HCI/CprE/ComS 575: Computational Perception
Homework 1 (part 1)
Out: Thursday Jan 25, 2024
Due: Sunday Feb 4, 2024

**For extra credit submit part 1 (only part 1) by Sunday Jan 28 before midnight.
(upload a scanned copy (as only 1 PDF) on Canvas)**

These problems are designed to test your background knowledge in linear algebra and math in general.
Please show the intermediary steps of your calculations (not just the final answers).

1. What is 1+1?

2. What is 1+1 in the binary system?

3. What is 2+2?

4. What is 2+2 in the ternary number system (i.e., one with base 3)?

5. Convert the decimal number 153 to binary.

6. In the hexadecimal system, what is the value of B + C?

7. Convert the hexadecimal number BEEF to binary.

8. Solve the following quadratic equations:

    a) $x^2 - 11x + 30 = 0$ b) $x^2 + 6x - 27 = 0$ c) $2x^2 + 3x - 20 = 0$

9. Solve the following equations for x.

    a) $4(2x - 3) = 8(2x + 5)$ b) $9(2x - 1) - 3x = 3(12 + x)$ c) $\dfrac{2}{7x + 3} = \dfrac{9}{2x - 5}$

10. Solve the following systems of equations:

    a)
    $$4x - 3y = -3$$
    $$2x - 5y = 9$$

    b)
    $$x + 2y + 3z = 5$$
    $$2x + 5y + 3z = 3$$
    $$x + 8z = 17$$

    c)
    $$x + 3y + z = 4$$
    $$2x + 2y + z = -1$$
    $$2x + 3y + z = 3$$

    d)
    $$-x - 2y - 3z = 0$$
    $$w + x + 4y + 4z = 7$$
    $$w + 3x + 7y + 9z = 4$$
    $$-w - 2x - 4y - 6z = 6$$

11. Two standard dice are rolled. One is red and one is green. What is the probability that the sum of the two numbers on top is equal to 7?

12. Compute the value of "n choose k", i.e., $\begin{pmatrix} n \\ k \end{pmatrix}$, for $n = 6$ and $k = 2$.

13. What are the numbers in the 12-th row of Pascal's triangle?

14. Derive a formula for the following sum $1 + 2 + 3 + \ldots + n$.

15. Use induction to prove that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$.

16. What is the value of $\log_2 1024$?

17. Compute $\displaystyle\int_0^1 dx$.

18. Compute $\displaystyle\int_0^1 x\,dx$.

19. Compute $\displaystyle\int_0^1 x^2\,dx$.

20. Compute $\displaystyle\int_0^1 (\cos^2 x + \sin^2 x)\,dx$.

21. Prove the Pythagorean theorem in two different ways.

22. Let A be an $m \times n$ matrix and let x be a column vector of size n. How many additions and multiplications are required to compute the matrix-vector product Ax?

23. Let A be an $m \times n$ matrix and let B be an $n \times p$ matrix. How many additions and multiplications are required to compute the matrix product AB?

24. What is the computational complexity, using big-O notation, for the following algorithms: Bubble sort, Insertion sort, Quicksort, Linear search, Binary search, Matrix-Vector product, Matrix-Matrix product, Matrix inversion, FFT, IFFT? Assume that all arrays and vectors are of size N and all matrices are $N \times N$.

25. Calculate the following determinants.

$$(a)\ \begin{vmatrix} 1 & 3 \\ -6 & 5 \end{vmatrix} \quad (b)\ \begin{vmatrix} \alpha - 3 & 7 \\ -4 & \alpha - 2 \end{vmatrix} \quad (c)\ \begin{vmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{vmatrix} \quad (d)\ \begin{vmatrix} -2 & 4 & 2 \\ 2 & 1 & -1 \\ 4 & 7 & 3 \end{vmatrix}$$

26. Show that the value of the determinant does not depend on θ.

$$\begin{vmatrix} \sin\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ \sin\theta - \cos\theta & \sin\theta + \cos\theta & 1 \end{vmatrix}$$

27. Plot the functions $\cos(x)$, $\cos(2x)$, and $2\cos(x)$ on the same plot.

28. Draw a table with the values of $\sin(x)$, $\cos(x)$, and $\tan(x)$ for $x = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$.

29. Calculate the inverse of the given matrix.

$$(a) \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad (b) \begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix} \quad (c) \begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix} \quad (d) \begin{bmatrix} 0 & 0 & 0 & k_1 \\ 0 & 0 & k_2 & 0 \\ 0 & k_3 & 0 & 0 \\ k_4 & 0 & 0 & 0 \end{bmatrix}$$

30. Let $u = (3, 2, -1)$, $v = (0, 2, -3)$, and $w = (2, 6, 7)$. Compute:

$$(a) \; u \cdot v \qquad (b) \|u\| \qquad (c) \; v \times w \qquad (d) \; u \times (v \times w) \qquad (e) \; (u \times v) \times w$$

31. Find the equation for the plane that passes through the points: $P(-4, -1, -1), Q(-2, 0, 1), R(-1, -2, -3)$.

32. Find all values of λ for which the determinant of the matrix is equal to 0.

$$(a) \begin{vmatrix} \lambda - 3 & 0 \\ 8 & \lambda + 1 \end{vmatrix} \quad (b) \begin{vmatrix} \lambda & 3 \\ 4 & \lambda \end{vmatrix} \quad (c) \begin{vmatrix} \lambda - 5 & 0 & 1 \\ 1 & \lambda - 1 & 0 \\ -7 & 1 & \lambda \end{vmatrix}$$

33. Find the eigenvalues of the following matrices.

$$(a) \begin{bmatrix} -1 & 6 \\ 0 & 5 \end{bmatrix} \quad (a) \begin{bmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{bmatrix} \quad (b) \begin{bmatrix} 3 & 0 & 0 \\ -2 & 7 & 0 \\ 4 & 8 & 1 \end{bmatrix}$$

34. Given the vector $v = [x, y]^T$ find a rotation matrix R that rotates the vector by $60°$ counterclockwise. Give the values for the vector $v' = [x', y']^T$ in terms of x and y, where $v' = Rv$.

35. By example, show that matrix multiplication is not commutative.

36. Prove that any pair of 3×3 diagonal matrices commute under multiplication.

37. Prove that any pair of 2×2 upper-triangular Toeplitz matrices commute under multiplication.

38. Give the formulas for the 1D and 2D probability density function of the normal distribution.

39. The figure below is constructed with safety matches. Move only four of them such that the new figure contains exactly 3 squares. Draw a picture of your solution. If it is not possible, say that it can't be done.
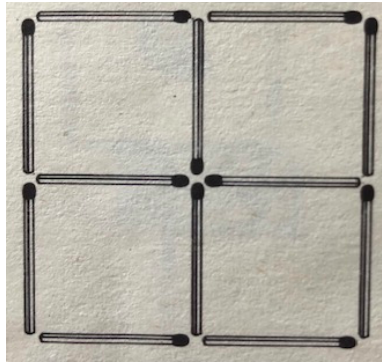
40. The numbers one through nine are represented by nine vegetables in the equations below. Each veggie represents the same number throughout. If the broccoli equals three, what is the identity of the carrot?
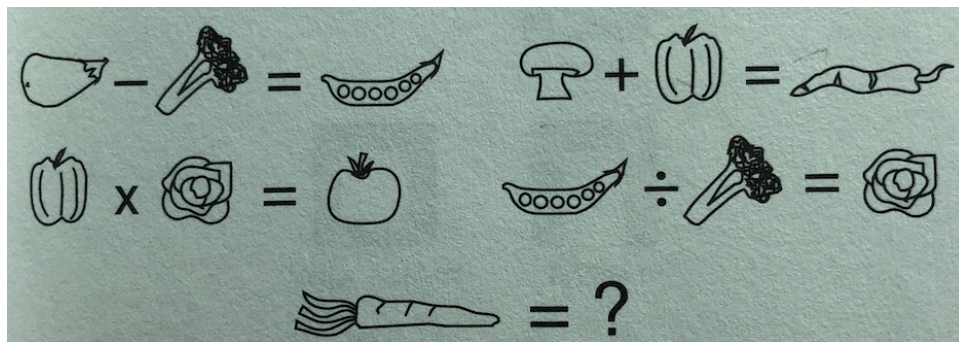
41. Solve the Connections puzzle shown in this picture (for rules see https://www.nytimes.com/games/connections).

Create four groups of four!

PRIME	TIME	AT	BAR
DEVISE	DOLLAR	FINE	POUND
TORCH	BUCK	COIN	QUALITY
CREATE	STERLING	PERCENT	INVENT

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Homework 1 (part 2)
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Download matlab on your computer. Then, take the "Matlab Onboarding" tutorial. As you progress through the tutorial, take several screenshots (start, middle, and end) to document that you did this task. See: `https://matlabacademy.mathworks.com/details/matlab-onramp/gettingstarted`

Then take the "Image Processing Onramp" Tutorial. As you progress through the tutorial, take several screenshots (start, middle, and end) to document that you did this task.

See: `https://matlabacademy.mathworks.com/details/image-processing-onramp/imageprocessing`

Attach these images to a PDF file and upload it on Canvas as your solutions for Part2.

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Homework 1 (part 3)
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The Matlab book, which is a recommended textbook for this class, shows several plot types on pages 170-174 (old edition of the book) or pages 184-188 (new edition).

Pick one plot type from each page and try to draw something similar, but it should not be the exact same plot as in the book. For each problem submit your matlab file and an output image. Name your files: myplot1.m, myplot2.m, ... , myplot5.m. Similarly, name your output images myoutput1.jpg, myoutput2.jpg, ... , myoutput5.jpg. (PNG or GIF file format is also acceptable).

Upload a ZIP file with all of your files to Canvas.