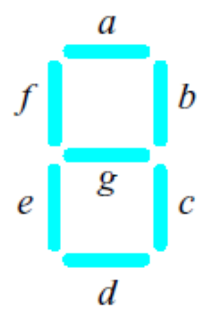
Code Converter (Definition)

- Converts from one type of input encoding to a different type of output encoding.
- A decoder does that as well, but its outputs are always one-hot encoded so the output code is really only one type of output code.
- A binary encoder does that as well but its inputs are always one-hot encoded so the input code is really only one type of input code.

A hex-to-7-segment display code converter



(a) Code converter



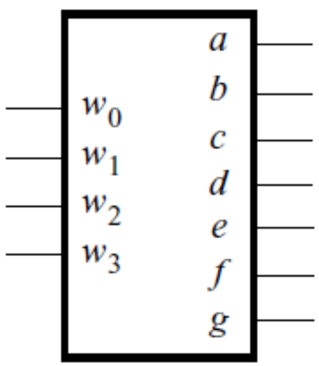
(b) 7-segment display

w_3	w_2	w_1	w_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	1	1	0	1	1	1
1	0	1	1	0	0	1	1	1	1	1
1	1	0	0	1	0	0	1	1	1	0
1	1	0	1	0	1	1	1	1	0	1
1	1	1	0	1	0	0	1	1	1	1
1	1	1	1	1	0	0	0	1	1	1

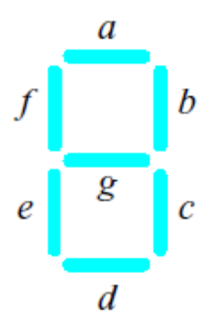
(c) Truth table

[Figure 4.21 from the textbook]

A hex-to-7-segment display code converter



(a) Code converter



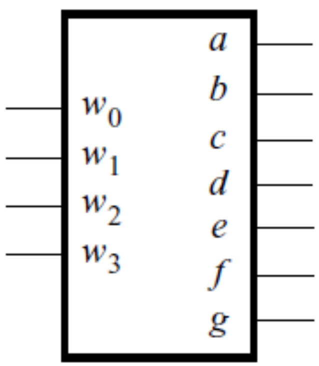
(b) 7-segment display

w_3	w_2	w_1	w_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	1	1	0	1	1	1
1	0	1	1	0	0	1	1	1	1	1
1	1	0	0	1	0	0	1	1	1	0
1	1	0	1	0	1	1	1	1	0	1
1	1	1	0	1	0	0	1	1	1	1
1	1	1	1	1	0	0	0	1	1	1

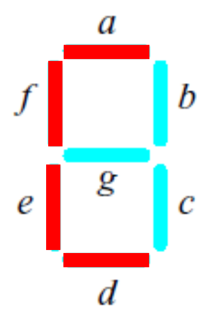
(c) Truth table

[Figure 4.21 from the textbook]

A hex-to-7-segment display code converter



(a) Code converter



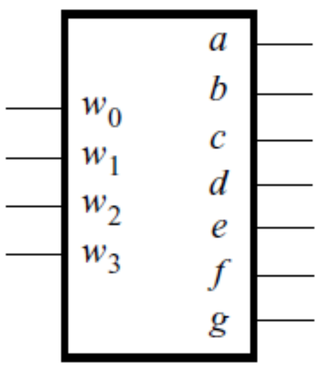
(b) 7-segment display

w_3	w_2	w_1	w_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	1	1	0	1	1	1
1	0	1	1	0	0	1	1	1	1	1
1	1	0	0	1	0	0	1	1	1	0
1	1	0	1	0	1	1	1	1	0	1
1	1	1	0	1	0	0	1	1	1	1
1	1	1	1	1	0	0	0	1	1	1

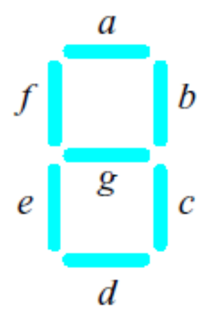
(c) Truth table

[Figure 4.21 from the textbook]

A hex-to-7-segment display code converter



(a) Code converter



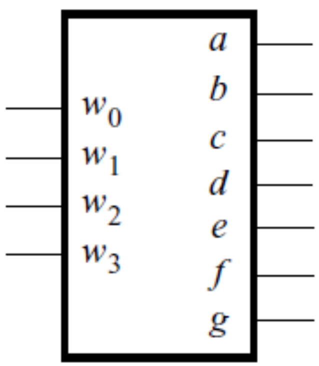
(b) 7-segment display

w_3	w_2	w_1	w_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	1	1	0	1	1	1
1	0	1	1	0	0	1	1	1	1	1
1	1	0	0	1	0	0	1	1	1	0
1	1	0	1	0	1	1	1	1	0	1
1	1	1	0	1	0	0	1	1	1	1
1	1	1	1	1	0	0	0	1	1	1

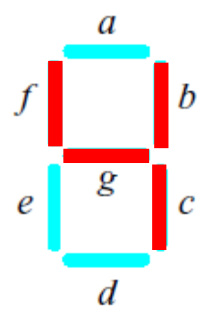
(c) Truth table

[Figure 4.21 from the textbook]

A hex-to-7-segment display code converter



(a) Code converter



(b) 7-segment display

w_3	w_2	w_1	w_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	1	1	0	1	1	1
1	0	1	1	0	0	1	1	1	1	1
1	1	0	0	1	0	0	1	1	1	0
1	1	0	1	0	1	1	1	1	0	1
1	1	1	0	1	0	0	1	1	1	1
1	1	1	1	1	0	0	0	1	1	1

(c) Truth table

[Figure 4.21 from the textbook]

What is the circuit for this code converter?

x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	1	1	0	1	1	1
1	0	1	1	0	0	1	1	1	1	1
1	1	0	0	1	0	0	1	1	1	0
1	1	0	1	0	1	1	1	1	0	1
1	1	1	0	1	0	0	1	1	1	1
1	1	1	1	1	0	0	0	1	1	1

What is the circuit for this code converter?

x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	1	1	0	1	1	1
1	0	1	1	0	0	1	1	1	1	1
1	1	0	0	1	0	0	1	1	1	0
1	1	0	1	0	1	1	1	1	0	1
1	1	1	0	1	0	0	1	1	1	1
1	1	1	1	1	0	0	0	1	1	1

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(0, 2, 3, 5, 6, 7, 8, 9, 10, 12, 14, 15)$$

What is the circuit for this code converter?

x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	1	1	0	1	1	1
1	0	1	1	0	0	1	1	1	1	1
1	1	0	0	1	0	0	1	1	1	0
1	1	0	1	0	1	1	1	1	0	1
1	1	1	0	1	0	0	1	1	1	1
1	1	1	1	1	0	0	0	1	1	1

$x_3 x_4$ \ $x_1 x_2$	00	01	11	10
00	1	0	1	1
01	0	1	0	1
11	1	1	1	0
10	1	1	1	1

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(0, 2, 3, 5, 6, 7, 8, 9, 10, 12, 14, 15)$$

What is the circuit for this code converter?

x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	1	1	0	1	1	1
1	0	1	1	0	0	1	1	1	1	1
1	1	0	0	1	0	0	1	1	1	0
1	1	0	1	0	1	1	1	1	0	1
1	1	1	0	1	0	0	1	1	1	1
1	1	1	1	1	0	0	0	1	1	1

What is the circuit for this code converter?

x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	1	1	0	1	1	1
1	0	1	1	0	0	1	1	1	1	1
1	1	0	0	1	0	0	1	1	1	0
1	1	0	1	0	1	1	1	1	0	1
1	1	1	0	1	0	0	1	1	1	1
1	1	1	1	1	0	0	0	1	1	1

$x_3 x_4$ \ $x_1 x_2$	00	01	11	10
00	1	0	1	1
01	0	1	1	1
11	1	0	0	1
10	1	1	1	0

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(0, 2, 3, 5, 6, 8, 9, 11, 12, 13, 14)$$

Example Problems from Chapter 4

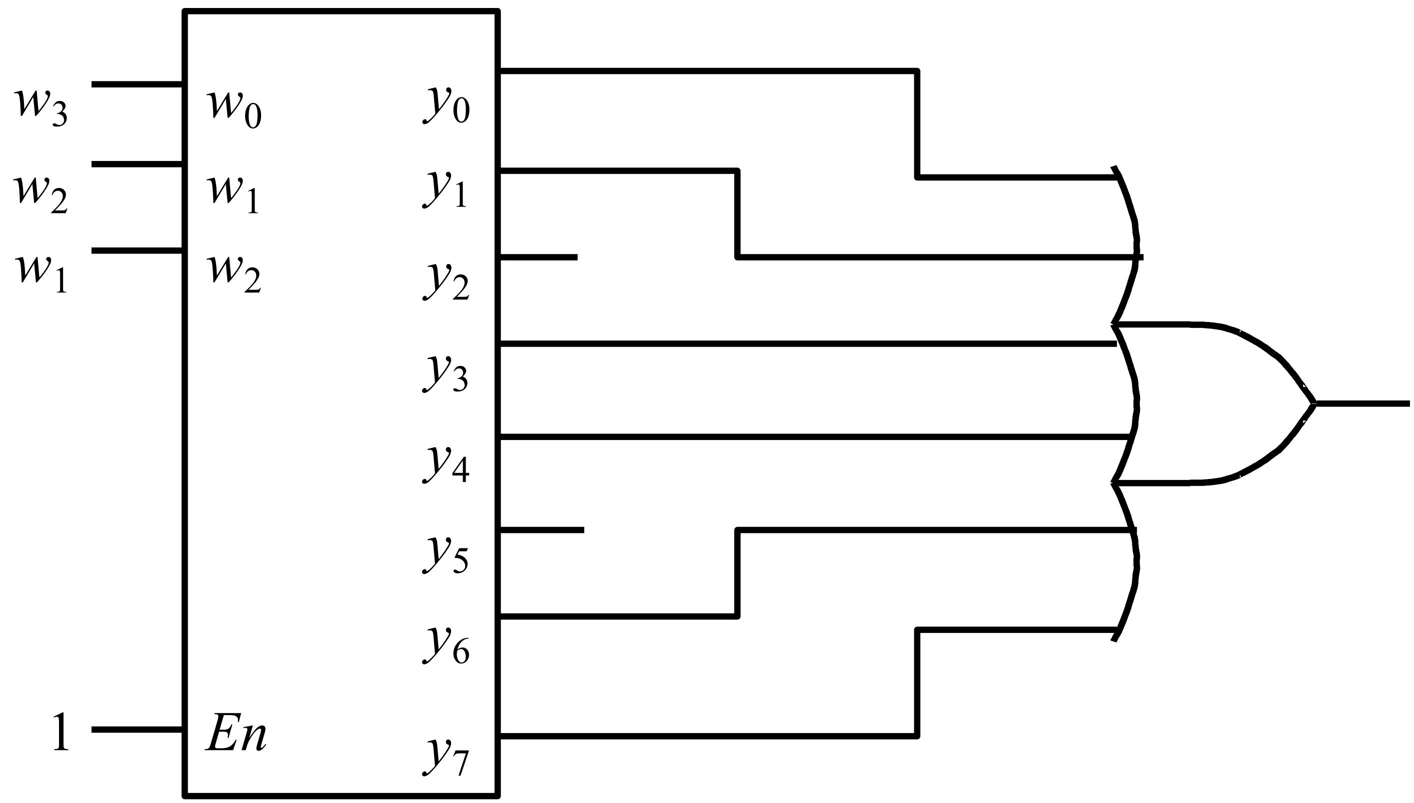
Example 1: SOP vs Decoders

Implement the function

$$f(w_1, w_2, w_3) = \Sigma\, m(0, 1, 3, 4, 6, 7)$$

by using a 3-to-8 binary decoder and one OR gate.

Solution Circuit

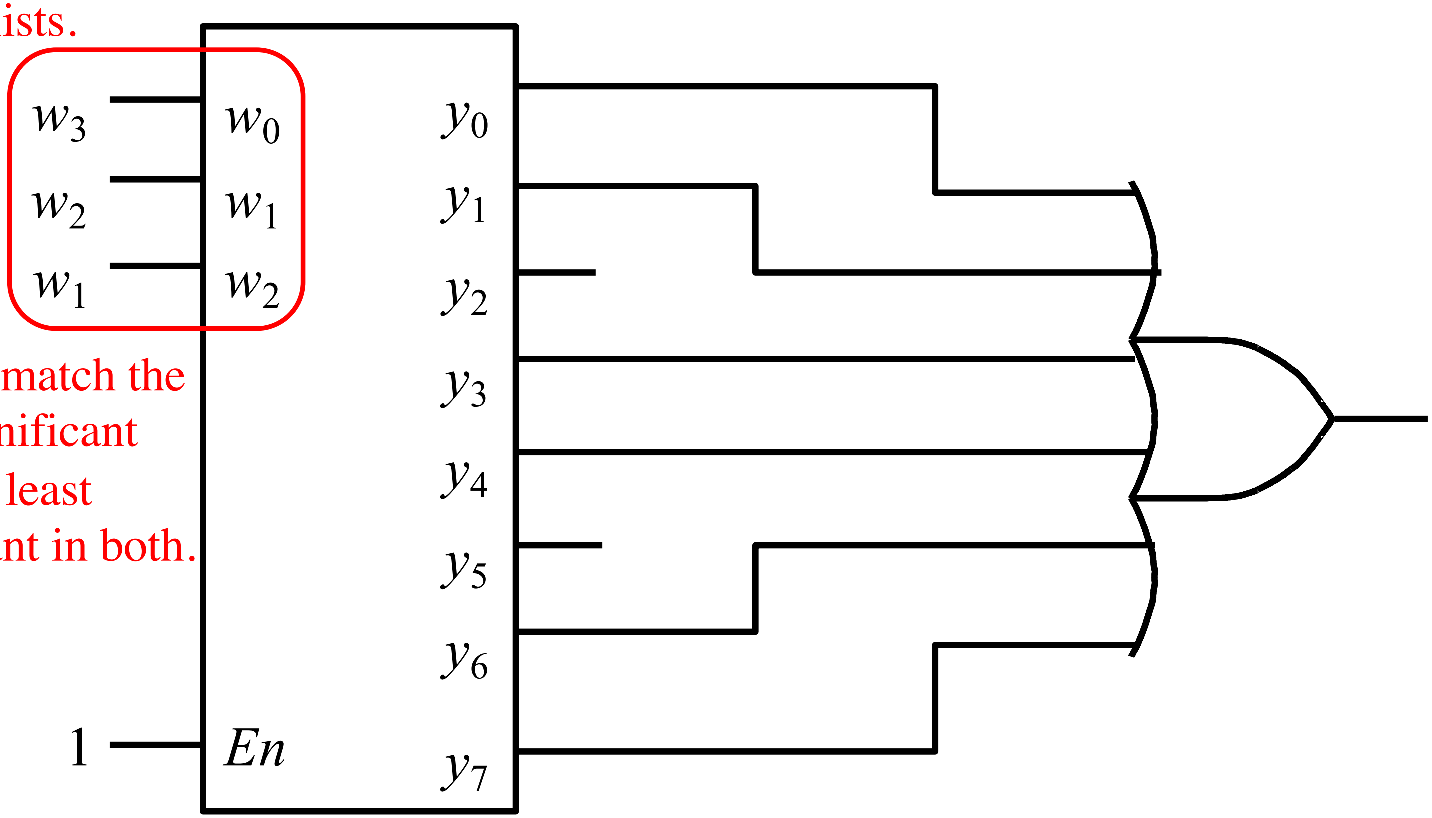


$$f(w_1, w_2, w_3) = \Sigma\, m(0, 1, 3, 4, 6, 7)$$

[Figure 4.44 from the textbook]

Solution Circuit

Notice this swap of variables in the two lists.



Need to match the least significant with the least significant in both.

$$f(w_1, w_2, w_3) = \Sigma\, m(0, 1, 3, 4, 6, 7)$$

[Figure 4.44 from the textbook]

Example 2: Implement an 8-to-3 binary encoder

w_7	w_6	w_5	w_4	w_3	w_2	w_1	w_0	y_2	y_1	y_0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	1	1	1

[Figure 4.45 from the textbook]

Example 2: Implement an 8-to-3 binary encoder

w_7	w_6	w_5	w_4	w_3	w_2	w_1	w_0	y_2	y_1	y_0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	1	1	1

These formulas work due to the one-hot encoded property of the inputs.

$$y_0 = w_1 + w_3 + w_5 + w_7$$
$$y_1 = w_2 + w_3 + w_6 + w_7$$
$$y_2 = w_4 + w_5 + w_6 + w_7$$

[Figure 4.45 from the textbook]

Example 2: Implement an 8-to-3 binary encoder

w_7	w_6	w_5	w_4	w_3	w_2	w_1	w_0	y_2	y_1	y_0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	1	1	1

$$y_0 = \boxed{w_1 + w_3 + w_5 + w_7}$$

$$y_1 = w_2 + w_3 + w_6 + w_7$$

$$y_2 = w_4 + w_5 + w_6 + w_7$$

Example 2: Implement an 8-to-3 binary encoder

w_7	w_6	w_5	w_4	w_3	w_2	w_1	w_0	y_2	y_1	y_0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	1	1	1

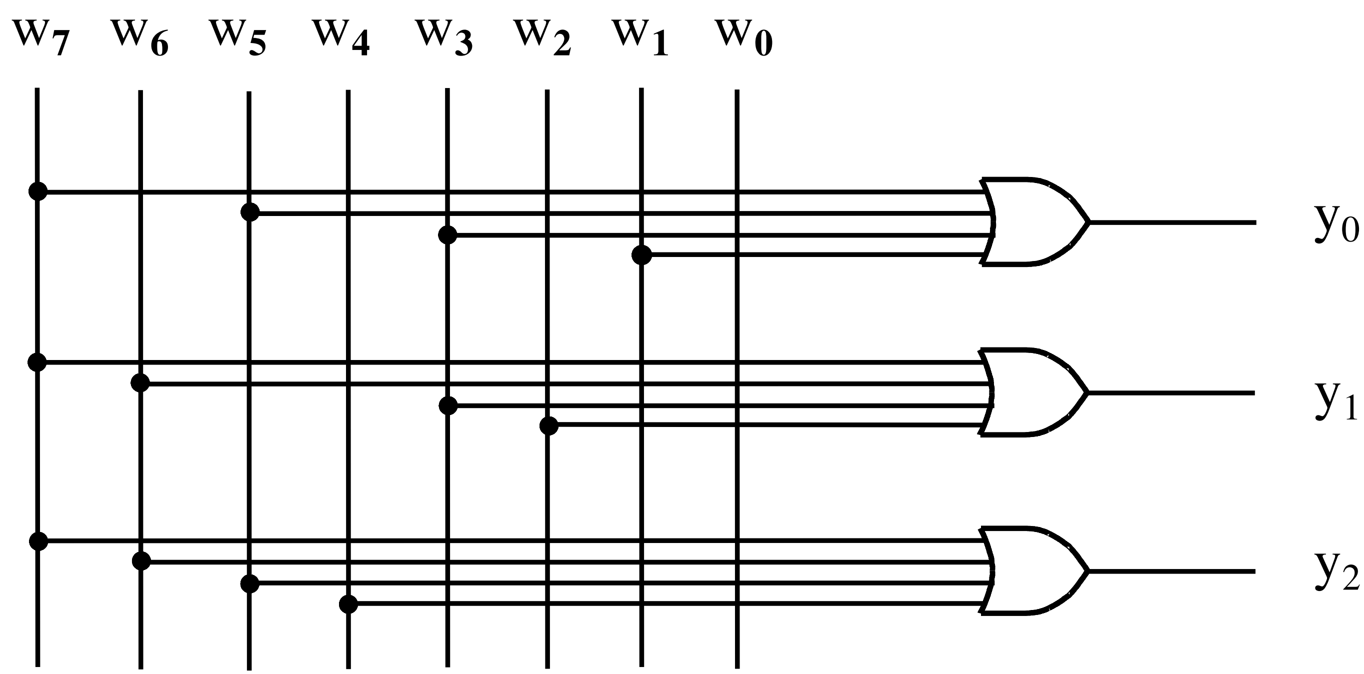
$$y_0 = w_1 + w_3 + w_5 + w_7$$
$$y_1 = \boxed{w_2 + w_3 + w_6 + w_7}$$
$$y_2 = w_4 + w_5 + w_6 + w_7$$

Example 2: Implement an 8-to-3 binary encoder

w_7	w_6	w_5	w_4	w_3	w_2	w_1	w_0	y_2	y_1	y_0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	1	1	1

$$y_0 = w_1 + w_3 + w_5 + w_7$$
$$y_1 = w_2 + w_3 + w_6 + w_7$$
$$y_2 = \boxed{w_4 + w_5 + w_6 + w_7}$$

Circuit for the 8-to-3 binary encoder



$$y_0 = w_1 + w_3 + w_5 + w_7$$
$$y_1 = w_2 + w_3 + w_6 + w_7$$
$$y_2 = w_4 + w_5 + w_6 + w_7$$

Example 3: Implement an 8-to-3 priority encoder

w_7	w_6	w_5	w_4	w_3	w_2	w_1	w_0	y_2	y_1	y_0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	1	1	1

Example 3: Implement an 8-to-3 priority encoder

w_7	w_6	w_5	w_4	w_3	w_2	w_1	w_0	y_2	y_1	y_0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	X	0	0	1
0	0	0	0	0	1	X	X	0	1	0
0	0	0	0	1	X	X	X	0	1	1
0	0	0	1	X	X	X	X	1	0	0
0	0	1	X	X	X	X	X	1	0	1
0	1	X	X	X	X	X	X	1	1	0
1	X	X	X	X	X	X	X	1	1	1

Example 3: Implement an 8-to-3 priority encoder

w_7	w_6	w_5	w_4	w_3	w_2	w_1	w_0	y_2	y_1	y_0	z
0	0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	0	1	x	0	0	1	1
0	0	0	0	0	1	x	x	0	1	0	1
0	0	0	0	1	x	x	x	0	1	1	1
0	0	0	1	x	x	x	x	1	0	0	1
0	0	1	x	x	x	x	x	1	0	1	1
0	1	x	x	x	x	x	x	1	1	0	1
1	x	x	x	x	x	x	x	1	1	1	1
0	0	0	0	0	0	0	0	d	d	d	0

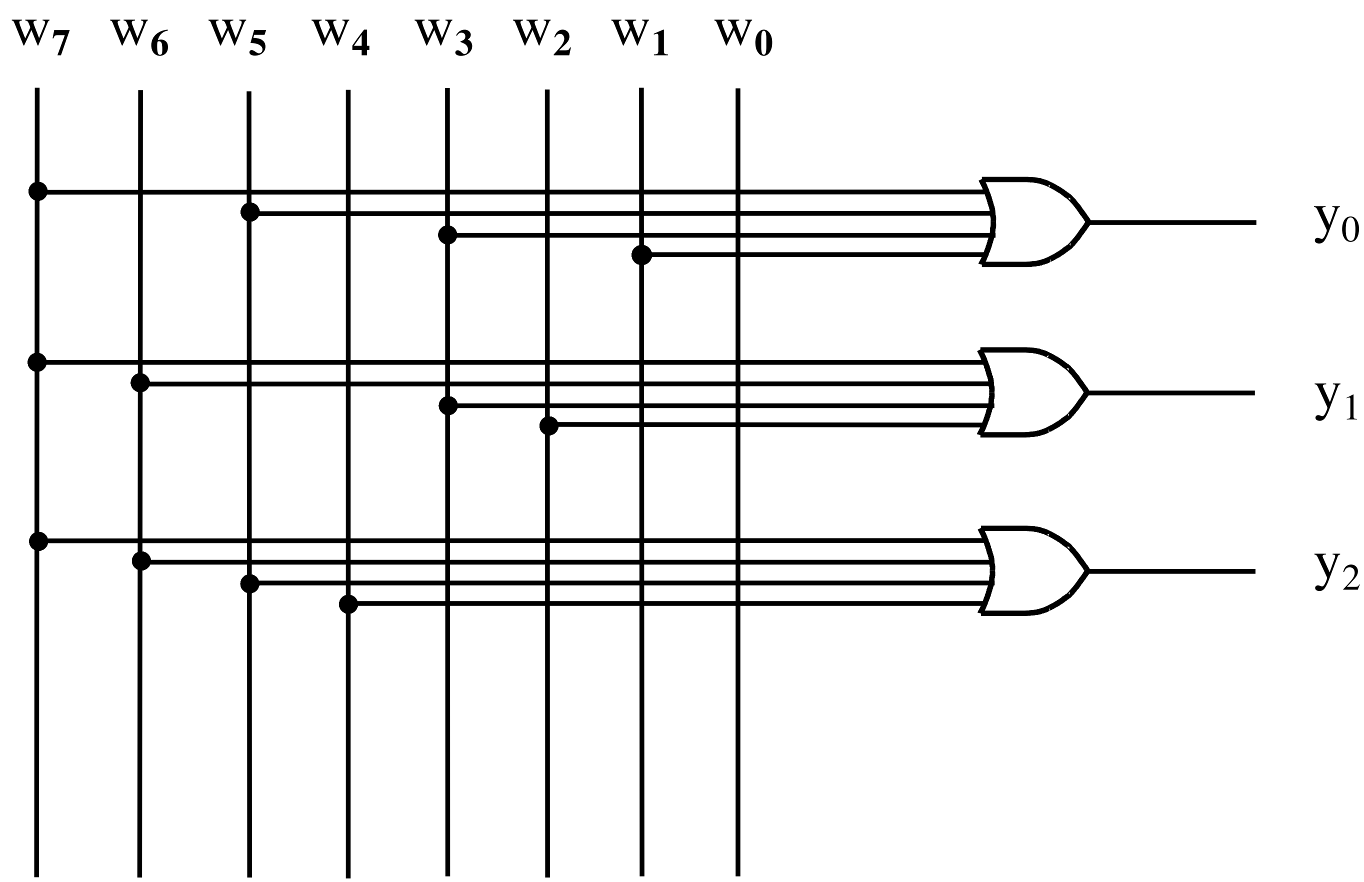
Example 3: Implement an 8-to-3 priority encoder

w_7	w_6	w_5	w_4	w_3	w_2	w_1	w_0	y_2	y_1	y_0	z
0	0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	0	1	x	0	0	1	1
0	0	0	0	0	1	x	x	0	1	0	1
0	0	0	0	1	x	x	x	0	1	1	1
0	0	0	1	x	x	x	x	1	0	0	1
0	0	1	x	x	x	x	x	1	0	1	1
0	1	x	x	x	x	x	x	1	1	0	1
1	x	x	x	x	x	x	x	1	1	1	1
0	0	0	0	0	0	0	0	d	d	d	0

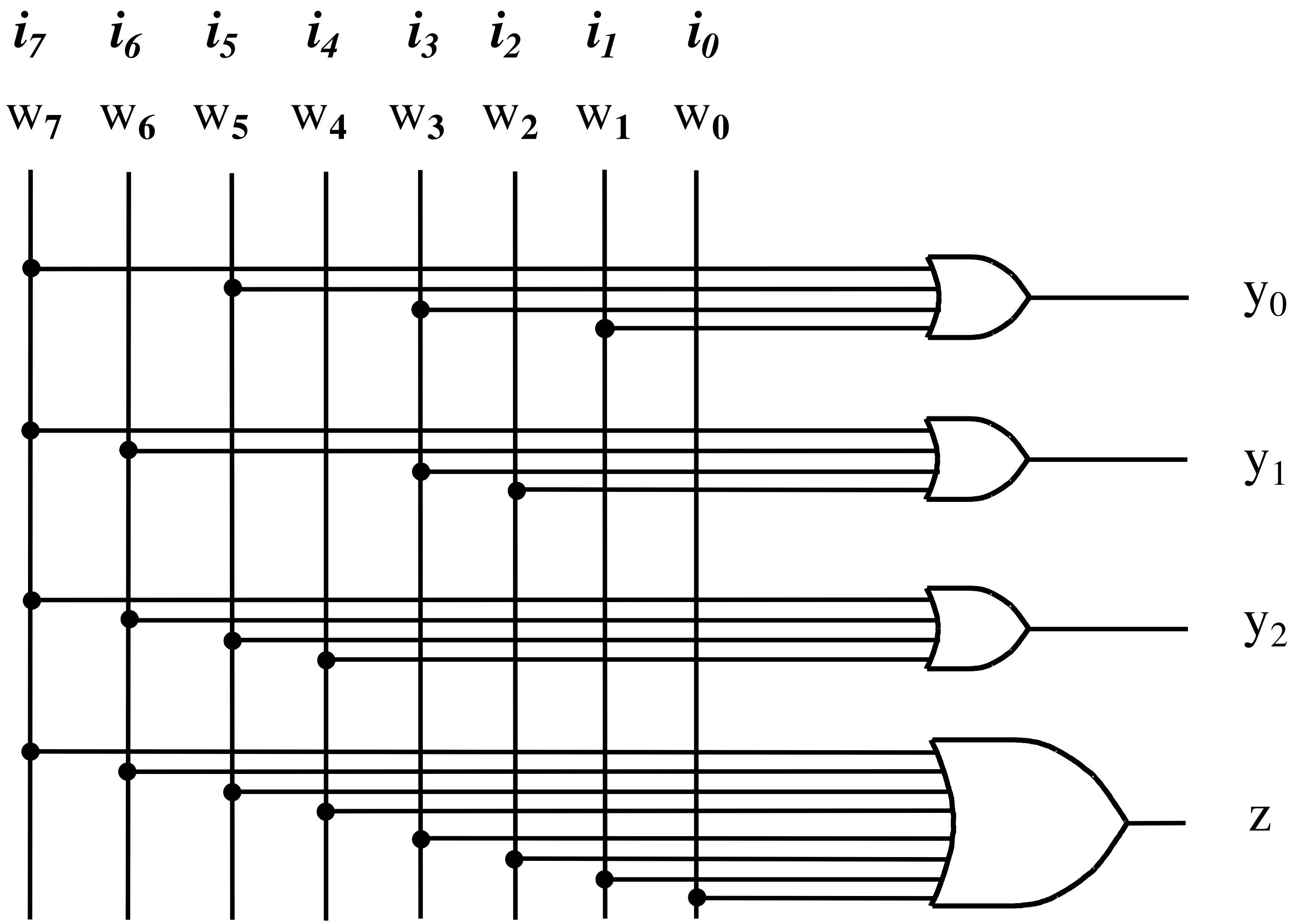
Example 3: Implement an 8-to-3 priority encoder

	w_7	w_6	w_5	w_4	w_3	w_2	w_1	w_0	y_2	y_1	y_0	z
$i_0 = \overline{w_7}\,\overline{w_6}\,\overline{w_5}\,\overline{w_4}\,\overline{w_3}\,\overline{w_2}\,\overline{w_1}\,w_0$	0	0	0	0	0	0	0	1	0	0	0	1
$i_1 = \overline{w_7}\,\overline{w_6}\,\overline{w_5}\,\overline{w_4}\,\overline{w_3}\,\overline{w_2}\,w_1$	0	0	0	0	0	0	1	x	0	0	1	1
$i_2 = \overline{w_7}\,\overline{w_6}\,\overline{w_5}\,\overline{w_4}\,\overline{w_3}\,w_2$	0	0	0	0	0	1	x	x	0	1	0	1
$i_3 = \overline{w_7}\,\overline{w_6}\,\overline{w_5}\,\overline{w_4}\,w_3$	0	0	0	0	1	x	x	x	0	1	1	1
$i_4 = \overline{w_7}\,\overline{w_6}\,\overline{w_5}\,w_4$	0	0	0	1	x	x	x	x	1	0	0	1
$i_5 = \overline{w_7}\,\overline{w_6}\,w_5$	0	0	1	x	x	x	x	x	1	0	1	1
$i_6 = \overline{w_7}\,w_6$	0	1	x	x	x	x	x	x	1	1	0	1
$i_7 = w_7$	1	x	x	x	x	x	x	x	1	1	1	1
$z = i_0+i_1+i_2+i_3+i_4+i_5+i_6+i_7$	0	0	0	0	0	0	0	0	d	d	d	0

Circuit for the 8-to-3 <u>binary</u> encoder



Circuit for the 8-to-3 priority encoder



Example 4:Circuit implementation using a multiplexer

Implement the function

$$f(w_1, w_2, w_3, w_4, w_5) = \overline{w}_1\overline{w}_2\overline{w}_4\overline{w}_5 + w_1w_2 + w_1w_3 + w_1w_4 + w_3w_4w_5$$

using a 4-to-1 multiplexer

Some Boolean Algebra Leads To

$$\bar{w}_1\bar{w}_2\bar{w}_4\bar{w}_5 + w_1w_2 + w_1w_3 + w_1w_4 + w_3w_4w_5$$

$$\bar{w}_1\bar{w}_4\,(\bar{w}_5\,\bar{w}_2) + w_4\,(w_3w_5) + w_1\,(w_2 + w_3) + w_1w_4\,(1)$$

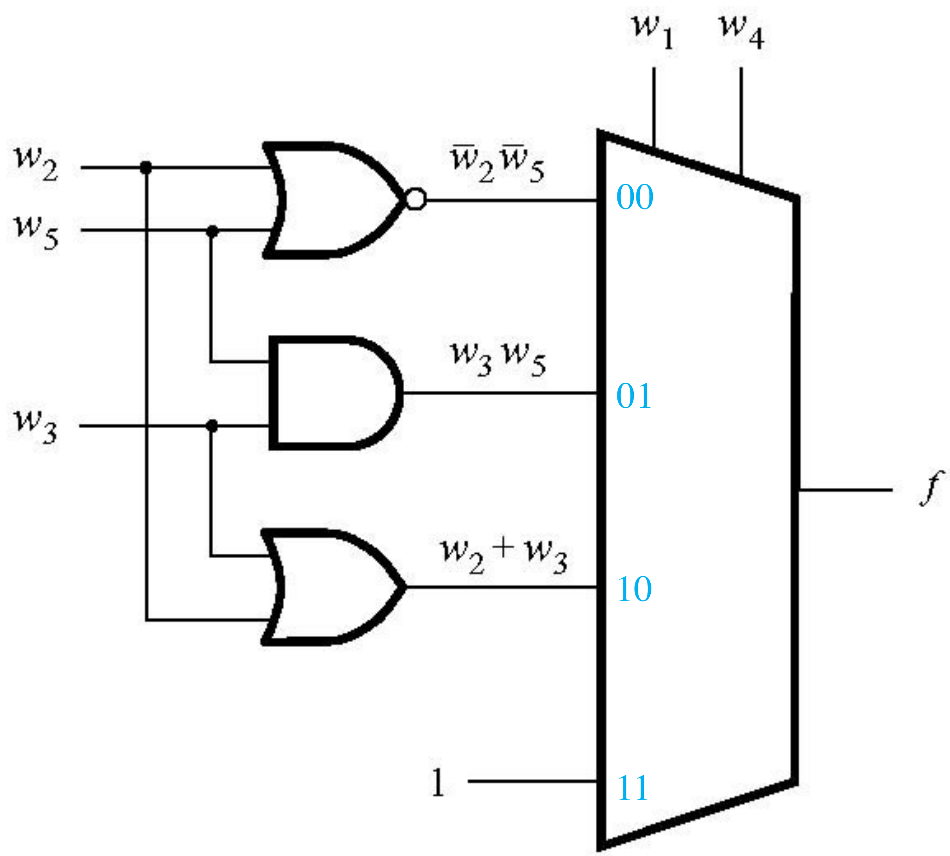
$$\bar{w}_1\bar{w}_4\,(\bar{w}_5\,\bar{w}_2) + (\,\bar{w}_1 + w_1)w_4\,(w_3w_5) + w_1(\,\bar{w}_4 + w_4)\,(w_2 + w_3) + w_1w_4\,(1)$$

$$\bar{w}_1\bar{w}_4\,(\bar{w}_5\,\bar{w}_2) + \bar{w}_1w_4\,(w_3w_5) + w_1\bar{w}_4\,(w_2 + w_3) + w_1w_4\,(\,w_3w_5 + (w_2 + w_3) + 1)$$

$$\bar{w}_1\bar{w}_4\,(\bar{w}_5\,\bar{w}_2) + \bar{w}_1w_4\,(w_3w_5) + w_1\bar{w}_4\,(w_2 + w_3) + w_1w_4\,(1)$$

Note that the split is by w_1 and w_4, not w_1 and w_2

Solution Circuit



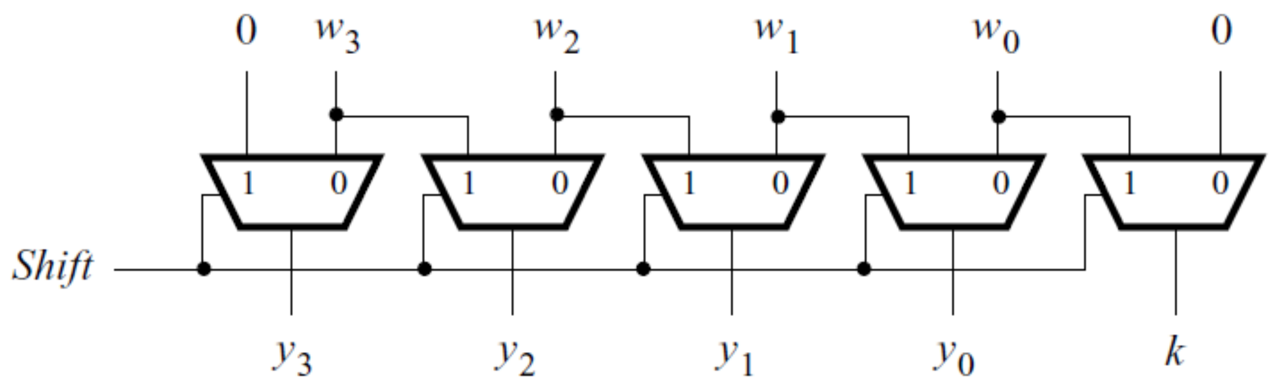
$$\overline{w}_1\overline{w}_4(\overline{w}_5\,\overline{w}_2) + \overline{w}_1w_4(w_3w_5) + w_1\overline{w}_4(w_2 + w_3) + w_1w_4(1)$$

[Figure 4.46 from the textbook]

Some Final Things from Chapter 4

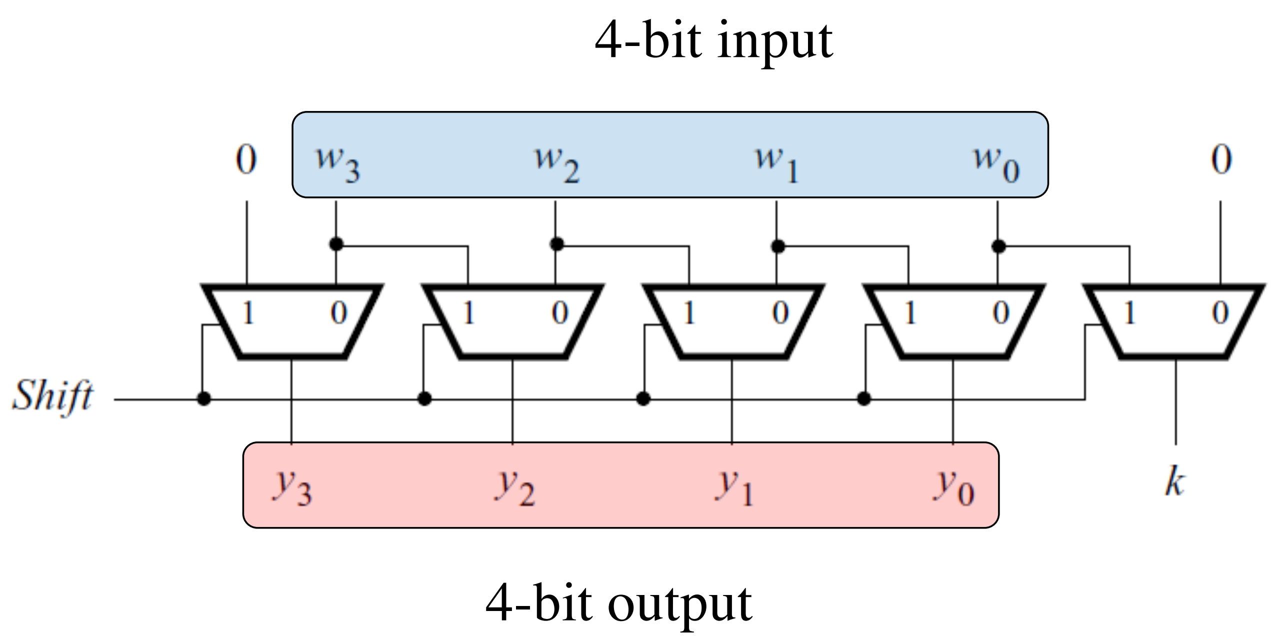
Shifter Circuit

Hold / Shift-Right Circuit



[Figure 4.50 from the textbook]

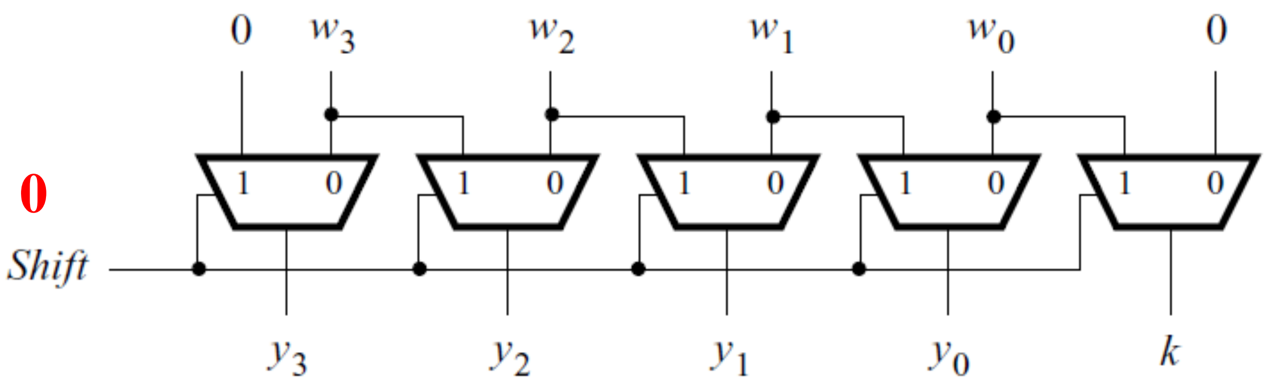
Hold / Shift-Right Circuit



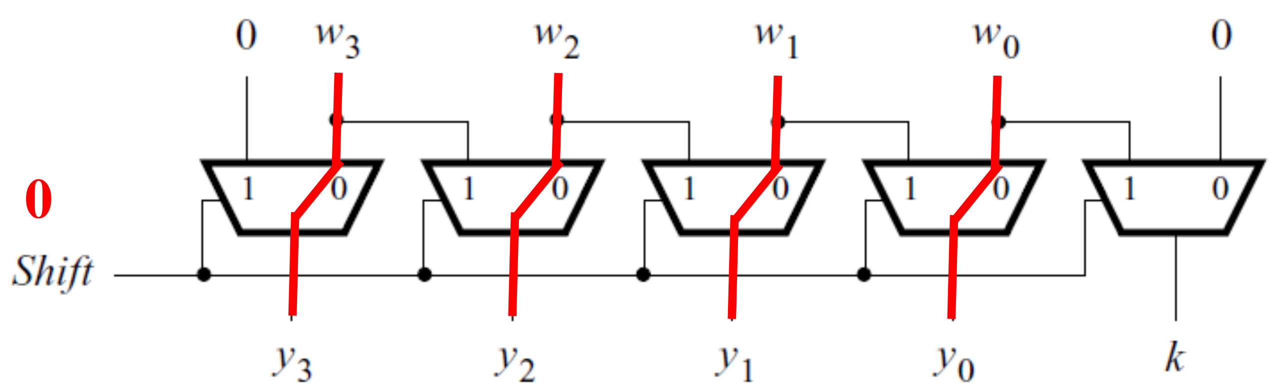
[Figure 4.50 from the textbook]

No shift if the control signal is 0

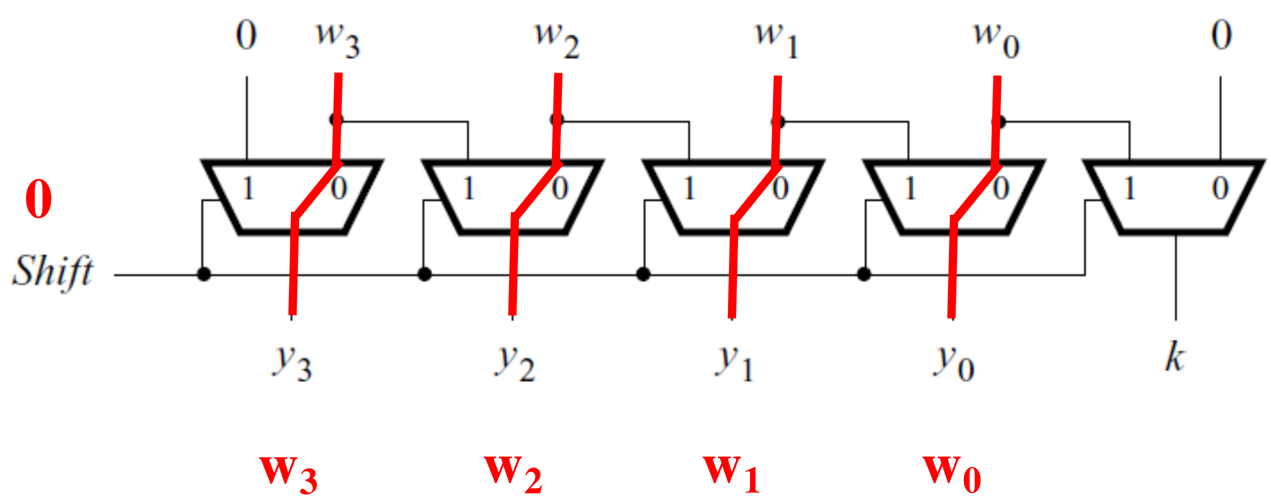
0



No shift if the control signal is 0

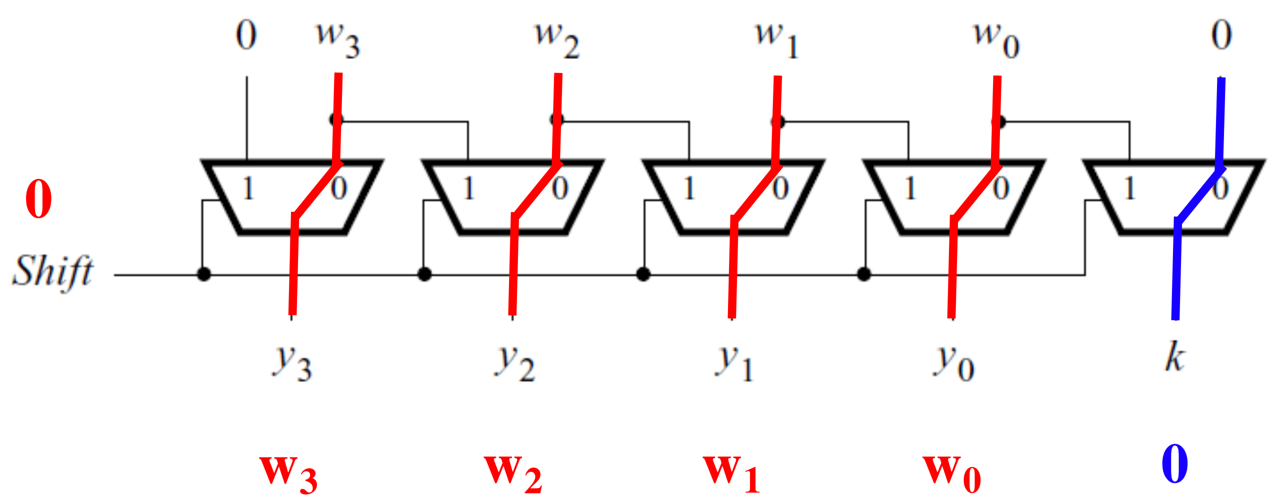


No shift if the control signal is 0



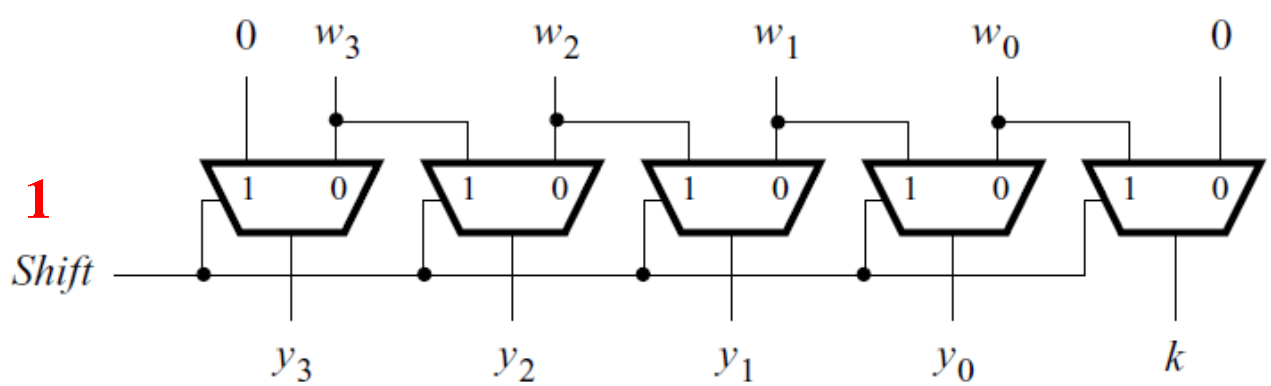
No shift in this case.

No shift if the control signal is 0

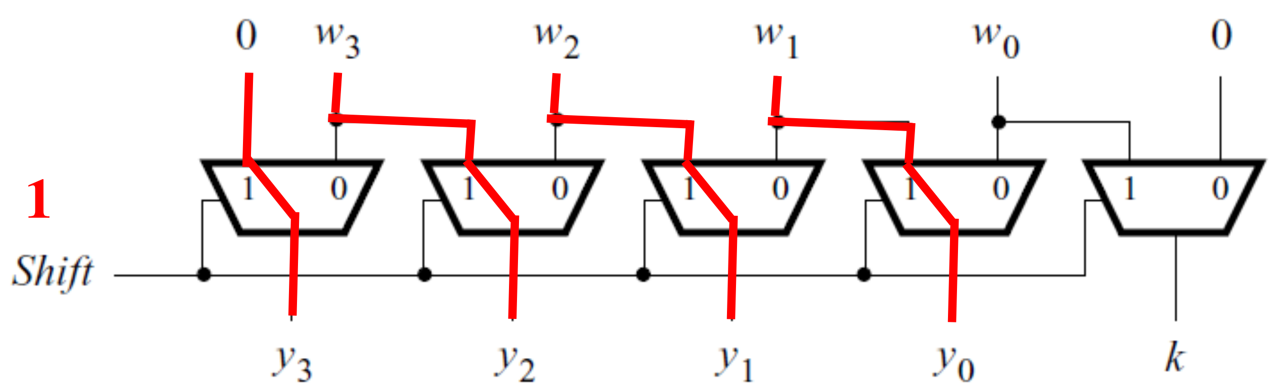


No shift in this case.

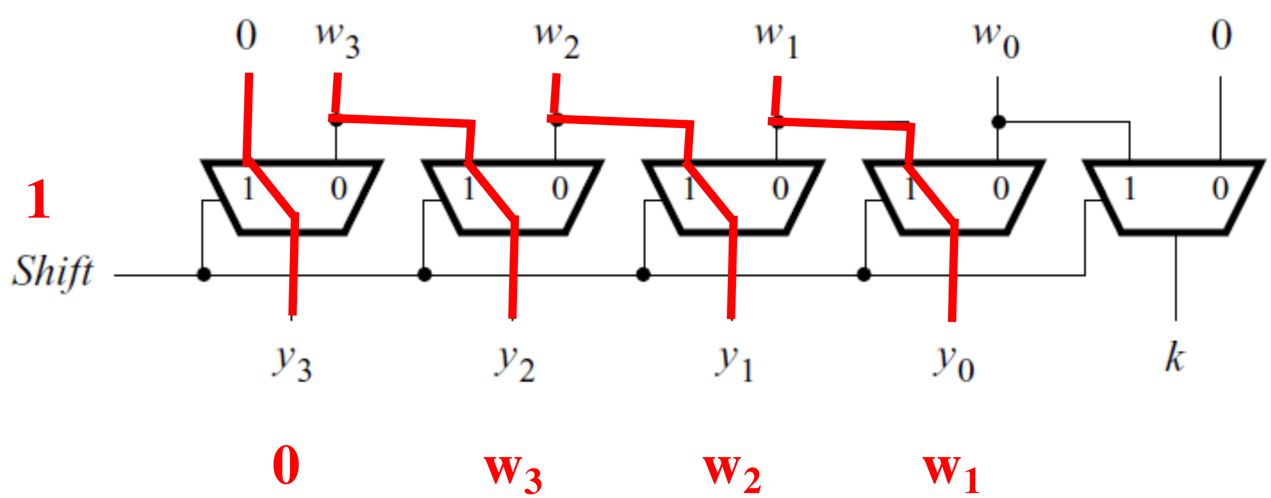
Shift right if the control signal is 1



Shift right if the control signal is 1

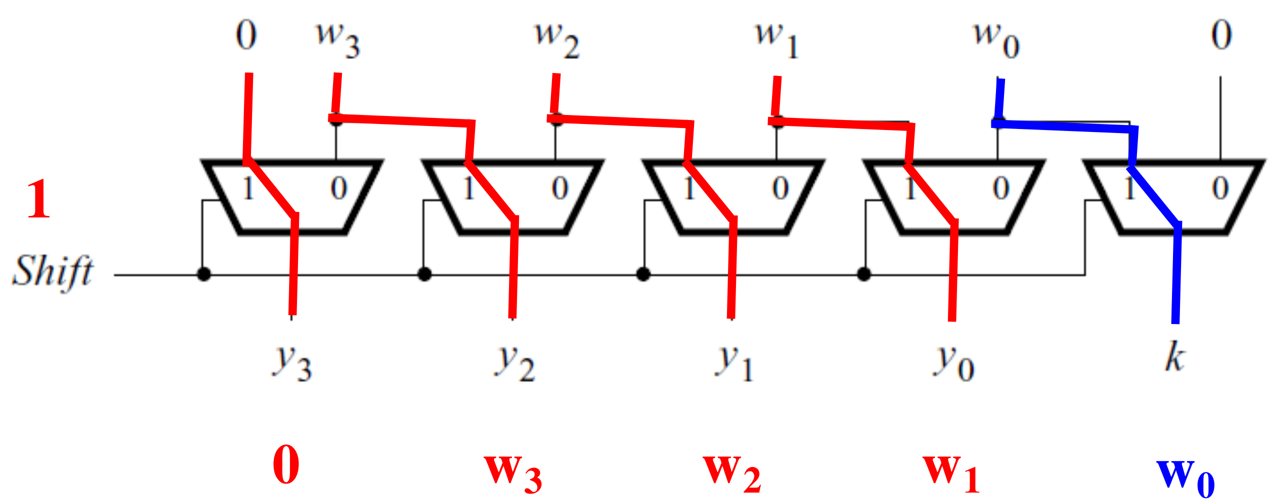


Shift right if the control signal is 1



Shift to the right by 1 bit

A shifter circuit



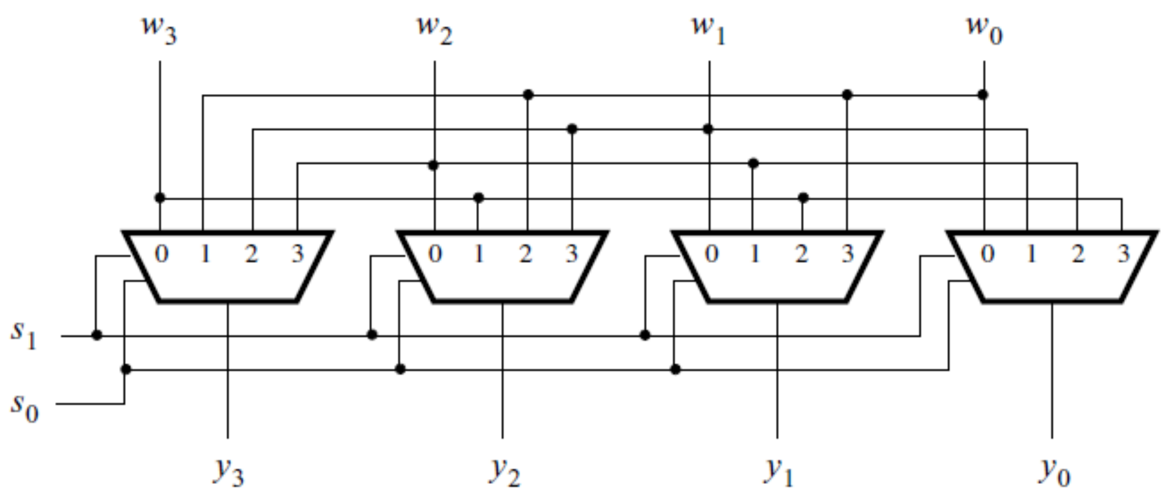
Shift to the right by 1 bit

Barrel Shifter

A barrel shifter circuit

s_1	s_0	y_3	y_2	y_1	y_0
0	0	w_3	w_2	w_1	w_0
0	1	w_0	w_3	w_2	w_1
1	0	w_1	w_0	w_3	w_2
1	1	w_2	w_1	w_0	w_3

(a) Truth table



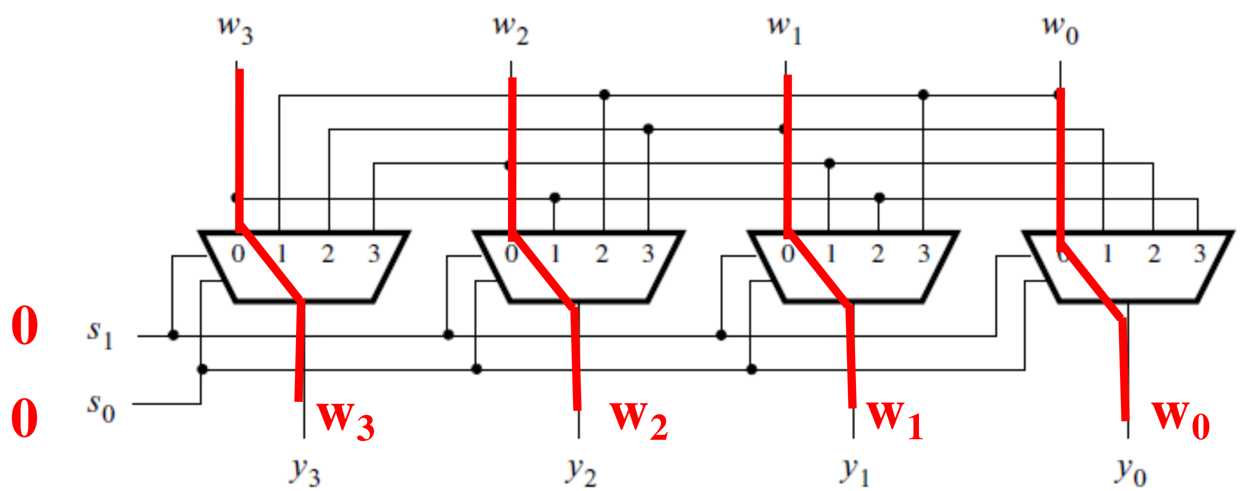
(b) Circuit

[Figure 4.51 from the textbook]

A barrel shifter circuit

s_1	s_0	y_3	y_2	y_1	y_0
0	0	w_3	w_2	w_1	w_0
0	1	w_0	w_3	w_2	w_1
1	0	w_1	w_0	w_3	w_2
1	1	w_2	w_1	w_0	w_3

(a) Truth table



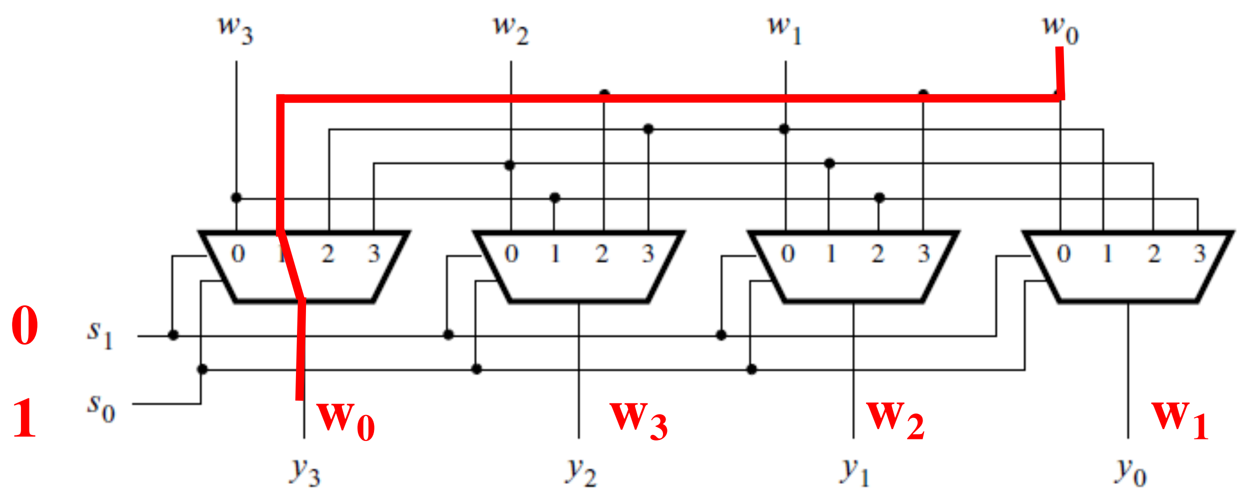
(b) Circuit

[Figure 4.51 from the textbook]

A barrel shifter circuit

s_1	s_0	y_3	y_2	y_1	y_0
0	0	w_3	w_2	w_1	w_0
0	1	w_0	w_3	w_2	w_1
1	0	w_1	w_0	w_3	w_2
1	1	w_2	w_1	w_0	w_3

(a) Truth table



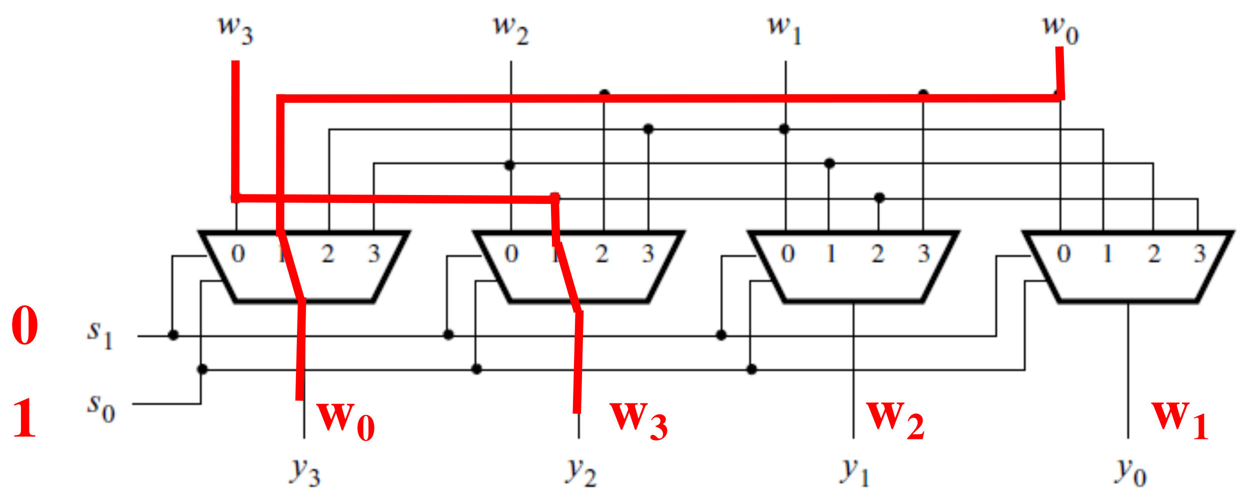
(b) Circuit

[Figure 4.51 from the textbook]

A barrel shifter circuit

s_1	s_0	y_3	y_2	y_1	y_0
0	0	w_3	w_2	w_1	w_0
0	1	w_0	w_3	w_2	w_1
1	0	w_1	w_0	w_3	w_2
1	1	w_2	w_1	w_0	w_3

(a) Truth table



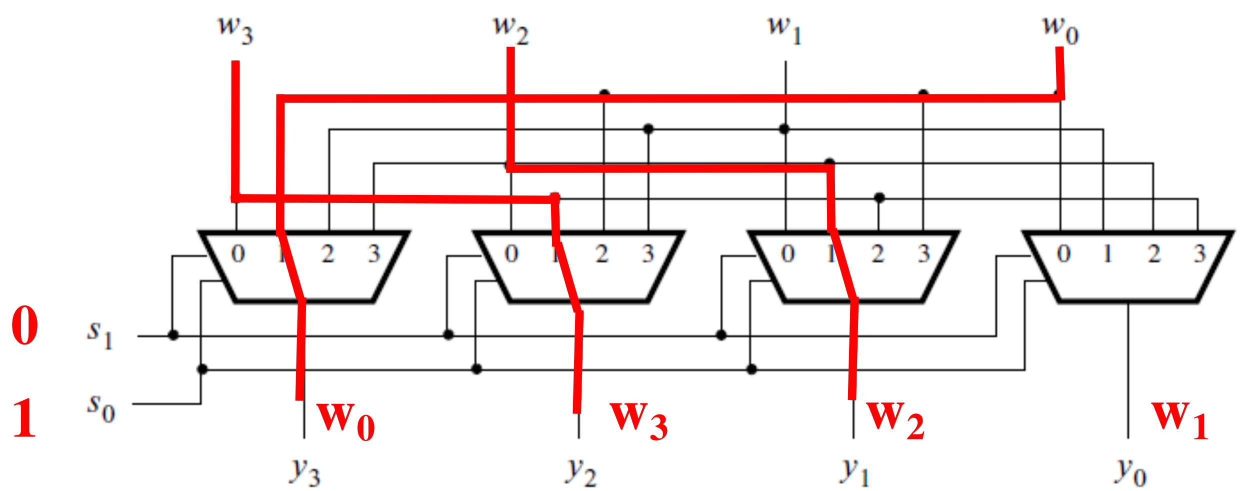
(b) Circuit

[Figure 4.51 from the textbook]

A barrel shifter circuit

s_1	s_0	y_3	y_2	y_1	y_0
0	0	w_3	w_2	w_1	w_0
0	1	w_0	w_3	w_2	w_1
1	0	w_1	w_0	w_3	w_2
1	1	w_2	w_1	w_0	w_3

(a) Truth table



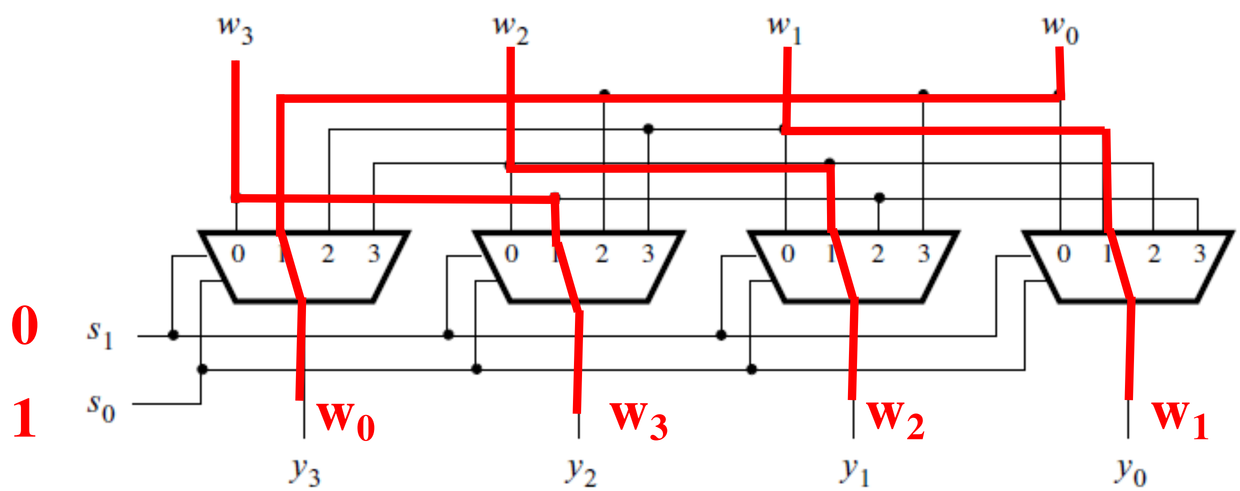
(b) Circuit

[Figure 4.51 from the textbook]

A barrel shifter circuit

s_1	s_0	y_3	y_2	y_1	y_0
0	0	w_3	w_2	w_1	w_0
0	1	w_0	w_3	w_2	w_1
1	0	w_1	w_0	w_3	w_2
1	1	w_2	w_1	w_0	w_3

(a) Truth table



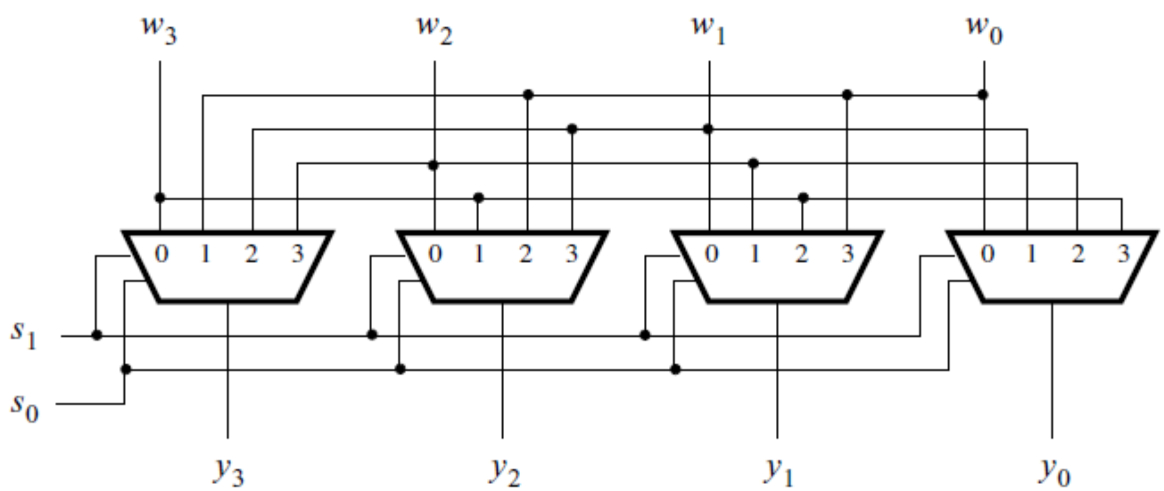
(b) Circuit

[Figure 4.51 from the textbook]

A barrel shifter circuit

s_1	s_0	y_3	y_2	y_1	y_0
0	0	w_3	w_2	w_1	w_0
0	1	w_0	w_3	w_2	w_1
1	0	w_1	w_0	w_3	w_2
1	1	w_2	w_1	w_0	w_3

(a) Truth table

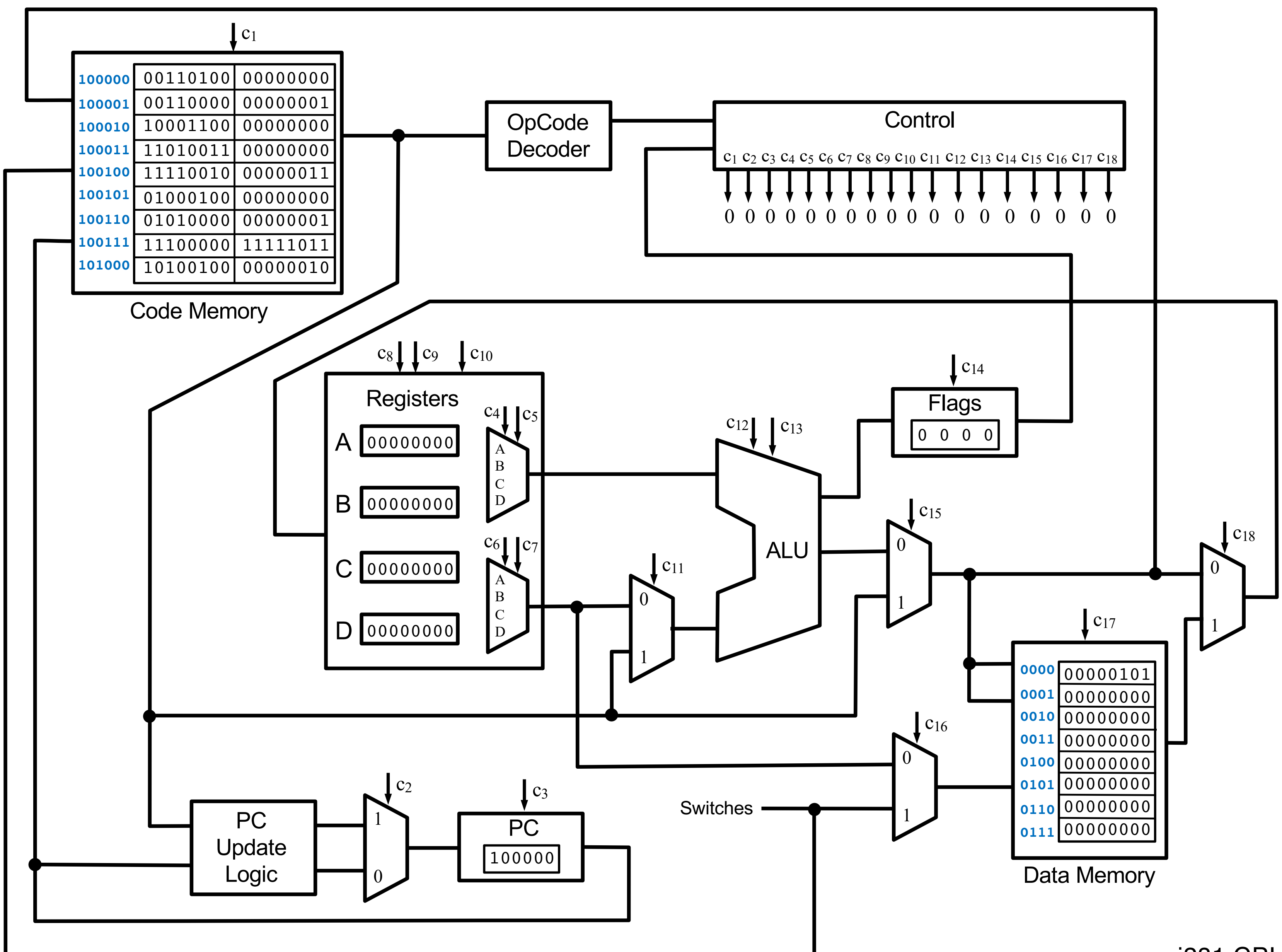


(b) Circuit

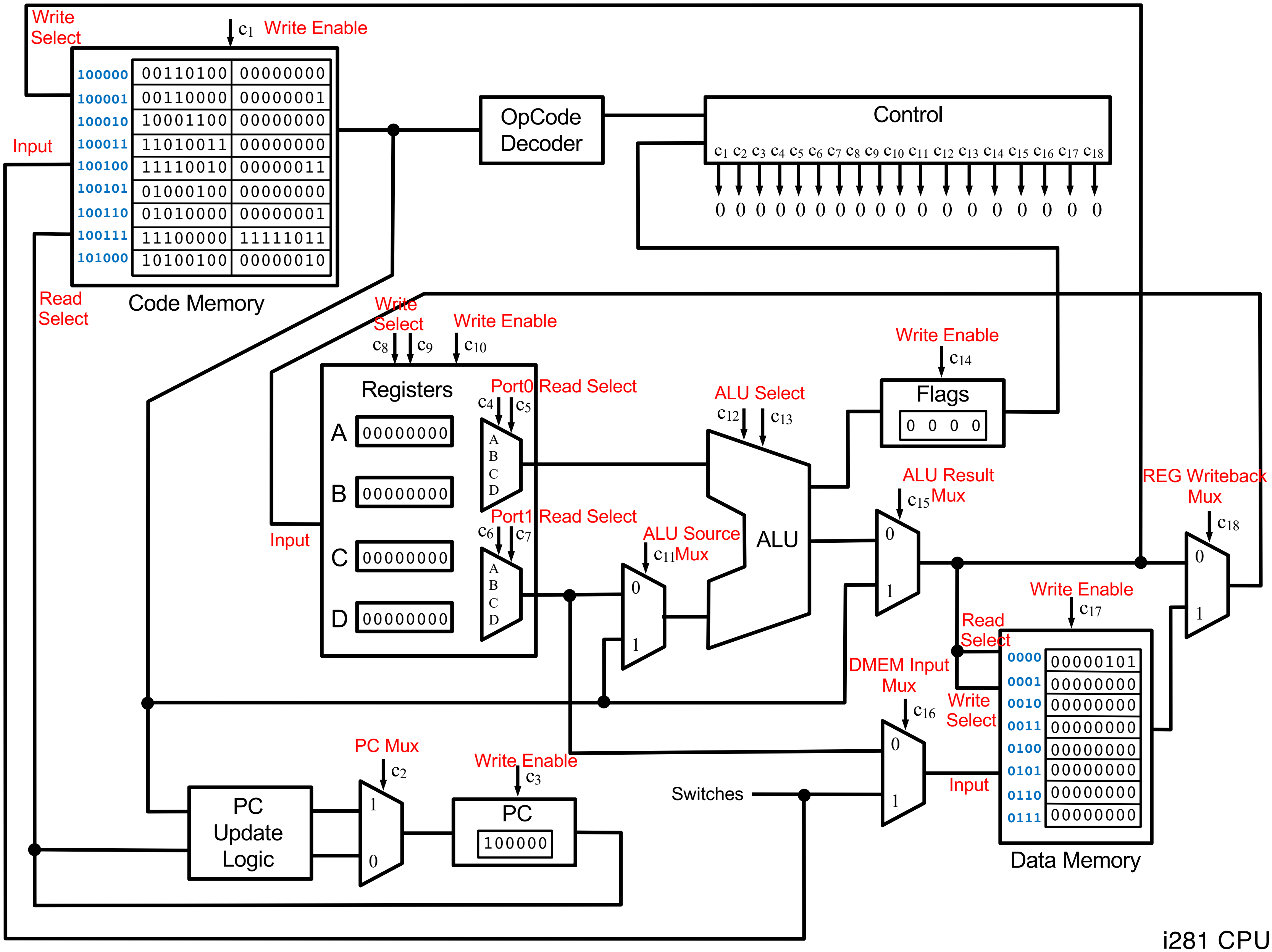
[Figure 4.51 from the textbook]

Arithmetic Logic Unit (ALU)

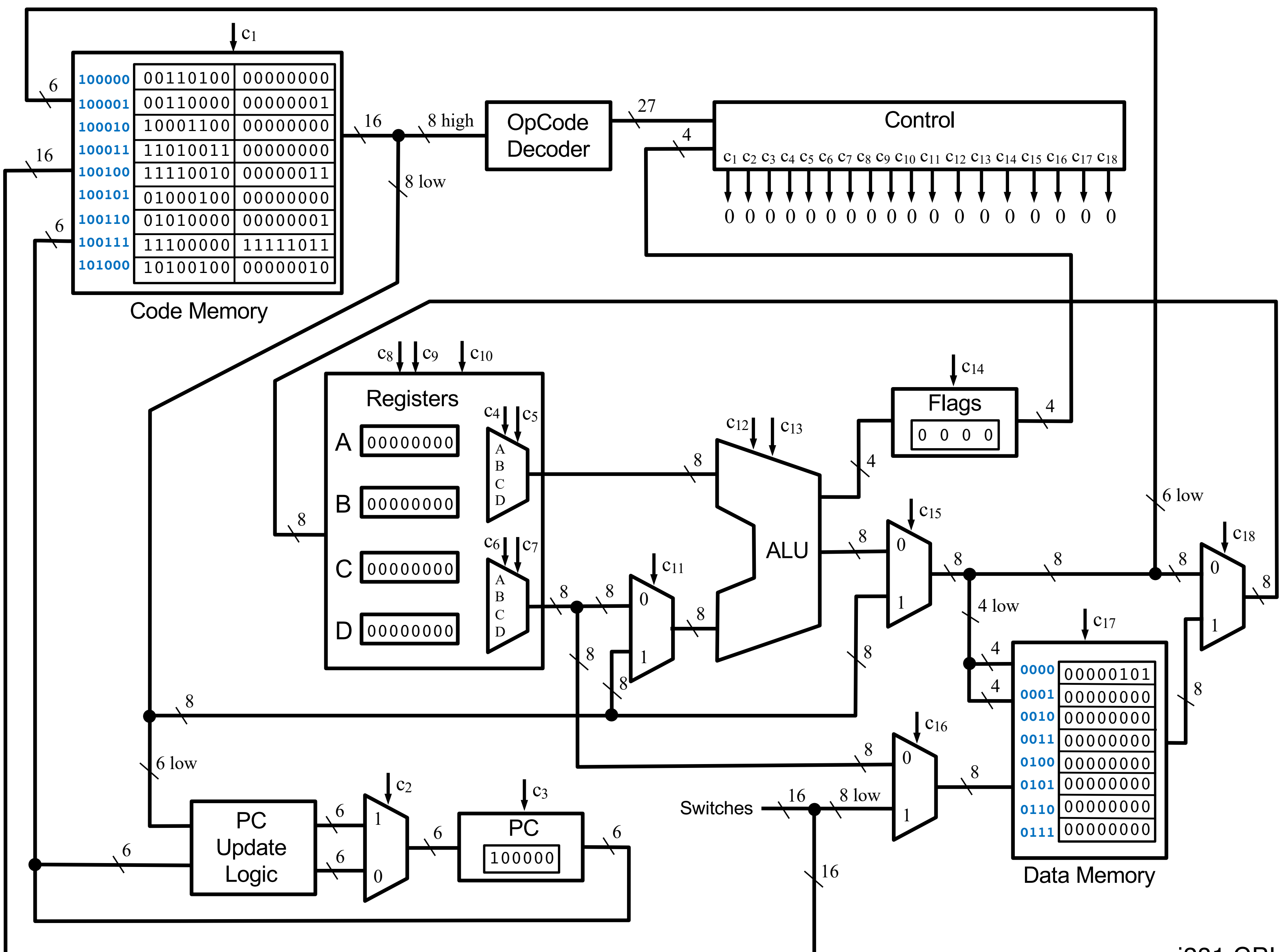
Shifter Circuit
of the i281 CPU


c1
100000 00110100 00000000
100001 00110000 00000001
100010 10001100 00000000
100011 11010011 00000000
100100 11110010 00000011
100101 01000100 00000000
100110 01010000 00000001
100111 11100000 11111011
101000 10100100 00000010
Code Memory
OpCode Decoder
Control
c1 c2 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14 c15 c16 c17 c18
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
c8 c9 c10
Registers
A 00000000
B 00000000
C 00000000
D 00000000
c4 c5
c6 c7
A B C D
A B C D
c11
0
1
c12 c13
ALU
c14
Flags
0 0 0 0
c15
0
1
c18
0
1
c17
0000 00000101
0001 00000000
0010 00000000
0011 00000000
0100 00000000
0101 00000000
0110 00000000
0111 00000000
Data Memory
c16
0
1
Switches
c2
1
0
c3
PC
100000
PC Update Logic

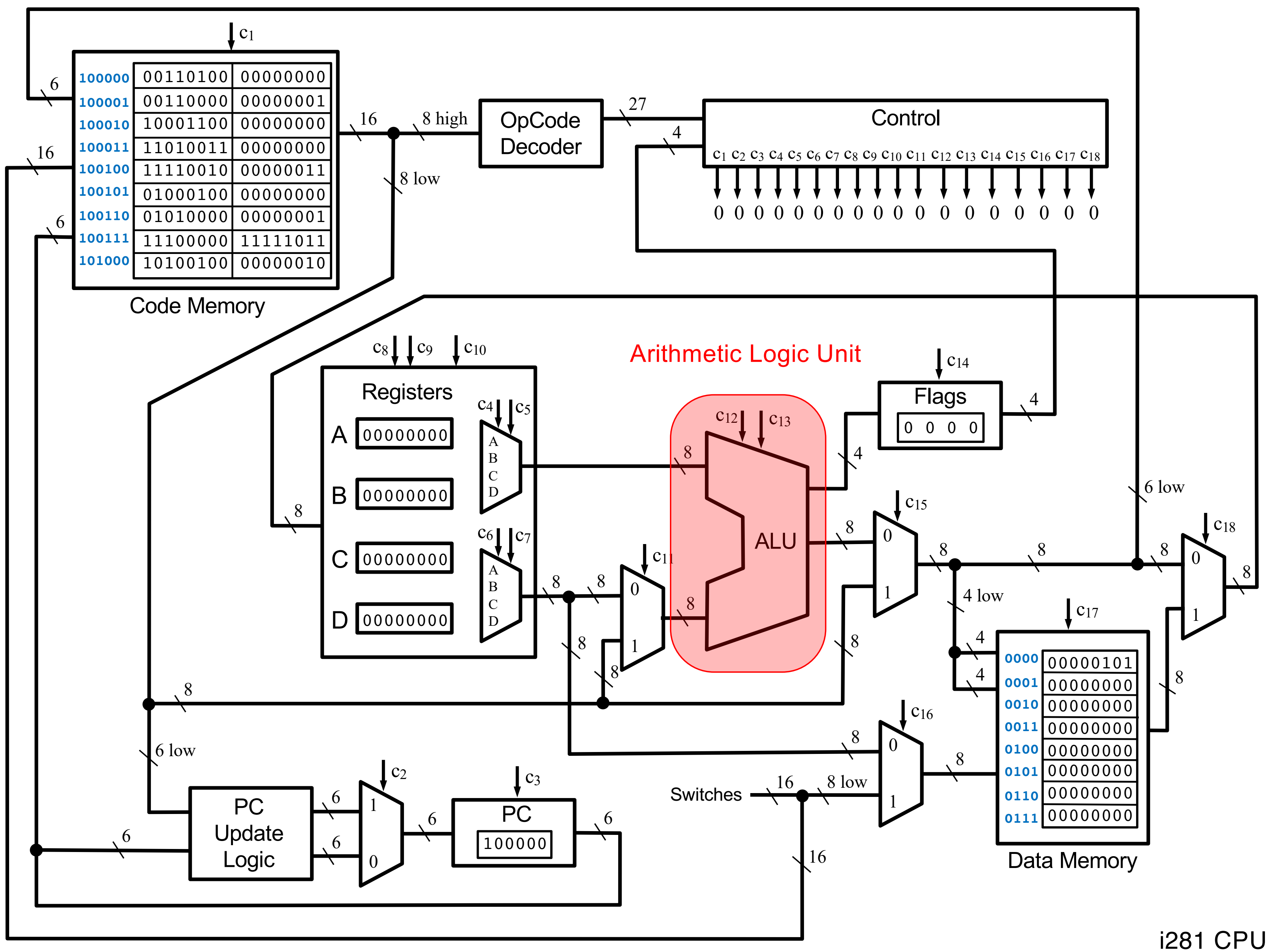
i281 CPU



Write Select
c1 Write Enable
Input
Read Select
Code Memory
100000 00110100 00000000
100001 00110000 00000001
100010 10001100 00000000
100011 11010011 00000000
100100 11110010 00000011
100101 01000100 00000000
100110 01010000 00000001
100111 11100000 11111011
101000 10100100 00000010
OpCode Decoder
Control
c1 c2 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14 c15 c16 c17 c18
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
Write Select
c8 c9
Write Enable c10
Registers
A 00000000
B 00000000
C 00000000
D 00000000
Input
Port0 Read Select
c4 c5
Port1 Read Select
c6 c7
A B C D
ALU Select
c12 c13
ALU
ALU Source Mux
c11
0 1
Write Enable
c14
Flags
0 0 0 0
ALU Result Mux
c15
REG Writeback Mux
c18
Write Enable
c17
Read Select
Write Select
DMEM Input Mux
c16
Input
Switches
Data Memory
0000 00000101
0001 00000000
0010 00000000
0011 00000000
0100 00000000
0101 00000000
0110 00000000
0111 00000000
PC Mux
c2
PC Update Logic
Write Enable
c3
PC
100000
i281 CPU

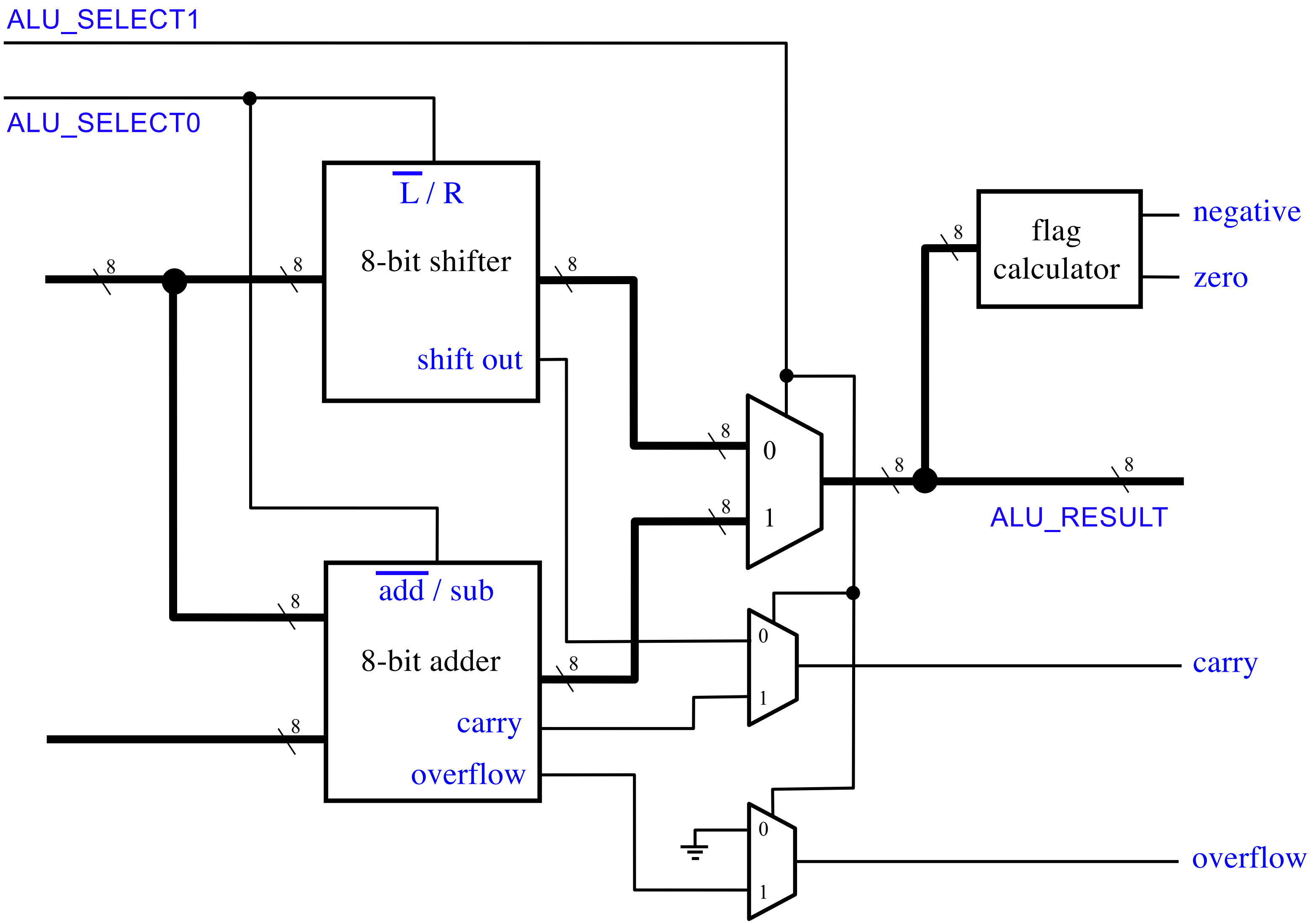

Code Memory
100000 00110100 00000000
100001 00110000 00000001
100010 10001100 00000000
100011 11010011 00000000
100100 11110010 00000011
100101 01000100 00000000
100110 01010000 00000001
100111 11100000 11111011
101000 10100100 00000010
OpCode Decoder
Control
c1 c2 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14 c15 c16 c17 c18
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
27
16
8 high
8 low
6 low
4 low
Registers
A 00000000
B 00000000
C 00000000
D 00000000
ALU
Flags
0 0 0 0
Data Memory
0000 00000101
0001 00000000
0010 00000000
0011 00000000
0100 00000000
0101 00000000
0110 00000000
0111 00000000
Switches
PC Update Logic
PC
100000

i281 CPU

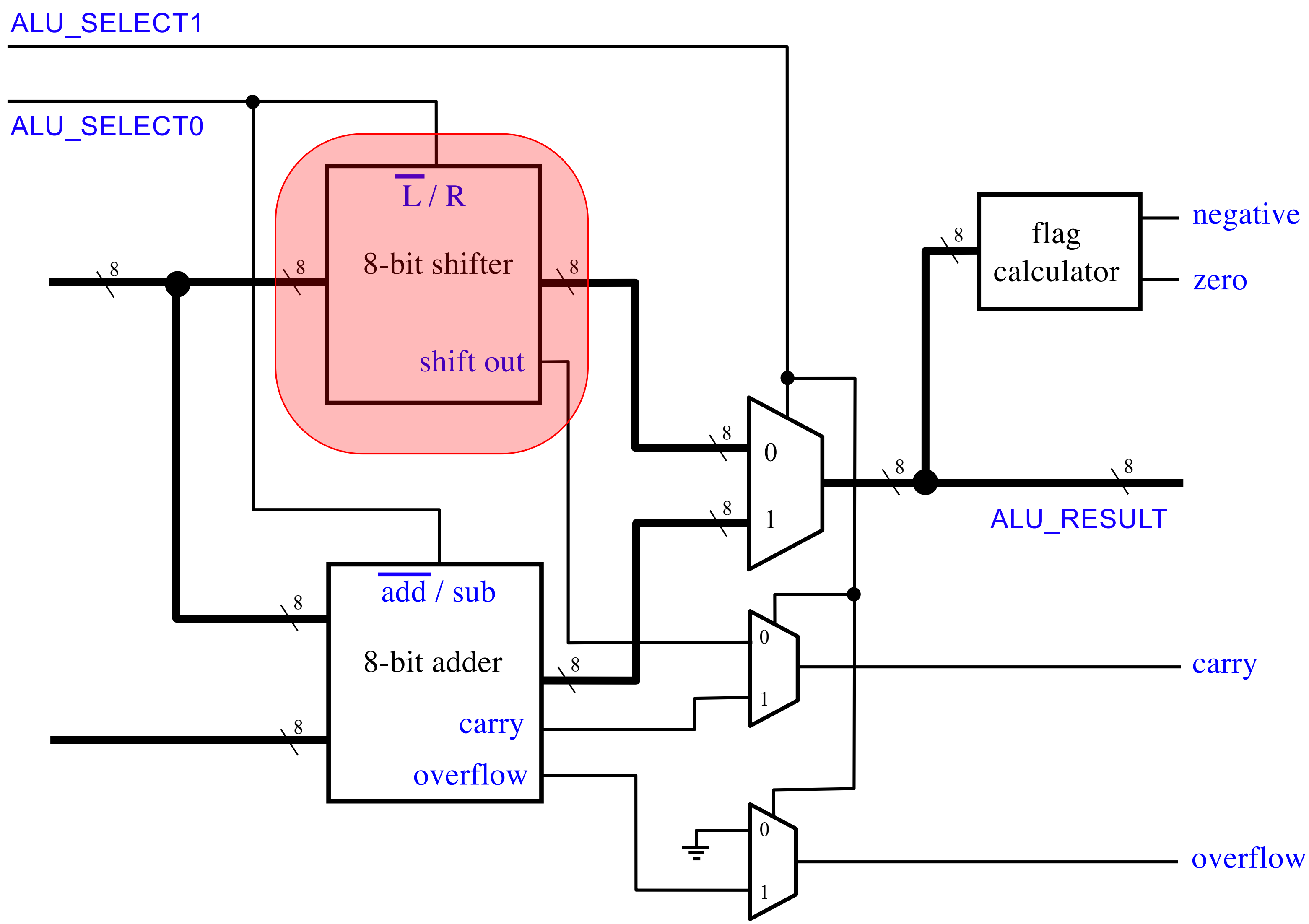


Code Memory
OpCode Decoder
Control
Registers
Arithmetic Logic Unit
ALU
Flags
PC Update Logic
PC
Switches
Data Memory
8 high
8 low
6 low
4 low
i281 CPU

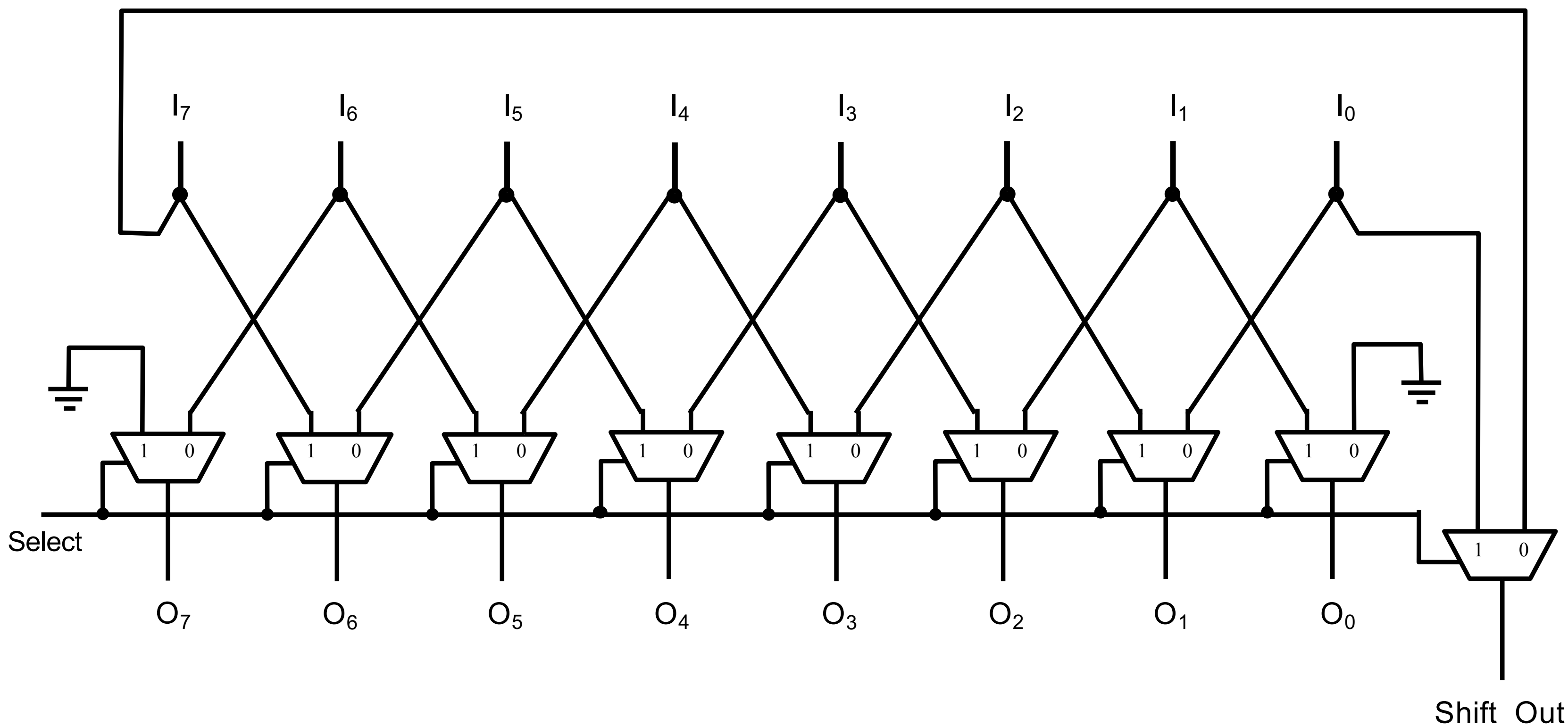
The ALU

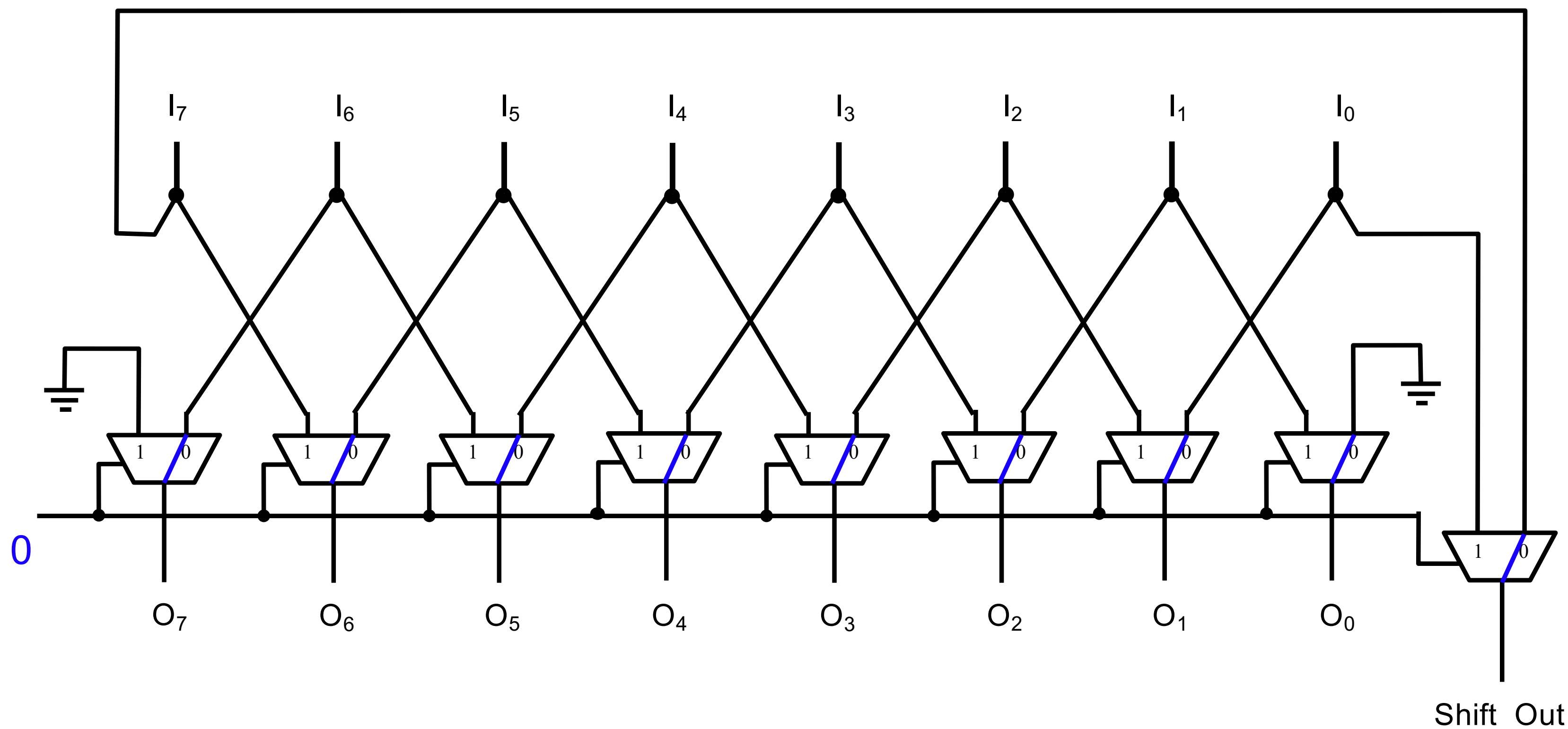

ALU_SELECT1
ALU_SELECT0
L / R
8-bit shifter
shift out
add / sub
8-bit adder
carry
overflow
flag
calculator
negative
zero
0
1
8
ALU_RESULT
carry
overflow

The Shifter Circuit

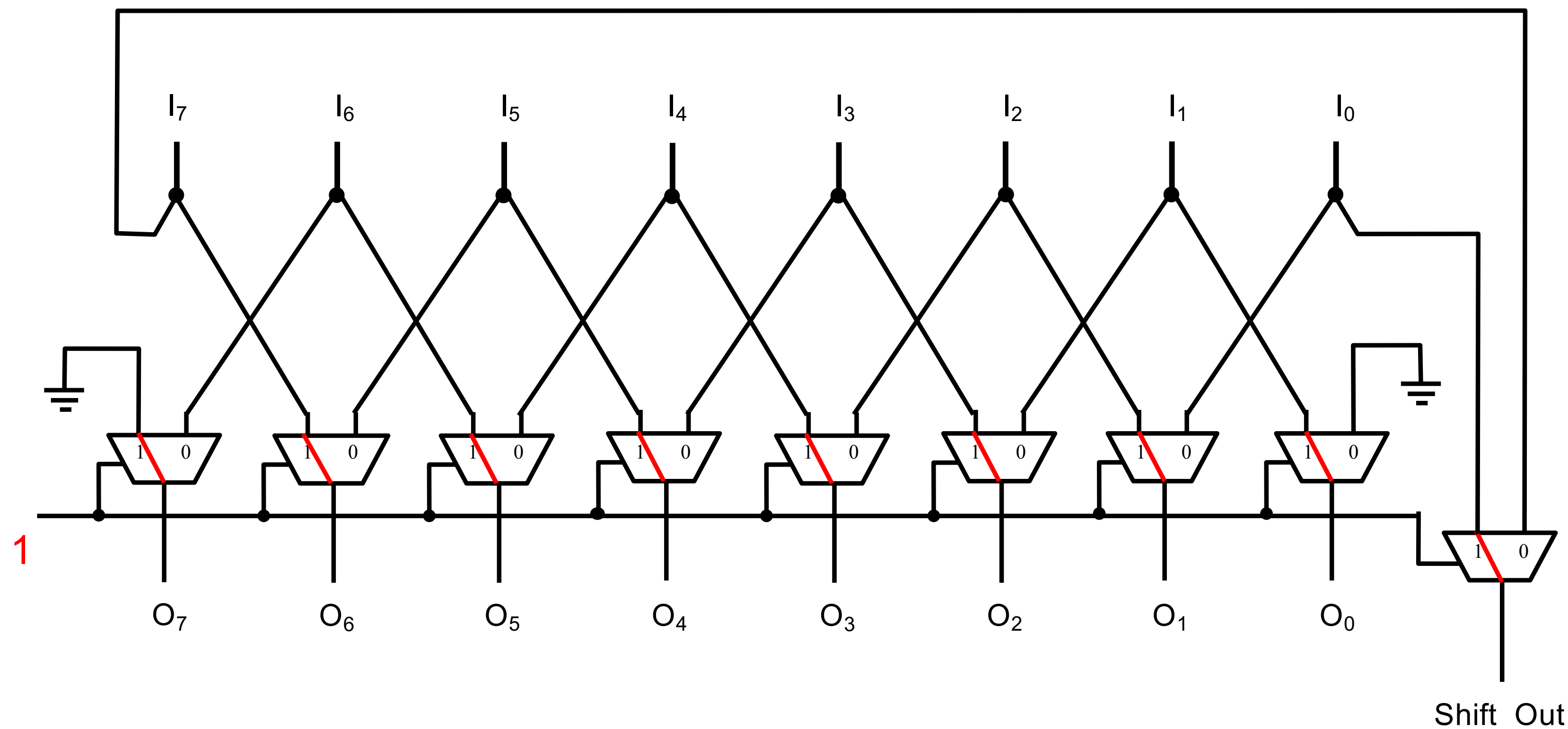


The i281 CPU Shifter Circuit



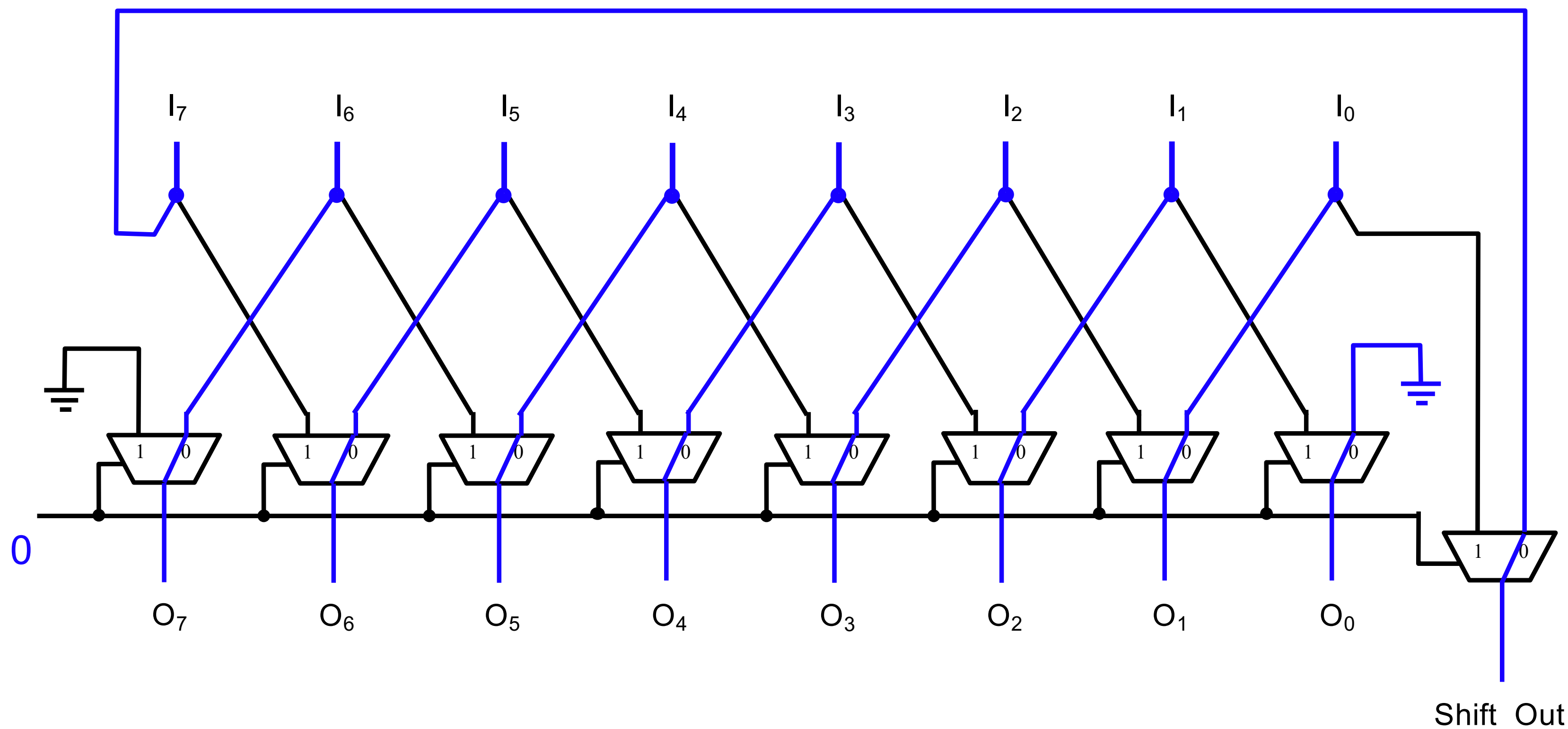


i281 CPU

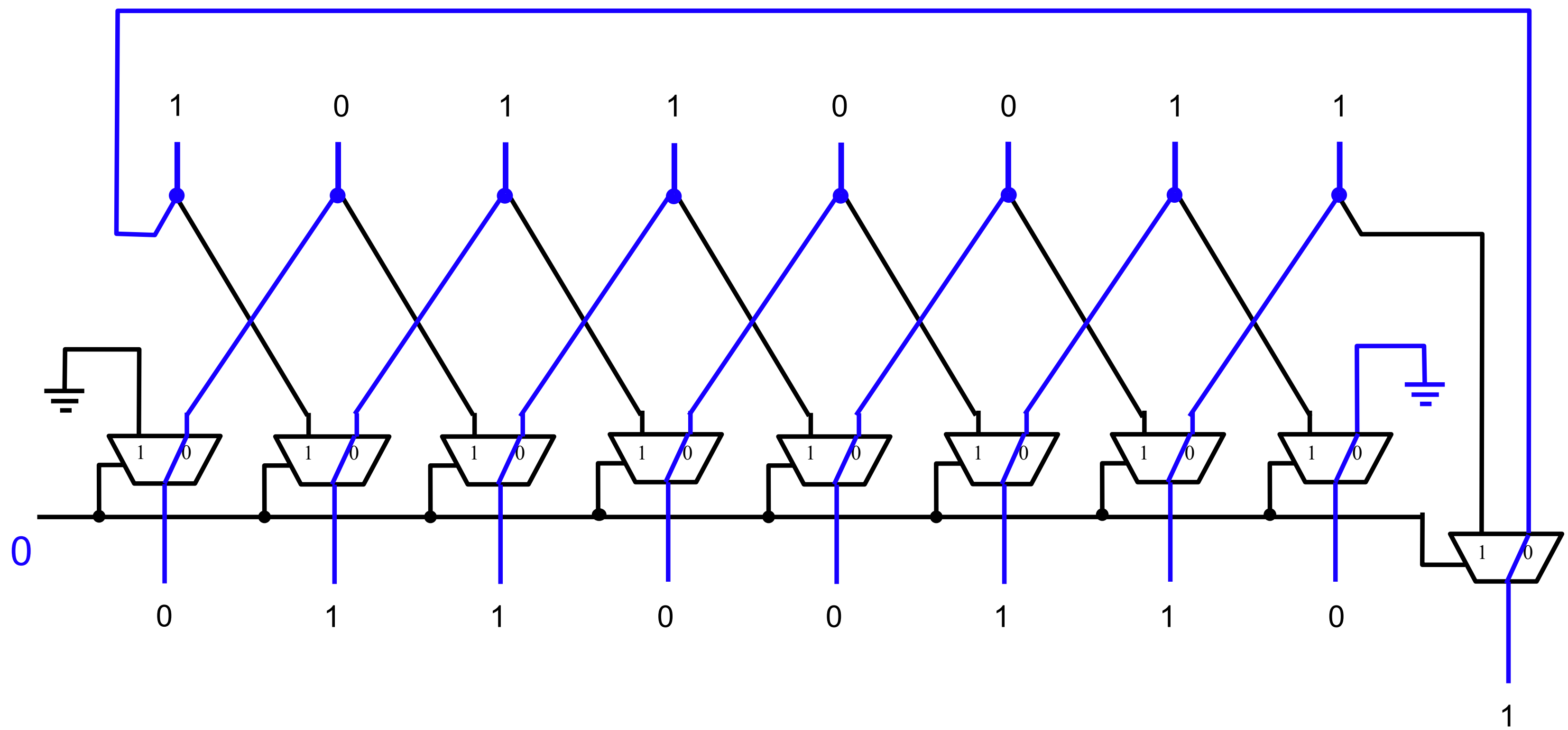


i281 CPU

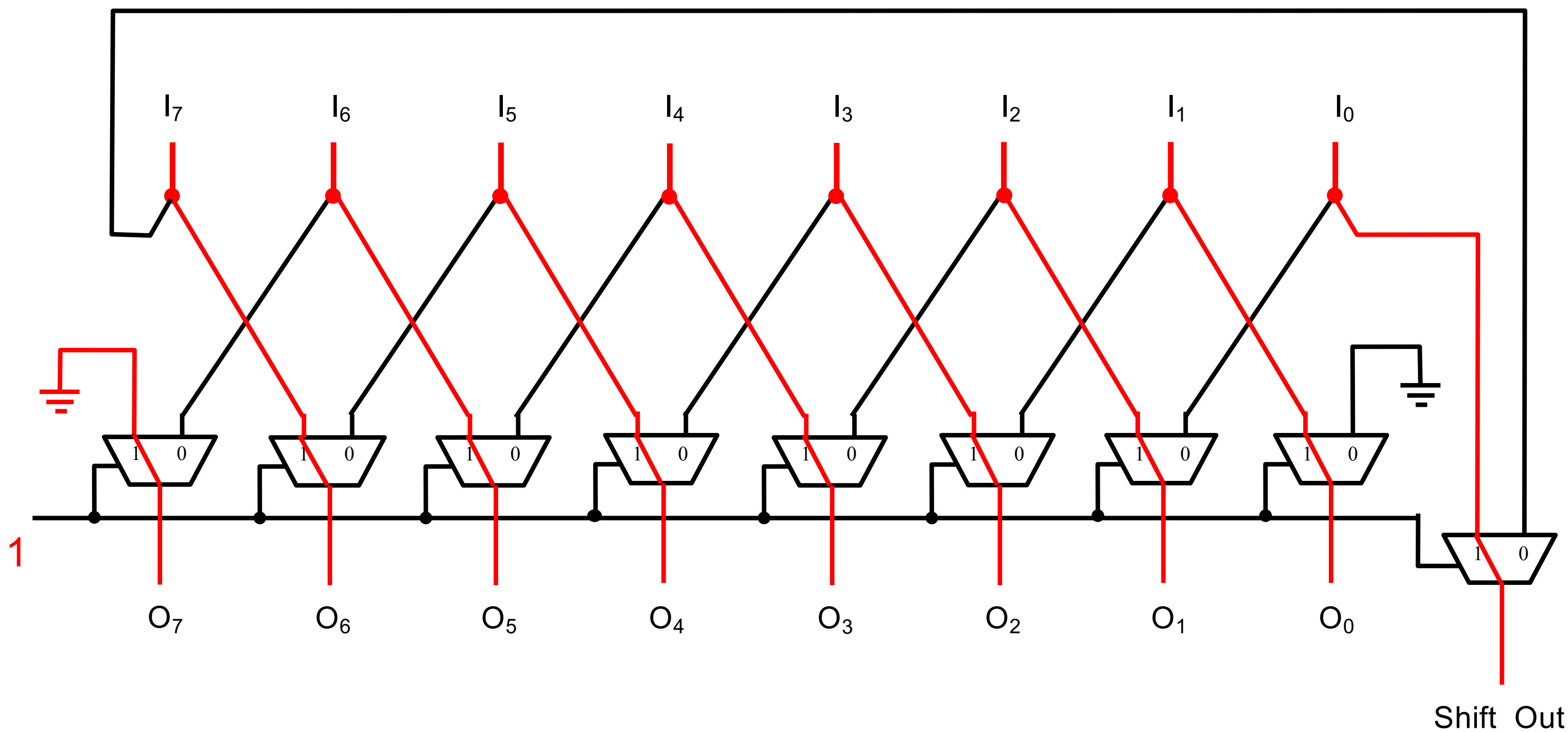
Shift Left



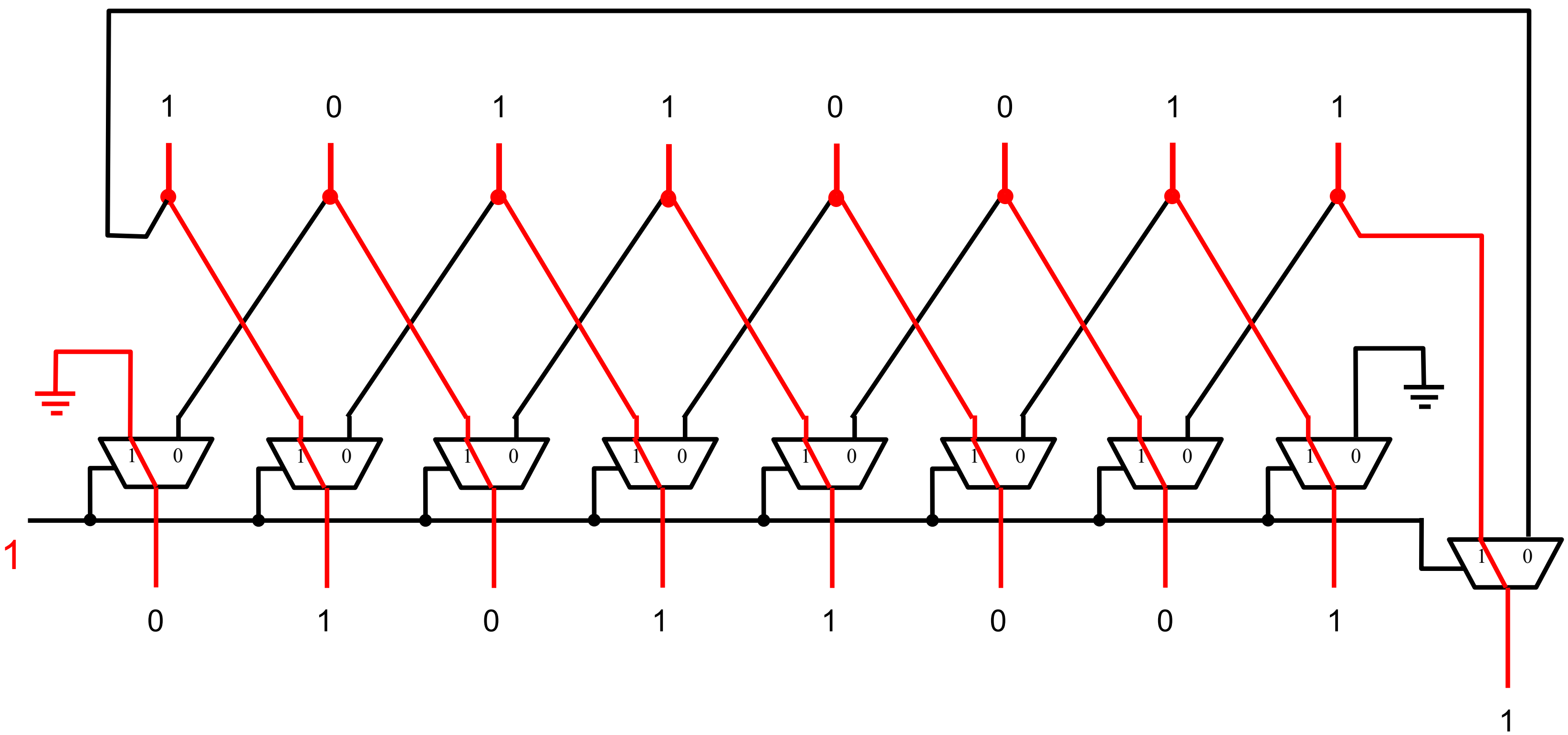
Shift Left



Shift Right



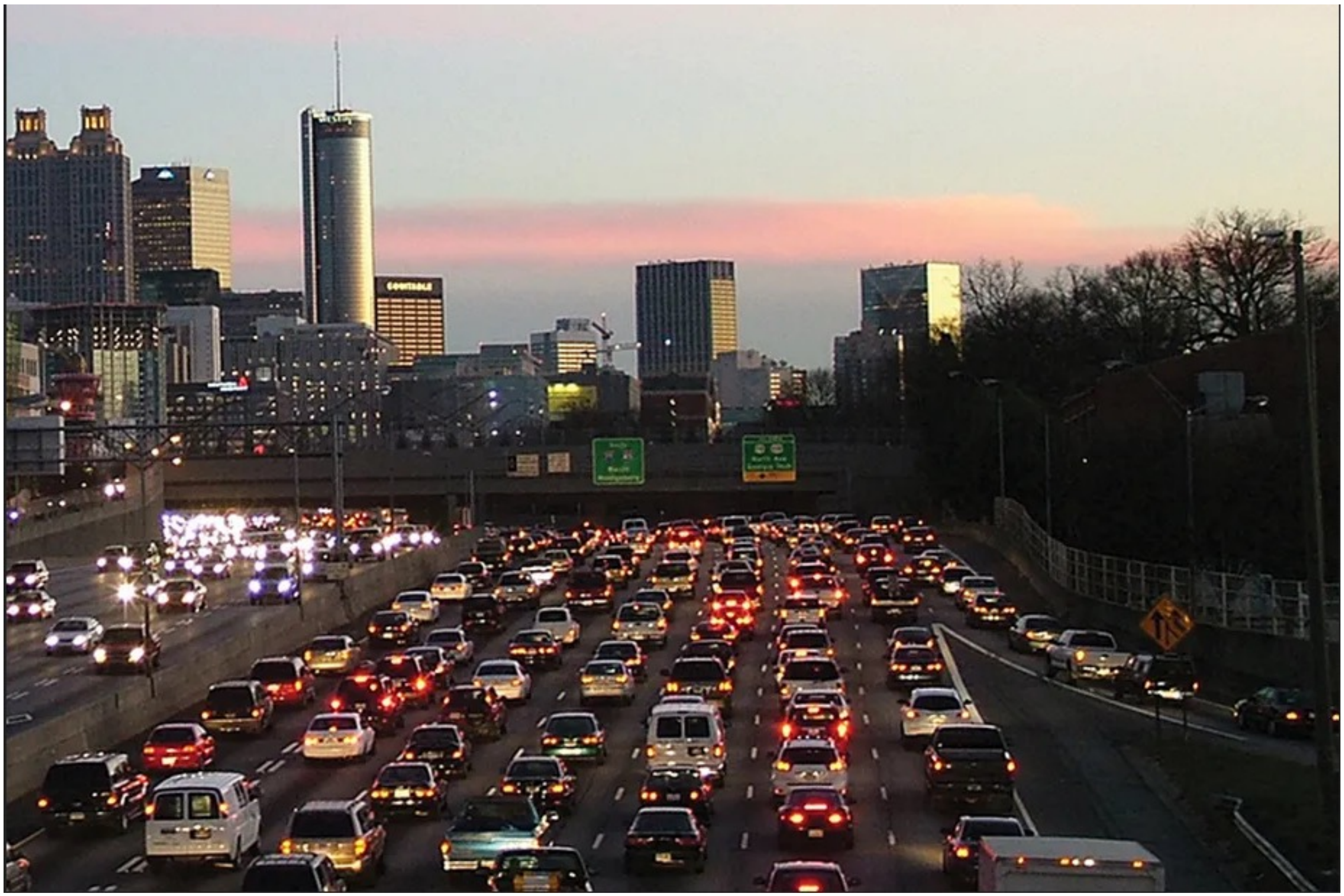
Shift Right



i281 CPU

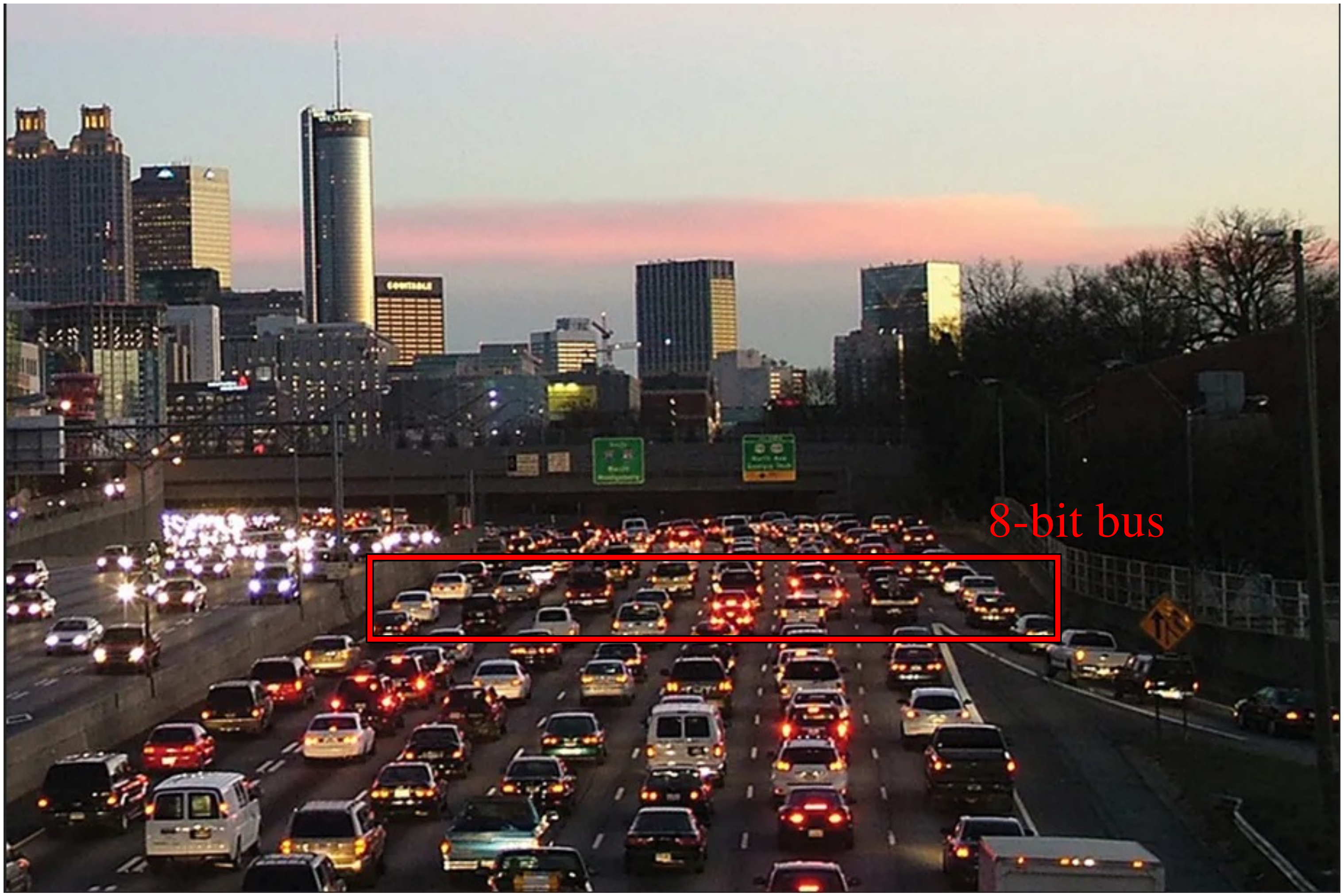
Bus Multiplexer

Bus Analogy



[https://atlanta.curbed.com/2018/10/8/17949734/study-evening-rush-hour-slows-traffic-23-percent]

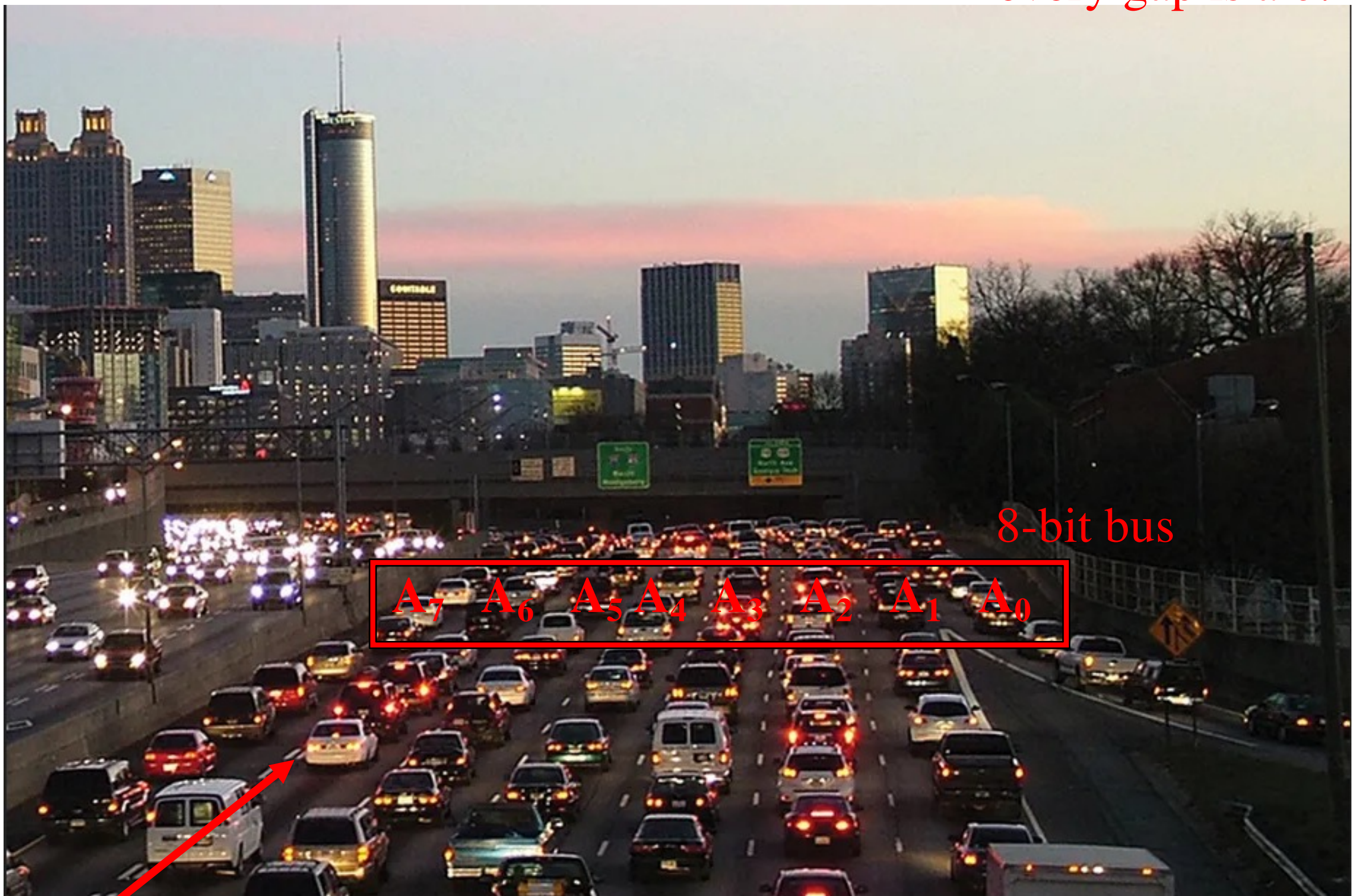
Bus Analogy



[https://atlanta.curbed.com/2018/10/8/17949734/study-evening-rush-hour-slows-traffic-23-percent]

Bus Analogy

For each lane:
every car is a 1,
every gap is a 0.



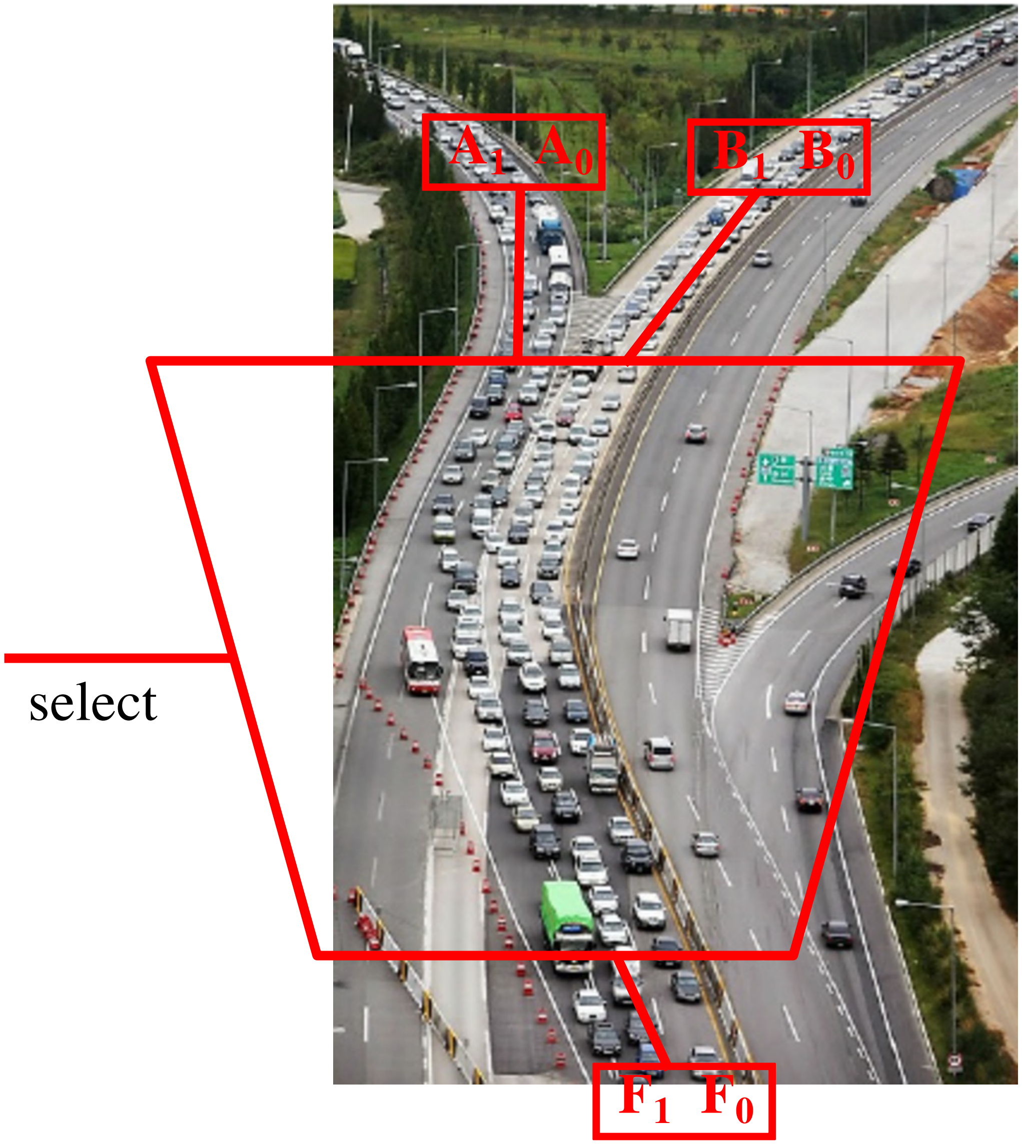
Time

[https://atlanta.curbed.com/2018/10/8/17949734/study-evening-rush-hour-slows-traffic-23-percent]

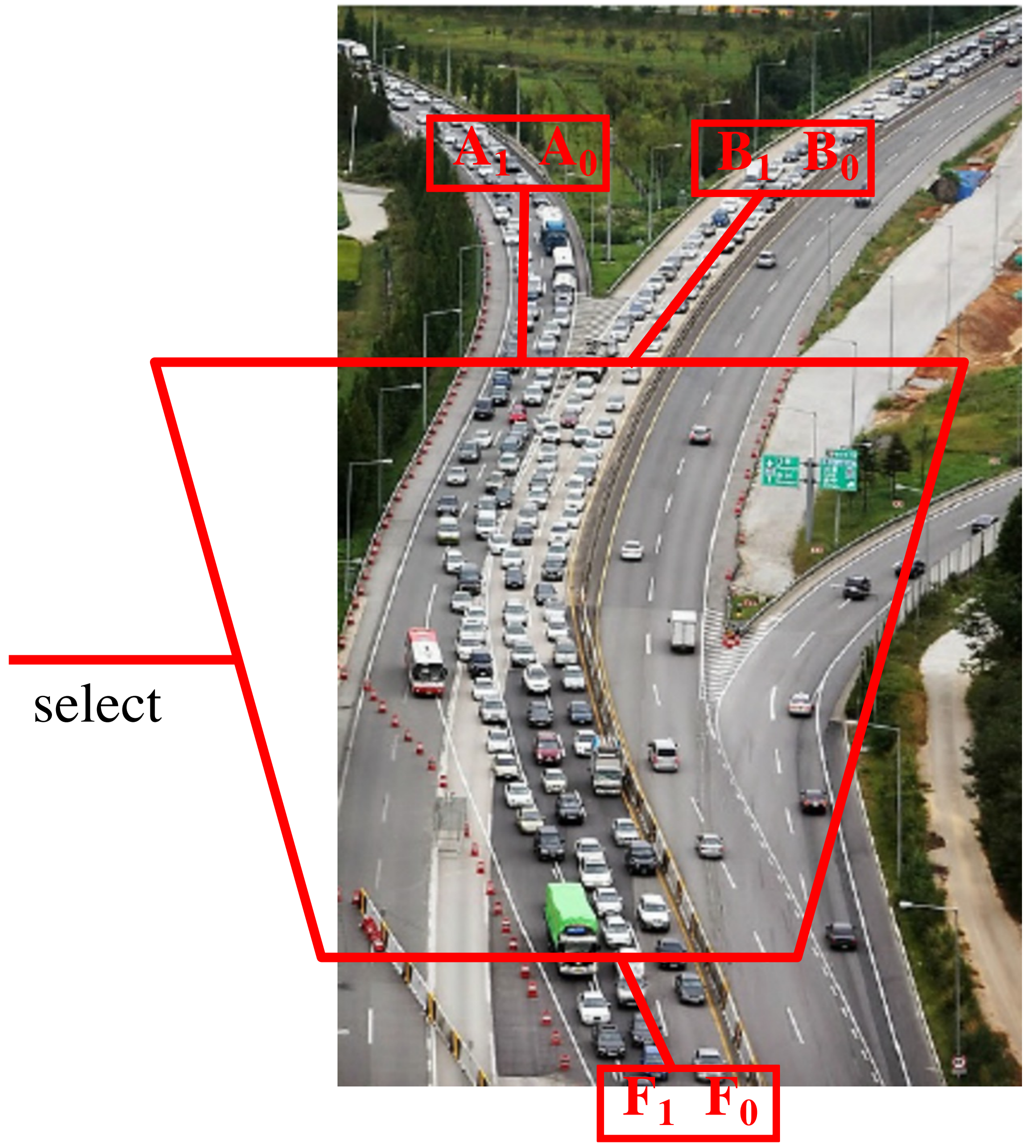
Bus Multiplexer Analogy

[https://www.researchgate.net/figure/An-example-of-traffic-jam-around-a-junction-where-several-lanes-join_fig1_221567960]

Bus Multiplexer Analogy

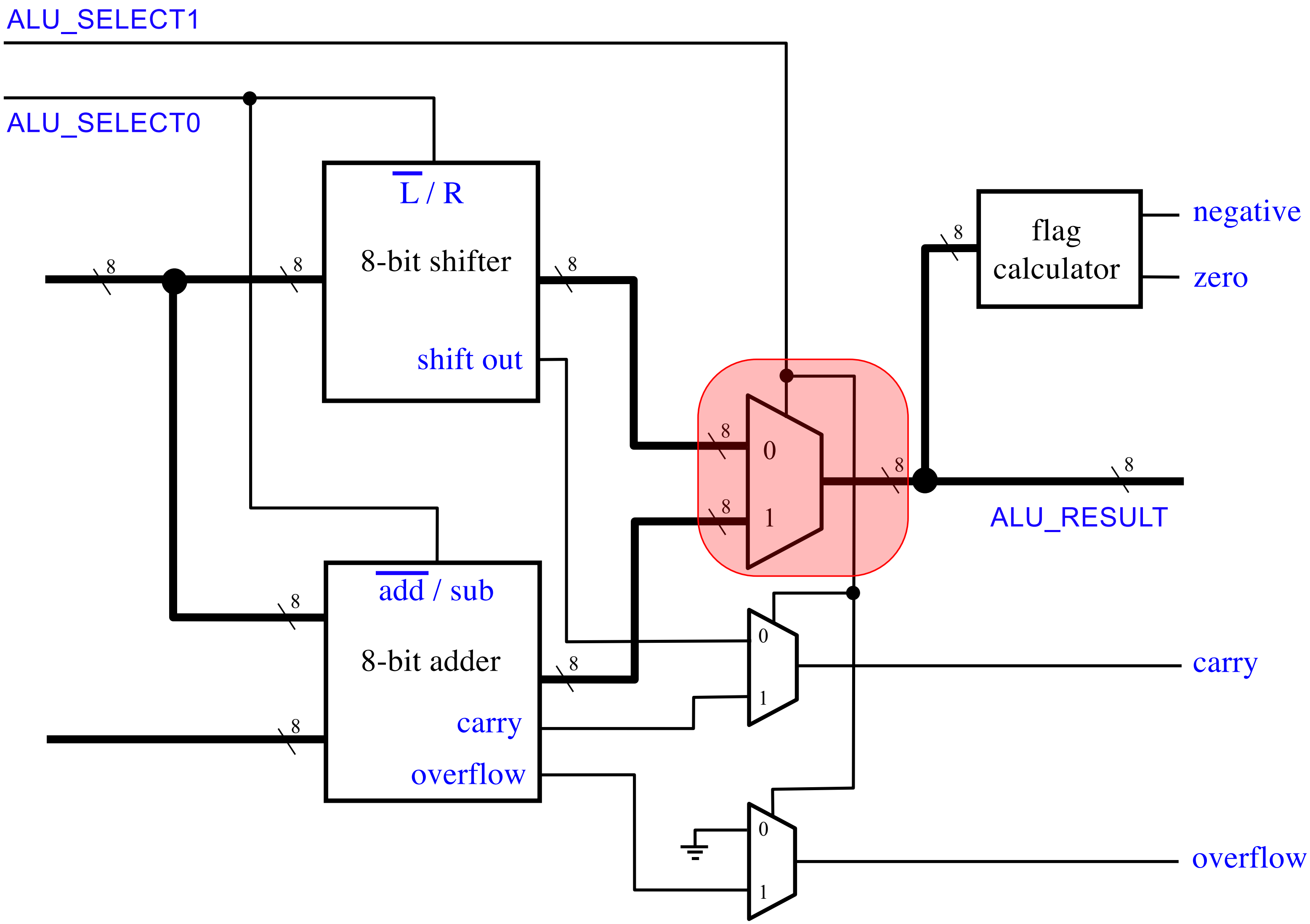


Bus Multiplexer Analogy

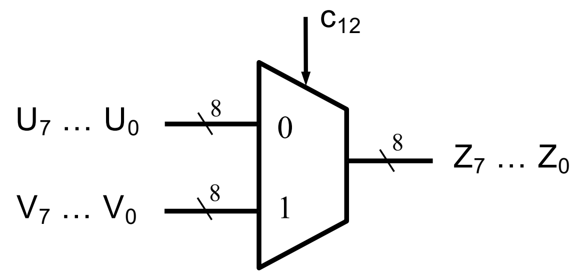


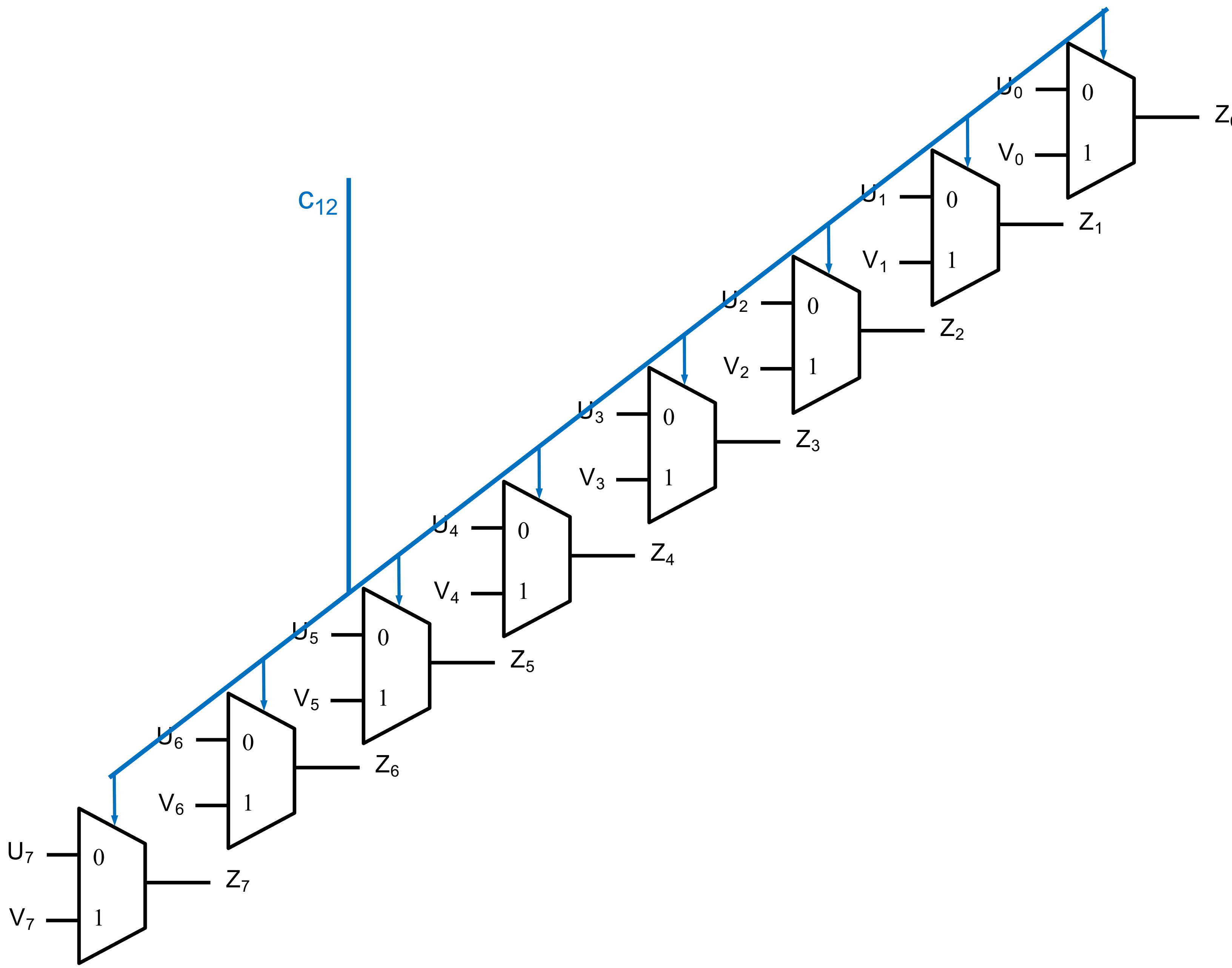
One restriction of digital multiplexers is that they cannot mix bits from A and B when outputting F. At each time step it is only A or only B.

The Internal ALU Bus Multiplexer

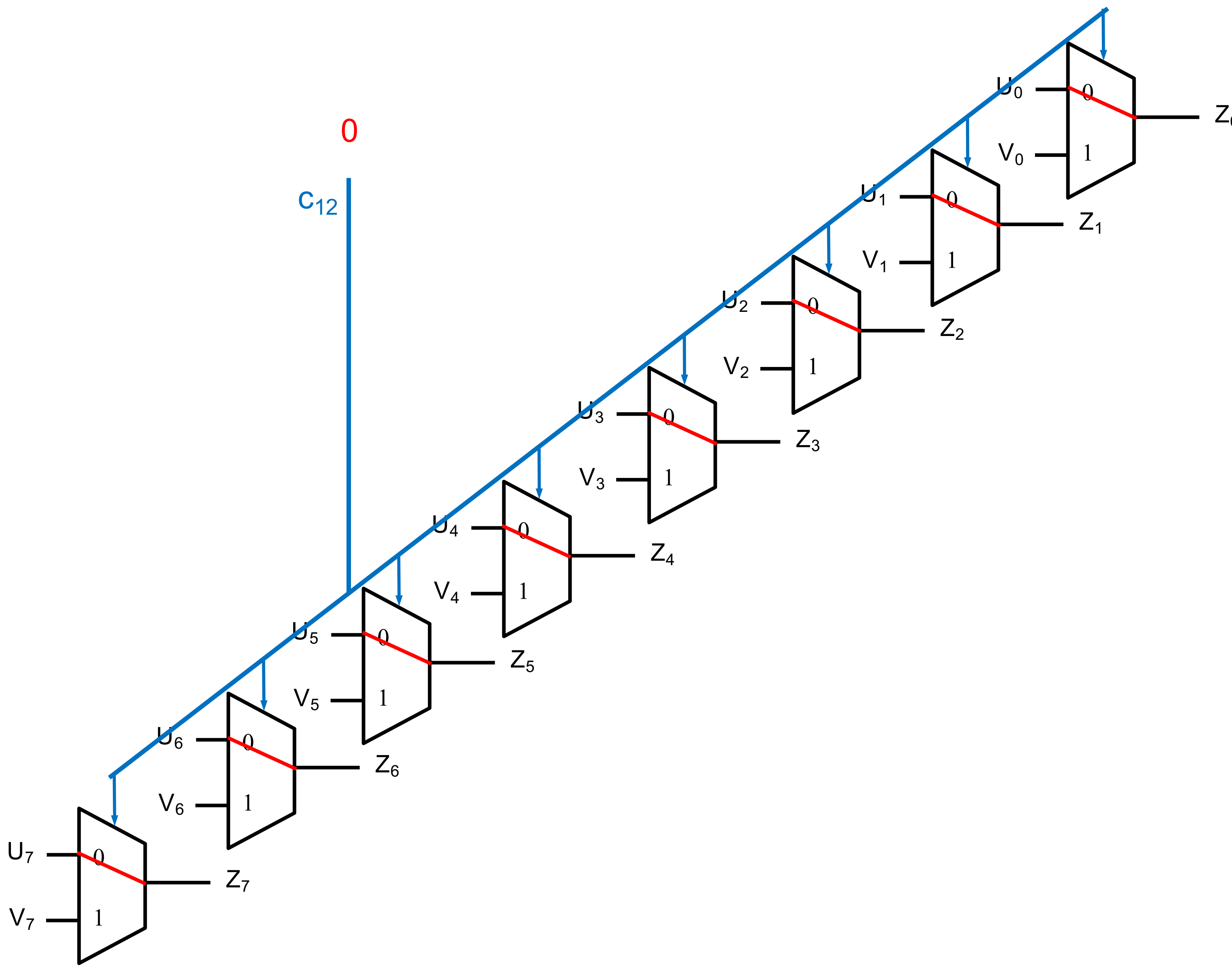


2-to-1 Bus Multiplexer (with 8-bit lines)

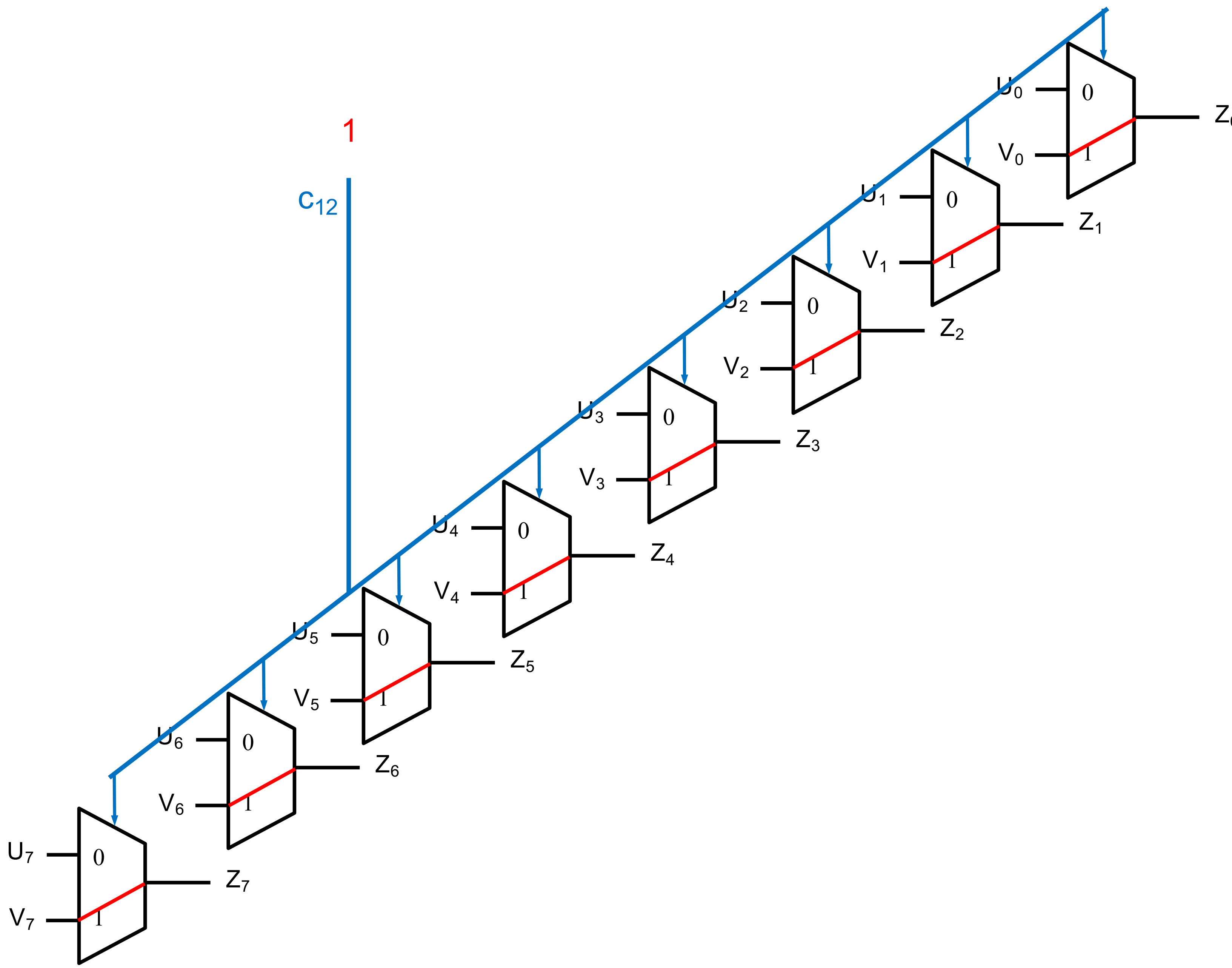




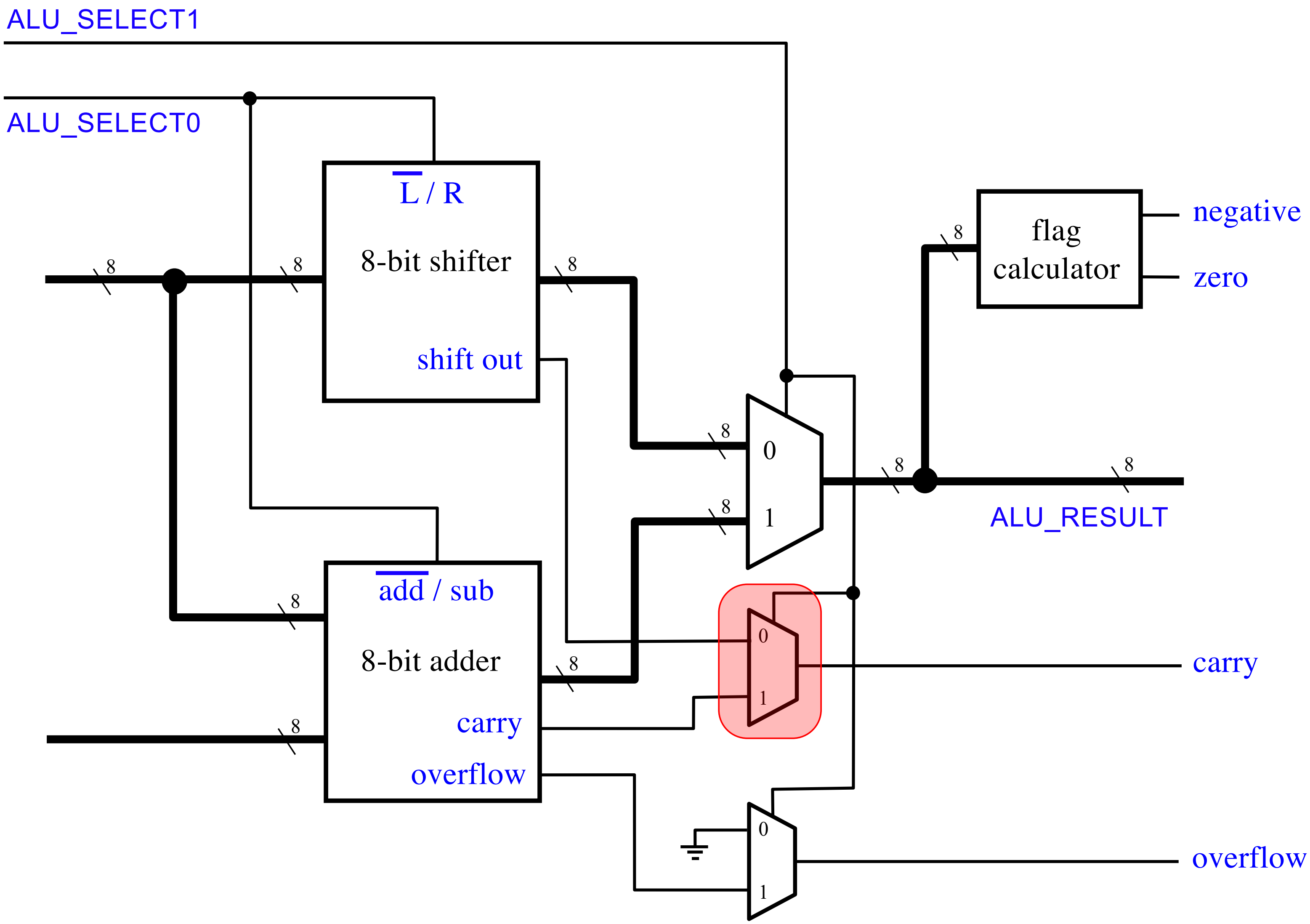
c12
U0
V0
0
1
Z0
U1
V1
Z1
U2
V2
Z2
U3
V3
Z3
U4
V4
Z4
U5
V5
Z5
U6
V6
Z6
U7
V7
Z7



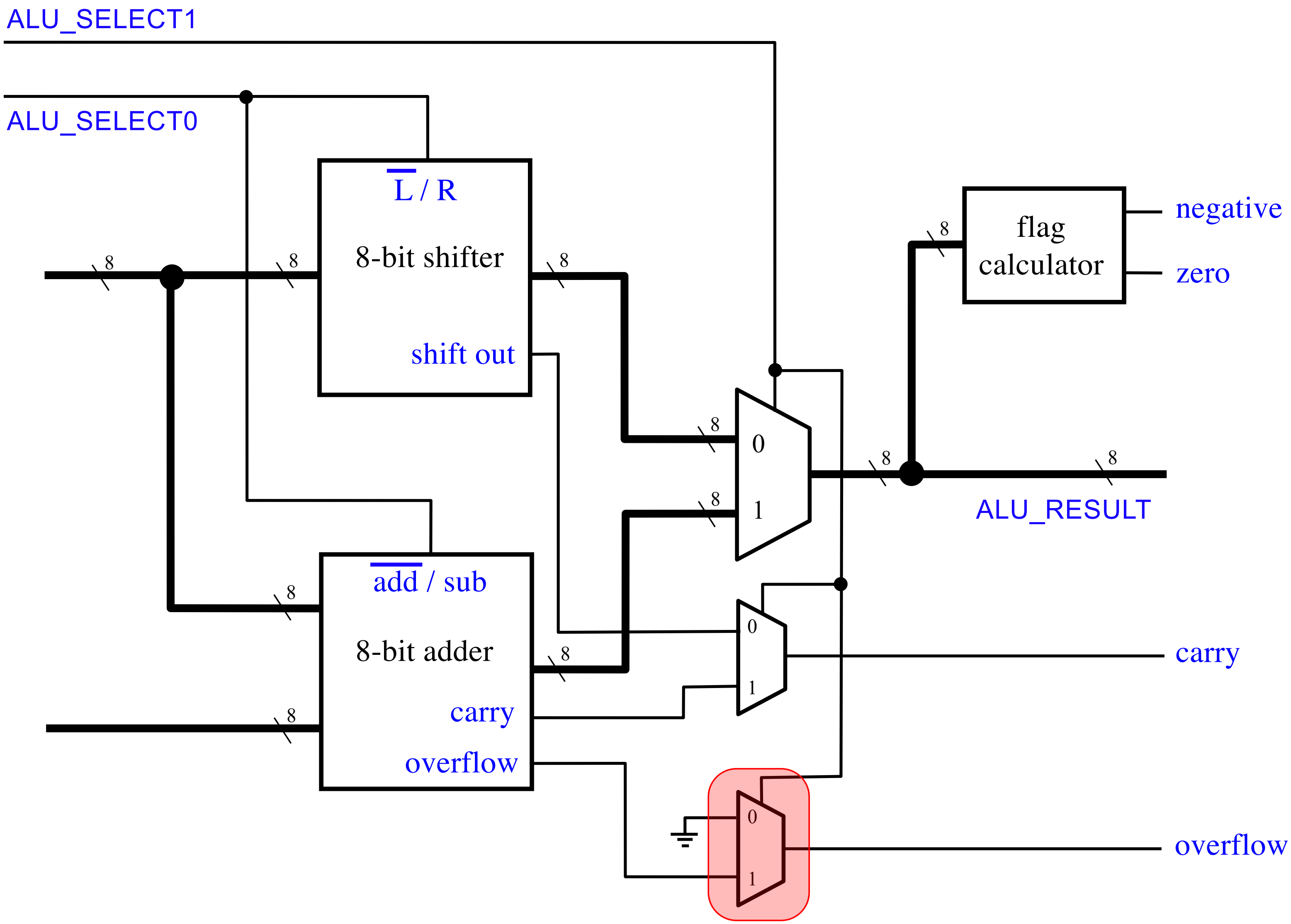
0
c12
U0
V0
Z0
U1
V1
Z1
U2
V2
Z2
U3
V3
Z3
U4
V4
Z4
U5
V5
Z5
U6
V6
Z6
U7
V7
Z7
0
1


1
c12
U0
V0
Z0
U1
V1
Z1
U2
V2
Z2
U3
V3
Z3
U4
V4
Z4
U5
V5
Z5
U6
V6
Z6
U7
V7
Z7

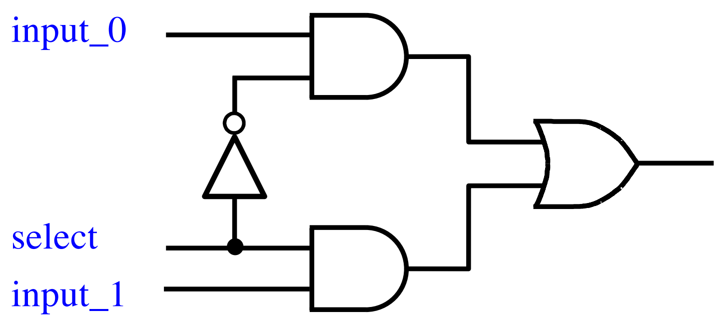
2-to-1 Multiplexer



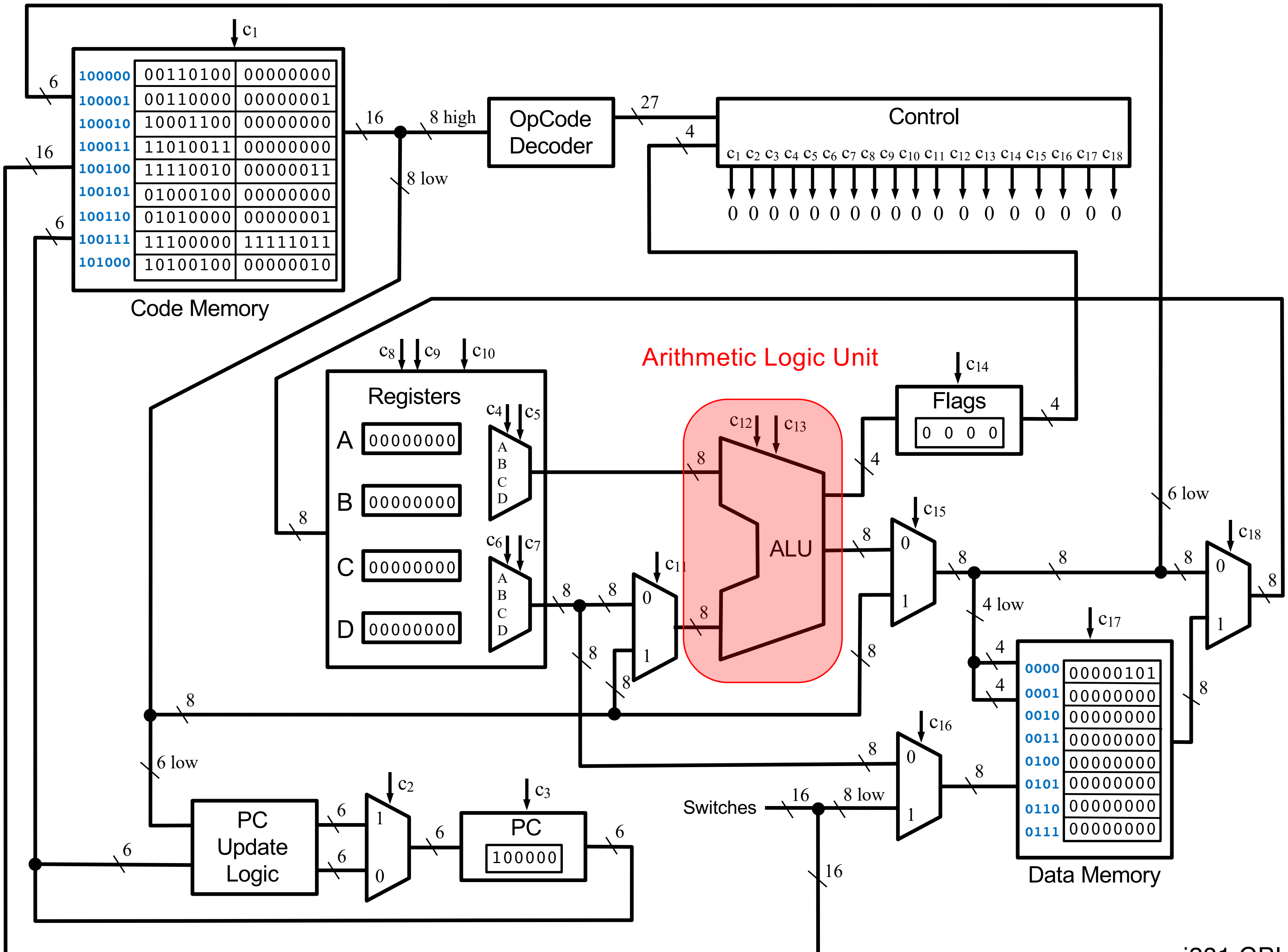
2-to-1 Multiplexer



2-to-1 Multiplexer



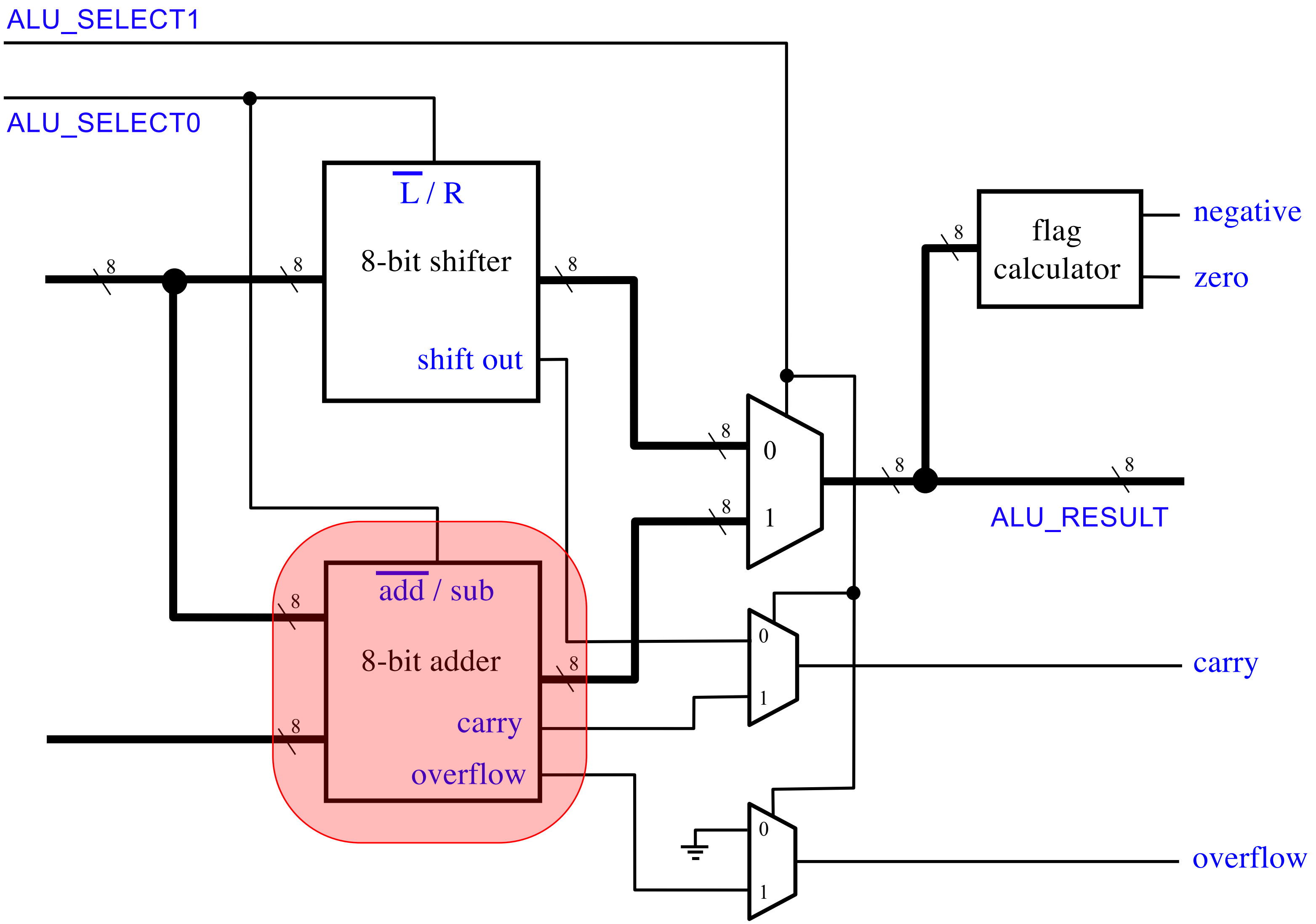
The adder/subtractor of the i281 CPU



c1
Code Memory
100000 00110100 00000000
100001 00110000 00000001
100010 10001100 00000000
100011 11010011 00000000
100100 11110010 00000011
100101 01000100 00000000
100110 01010000 00000001
100111 11100000 11111011
101000 10100100 00000010
16
8 high
8 low
OpCode Decoder
27
Control
c1 c2 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14 c15 c16 c17 c18
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
Registers
c8 c9 c10
A 00000000
B 00000000
C 00000000
D 00000000
c4 c5
c6 c7
Arithmetic Logic Unit
c12 c13
ALU
c11
Flags
0 0 0 0
c14
c15
c18
6 low
4 low
c17
Data Memory
0000 00000101
0001 00000000
0010 00000000
0011 00000000
0100 00000000
0101 00000000
0110 00000000
0111 00000000
c16
Switches
8 low
PC Update Logic
c2
c3
PC
100000
6 low

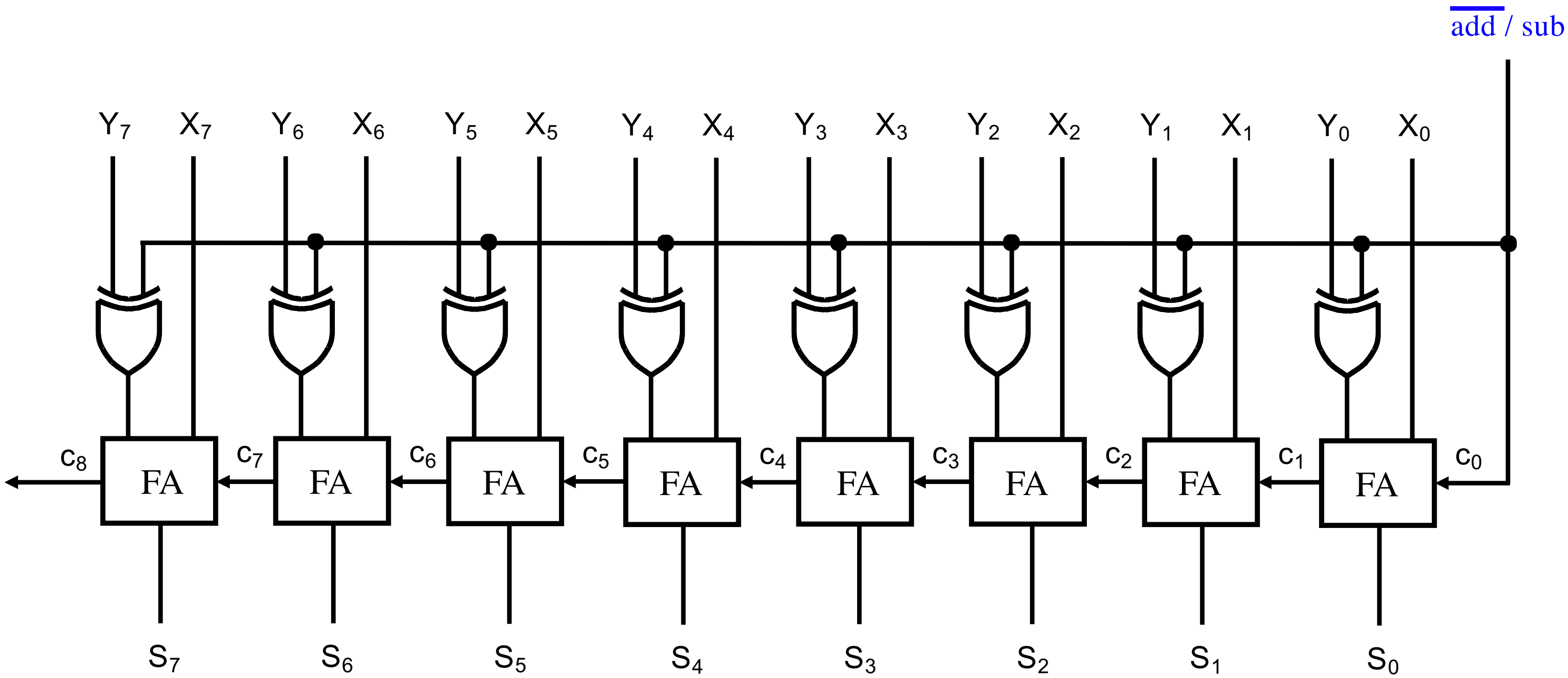
i281 CPU

The Adder / Subtractor



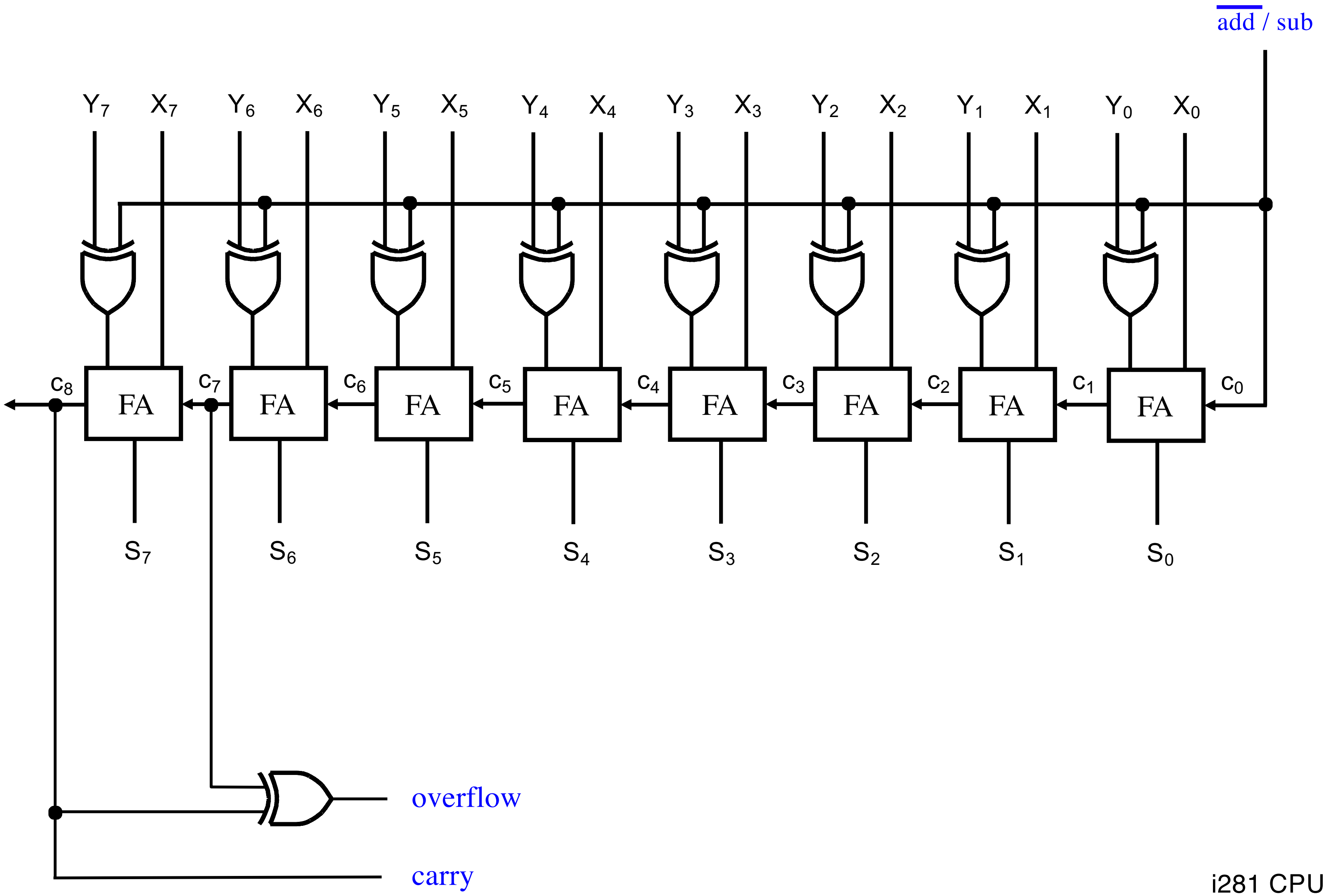
i281 CPU

The Adder / Subtractor

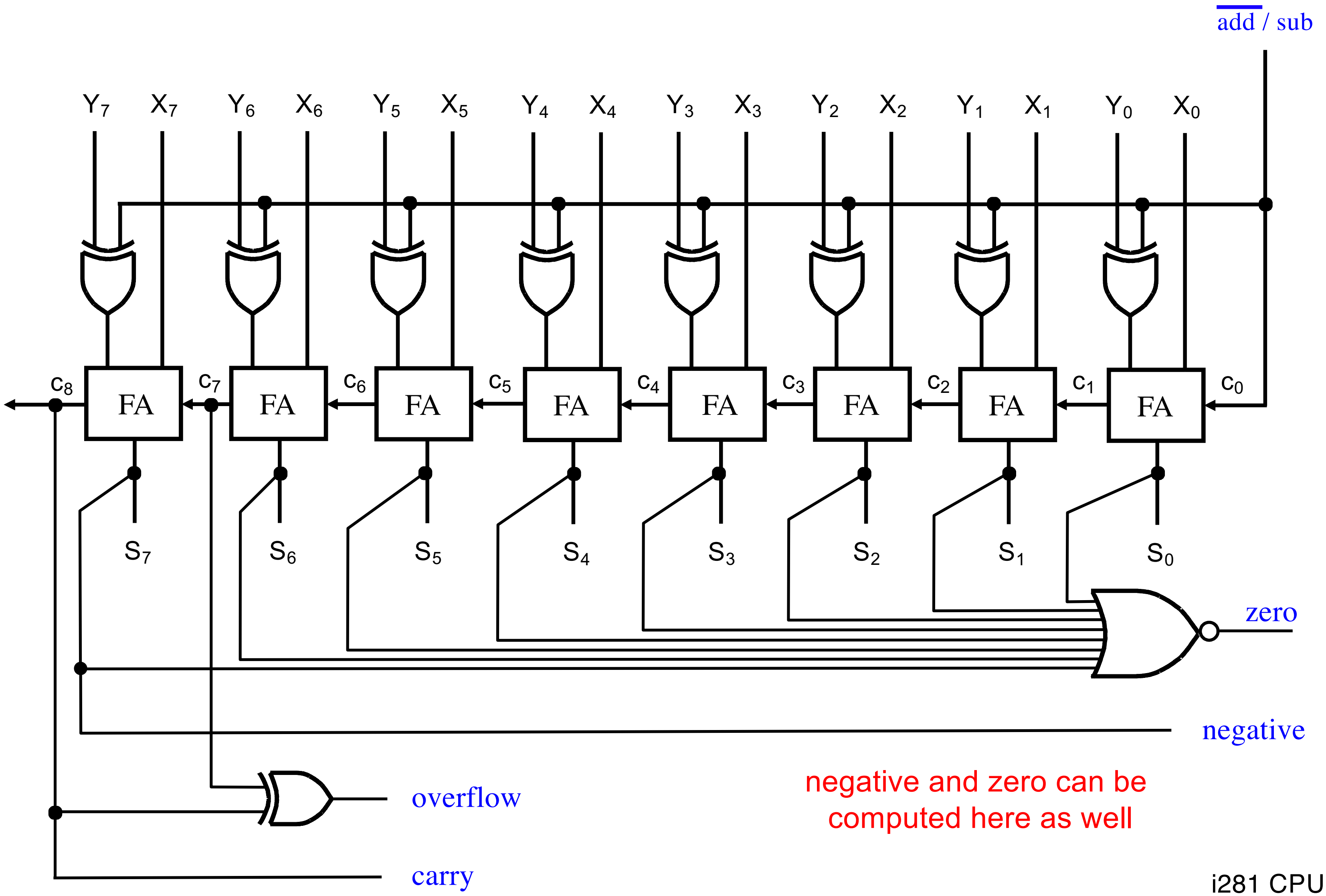


This is an 8-bit ripple-carry adder. Note that the X and Y lines are swapped.

The Adder / Subtractor



The Adder / Subtractor



negative and zero can be computed here as well

Abbreviations for the Flags

- Carry Flag (CF)
- Overflow Flag (OF)
- Negative Flag (NF)
- Zero Flag (ZF)

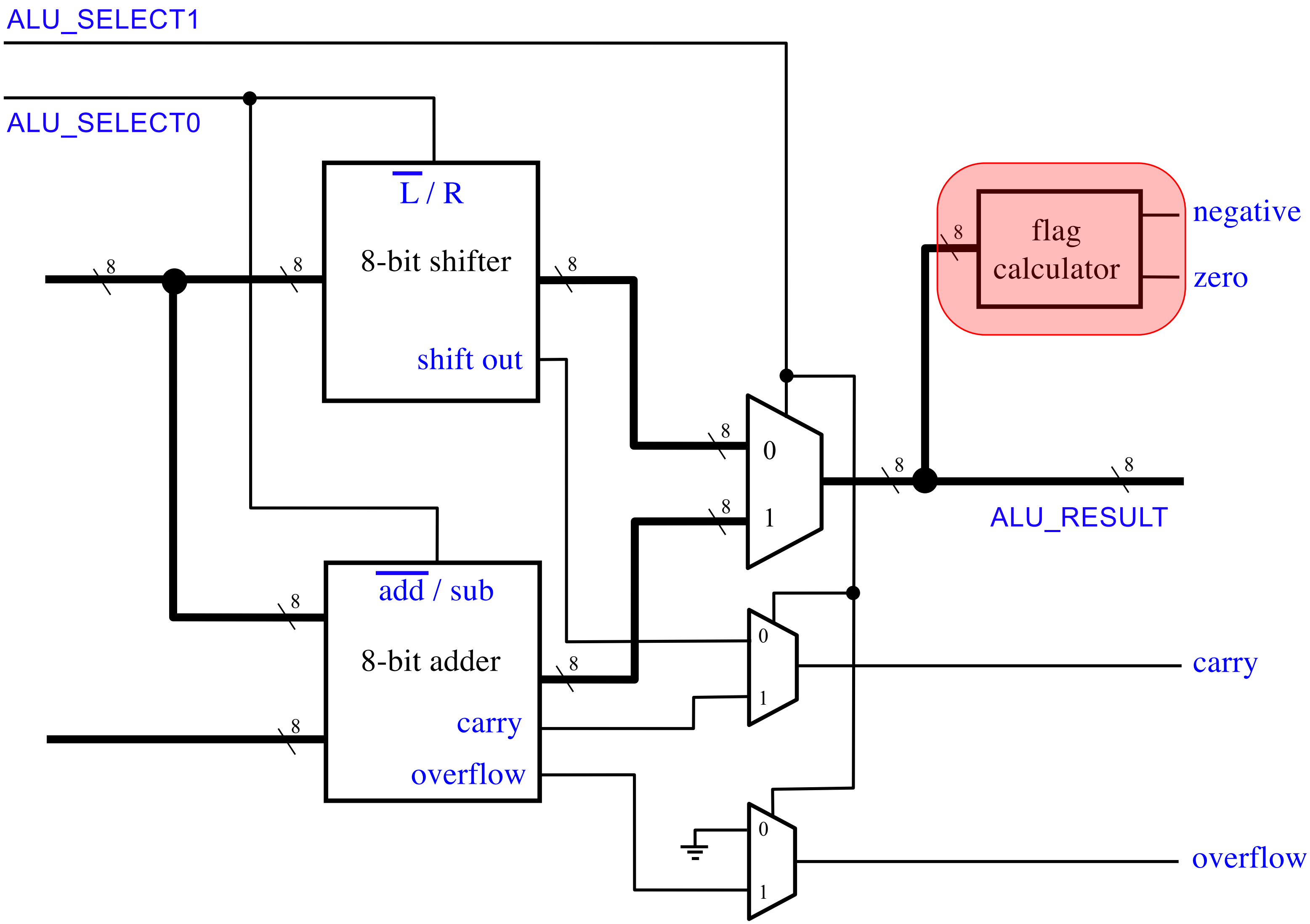
Abbreviations for the Flags

- **Carry Flag (CF)**
- **Overflow Flag (OF)**
- **Negative Flag (NF)**
- **Zero Flag (ZF)**

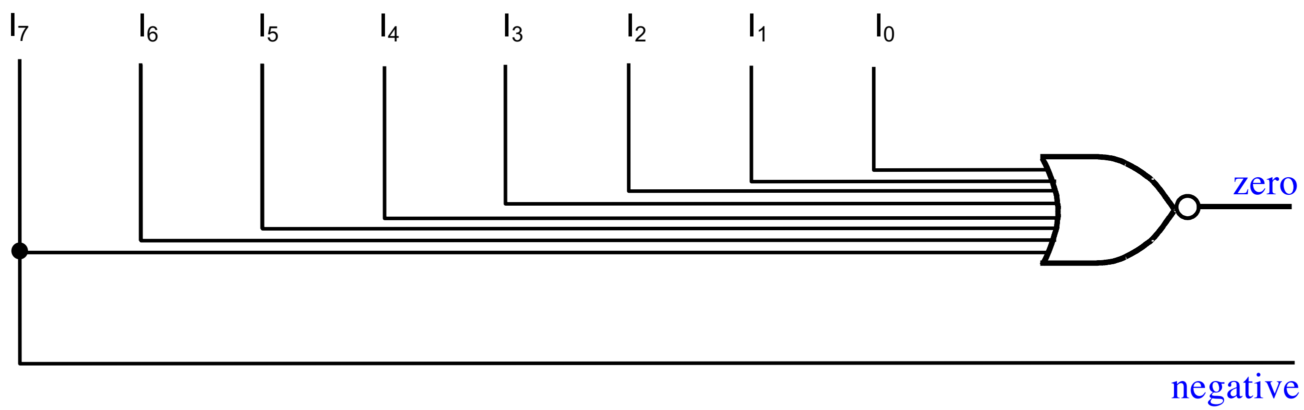
In some CPU architectures the carry flag means borrow. And it could be inverted relative to the previous diagram.

The flag calculator of the i281 CPU

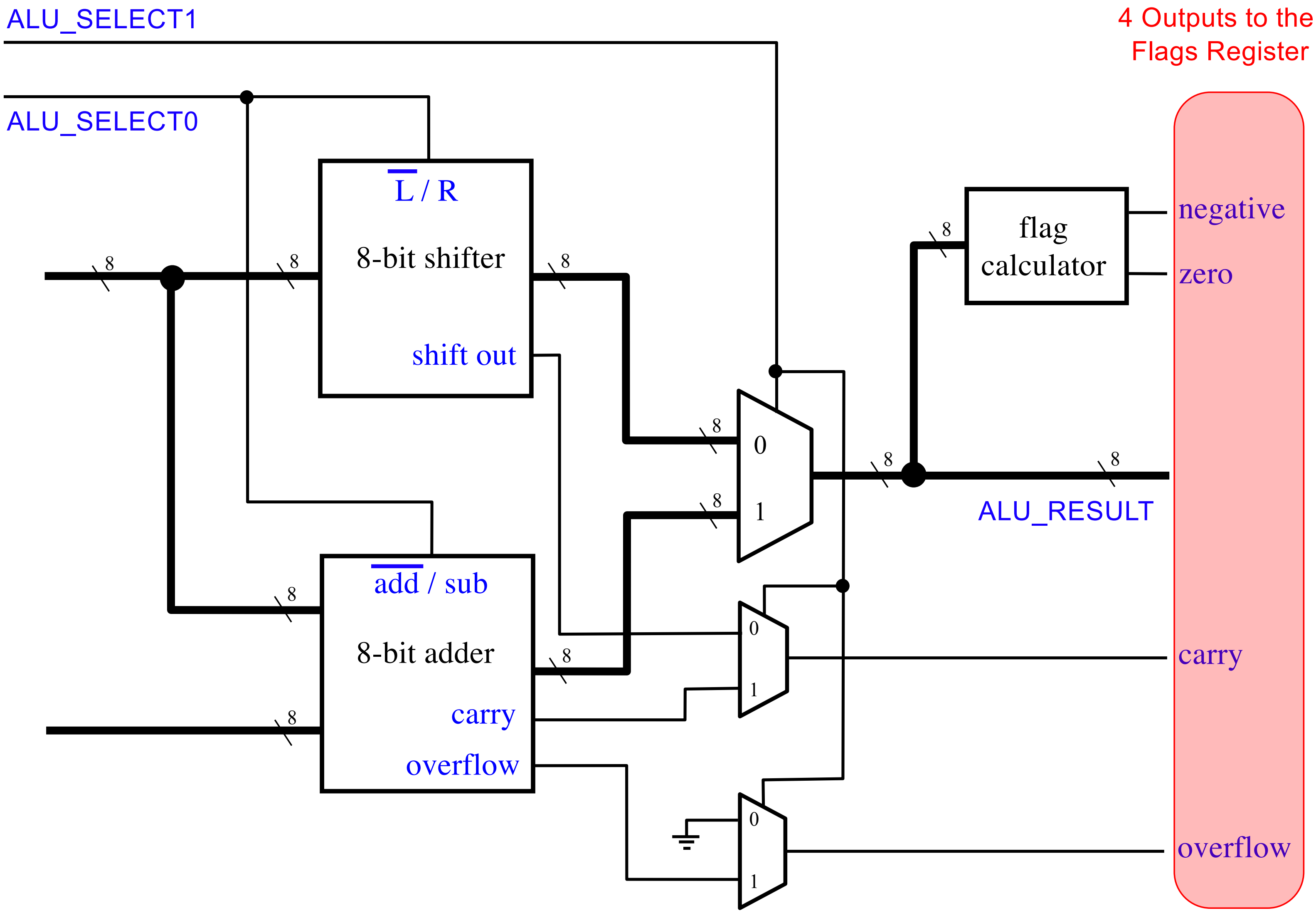
The ALU Flag Calculator



The ALU Flag Calculator



ALU Outputs to the Flags Register



Questions?

THE END