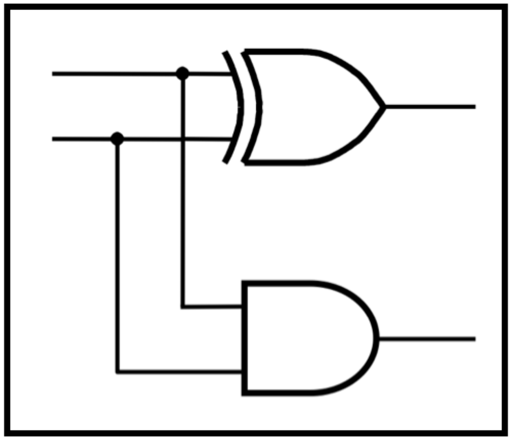


CprE 2810: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

Synthesis
Using AND, OR, and NOT Gates

CprE 2810: Digital Logic
Iowa State University, Ames, IA

Administrative Stuff

- HW2 is due on Monday Sep 8 @ 10pm

- Please write clearly on the first page the following three things:

  - Your First and Last Name
  - Your Student ID Number
  - Your Lab Section Letter

- Submit on Canvas as *one* PDF file.

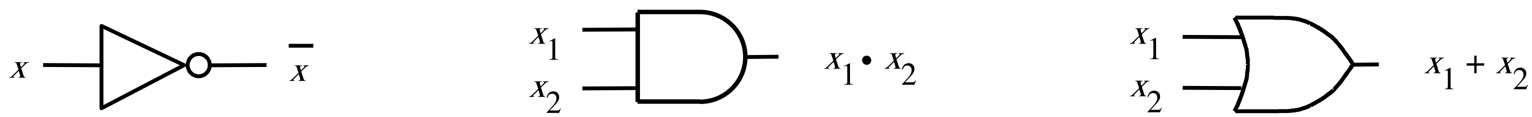
- Please orient your pages such that the text can be read without the need to rotate the page.

Administrative Stuff

- **Next week we will start with Lab2**
- **Read the lab assignment and do the prelab at home.**
- **Complete the prelab on paper before you go to the lab. Otherwise you'll lose 20% of your grade for that lab.**

Quick Review

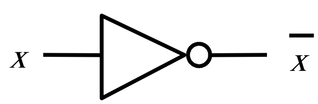
The Three Basic Logic Gates



NOT gate AND gate OR gate

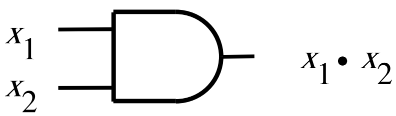
[Figure 2.8 from the textbook]

Truth Table for NOT



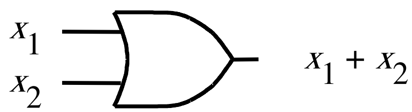
x	$\overline{x}$
0	1
1	0

Truth Table for AND



x_1	x_2	$x_1 \cdot x_2$
0	0	0
0	1	0
1	0	0
1	1	1

Truth Table for OR



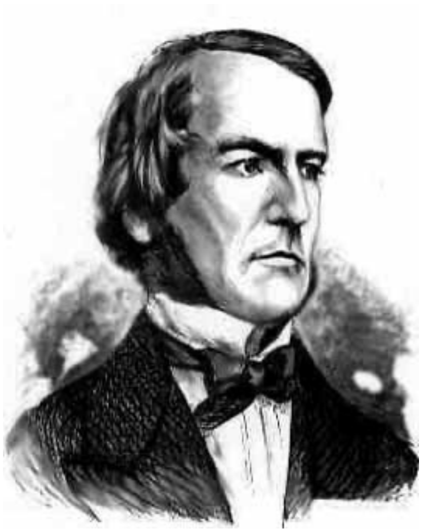
x_1	x_2	$x_1 + x_2$
0	0	0
0	1	1
1	0	1
1	1	1

Truth Tables for AND and OR

x_1	x_2	$x_1 \cdot x_2$	$x_1 + x_2$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1
		AND	OR

[Figure 2.6b from the textbook]

Boolean Algebra



George Boole
1815-1864

- **An algebraic structure consists of**
  - **a set of elements {0, 1}**
  - **binary operators {+, •}**
  - **and a unary operator { ʹ } or { ¯ } or { ~ }**

- **Introduced by George Boole in 1854**

- **An effective means of describing circuits built with switches**
- **A powerful tool that can be used for designing and analyzing logic circuits**

Different Notations for Negation

- **All three of these mean "negate x"**
  - x'
  - $\overline{x}$
  - $\sim x$

Operator Precedence

- In regular arithmetic and algebra, multiplication takes precedence over addition.
- This is also true in Boolean algebra.
- For example, $x + y \cdot z$ means
  multiply y by z and add the product to x .
- In other words, $x + y \cdot z$ is equal to $x + (y \cdot z)$,
  not $(x + y) \cdot z$.

The multiplication dot is optional

- **In regular algebra, the multiplication operator is often omitted to shorten the equations.**
- **This is also true in Boolean algebra.**
- **Both of these mean the same thing:**

  xy   is equal to   x • y

Operator Precedence
(three different ways to write the same)

$$x_1 \cdot x_2 + \overline{x}_1 \cdot \overline{x}_2$$

$$(x_1 \cdot x_2) + ((\overline{x}_1) \cdot (\overline{x}_2))$$

$$x_1 x_2 + \overline{x}_1 \overline{x}_2$$

Operator Precedence

- **Negation of a single variable takes precedence over multiplication of that variable with another variable.**
- **For example,**

  $\overline{A}\,B$ means negate A first and then multiply $\overline{A}$ by B

Operator Precedence

- However, a horizontal bar over a product of two variables means that the negation is performed after the product is computed.
- For example,

  $\overline{A\,B}$ means multiply A and B and then negate

Operator Precedence

- **Note that these two expressions are different:**

$\overline{A\ B}$ is not equal to $\overline{A}\ \overline{B}$

$\overline{A\ B}$ means multiply A and B and then negate

$\overline{A}\ \overline{B}$ means negate A and B separately and then multiply

Operator Precedence

- **Note that these two expressions are different:**

$$\overline{A\,B} \text{ is not equal to } \overline{A}\,\overline{B}$$

A	B	$\overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

A	B	$\overline{A}\overline{B}$
0	0	1
0	1	0
1	0	0
1	1	0

DeMorgan's Theorem

15a. $\overline{x \cdot y} = \overline{x} + \overline{y}$

15b. $\overline{x + y} = \overline{x} \cdot \overline{y}$

Proof of DeMorgan's theorem

15a. $\overline{x \cdot y} = \overline{x} + \overline{y}$

x	y	$x \cdot y$	$\overline{x \cdot y}$	$\overline{x}$	$\overline{y}$	$\overline{x} + \overline{y}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

LHS ($x \cdot y$, $\overline{x \cdot y}$) RHS ($\overline{x}$, $\overline{y}$, $\overline{x} + \overline{y}$)

Proof of DeMorgan's theorem

15a. $\overline{x \cdot y} = \overline{x} + \overline{y}$

x	y	$x \cdot y$	$\overline{x \cdot y}$	$\overline{x}$	$\overline{y}$	$\overline{x} + \overline{y}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

LHS RHS

These two columns are equal. Therefore, the theorem is true.

How to remember DeMorgan's theorem

$$\overline{x \cdot y}$$

start with the
left-hand side

How to remember DeMorgan's theorem

$$\overline{x \cdot y}$$

divide the bar
into 3 equal parts

How to remember DeMorgan's theorem

$$\overline{x} \cdot \overline{y}$$

erase the
middle segment

How to remember DeMorgan's theorem

$$\overline{x} + \overline{y}$$

change the product to a sum

How to remember DeMorgan's theorem

$$\overline{x} + \overline{y}$$

this is the
right-hand side

How to remember DeMorgan's theorem

$$\overline{x \bullet y} = \overline{x} + \overline{y}$$

Proof of the other DeMorgan's theorem

15b. $\overline{x + y} = \overline{x} \bullet \overline{y}$

x	y	$x + y$	$\overline{x + y}$	$\overline{x}$	$\overline{y}$	$\overline{x} \bullet \overline{y}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0
		LHS		RHS		

Proof of the other DeMorgan's theorem

15b. $\overline{x + y} = \overline{x} \bullet \overline{y}$

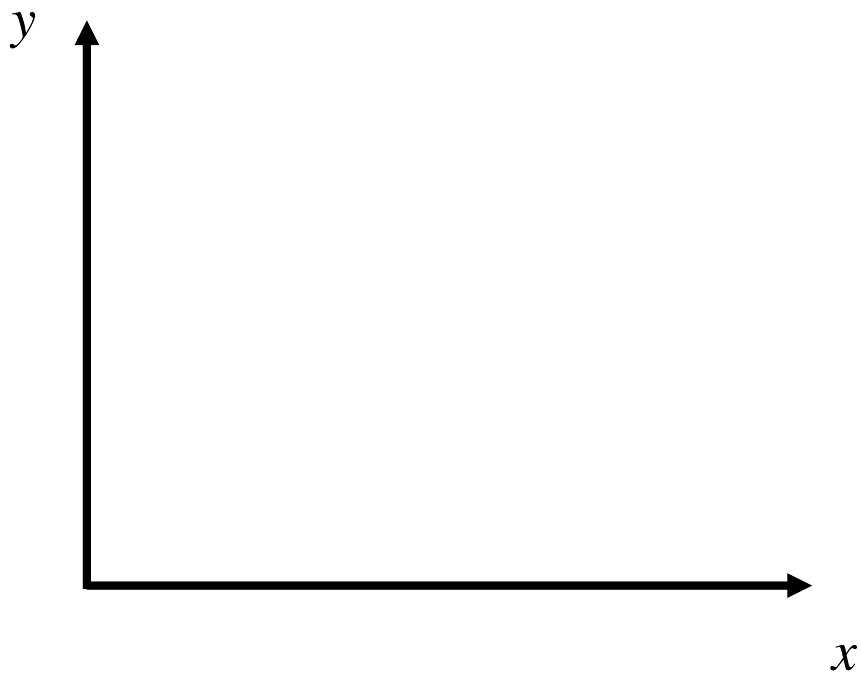
x	y	$x + y$	$\overline{x + y}$	$\overline{x}$	$\overline{y}$	$\overline{x} \bullet \overline{y}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

LHS (columns $x + y$, $\overline{x + y}$) RHS (columns $\overline{x}$, $\overline{y}$, $\overline{x} \bullet \overline{y}$)

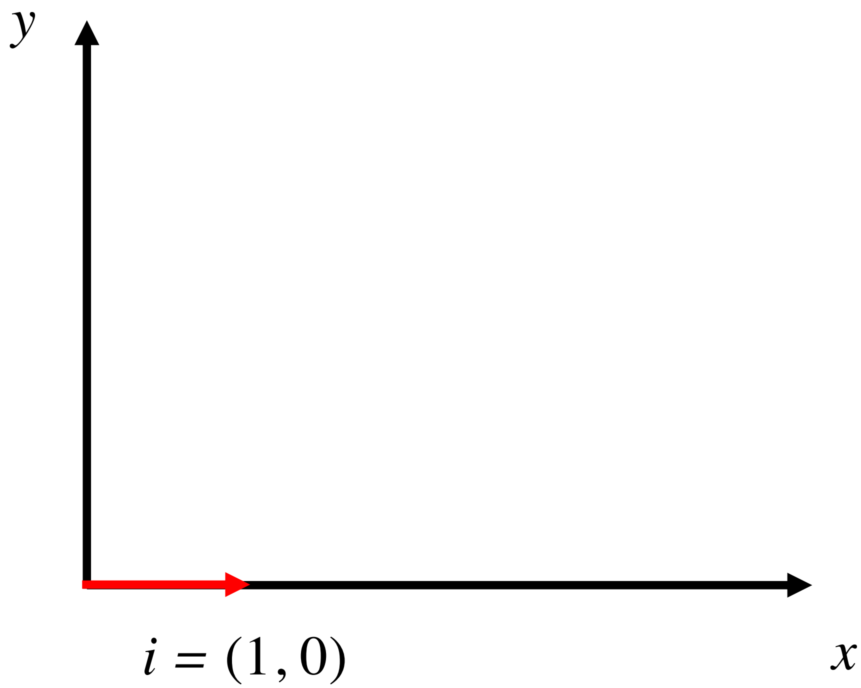
These two columns are equal. Therefore, the theorem is true.

A Short Digression

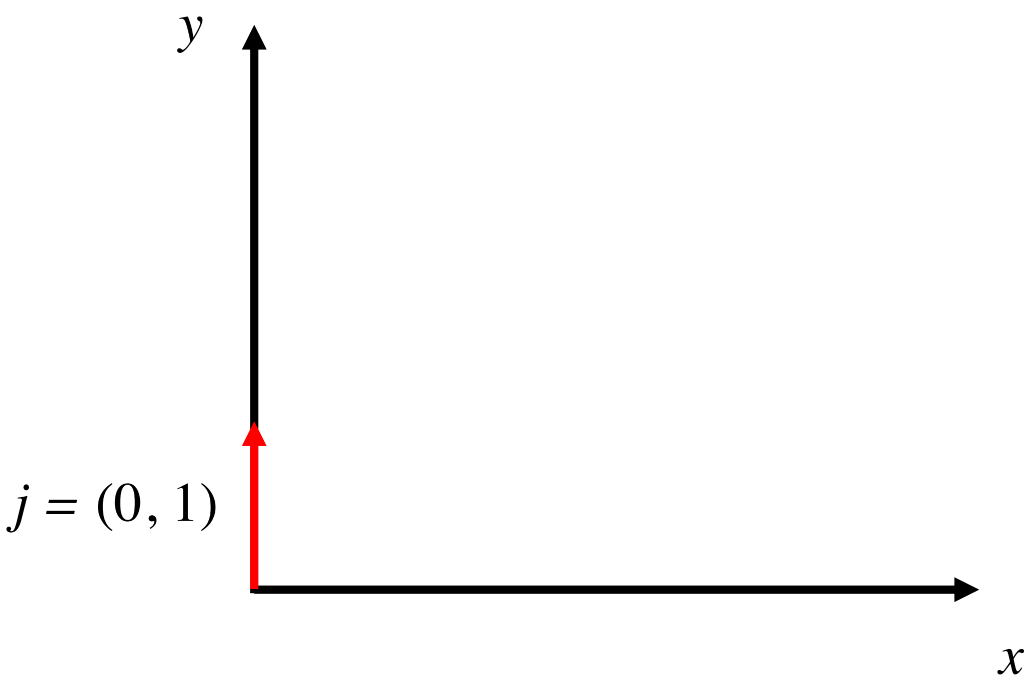
The 2D Plane



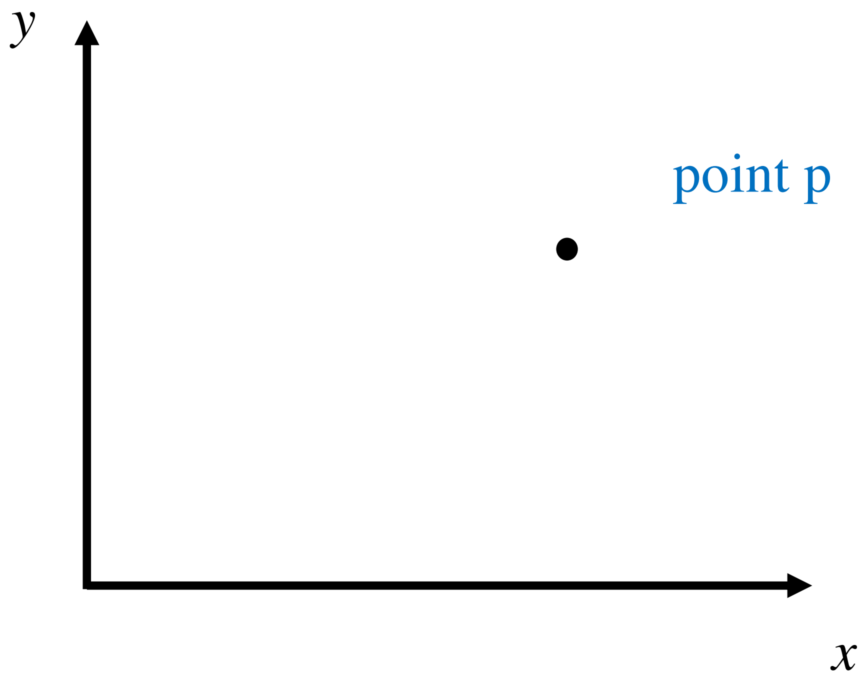
The 2D Plane



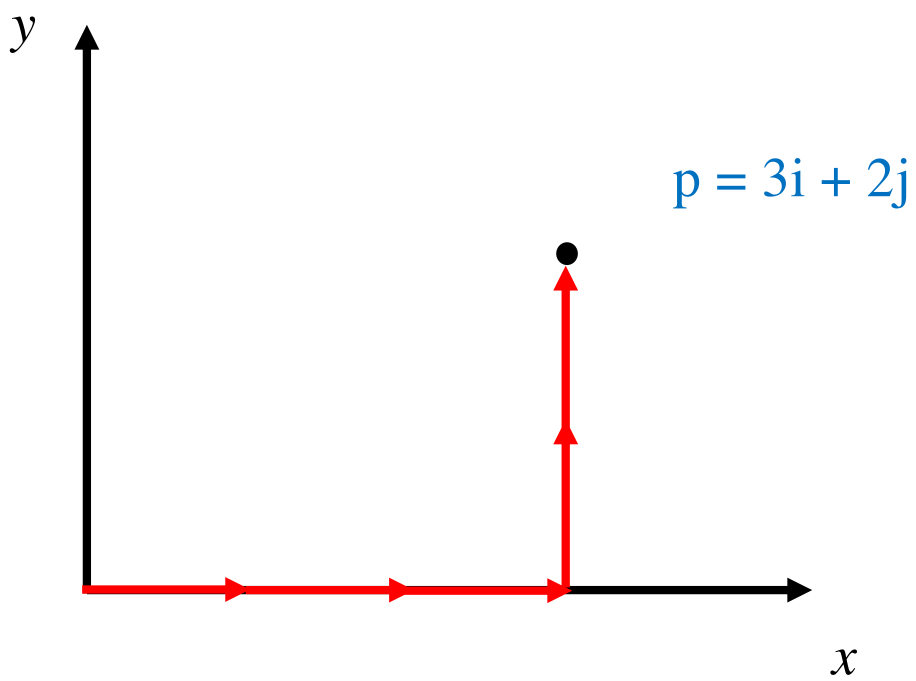
The 2D Plane



The 2D Plane



The 2D Plane



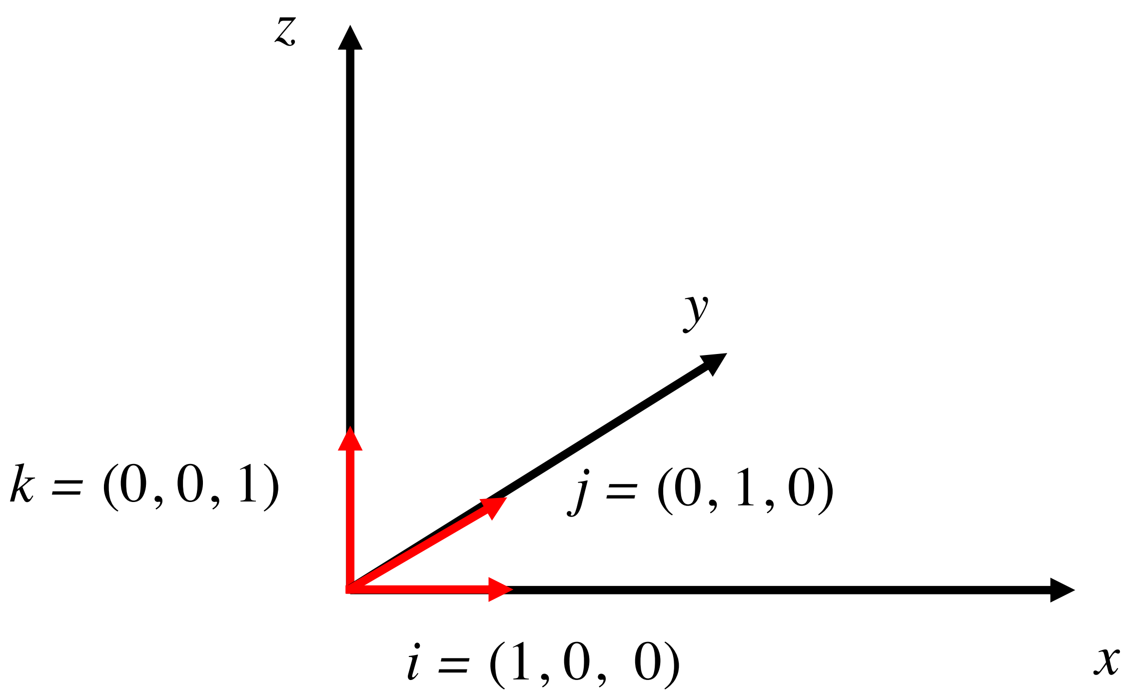
The unit vectors `i` and `j` form a basis

- Any point in the 2D plane can be represented as a linear combination of these two vectors.

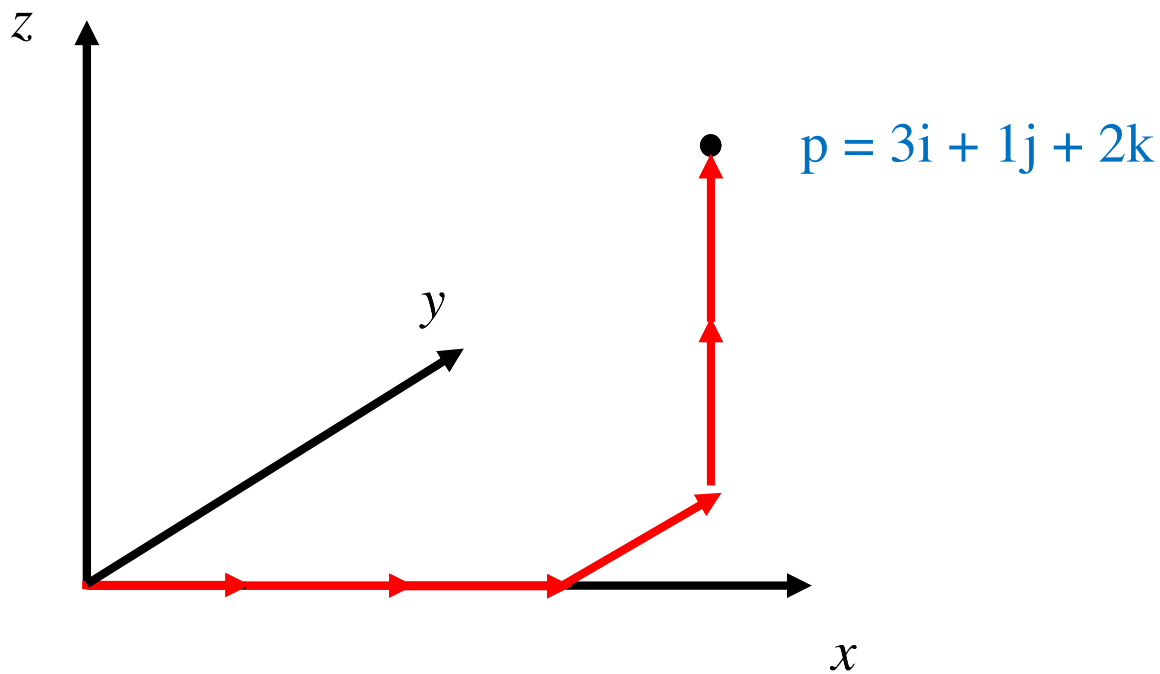
```
i=(1, 0)
j=(0, 1)
```

Note that there is only one 1 in each.

3D Space



3D Space



The 3D Basis

- In 3D we have `i, j,` and `k`

```
i=(1, 0, 0)
j=(0, 1, 0)
k=(0, 0, 1)
```

Note that there is only one 1 in each.

Any point in the 3D space can be represented as a linear combination of these three basis vectors.

The 4D Basis

- In 4D we have `four vectors`

$$\mathbf{x}^1 = (1,\ 0,\ 0,\ 0)$$
$$\mathbf{x}^2 = (0,\ 1,\ 0,\ 0)$$
$$\mathbf{x}^3 = (0,\ 0,\ 1,\ 0)$$
$$\mathbf{x}^4 = (0,\ 0,\ 0,\ 1)$$

Note that there is only one 1 in each.

Any point in this 4D space can be represented as a linear combination of these four basis vectors.

Basis Functions
(for two variables)

The Four Basis Functions

x	y	f_{00}
0	0	1
0	1	0
1	0	0
1	1	0

$f_{00}(x, y)$

x	y	f_{01}
0	0	0
0	1	1
1	0	0
1	1	0

$f_{01}(x, y)$

x	y	f_{10}
0	0	0
0	1	0
1	0	1
1	1	0

$f_{10}(x, y)$

x	y	f_{11}
0	0	0
0	1	0
1	0	0
1	1	1

$f_{11}(x, y)$

The Four Basis Functions

x	y	f_{00}
0	0	1
0	1	0
1	0	0
1	1	0

$f_{00}(x, y)$

x	y	f_{01}
0	0	0
0	1	1
1	0	0
1	1	0

$f_{01}(x, y)$

x	y	f_{10}
0	0	0
0	1	0
1	0	1
1	1	0

$f_{10}(x, y)$

x	y	f_{11}
0	0	0
0	1	0
1	0	0
1	1	1

$f_{11}(x, y)$

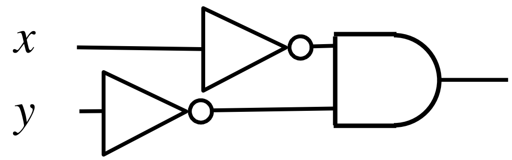
The Four Basis Functions

x	y		$f_{00}(x, y)$	$f_{01}(x, y)$	$f_{10}(x, y)$	$f_{11}(x, y)$
0	0		1	0	0	0
0	1		0	1	0	0
1	0		0	0	1	0
1	1		0	0	0	1

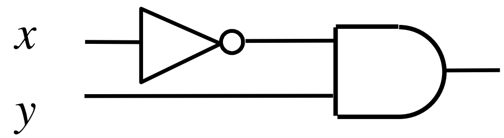
The Four Basis Functions

x	y		$\overline{x}\,\overline{y}$	$\overline{x}\,y$	$x\,\overline{y}$	$x\,y$
0	0		1	0	0	0
0	1		0	1	0	0
1	0		0	0	1	0
1	1		0	0	0	1

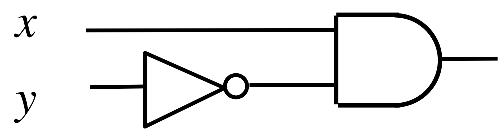
Circuits for the four basis functions



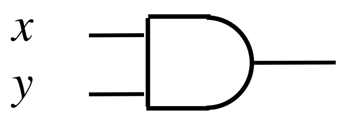
$f_{00}(x, y) = \bar{x}\,\bar{y}$



$f_{01}(x, y) = \bar{x}\,y$



$f_{10}(x, y) = x\,\bar{y}$



$f_{11}(x, y) = x\,y$

The Four Basis Functions

x	y	f_{00}
0	0	1
0	1	0
1	0	0
1	1	0

x	y	f_{01}
0	0	0
0	1	1
1	0	0
1	1	0

x	y	f_{10}
0	0	0
0	1	0
1	0	1
1	1	0

x	y	f_{11}
0	0	0
0	1	0
1	0	0
1	1	1

$f_{00}(x, y) = \overline{x}\,\overline{y}$

$f_{01}(x, y) = \overline{x}\,y$

$f_{10}(x, y) = x\,\overline{y}$

$f_{11}(x, y) = x\,y$

The Four Basis Functions

x	y	f_{00}
0	0	1
0	1	0
1	0	0
1	1	0

x	y	f_{01}
0	0	0
0	1	1
1	0	0
1	1	0

x	y	f_{10}
0	0	0
0	1	0
1	0	1
1	1	0

x	y	f_{11}
0	0	0
0	1	0
1	0	0
1	1	1

$f_{00}(x, y) = \overline{x}\ \overline{y}$ $f_{01}(x, y) = \overline{x}\ y$ $f_{10}(x, y) = x\ \overline{y}$ $f_{11}(x, y) = x\ y$

Negate the variable if the corresponding subscript of f is 0.

The Four Basis Functions

x	y	f_{00}
0	0	1
0	1	0
1	0	0
1	1	0

x	y	f_{01}
0	0	0
0	1	1
1	0	0
1	1	0

x	y	f_{10}
0	0	0
0	1	0
1	0	1
1	1	0

x	y	f_{11}
0	0	0
0	1	0
1	0	0
1	1	1

$f_{00}(x, y) = \overline{x}\ \overline{y}$ $\quad f_{01}(x, y) = \overline{x}\ y$ $\quad f_{10}(x, y) = x\ \overline{y}$ $\quad f_{11}(x, y) = x\ y$

Negate the variable if the corresponding subscript of f is 0.

The Four Basis Functions

x	y	f_{00}
0	0	1
0	1	0
1	0	0
1	1	0

x	y	f_{01}
0	0	0
0	1	1
1	0	0
1	1	0

x	y	f_{10}
0	0	0
0	1	0
1	0	1
1	1	0

x	y	f_{11}
0	0	0
0	1	0
1	0	0
1	1	1

$f_{00}(x, y) = \overline{x}\,\overline{y}$ $f_{01}(x, y) = \overline{x}\,y$ $f_{10}(x, y) = x\,\overline{y}$ $f_{11}(x, y) = x\,y$

Negate the variable if the corresponding subscript of f is 0.

minterms
(an alternative name for
the set of basis functions)

The Four Basis Functions

x	y	f_{00}
0	0	1
0	1	0
1	0	0
1	1	0

x	y	f_{01}
0	0	0
0	1	1
1	0	0
1	1	0

x	y	f_{10}
0	0	0
0	1	0
1	0	1
1	1	0

x	y	f_{11}
0	0	0
0	1	0
1	0	0
1	1	1

$f_{00}(x, y) = \overline{x}\ \overline{y}$

$f_{01}(x, y) = \overline{x}\ y$

$f_{10}(x, y) = x\ \overline{y}$

$f_{11}(x, y) = x\ y$

The Four Basis Functions
(alternative names)

x	y	f_{00}
0	0	1
0	1	0
1	0	0
1	1	0

x	y	f_{01}
0	0	0
0	1	1
1	0	0
1	1	0

x	y	f_{10}
0	0	0
0	1	0
1	0	1
1	1	0

x	y	f_{11}
0	0	0
0	1	0
1	0	0
1	1	1

$f_{00}(x, y) = \overline{x}\,\overline{y}$ $f_{01}(x, y) = \overline{x}\,y$ $f_{10}(x, y) = x\,\overline{y}$ $f_{11}(x, y) = x\,y$

m_0 m_1 m_2 m_3

The Four Basis Functions
(minterms)

x	y	m_0
0	0	1
0	1	0
1	0	0
1	1	0

$f_{00}(x, y) = \overline{x}\ \overline{y}$

m_0

x	y	m_1
0	0	0
0	1	1
1	0	0
1	1	0

$f_{01}(x, y) = \overline{x}\ y$

m_1

x	y	m_2
0	0	0
0	1	0
1	0	1
1	1	0

$f_{10}(x, y) = x\ \overline{y}$

m_2

x	y	m_3
0	0	0
0	1	0
1	0	0
1	1	1

$f_{11}(x, y) = x\ y$

m_3

The Four Basis Functions
(minterms)

x	y		$\overline{x}\,\overline{y}$	$\overline{x}\,y$	$x\,\overline{y}$	$x\,y$
0	0		1	0	0	0
0	1		0	1	0	0
1	0		0	0	1	0
1	1		0	0	0	1

Expressions for the minterms

$$m_0 = \overline{x}\ \overline{y}$$

$$m_1 = \overline{x}\ y$$

$$m_2 = x\ \overline{y}$$

$$m_3 = x\ y$$

Expressions for the minterms

0 0	$m_0 = \overline{x}\ \overline{y}$	
0 1	$m_1 = \overline{x}\ y$	
1 0	$m_2 = x\ \overline{y}$	
1 1	$m_3 = x\ y$	

The bars coincide
with the 0's
in the binary expansion
of the minterm sub-index

Expressions for the minterms

0	0	$m_0 = \bar{x}\ \bar{y}$	
0	1	$m_1 = \bar{x}\ y$	
1	0	$m_2 = x\ \bar{y}$	
1	1	$m_3 = x\ y$	

The bars coincide
with the 0's
in the binary expansion
of the minterm sub-index

Function Synthesis Example
(with two variables)

Synthesize the Following Function

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

1) Split the function into a sum of 4 functions

x_1	x_2	$f(x_1, x_2)$		$f_{00}(x_1, x_2)$	$f_{01}(x_1, x_2)$	$f_{10}(x_1, x_2)$	$f_{11}(x_1, x_2)$
0	0	1		1	0	0	0
0	1	1		0	1	0	0
1	0	0		0	0	1	0
1	1	1		0	0	0	1

1) Split the function into a sum of 4 functions

x_1	x_2	$f(x_1, x_2)$		$f_{00}(x_1, x_2)$	$f_{01}(x_1, x_2)$	$f_{10}(x_1, x_2)$	$f_{11}(x_1, x_2)$
0	0	1		1	0	0	0
0	1	1		0	1	0	0
1	0	0		0	0	1	0
1	1	1		0	0	0	1

$$f(x_1, x_2) = 1 \bullet f_{00} + 1 \bullet f_{01} + 0 \bullet f_{10} + 1 \bullet f_{11}$$

2) Write the expressions for all four

x_1	x_2	$f(x_1, x_2)$		$f_{00}(x_1, x_2)$	$f_{01}(x_1, x_2)$	$f_{10}(x_1, x_2)$	$f_{11}(x_1, x_2)$
0	0	1		1	0	0	0
0	1	1		0	1	0	0
1	0	0		0	0	1	0
1	1	1		0	0	0	1

$$f(x_1, x_2) = \underbrace{1 \bullet f_{00}} + \underbrace{1 \bullet f_{01}} + \underbrace{0 \bullet f_{10}} + \underbrace{1 \bullet f_{11}}$$

2) Write the expressions for all four

x_1	x_2	$f(x_1, x_2)$	$f_{00}(x_1, x_2)$	$f_{01}(x_1, x_2)$	$f_{10}(x_1, x_2)$	$f_{11}(x_1, x_2)$
0	0	1	1	0	0	0
0	1	1	0	1	0	0
1	0	0	0	0	1	0
1	1	1	0	0	0	1

$$f(x_1, x_2) = \underbrace{1 \bullet f_{00}}_{\bar{x}_1\bar{x}_2} + \underbrace{1 \bullet f_{01}}_{\bar{x}_1 x_2} + \underbrace{0 \bullet f_{10}}_{0} + \underbrace{1 \bullet f_{11}}_{x_1 x_2}$$

3) Then just add them together

x_1	x_2	$f(x_1, x_2)$		$f_{00}(x_1, x_2)$	$f_{01}(x_1, x_2)$	$f_{10}(x_1, x_2)$	$f_{11}(x_1, x_2)$
0	0	1		1	0	0	0
0	1	1		0	1	0	0
1	0	0		0	0	1	0
1	1	1		0	0	0	1

$$f(x_1, x_2) = \underbrace{1 \bullet f_{00}} + \underbrace{1 \bullet f_{01}} + \underbrace{0 \bullet f_{10}} + \underbrace{1 \bullet f_{11}}$$

$$f(x_1, x_2) = \bar{x}_1\bar{x}_2 + \bar{x}_1 x_2 + 0 + x_1 x_2$$

3) Then just add them together

x_1	x_2	$f(x_1, x_2)$		$f_{00}(x_1, x_2)$	$f_{01}(x_1, x_2)$	$f_{10}(x_1, x_2)$	$f_{11}(x_1, x_2)$
0	0	1		1	0	0	0
0	1	1		0	1	0	0
1	0	0		0	0	1	0
1	1	1		0	0	0	1

$$f(x_1, x_2) = \bar{x}_1\bar{x}_2 + \bar{x}_1 x_2 + 0 + x_1 x_2$$

A function to be synthesized

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

[Figure 2.19 from the textbook]

Let's look at it row by row.
How can we express the last row?

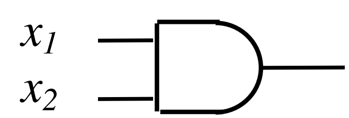
x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

Let's look at it row by row.
How can we express the last row?

x_1	x_2	$f(x_1, x_2)$	
0	0	1	
0	1	1	
1	0	0	
1	1	1	$x_1 x_2$

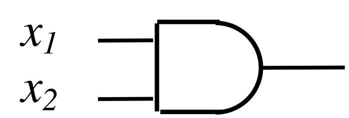
Let’s look at it row by row.
How can we express the last row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1



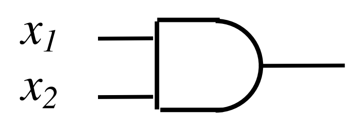
What about this row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1



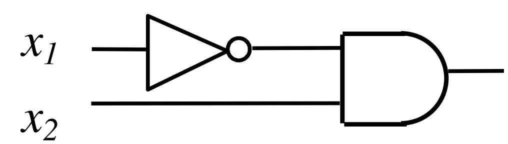
What about this row?

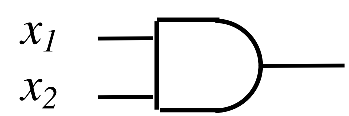
x_1	x_2	$f(x_1, x_2)$	
0	0	1	
0	1	1	$\bar{x}_1 x_2$
1	0	0	
1	1	1	



What about this row?

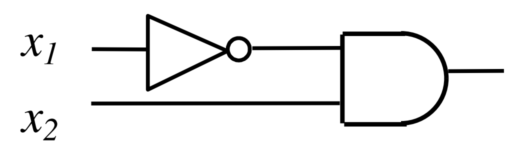
x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

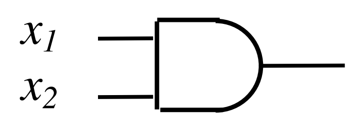




What about the first row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

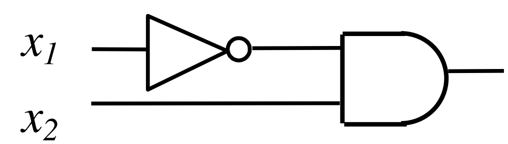


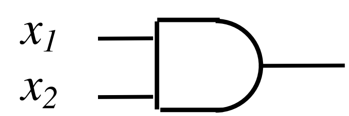


What about the first row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

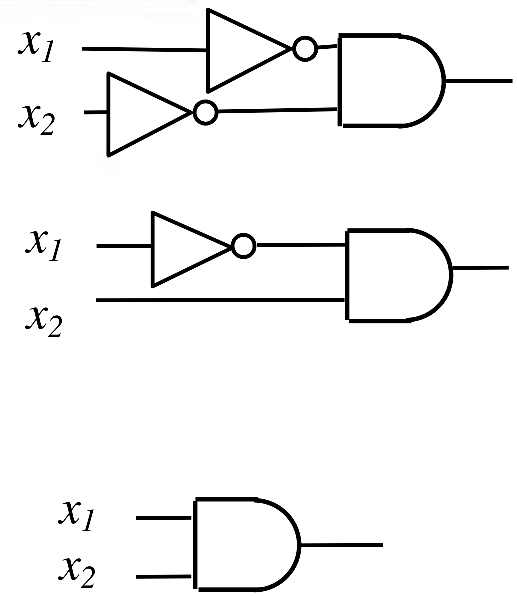
$\overline{x}_1\overline{x}_2$





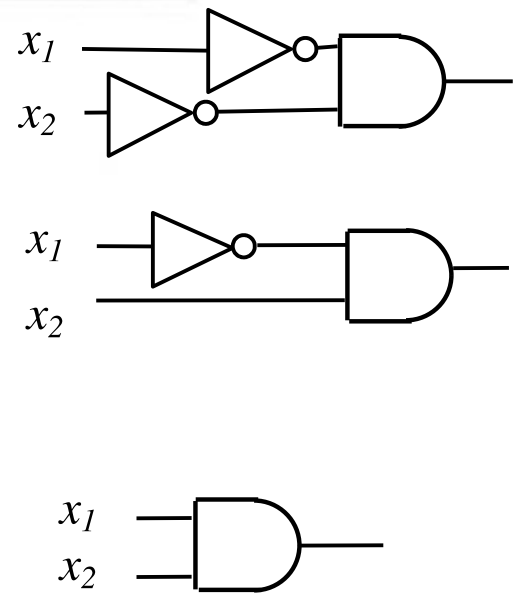
What about the first row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

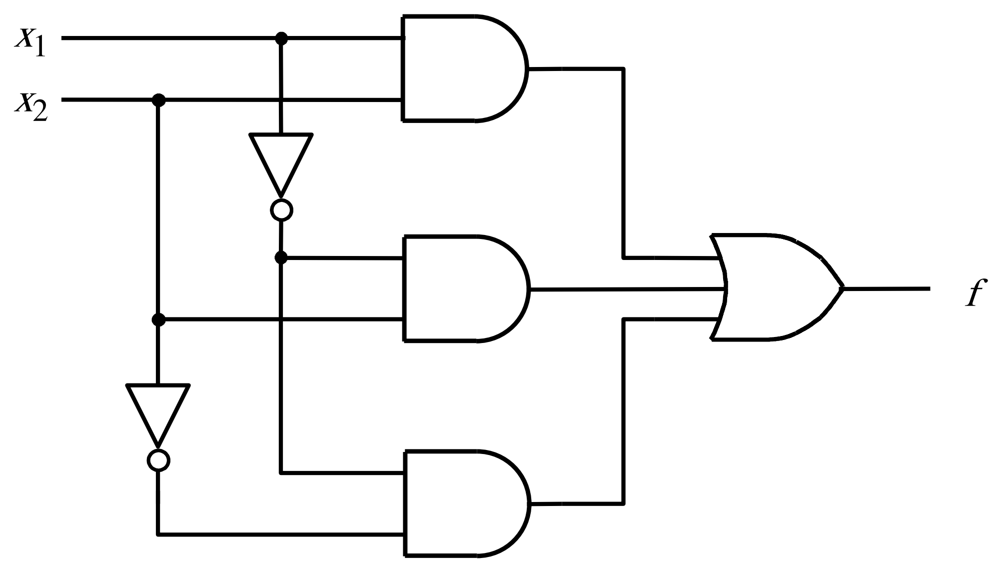


Finally, what about the zero?

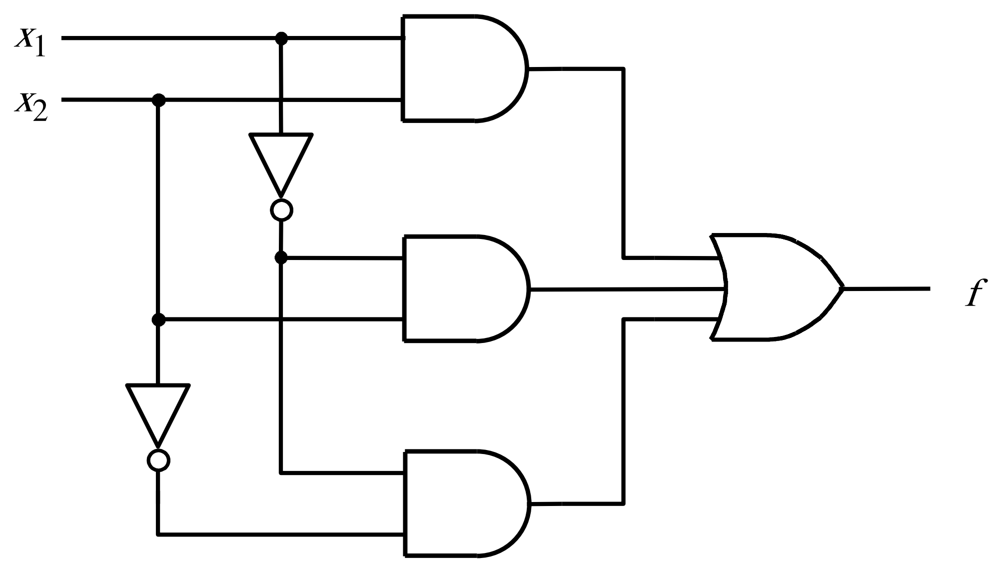
x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1



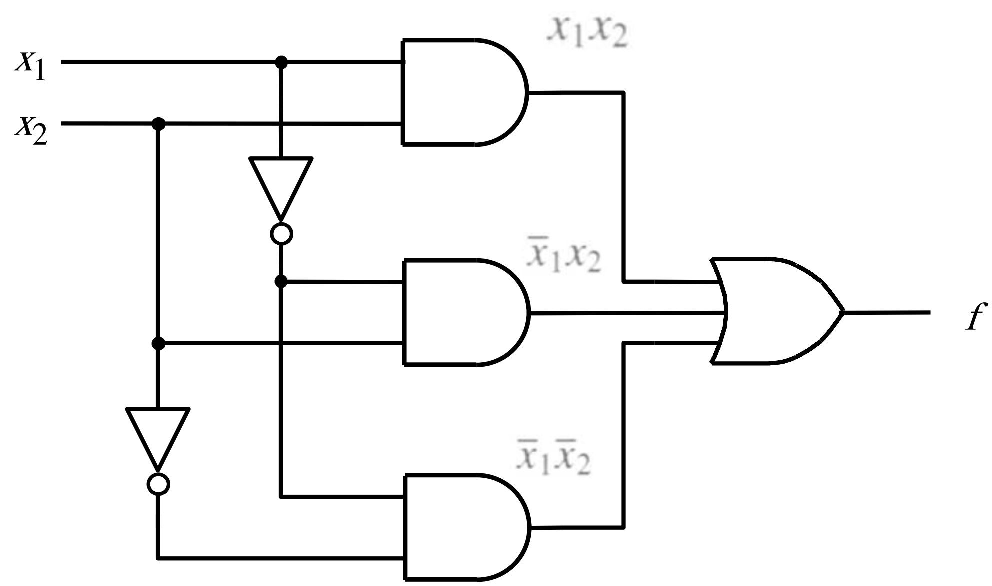
Putting it all together



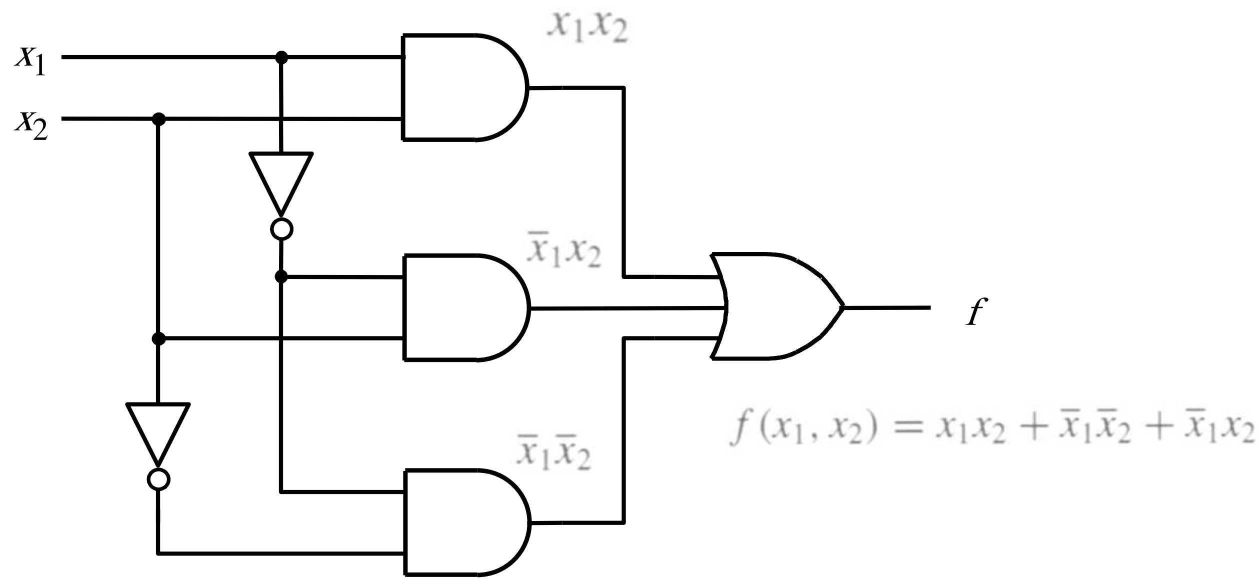
Let's verify that this circuit implements correctly the target truth table



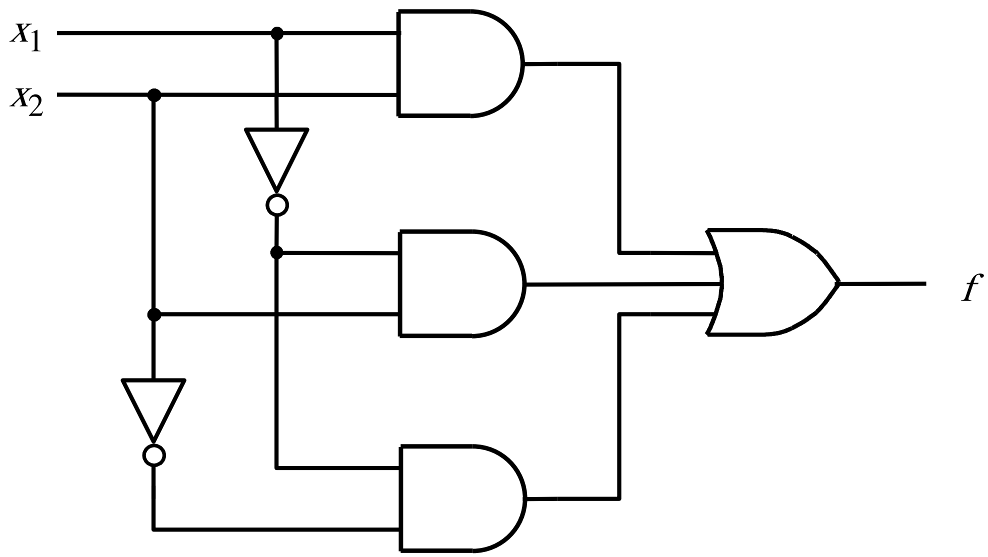
Let’s verify that this circuit implements correctly the target truth table



Putting it all together



Canonical Sum-Of-Products (SOP)



$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

[Figure 2.20a from the textbook]

Summary of This Procedure

- **Get the truth table of the function**
- **Form a product term (AND gate) for each row of the table for which the function is 1**
- **Each product term contains all input variables**
- **In each row, if x_i=1 enter it as x_i, otherwise use $\overline{x_i}$**
- **Sum all of these products (OR gate) to get the function**

Simplification Steps

$$f(x_1, x_2) = x_1 x_2 + \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2$$

Simplification Steps

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

replicate
this term

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2 + \bar{x}_1x_2$$

Simplification Steps

$$f(x_1, x_2) = x_1 x_2 + \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2$$

group
these terms

$$f(x_1, x_2) = x_1 x_2 + \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2 + \bar{x}_1 x_2$$

$$f(x_1, x_2) = (x_1 + \bar{x}_1) x_2 + \bar{x}_1 (\bar{x}_2 + x_2)$$

Simplification Steps

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2 + \bar{x}_1x_2$$

These two terms are trivially equal to 1

$$f(x_1, x_2) = (x_1 + \bar{x}_1)x_2 + \bar{x}_1(\bar{x}_2 + x_2)$$

$$f(x_1, x_2) = 1 \cdot x_2 + \bar{x}_1 \cdot 1$$

Simplification Steps

$$f(x_1, x_2) = x_1 x_2 + \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2$$

$$f(x_1, x_2) = x_1 x_2 + \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2 + \bar{x}_1 x_2$$

$$f(x_1, x_2) = (x_1 + \bar{x}_1) x_2 + \bar{x}_1 (\bar{x}_2 + x_2)$$

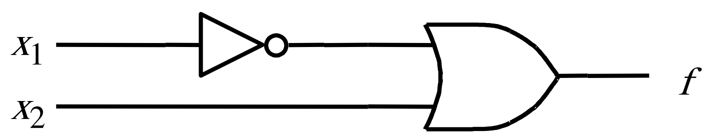
Drop the 1's

$$f(x_1, x_2) = 1 \cdot x_2 + \bar{x}_1 \cdot 1$$

$$f(x_1, x_2) = x_2 + \bar{x}_1$$

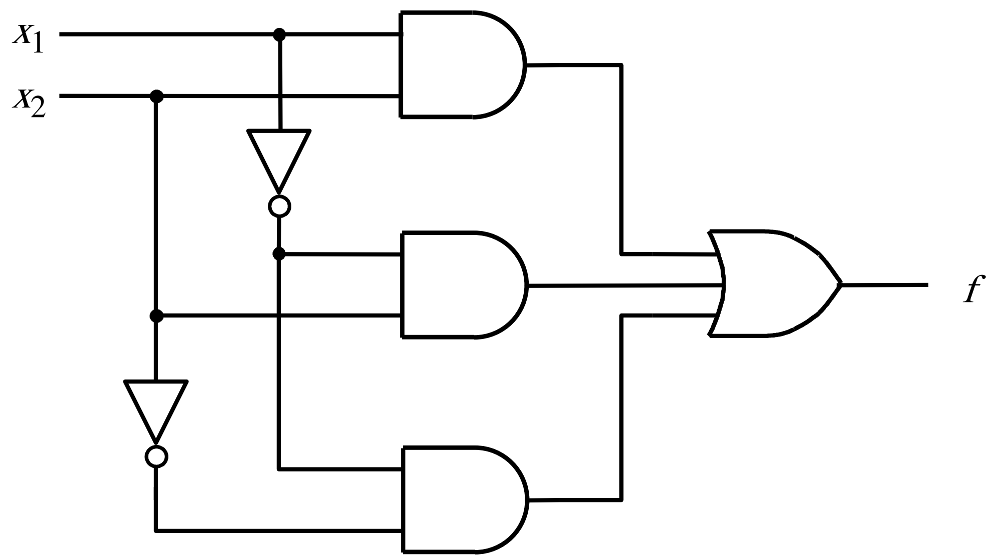
Minimal-cost realization

$$f(x_1, x_2) = x_2 + \bar{x}_1$$

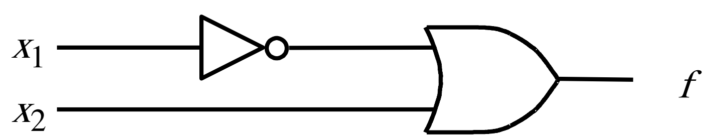


[Figure 2.20b from the textbook]

Two implementations for the same function



(a) Canonical sum-of-products



(b) Minimal-cost realization

[Figure 2.20 from the textbook]

Basis Functions / minterms (for three variables)

The Eight Basis Functions

x	y	z	f_{000}	f_{001}	f_{010}	f_{011}	f_{100}	f_{101}	f_{110}	f_{111}
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

The Eight Basis Functions

x	y	z	f_{000}	f_{001}	f_{010}	f_{011}	f_{100}	f_{101}	f_{110}	f_{111}
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

The Eight minterms

x	y	z	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

Expressions for the minterms

$$m_0 = \bar{x}\,\bar{y}\,\bar{z}$$

$$m_1 = \bar{x}\,\bar{y}\,z$$

$$m_2 = \bar{x}\,y\,\bar{z}$$

$$m_3 = \bar{x}\,y\,z$$

$$m_4 = x\,\bar{y}\,\bar{z}$$

$$m_5 = x\,\bar{y}\,z$$

$$m_6 = x\,y\,\bar{z}$$

$$m_7 = x\,y\,z$$

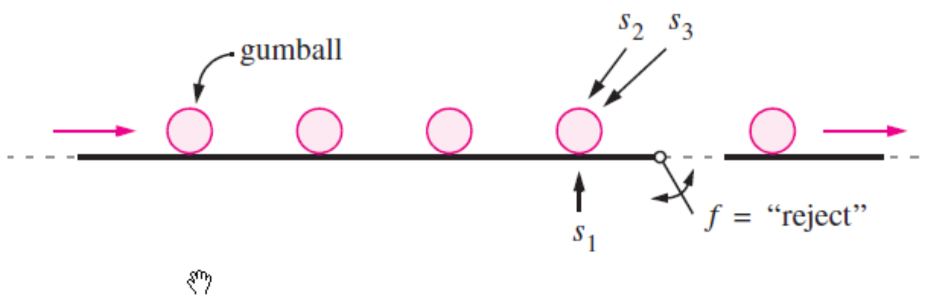
Expressions for the minterms

0	0	0	$m_0 = \bar{x}\,\bar{y}\,\bar{z}$
0	0	1	$m_1 = \bar{x}\,\bar{y}\,z$
0	1	0	$m_2 = \bar{x}\,y\,\bar{z}$
0	1	1	$m_3 = \bar{x}\,y\,z$
1	0	0	$m_4 = x\,\bar{y}\,\bar{z}$
1	0	1	$m_5 = x\,\bar{y}\,z$
1	1	0	$m_6 = x\,y\,\bar{z}$
1	1	1	$m_7 = x\,y\,z$

The bars coincide
with the 0's
in the binary expansion
of the minterm sub-index

Function Synthesis Example (with three variables)

Let's look at another problem



(a) Conveyor and sensors

s_1	s_2	s_3	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(b) Truth table

[Figure 2.21 from the textbook]

Let's look at another problem

s_1	s_2	s_3	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

[Figure 2.21b from the textbook]

Let's look at another problem

s_1	s_2	s_3	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Let's look at another problem

s_1	s_2	s_3	f	
0	0	0	0	
0	0	1	1	$\bar{s}_1 \bar{s}_2 s_3$
0	1	0	0	
0	1	1	1	$\bar{s}_1 s_2 s_3$
1	0	0	0	
1	0	1	1	$s_1 \bar{s}_2 s_3$
1	1	0	1	$s_1 s_2 \bar{s}_3$
1	1	1	1	$s_1 s_2 s_3$

Let's look at another problem

s_1	s_2	s_3	f	
0	0	0	0	
$\bar{0}$	$\bar{0}$	1	1	$\bar{s}_1\bar{s}_2s_3$
0	1	0	0	
0	1	1	1	$\bar{s}_1s_2s_3$
1	0	0	0	
1	0	1	1	$s_1\bar{s}_2s_3$
1	1	0	1	$s_1s_2\bar{s}_3$
1	1	1	1	$s_1s_2s_3$

Let's look at another problem

s_1	s_2	s_3	f	
0	0	0	0	
$\overline{0}$	$\overline{0}$	1	1	$\overline{s}_1\overline{s}_2s_3$
0	1	0	0	
$\overline{0}$	1	1	1	$\overline{s}_1s_2s_3$
1	0	0	0	
1	0	1	1	$s_1\overline{s}_2s_3$
1	1	0	1	$s_1s_2\overline{s}_3$
1	1	1	1	$s_1s_2s_3$

Let's look at another problem

s_1	s_2	s_3	f	
0	0	0	0	
$\overline{0}$	$\overline{0}$	1	1	$\overline{s}_1 \overline{s}_2 s_3$
0	1	0	0	
$\overline{0}$	1	1	1	$\overline{s}_1 s_2 s_3$
1	0	0	0	
1	$\overline{0}$	1	1	$s_1 \overline{s}_2 s_3$
1	1	0	1	$s_1 s_2 \overline{s}_3$
1	1	1	1	$s_1 s_2 s_3$

Let's look at another problem

s_1	s_2	s_3	f	
0	0	0	0	
$\overline{0}$	$\overline{0}$	1	1	$\overline{s}_1\overline{s}_2s_3$
0	1	0	0	
$\overline{0}$	1	1	1	$\overline{s}_1s_2s_3$
1	0	0	0	
1	$\overline{0}$	1	1	$s_1\overline{s}_2s_3$
1	1	$\overline{0}$	1	$s_1s_2\overline{s}_3$
1	1	1	1	$s_1s_2s_3$

Let's look at another problem

s_1	s_2	s_3	f	
0	0	0	0	
0	0	1	1	$\bar{s}_1\bar{s}_2s_3$
0	1	0	0	
0	1	1	1	$\bar{s}_1s_2s_3$
1	0	0	0	
1	0	1	1	$s_1\bar{s}_2s_3$
1	1	0	1	$s_1s_2\bar{s}_3$
1	1	1	1	$s_1s_2s_3$

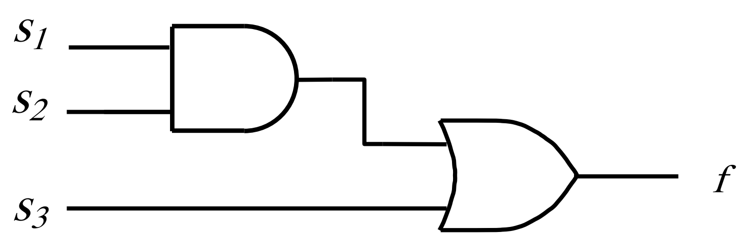
$$f = \bar{s}_1\bar{s}_2s_3 + \bar{s}_1s_2s_3 + s_1\bar{s}_2s_3 + s_1s_2\bar{s}_3 + s_1s_2s_3$$

Let's look at another problem (minimization)

$$
\begin{aligned}
f &= \bar{s}_1\bar{s}_2s_3 + \bar{s}_1s_2s_3 + s_1\bar{s}_2s_3 + s_1s_2s_3 + s_1s_2\bar{s}_3 + s_1s_2s_3 \\
&= \bar{s}_1s_3(\bar{s}_2 + s_2) + s_1s_3(\bar{s}_2 + s_2) + s_1s_2(\bar{s}_3 + s_3) \\
&= \bar{s}_1s_3 + s_1s_3 + s_1s_2 \\
&= s_3 + s_1s_2
\end{aligned}
$$

Let's look at another problem (minimization)

$$\begin{aligned} f &= \bar{s}_1\bar{s}_2s_3 + \bar{s}_1s_2s_3 + s_1\bar{s}_2s_3 + s_1s_2s_3 + s_1s_2\bar{s}_3 + s_1s_2s_3 \\ &= \bar{s}_1s_3(\bar{s}_2 + s_2) + s_1s_3(\bar{s}_2 + s_2) + s_1s_2(\bar{s}_3 + s_3) \\ &= \bar{s}_1s_3 + s_1s_3 + s_1s_2 \\ &= s_3 + s_1s_2 \end{aligned}$$



Maxterms
(an alternative set of basis functions)

The Four Maxterms

x	y	M_0
0	0	0
0	1	1
1	0	1
1	1	1

$M_0(x, y)$

x	y	M_1
0	0	1
0	1	0
1	0	1
1	1	1

$M_1(x, y)$

x	y	M_2
0	0	1
0	1	1
1	0	0
1	1	1

$M_2(x, y)$

x	y	M_3
0	0	1
0	1	1
1	0	1
1	1	0

$M_3(x, y)$

The Four Maxterms

x	y	M_0
0	0	0
0	1	1
1	0	1
1	1	1

$M_0(x, y)$

x	y	M_1
0	0	1
0	1	0
1	0	1
1	1	1

$M_1(x, y)$

x	y	M_2
0	0	1
0	1	1
1	0	0
1	1	1

$M_2(x, y)$

x	y	M_3
0	0	1
0	1	1
1	0	1
1	1	0

$M_3(x, y)$

The Four Maxterms

x	y		$M_0(x, y)$	$M_1(x, y)$	$M_2(x, y)$	$M_3(x, y)$
0	0		0	1	1	1
0	1		1	0	1	1
1	0		1	1	0	1
1	1		1	1	1	0

The Four Maxterms

x	y		x + y	$x + \overline{y}$	$\overline{x} + y$	$\overline{x} + \overline{y}$
0	0		0	1	1	1
0	1		1	0	1	1
1	0		1	1	0	1
1	1		1	1	1	0

Expressions for the Maxterms

$$M_0 = x + y$$

$$M_1 = x + \overline{y}$$

$$M_2 = \overline{x} + y$$

$$M_3 = \overline{x} + \overline{y}$$

Expressions for the Maxterms

$M_0 = x + y$

$M_1 = x + \overline{y}$

$M_2 = \overline{x} + y$

$M_3 = \overline{x} + \overline{y}$

Note that these are now sums, not products.

Expressions for the Maxterms

0 0	$M_0 = x + y$	
0 1	$M_1 = x + \overline{y}$	
1 0	$M_2 = \overline{x} + y$	
1 1	$M_3 = \overline{x} + \overline{y}$	

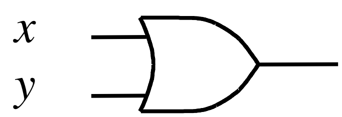
The bars coincide
with the 1's
in the binary expansion
of the maxterm sub-index

Expressions for the Maxterms

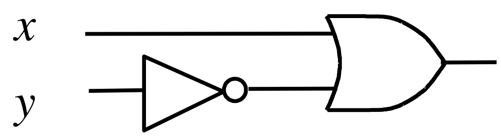
0 0	$M_0 = x + y$	
0 1	$M_1 = x + \overline{y}$	
1 0	$M_2 = \overline{x} + y$	
1 1	$M_3 = \overline{x} + \overline{y}$	

The bars coincide
with the 1's
in the binary expansion
of the maxterm sub-index

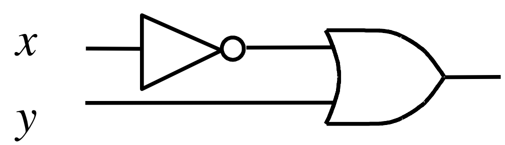
Circuits for the four Maxterms



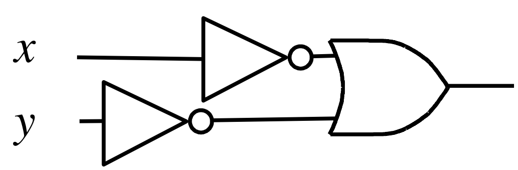
$M_0(x, y) = x + y$



$M_1(x, y) = x + \overline{y}$



$M_2(x, y) = \overline{x} + y$



$M_3(x, y) = \overline{x} + \overline{y}$

Minterms and Maxterms

Row number	x_1	x_2	Minterm	Maxterm
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	$M_0 = x_1 + x_2$
1	0	1	$m_1 = \overline{x}_1 x_2$	$M_1 = x_1 + \overline{x}_2$
2	1	0	$m_2 = x_1\overline{x}_2$	$M_2 = \overline{x}_1 + x_2$
3	1	1	$m_3 = x_1 x_2$	$M_3 = \overline{x}_1 + \overline{x}_2$

Minterms and Maxterms

Row number	x_1	x_2	Minterm	Maxterm
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	$M_0 = x_1 + x_2$
1	0	1	$m_1 = \overline{x}_1 x_2$	$M_1 = x_1 + \overline{x}_2$
2	1	0	$m_2 = x_1\overline{x}_2$	$M_2 = \overline{x}_1 + x_2$
3	1	1	$m_3 = x_1 x_2$	$M_3 = \overline{x}_1 + \overline{x}_2$

Use these for Sum-of-Products Minimization
(1's of the function)

Use these for Product-of-Sums Minimization
(0's of the function)

Minterms and Maxterms

Row number	x_1	x_2	Minterm	Maxterm
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	$M_0 = x_1 + x_2$
1	0	1	$m_1 = \overline{x}_1 x_2$	$M_1 = x_1 + \overline{x}_2$
2	1	0	$m_2 = x_1\overline{x}_2$	$M_2 = \overline{x}_1 + x_2$
3	1	1	$m_3 = x_1 x_2$	$M_3 = \overline{x}_1 + \overline{x}_2$

Use these for
Sum-of-Products
Minimization
(1's of the function)

Use these for
Product-of-Sums
Minimization
(0's of the function)

Sum-of-Products Form

(uses the ones of the function)

Sum-of-Products Form

(for the AND logic function)

Row number	x_1	x_2	Minterm	$f(x_1, x_2)$
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	0
1	0	1	$m_1 = \overline{x}_1 x_2$	0
2	1	0	$m_2 = x_1\overline{x}_2$	0
3	1	1	$m_3 = x_1 x_2$	1

Sum-of-Products Form

(for the AND logic function)

Row number	x_1	x_2	Minterm	$f(x_1, x_2)$
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	0
1	0	1	$m_1 = \overline{x}_1 x_2$	0
2	1	0	$m_2 = x_1\overline{x}_2$	0
3	1	1	$m_3 = x_1 x_2$	1

Sum-of-Products Form

(for the AND logic function)

Row number	x_1	x_2	Minterm	$f(x_1, x_2)$
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	0
1	0	1	$m_1 = \overline{x}_1 x_2$	0
2	1	0	$m_2 = x_1\overline{x}_2$	0
3	1	1	$m_3 = x_1 x_2$	1

$$f(x_1, x_2) = m_3 = x_1 x_2$$

(In this case there is just one product and there is no need for a sum)

Another Example

Sum-of-Products Form

Row number	x_1	x_2	Minterm	$f(x_1, x_2)$
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	1
1	0	1	$m_1 = \overline{x}_1 x_2$	1
2	1	0	$m_2 = x_1\overline{x}_2$	0
3	1	1	$m_3 = x_1 x_2$	1

Sum-of-Products Form

Row number	x_1	x_2	Minterm	$f(x_1, x_2)$
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	1
1	0	1	$m_1 = \overline{x}_1 x_2$	1
2	1	0	$m_2 = x_1\overline{x}_2$	0
3	1	1	$m_3 = x_1 x_2$	1

Sum-of-Products Form

Row number	x_1	x_2	Minterm	$f(x_1, x_2)$
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	1
1	0	1	$m_1 = \overline{x}_1 x_2$	1
2	1	0	$m_2 = x_1\overline{x}_2$	0
3	1	1	$m_3 = x_1 x_2$	1

$$\begin{aligned} f &= m_0 \cdot 1 + m_1 \cdot 1 + m_2 \cdot 0 + m_3 \cdot 1 \\ &= m_0 + m_1 + m_3 \\ &= \overline{x}_1\overline{x}_2 + \overline{x}_1 x_2 + x_1 x_2 \end{aligned}$$

Product-of-Sums Form

(uses the zeros of the function)

Product-of-Sums Form
(for the OR logic function)

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1
2	1	0	$M_2 = \overline{x}_1 + x_2$	1
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1

Product-of-Sums Form

(for the OR logic function)

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1
2	1	0	$M_2 = \overline{x}_1 + x_2$	1
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1

Product-of-Sums Form
(for the OR logic function)

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1
2	1	0	$M_2 = \overline{x}_1 + x_2$	1
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1

$$f(x_1, x_2) = M_0 = x_1 + x_2$$

(In this case there is just one sum and there is no need for a product)

Another Example

Product-of-Sums Form
(for this logic function)

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1
2	1	0	$M_2 = \overline{x}_1 + x_2$	0
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1

Product-of-Sums Form
(for this logic function)

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1
2	1	0	$M_2 = \overline{x}_1 + x_2$	0
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1

Product-of-Sums Form
(for this logic function)

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1
2	1	0	$M_2 = \overline{x}_1 + x_2$	0
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1

$$f(x_1, x_2) = M_0 \bullet M_2 = (x_1 + x_2) \bullet (\overline{x}_1 + x_2)$$

Yet Another Example

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1
1	0	1	$M_1 = x_1 + \overline{x}_2$	1
2	1	0	$M_2 = \overline{x}_1 + x_2$	0
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1

We need to minimize using the zeros of the function f.
But let's first minimize the inverse of f, i.e., $\overline{f}$.

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$	$\overline{f}(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1	0
2	1	0	$M_2 = \overline{x}_1 + x_2$	0	1
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1	0

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$	$\overline{f}(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1	0
2	1	0	$M_2 = \overline{x}_1 + x_2$	0	1
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1	0

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$	$\overline{f}(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1	0
2	1	0	$M_2 = \overline{x}_1 + x_2$	0	1
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1	0

$$\begin{aligned}\overline{f}(x_1, x_2) &= m_2 \\ &= x_1\overline{x}_2\end{aligned}$$

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$	$\overline{f}(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1	0
2	1	0	$M_2 = \overline{x}_1 + x_2$	0	1
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1	0

$$\overline{\overline{f}} = f = \overline{x_1 \overline{x}_2} = \overline{x}_1 + x_2$$

$$\overline{f}(x_1, x_2) = m_2 = x_1 \overline{x}_2$$

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$	$\overline{f}(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1	0
2	1	0	$M_2 = \overline{x}_1 + x_2$	0	1
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1	0

$$\overline{\overline{f}} = f = \overline{x_1 \overline{x}_2} = \overline{x}_1 + x_2$$

$$\overline{f}(x_1, x_2) = m_2 = x_1 \overline{x}_2$$

$$f = \overline{m}_2 = M_2$$

minterms
(for three variables)

The Eight minterms

x	y	z	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

The Eight minterms

x	y	z	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

Expressions for the minterms

$$m_0 = \bar{x}\ \bar{y}\ \bar{z}$$

$$m_1 = \bar{x}\ \bar{y}\ z$$

$$m_2 = \bar{x}\ y\ \bar{z}$$

$$m_3 = \bar{x}\ y\ z$$

$$m_4 = x\ \bar{y}\ \bar{z}$$

$$m_5 = x\ \bar{y}\ z$$

$$m_6 = x\ y\ \bar{z}$$

$$m_7 = x\ y\ z$$

Expressions for the minterms

0 0 0	$m_0 = \bar{x}\,\bar{y}\,\bar{z}$
0 0 1	$m_1 = \bar{x}\,\bar{y}\,z$
0 1 0	$m_2 = \bar{x}\,y\,\bar{z}$
0 1 1	$m_3 = \bar{x}\,y\,z$
1 0 0	$m_4 = x\,\bar{y}\,\bar{z}$
1 0 1	$m_5 = x\,\bar{y}\,z$
1 1 0	$m_6 = x\,y\,\bar{z}$
1 1 1	$m_7 = x\,y\,z$

The bars coincide
with the 0's
in the binary expansion
of the minterm sub-index

Maxterms
(for three variables)

The Eight Maxterms

x	y	z	M_0	M_1	M_2	M_3	M_4	M_5	M_6	M_7
0	0	0	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1	1	1
0	1	1	1	1	1	0	1	1	1	1
1	0	0	1	1	1	1	0	1	1	1
1	0	1	1	1	1	1	1	0	1	1
1	1	0	1	1	1	1	1	1	0	1
1	1	1	1	1	1	1	1	1	1	0

The Eight Maxterms

x	y	z	M_0	M_1	M_2	M_3	M_4	M_5	M_6	M_7
0	0	0	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1	1	1
0	1	1	1	1	1	0	1	1	1	1
1	0	0	1	1	1	1	0	1	1	1
1	0	1	1	1	1	1	1	0	1	1
1	1	0	1	1	1	1	1	1	0	1
1	1	1	1	1	1	1	1	1	1	0

Expressions for the Maxterms

$M_0 = x + y + z$

$M_1 = x + y + \overline{z}$

$M_2 = x + \overline{y} + z$

$M_3 = x + \overline{y} + \overline{z}$

$M_4 = \overline{x} + y + z$

$M_5 = \overline{x} + y + \overline{z}$

$M_6 = \overline{x} + \overline{y} + z$

$M_7 = \overline{x} + \overline{y} + \overline{z}$

Expressions for the Maxterms

0	0	0	$M_0 = x + y + z$
0	0	1	$M_1 = x + y + \overline{z}$
0	1	0	$M_2 = x + \overline{y} + z$
0	1	1	$M_3 = x + \overline{y} + \overline{z}$
1	0	0	$M_4 = \overline{x} + y + z$
1	0	1	$M_5 = \overline{x} + y + \overline{z}$
1	1	0	$M_6 = \overline{x} + \overline{y} + z$
1	1	1	$M_7 = \overline{x} + \overline{y} + \overline{z}$

The bars coincide
with the 1's
in the binary expansion
of the maxterm sub-index

minterms and Maxterms
(for three variables)

minterms and Maxterms

$m_0 = \bar{x}\ \bar{y}\ \bar{z}$ $M_0 = x + y + z$

$m_1 = \bar{x}\ \bar{y}\ z$ $M_1 = x + y + \bar{z}$

$m_2 = \bar{x}\ y\ \bar{z}$ $M_2 = x + \bar{y} + z$

$m_3 = \bar{x}\ y\ z$ $M_3 = x + \bar{y} + \bar{z}$

$m_4 = x\ \bar{y}\ \bar{z}$ $M_4 = \bar{x} + y + z$

$m_5 = x\ \bar{y}\ z$ $M_5 = \bar{x} + y + \bar{z}$

$m_6 = x\ y\ \bar{z}$ $M_6 = \bar{x} + \bar{y} + z$

$m_7 = x\ y\ z$ $M_7 = \bar{x} + \bar{y} + \bar{z}$

minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \overline{x}_1\overline{x}_2x_3$	$M_1 = x_1 + x_2 + \overline{x}_3$
2	0	1	0	$m_2 = \overline{x}_1x_2\overline{x}_3$	$M_2 = x_1 + \overline{x}_2 + x_3$
3	0	1	1	$m_3 = \overline{x}_1x_2x_3$	$M_3 = x_1 + \overline{x}_2 + \overline{x}_3$
4	1	0	0	$m_4 = x_1\overline{x}_2\overline{x}_3$	$M_4 = \overline{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\overline{x}_2x_3$	$M_5 = \overline{x}_1 + x_2 + \overline{x}_3$
6	1	1	0	$m_6 = x_1x_2\overline{x}_3$	$M_6 = \overline{x}_1 + \overline{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$

[Figure 2.22 from the textbook]

Examples with three-variable functions

A three-variable function

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

[Figure 2.23 from the textbook]

Sum-of-Products (SOP)

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

Sum-of-Products (SOP)

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$f(x_1, x_2, x_3) = \bar{x}_1\bar{x}_2x_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2x_3 + x_1x_2\bar{x}_3$$

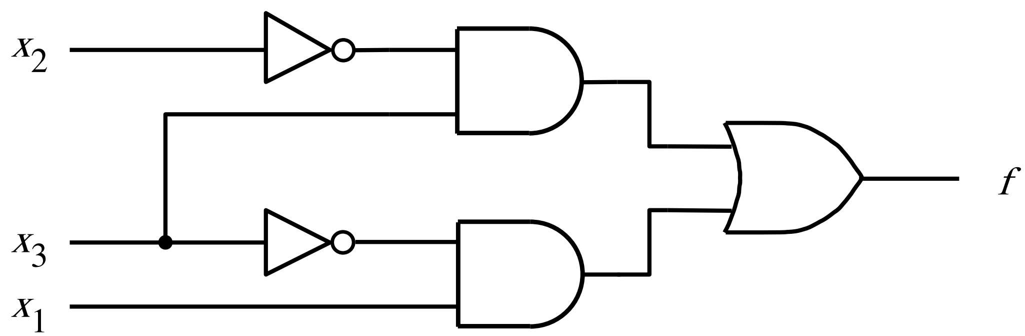
Sum-of-Products (SOP)

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$f(x_1, x_2, x_3) = \bar{x}_1\bar{x}_2x_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2x_3 + x_1x_2\bar{x}_3$$

$$\begin{aligned} f &= (\bar{x}_1 + x_1)\bar{x}_2x_3 + x_1(\bar{x}_2 + x_2)\bar{x}_3 \\ &= 1 \cdot \bar{x}_2x_3 + x_1 \cdot 1 \cdot \bar{x}_3 \\ &= \bar{x}_2x_3 + x_1\bar{x}_3 \end{aligned}$$

Sum-of-products realization of this function



$$f = \overline{x_2}\, x_3 + x_1\, \overline{x_3}$$

[Figure 2.24a from the textbook]

A three-variable function

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

[Figure 2.23 from the textbook]

Product-of-Sums (POS)

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

Product-of-Sums (POS)

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$\begin{aligned} f &= \overline{m_0 + m_2 + m_3 + m_7} \\ &= \overline{m}_0 \cdot \overline{m}_2 \cdot \overline{m}_3 \cdot \overline{m}_7 \\ &= M_0 \cdot M_2 \cdot M_3 \cdot M_7 \\ &= (x_1 + x_2 + x_3)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + \overline{x}_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3) \end{aligned}$$

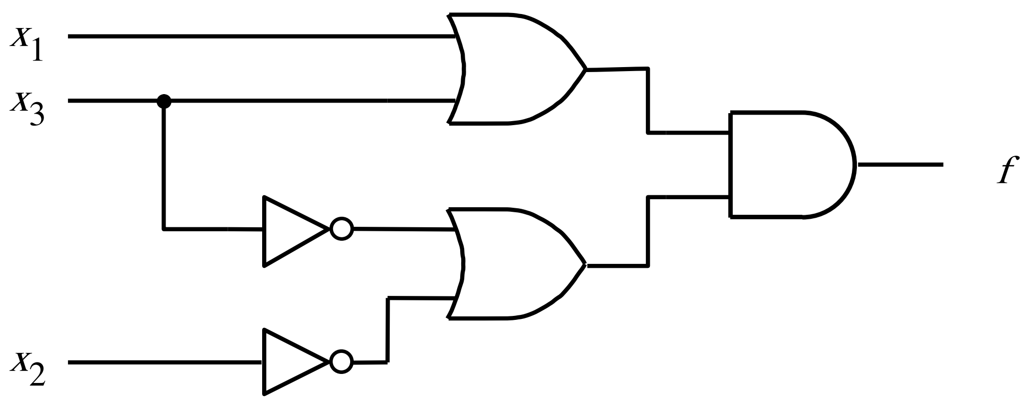
Product-of-Sums (POS)

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$f = ((x_1 + x_3) + x_2)((x_1 + x_3) + \overline{x}_2)(x_1 + (\overline{x}_2 + \overline{x}_3))(\overline{x}_1 + (\overline{x}_2 + \overline{x}_3))$$

$$f = (x_1 + x_3)(\overline{x}_2 + \overline{x}_3)$$

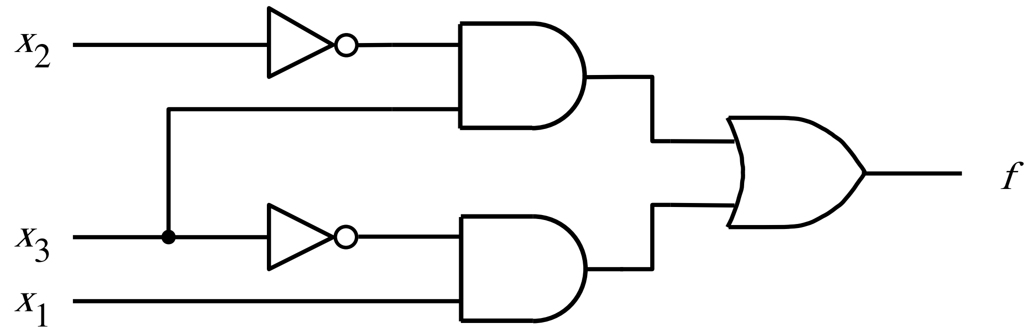
Product-of-sums realization of this function



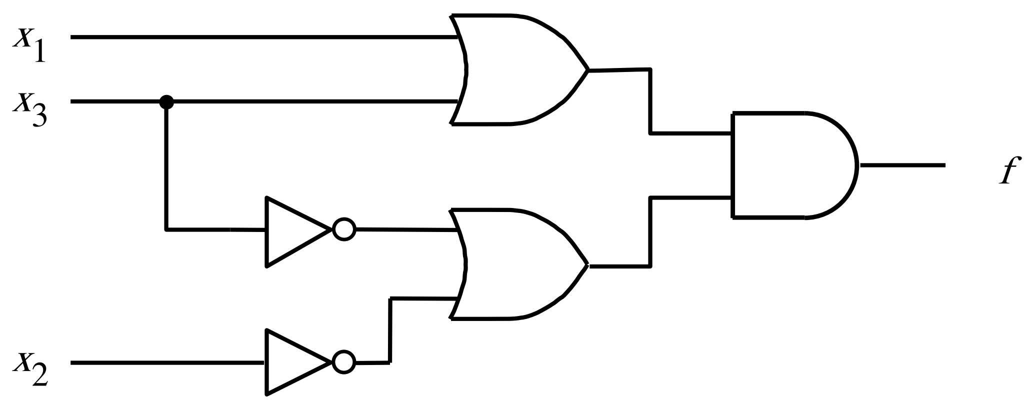
$$f = (x_1 + x_3) \bullet (\overline{x_2} + \overline{x_3})$$

[Figure 2.24b from the textbook]

Two realizations of this function



(a) A minimal sum-of-products realization



(b) A minimal product-of-sums realization

[Figure 2.24 from the textbook]

Shorthand Notation for SOP

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$f(x_1, x_2, x_3) = \sum (m_1, m_4, m_5, m_6)$$

or

$$f(x_1, x_2, x_3) = \sum m(1, 4, 5, 6)$$

Shorthand Notation for POS

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$f(x_1, x_2, x_3) = \Pi(M_0, M_2, M_3, M_7)$$

or

$$f(x_1, x_2, x_3) = \Pi M(0, 2, 3, 7)$$

Shorthand Notation

- **Sum-of-Products (SOP)**

$$f(x_1, x_2, x_3) = \sum (m_1, m_4, m_5, m_6)$$

or

$$f(x_1, x_2, x_3) = \sum m(1, 4, 5, 6)$$

- **Product-of-Sums (POS)**

$$f(x_1, x_2, x_3) = \Pi (M_0, M_2, M_3, M_7)$$

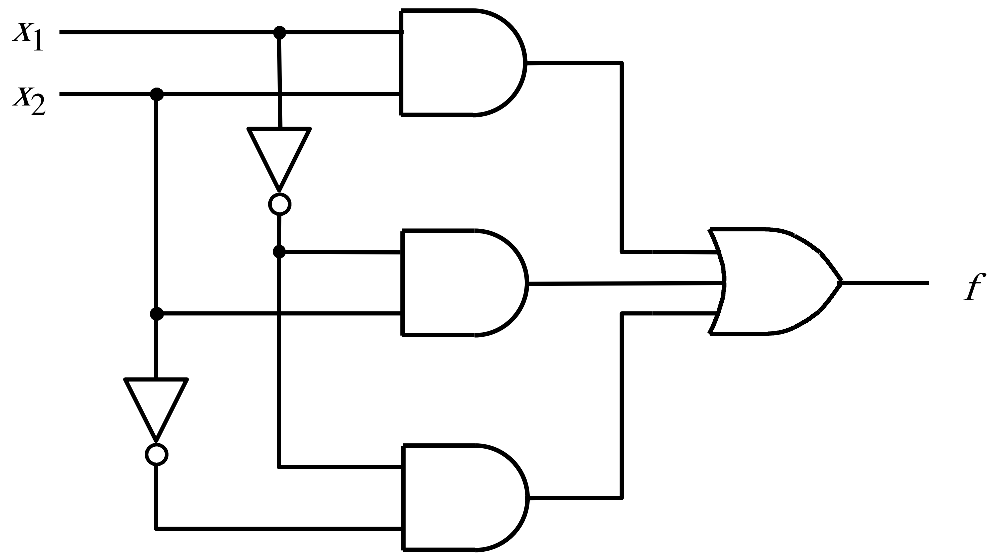
or

$$f(x_1, x_2, x_3) = \Pi M (0, 2, 3, 7)$$

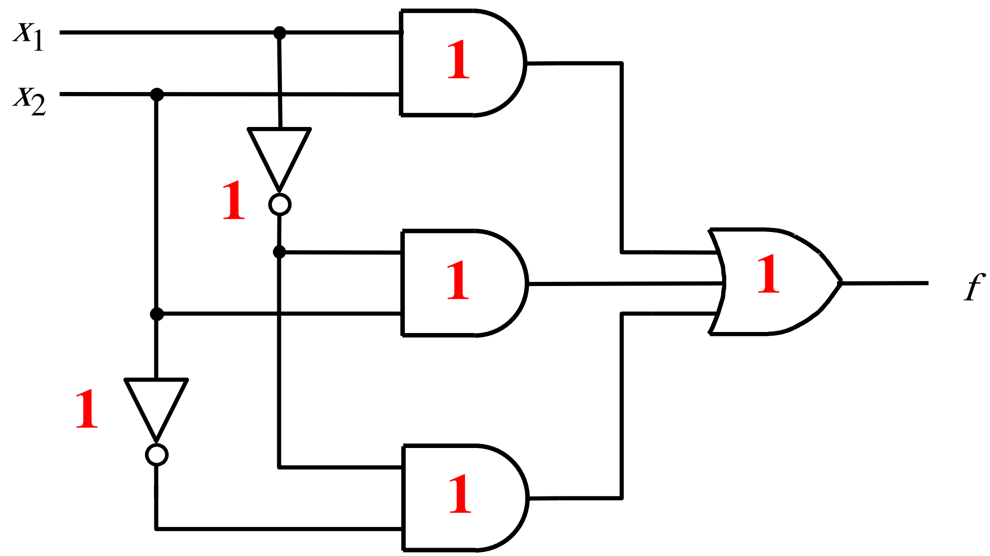
The Cost of a Circuit

- **Count all gates**
- **Count all inputs/wires to the gates**
- **Add the two partial counts. That is the cost.**

What is the cost of this circuit?

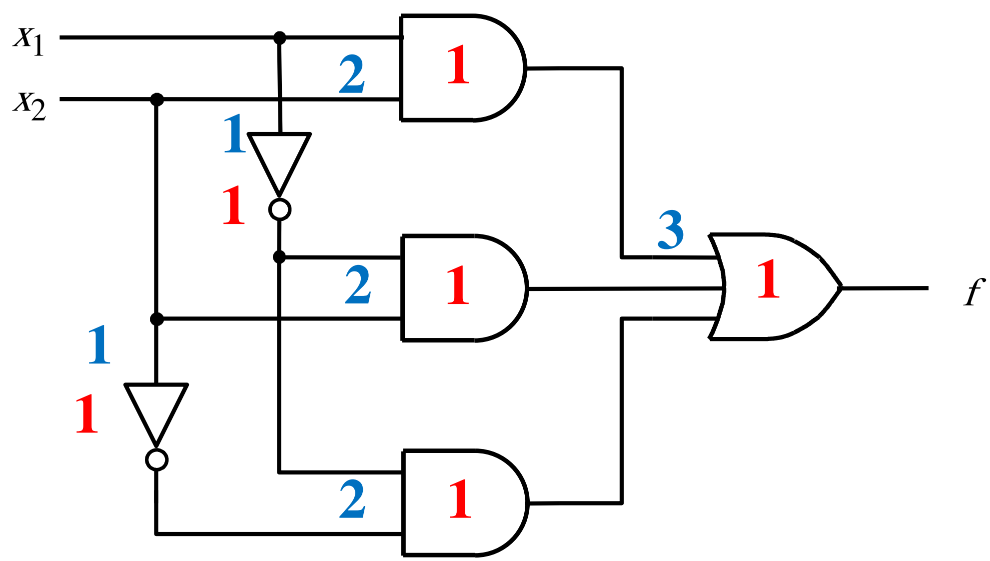


What is the cost of this circuit?



There are 6 gates.

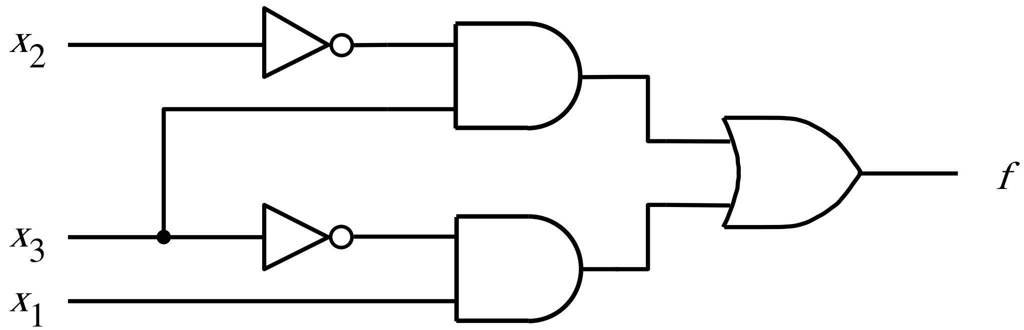
What is the cost of this circuit?



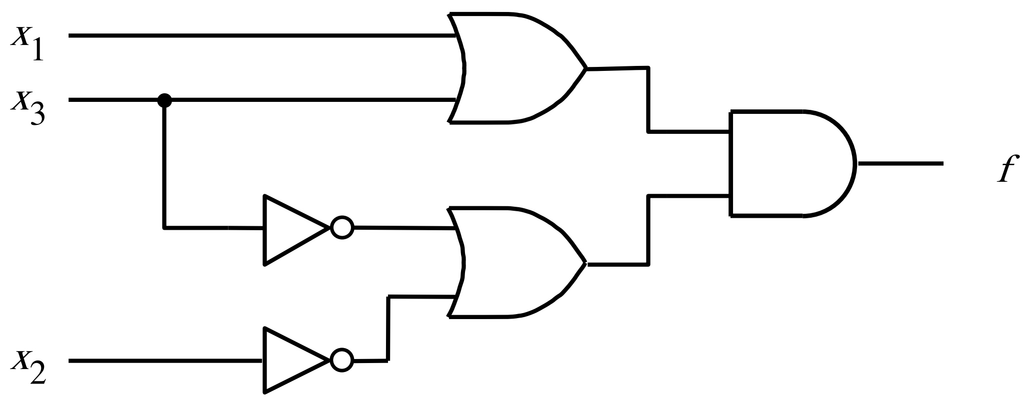
There are 6 gates and 11 inputs.

The total cost is 17.

What is the cost of each circuit?



(a) A minimal sum-of-products realization



(b) A minimal product-of-sums realization

Questions?

THE END