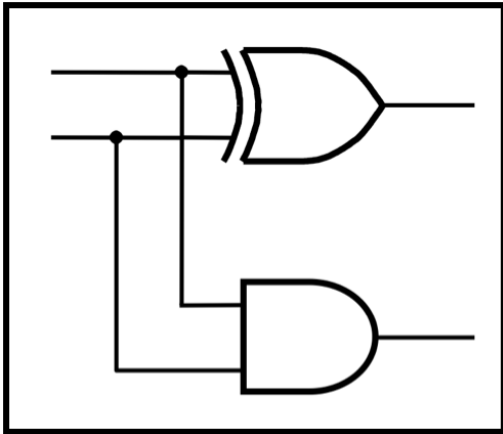




CprE 2810: Digital Logic

Instructor: Alexander Stoytchev

<http://www.ece.iastate.edu/~alexs/classes/>

Boolean Algebra

CprE 2810: Digital Logic
Iowa State University, Ames, IA
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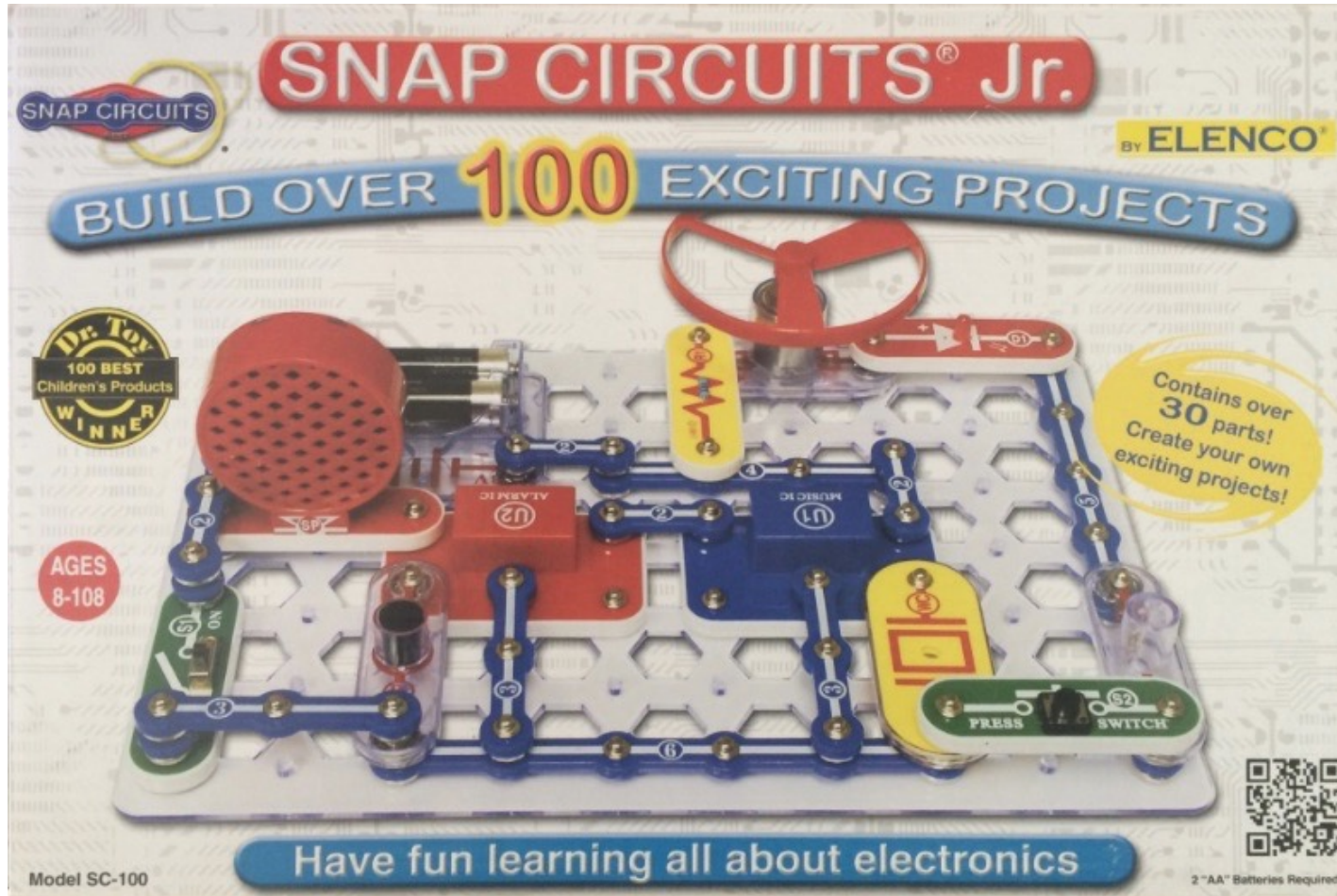
Administrative Stuff

- **HW1 is due today @ 10 pm**
- **Sample solutions will be posted on Canvas after the deadline.
Look for the solutions under the “Files” tab.**
- **No late homeworks will be accepted.**

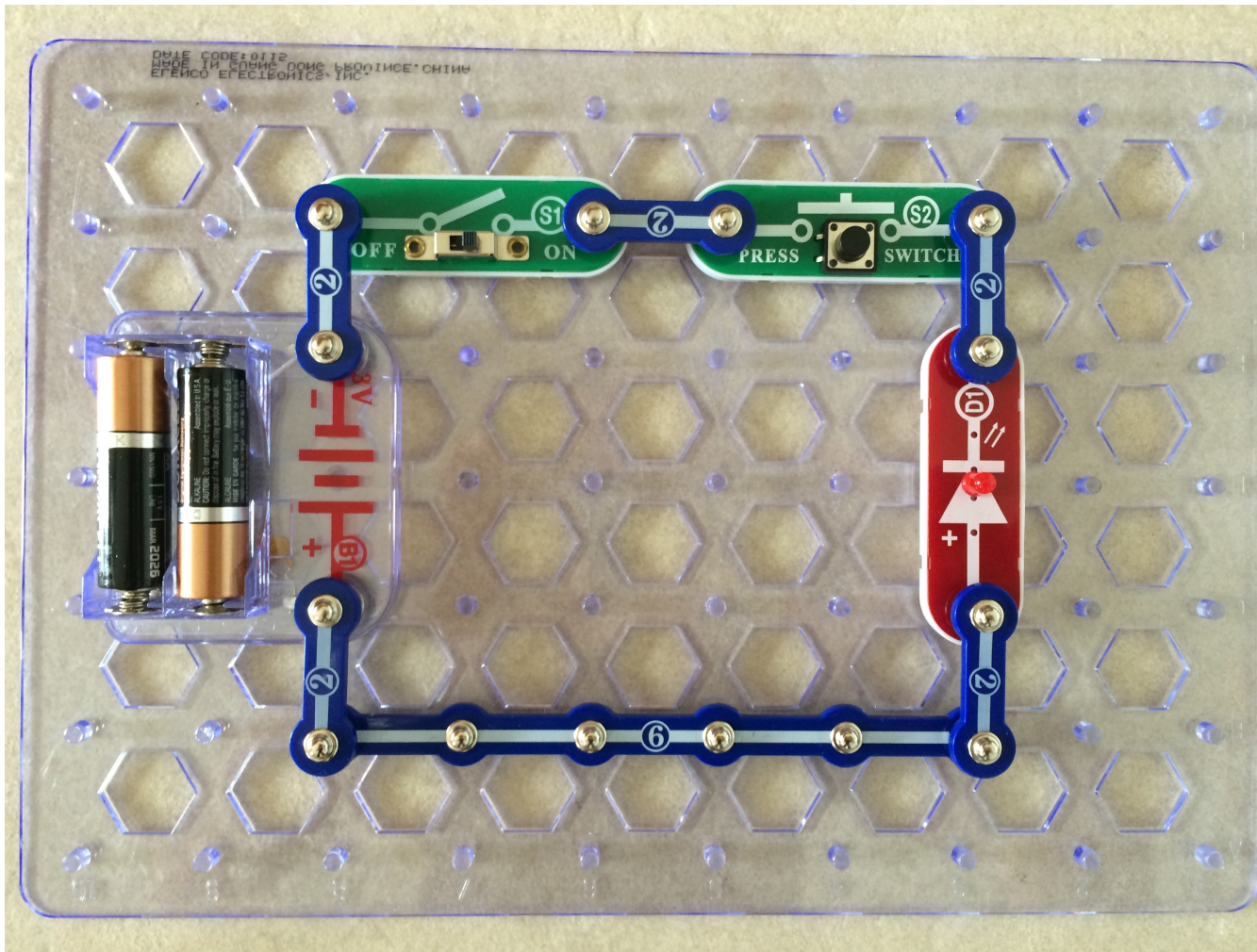
Administrative Stuff

- **HW2 is out**
- **It is due on Monday Sep 8 @ 10pm.**
- **Submit it on Canvas before the deadline.**

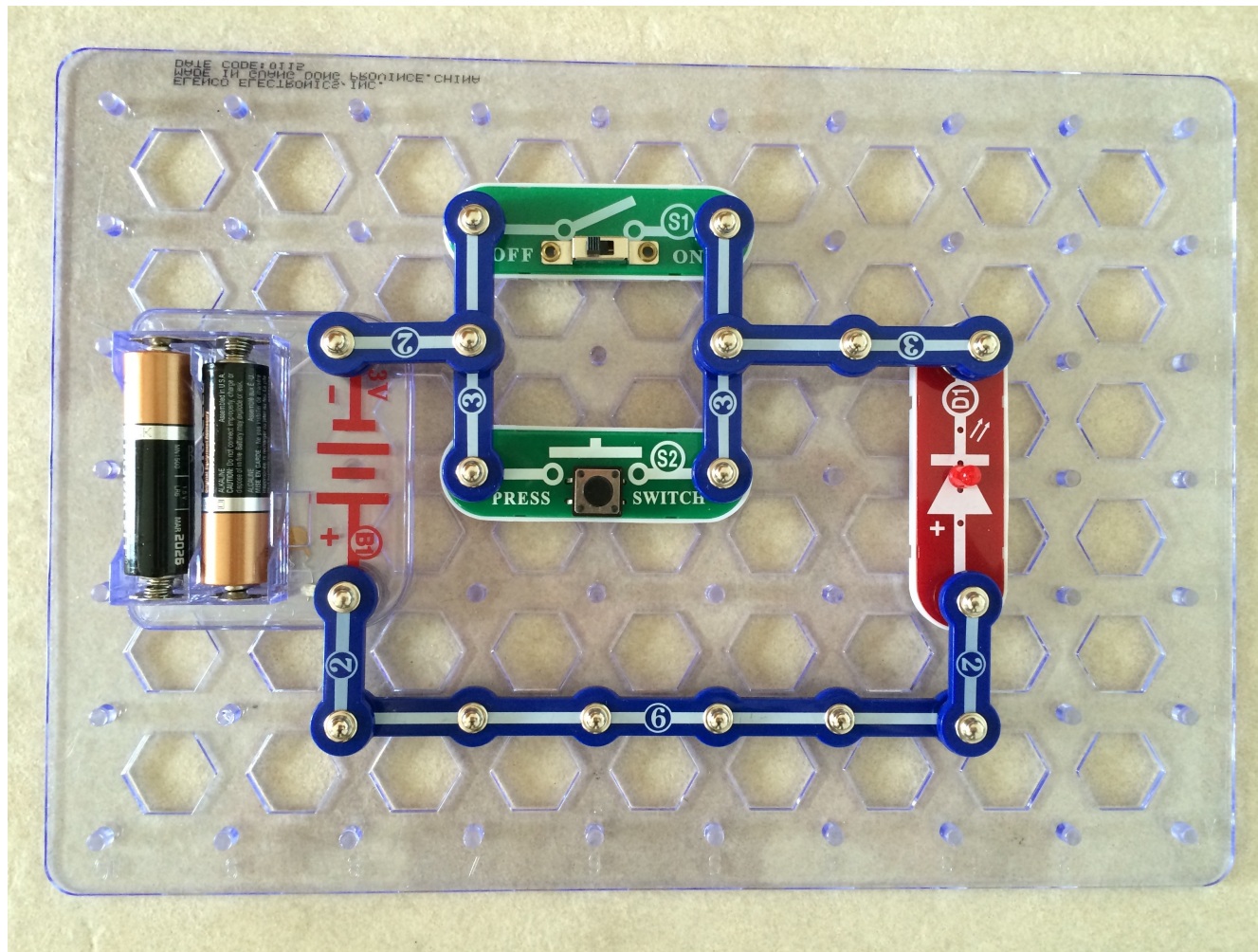
Did you play with this toy?



AND Gate

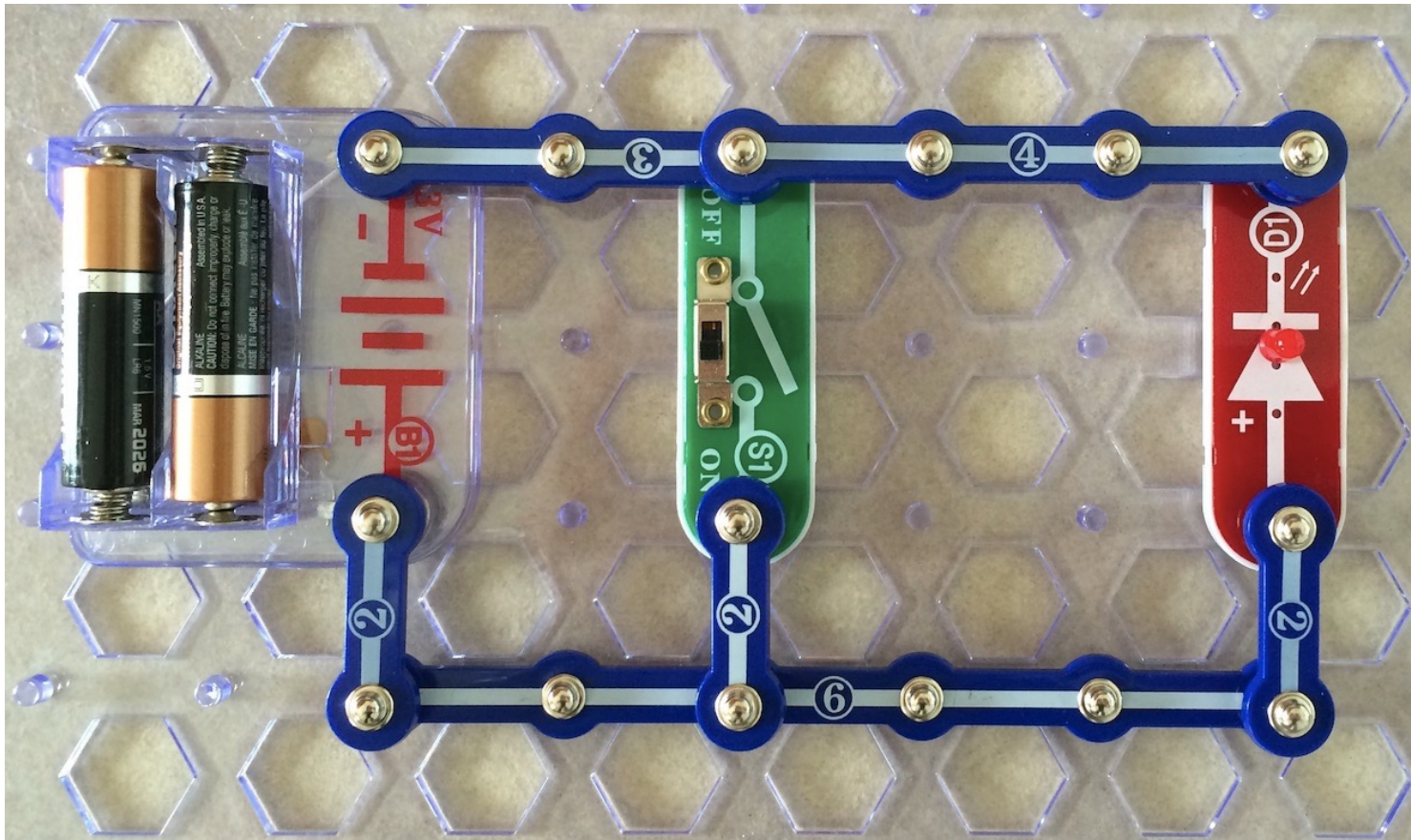


OR Gate



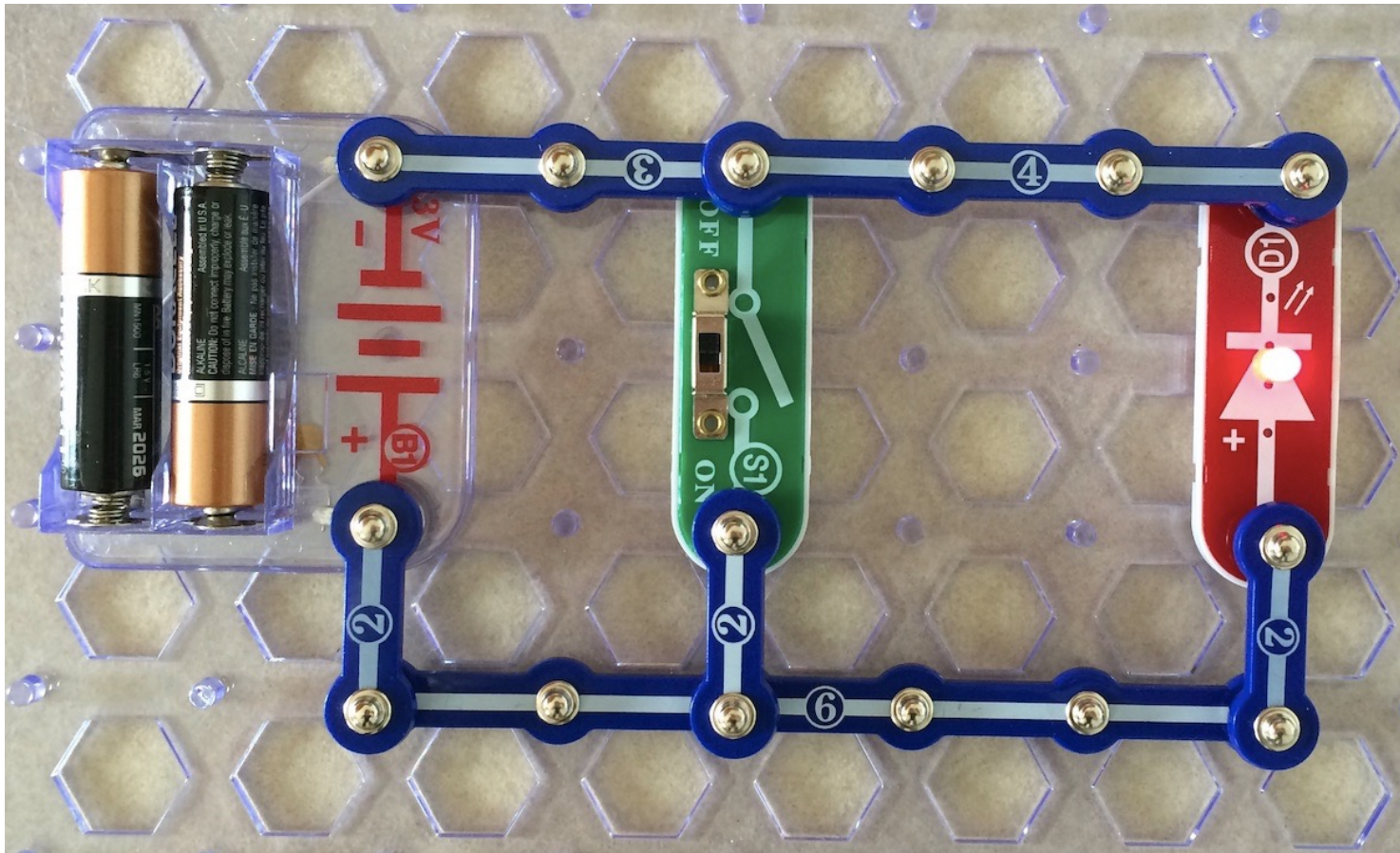
NOT Gate

(the switch is ON but the light is OFF)

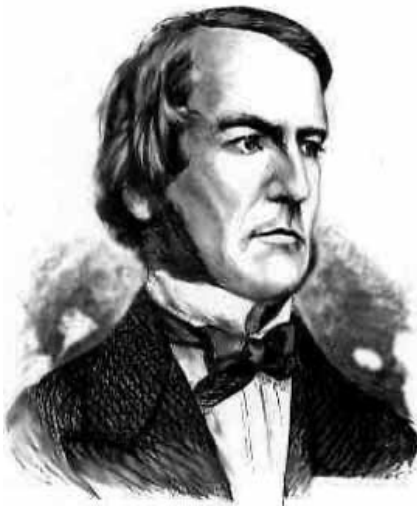


NOT Gate

(the switch is OFF but the light is ON)



Boolean Algebra



George Boole
1815-1864

- **An algebraic structure consists of**
 - a set of elements $\{0, 1\}$
 - binary operators $\{+, \bullet\}$
 - and a unary operator $\{ ' \}$ or $\{ \neg \}$ or $\{ \sim \}$
- **Introduced by George Boole in 1854**
- **An effective means of describing circuits built with switches**
- **A powerful tool that can be used for designing and analyzing logic circuits**

Different Notations for Negation

- All three of these mean “negate x”

- x'

- $\overline{x}$

- $\sim x$

Axioms of Boolean Algebra

1a. $0 \cdot 0 = 0$

1b. $1 + 1 = 1$

2a. $1 \cdot 1 = 1$

2b. $0 + 0 = 0$

3a. $0 \cdot 1 = 1 \cdot 0 = 0$

3b. $1 + 0 = 0 + 1 = 1$

4a. If $x=0$, then $\overline{x} = 1$

4b. If $x=1$, then $\overline{x} = 0$

Single-Variable Theorems

$$5a. \quad x \cdot 0 = 0$$

$$5b. \quad x + 1 = 1$$

$$6a. \quad x \cdot 1 = x$$

$$6b. \quad x + 0 = x$$

$$7a. \quad x \cdot x = x$$

$$7b. \quad x + x = x$$

$$8a. \quad x \cdot \overline{x} = 0$$

$$8b. \quad x + \overline{x} = 1$$

$$9. \quad \overline{\overline{x}} = x$$

Two- and Three-Variable Properties

10a. $x \cdot y = y \cdot x$

Commutative

10b. $x + y = y + x$

11a. $x \cdot (y \cdot z) = (x \cdot y) \cdot z$

Associative

11b. $x + (y + z) = (x + y) + z$

12a. $x \cdot (y + z) = x \cdot y + x \cdot z$

Distributive

12b. $x + y \cdot z = (x + y) \cdot (x + z)$

13a. $x + x \cdot y = x$

Absorption

13b. $x \cdot (x + y) = x$

Two- and Three-Variable Properties

$$14a. \quad x \cdot y + x \cdot \bar{y} = x$$

Combining

$$14b. \quad (x + y) \cdot (x + \bar{y}) = x$$

$$15a. \quad \overline{x \cdot y} = \bar{x} + \bar{y}$$

DeMorgan's
theorem

$$15b. \quad \overline{x + y} = \bar{x} \cdot \bar{y}$$

$$16a. \quad x + \bar{x} \cdot y = x + y$$

$$16b. \quad x \cdot (\bar{x} + y) = x \cdot y$$

$$17a. \quad x \cdot y + y \cdot z + \bar{x} \cdot z = x \cdot y + \bar{x} \cdot z$$

Consensus

$$17b. \quad (x+y) \cdot (y+z) \cdot (\bar{x}+z) = (x+y) \cdot (\bar{x}+z)$$

Now, let's prove all of these

The First Four are Axioms (i.e., they don't require a proof)

1a. $0 \cdot 0 = 0$

1b. $1 + 1 = 1$

2a. $1 \cdot 1 = 1$

2b. $0 + 0 = 0$

3a. $0 \cdot 1 = 1 \cdot 0 = 0$

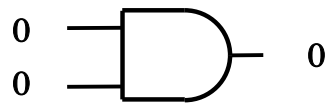
3b. $1 + 0 = 0 + 1 = 1$

4a. If $x=0$, then $\overline{x} = 1$

4b. If $x=1$, then $\overline{x} = 0$

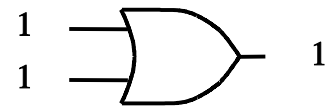
**But here are some other ways
to think about them**

1a. $0 \cdot 0 = 0$



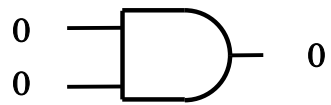
AND gate

1b. $1 + 1 = 1$



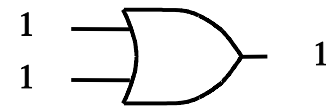
OR gate

1a. $0 \cdot 0 = 0$



AND gate

1b. $1 + 1 = 1$

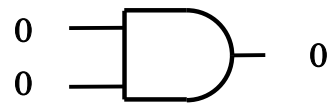


OR gate

x_1	x_2	f
0	0	0
0	1	0
1	0	0
1	1	1

x_1	x_2	f
0	0	0
0	1	1
1	0	1
1	1	1

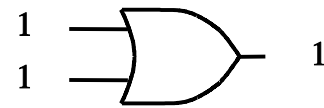
1a. $0 \cdot 0 = 0$



AND gate

x_1	x_2	f
0	0	0
0	1	0
1	0	0
1	1	1

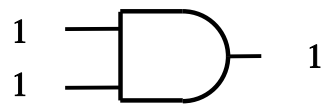
1b. $1 + 1 = 1$



OR gate

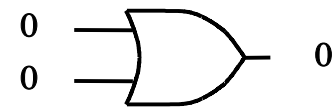
x_1	x_2	f
0	0	0
0	1	1
1	0	1
1	1	1

2a. $1 \cdot 1 = 1$



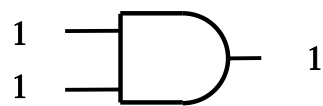
AND gate

2b. $0 + 0 = 0$



OR gate

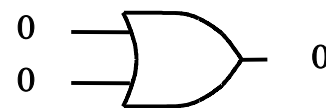
2a. $1 \cdot 1 = 1$



AND gate

x_1	x_2	f
0	0	0
0	1	0
1	0	0
1	1	1

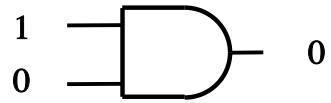
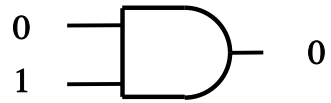
2b. $0 + 0 = 0$



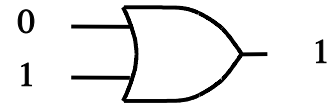
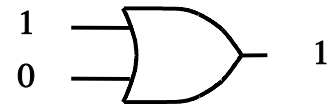
OR gate

x_1	x_2	f
0	0	0
0	1	1
1	0	1
1	1	1

3a. $0 \cdot 1 = 1 \cdot 0 = 0$

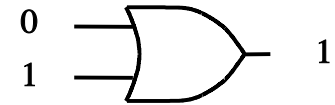
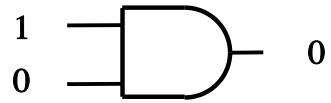
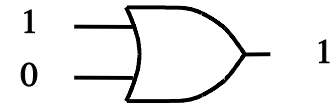
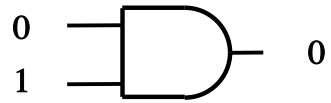


3b. $1 + 0 = 0 + 1 = 1$



3a. $0 \cdot 1 = 1 \cdot 0 = 0$

3b. $1 + 0 = 0 + 1 = 1$



AND gate

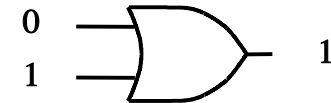
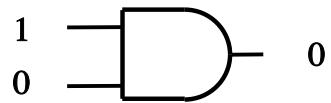
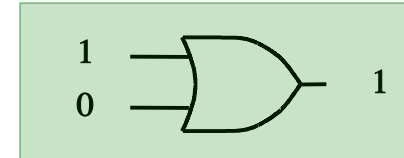
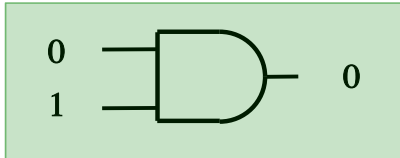
x_1	x_2	f
0	0	0
0	1	0
1	0	0
1	1	1

OR gate

x_1	x_2	f
0	0	0
0	1	1
1	0	1
1	1	1

3a. $0 \cdot 1 = 1 \cdot 0 = 0$

3b. $1 + 0 = 0 + 1 = 1$



AND gate

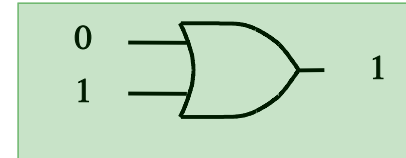
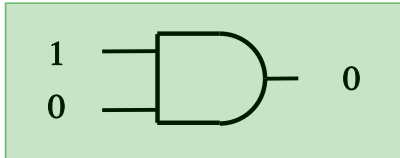
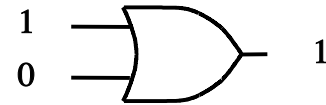
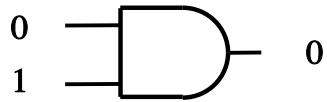
x_1	x_2	f
0	0	0
0	1	0
1	0	0
1	1	1

OR gate

x_1	x_2	f
0	0	0
0	1	1
1	0	1
1	1	1

3a. $0 \cdot 1 = 1 \cdot 0 = 0$

3b. $1 + 0 = 0 + 1 = 1$



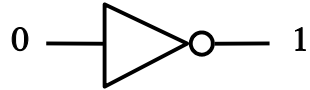
AND gate

x_1	x_2	f
0	0	0
0	1	0
1	0	0
1	1	1

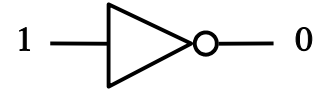
OR gate

x_1	x_2	f
0	0	0
0	1	1
1	0	1
1	1	1

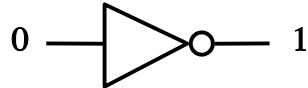
4a. If $x=0$, then $\bar{x} = 1$



4b. If $x=1$, then $\bar{x} = 0$



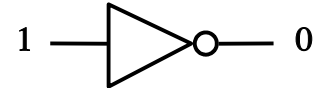
4a. If $x=0$, then $\bar{x} = 1$



NOT gate

x	$\bar{x}$
0	1
1	0

4b. If $x=1$, then $\bar{x} = 0$



NOT gate

x	$\bar{x}$
0	1
1	0

Single-Variable Theorems

$$5a. \quad x \cdot 0 = 0$$

$$5b. \quad x + 1 = 1$$

$$6a. \quad x \cdot 1 = x$$

$$6b. \quad x + 0 = x$$

$$7a. \quad x \cdot x = x$$

$$7b. \quad x + x = x$$

$$8a. \quad x \cdot \overline{x} = 0$$

$$8b. \quad x + \overline{x} = 1$$

$$9. \quad \overline{\overline{x}} = x$$

5a. $\mathbf{x} \cdot \mathbf{0} = \mathbf{0}$

$$\mathbf{5a. \quad x \cdot 0 = 0}$$

**The Boolean variable x can have only two possible values:
0 or 1. Let's look at each case separately.**

$$\mathbf{5a. \quad x \cdot 0 = 0}$$

**The Boolean variable x can have only two possible values:
0 or 1. Let's look at each case separately.**

i) If $x = 0$, then we have

$$0 \cdot 0 = 0 \qquad \text{// axiom 1a}$$

$$\mathbf{5a. \quad x \cdot 0 = 0}$$

**The Boolean variable x can have only two possible values:
0 or 1. Let's look at each case separately.**

i) If $x = 0$, then we have

$$0 \cdot 0 = 0 \qquad \text{// axiom 1a}$$

ii) If $x = 1$, then we have

$$1 \cdot 0 = 0 \qquad \text{// axiom 3a}$$

5b. $x + 1 = 1$

$$\mathbf{5b. \quad x + 1 = 1}$$

**The Boolean variable x can have only two possible values:
0 or 1. Let's look at each case separately.**

i) If $x = 0$, then we have

$$0 + 1 = 1 \qquad \text{// axiom 3b}$$

$$\mathbf{5b. \quad x + 1 = 1}$$

**The Boolean variable x can have only two possible values:
0 or 1. Let's look at each case separately.**

i) If $x = 0$, then we have

$$0 + 1 = 1 \quad \text{// axiom 3b}$$

ii) If $x = 1$, then we have

$$1 + 1 = 1 \quad \text{// axiom 1b}$$

$$\mathbf{6a. \quad x \cdot 1 = x}$$

**The Boolean variable x can have only two possible values:
0 or 1. Let's look at each case separately.**

i) If $x = 0$, then we have

$$0 \cdot 1 = 0 \qquad \text{// axiom 3a}$$

ii) If $x = 1$, then we have

$$1 \cdot 1 = 1 \qquad \text{// axiom 2a}$$

$$6a. \quad \boxed{x} \cdot 1 = \boxed{x}$$

The Boolean variable x can have only two possible values:
0 or 1. Let's look at each case separately.

i) If $x = 0$, then we have

$$\boxed{0} \cdot 1 = \boxed{0}$$

// axiom 3a

ii) If $x = 1$, then we have

$$\boxed{1} \cdot 1 = \boxed{1}$$

// axiom 2a

$$\mathbf{6b. \quad x + 0 = x}$$

**The Boolean variable x can have only two possible values:
0 or 1. Let's look at each case separately.**

i) If $x = 0$, then we have

$$0 + 0 = 0 \qquad \text{// axiom 2b}$$

ii) If $x = 1$, then we have

$$1 + 0 = 1 \qquad \text{// axiom 3b}$$

$$6b. \quad \boxed{x} + 0 = \boxed{x}$$

The Boolean variable x can have only two possible values:
0 or 1. Let's look at each case separately.

i) If $x = 0$, then we have

$$\boxed{0} + 0 = \boxed{0}$$

// axiom 2b

ii) If $x = 1$, then we have

$$\boxed{1} + 0 = \boxed{1}$$

// axiom 3b

$$7a. \quad \mathbf{x} \cdot \mathbf{x} = \mathbf{x}$$

i) If $x = 0$, then we have

$$0 \cdot 0 = 0 \quad // \text{ axiom 1a}$$

ii) If $x = 1$, then we have

$$1 \cdot 1 = 1 \quad // \text{ axiom 2a}$$

$$7a. \quad \boxed{x} \cdot \boxed{x} = \boxed{x}$$

i) If $x = 0$, then we have

$$\boxed{0} \cdot \boxed{0} = \boxed{0}$$

// axiom 1a

ii) If $x = 1$, then we have

$$\boxed{1} \cdot \boxed{1} = \boxed{1}$$

// axiom 2a

$$\mathbf{7b. \quad x + x = x}$$

i) If $x = 0$, then we have

$$0 + 0 = 0 \quad // \text{ axiom 2b}$$

ii) If $x = 1$, then we have

$$1 + 1 = 1 \quad // \text{ axiom 1b}$$

7b. $\boxed{x} + \boxed{x} = \boxed{x}$

i) If $x = 0$, then we have

$$\boxed{0} + \boxed{0} = \boxed{0}$$

// axiom 2b

ii) If $x = 1$, then we have

$$\boxed{1} + \boxed{1} = \boxed{1}$$

// axiom 1b

$$\mathbf{8a.} \quad \mathbf{x \cdot \overline{x} = 0}$$

i) If $x = 0$, then we have

$$0 \cdot 1 = 0$$

// axiom 3a

ii) If $x = 1$, then we have

$$1 \cdot 0 = 0$$

// axiom 3a

$$8a. \quad x \cdot \overline{x} = 0$$

i) If $x = 0$, then we have

$$0 \cdot 1 = 0 \quad // \text{ axiom 3a}$$

ii) If $x = 1$, then we have

$$1 \cdot 0 = 0 \quad // \text{ axiom 3a}$$

$$\mathbf{8b.} \quad \mathbf{x + \overline{x} = 1}$$

i) If $x = 0$, then we have

$$0 + 1 = 1 \quad // \text{ axiom 3b}$$

ii) If $x = 1$, then we have

$$1 + 0 = 1 \quad // \text{ axiom 3b}$$

$$\mathbf{8b.} \quad \mathbf{x + \overline{x} = 1}$$

i) If $x = 0$, then we have

$$0 + 1 = 1$$

// axiom 3b

ii) If $x = 1$, then we have

$$1 + 0 = 1$$

// axiom 3b

$$9. \quad \overline{\overline{x}} = x$$

i) If $x = 0$, then we have

$$\overline{x} = 1 \quad // \text{ axiom 4a}$$

let $y = \overline{x} = 1$, then we have

$$\overline{y} = 0 \quad // \text{ axiom 4b}$$

Therefore,

$$\overline{\overline{x}} = x \quad (\text{when } x = 0)$$

$$9. \quad \overline{\overline{x}} = x$$

ii) If $x = 1$, then we have

$$\overline{x} = 0 \quad // \text{ axiom 4b}$$

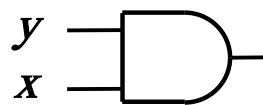
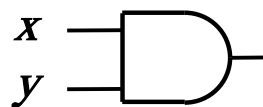
let $y = \overline{x} = 0$, then we have

$$\overline{y} = 1 \quad // \text{ axiom 4a}$$

Therefore,

$$\overline{\overline{x}} = x \quad (\text{when } x = 1)$$

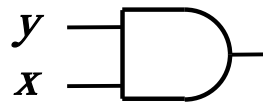
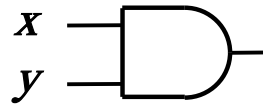
10a. $\mathbf{x} \cdot \mathbf{y} = \mathbf{y} \cdot \mathbf{x}$



10b. $\mathbf{x} + \mathbf{y} = \mathbf{y} + \mathbf{x}$



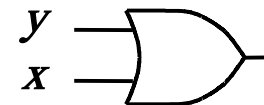
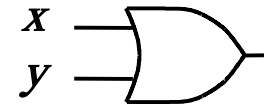
10a. $x \cdot y = y \cdot x$



AND gate

x	y	f
0	0	0
0	1	0
1	0	0
1	1	1

10b. $x + y = y + x$



OR gate

x	y	f
0	0	0
0	1	1
1	0	1
1	1	1

The order of the inputs does not matter.

$$11a. \quad \mathbf{x \cdot (y \cdot z) = (x \cdot y) \cdot z}$$

x	y	z	x	y • z	x•(y•z)
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

Truth table for the left-hand side

$$11a. \quad x \cdot (y \cdot z) = (x \cdot y) \cdot z$$

x	y	z	x	y • z	x•(y•z)
0	0	0	0	0	
0	0	1	0	0	
0	1	0	0	0	
0	1	1	0	1	
1	0	0	1	0	
1	0	1	1	0	
1	1	0	1	0	
1	1	1	1	1	

Truth table for the left-hand side

$$11a. \quad \mathbf{x \cdot (y \cdot z) = (x \cdot y) \cdot z}$$

x	y	z	x	y • z	x•(y•z)
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	1	0
1	0	0	1	0	0
1	0	1	1	0	0
1	1	0	1	0	0
1	1	1	1	1	1

Truth table for the left-hand side

$$11a. \quad \mathbf{x \cdot (y \cdot z) = (x \cdot y) \cdot z}$$

x	y	z	$x \cdot y$	z	$(x \cdot y) \cdot z$
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	0
0	1	1	0	1	0
1	0	0	0	0	0
1	0	1	0	1	0
1	1	0	1	0	0
1	1	1	1	1	1

Truth table for the right-hand side

$$11a. \quad x \cdot (y \cdot z) = (x \cdot y) \cdot z$$

$x \cdot (y \cdot z)$
0
0
0
0
0
0
0
1

$(x \cdot y) \cdot z$
0
0
0
0
0
0
0
1

These two are identical, which concludes the proof.

11b. $x + (y + z) = (x + y) + z$

x	y	z	x	y + z	x+(y+z)
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

Truth table for the left-hand side

11b. $x + (y + z) = (x + y) + z$

x	y	z	x	y + z	x+(y+z)
0	0	0	0	0	
0	0	1	0	1	
0	1	0	0	1	
0	1	1	0	1	
1	0	0	1	0	
1	0	1	1	1	
1	1	0	1	1	
1	1	1	1	1	

Truth table for the left-hand side

11b. $x + (y + z) = (x + y) + z$

x	y	z	x	y + z	x+(y+z)
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	1	1
1	0	0	1	0	1
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

Truth table for the left-hand side

11b. $x + (y + z) = (x + y) + z$

x	y	z	x + y	z	(x+y)+z
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	1	0	1
1	0	1	1	1	1
1	1	0	1	0	1
1	1	1	1	1	1

Truth table for the right-hand side

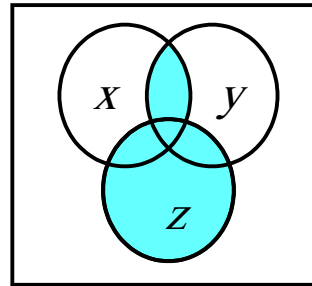
11b. $x + (y + z) = (x + y) + z$

$x+(y+z)$
0
1
1
1
1
1
1
1
1

$(x+y)+z$
0
1
1
1
1
1
1
1
1

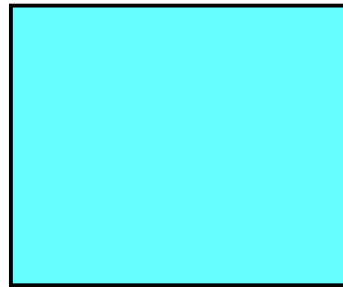
These two are identical, which concludes the proof.

The Venn Diagram Representation



$$xy + z$$

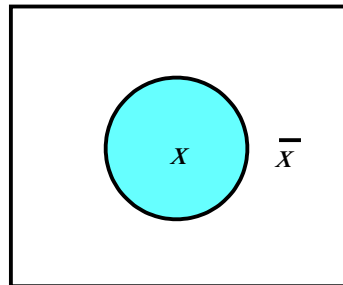
Venn Diagram Basics



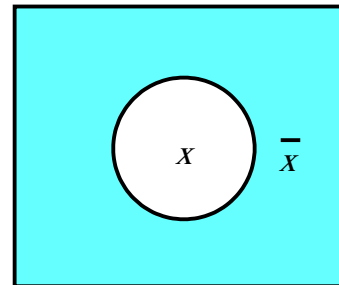
(a) Constant 1



(b) Constant 0

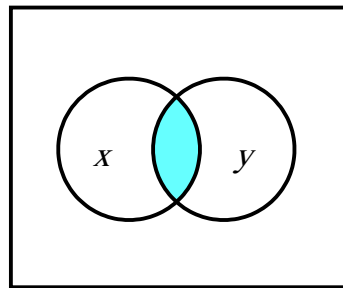


(c) Variable x

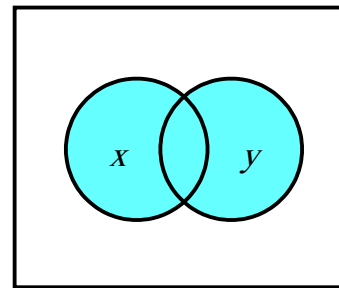


(d) $\bar{x}$

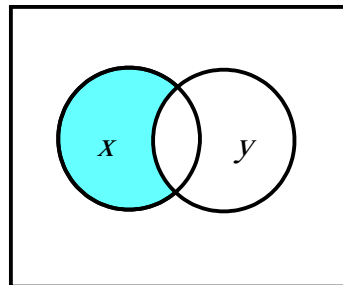
Venn Diagram Basics



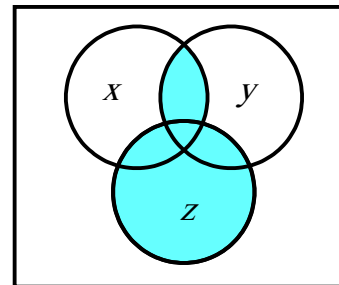
(e) $x \cap y$



(f) $x \cup y$



(g) $x \cup \bar{y}$



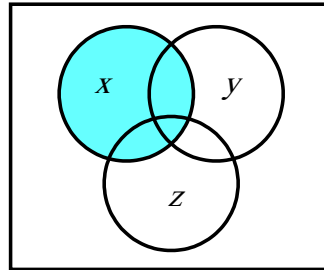
(h) $(x \cap y) \cup z$

Let's Prove the Distributive Properties

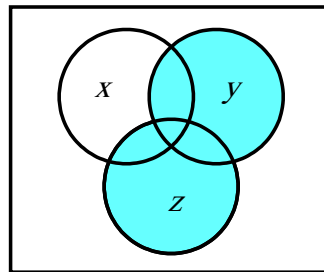
$$12a. \quad x \cdot (y + z) = x \cdot y + x \cdot z$$

$$12b. \quad x + y \cdot z = (x + y) \cdot (x + z)$$

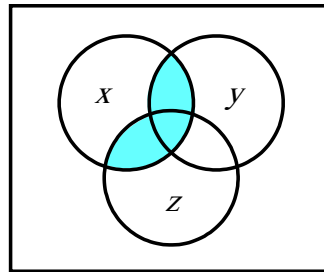
$$12a. \quad x \cdot (y + z) = x \cdot y + x \cdot z$$



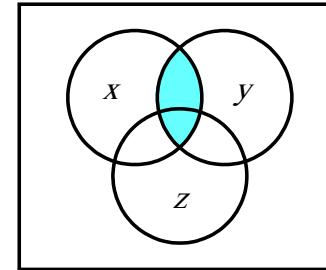
(a) x



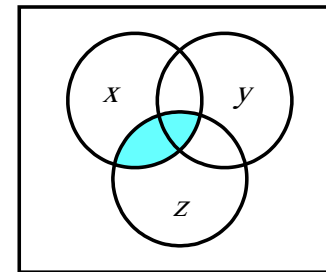
(b) $y + z$



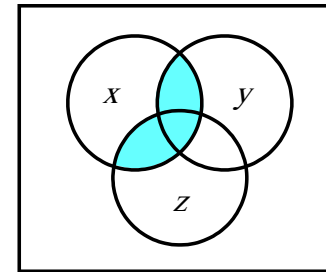
(c) $x \cdot (y + z)$



(d) $x \cdot y$

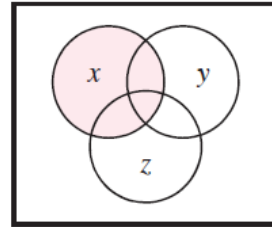


(e) $x \cdot z$

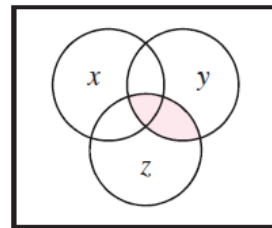


(f) $x \cdot y + x \cdot z$

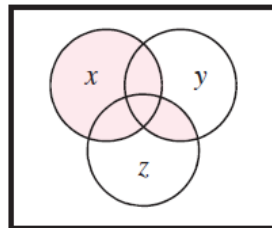
$$12b. \quad x + y \cdot z = (x + y) \cdot (x + z)$$



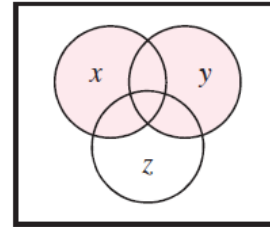
(a) x



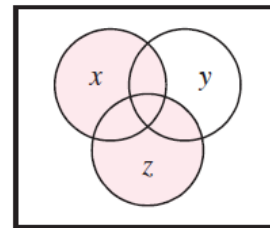
(b) $y \cdot z$



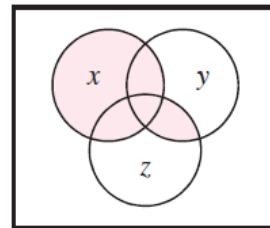
(c) $x + y \cdot z$



(d) $x + y$



(e) $x + z$



(f) $(x + y) \cdot (x + z)$

[Figure 2.17 from the textbook]

Try to prove these ones at home

$$**13a. \quad x + x \cdot y = x**$$

$$**13b. \quad x \cdot (x + y) = x**$$

$$**14a. \quad x \cdot y + x \cdot \overline{y} = x**$$

$$**14b. \quad (x + y) \cdot (x + \overline{y}) = x**$$

DeMorgan's Theorem

$$15a. \quad \overline{x \cdot y} = \bar{x} + \bar{y}$$

$$15b. \quad \overline{x + y} = \bar{x} \cdot \bar{y}$$

Proof of DeMorgan's theorem

$$15a. \quad \overline{x \cdot y} = \bar{x} + \bar{y}$$

Proof of DeMorgan's theorem

15a. $\overline{x \cdot y} = \bar{x} + \bar{y}$

x	y	$x \cdot y$	$\overline{x \cdot y}$	$\bar{x}$	$\bar{y}$	$\bar{x} + \bar{y}$
0	0					
0	1					
1	0					
1	1					



LHS



RHS

Proof of DeMorgan's theorem

15a. $\overline{x \cdot y} = \bar{x} + \bar{y}$

x	y	$x \cdot y$	$\overline{x \cdot y}$	$\bar{x}$	$\bar{y}$	$\bar{x} + \bar{y}$
0	0	0				
0	1	0				
1	0	0				
1	1	1				



LHS



RHS

Proof of DeMorgan's theorem

15a. $\overline{x \cdot y} = \bar{x} + \bar{y}$

x	y	$x \cdot y$	$\overline{x \cdot y}$	$\bar{x}$	$\bar{y}$	$\bar{x} + \bar{y}$
0	0	0	1			
0	1	0	1			
1	0	0	1			
1	1	1	0			



LHS



RHS

Proof of DeMorgan's theorem

$$\mathbf{15a.} \quad \overline{x \cdot y} = \bar{x} + \bar{y}$$

x	y	$x \cdot y$	$\overline{x \cdot y}$	$\bar{x}$	$\bar{y}$	$\bar{x} + \bar{y}$
0	0	0	1	1		
0	1	0	1	1		
1	0	0	1	0		
1	1	1	0	0		



LHS



RHS

Proof of DeMorgan's theorem

15a. $\overline{x \cdot y} = \bar{x} + \bar{y}$

x	y	$x \cdot y$	$\overline{x \cdot y}$	$\bar{x}$	$\bar{y}$	$\bar{x} + \bar{y}$
0	0	0	1	1	1	
0	1	0	1	1	0	
1	0	0	1	0	1	
1	1	1	0	0	0	

⏟

LHS

⏟

RHS

Proof of DeMorgan's theorem

15a. $\overline{x \cdot y} = \bar{x} + \bar{y}$

x	y	$x \cdot y$	$\overline{x \cdot y}$	$\bar{x}$	$\bar{y}$	$\bar{x} + \bar{y}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

⏟

LHS

⏟

RHS

Proof of DeMorgan's theorem

15a. $\overline{x \cdot y} = \bar{x} + \bar{y}$

x	y	$x \cdot y$	$\overline{x \cdot y}$	$\bar{x}$	$\bar{y}$	$\bar{x} + \bar{y}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0



LHS

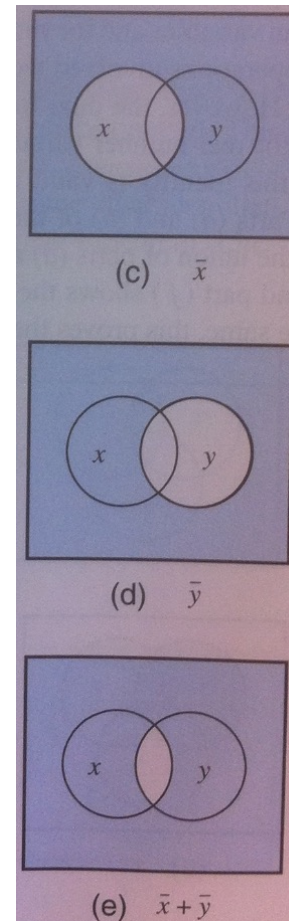
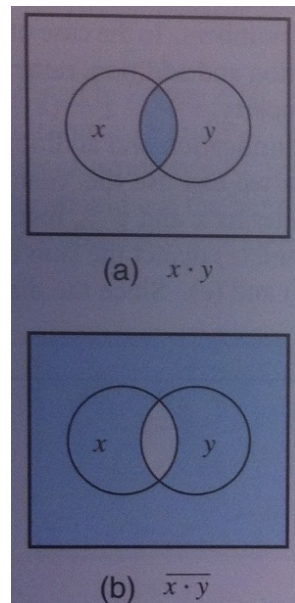


RHS

These two columns are equal. Therefore, the theorem is true.

Alternative proof using Venn Diagrams

15a. $\overline{x \cdot y} = \bar{x} + \bar{y}$



[Figure 2.18 from the textbook]

Let's prove the other DeMorgan's theorem

$$\mathbf{15b.} \quad \overline{\mathbf{x + y}} = \overline{\mathbf{x}} \cdot \overline{\mathbf{y}}$$

Let's prove the other DeMorgan's theorem

$$15b. \quad \overline{x + y} = \bar{x} \cdot \bar{y}$$

x	y	$x + y$	$\overline{x + y}$	$\bar{x}$	$\bar{y}$	$\bar{x} \cdot \bar{y}$
0	0					
0	1					
1	0					
1	1					



LHS



RHS

Let's prove the other DeMorgan's theorem

$$15b. \quad \overline{x + y} = \bar{x} \cdot \bar{y}$$

x	y	$x + y$	$\overline{x + y}$	$\bar{x}$	$\bar{y}$	$\bar{x} \cdot \bar{y}$
0	0	0				
0	1	1				
1	0	1				
1	1	1				



LHS



RHS

Let's prove the other DeMorgan's theorem

$$\mathbf{15b.} \quad \overline{x + y} = \bar{x} \cdot \bar{y}$$

x	y	$x + y$	$\overline{x + y}$	$\bar{x}$	$\bar{y}$	$\bar{x} \cdot \bar{y}$
0	0	0	1			
0	1	1	0			
1	0	1	0			
1	1	1	0			



LHS



RHS

Let's prove the other DeMorgan's theorem

$$15b. \quad \overline{x + y} = \bar{x} \cdot \bar{y}$$

x	y	$x + y$	$\overline{x + y}$	$\bar{x}$	$\bar{y}$	$\bar{x} \cdot \bar{y}$
0	0	0	1	1		
0	1	1	0	1		
1	0	1	0	0		
1	1	1	0	0		



LHS



RHS

Let's prove the other DeMorgan's theorem

$$\mathbf{15b.} \quad \overline{x + y} = \bar{x} \cdot \bar{y}$$

x	y	$x + y$	$\overline{x + y}$	$\bar{x}$	$\bar{y}$	$\bar{x} \cdot \bar{y}$
0	0	0	1	1	1	
0	1	1	0	1	0	
1	0	1	0	0	1	
1	1	1	0	0	0	



LHS



RHS

Let's prove the other DeMorgan's theorem

$$15b. \quad \overline{x + y} = \bar{x} \cdot \bar{y}$$

x	y	$x + y$	$\overline{x + y}$	$\bar{x}$	$\bar{y}$	$\bar{x} \cdot \bar{y}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0



LHS



RHS

Let's prove the other DeMorgan's theorem

$$15b. \quad \overline{x + y} = \bar{x} \cdot \bar{y}$$

These two columns are equal, so the theorem is true.

x	y	$x + y$	$\overline{x + y}$	$\bar{x}$	$\bar{y}$	$\bar{x} \cdot \bar{y}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0



LHS



RHS

DeMorgan's Theorem

Generalizes to more than 2 variables

$$\overline{x \cdot y \cdot z} = \overline{x} + \overline{y} + \overline{z}$$

$$\overline{x + y + z} = \overline{x} \cdot \overline{y} \cdot \overline{z}$$

DeMorgan's Theorem

Generalizes to more than 2 variables

$$\overline{a \cdot b \cdot c \cdot d} = \overline{a} + \overline{b} + \overline{c} + \overline{d}$$

$$\overline{a + b + c + d} = \overline{a} \cdot \overline{b} \cdot \overline{c} \cdot \overline{d}$$

Try to prove these ones at home

$$16a. \quad x + \bar{x} \cdot y = x + y$$

$$16b. \quad x \cdot (\bar{x} + y) = x \cdot y$$

$$17a. \quad x \cdot y + y \cdot z + \bar{x} \cdot z = x \cdot y + \bar{x} \cdot z$$

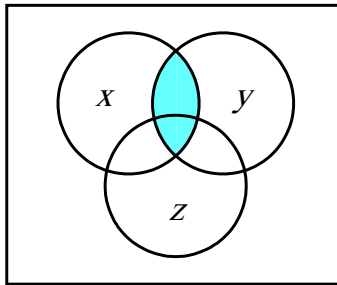
$$17b. \quad (x+y) \cdot (y+z) \cdot (\bar{x}+z) = (x+y) \cdot (\bar{x}+z)$$

Venn Diagram Example

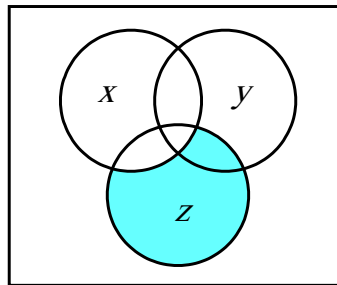
Proof of Property 17a

$$17a. \quad x \cdot y + y \cdot z + \overline{x} \cdot z = x \cdot y + \overline{x} \cdot z$$

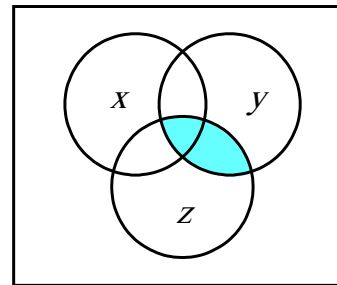
Left-Hand Side



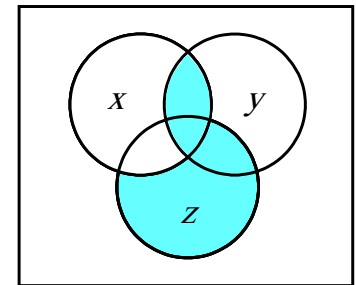
$$x y$$



$$\bar{x} z$$

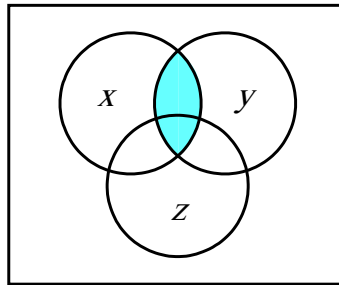


$$y z$$

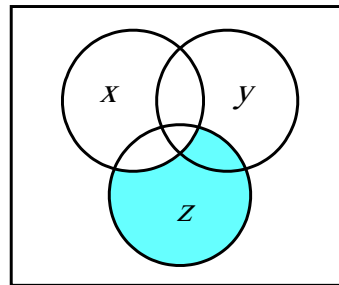


$$x y + \bar{x} z + y z$$

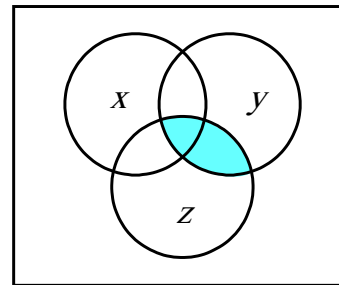
Left-Hand Side



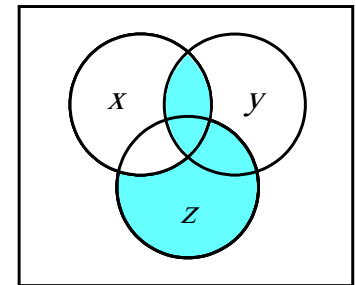
$$x y$$



$$\bar{x} z$$

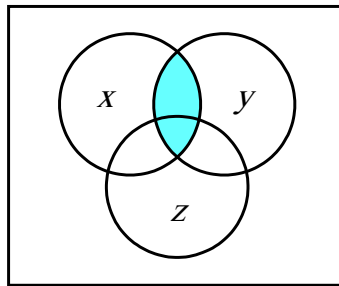


$$y z$$

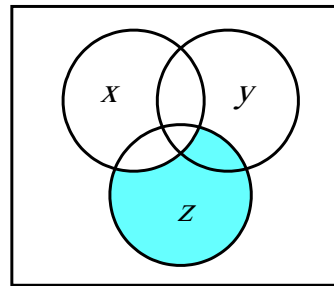


$$x y + \bar{x} z + y z$$

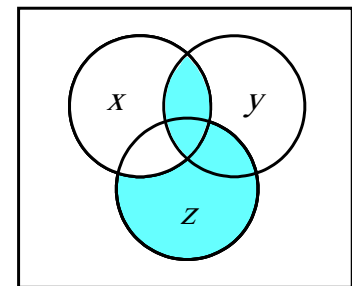
Right-Hand Side



$$x \bar{y}$$

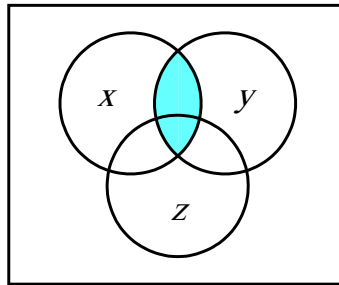


$$\bar{x} z$$

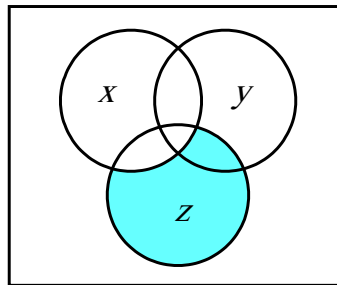


$$x \bar{y} + \bar{x} z$$

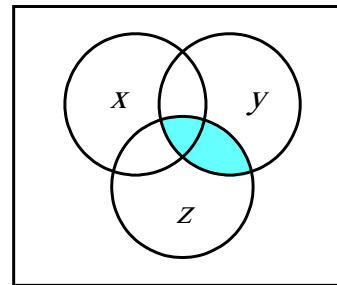
Left-Hand Side



$$x y$$

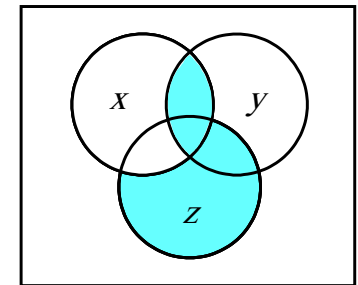


$$\bar{x} z$$



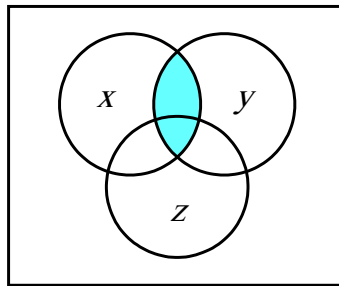
$$y z$$

These two are equal

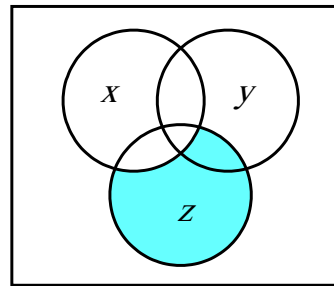


$$x y + \bar{x} z + y z$$

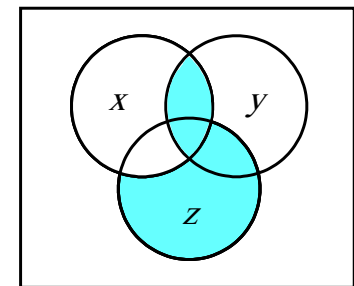
Right-Hand Side



$$\bar{x} y$$



$$\bar{x} z$$



$$\bar{x} y + \bar{x} z$$

[Figure 2.16 from the textbook]

Questions?

THE END