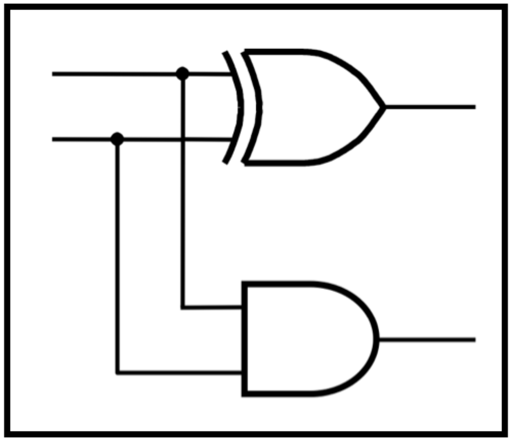


CprE 2810: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

Incompletely Specified Functions & Multiple-Output Circuits

CprE 2810: Digital Logic
Iowa State University, Ames, IA

Administrative Stuff

- HW4 is due on Monday Sep 22 @ 10 pm
- It is posted on the class web page
- I also sent you an e-mail with the link.

Administrative Stuff

- HW5 is due on Monday Sep 29 @ 10 pm
- It is posted on the class web page
- I also sent you an e-mail with the link.

Administrative Stuff

- Midterm Exam #1
- When: Friday Sep 26.
- What: Chapter 1 and Chapter 2 plus number systems
- The exam will be closed book but open notes
  (you can bring up to 3 pages of handwritten notes).
- It will be in this room.
- Sample exams are posted on the class web page.

Topics for the Midterm Exam

- Binary Numbers
- Octal Numbers
- Hexadecimal Numbers
- Conversion between the different number systems
- Truth Tables
- Boolean Algebra
- Logic Gates
- Circuit Synthesis with AND, OR, NOT
- Circuit Synthesis with NAND, NOR
- Converting an AND/OR/NOT circuit to NAND circuit
- Converting an AND/OR/NOT circuit to NOR circuit
- SOP and POS expressions

Topics for the Midterm Exam

- Mapping a Circuit to Verilog code
- Mapping Verilog code to a circuit

- Multiplexers
- Venn Diagrams
- K-maps for 2, 3, and 4 variables

- Minimization of Boolean expressions using theorems
- Minimization of Boolean expressions with K-maps

- Incompletely specified functions (with don't cares)
- Functions with multiple outputs

- Something from Star Wars

One-Variable K-Map

One-Variable K-map

x_1	f
0	m_0
1	m_1

(a) Truth table

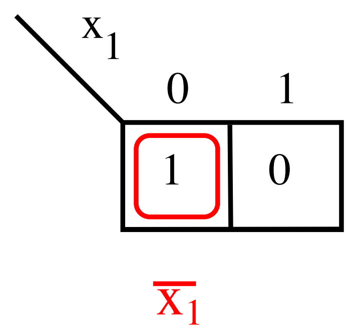
x_1	0	1
	m_0	m_1

(b) Karnaugh map

One-Variable K-map

x_1	f
0	1
1	0

(a) Truth table



(b) Karnaugh map

Two-Variable K-Map

Two-Variable K-map

x_1	x_2	
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

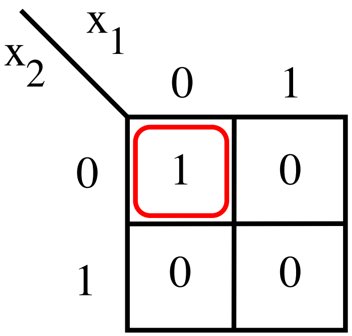
(a) Truth table

x_2 \ x_1	0	1
0	m_0	m_2
1	m_1	m_3

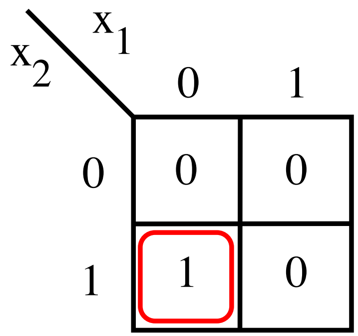
(b) Karnaugh map

[Figure 2.49 from the textbook]

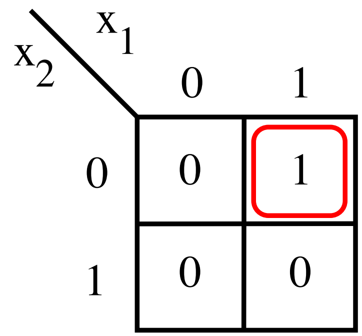
These are all valid groupings



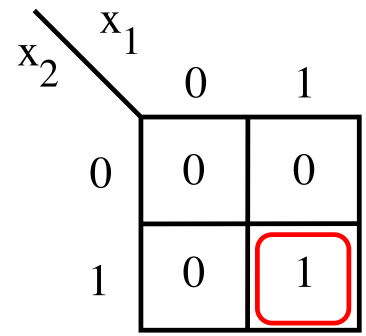
$\bar{x}_1 \bar{x}_2$



$\bar{x}_1 x_2$

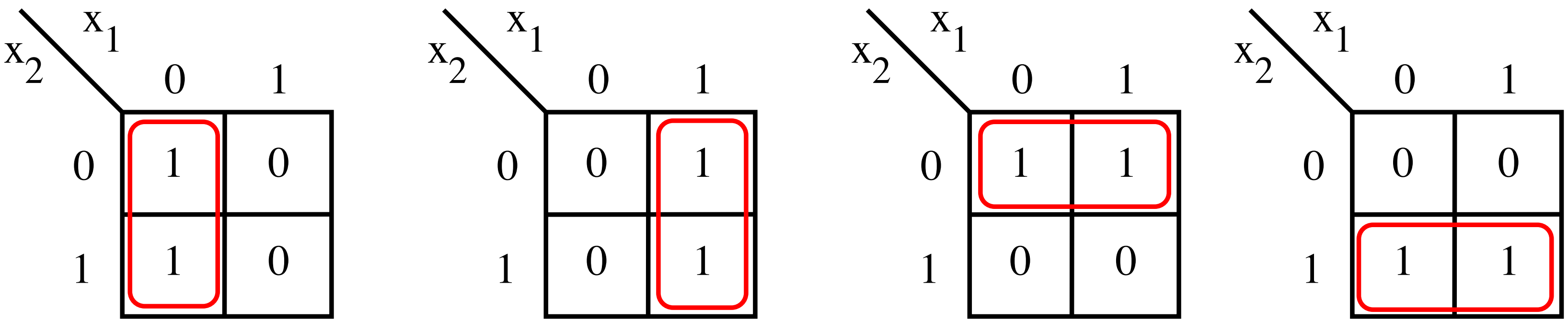


$x_1 \bar{x}_2$



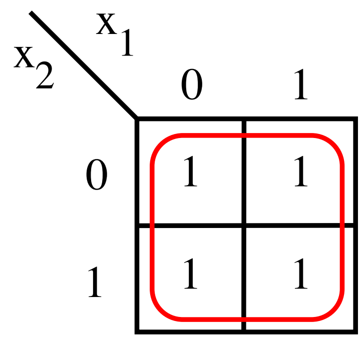
$x_1 x_2$

These are all valid groupings



$\bar{x}_1$ x_1 $\bar{x}_2$ x_2

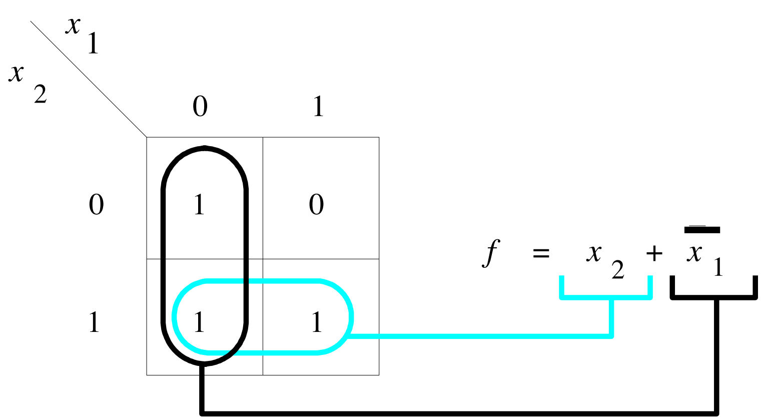
This one is valid too



In this case the result is the constant function 1.

Why are these two not valid?

Minimization Example with a two-variable K-map



[Figure 2.50 from the textbook]

Three-Variable K-Map

Three-Variable K-map

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3 \ $x_1 x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

[Figure 2.51 from the textbook]

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

$x_3 \backslash x_1x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

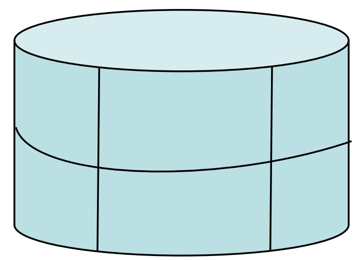
Notice the placement of

- **Variables**
- **Binary pair values**
- **Minterms**

Adjacency Rules

$x_3 \backslash x_1x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

adjacent
columns

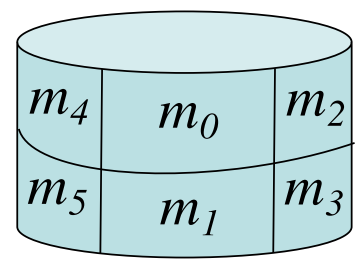


As if the K-map were
drawn on a cylinder

Adjacency Rules

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

adjacent
columns



As if the K-map were
drawn on a cylinder

Some Invalid Groupings

x_3 \ x_1x_2	00	01	11	10
0	0	1	0	0
1	0	0	1	0

x_3 \ x_1x_2	00	01	11	10
0	0	0	1	0
1	0	1	0	0

Can’t group diagonally.

Some Invalid Groupings

x_3 \ x_1x_2	00	01	11	10
0	1	1	1	0
1	0	0	0	0

x_3 \ x_1x_2	00	01	11	10
0	0	0	0	0
1	0	1	1	1

Can't group three in a row.
Each side must be a power of 2.

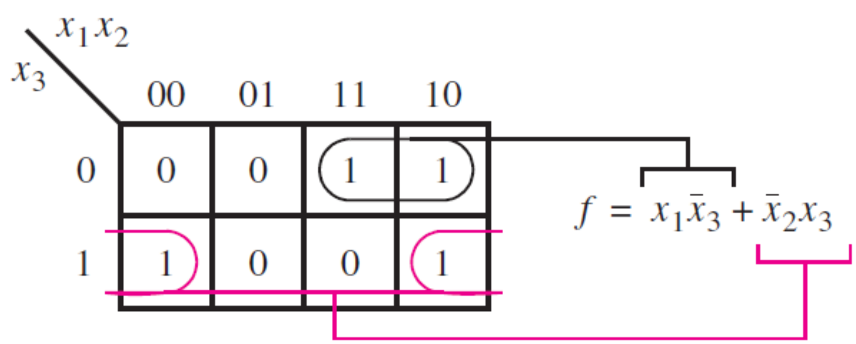
Some Invalid Groupings

x_3 \ x_1x_2	00	01	11	10
0	1	**0**	1	1
1	0	0	0	0

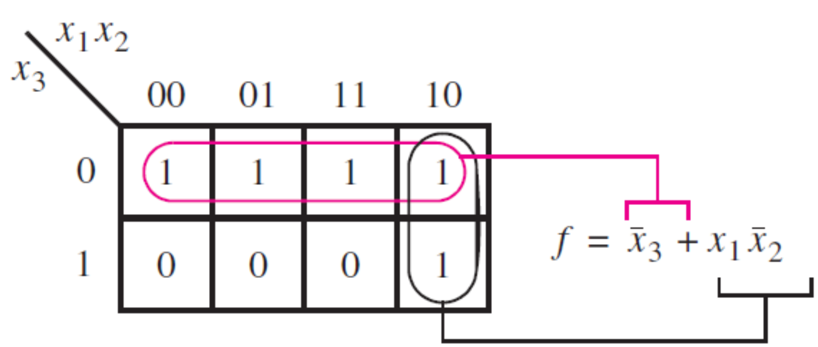
x_3 \ x_1x_2	00	01	11	10
0	0	**0**	1	0
1	0	1	1	0

Can’t group zeros and ones together.

Three-Variable K-map



(a) The function of Figure 2.23



(b) The function of Figure 2.48

[Figure 2.52 from the textbook]

From Boolean Expression to K-map

From Boolean Expression to K-map

$$F = \bar{A}\,\bar{C} + \bar{A}\,\bar{B} + \bar{A}\,B\,C$$

C \ AB	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

From Boolean Expression to K-map

$$F = \boxed{\overline{A}\,\overline{C}} + \overline{A}\,\overline{B} + \overline{A}\,B\,C$$

C \ AB	00	01	11	10
0				
1				

From Boolean Expression to K-map

$$F = \boxed{\overline{A}\,\overline{C}} + \overline{A}\,\overline{B} + \overline{A}\,B\,C$$

C \ AB	00	01	11	10
0	1	1		
1				

From Boolean Expression to K-map

$$F = \overline{A}\,\overline{C} + \boxed{\overline{A}\,\overline{B}} + \overline{A}\,B\,C$$

C \ AB	00	01	11	10
0	1	1		
1				

From Boolean Expression to K-map

$$F = \bar{A}\,\bar{C} + \boxed{\bar{A}\,\bar{B}} + \bar{A}\,B\,C$$

C \ AB	00	01	11	10
0	1	1		
1	1			

From Boolean Expression to K-map

$$F = \bar{A}\,\bar{C} + \bar{A}\,\bar{B} + \boxed{\bar{A}\,B\,C}$$

C \ AB	00	01	11	10
0	1	1		
1	1			

From Boolean Expression to K-map

$$F = \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \boxed{\overline{A}\,B\,C}$$

C \ AB	00	01	11	10
0	1	1		
1	1	1		

From Boolean Expression to K-map

$$F = \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{A}\,B\,C$$

C \ AB	00	01	11	10
0	1	1	0	0
1	1	1	0	0

From Boolean Expression to K-map

$$F = \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{A}\,B\,C$$

C \ AB	00	01	11	10
0	1	1	0	0
1	1	1	0	0

$\overline{A}$

From Boolean Expression to K-map

$$F = \bar{A}\,\bar{C} + \bar{A}\,\bar{B} + \bar{A}\,B\,C = \bar{A}$$

C \ AB	00	01	11	10
0	1	1	0	0
1	1	1	0	0

$\bar{A}$

Different Ways to Draw the K-map

Two Different Ways to Draw the K-map

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3 \ $x_1 x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

x_1 \ $x_2 x_3$	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Another Way to Draw 3-variable K-map

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3 \ $x_1 x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

x_2x_3 \ x_1	0	1
00	m_0	m_4
01	m_1	m_5
11	m_3	m_7
10	m_2	m_6

There are 4 different versions!

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

x_1 \ x_2x_3	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

x_1x_2 \ x_3	0	1
00	m_0	m_1
01	m_2	m_3
11	m_6	m_7
10	m_4	m_5

x_2x_3 \ x_1	0	1
00	m_0	m_4
01	m_1	m_5
11	m_3	m_7
10	m_2	m_6

Why is it OK to combine a group of four ones?

Why can we group these four ones?

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

Theorem 14a is behind the K-Map theory.
But that theorem is just for two variables.
Why is this grouping of four ones possible?

The K-Map theory uses the combining theorems of Boolean algebra

14a. $x \bullet y + x \bullet \overline{y} = x$

14b. $(x + y) \bullet (x + \overline{y}) = x$

The K-Map theory uses the combining theorems of Boolean algebra

optimization by 1's

14a. $x \bullet y + x \bullet \overline{y} = x$

14b. $(x + y) \bullet (x + \overline{y}) = x$

The K-Map theory uses the combining theorems of Boolean algebra

14a. $x \bullet y + x \bullet \overline{y} = x$

14b. $(x + y) \bullet (x + \overline{y}) = x$

optimization by 0's

Why can we group these four ones?

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

Why can we group these four ones?

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

$$f = \bar{x}\, y\, \bar{z} + x\, y\, \bar{z} + \bar{x}\, y\, z + x\, y\, z$$

Why can we group these four ones?

z \ x y	00	01	11	10
0	0	$\boxed{1}$	$\boxed{1}$	0
1	0	$\boxed{1}$	$\boxed{1}$	0

$$f = \underbrace{\bar{x}\, y\, \bar{z} + x\, y\, \bar{z}}_{(\bar{x}\, y + x\, y)\, \bar{z}} + \underbrace{\bar{x}\, y\, z + x\, y\, z}_{(\bar{x}\, y + x\, y)\, z}$$

Why can we group these four ones?

z \ x y	00	01	11	10
0	0	$\boxed{1}$	$\boxed{1}$	0
1	0	$\boxed{1}$	$\boxed{1}$	0

$$f = (\overline{x}\, y + x\, y)\, \overline{z} + (\overline{x}\, y + x\, y)\, z$$

Why can we group these four ones?

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

$$f = \underbrace{(\bar{x}\ y + x\ y)\ \bar{z}}_{y\ \bar{z}\ \text{(by 14a)}} + \underbrace{(\bar{x}\ y + x\ y)\ z}_{y\ z\ \text{(by 14a)}}$$

Why can we group these four ones?

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

$$f = y\,\overline{z} + y\,z$$

Why can we group these four ones?

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

$$f = \underbrace{y\,\bar{z} + y\,z}_{y \text{ (by 14a)}}$$

Why can we group these four ones?

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

$$f = y$$

Why can we group these four ones?

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

Answer: We can combine them because Theorem 14a is applied three times, not just once.

Alternative Derivation

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

Alternative Derivation

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

$$f = \bar{x}\, y\, \bar{z} + x\, y\, \bar{z} + \bar{x}\, y\, z + x\, y\, z$$

Alternative Derivation

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

$$f = \bar{x}\,y\,\bar{z} + \bar{x}\,y\,z + x\,y\,\bar{z} + x\,y\,z$$

Alternative Derivation

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

$$f = \underbrace{\bar{x}\, y\, \bar{z} + \bar{x}\, y\, z}_{\bar{x}\,(y\,\bar{z} + y\,z)} + \underbrace{x\, y\, \bar{z} + x\, y\, z}_{x\,(y\,\bar{z} + y\,z)}$$

Alternative Derivation

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

$$f = \overline{x}\,(y\,\overline{z} + y\,z) + x\,(y\,\overline{z} + y\,z)$$

Alternative Derivation

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

$$f = \underbrace{\bar{x}\,(y\,\bar{z} + y\,z)}_{\bar{x}\,y\ \text{(by 14a)}} + \underbrace{x\,(y\,\bar{z} + y\,z)}_{x\,y\ \text{(by 14a)}}$$

Alternative Derivation

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

$$f = \bar{x}\, y + x\, y$$

Alternative Derivation

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

$$f = \underbrace{\bar{x}\,y + x\,y}_{y \text{ (by 14a)}}$$

Alternative Derivation

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

$$f = y$$

Alternative Derivation

z \ x y	00	01	11	10
0	0	1	1	0
1	0	1	1	0

Answer: We can combine them because Theorem 14a is applied three times, not just once.

Four-Variable K-Map

A four-variable Karnaugh map

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

[Figure 2.53 from the textbook]

A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

x_1 x_2 x_3 x_4

Gray Code & K-map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15

$x_3 x_4$ \ $x_1 x_2$	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

x_1 x_2 x_3 x_4

Gray Code & K-map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15

x_3x_4 \ x_1x_2	00	01	11	10
00	0000	0100	1100	1000
01	0001	0101	1101	1001
11	0011	0111	1111	1011
10	0010	0110	1110	1010

Adjacency Rules

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

adjacent columns

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

adjacent rows

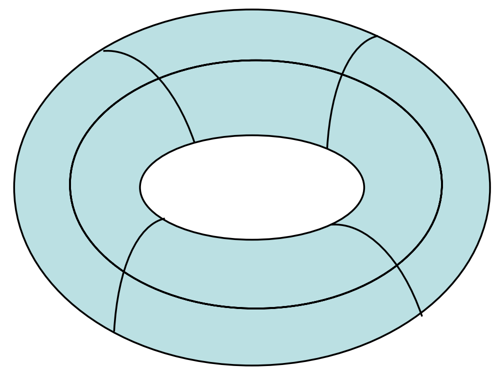
adjacent columns

Adjacency Rules

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

adjacent rows

adjacent columns



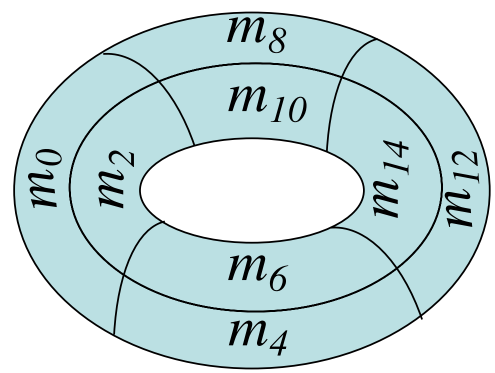
As if the K-map were drawn on a torus

Adjacency Rules

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

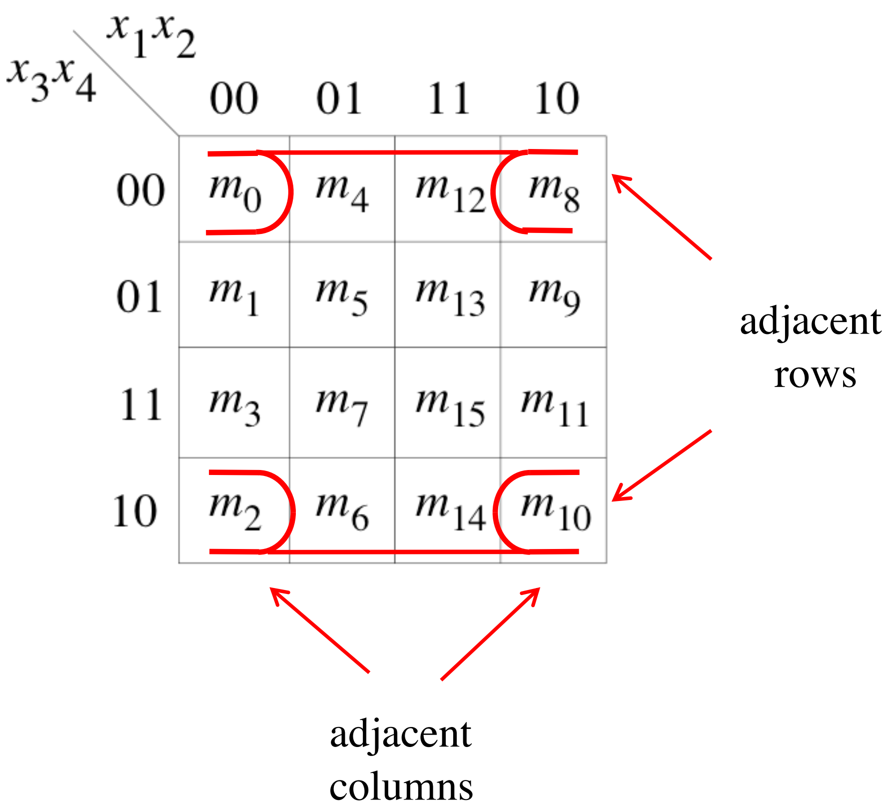
adjacent rows

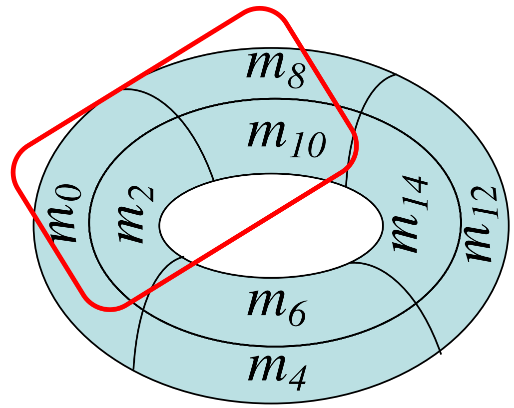
adjacent columns



As if the K-map were drawn on a torus

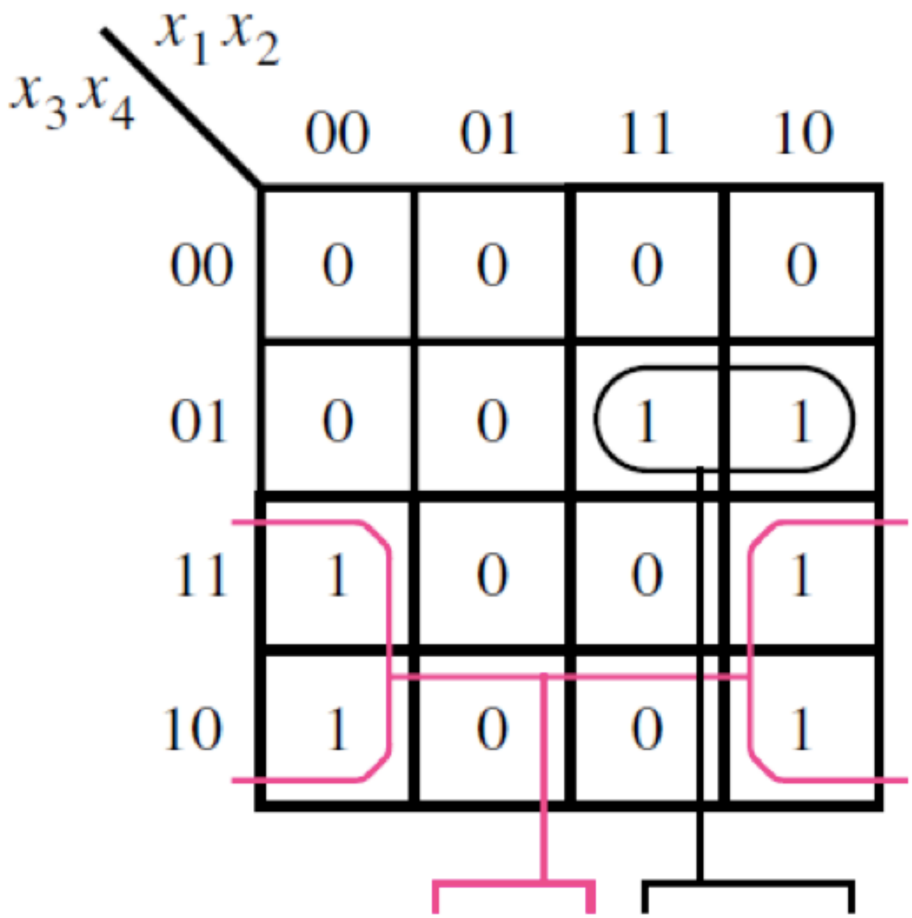
Adjacency Rules





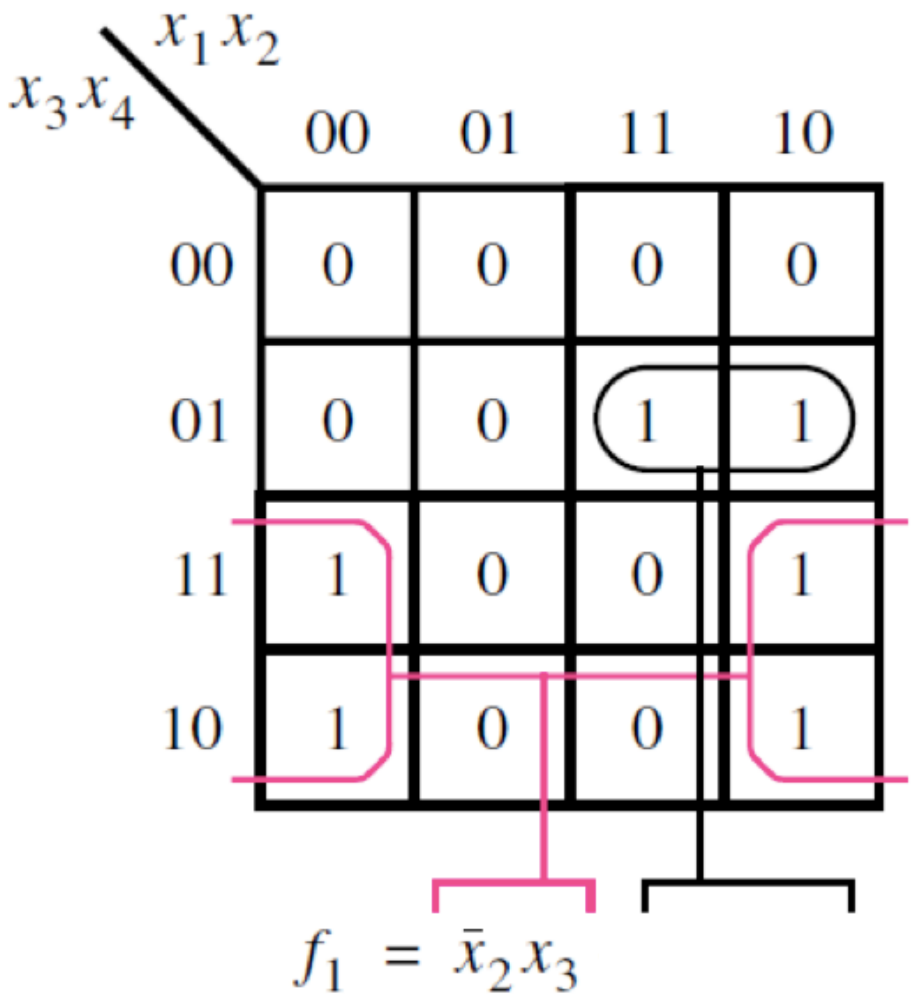
As if the K-map were drawn on a torus

Example of a four-variable Karnaugh map



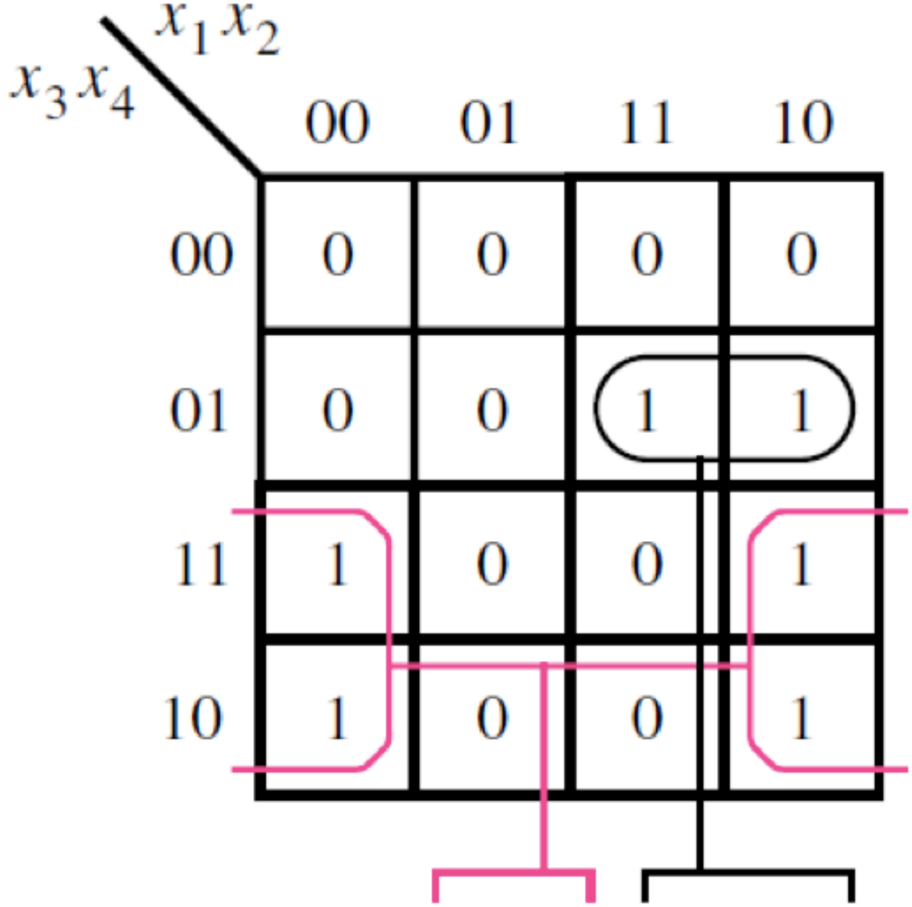
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



[Figure 2.54 from the textbook]

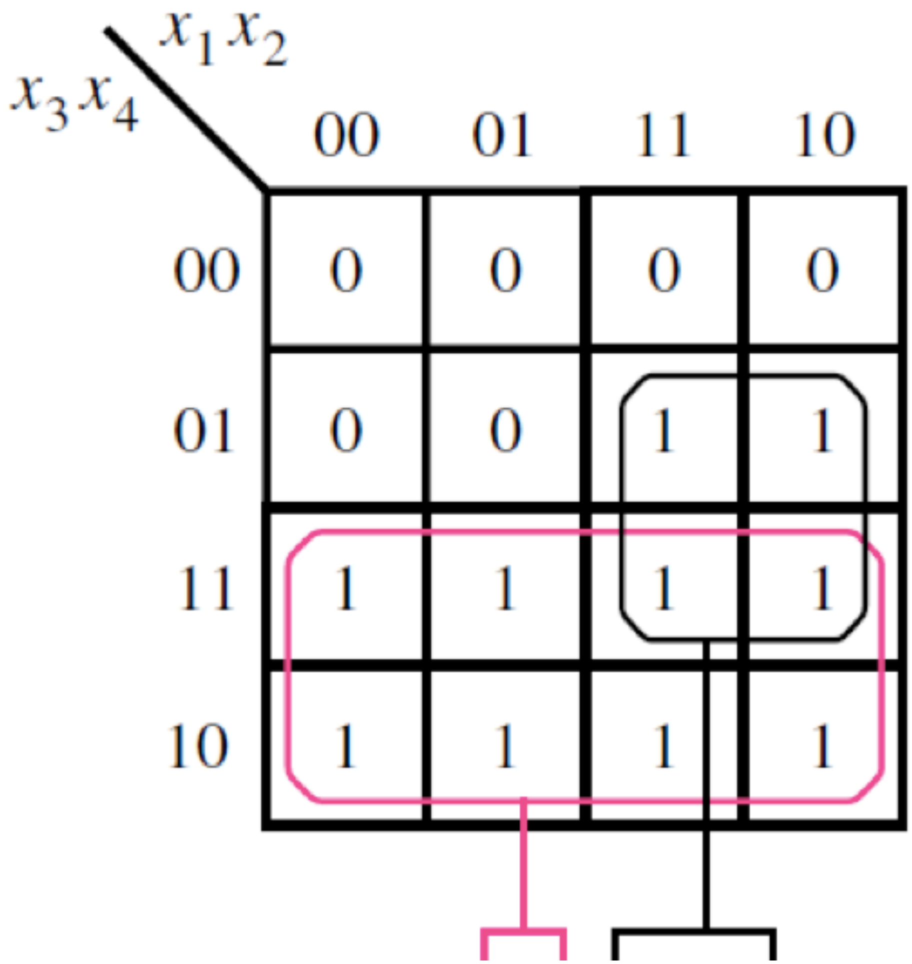
Example of a four-variable Karnaugh map



$$f_1 = \bar{x}_2 x_3 + x_1 \bar{x}_3 x_4$$

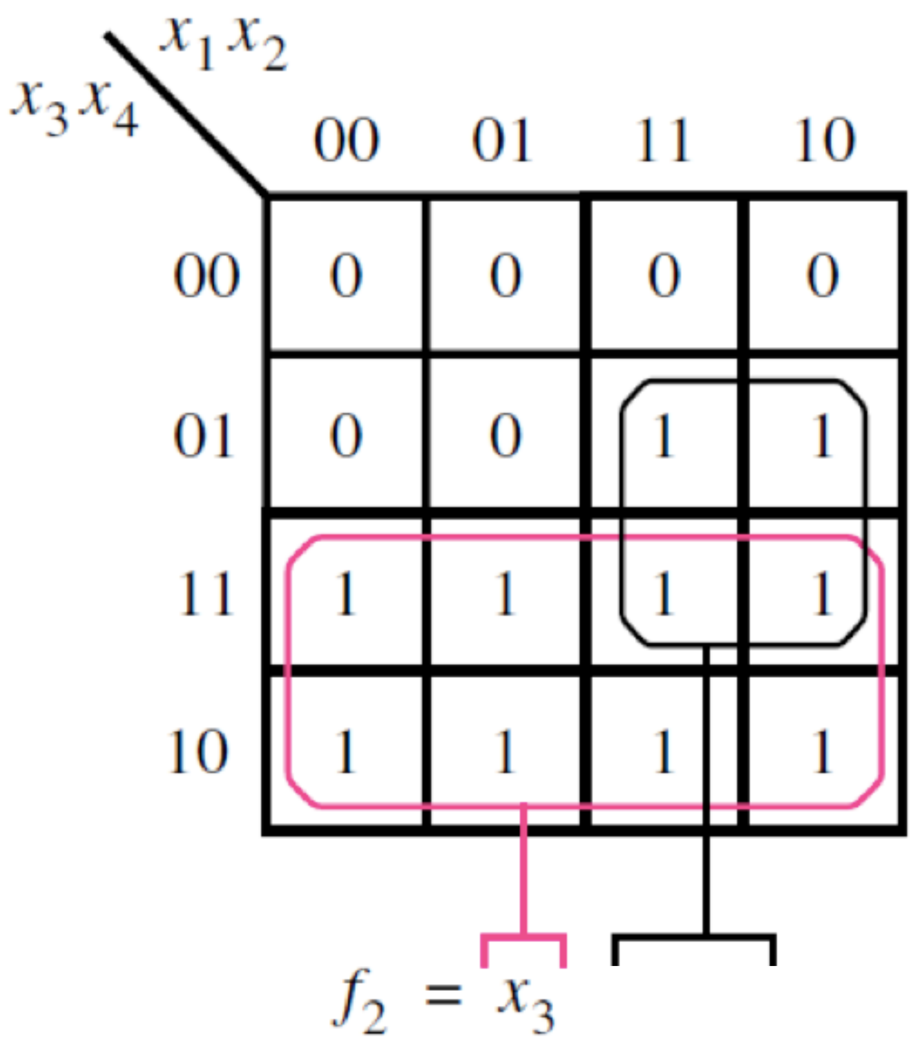
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



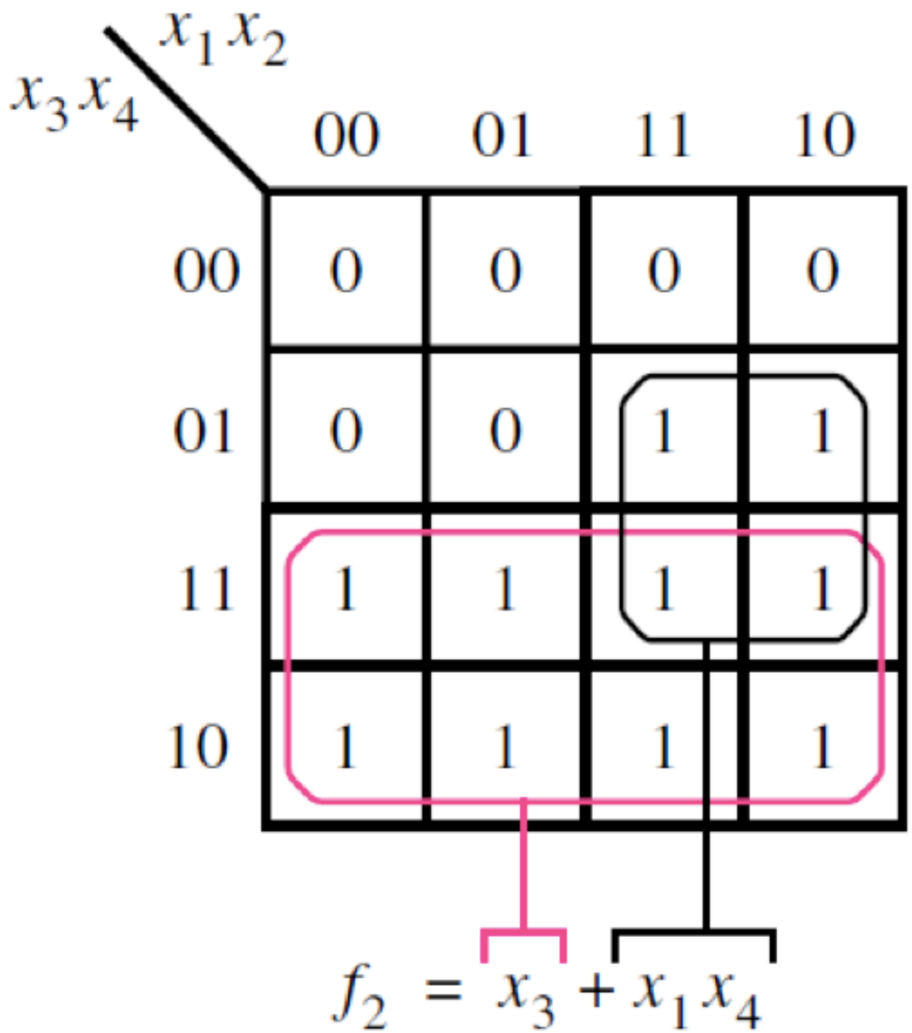
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



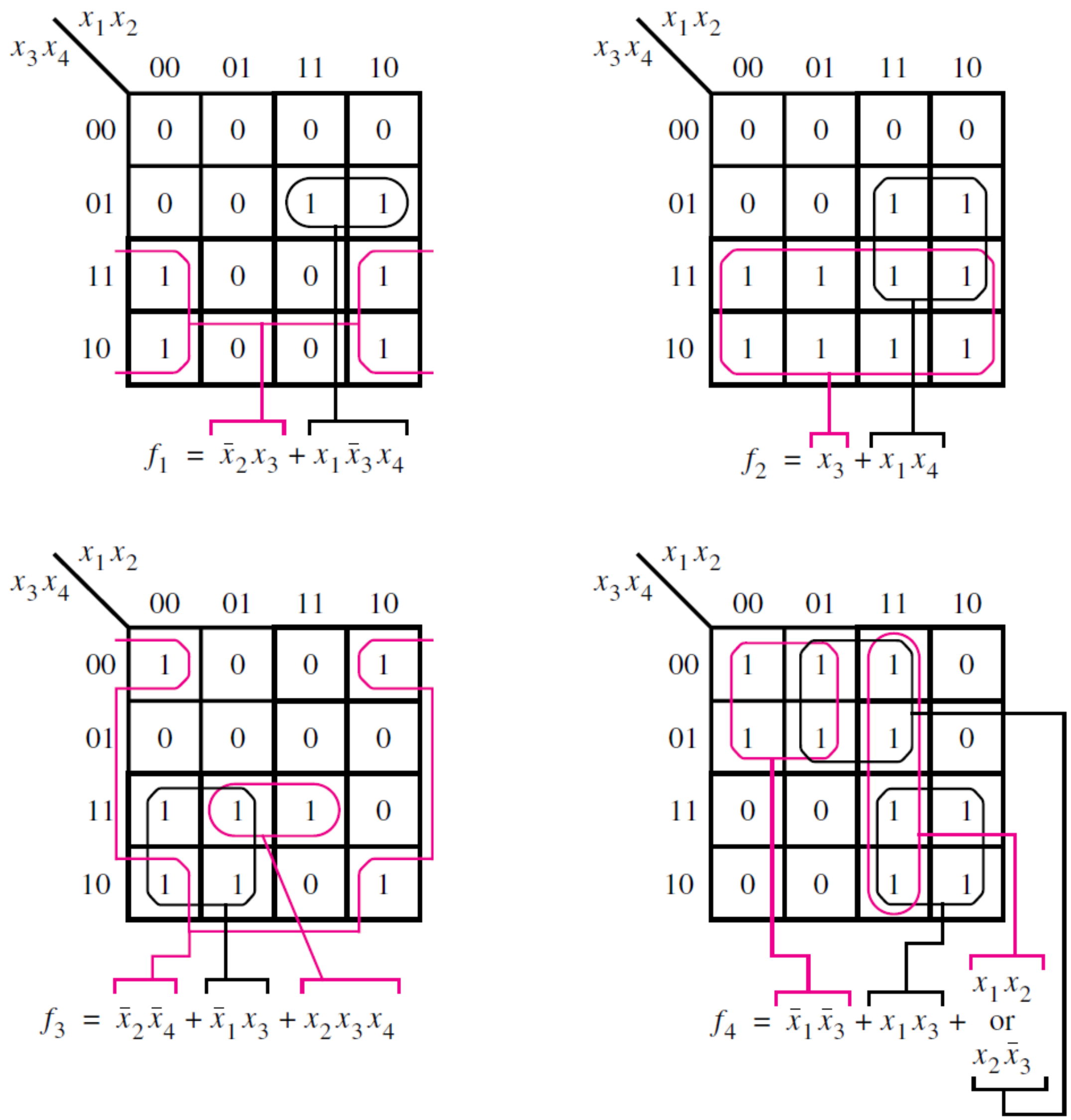
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



[Figure 2.54 from the textbook]

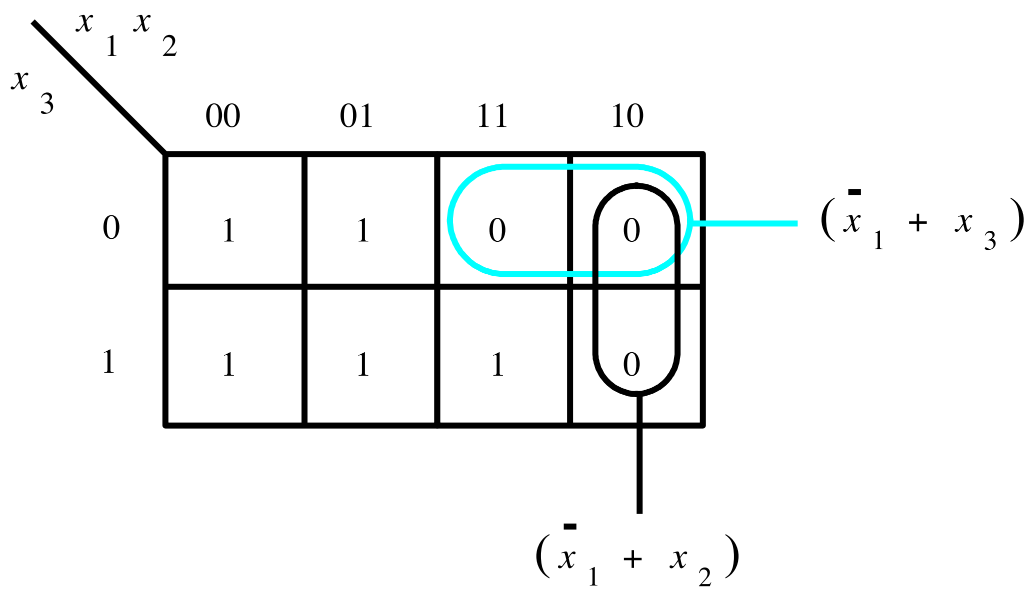
Other Four-Variable K-map Examples



[Figure 2.54 from the textbook]

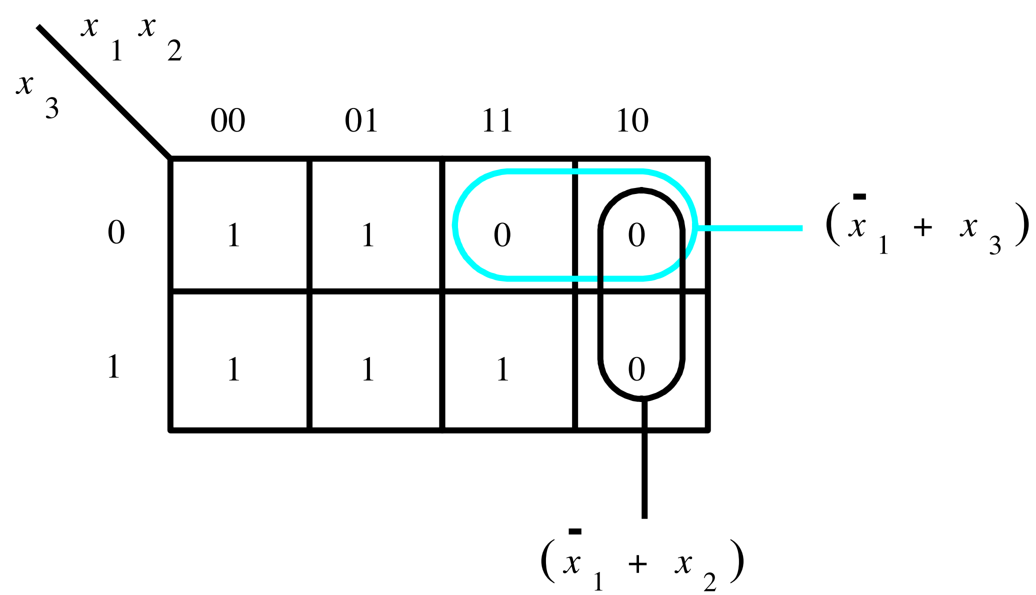
POS Minimization

POS minimization of $f(x_1, x_2, x_3) = \prod M(4, 5, 6)$



[Figure 2.60 from the textbook]

POS minimization of $f(x_1, x_2, x_3) = \prod M(4, 5, 6)$



$$f(x_1, x_2, x_3) = (\bar{x}_1 + x_3)(\bar{x}_1 + x_2)$$

[Figure 2.60 from the textbook]

POS minimization of $f(x_1,\ldots,x_4) = \prod M(0, 1, 4, 8, 9, 12, 15)$

x_3x_4 \ x_1x_2	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	1	1	0	1
10	1	1	1	1

$(x_3 + x_4)$

$(x_2 + x_3)$

$(\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \bar{x}_4)$

[Figure 2.61 from the textbook]

POS minimization of $f(x_1, \ldots, x_4) = \prod M(0, 1, 4, 8, 9, 12, 15)$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	1	1	0	1
10	1	1	1	1

$(x_3 + x_4)$

$(x_2 + x_3)$

$(\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \bar{x}_4)$

$$f(x_1, x_2, x_3, x_4) = (x_3 + x_4)(x_2 + x_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3 + \overline{x}_4)$$

[Figure 2.61 from the textbook]

Example:
Incompletely Specified Function

Three Ways to Specify the Function

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

x_1 x_2 x_3 x_4	f
0 0 0 0	m_0
0 0 0 1	m_1
0 0 1 0	m_2
0 0 1 1	m_3
0 1 0 0	m_4
0 1 0 1	m_5
0 1 1 0	m_6
0 1 1 1	m_7
1 0 0 0	m_8
1 0 0 1	m_9
1 0 1 0	m_{10}
1 0 1 1	m_{11}
1 1 0 0	m_{12}
1 1 0 1	m_{13}
1 1 1 0	m_{14}
1 1 1 1	m_{15}

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

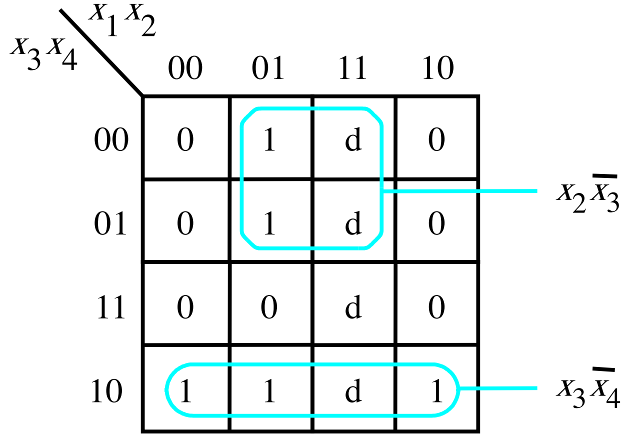
Three Ways to Specify the Function

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

x_1 x_2 x_3 x_4	f	
0 0 0 0	m_0	0
0 0 0 1	m_1	0
0 0 1 0	m_2	1
0 0 1 1	m_3	0
0 1 0 0	m_4	1
0 1 0 1	m_5	1
0 1 1 0	m_6	1
0 1 1 1	m_7	1
1 0 0 0	m_8	0
1 0 0 1	m_9	0
1 0 1 0	m_{10}	1
1 0 1 1	m_{11}	0
1 1 0 0	m_{12}	d
1 1 0 1	m_{13}	d
1 1 1 0	m_{14}	d
1 1 1 1	m_{15}	d

x_3x_4 \ x_1x_2	00	01	11	10
00	0	1	d	0
01	0	1	d	0
11	0	0	d	0
10	1	1	d	1

SOP implementation



(a) SOP implementation

[Figure 2.62 from the textbook]

POS implementation

$x_3 x_4$ \ $x_1 x_2$	00	01	11	10
00	0	1	d	0
01	0	1	d	0
11	0	0	d	0
10	1	1	d	1

$(x_2 + x_3)$

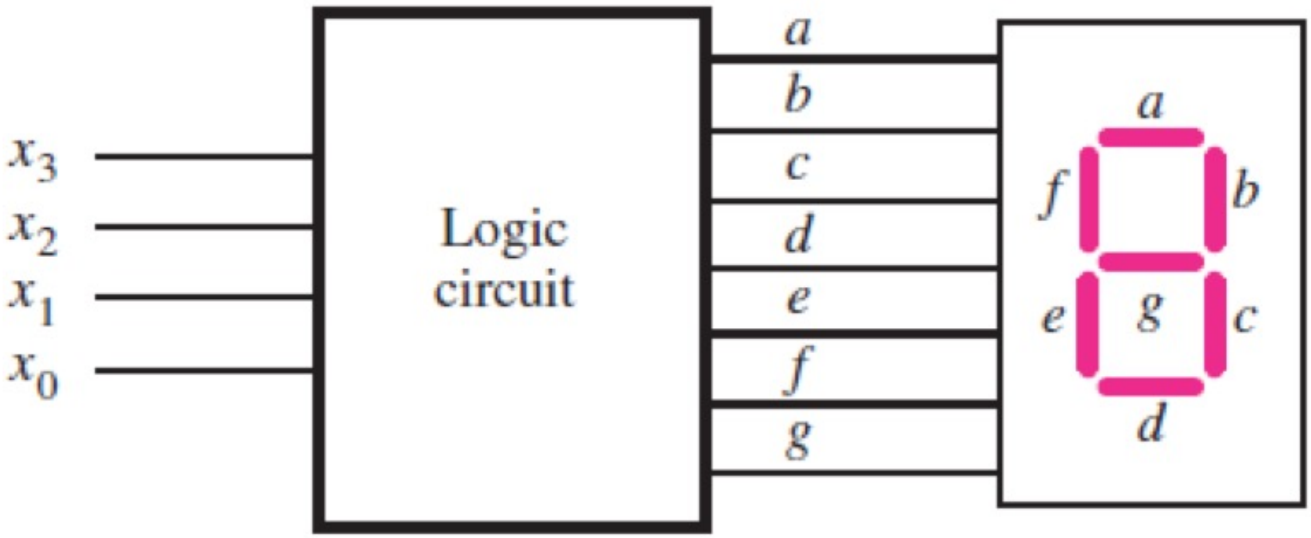
$(\overline{x_3} + \overline{x_4})$

(b) POS implementation

[Figure 2.62 from the textbook]

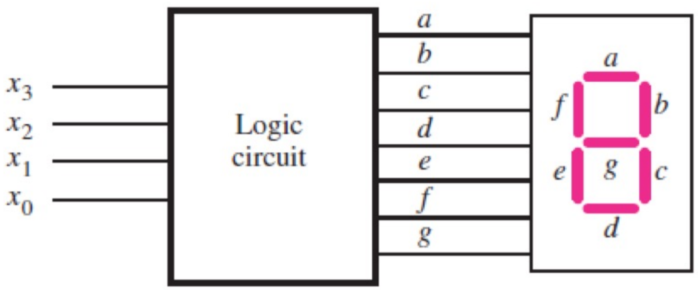
Example:
A circuit with multiple outputs

Seven-Segment Indicator



(a) Logic circuit and 7-segment display

Seven-Segment Indicator



(a) Logic circuit and 7-segment display

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1

(b) Truth table

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0							
	1	0	1	1							
	1	1	0	0							
	1	1	0	1							
	1	1	1	0							
	1	1	1	1							

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	d	d	d	d	d	d	d
	1	0	1	1	d	d	d	d	d	d	d
	1	1	0	0	d	d	d	d	d	d	d
	1	1	0	1	d	d	d	d	d	d	d
	1	1	1	0	d	d	d	d	d	d	d
	1	1	1	1	d	d	d	d	d	d	d

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	d	d	d	d	d	d	d
	1	0	1	1	d	d	d	d	d	d	d
	1	1	0	0	d	d	d	d	d	d	d
	1	1	0	1	d	d	d	d	d	d	d
	1	1	1	0	d	d	d	d	d	d	d
	1	1	1	1	d	d	d	d	d	d	d

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	d	d	d	d	d	d	d
	1	0	1	1	d	d	d	d	d	d	d
	1	1	0	0	d	d	d	d	d	d	d
	1	1	0	1	d	d	d	d	d	d	d
	1	1	1	0	d	d	d	d	d	d	d
	1	1	1	1	d	d	d	d	d	d	d

$x_1x_0 \backslash x_3x_2$	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

$x_1x_0 \backslash x_3x_2$	00	01	11	10
00				
01				
11				
10				

Seven-Segment Indicator

x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	d	d	d	d	d	d	d
1	0	1	1	d	d	d	d	d	d	d
1	1	0	0	d	d	d	d	d	d	d
1	1	0	1	d	d	d	d	d	d	d
1	1	1	0	d	d	d	d	d	d	d
1	1	1	1	d	d	d	d	d	d	d

x_1

x_1x_0 \ x_3x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

x_4

x_3

x_2

x_1x_0 \ x_3x_2	00	01	11	10
00	1	0	d	1
01	0	1	d	1
11	1	1	d	d
10	1	1	d	d

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	d	d	d	d	d	d	d
	1	0	1	1	d	d	d	d	d	d	d
	1	1	0	0	d	d	d	d	d	d	d
	1	1	0	1	d	d	d	d	d	d	d
	1	1	1	0	d	d	d	d	d	d	d
	1	1	1	1	d	d	d	d	d	d	d

x_1

x_1x_0 \ x_3x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

x_4

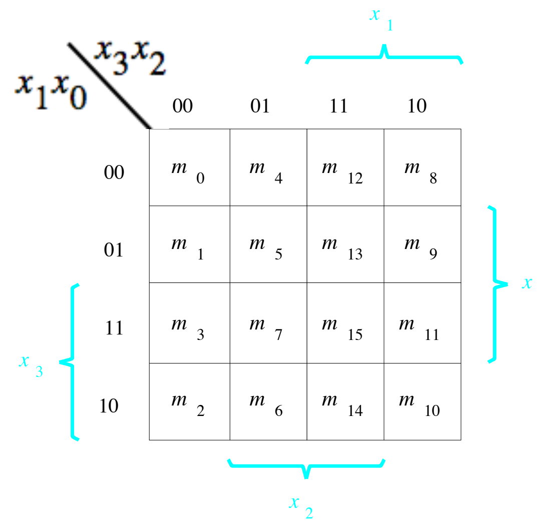
x_3

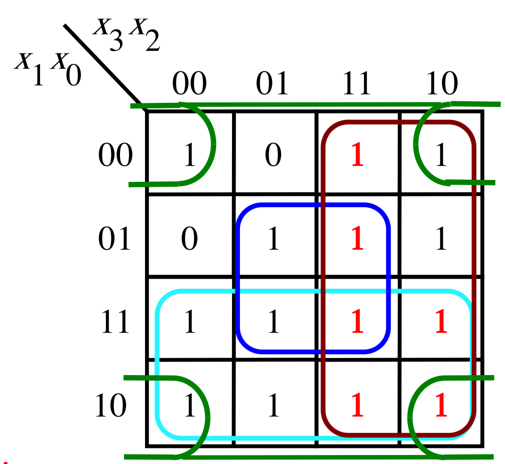
x_2

x_1x_0 \ x_3x_2	00	01	11	10
00	1	0	d	1
01	0	1	d	1
11	1	1	d	d
10	1	1	d	d

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	1	d	d	d	d	d	d
	1	0	1	1	1	d	d	d	d	d	d
	1	1	0	0	1	d	d	d	d	d	d
	1	1	0	1	1	d	d	d	d	d	d
	1	1	1	0	1	d	d	d	d	d	d
	1	1	1	1	1	d	d	d	d	d	d





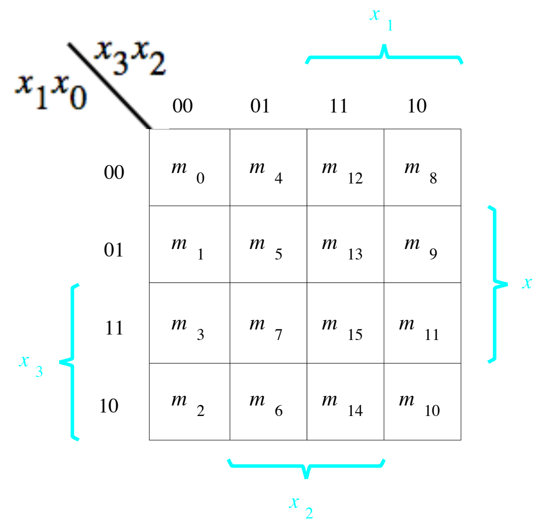
In this case all d's were treated as 1's.

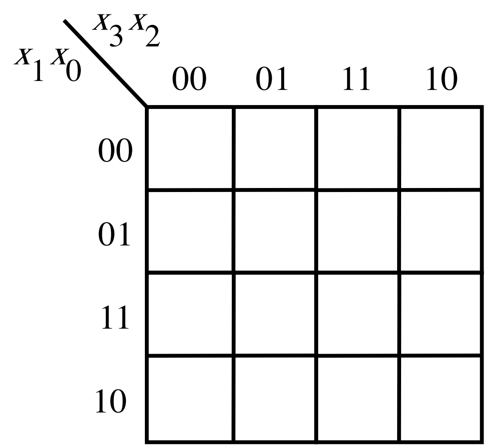
Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	1	d	d	d	d	d	d
	1	0	1	1	1	d	d	d	d	d	d
	1	1	0	0	1	d	d	d	d	d	d
	1	1	0	1	1	d	d	d	d	d	d
	1	1	1	0	1	d	d	d	d	d	d
	1	1	1	1	1	d	d	d	d	d	d

Seven-Segment Indicator

x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	d	d	d	d	d	d
1	0	1	1	1	d	d	d	d	d	d
1	1	0	0	1	d	d	d	d	d	d
1	1	0	1	1	d	d	d	d	d	d
1	1	1	0	1	d	d	d	d	d	d
1	1	1	1	1	d	d	d	d	d	d





Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	1	d	d	d	d	d	d
	1	0	1	1	1	d	d	d	d	d	d
	1	1	0	0	1	d	d	d	d	d	d
	1	1	0	1	1	d	d	d	d	d	d
	1	1	1	0	1	d	d	d	d	d	d
	1	1	1	1	1	d	d	d	d	d	d

x_1x_0 \ x_3x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

x_1 x_4 x_3 x_2

x_1x_0 \ x_3x_2	00	01	11	10
00	1	0	d	1
01	0	0	d	0
11	0	0	d	d
10	1	1	d	d

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	1	d	d	d	d	d	d
	1	0	1	1	1	d	d	d	d	d	d
	1	1	0	0	1	d	d	d	d	d	d
	1	1	0	1	1	d	d	d	d	d	d
	1	1	1	0	1	d	d	d	d	d	d
	1	1	1	1	1	d	d	d	d	d	d

x_1x_0 \ x_3x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

x_1x_0 \ x_3x_2	00	01	11	10
00	1	0	d	1
01	0	0	d	0
11	0	0	d	d
10	1	1	d	d

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	1	d	d	d	**1**	d	d
	1	0	1	1	1	d	d	d	**0**	d	d
	1	1	0	0	1	d	d	d	**0**	d	d
	1	1	0	1	1	d	d	d	**0**	d	d
	1	1	1	0	1	d	d	d	**1**	d	d
	1	1	1	1	1	d	d	d	**0**	d	d

x_1x_0 \ x_3x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

x_1 x_4 x_3 x_2

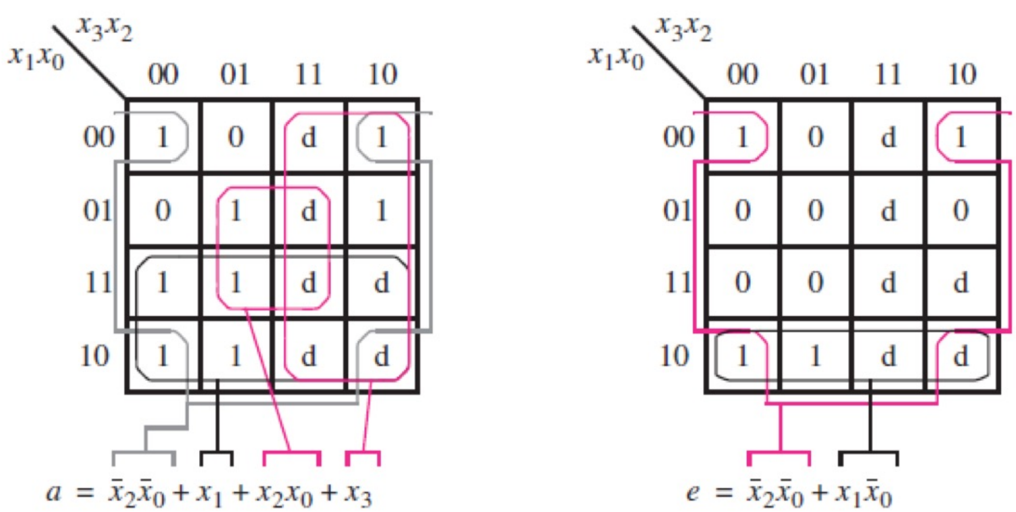
x_1x_0 \ x_3x_2	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	1	1	1

In this case some d's were treated as 1's, others as 0's.

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1

(b) Truth table



(c) The Karnaugh maps for outputs a and e.

Another Example

x_3x_4 \ x_1x_2	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

x_3x_4 \ x_1x_2	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function f_2

[Figure 2.64 from the textbook]

(a) Function f_1 (b) Function f_2

x_3x_4 \ x_1x_2	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$\bar{x}_1 \; x_3$

x_3x_4 \ x_1x_2	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function f_2

x_3x_4 \ x_1x_2	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$\bar{x}_1 \; x_3$

x_3x_4 \ x_1x_2	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function f_2

x_3x_4 \ x_1x_2	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

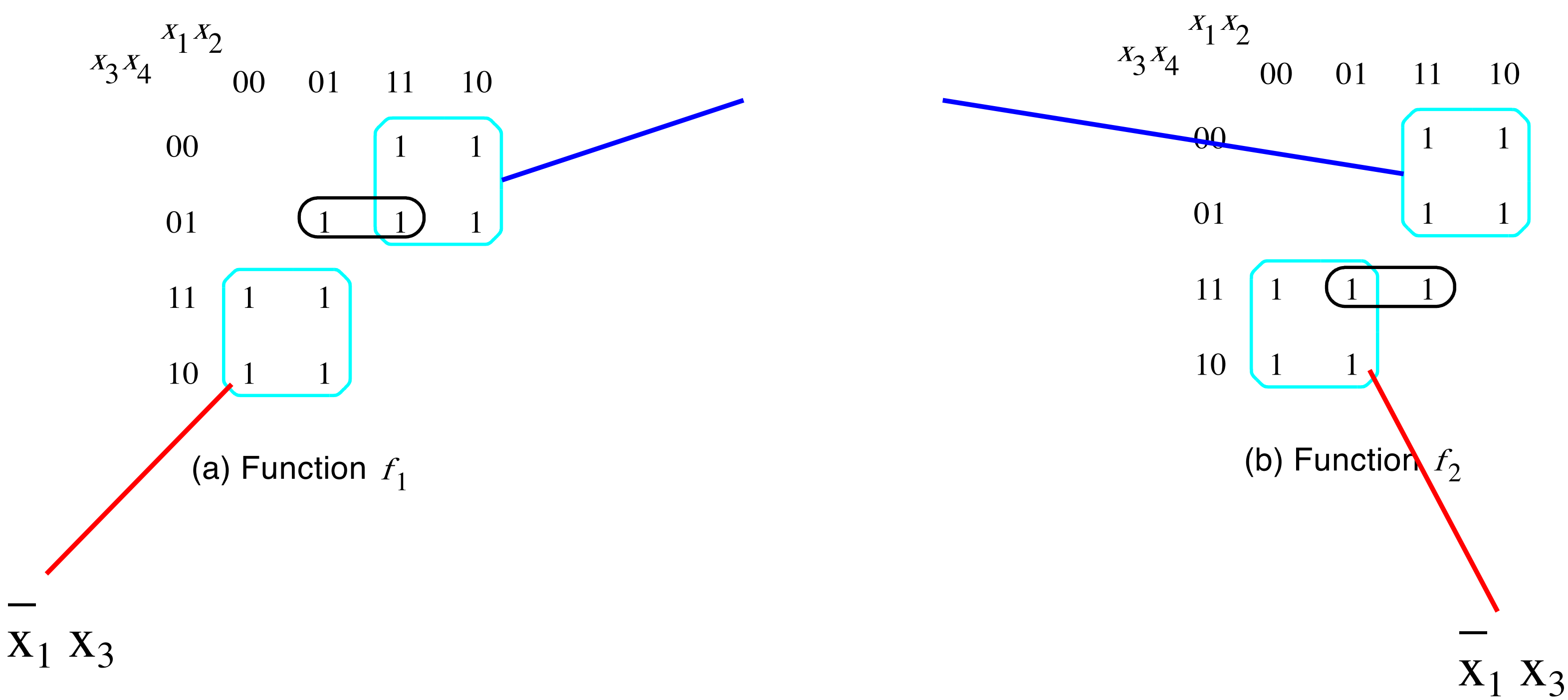
(a) Function f_1

$\bar{x}_1 \; x_3$

x_3x_4 \ x_1x_2	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

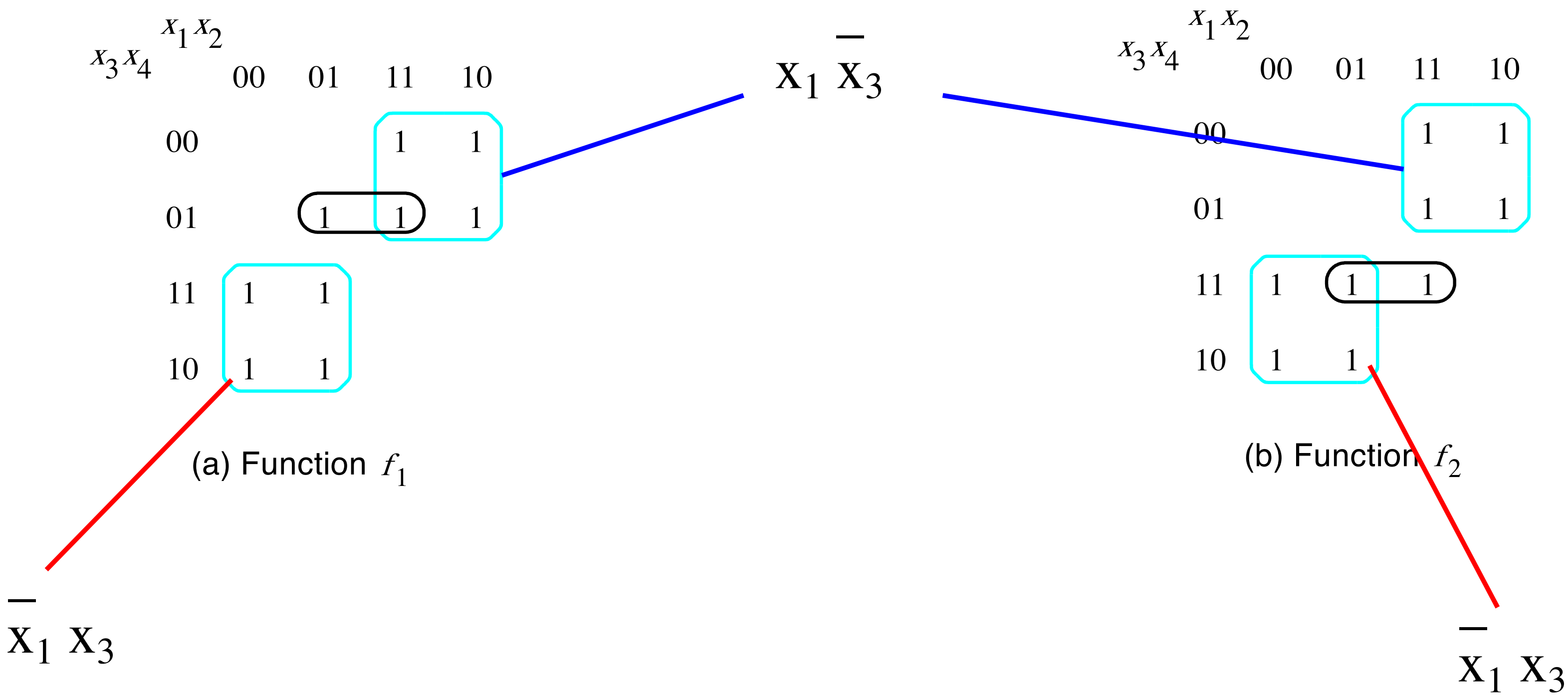
(b) Function f_2

$\bar{x}_1 \; x_3$



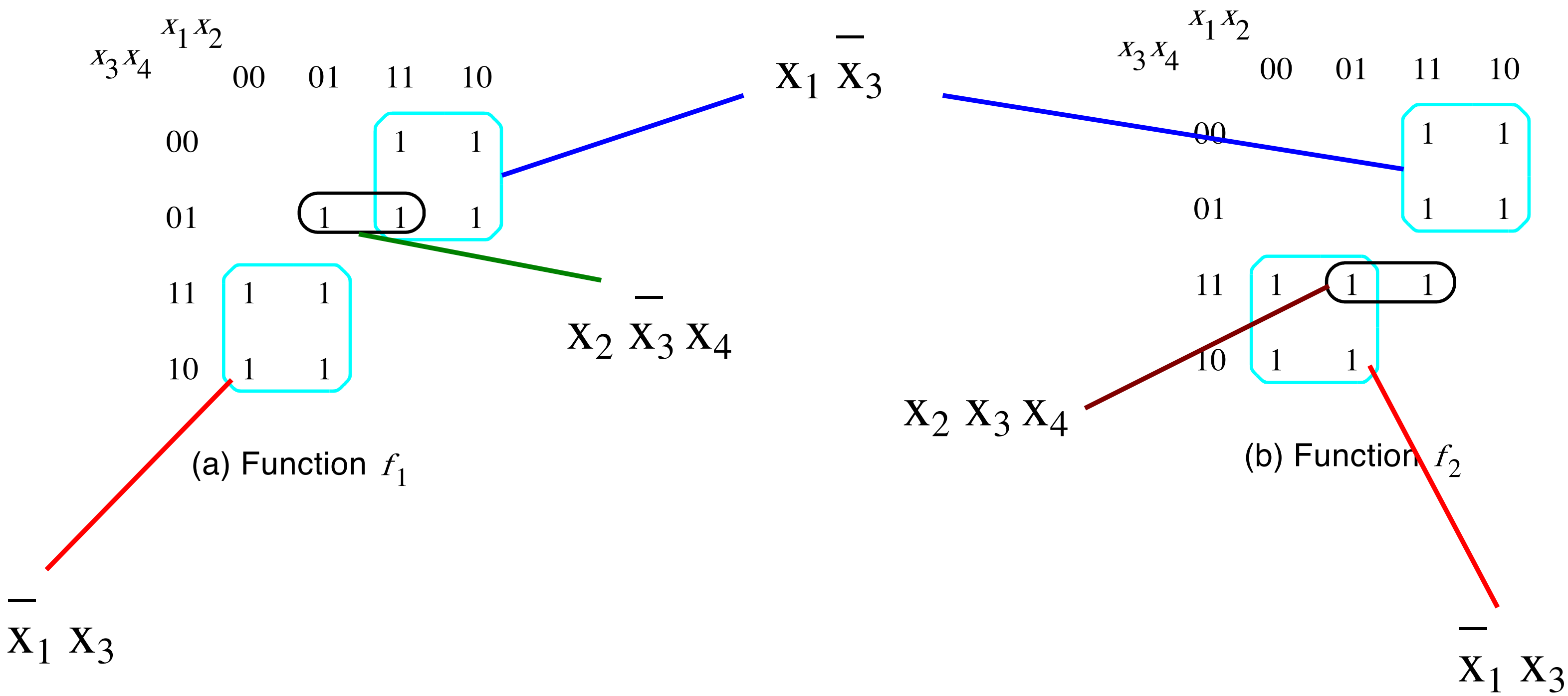
(a) Function f_1

(b) Function f_2



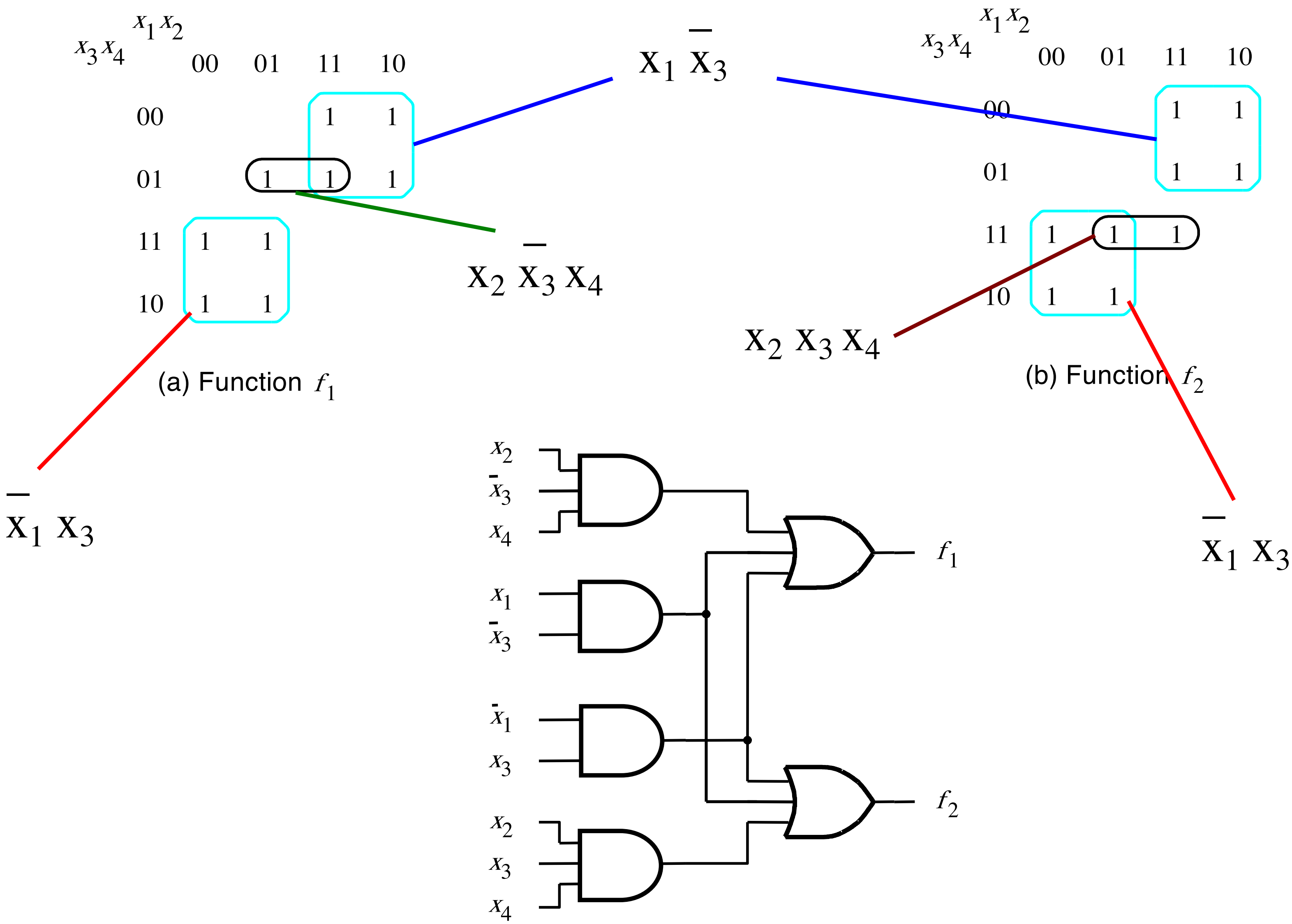
(a) Function f_1

(b) Function f_2



(a) Function f_1

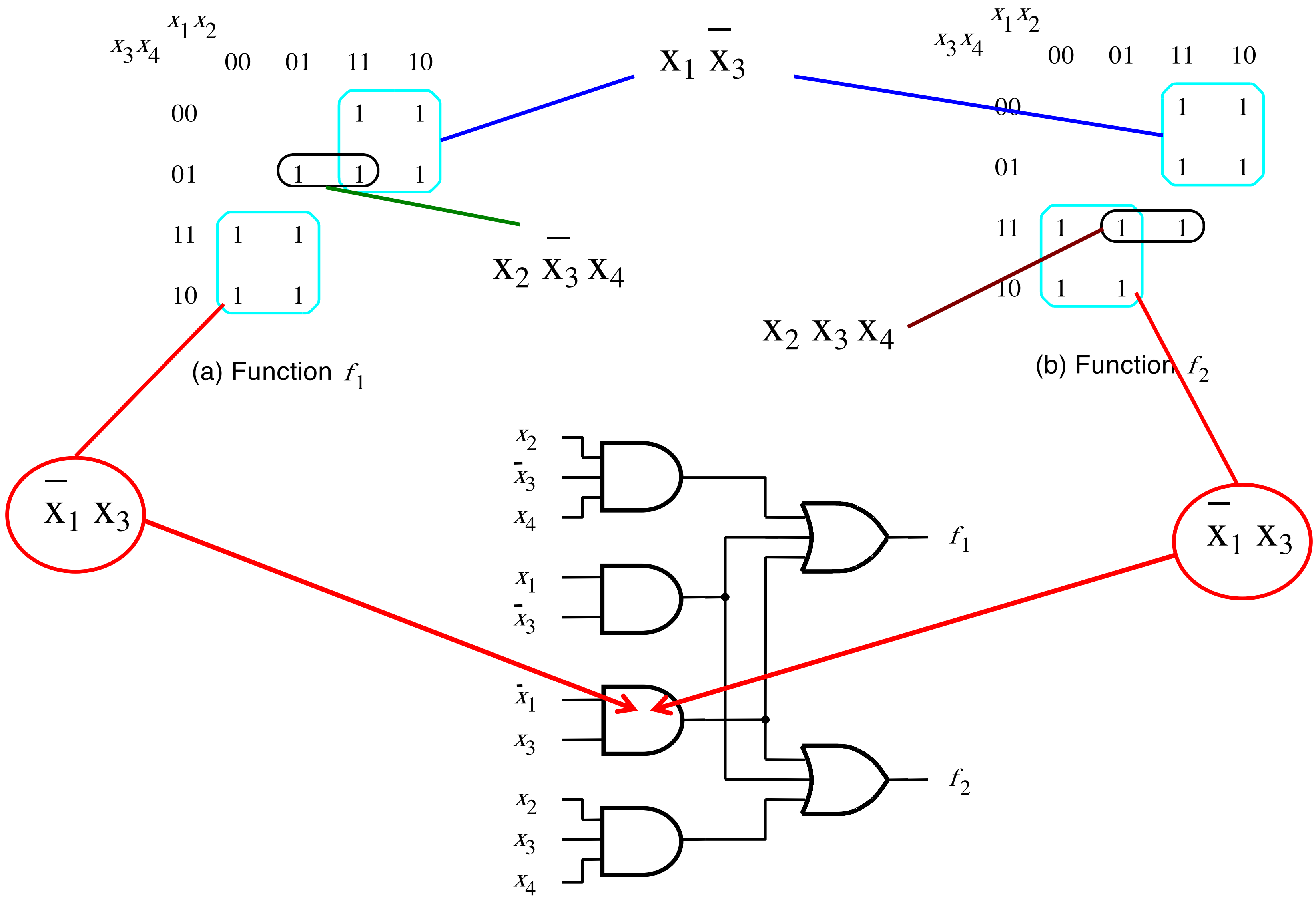
(b) Function f_2



(a) Function f_1

(b) Function f_2

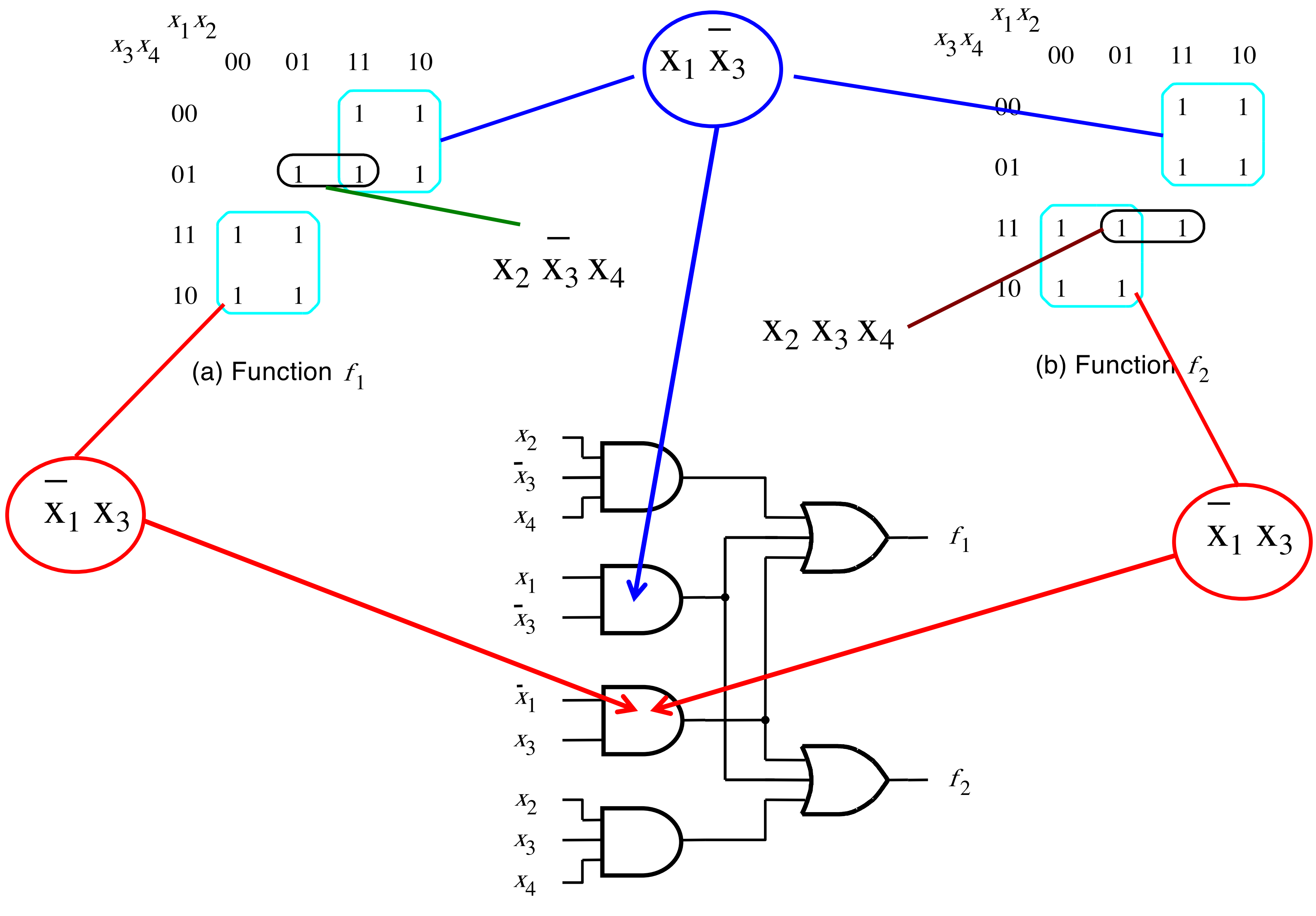
(c) Combined circuit for f_1 and f_2



(a) Function f_1

(b) Function f_2

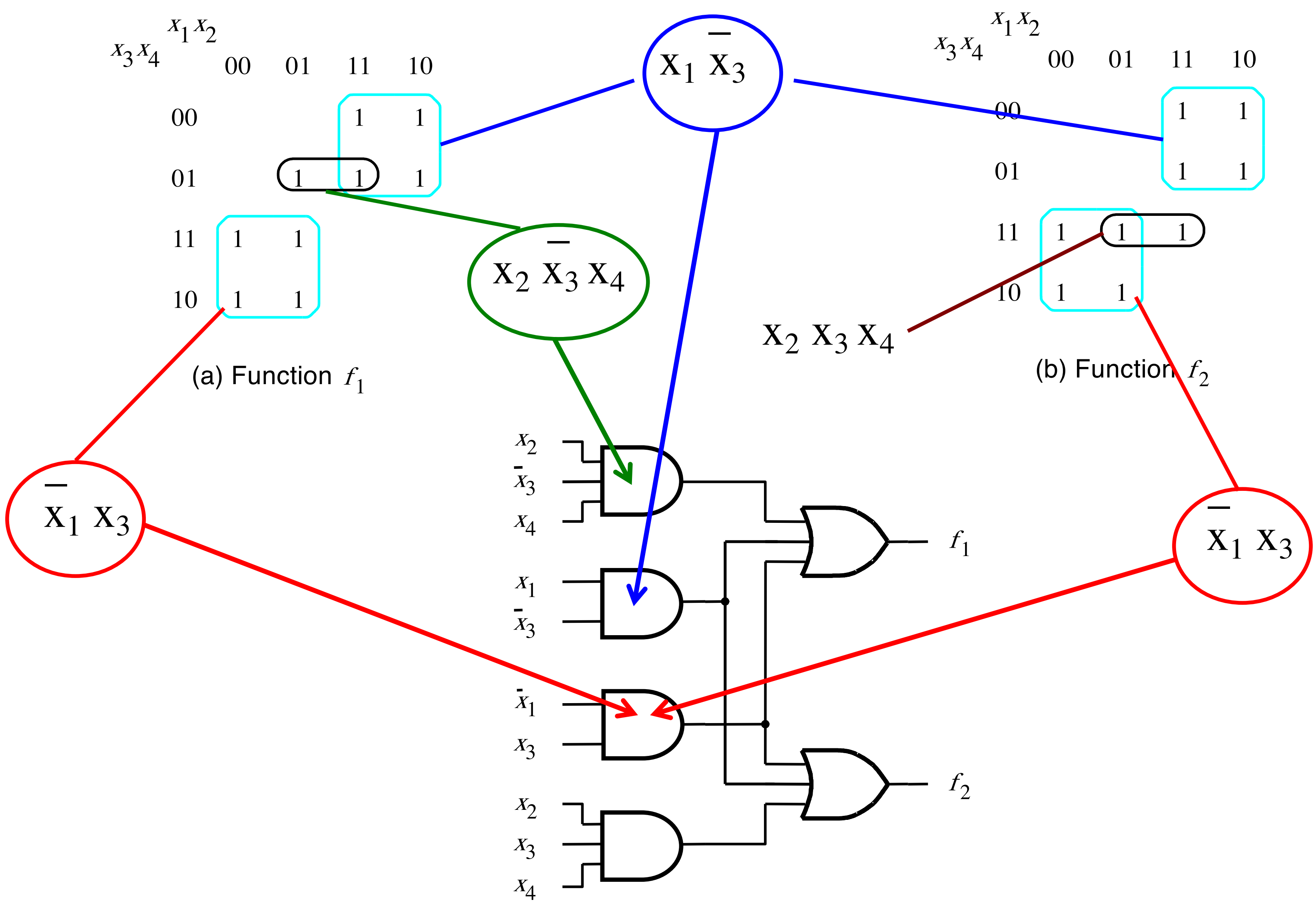
(c) Combined circuit for f_1 and f_2



(a) Function f_1

(b) Function f_2

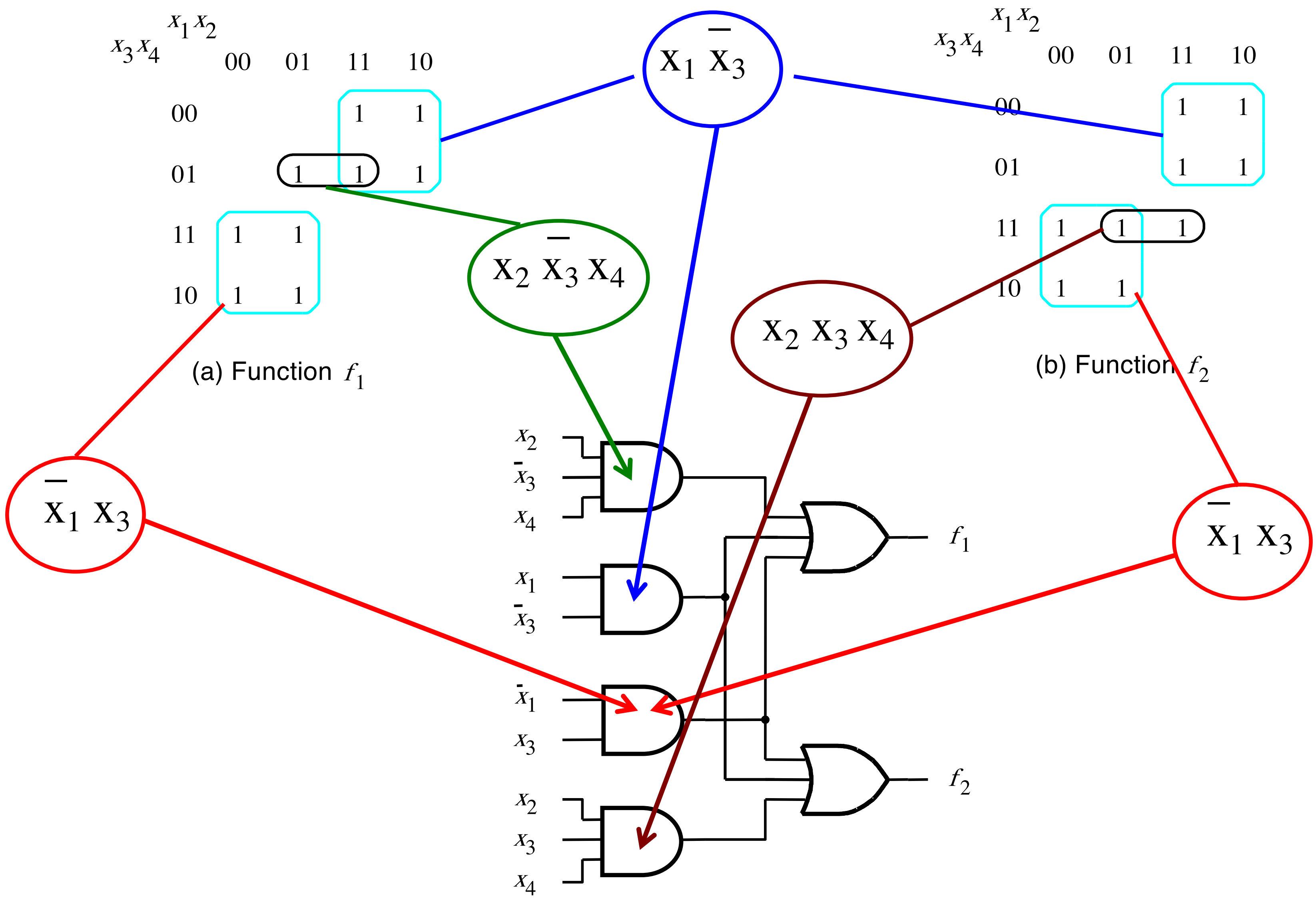
(c) Combined circuit for f_1 and f_2



(a) Function f_1

(b) Function f_2

(c) Combined circuit for f_1 and f_2



(a) Function f_1

(b) Function f_2

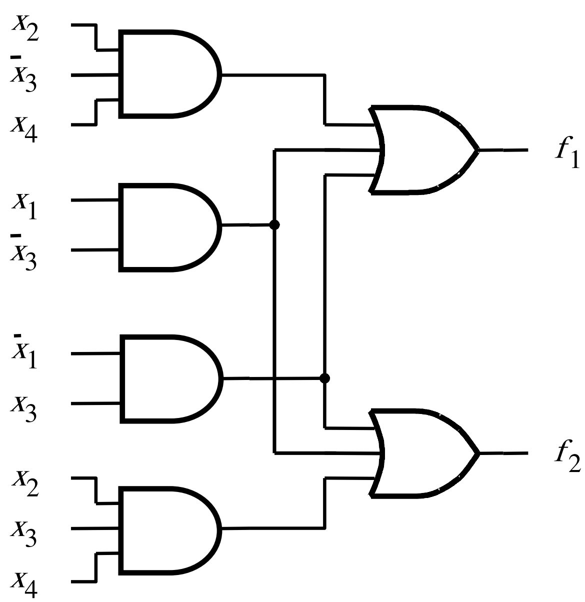
(c) Combined circuit for f_1 and f_2

x_3x_4 \ x_1x_2	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

x_3x_4 \ x_1x_2	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function f_2

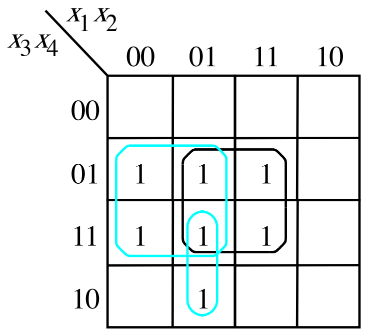


(c) Combined circuit for f_1 and f_2

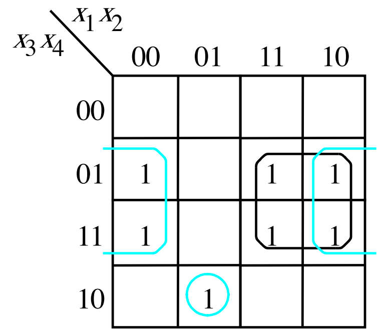
[Figure 2.64 from the textbook]

Individual vs Joint Optimization

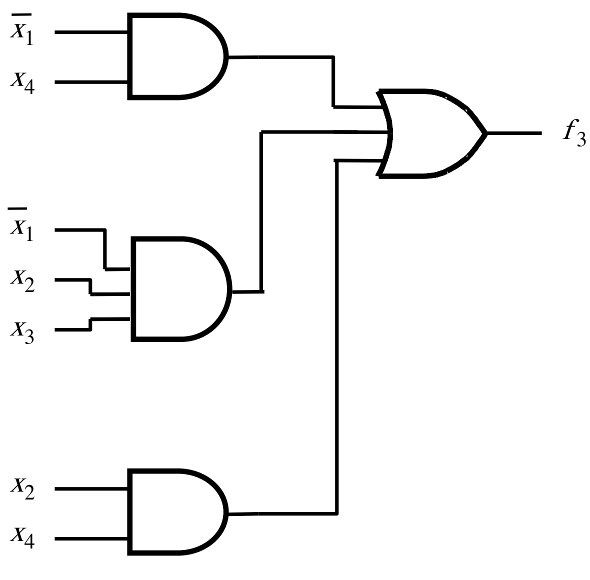
Individual Optimization



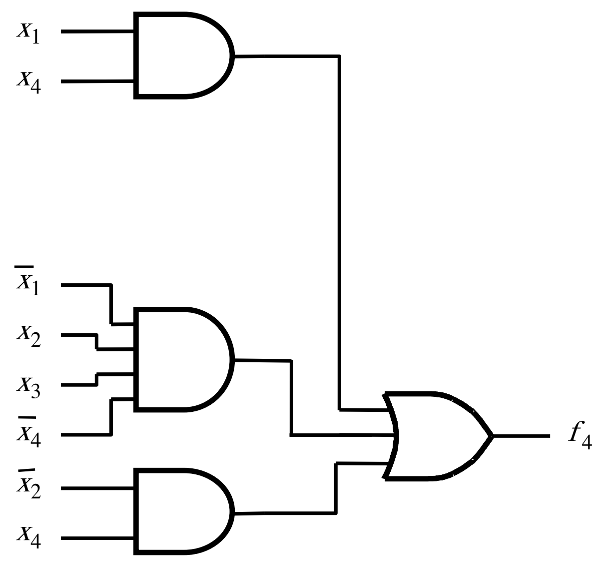
(a) Optimal realization of f_3



(b) Optimal realization of f_4

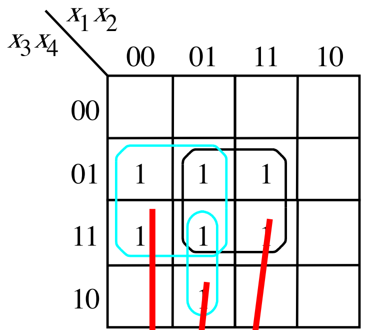


Circuit only for f_3

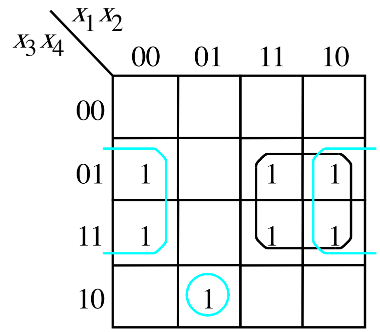


Circuit only for f_4

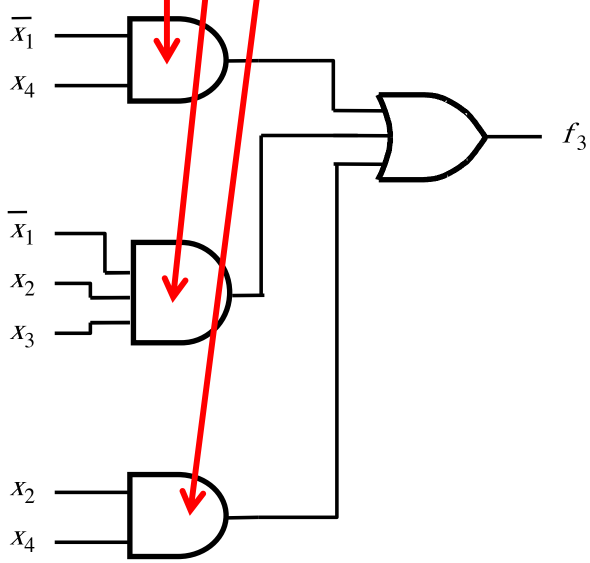
Individual Optimization



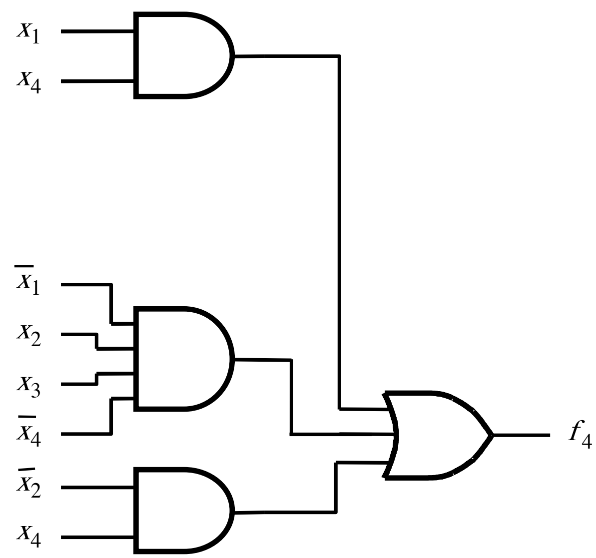
(a) Optimal realization of f_3



(b) Optimal realization of f_4

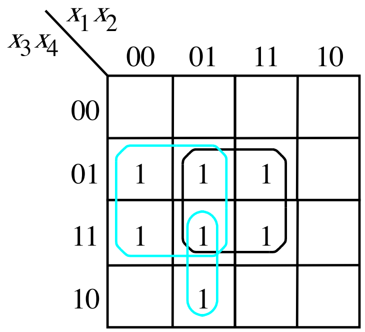


Circuit only for f_3

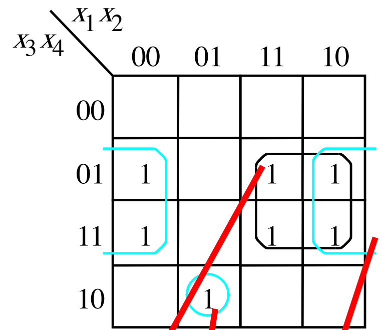


Circuit only for f_4

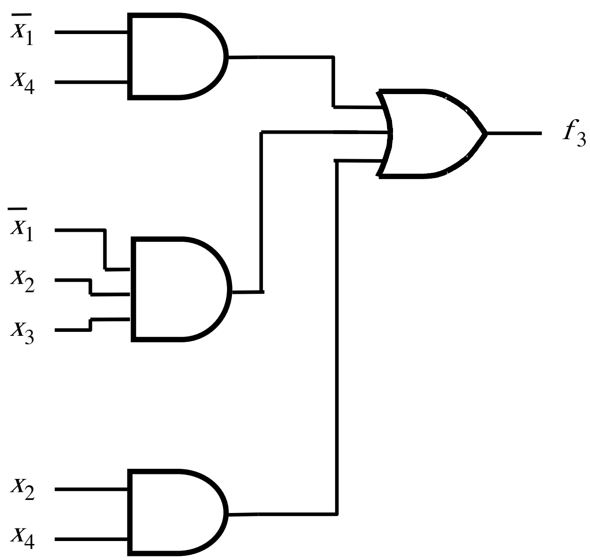
Individual Optimization



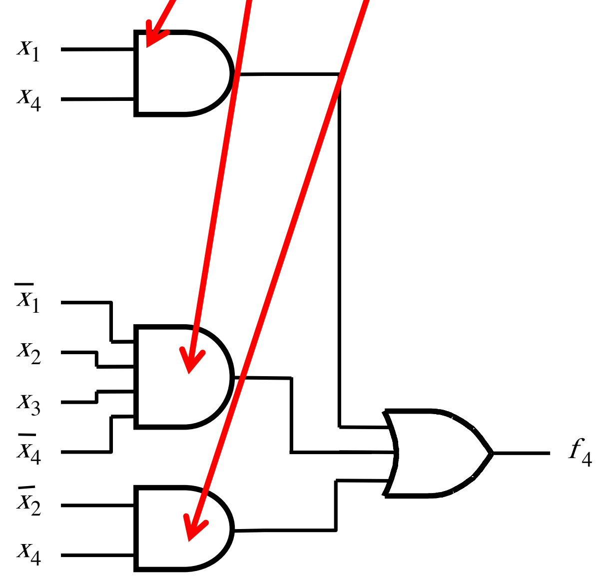
(a) Optimal realization of f_3



(b) Optimal realization of f_4

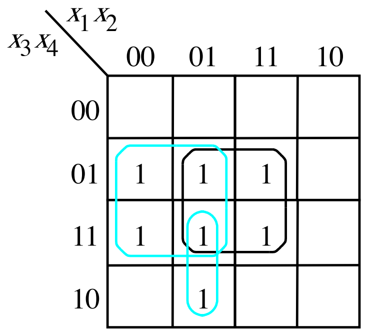


Circuit only for f_3

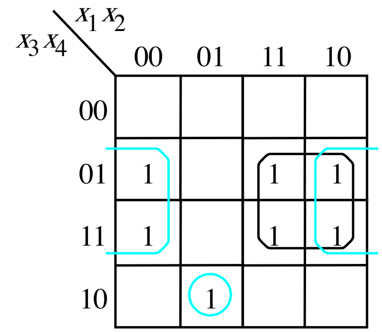


Circuit only for f_4

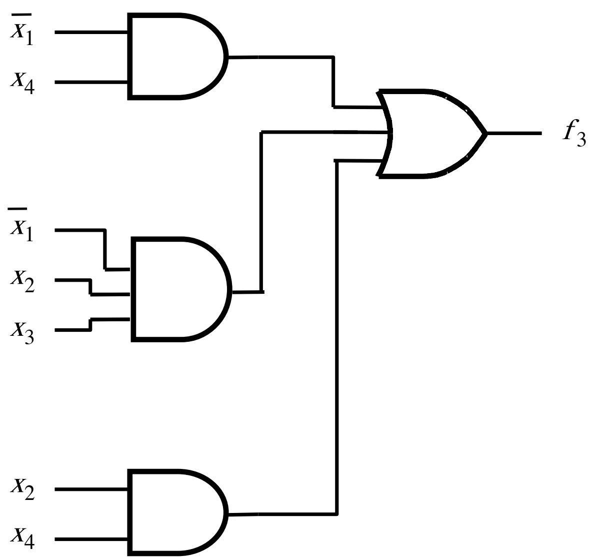
Individual Optimization



(a) Optimal realization of f_3

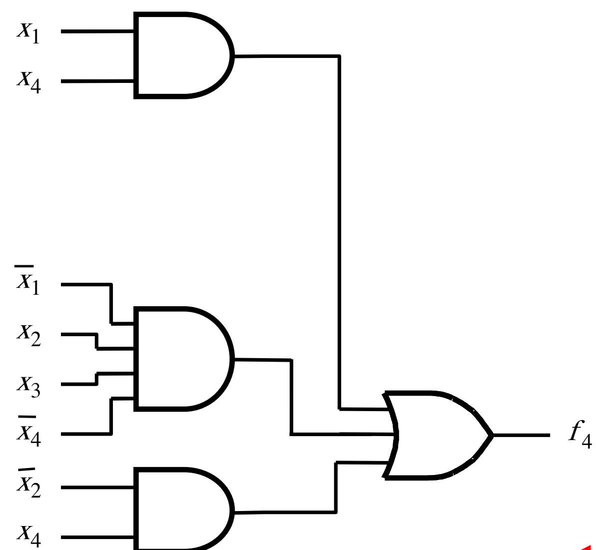


(b) Optimal realization of f_4



cost = 14

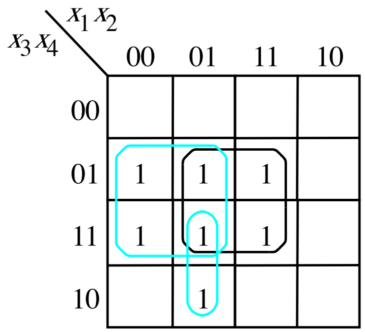
Circuit only for f_3



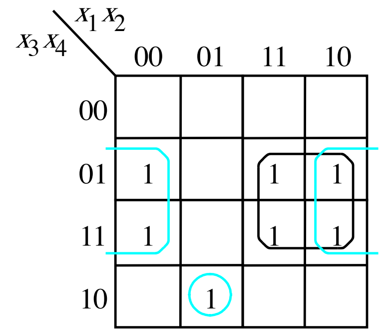
cost = 15

Circuit only for f_4

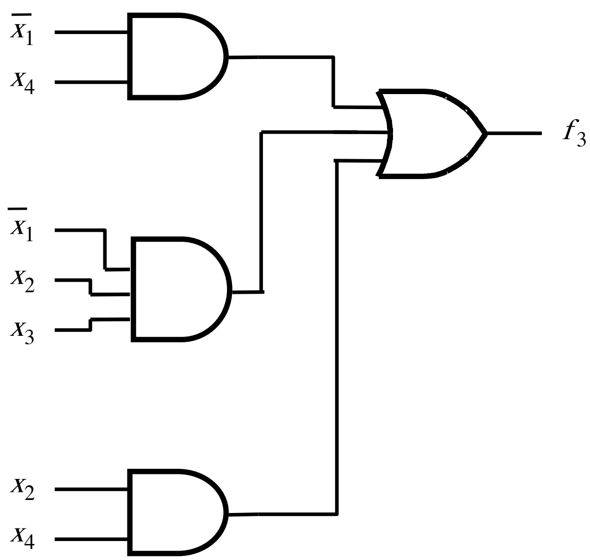
Individual Optimization



(a) Optimal realization of f_3

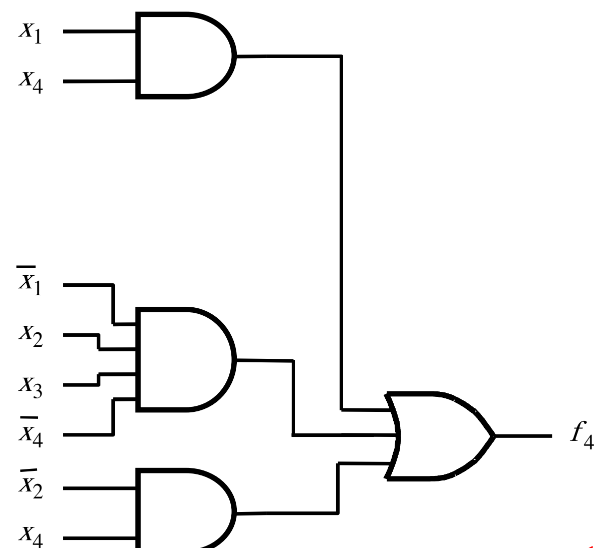


(b) Optimal realization of f_4



cost = 14

Circuit only for f_3



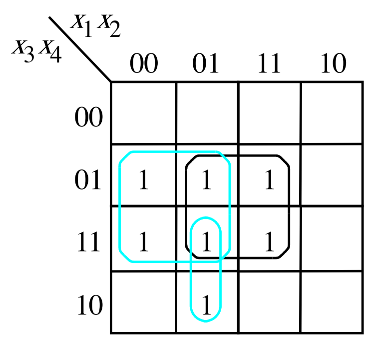
cost = 15

Circuit only for f_4

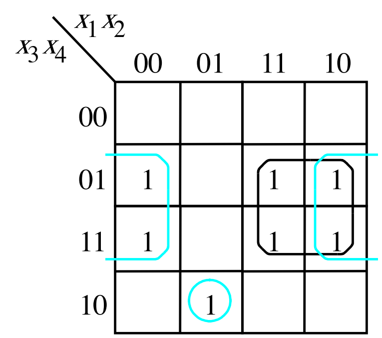
TOTAL cost: 29

Individual vs Joint Optimization

Individual

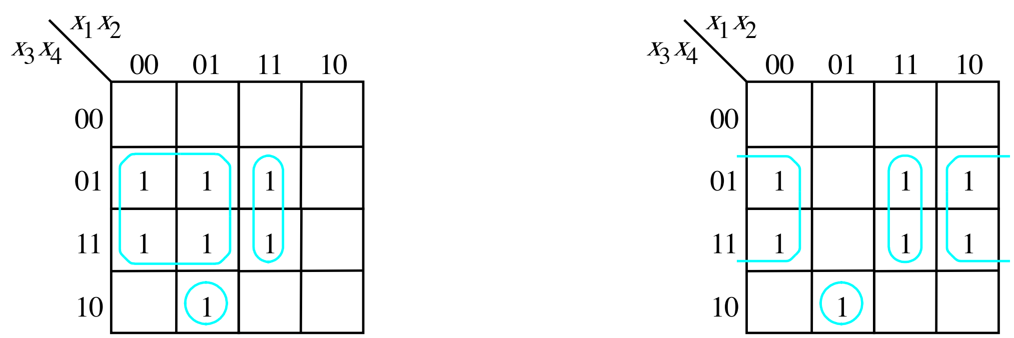


(a) Optimal realization of f_3



(b) Optimal realization of f_4

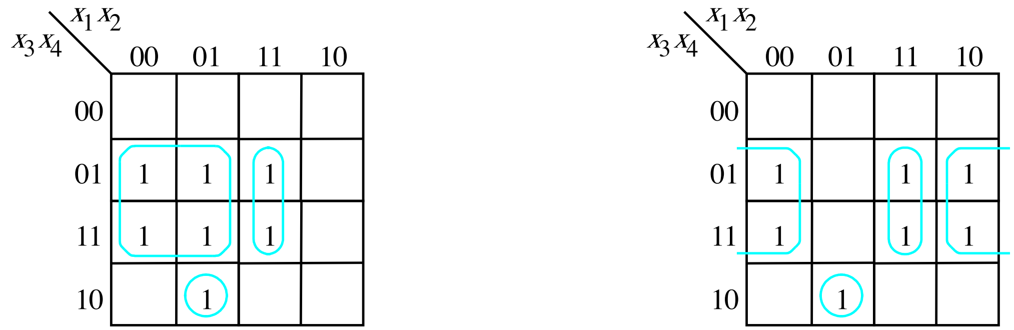
Joint



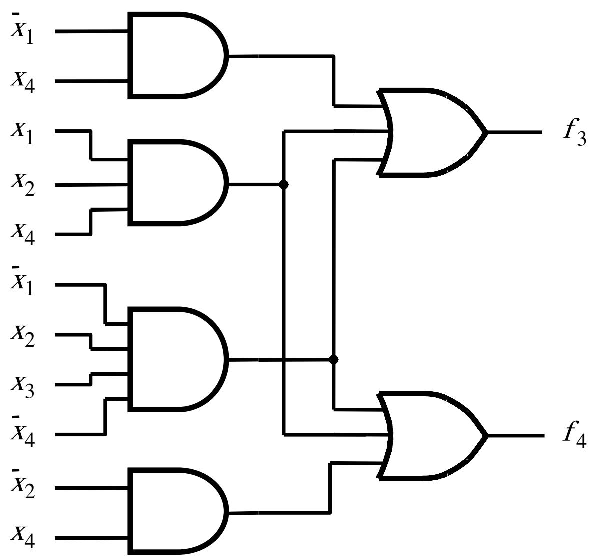
(c) Optimal realization of f_3 and f_4 together

[Figure 2.65 from the textbook]

Joint Optimization



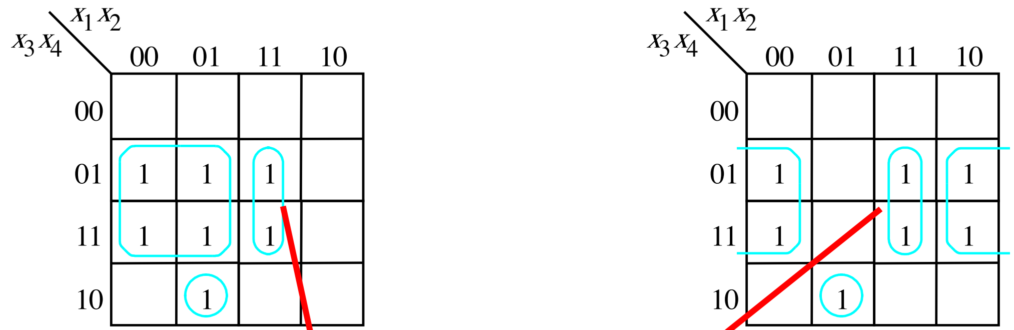
(c) Optimal realization of f_3 and f_4 together



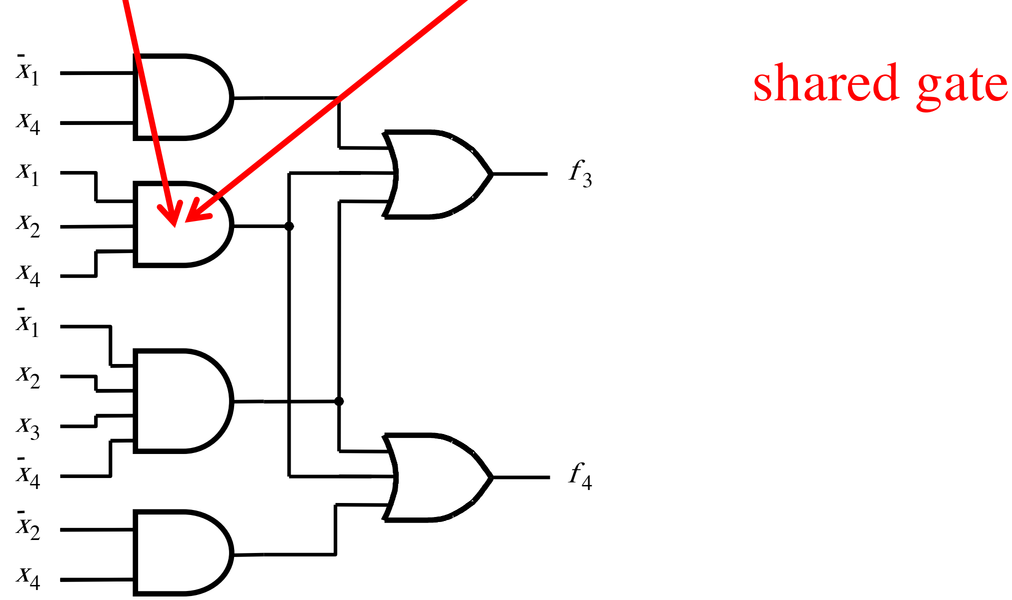
(d) Combined circuit for f_3 and f_4

[Figure 2.65 from the textbook]

Joint Optimization



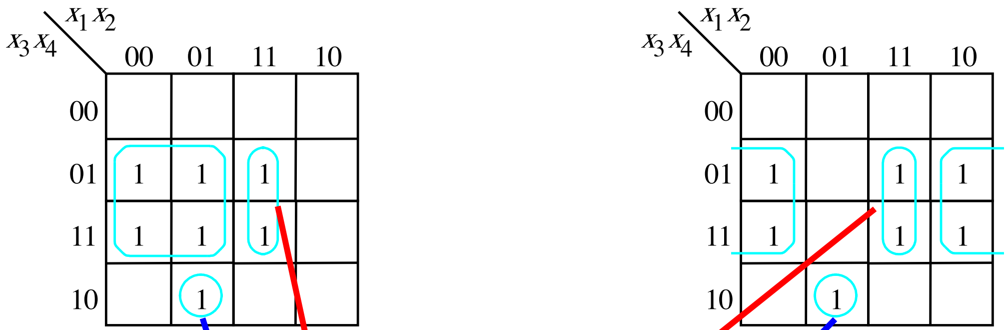
(c) Optimal realization of f_3 and f_4 together



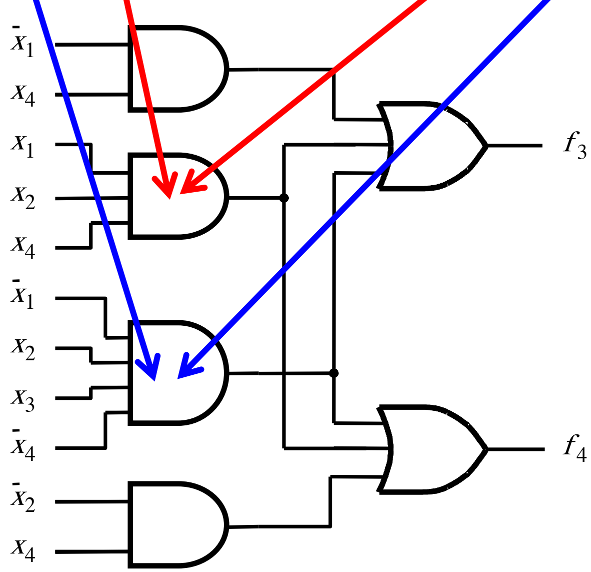
(d) Combined circuit for f_3 and f_4

[Figure 2.65 from the textbook]

Joint Optimization



(c) Optimal realization of f_3 and f_4 together

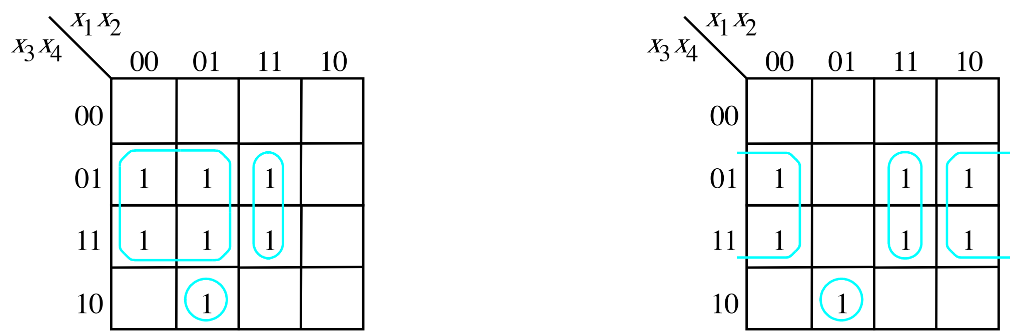


another shared gate

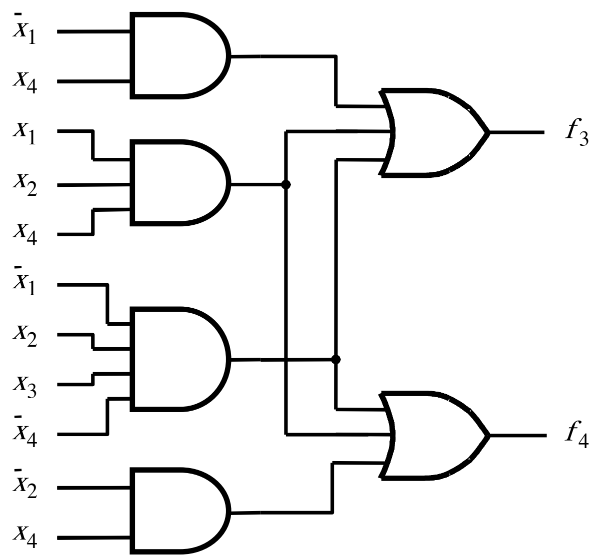
(d) Combined circuit for f_3 and f_4

[Figure 2.65 from the textbook]

Joint Optimization



(c) Optimal realization of f_3 and f_4 together



(d) Combined circuit for f_3 and f_4

cost = 23

[Figure 2.65 from the textbook]

Questions?

THE END