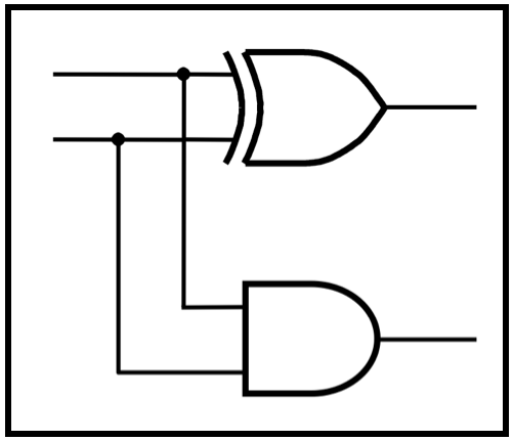


CprE 2810: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

Multiplication

CprE 2810: Digital Logic
Iowa State University, Ames, IA

Administrative Stuff

- **No HW is due today**
- **HW 6 is due on Monday Oct. 13 @ 10 pm.**
- **Posted on the class web page.**

Administrative Stuff

- **Labs this week**
- **Mini-Project**
- **This is worth 3% of your grade (x2 labs)**
- **https://www.ece.iastate.edu/~alexs/classes/2025_Fall_2810/labs/Project-Mini/**

Multiplication and division by 10 in the decimal system

Decimal Multiplication by 10

What happens when we multiply a number by 10?

4 x 10 = ?

542 x 10 = ?

1245 x 10 = ?

Decimal Multiplication by 10

What happens when we multiply a number by 10?

4 x 10 = 40

542 x 10 = 5420

1245 x 10 = 12450

Decimal Multiplication by 10

What happens when we multiply a number by 10?

4 x 10 = 40

542 x 10 = 5420

1245 x 10 = 12450

You simply add a zero as the rightmost number

Decimal Division by 10

What happens when we divide a number by 10?

14 / 10 = ?

540 / 10 = ?

1240 / 10 = ?

Decimal Division by 10

What happens when we divide a number by 10?

14 / 10 = 1 //integer division

540 / 10 = 54

1240 / 10 = 124

You simply delete the rightmost number

Multiplication and division by 2 in the binary system

Binary Multiplication by 2

What happens when we multiply a number by 2?

011 times 2 = ?

101 times 2 = ?

110011 times 2 = ?

Binary Multiplication by 2

What happens when we multiply a number by 2?

011 times 2 = 0110

101 times 2 = 1010

110011 times 2 = 1100110

You simply add a zero as the rightmost number

Binary Multiplication by 4

What happens when we multiply a number by 4?

011 times 4 = ?

101 times 4 = ?

110011 times 4 = ?

Binary Multiplication by 4

What happens when we multiply a number by 4?

011 times 4 = 01100

101 times 4 = 10100

110011 times 4 = 11001100

add two zeros in the last two bits and shift everything else to the left

Binary Multiplication by 2^N

What happens when we multiply a number by 2^N?

011 times 2^N = 011 00…0 // add N zeros

101 times 4 = 101 00…0 // add N zeros

110011 times 4 = 110011 00…0 // add N zeros

Binary Division by 2

What happens when we divide a number by 2?

0110 divided by 2 = ?

1010 divides by 2 = ?

110011 divides by 2 = ?

Binary Division by 2

What happens when we divide a number by 2?

0110 divided by 2 = 011

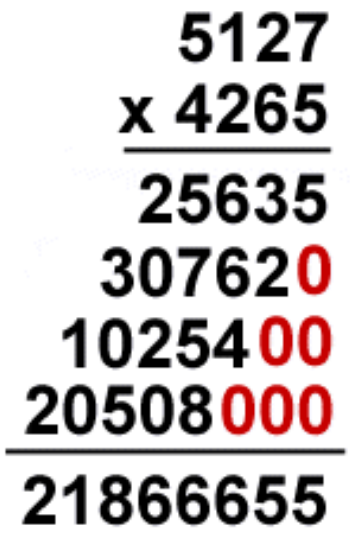
1010 divides by 2 = 101

110011 divides by 2 = 11001

You simply delete the rightmost number

Multiplication of two unsigned binary numbers

Decimal Multiplication By Hand



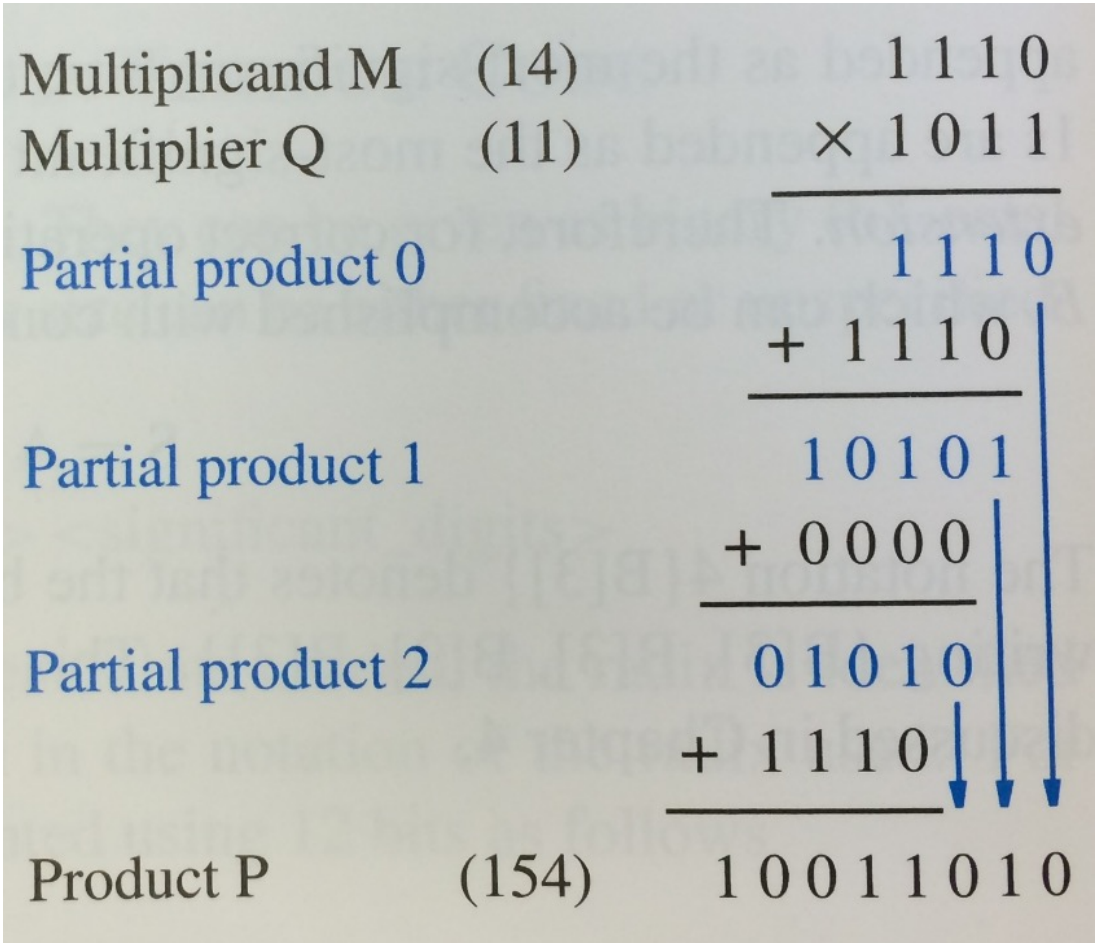
[http://www.ducksters.com/kidsmath/long_multiplication.php]

Binary Multiplication By Hand

Multiplicand M	(14)	1 1 1 0
Multiplier Q	(11)	x 1 0 1 1
		1 1 1 0
		1 1 1 0
		0 0 0 0
		1 1 1 0
Product P	(154)	1 0 0 1 1 0 1 0

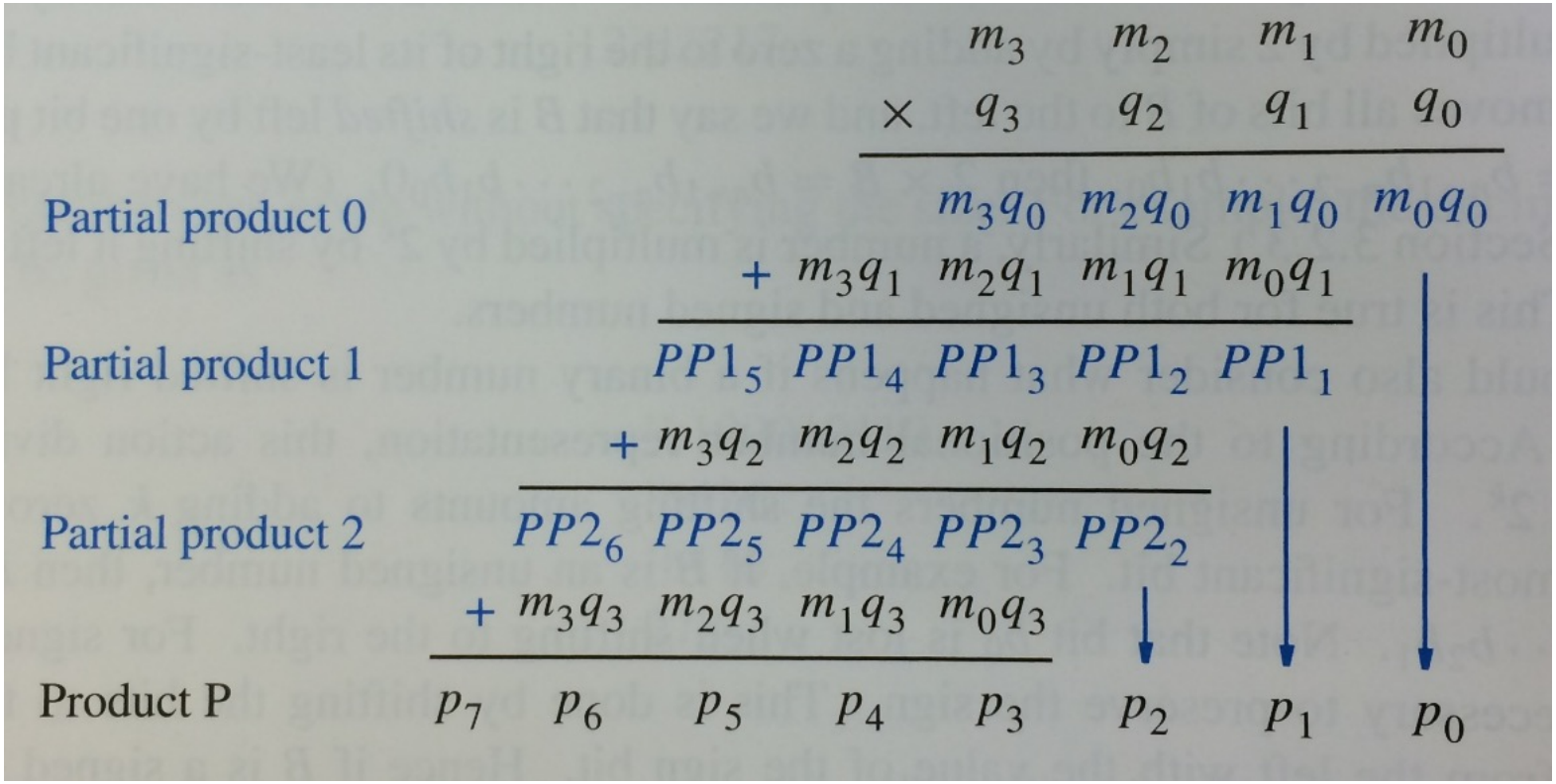
[Figure 3.34a from the textbook]

Binary Multiplication By Hand



[Figure 3.34b from the textbook]

Binary Multiplication By Hand



[Figure 3.34c from the textbook]

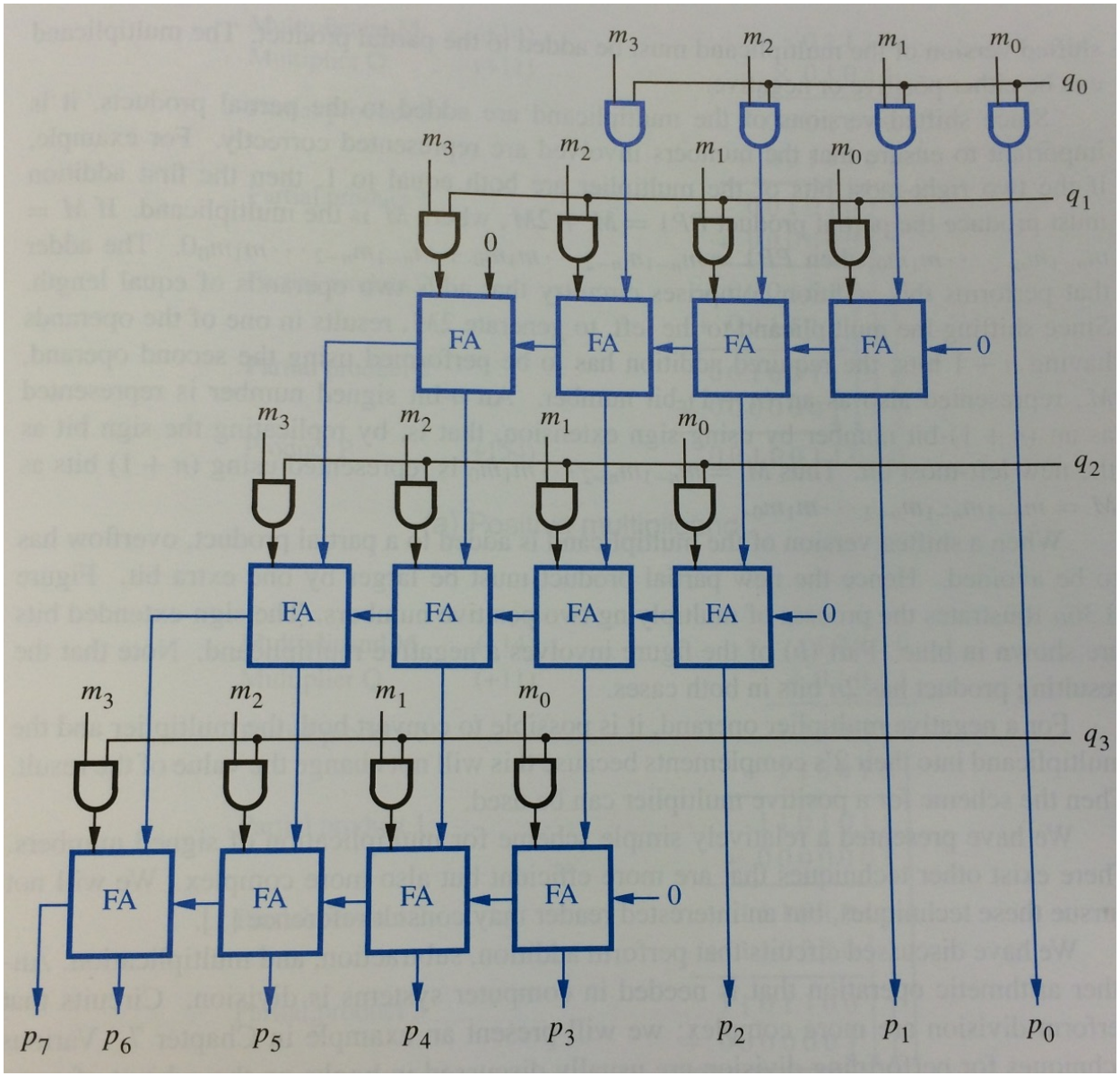


Figure 3.35. A 4x4 multiplier circuit.

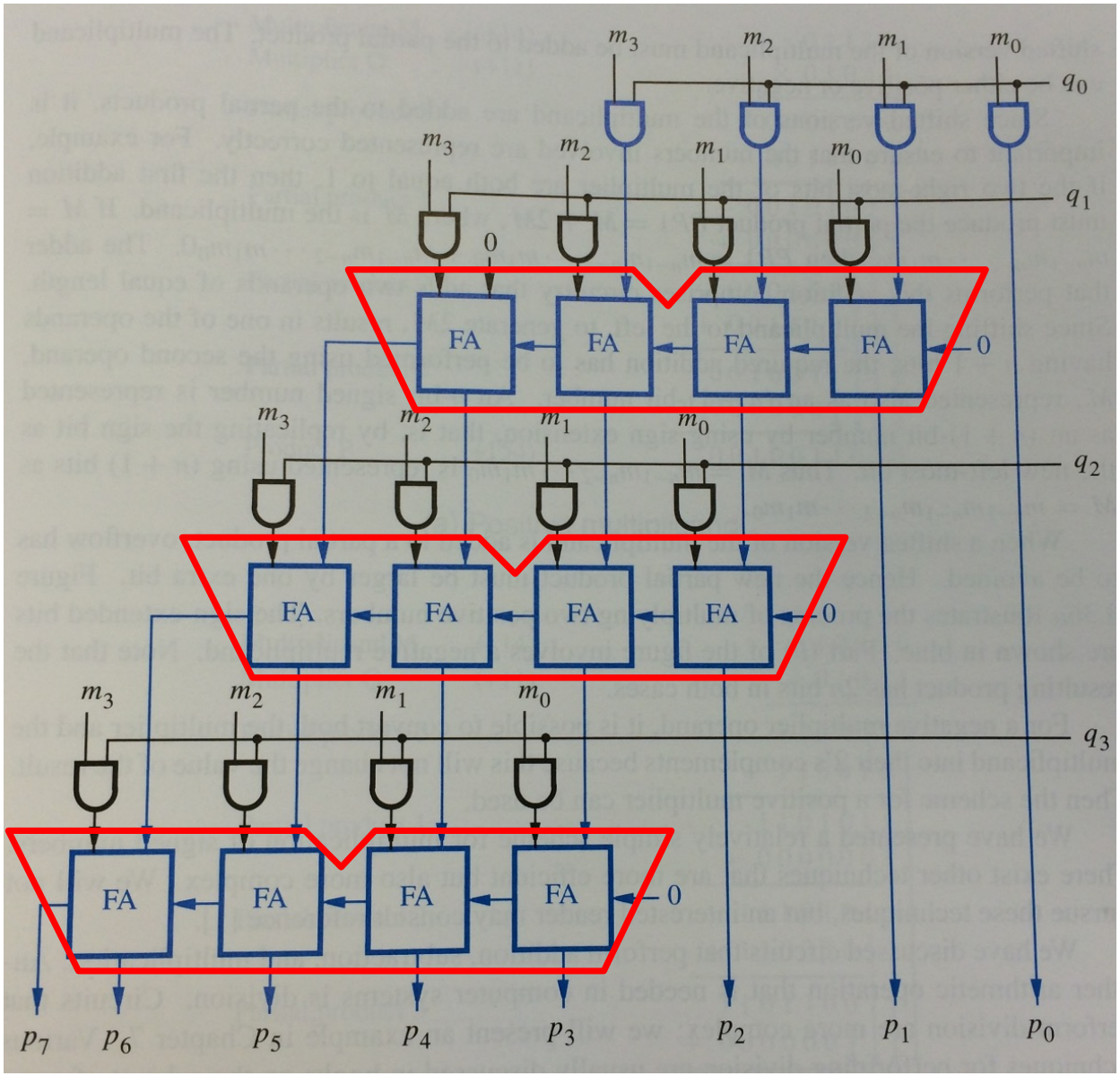


Figure 3.35. A 4x4 multiplier circuit.

Sign Extension

Sign extension for positive numbers

- **If we want to represent the same positive number with more bits, we simply pad it on the left with zeros.**

- **For example:**

0110	**(+6 with 4-bits)**
00110	**(+6 with 5-bits)**
000110	**(+6 with 6-bits)**

Sign extension for negative numbers

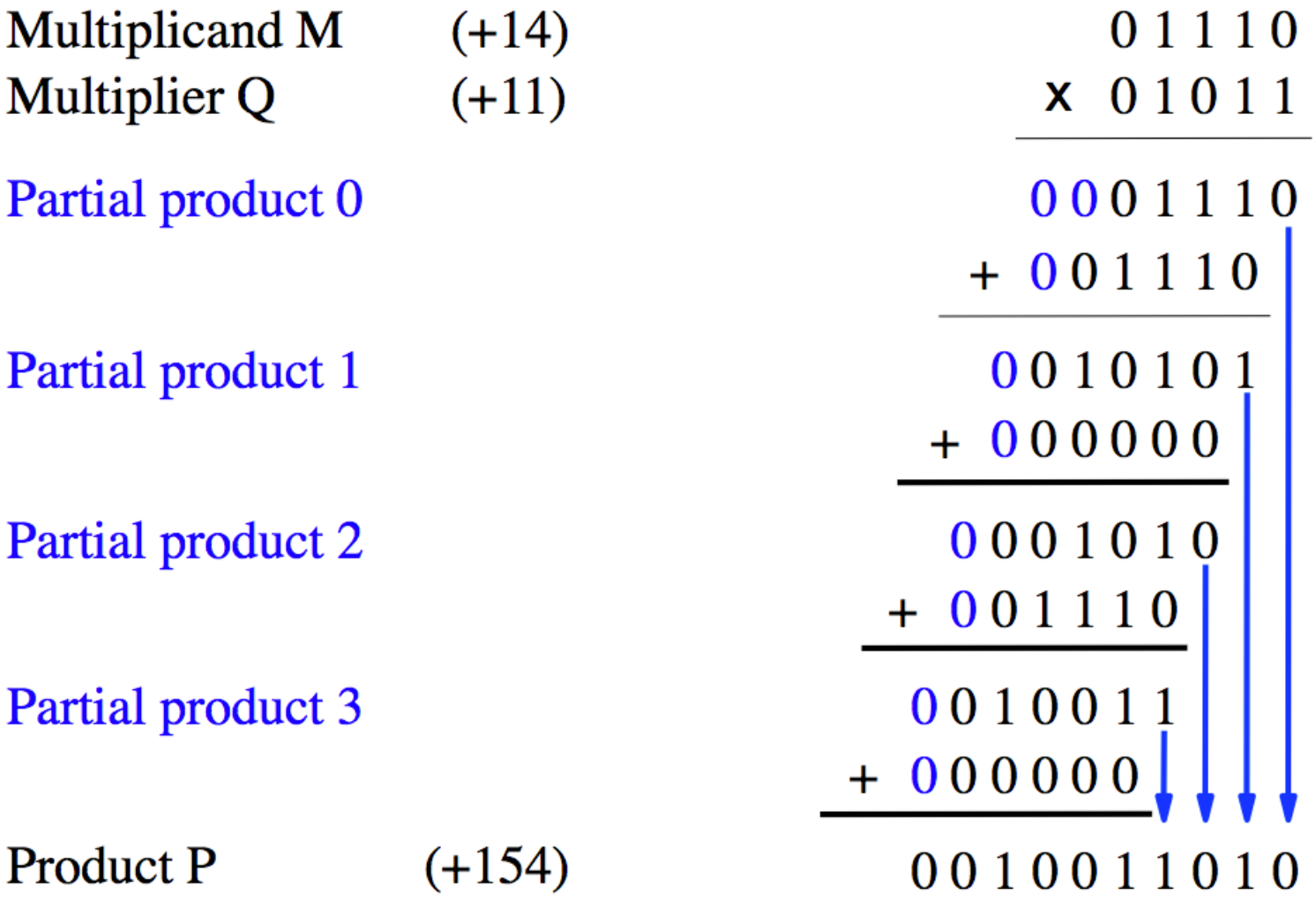
- **If we want to represent the same negative number with more bits, we simply pad it on the left with ones.**

- **For example:**

1011	**(-5 with 4-bits)**
11011	**(-5 with 5-bits)**
111011	**(-5 with 6-bits)**

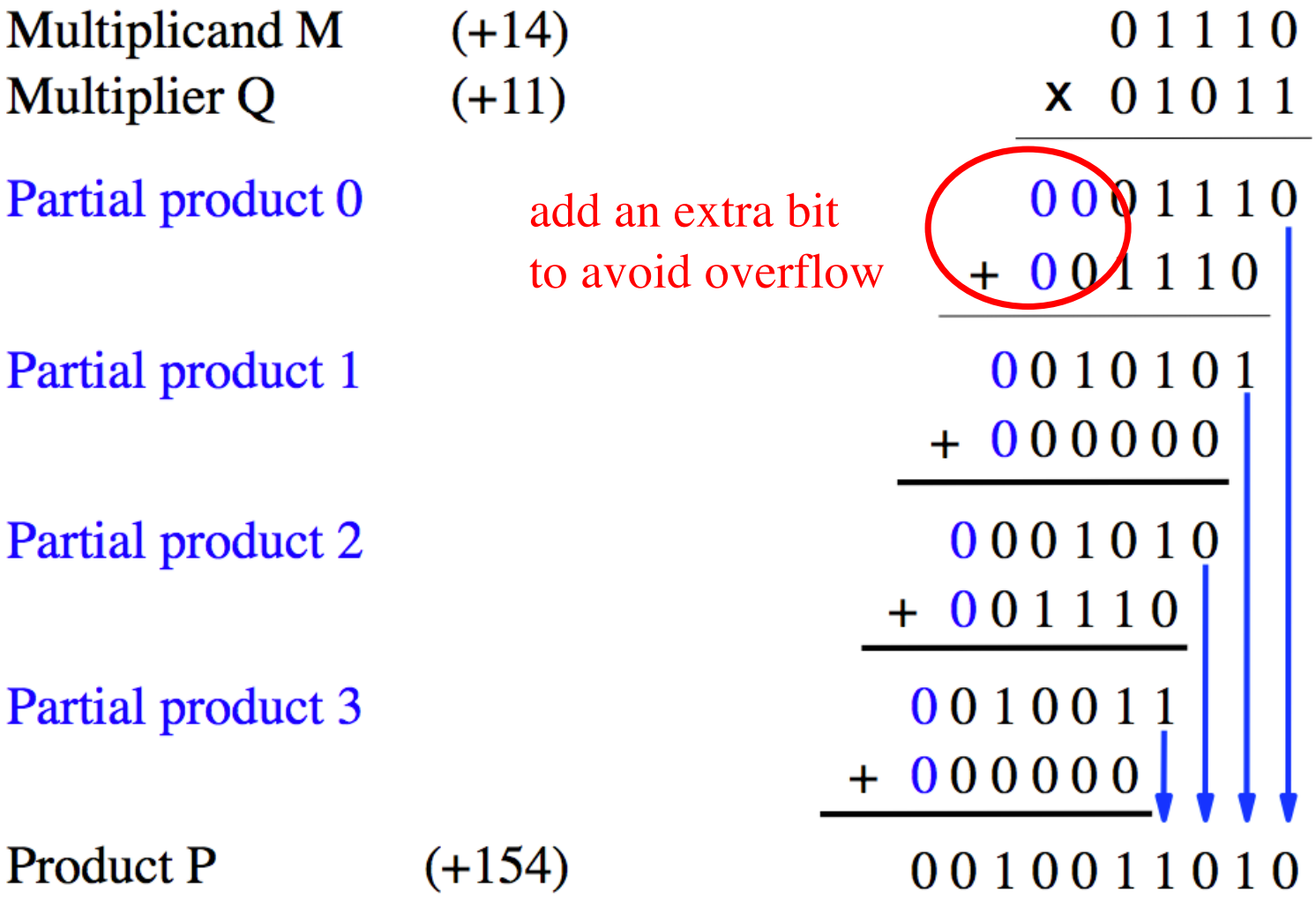
Multiplication of two signed binary numbers

Positive Multiplicand Example



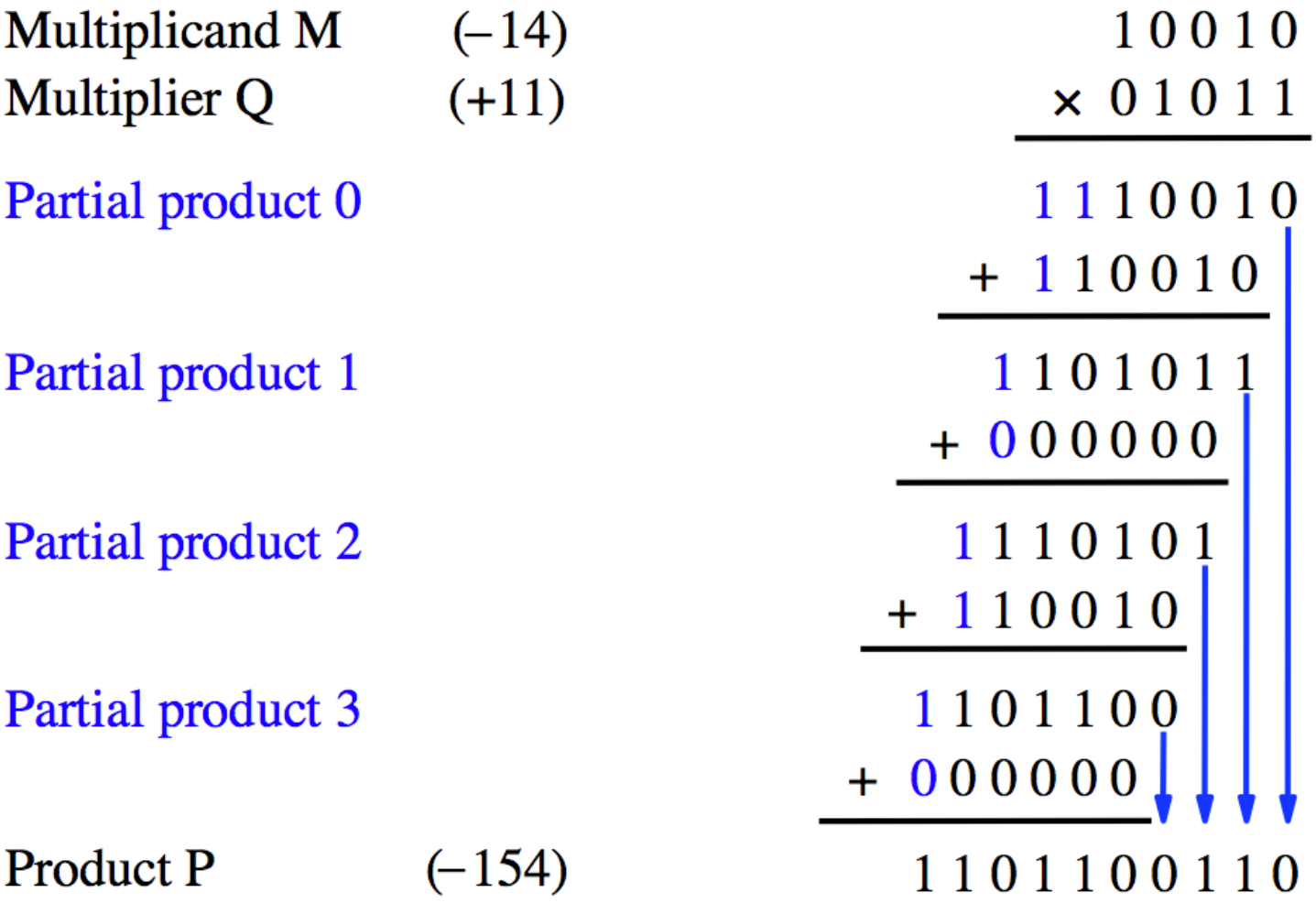
[Figure 3.36a in the textbook]

Positive Multiplicand Example



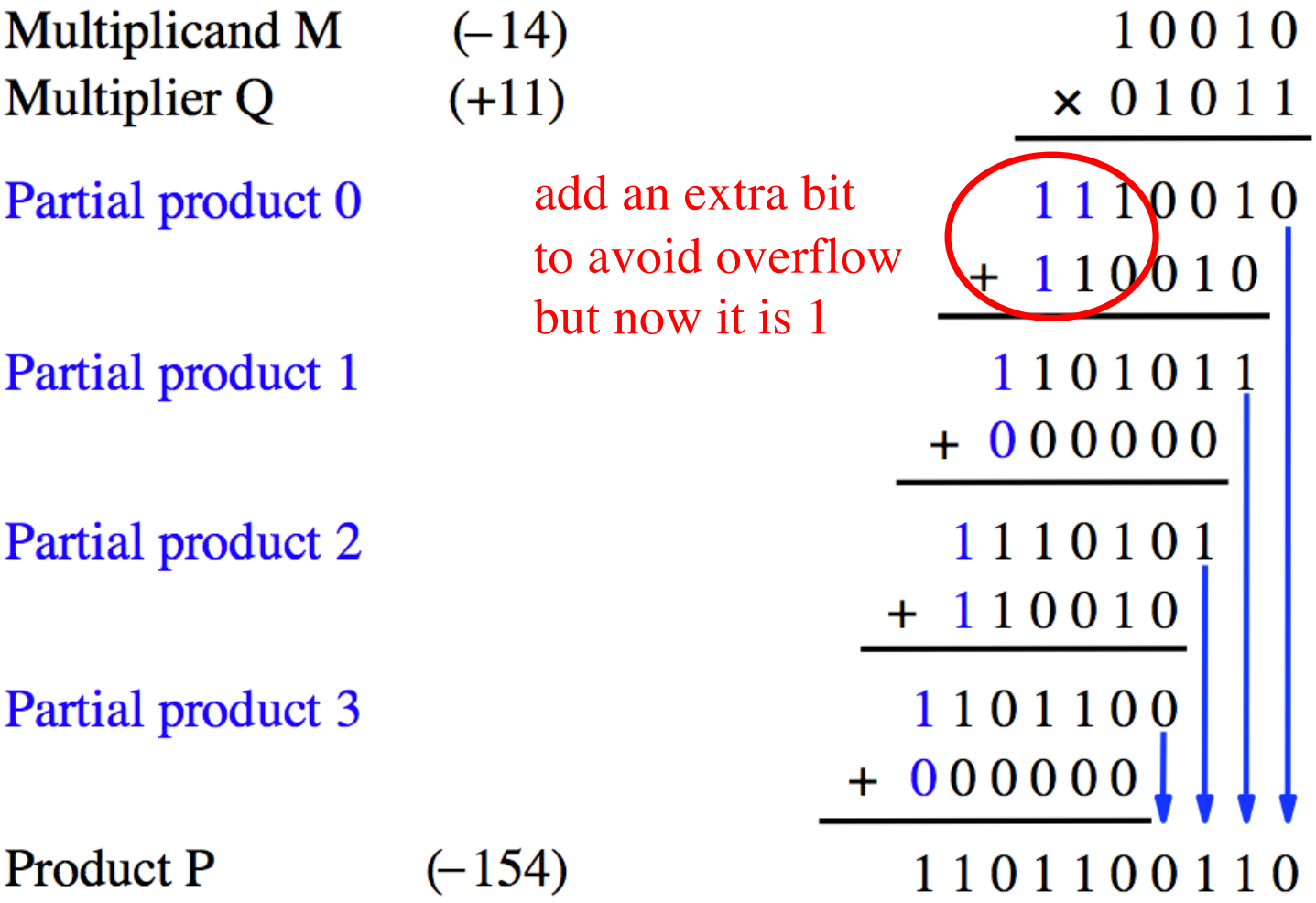
[Figure 3.36a in the textbook]

Negative Multiplicand Example



[Figure 3.36b in the textbook]

Negative Multiplicand Example



[Figure 3.36b in the textbook]

What if the Multiplier is Negative?

- **Negate both numbers.**
- **This will make the multiplier positive.**
- **Then proceed as normal.**
- **This will not affect the result.**
- **Example: 5*(- 4) = (- 5)*(4)= - 20**

Arithmetic Comparison Circuits

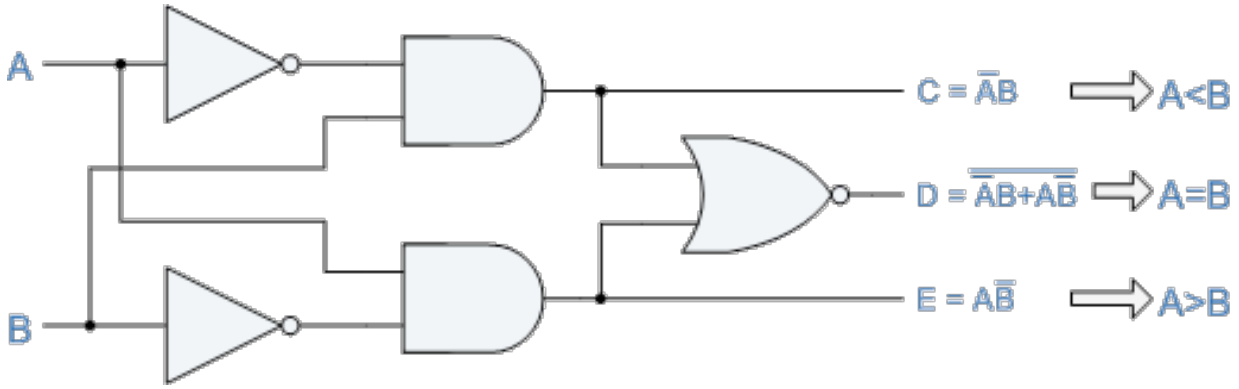
Truth table for a one-bit digital comparator

Inputs		Outputs		
A	B	$A > B$	$A = B$	$A < B$
0	0	0	1	0
0	1	0	0	1
1	0	1	0	0
1	1	0	1	0

[http://en.wikipedia.org/wiki/Digital_comparator]

A one-bit digital comparator circuit

Inputs		Outputs		
A	B	$A > B$	$A = B$	$A < B$
0	0	0	1	0
0	1	0	0	1
1	0	1	0	0
1	1	0	1	0



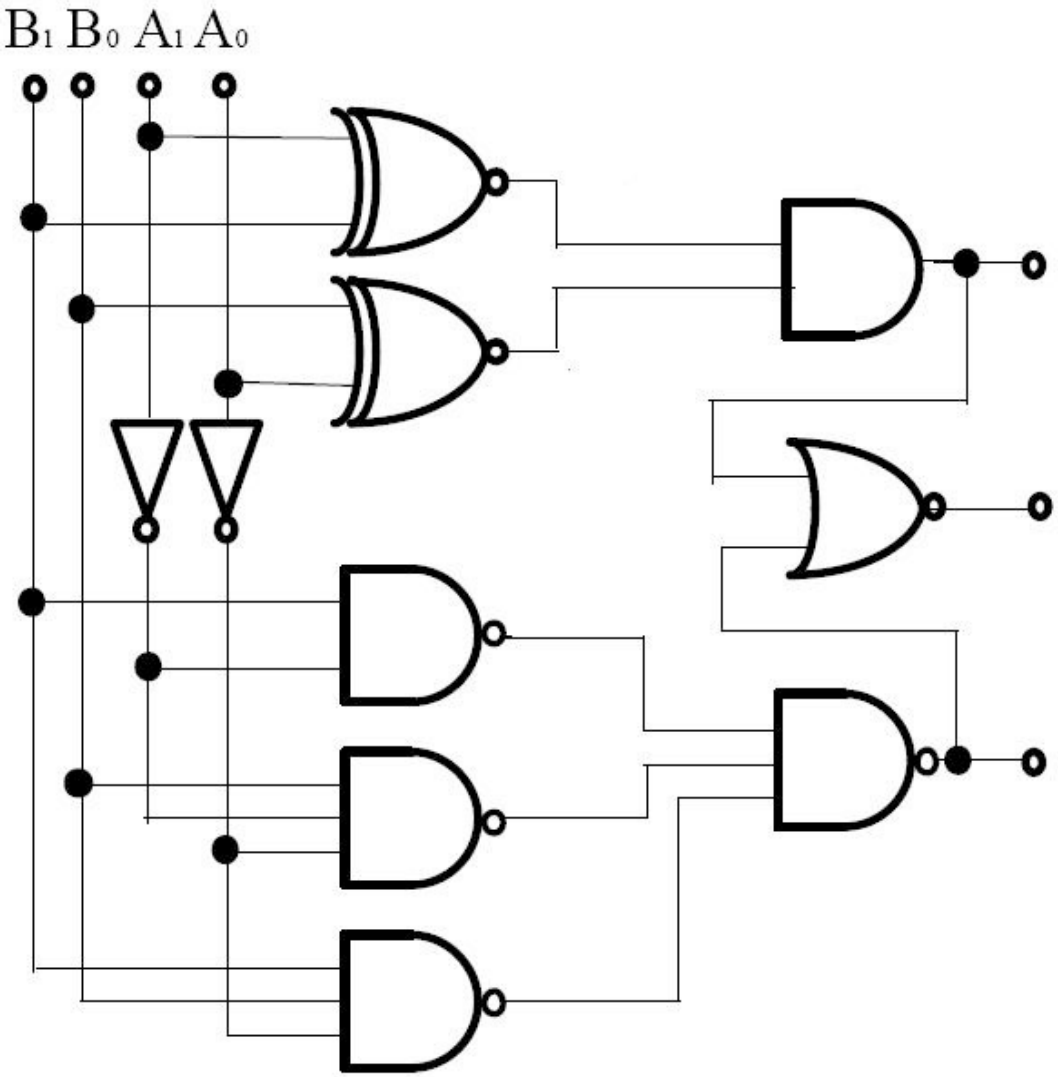
[http://www.personal.psu.edu/users///c/w/cwb5096/Old%20Site/index_files/cmpenlab4.htm]

Truth table for a two-bit digital comparator

Inputs				Outputs		
A_1	A_0	B_1	B_0	$A < B$	$A = B$	$A > B$
0	0	0	0	0	1	0
0	0	0	1	1	0	0
0	0	1	0	1	0	0
0	0	1	1	1	0	0
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	0	1	0
1	0	1	1	1	0	0
1	1	0	0	0	0	1
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	0	1	0

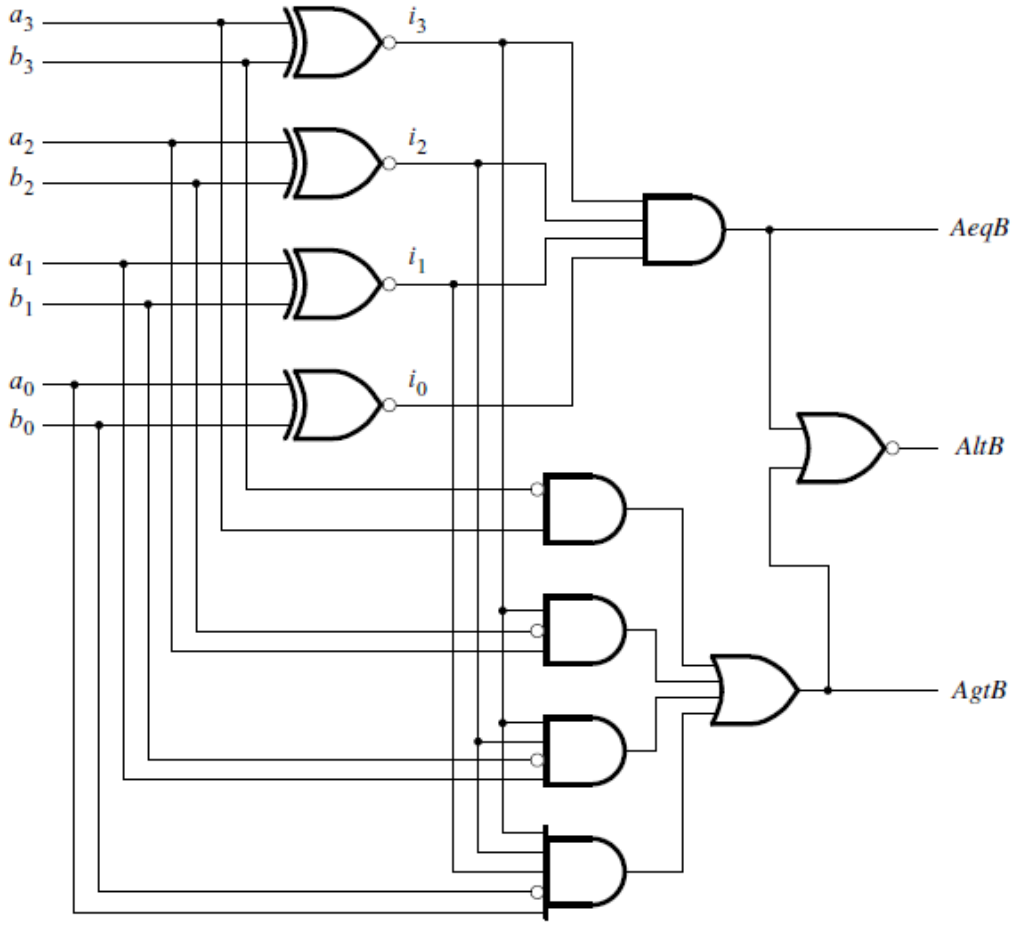
[http://en.wikipedia.org/wiki/Digital_comparator]

A two-bit digital comparator circuit



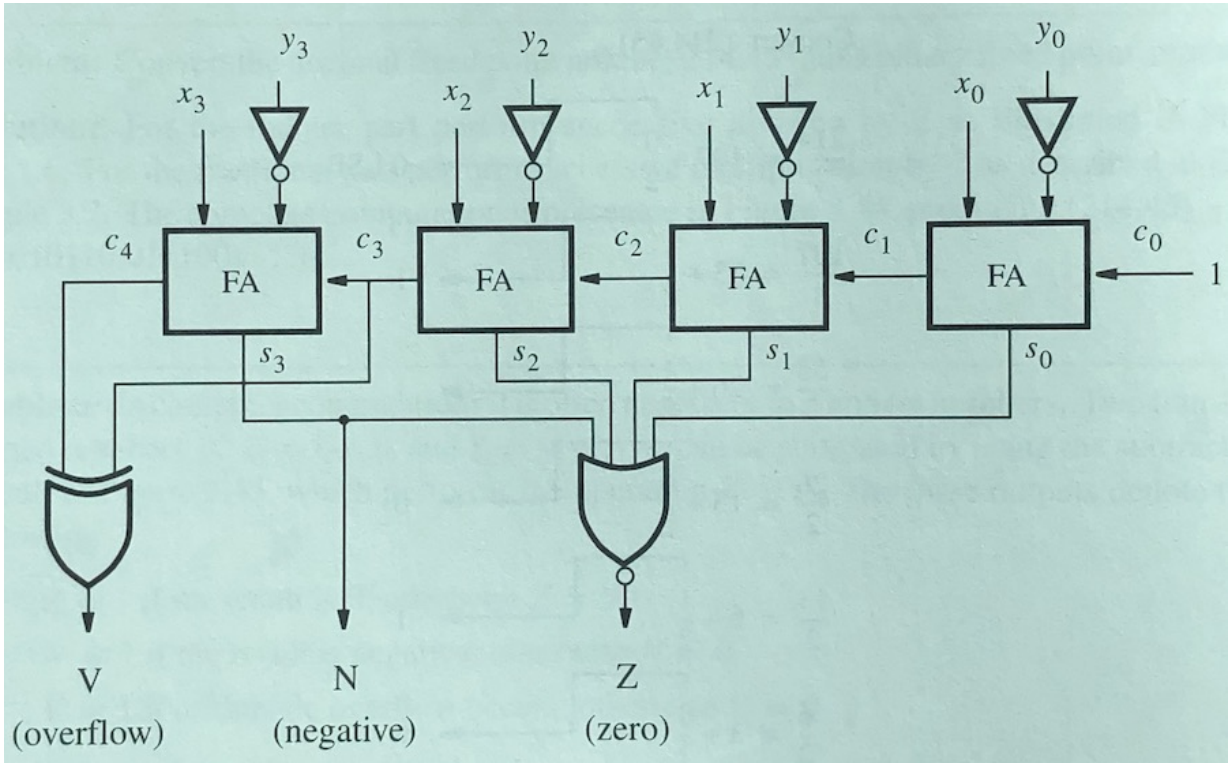
[http://forum.allaboutcircuits.com/showthread.php?t=10561]

A four-bit comparator circuit



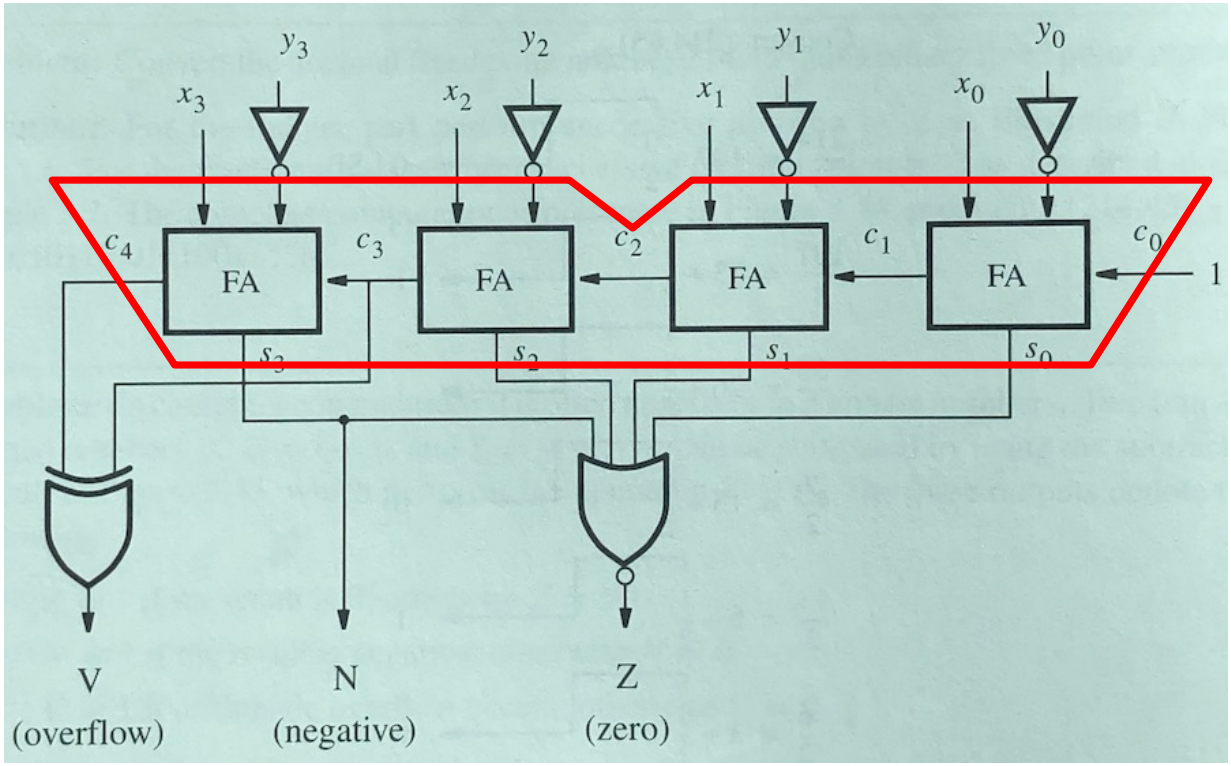
[Figure 4.22 from the textbook]

Another four-bit comparator circuit



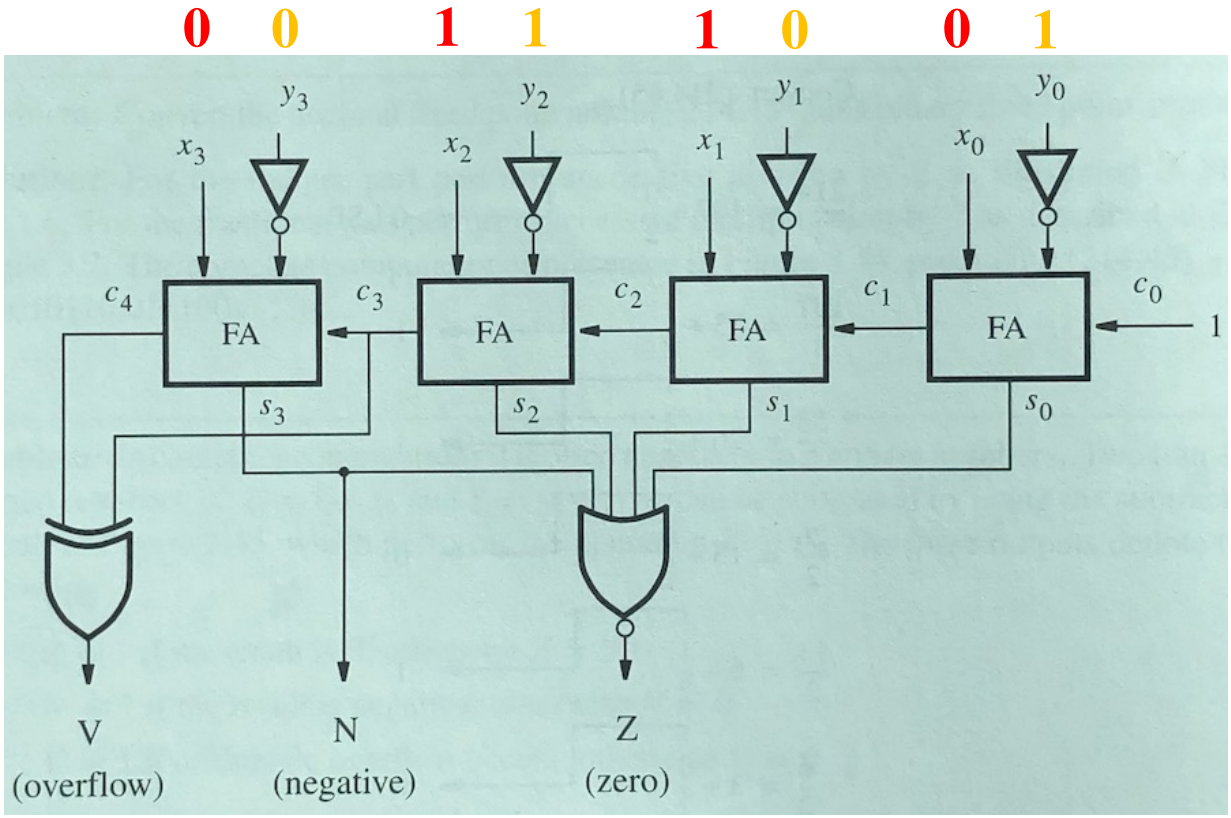
[Figure 3.45 from the textbook]

Another four-bit comparator circuit



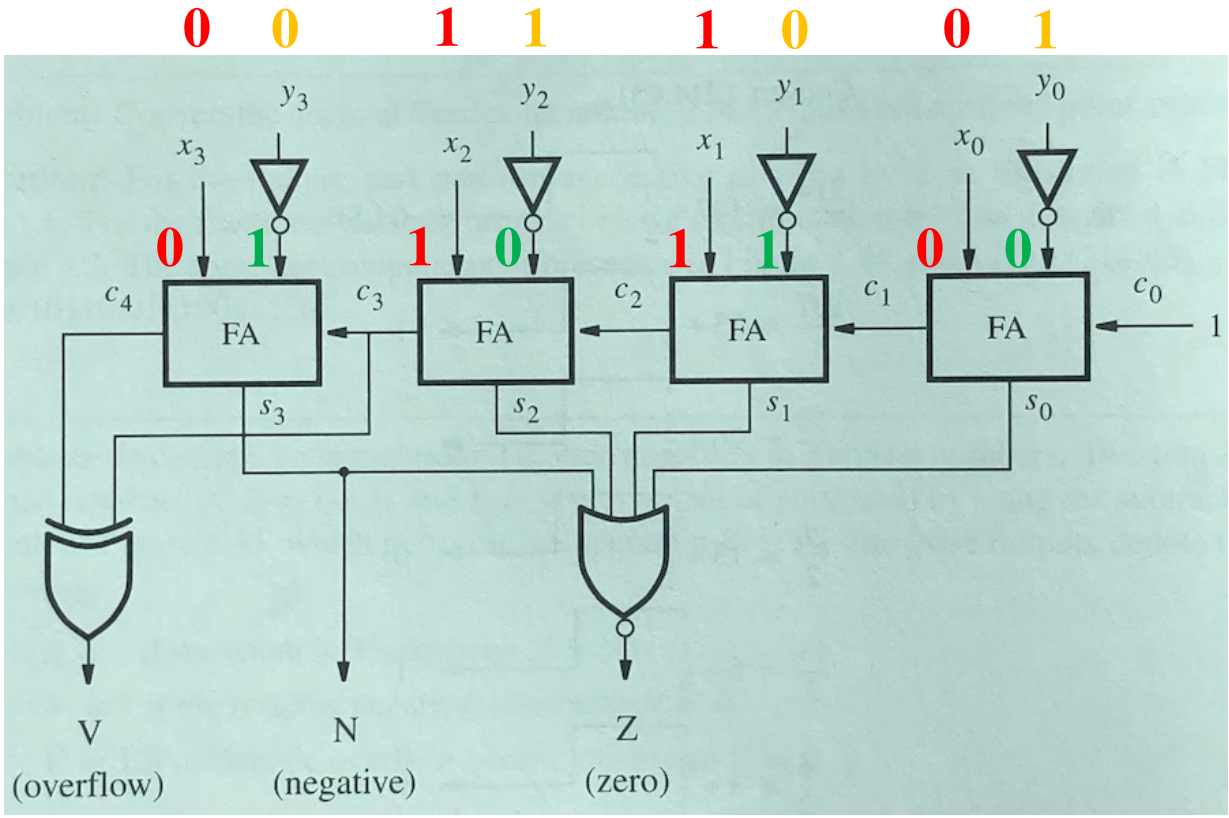
[Figure 3.45 from the textbook]

Another four-bit comparator circuit

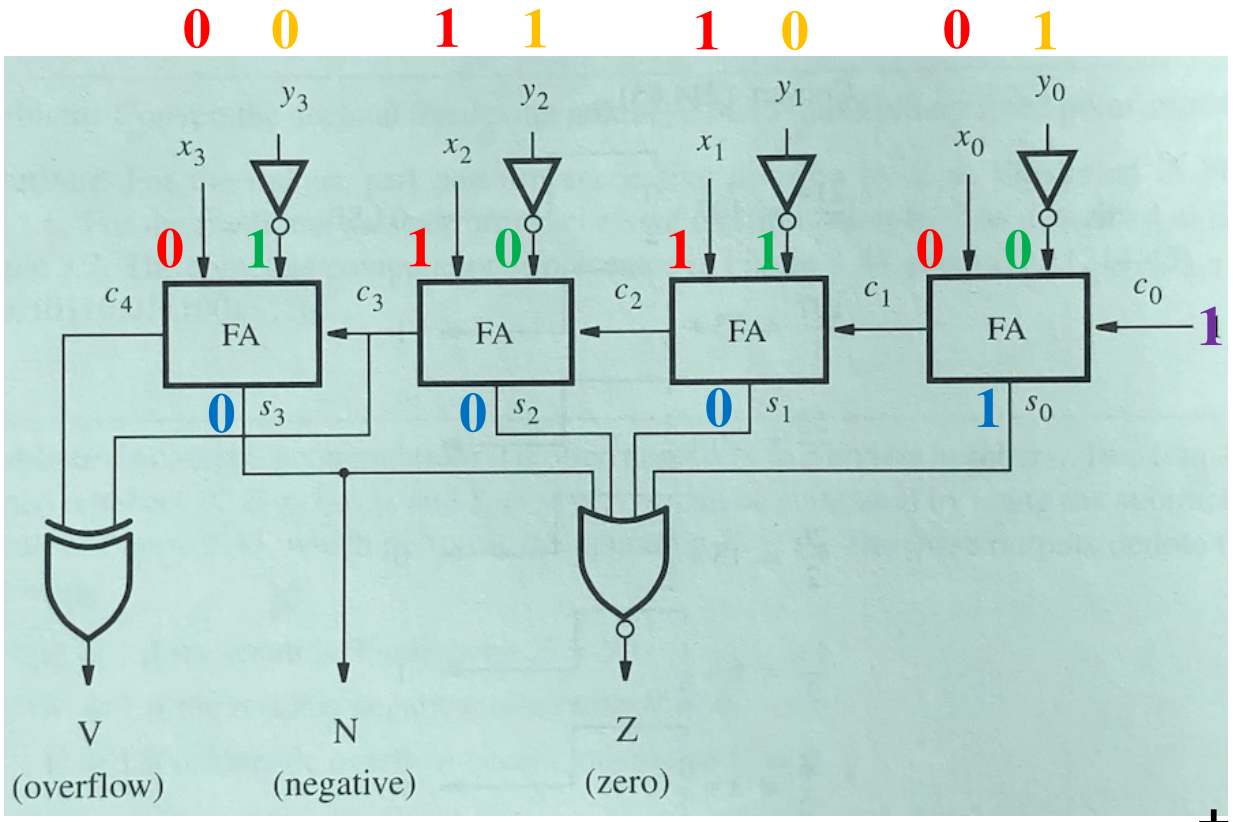


Compare 6 with 5 by subtraction (6-5).

Another four-bit comparator circuit

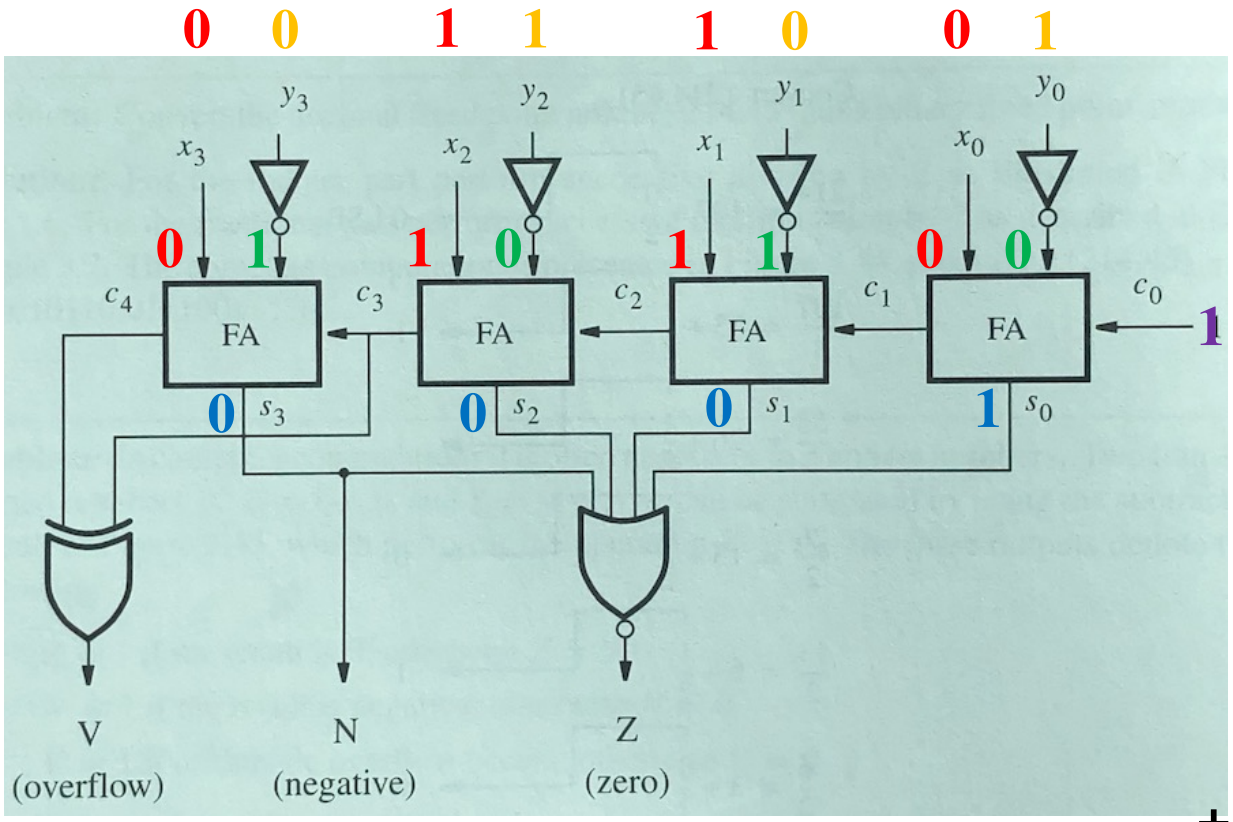


Another four-bit comparator circuit



$$
\begin{array}{r}
\ 1 \\
+\ 0\,1\,1\,0 \\
\ 1\,0\,1\,0 \\
\hline
1\,0\,0\,0\,1
\end{array}
$$

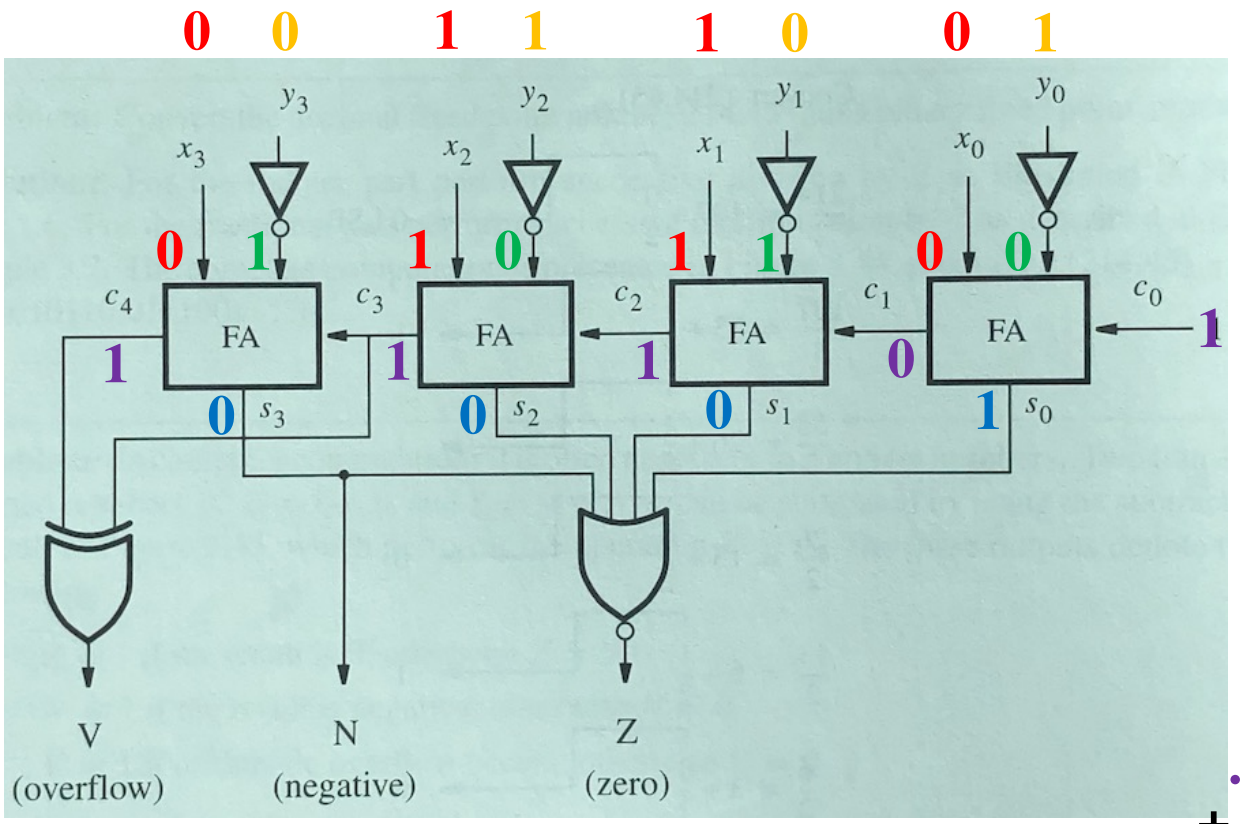
Another four-bit comparator circuit



$$\begin{array}{r} 1 \\ 0\,1\,1\,0 \\ +\ 1\,0\,1\,0 \\ \hline 1\,0\,0\,0\,1 \end{array}$$

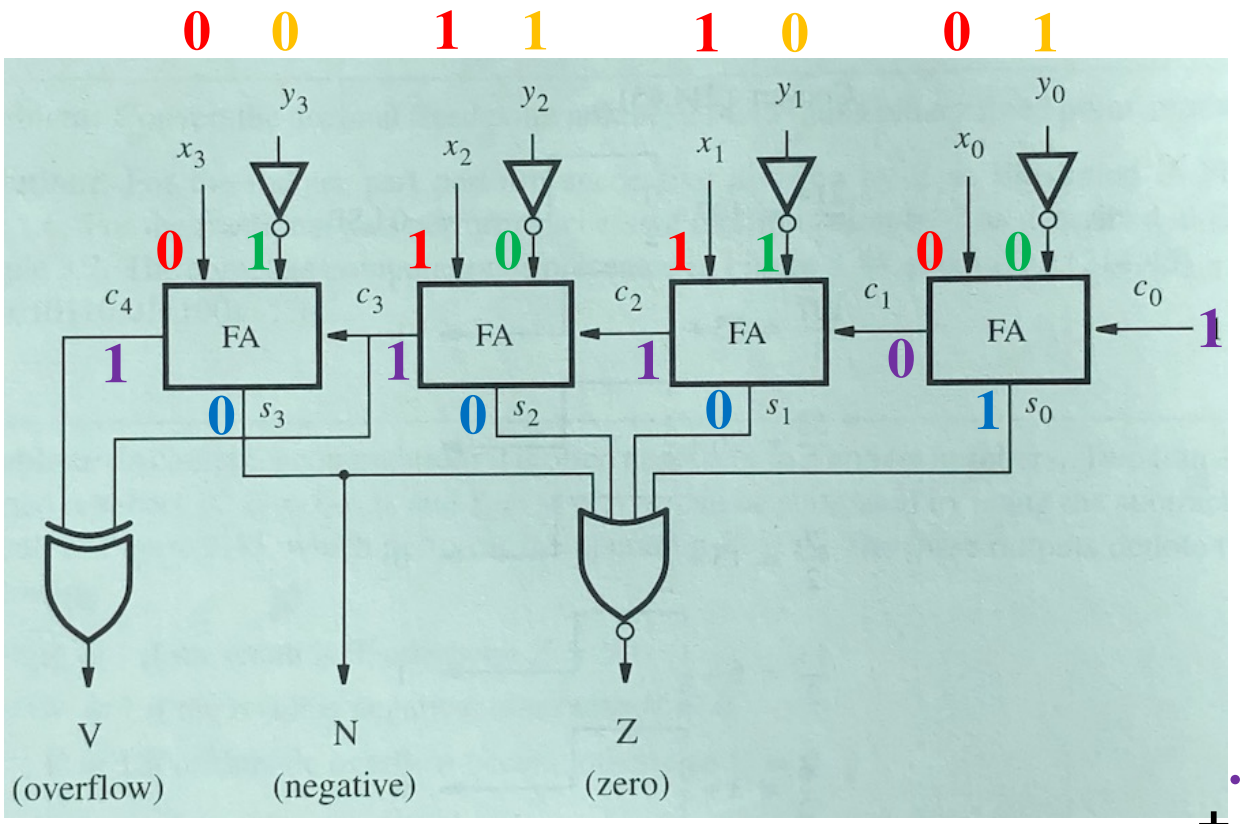
Ignore

Another four-bit comparator circuit




```
. . .  1
+ 0 1 1 0
  1 0 1 0
-------------
1 0 0 0 1
```

Another four-bit comparator circuit



$$
\begin{array}{r}
\ldots\ 1 \\
+\ 0\,1\,1\,0 \\
\quad 1\,0\,1\,0 \\
\hline
1\,0\,0\,0\,1
\end{array}
$$

Binary Coded Decimal (BCD)

Table of Binary-Coded Decimal Digits

Decimal digit	BCD code
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Addition of BCD digits

X	0 1 1 1	7
+ Y	+ 0 1 0 1	+ 5
Z	1 1 0 0	12

[Figure 3.38a in the textbook]

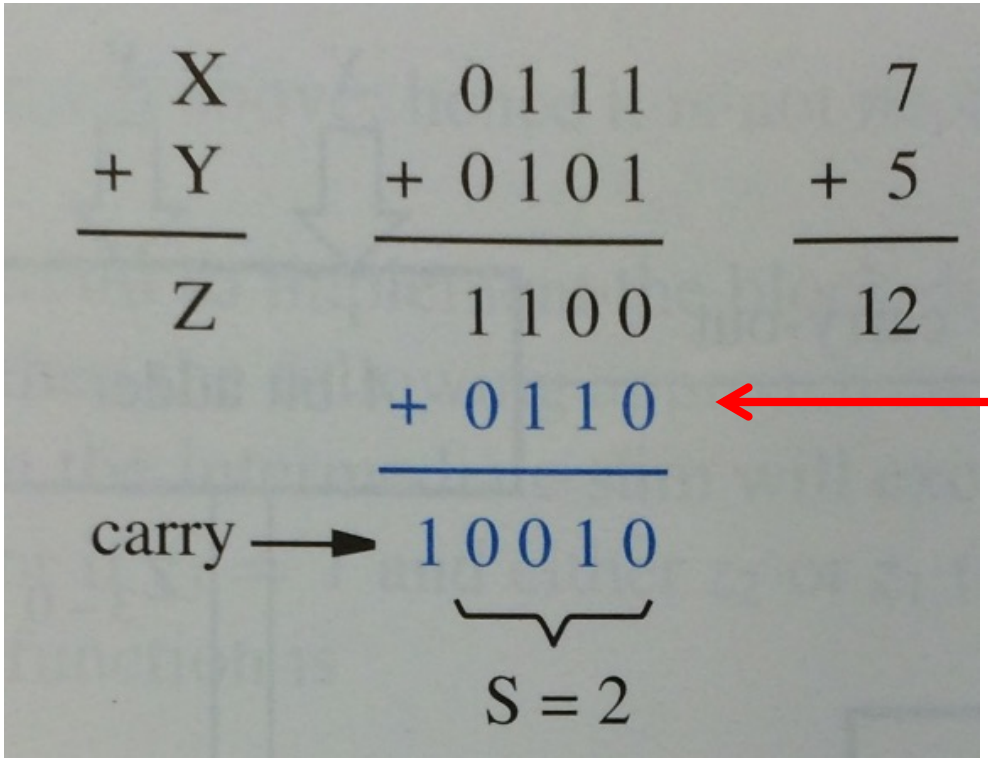
Addition of BCD digits

X	0 1 1 1	7
+ Y	+ 0 1 0 1	+ 5
Z	1 1 0 0	12

The result is greater than 9, which is not a valid BCD number

[Figure 3.38a in the textbook]

Addition of BCD digits



[Figure 3.38a in the textbook]

Addition of BCD digits

X	1000	8
+ Y	+ 1001	+ 9
Z	10001	17

[Figure 3.38b in the textbook]

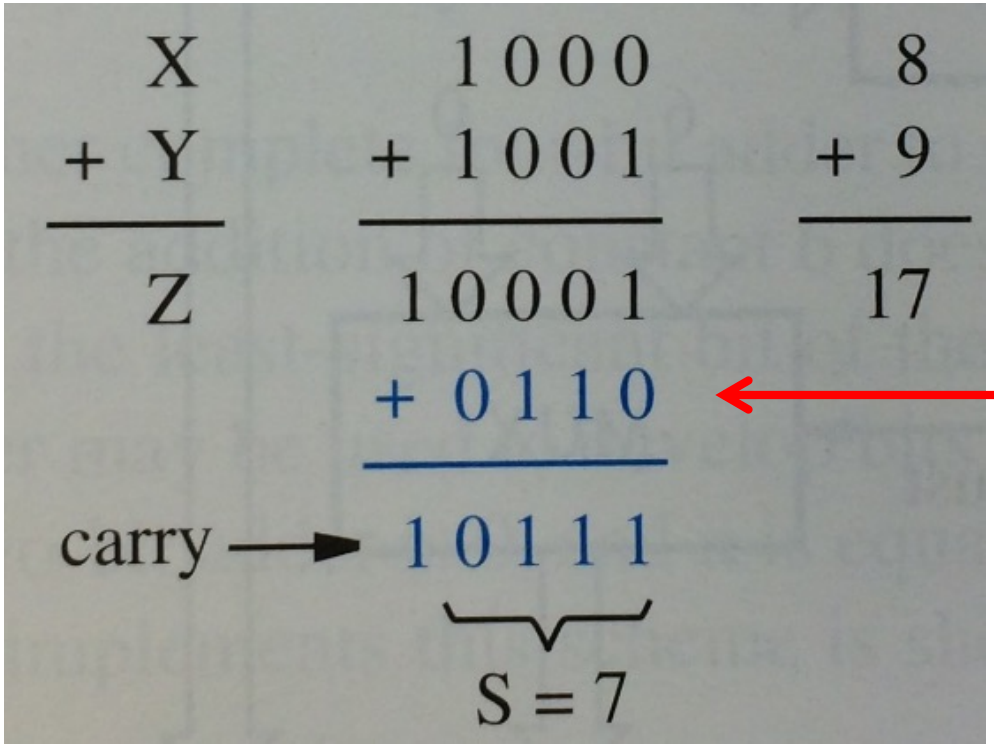
Addition of BCD digits

X	1000	8
+ Y	+ 1001	+ 9
Z	10001	17

The result is 1, but it should be 7

[Figure 3.38b in the textbook]

Addition of BCD digits



[Figure 3.38b in the textbook]

Why add 6?

- **Think of BCD addition as a mod 16 operation**

- **Decimal addition is mod 10 operation**

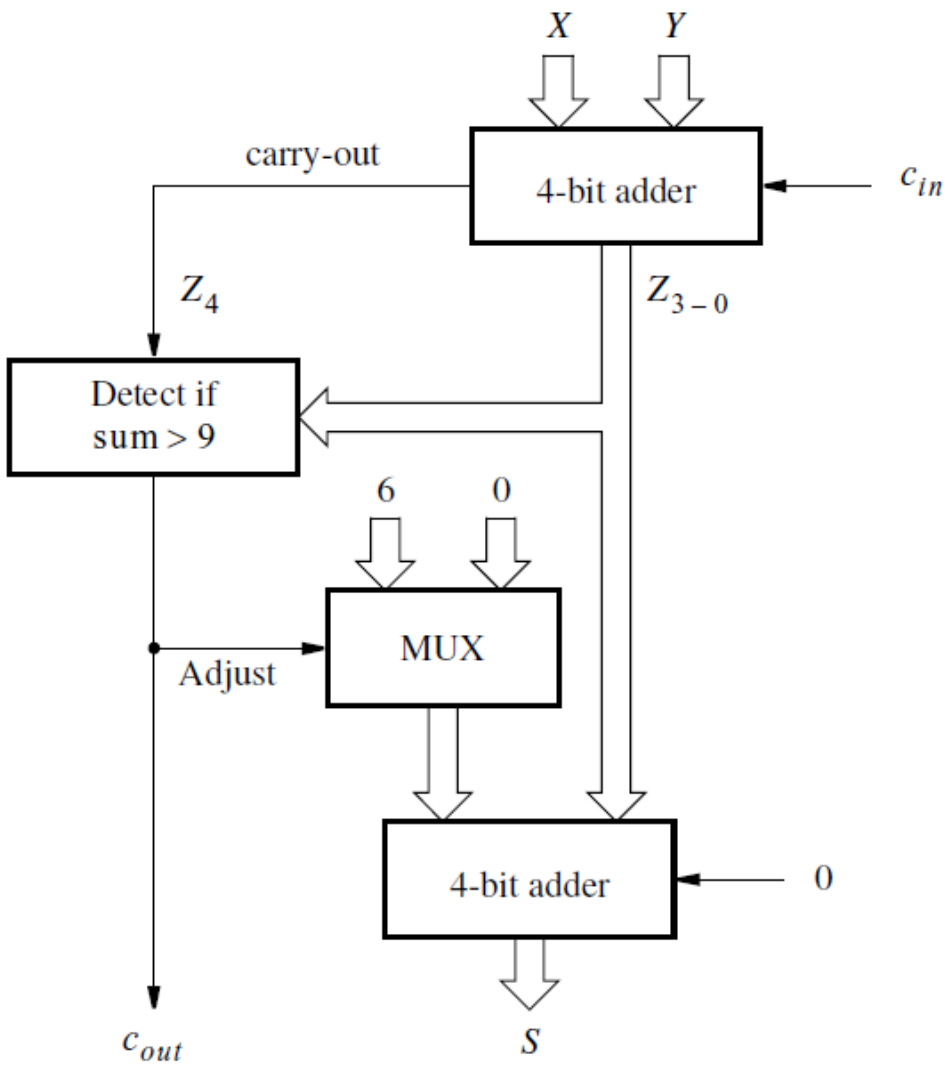
BCD Arithmetic Rules

Z = X + Y

If Z <= 9, then S=Z and carry-out = 0

If Z > 9, then S=Z+6 and carry-out =1

Block diagram for a one-digit BCD adder



[Figure 3.39 in the textbook]

How to check if the number is > 9?

7 - 0111

8 - 1000

9 - 1001

10 - 1010

11 - 1011

12 - 1100

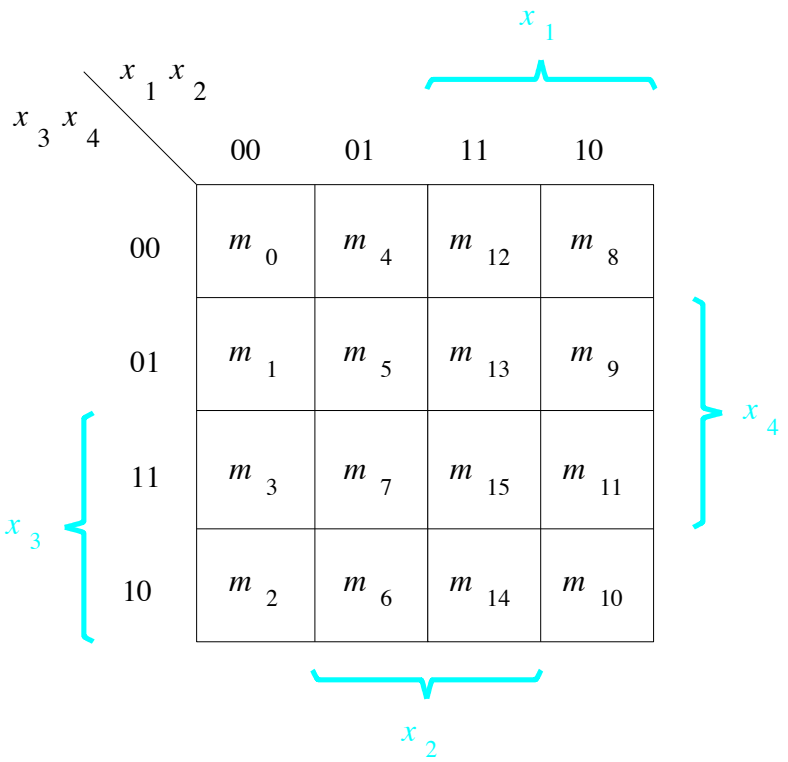
13 - 1101

14 - 1110

15 - 1111

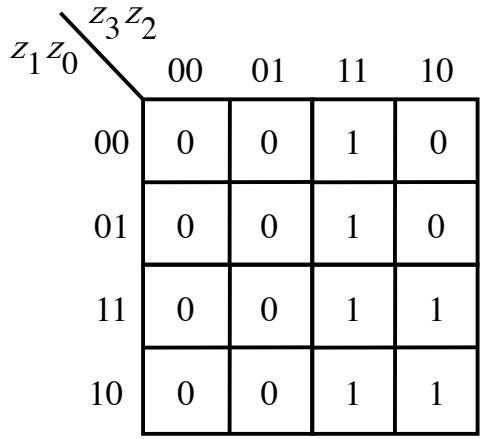
A four-variable Karnaugh map

x1	x2	x3	x4		
0	0	0	0	m0	0
0	0	0	1	m1	0
0	0	1	0	m2	0
0	0	1	1	m3	0
0	1	0	0	m4	0
0	1	0	1	m5	0
0	1	1	0	m6	0
0	1	1	1	m7	0
1	0	0	0	m8	0
1	0	0	1	m9	0
1	0	1	0	m10	1
1	0	1	1	m11	1
1	1	0	0	m12	1
1	1	0	1	m13	1
1	1	1	0	m14	1
1	1	1	1	m15	1



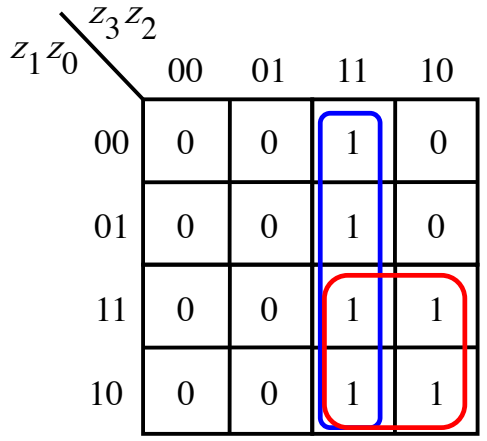
How to check if the number is > 9?

z3	z2	z1	z0		
0	0	0	0	m0	0
0	0	0	1	m1	0
0	0	1	0	m2	0
0	0	1	1	m3	0
0	1	0	0	m4	0
0	1	0	1	m5	0
0	1	1	0	m6	0
0	1	1	1	m7	0
1	0	0	0	m8	0
1	0	0	1	m9	0
1	0	1	0	m10	1
1	0	1	1	m11	1
1	1	0	0	m12	1
1	1	0	1	m13	1
1	1	1	0	m14	1
1	1	1	1	m15	1



How to check if the number is > 9?

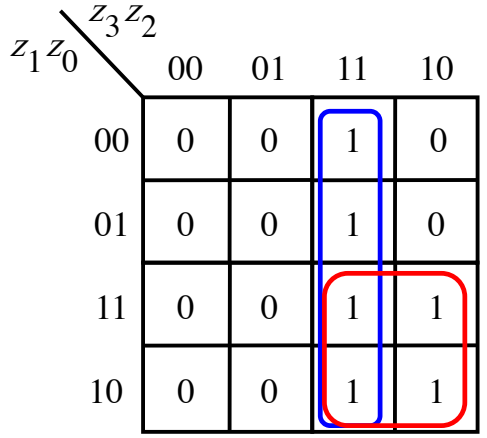
z3	z2	z1	z0		
0	0	0	0	m0	0
0	0	0	1	m1	0
0	0	1	0	m2	0
0	0	1	1	m3	0
0	1	0	0	m4	0
0	1	0	1	m5	0
0	1	1	0	m6	0
0	1	1	1	m7	0
1	0	0	0	m8	0
1	0	0	1	m9	0
1	0	1	0	m10	1
1	0	1	1	m11	1
1	1	0	0	m12	1
1	1	0	1	m13	1
1	1	1	0	m14	1
1	1	1	1	m15	1



$f = z_3z_2 + z_3z_1$

How to check if the number is > 9?

z3	z2	z1	z0		
0	0	0	0	m0	0
0	0	0	1	m1	0
0	0	1	0	m2	0
0	0	1	1	m3	0
0	1	0	0	m4	0
0	1	0	1	m5	0
0	1	1	0	m6	0
0	1	1	1	m7	0
1	0	0	0	m8	0
1	0	0	1	m9	0
1	0	1	0	m10	1
1	0	1	1	m11	1
1	1	0	0	m12	1
1	1	0	1	m13	1
1	1	1	0	m14	1
1	1	1	1	m15	1



$$f = z_3z_2 + z_3z_1$$

In addition, also check if there was a carry

$$f = \text{carry-out} + z_3z_2 + z_3z_1$$

Verilog code for a one-digit BCD adder

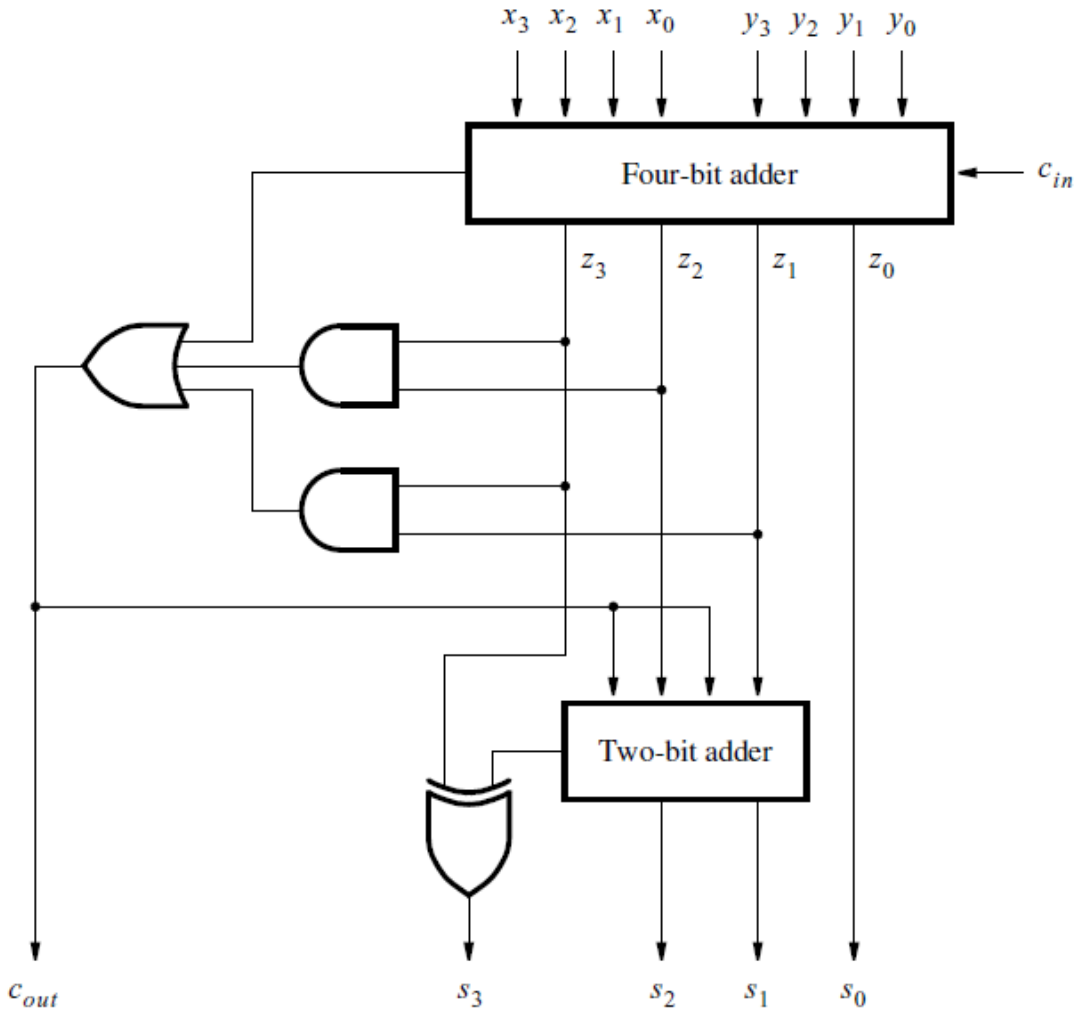
```
module bcdadd(Cin, X, Y, S, Cout);
    input Cin;
    input [3:0] X,Y;
    output reg [3:0] S;
    output reg Cout;
    reg [4:0] Z;

    always@ (X, Y, Cin)
    begin
        Z = X + Y + Cin;
        if (Z < 10)
            {Cout,S} = Z;
        else
            {Cout, S} = Z + 6;
    end

endmodule
```

[Figure 3.40 in the textbook]

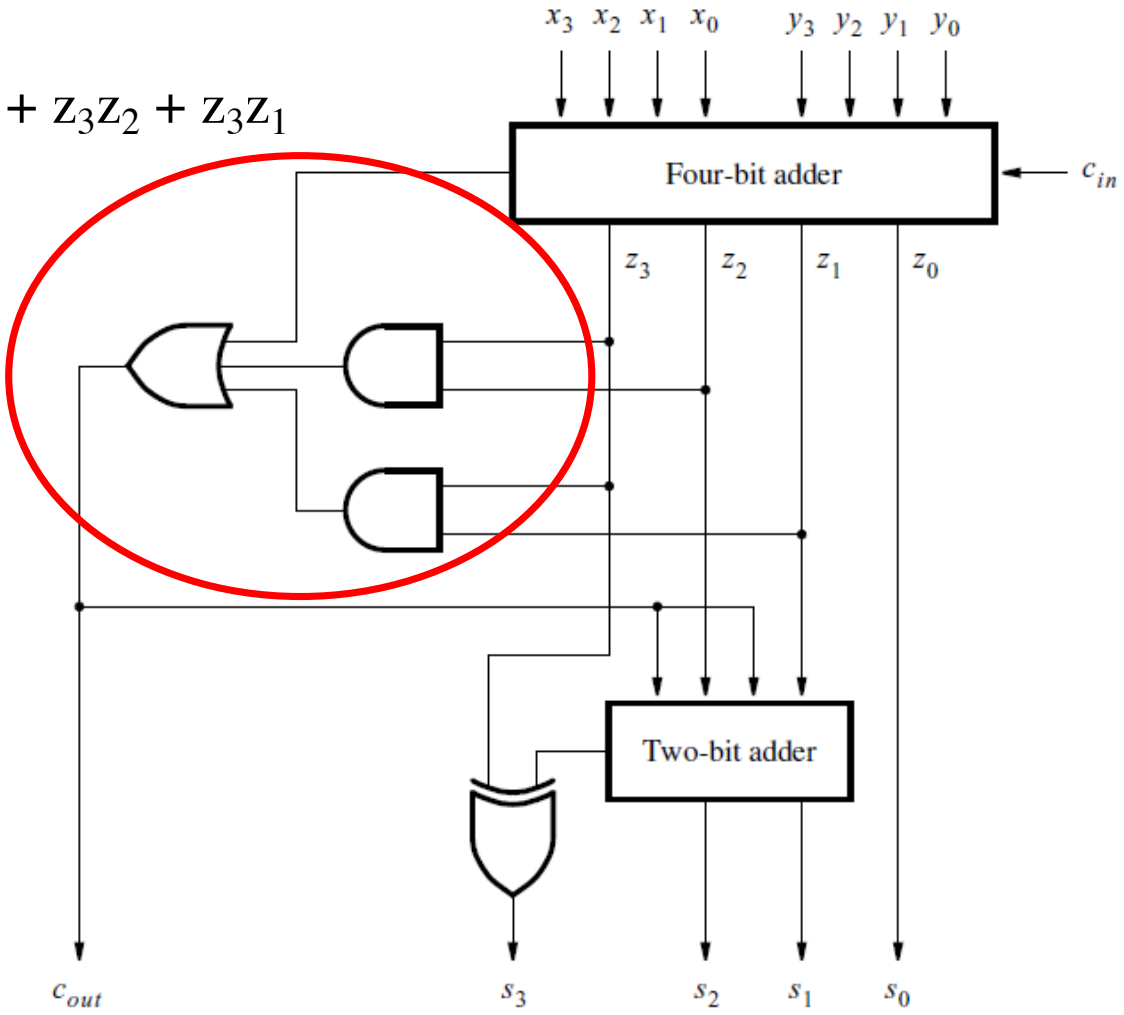
Circuit for a one-digit BCD adder



[Figure 3.41 in the textbook]

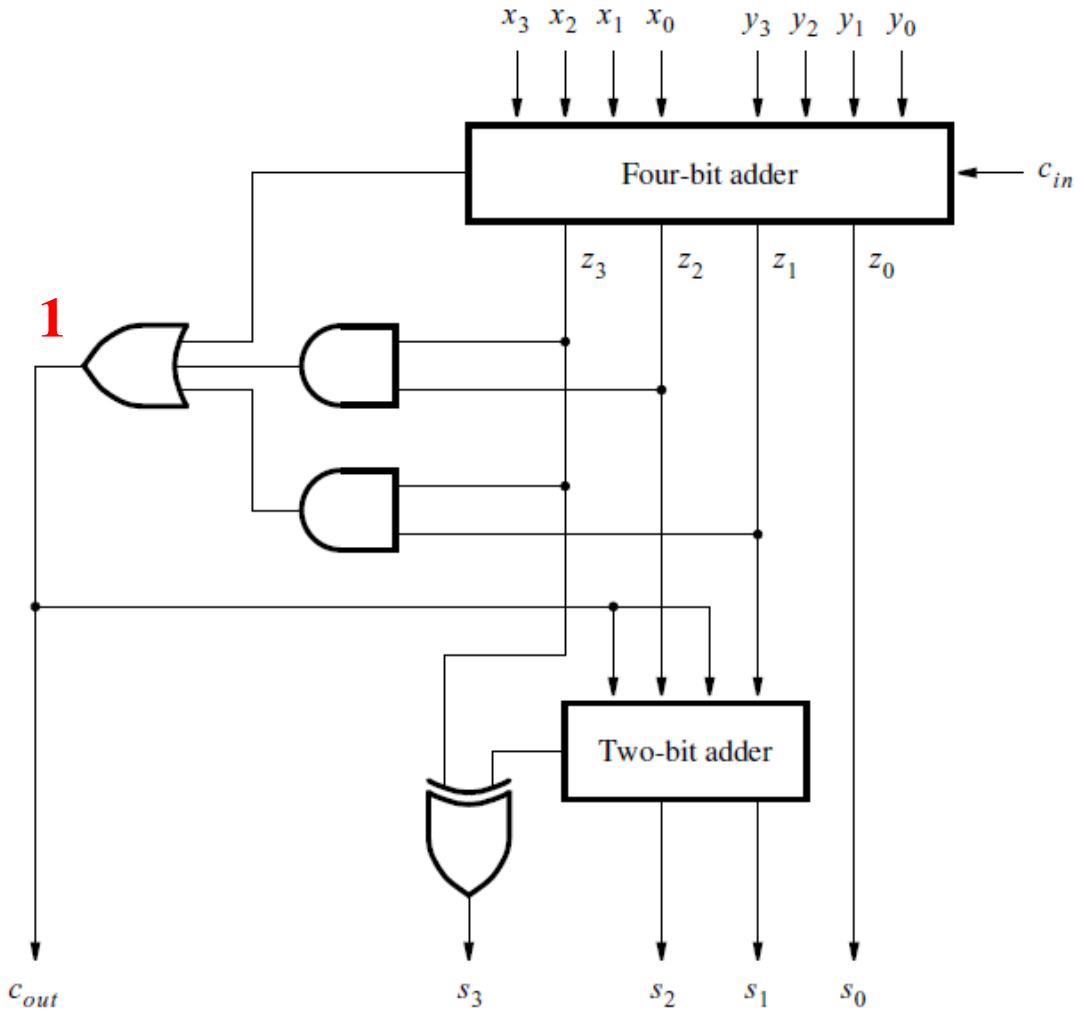
Circuit for a one-digit BCD adder

carry-out + z_3z_2 + z_3z_1



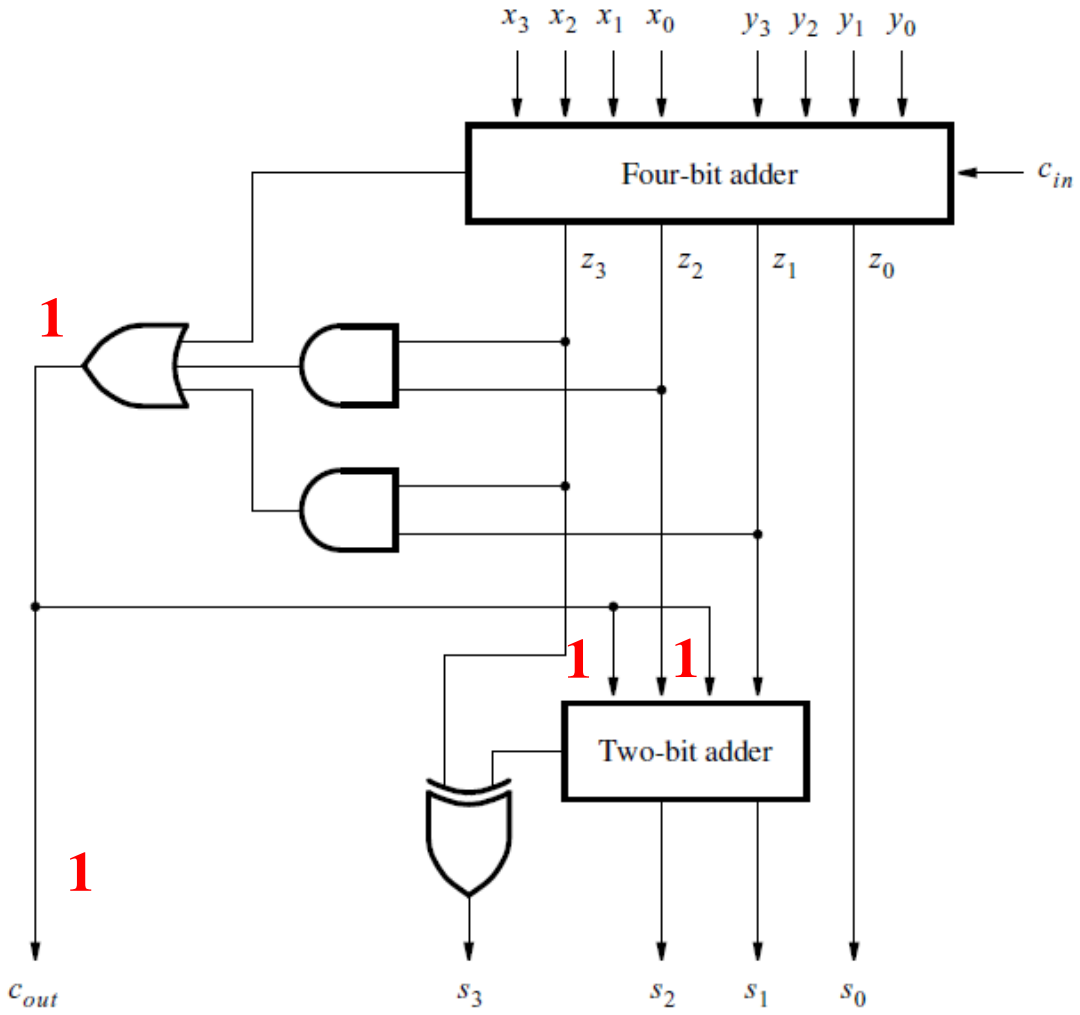
[Figure 3.41 in the textbook]

Circuit for a one-digit BCD adder



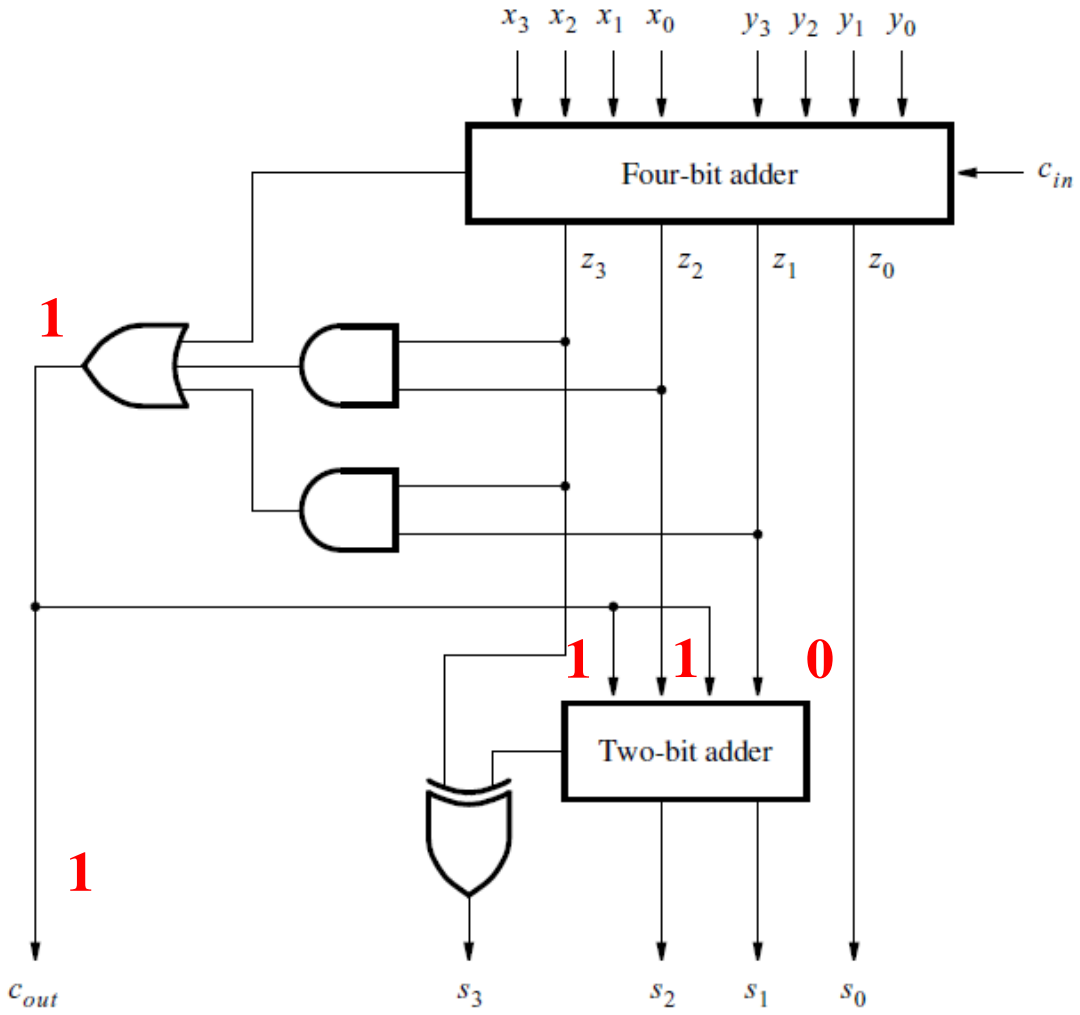
[Figure 3.41 in the textbook]

Circuit for a one-digit BCD adder



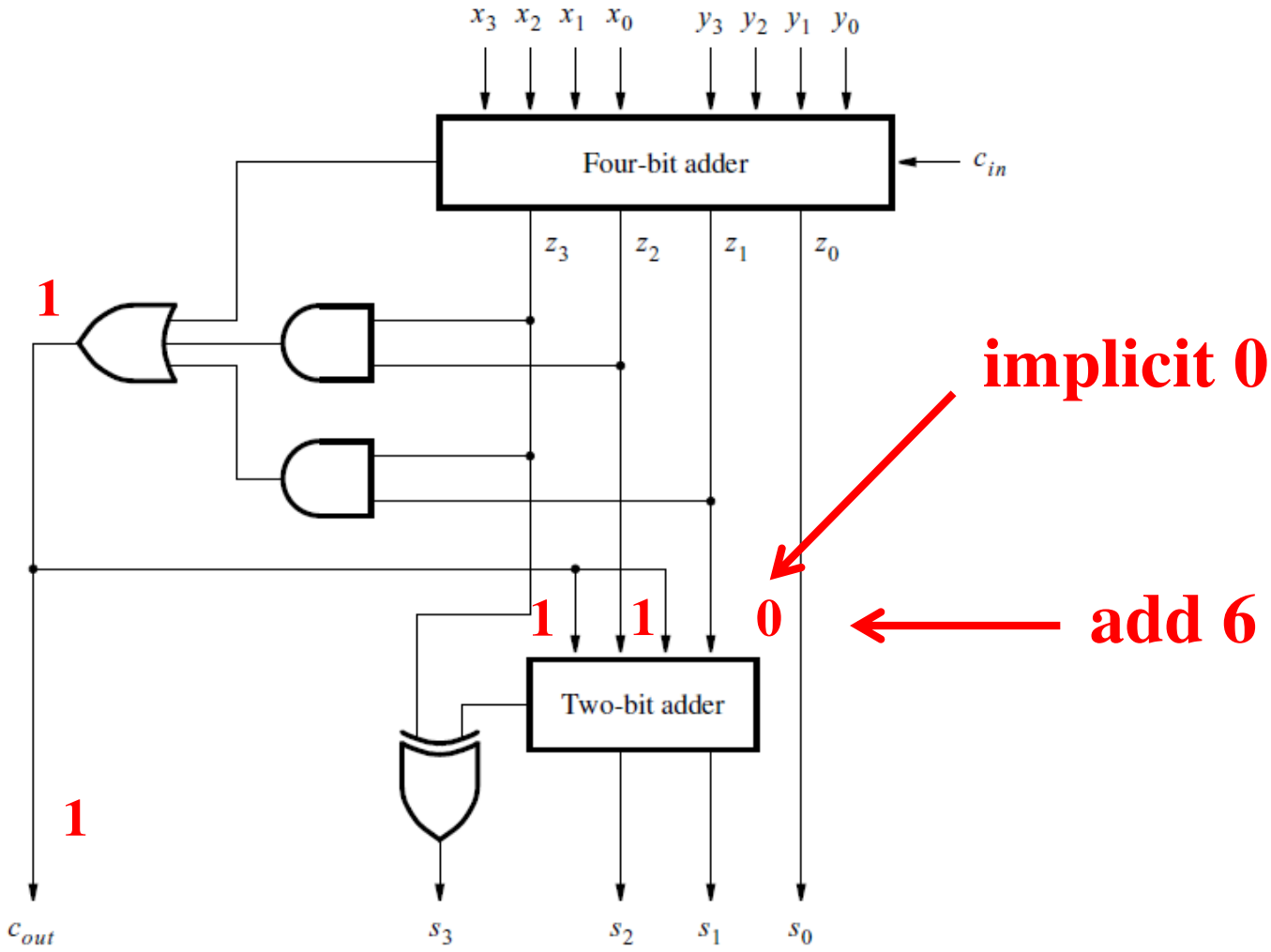
[Figure 3.41 in the textbook]

Circuit for a one-digit BCD adder



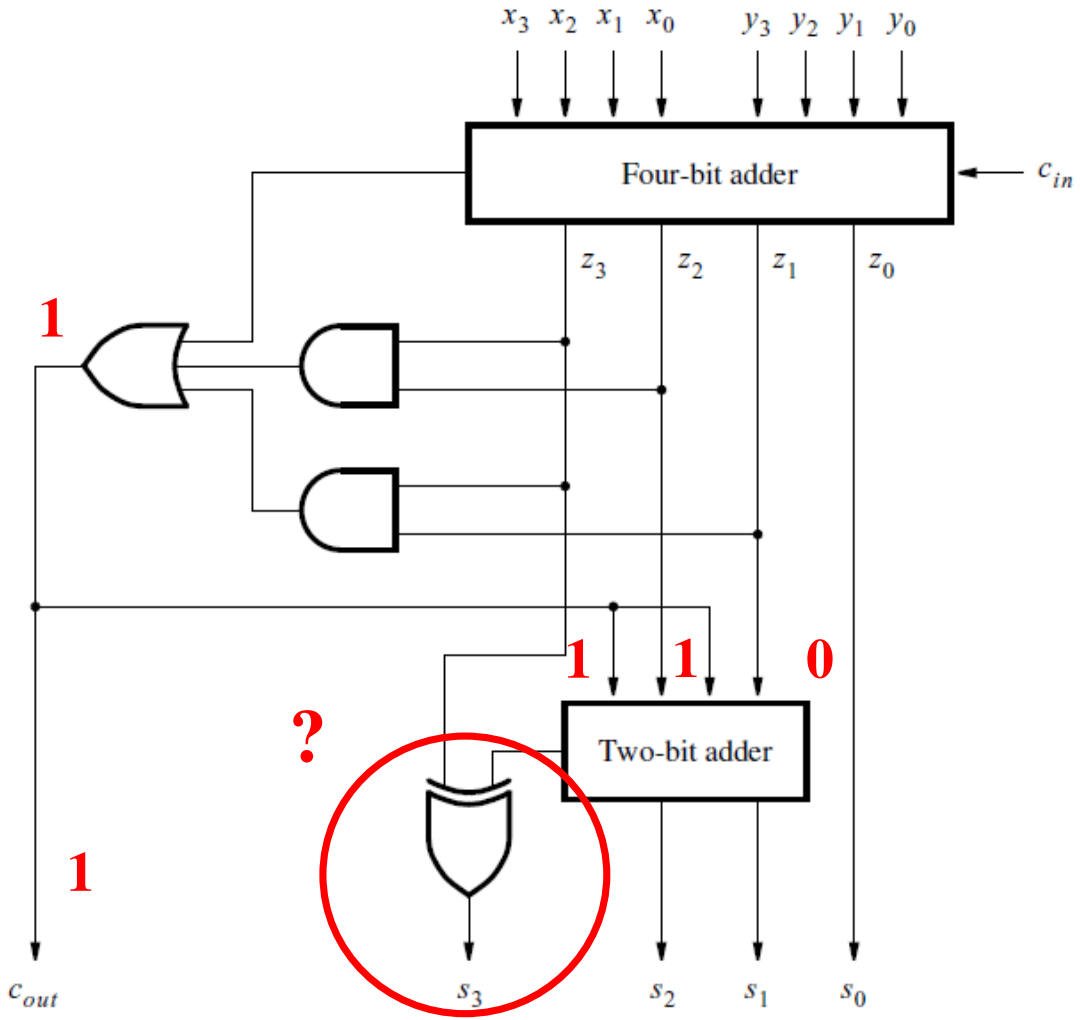
[Figure 3.41 in the textbook]

Circuit for a one-digit BCD adder



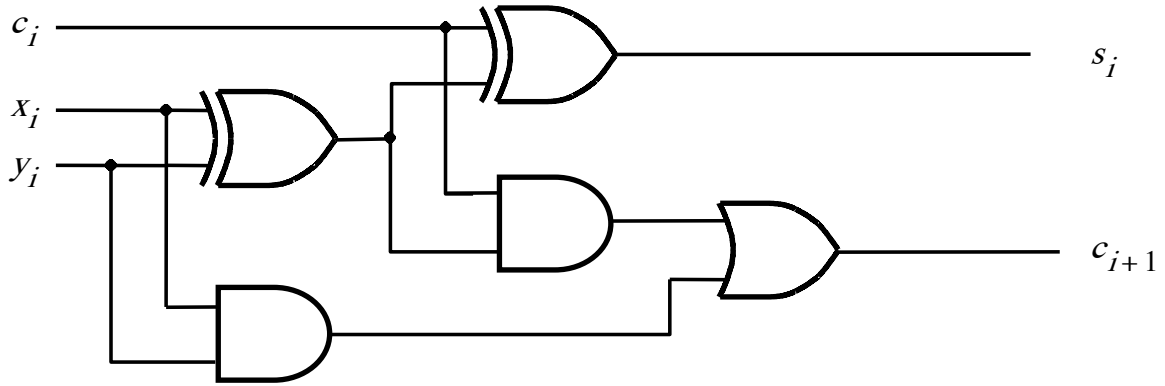
[Figure 3.41 in the textbook]

Circuit for a one-digit BCD adder



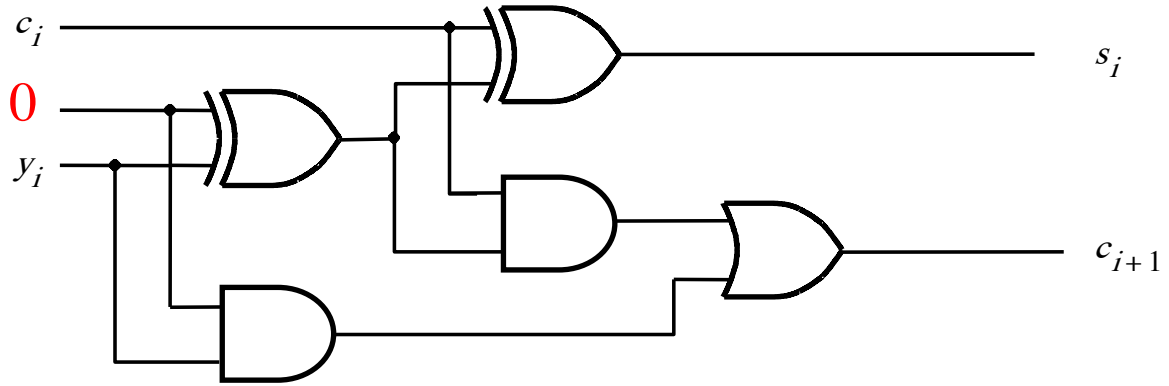
[Figure 3.41 in the textbook]

Simplification of the Full-Adder circuit when x_i=0

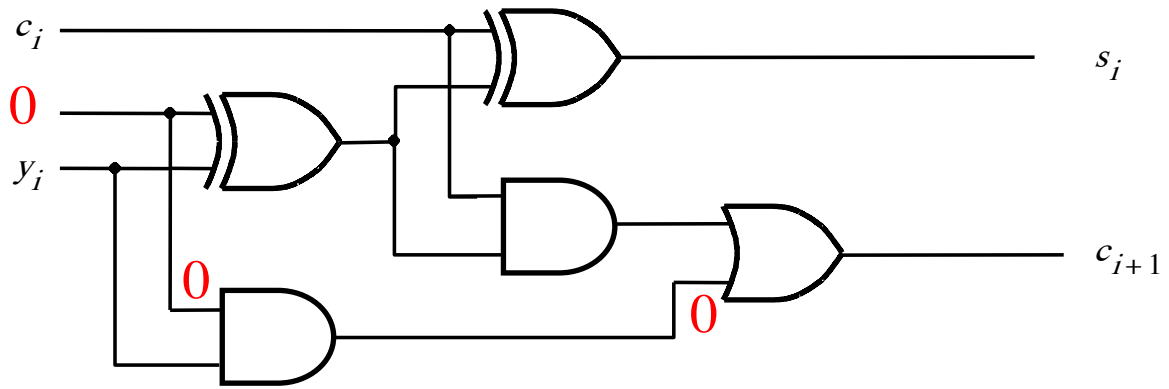


[Figure 3.4b from the textbook]

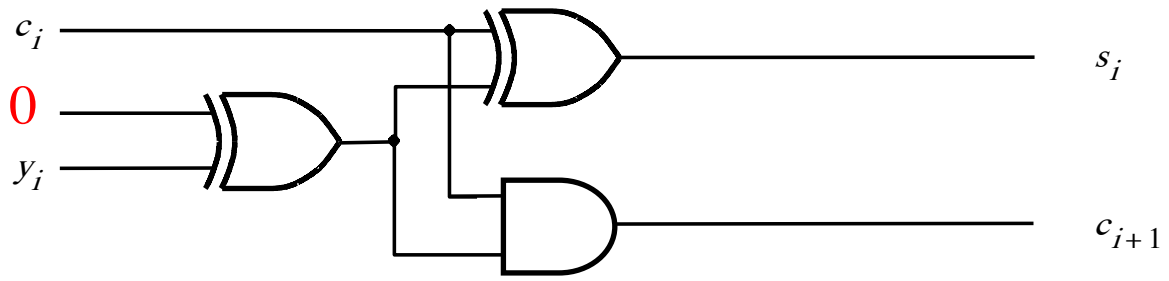
Simplification of the Full-Adder circuit when x_i=0



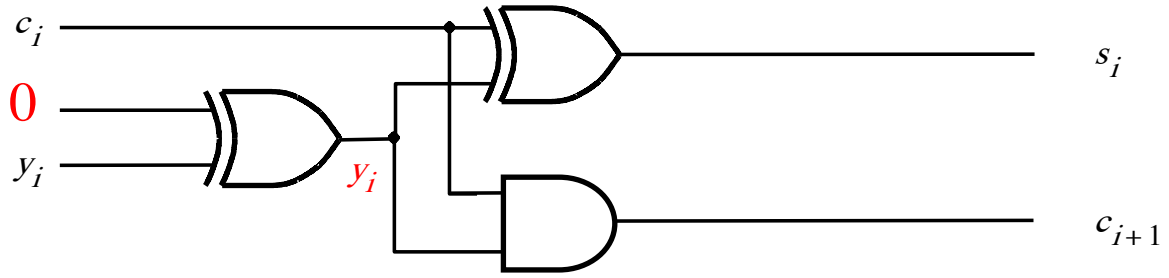
Simplification of the Full-Adder circuit when x_i=0



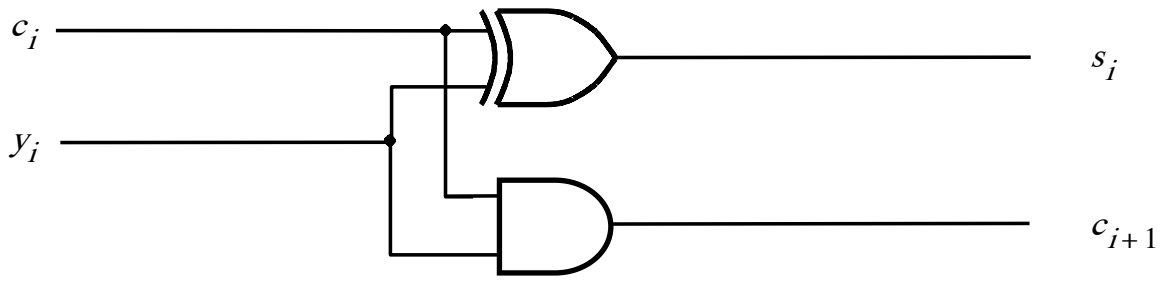
Simplification of the Full-Adder circuit when x_i=0



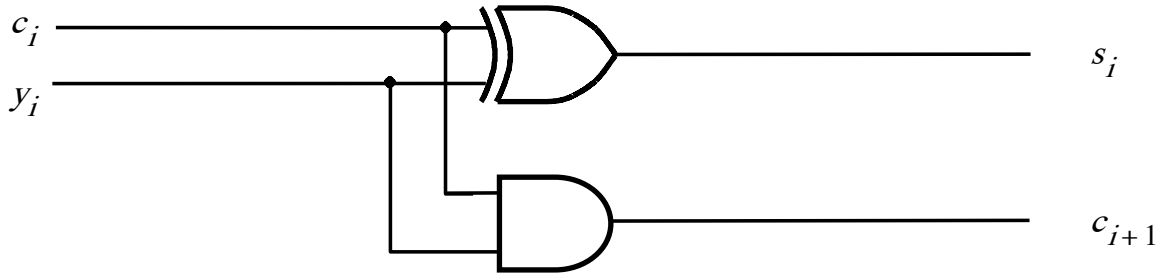
Simplification of the Full-Adder circuit when x_i=0



Simplification of the Full-Adder circuit when x_i=0

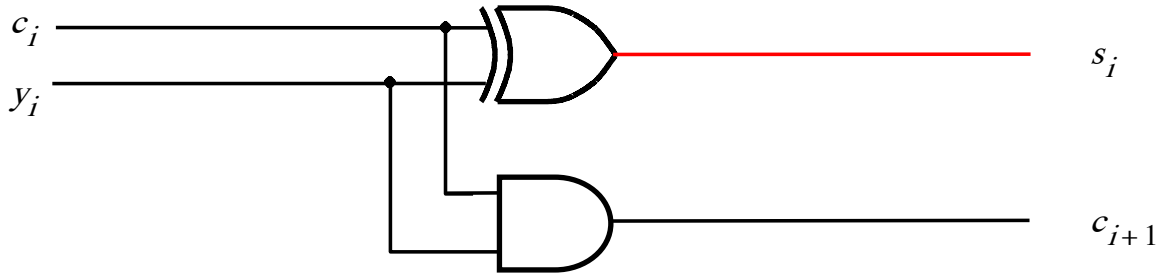


Simplification of the Full-Adder circuit when x_i=0



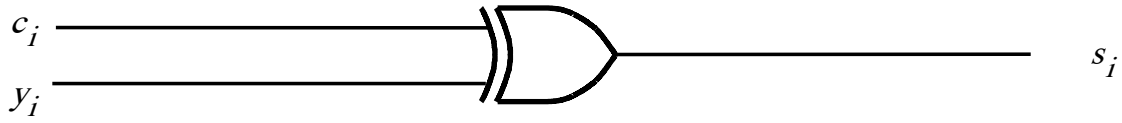
It reduces to a half-adder.

Simplification of the Full-Adder circuit when x_i=0



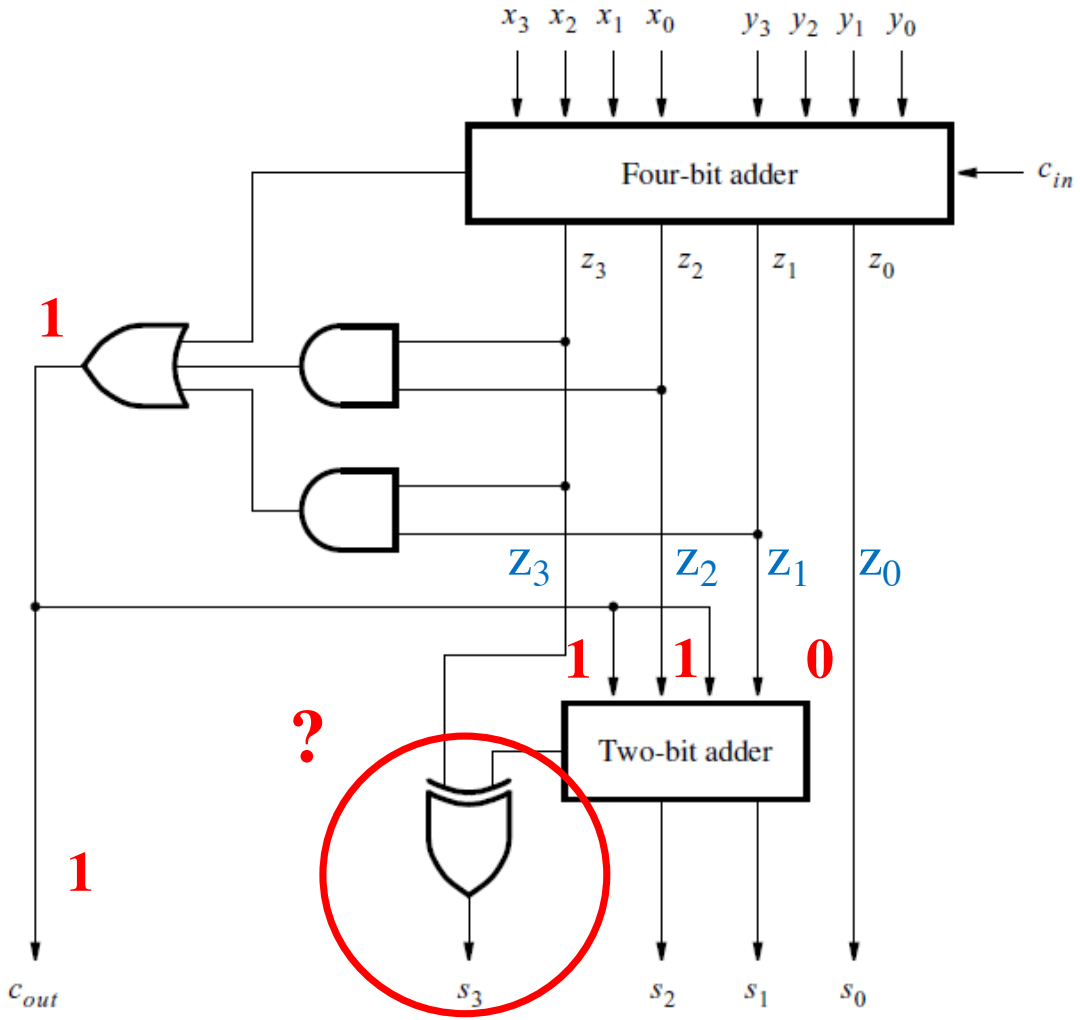
But if we only need the sum bit …

Simplification of the Full-Adder circuit when $x_i=0$



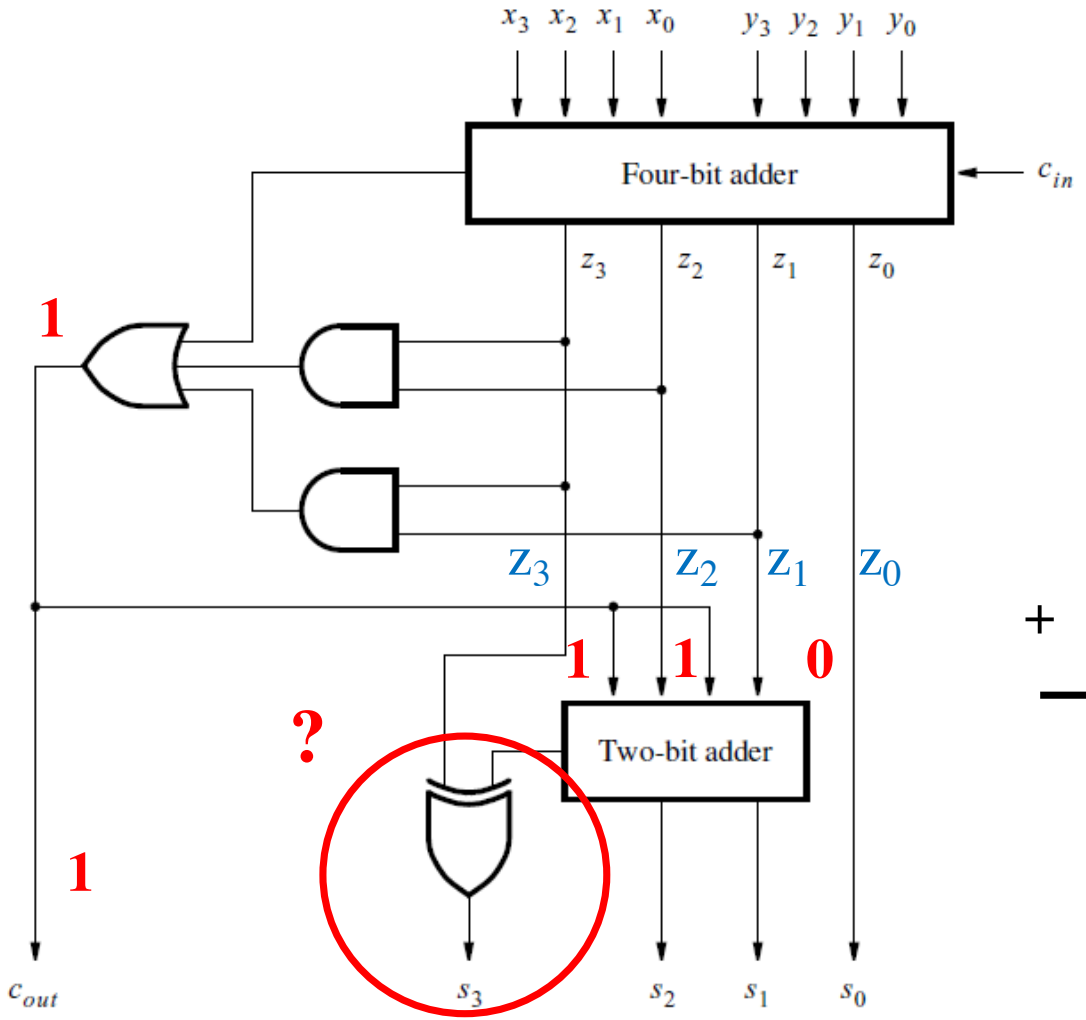
… it reduces to an XOR.

Circuit for a one-digit BCD adder



[Figure 3.41 in the textbook]

Circuit for a one-digit BCD adder



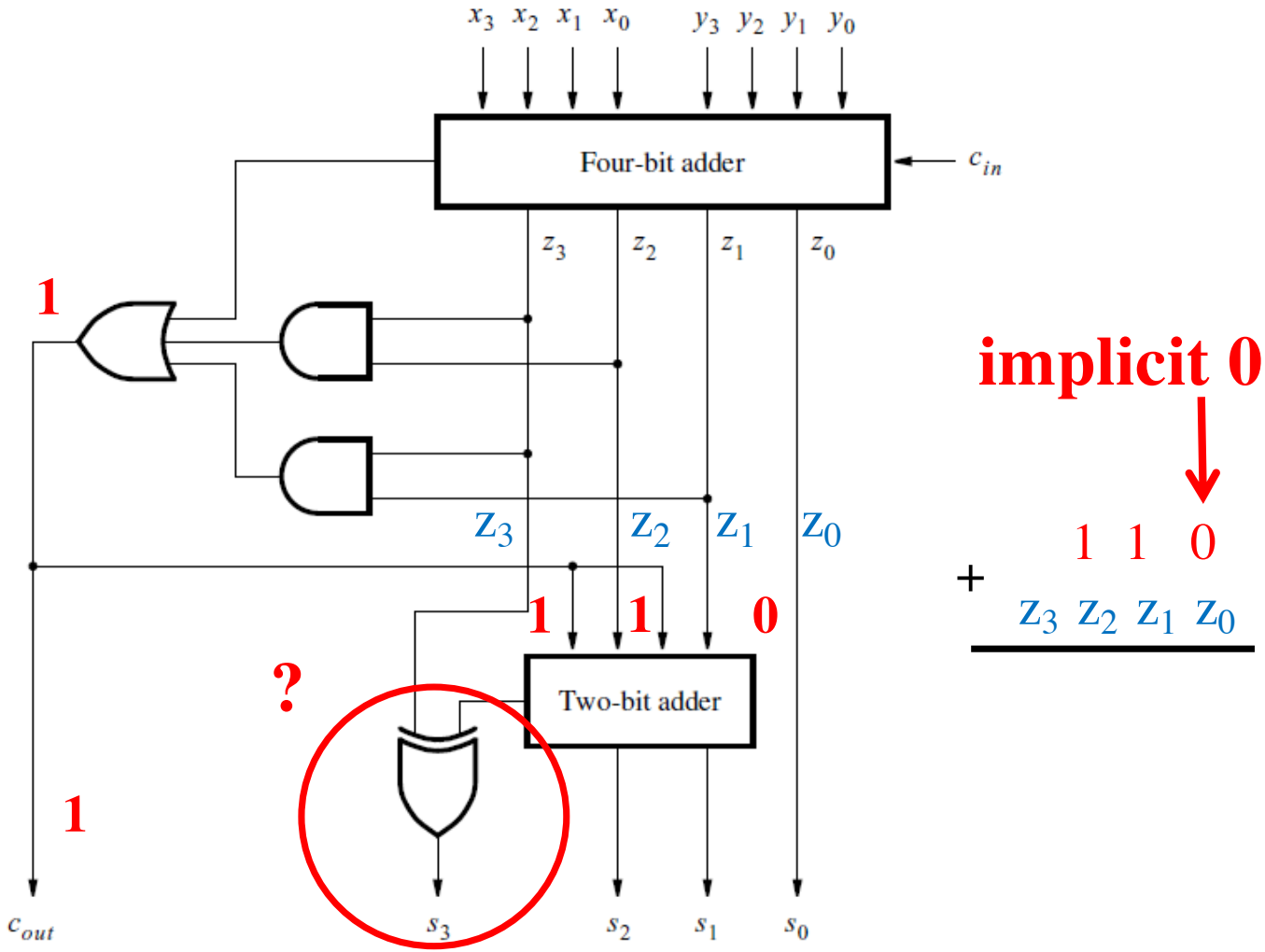
+

1 1 0

Z_3 Z_2 Z_1 Z_0

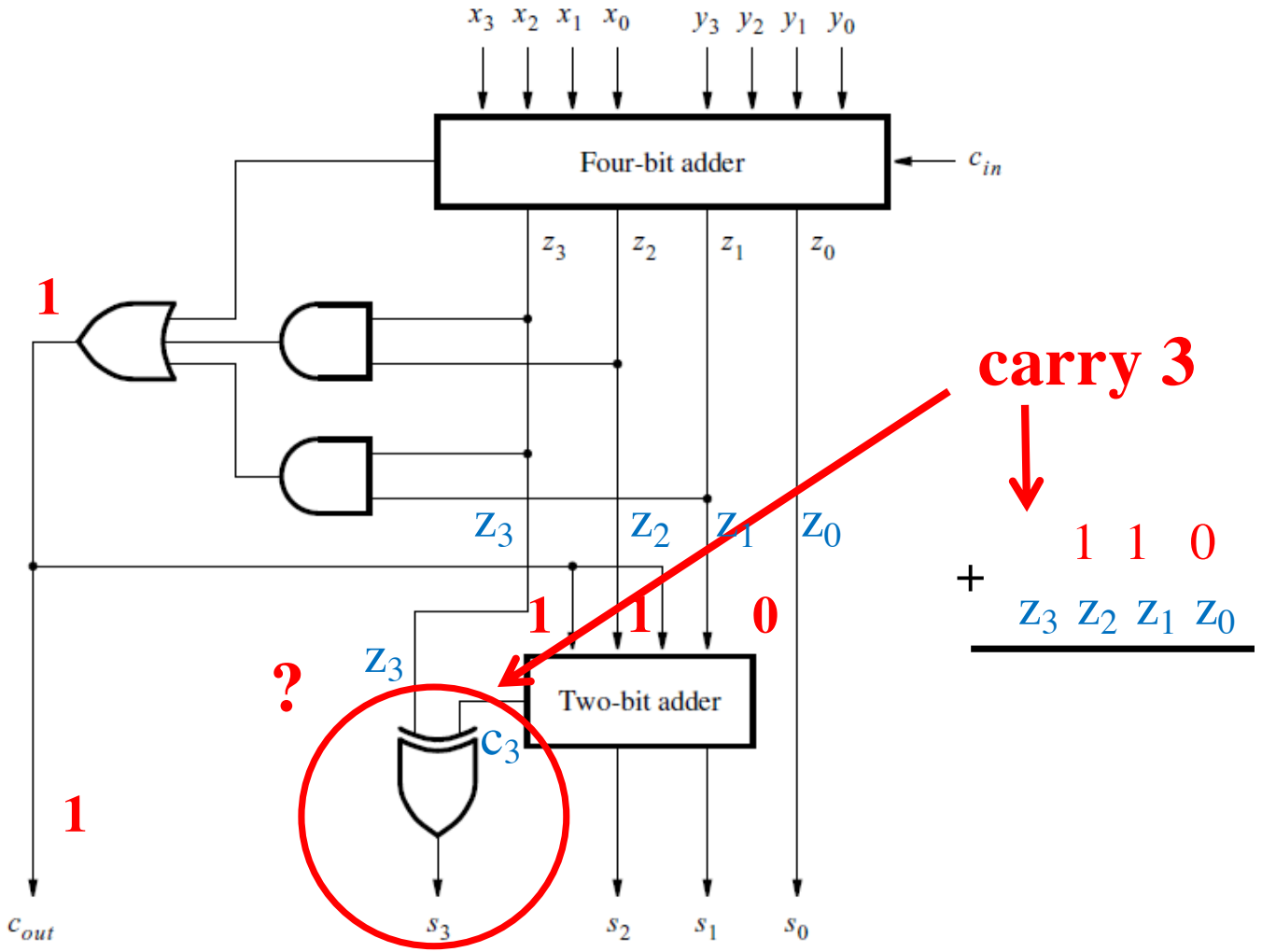
[Figure 3.41 in the textbook]

Circuit for a one-digit BCD adder



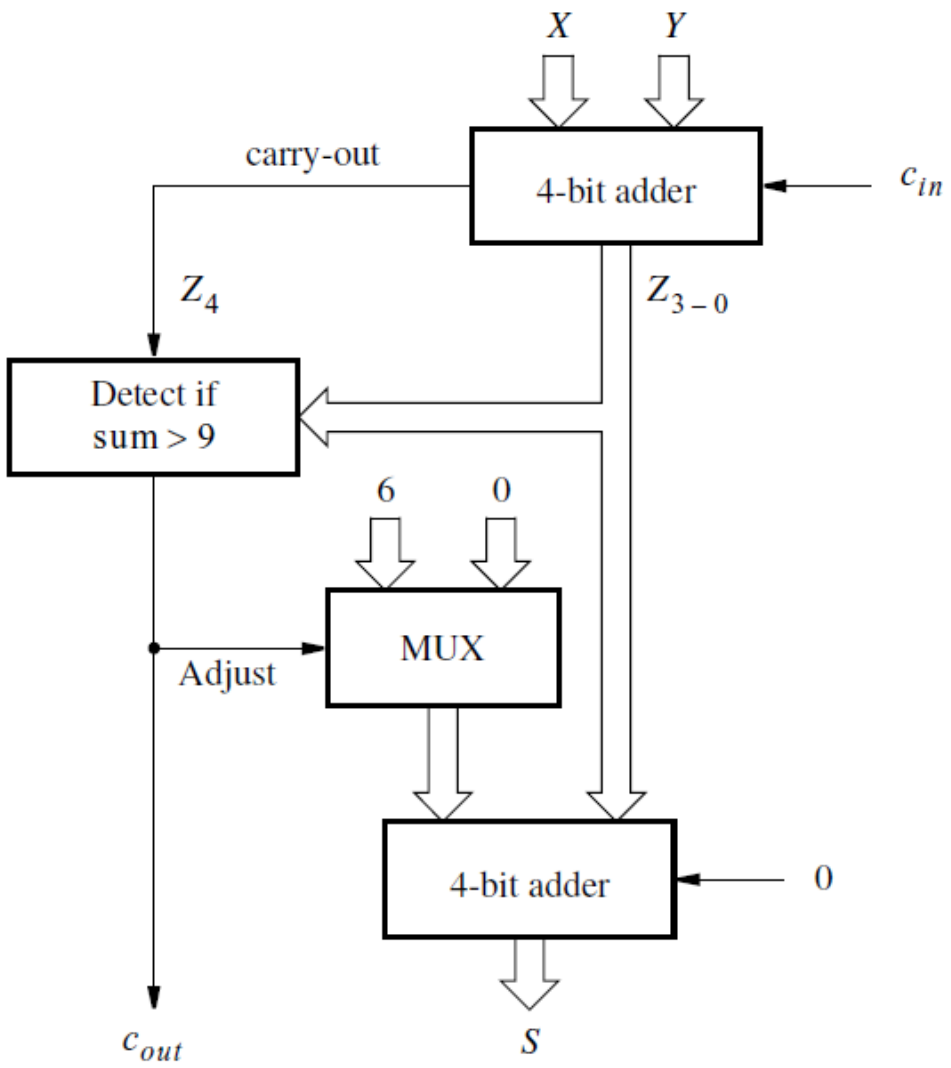
[Figure 3.41 in the textbook]

Circuit for a one-digit BCD adder



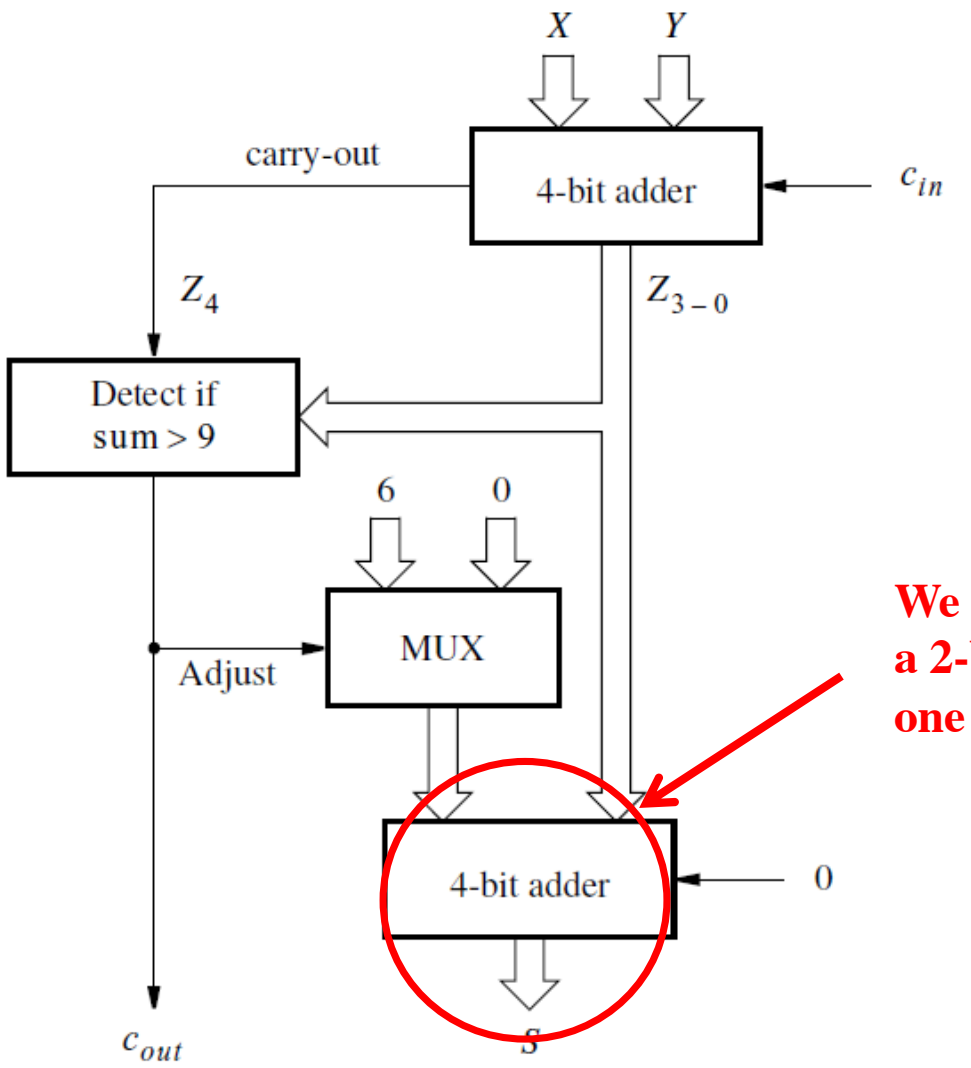
[Figure 3.41 in the textbook]

Block diagram for a one-digit BCD adder



[Figure 3.39 in the textbook]

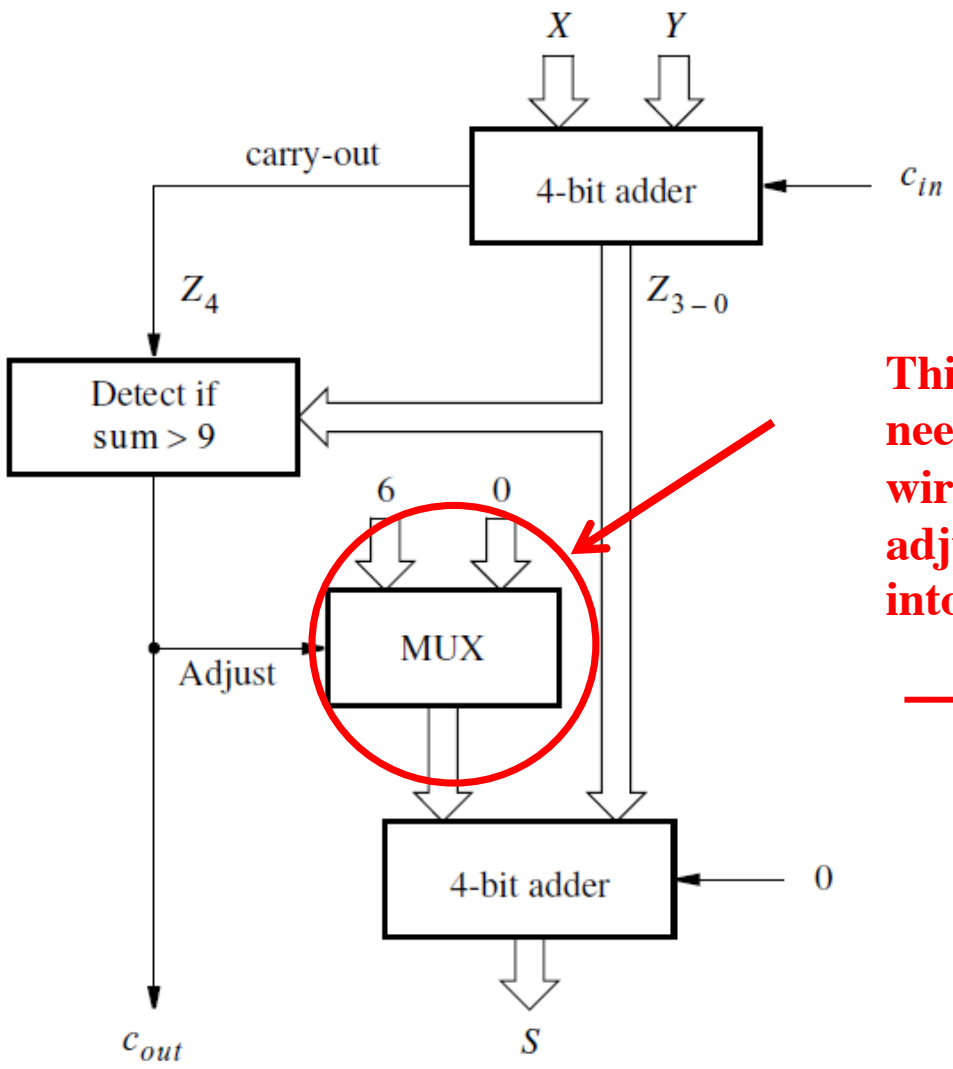
Block diagram for a one-digit BCD adder



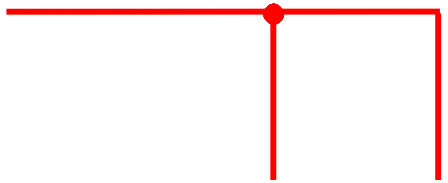
We can get by with a 2-bit adder and one XOR gate.

[Figure 3.39 in the textbook]

Block diagram for a one-digit BCD adder

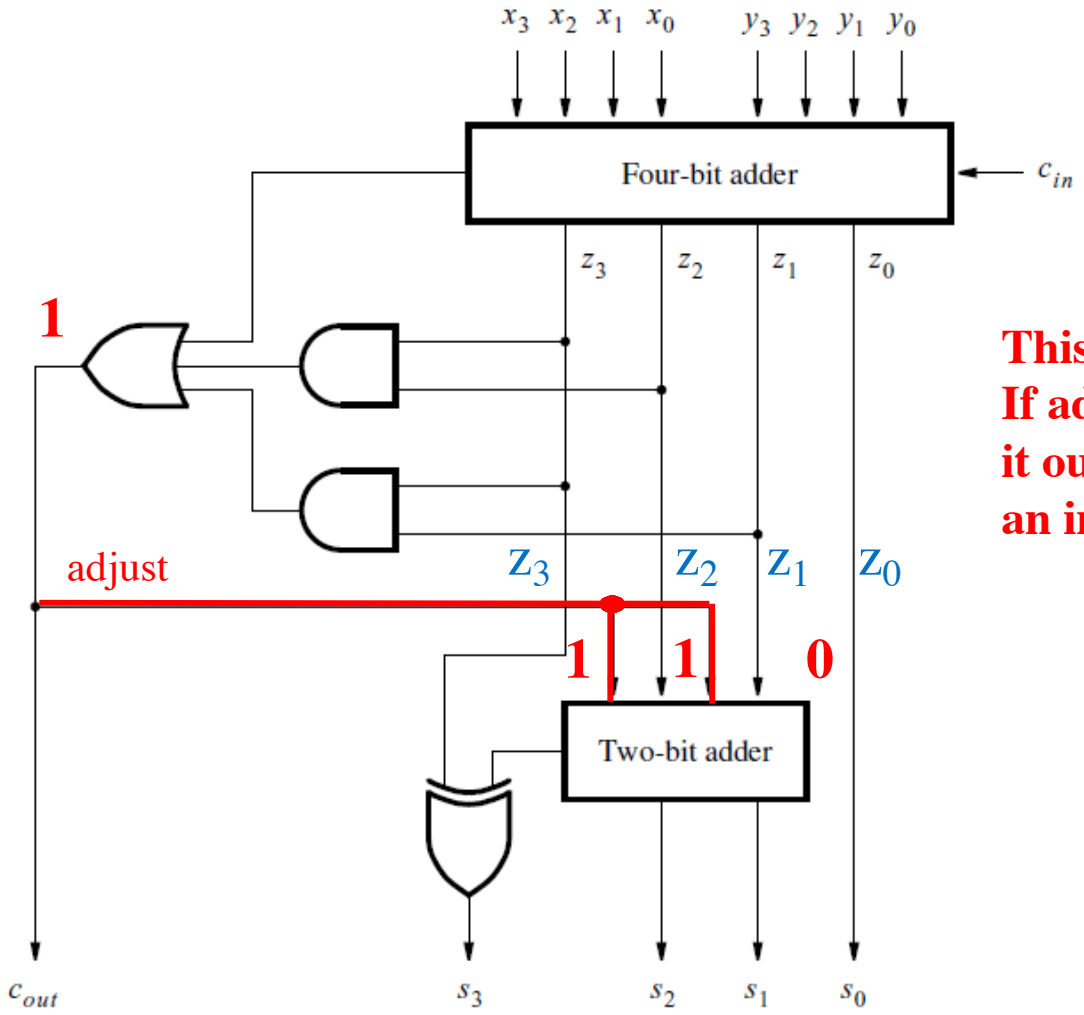


This bus mux is not needed. It reduces to 2 wires that carry the adjust signal and feed into the 2-bit adder.



[Figure 3.39 in the textbook]

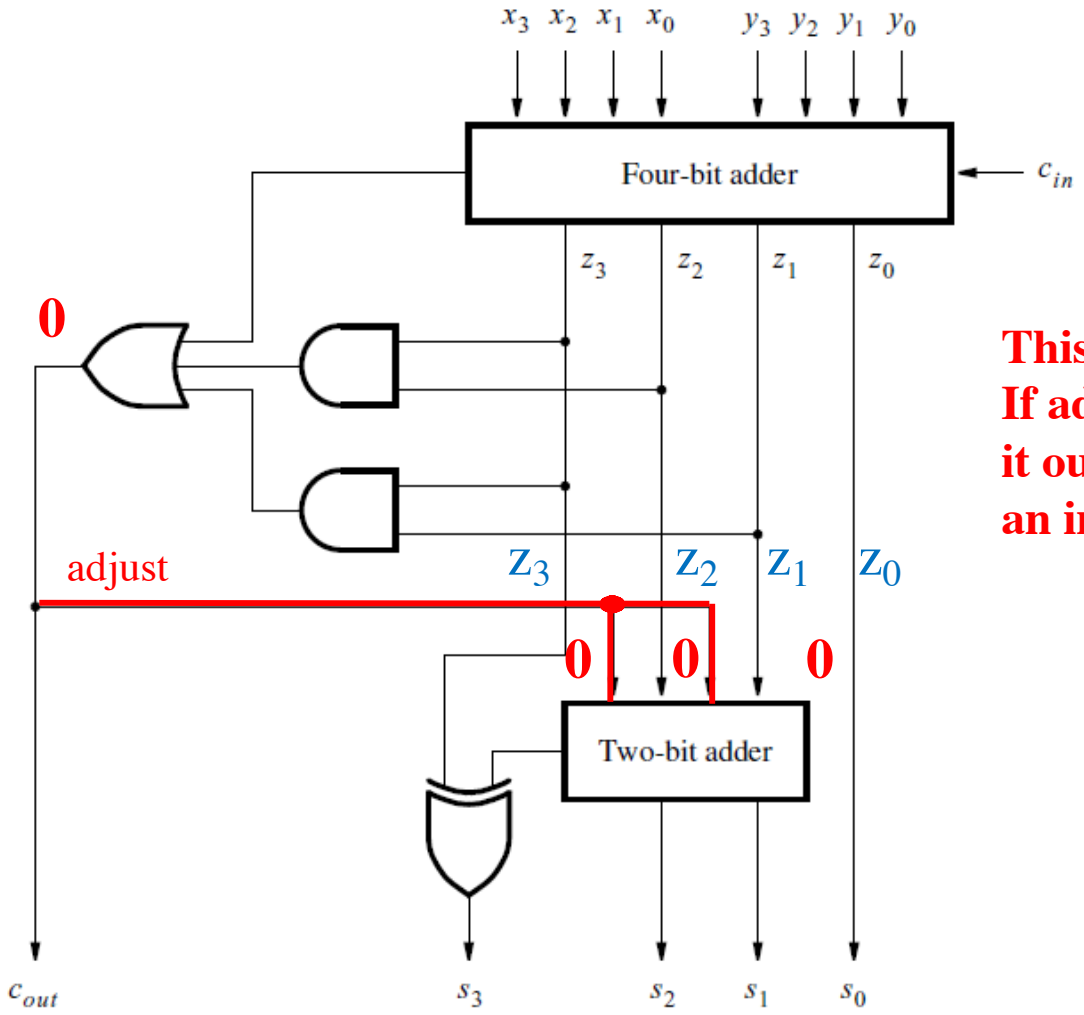
Circuit for a one-digit BCD adder



This is the bus mux!
If adjust is equal to one it outputs +6, i.e., 11 (with an implicit zero).

[Figure 3.41 in the textbook]

Circuit for a one-digit BCD adder



This is the bus mux!
If adjust is equal to zero it outputs 0, i.e., 00 (with an implicit zero).

[Figure 3.41 in the textbook]

Questions?

THE END