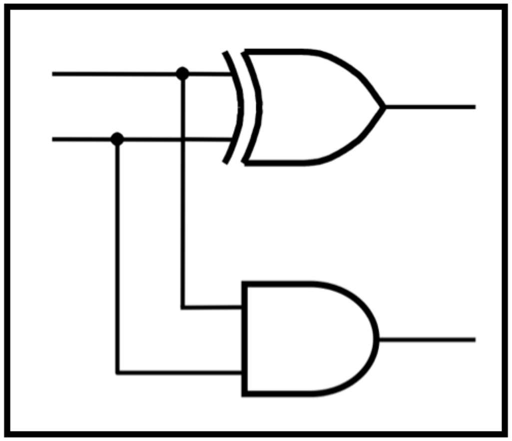


# CprE 2810: Digital Logic

**Instructor: Alexander Stoytchev**

**http://www.ece.iastate.edu/~alexs/classes/**

# Addition of Unsigned Numbers

*CprE 281: Digital Logic*
*Iowa State University, Ames, IA*

# Administrative Stuff

- We are now starting with Chapter 3

# Administrative Stuff

- HW5 is due today @ 10 pm

# Administrative Stuff

- No homework due next week

- HW6 will be due on Monday, Oct 14

# Quick Review

# Number Systems

$$N = d_n B^n + d_{n-1} B^{n-1} + \cdots + d_1 B^1 + d_0 B^0$$

# Number Systems

$$N = d_n B^n + d_{n-1} B^{n-1} + \cdots + d_1 B^1 + d_0 B^0$$

n-th digit
(most significant)

0-th digit
(least significant)

# Number Systems

base

power

$$N = d_n B^n + d_{n-1} B^{n-1} + \cdots + d_1 B^1 + d_0 B^0$$

n-th digit
(most significant)

0-th digit
(least significant)

# The Decimal System

$$524_{10} = 5\times10^2+2\times10^1+4\times10^0$$

# The Decimal System

$$524_{10} = 5\times10^2+2\times10^1+4\times10^0$$

$$= 5\times100+2\times10+4\times1$$

$$= 500+20+4$$

$$= 524_{10}$$

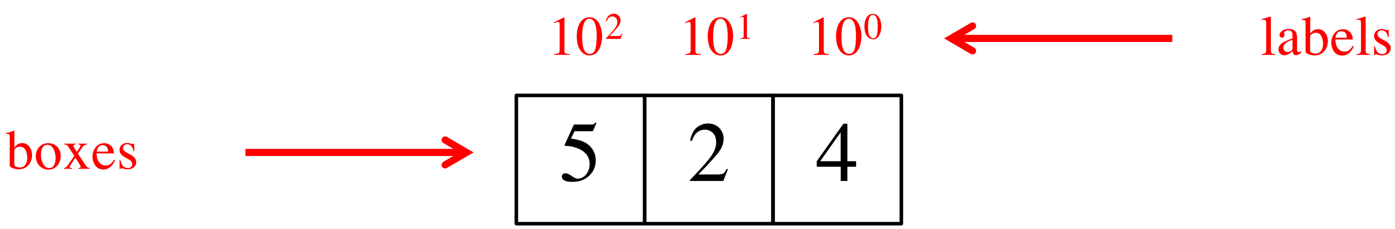
# Another Way to Look at This

| | | |
|---|---|---|
| 5 | 2 | 4 |

# Another Way to Look at This

| $10^2$ | $10^1$ | $10^0$ |
|---|---|---|
| 5 | 2 | 4 |

# Another Way to Look at This




Each box can contain only one digit and has only one label. From right to left, the labels are increasing powers of the base, starting from 0.

# Base 7

$$524_7 = 5 \times 7^2 + 2 \times 7^1 + 4 \times 7^0$$

# Base 7

base power

$$524_7 = 5 \times 7^2 + 2 \times 7^1 + 4 \times 7^0$$

# Base 7

base

power

$$524_7 = 5 \times 7^2 + 2 \times 7^1 + 4 \times 7^0$$

most significant digit

least significant digit

# Base 7

$$
\begin{aligned}
524_7 &= 5 \times 7^2 + 2 \times 7^1 + 4 \times 7^0 \\
&= 5 \times 49 + 2 \times 7 + 4 \times 1 \\
&= 245 + 14 + 4 \\
&= 263_{10}
\end{aligned}
$$

# Another Way to Look at This

| $7^2$ | $7^1$ | $7^0$ | | $10^2$ | $10^1$ | $10^0$ |
|---|---|---|---|---|---|---|
| 5 | 2 | 4 | = | 2 | 6 | 3 |

# Binary Numbers (Base 2)

$$1001_2 = \; 1 \times 2^3 \; + \; 0 \times 2^2 \; + \; 0 \times 2^1 \; + \; 1 \times 2^0$$

# Binary Numbers (Base 2)

base

power

$$1001_2 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

most significant bit

least significant bit

# Binary Numbers (Base 2)

$$\begin{aligned}
1001_2 = & \; 1 \times 2^3 & + & \; 0 \times 2^2 & + & \; 0 \times 2^1 & + & \; 1 \times 2^0 & = \\
= & \; 1 \times 8 & + & \; 0 \times 4 & + & \; 0 \times 2 & + & \; 1 \times 1 & = \\
= & \; 8 & + & \; 0 & + & \; 0 & + & \; 1 & = \\
= & \; 9_{10}
\end{aligned}$$

# Another Example

$$
\begin{array}{rlllllllllll}
11101_2 = & 1 \times 2^4 & + & 1 \times 2^3 & + & 1 \times 2^2 & + & 0 \times 2^1 & + & 1 \times 2^0 & = & \\
= & 1 \times 16 & + & 1 \times 8 & + & 1 \times 4 & + & 0 \times 2 & + & 1 \times 1 & = & \\
= & 16 & + & 8 & + & 4 & + & 0 & + & 1 & = & 29_{10}
\end{array}
$$

# Powers of 2

$$
\begin{aligned}
2^{10} &= 1024 \\
2^{9} &= 512 \\
2^{8} &= 256 \\
2^{7} &= 128 \\
2^{6} &= 64 \\
2^{5} &= 32 \\
2^{4} &= 16 \\
2^{3} &= 8 \\
2^{2} &= 4 \\
2^{1} &= 2 \\
2^{0} &= 1
\end{aligned}
$$

# What is the value of this binary number?

- 0 0 1 0 1 1 0 0
- 0 0 1 0 1 1 0 0
- $0*2^7 + 0*2^6 + 1*2^5 + 0*2^4 + 1*2^3 + 1*2^2 + 0*2^1 + 0*2^0$
- 0*128 + 0*64 + 1*32 + 0*16 + 1*8 + 1*4 + 0*2 + 0*1
- 0*128 + 0*64 + 1*32 + 0*16 + 1*8 + 1*4 + 0*2 + 0*1
- 32+ 8 + 4 = 44 (in decimal)

# Another Way to Look at This

| $2^7$ | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |

# Another Way to Look at This

| $2^7$ | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |
| | | 32 | | 8 | 4 | | |

# Signed v.s. Unsigned Numbers

# Signed v.s. Unsigned Numbers

Signed: positive and negative integers

Unsigned: only positive integers

# Signed v.s. Unsigned Numbers

| Signed | Unsigned |
| --- | --- |
| positive and negative integers and zero | only positive integers and zero |

# Two Different Types of Binary Numbers

## Unsigned numbers

- All bits jointly represent a positive integer.
- Negative numbers cannot be represented this way.

## Signed numbers

- The left-most bit represents the sign of the number.
- If that bit is 0, then the number is positive.
- If that bit is 1, then the number is negative.
- The magnitude of the largest number that can be represented in this way is twice smaller than the largest number in the unsigned representation.

# Two Different Types of Binary Numbers

## Unsigned numbers

- All bits jointly represent a positive integer.
- Negative numbers cannot be represented this way.

There are 3 different ways to represent signed numbers. They will be introduced next time.

## Signed numbers

- The left-most bit represents the sign of the number.
- If that bit is 0, then the number is positive.
- If that bit is 1, then the number is negative.
- The magnitude of the largest number that can be represented in this way is twice smaller than the largest number in the unsigned representation.

# Unsigned Representation

| $2^7$ | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |

This represents + 44.

# Unsigned Representation

| $2^7$ | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |

This represents + 172.

# Signed Representation
# (using the left-most bit as the sign)

| sign | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |

This represents + 44.

# Signed Representation (using the left-most bit as the sign)

| sign | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |

This represents – 44.

# Today's Lecture is About
# Addition of Unsigned Numbers

# Important Clarification: There are two types of addition

- **Addition of Boolean variables, e.g.,**

  $x + y$ where $x, y \in \{0, 1\}$

- **Addition of n-bit Binary numbers, e.g.,**

  $x_4x_3x_2x_1x_0 + y_4y_3y_2y_1y_0$ where each $x_k, y_k \in \{0, 1\}$

# Important Clarification: There are two types of addition

- **Addition of Boolean variables, e.g.,**

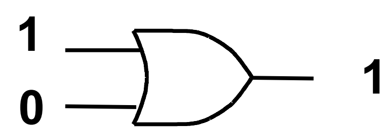
  **1 + 0 = 1**

- **Addition of n-bit Binary numbers, e.g.,**

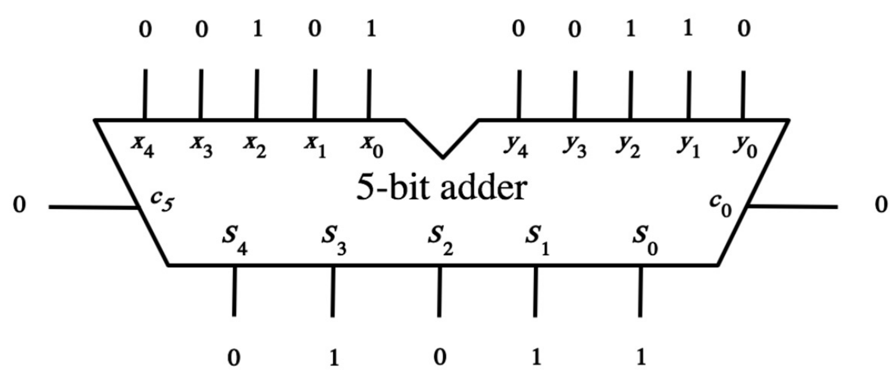
  **00101 + 00110 = 01011**

# Important Clarification:
# There are two types of addition

- **Addition of Boolean variables, e.g.,**

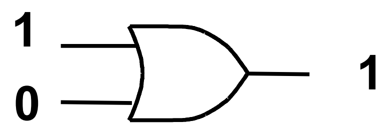



- **Addition of n-bit Binary numbers, e.g.,**

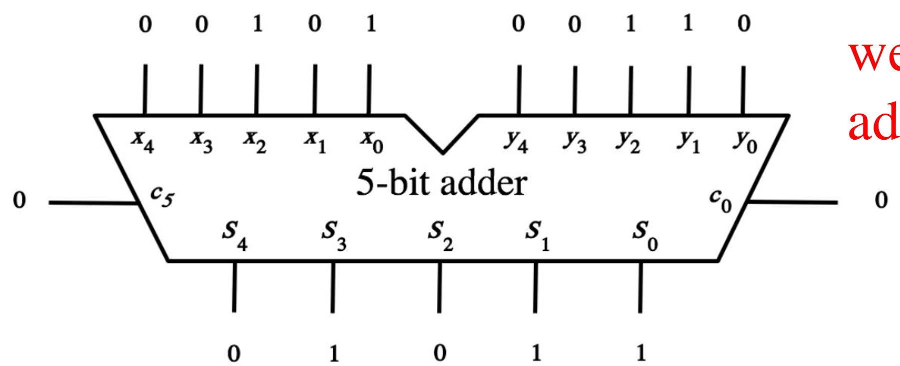


# Important Clarification:
# There are two types of addition

- **Addition of Boolean variables, e.g.,**




- **Addition of n-bit Binary numbers, e.g.,**




we will derive this adder circuit today

# Important Clarification: There are two types of addition

- **Addition of Boolean variables, e.g.,**

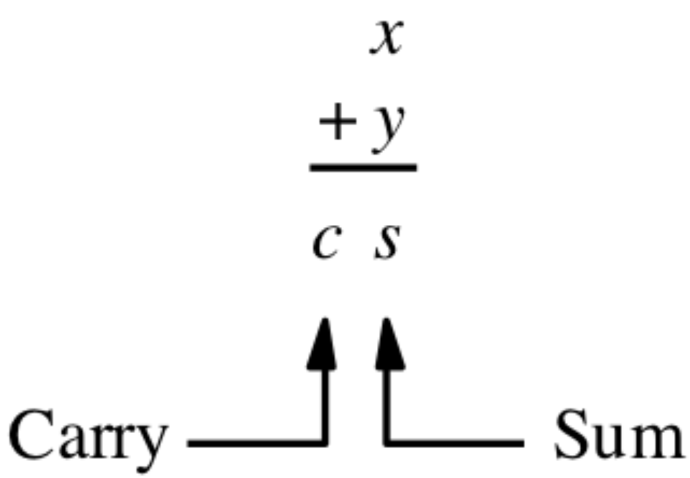
  **1 + 1 = 1 (according to the rules of Boolean algebra)**

- **Addition of n-bit Binary numbers, e.g.,**

  **1 + 1 = 10 (because in decimal 1 + 1 = 2 )**

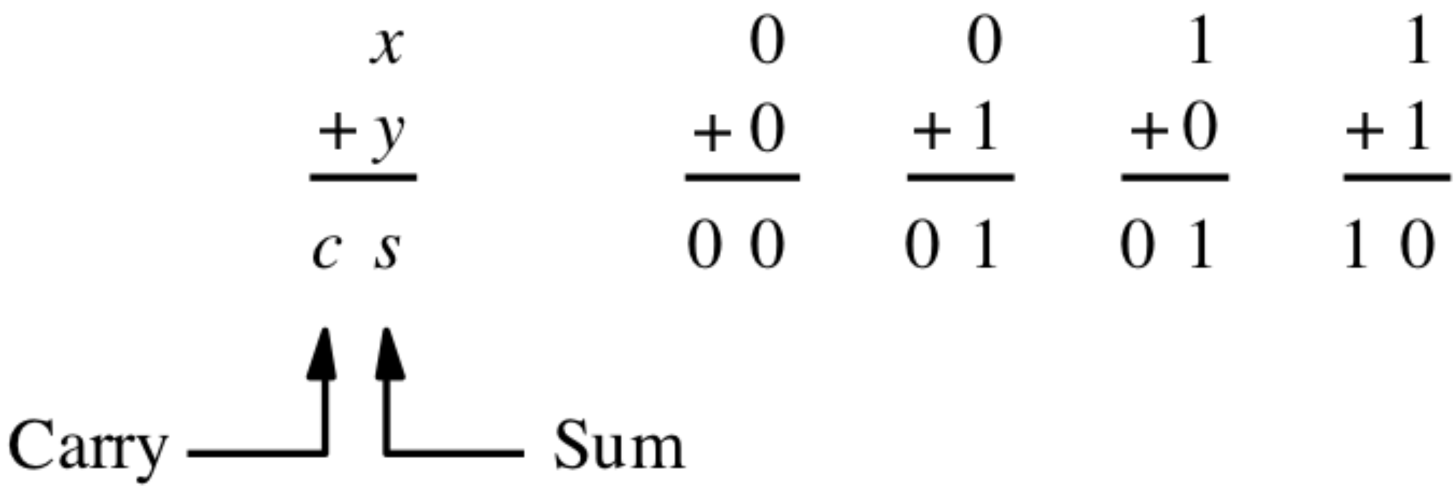
# Addition of 1-bit Unsigned Numbers

# Addition of two 1-bit numbers



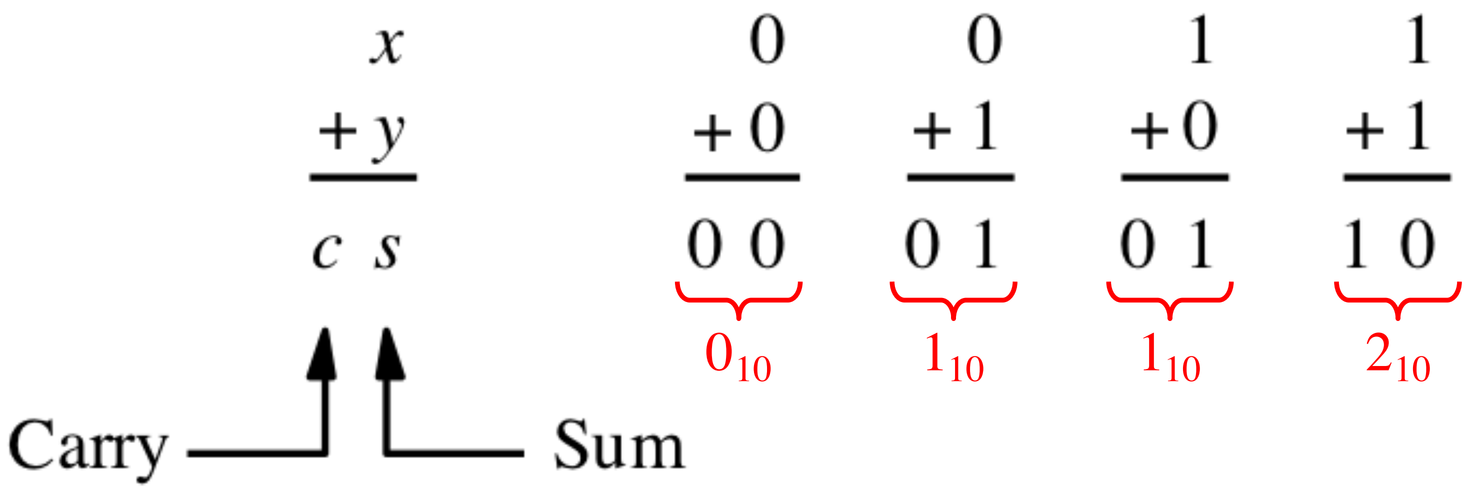

[ Figure 3.1a from the textbook ]

# Addition of two 1-bit numbers
## (there are four possible cases)




[ Figure 3.1a from the textbook ]

# Addition of two 1-bit numbers
## (there are four possible cases)




[ Figure 3.1a from the textbook ]

# Addition of two 1-bit numbers (the truth table)

| $x$ | $y$ | Carry $c$ | Sum $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

[ Figure 3.1b from the textbook ]

# Addition of two 1-bit numbers

$$
\begin{array}{r}
x \\
+\,y \\
\hline
c\ \ s
\end{array}
\qquad\qquad
\begin{array}{r}
0 \\
+\,0 \\
\hline
0\ \ 0
\end{array}
\qquad
\begin{array}{r}
0 \\
+\,1 \\
\hline
0\ \ 1
\end{array}
\qquad
\begin{array}{r}
1 \\
+\,0 \\
\hline
0\ \ 1
\end{array}
\qquad
\begin{array}{r}
1 \\
+\,1 \\
\hline
1\ \ 0
\end{array}
$$

[ Figure 2.12 from the textbook ]

# Addition of two 1-bit numbers

$$
\begin{array}{r}
x \\
+\,y \\
\hline
c\ s
\end{array}
\qquad
\begin{array}{r}
0 \\
+\,0 \\
\hline
0\ 0
\end{array}
\qquad
\begin{array}{r}
0 \\
+\,1 \\
\hline
0\ 1
\end{array}
\qquad
\begin{array}{r}
1 \\
+\,0 \\
\hline
0\ 1
\end{array}
\qquad
\begin{array}{r}
1 \\
+\,1 \\
\hline
1\ 0
\end{array}
$$

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

| | | | | |
|---|---|---|---|---|
| $x$ | 0 | 0 | 1 | 1 |
| $+\,y$ | $+\,0$ | $+\,1$ | $+\,0$ | $+\,1$ |
| $c\ \ s$ | 0 0 | 0 1 | 0 1 | 1 0 |

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

| $x$ | 0 | 0 | 1 | 1 |
|---|---|---|---|---|
| $+y$ | +0 | +1 | +0 | +1 |
| $c\ s$ | 0 0 | 0 1 | 0 1 | 1 0 |

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

$$
\begin{array}{r}
x \\
+\,y \\
\hline
c\ s
\end{array}
\qquad
\begin{array}{r}
0 \\
+\,0 \\
\hline
0\ 0
\end{array}
\qquad
\begin{array}{r}
0 \\
+\,1 \\
\hline
0\ 1
\end{array}
\qquad
\begin{array}{r}
1 \\
+\,0 \\
\hline
0\ 1
\end{array}
\qquad
\begin{array}{r}
1 \\
+\,1 \\
\hline
1\ 0
\end{array}
$$

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

| | | | | |
|---|---|---|---|---|
| $x$ | 0 | 0 | 1 | 1 |
| $+\,y$ | $+\,0$ | $+\,1$ | $+\,0$ | $+\,1$ |
| $c\ s$ | 0 0 | 0 1 | 0 1 | 1 0 |

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

| | | | | |
|---|---|---|---|---|
| $x$ | 0 | 0 | 1 | 1 |
| $+\,y$ | $+\,0$ | $+\,1$ | $+\,0$ | $+\,1$ |
| $c\ s$ | 0 0 | 0 1 | 0 1 | 1 0 |

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

$$\begin{array}{r} x \\ +\,y \\ \hline c\ s \end{array} \qquad \begin{array}{r} 0 \\ +\,0 \\ \hline 0\ 0 \end{array} \qquad \begin{array}{r} 0 \\ +\,1 \\ \hline 0\ 1 \end{array} \qquad \begin{array}{r} 1 \\ +\,0 \\ \hline 0\ 1 \end{array} \qquad \begin{array}{r} 1 \\ +\,1 \\ \hline 1\ 0 \end{array}$$

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

$$
\begin{array}{r} x \\ +\,y \\ \hline c\;s \end{array}
\qquad
\begin{array}{r} 0 \\ +\,0 \\ \hline 0\;0 \end{array}
\qquad
\begin{array}{r} 0 \\ +\,1 \\ \hline 0\;1 \end{array}
\qquad
\begin{array}{r} 1 \\ +\,0 \\ \hline 0\;1 \end{array}
\qquad
\begin{array}{r} 1 \\ +\,1 \\ \hline 1\;0 \end{array}
$$

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

$$\begin{array}{r} x \\ +\,y \\ \hline c\ s \end{array} \qquad \begin{array}{r} 0 \\ +\,0 \\ \hline 0\ 0 \end{array} \qquad \begin{array}{r} 0 \\ +\,1 \\ \hline 0\ 1 \end{array} \qquad \begin{array}{r} 1 \\ +\,0 \\ \hline 0\ 1 \end{array} \qquad \begin{array}{r} 1 \\ +\,1 \\ \hline 1\ 0 \end{array}$$

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

$$\begin{array}{r} x \\ +\,y \\ \hline c\ s \end{array} \qquad \begin{array}{r} 0 \\ +\,0 \\ \hline 0\ 0 \end{array} \qquad \begin{array}{r} 0 \\ +\,1 \\ \hline 0\ 1 \end{array} \qquad \begin{array}{r} 1 \\ +\,0 \\ \hline 0\ 1 \end{array} \qquad \begin{array}{r} 1 \\ +\,1 \\ \hline 1\ 0 \end{array}$$

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

$$\begin{array}{r} x \\ +\,y \\ \hline c\;\boxed{s} \end{array} \qquad \begin{array}{r} 0 \\ +\,0 \\ \hline 0\;\boxed{0} \end{array} \qquad \begin{array}{r} 0 \\ +\,1 \\ \hline 0\;\boxed{1} \end{array} \qquad \begin{array}{r} 1 \\ +\,0 \\ \hline 0\;\boxed{1} \end{array} \qquad \begin{array}{r} 1 \\ +\,1 \\ \hline 1\;\boxed{0} \end{array}$$

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

$$
\begin{array}{r} x \\ +\,y \\ \hline c\;\boxed{s} \end{array}
\qquad
\begin{array}{r} 0 \\ +\,0 \\ \hline 0\;\boxed{0} \end{array}
\qquad
\begin{array}{r} 0 \\ +\,1 \\ \hline 0\;\boxed{1} \end{array}
\qquad
\begin{array}{r} 1 \\ +\,0 \\ \hline 0\;\boxed{1} \end{array}
\qquad
\begin{array}{r} 1 \\ +\,1 \\ \hline 1\;\boxed{0} \end{array}
$$

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

| | | | | |
|---|---|---|---|---|
| $x$ | 0 | 0 | 1 | 1 |
| $+y$ | $+0$ | $+1$ | $+0$ | $+1$ |
| $c\ s$ | 0 0 | 0 1 | 0 1 | 1 0 |

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

| $x$ | | $y$ | | $c$ | $s$ | |
|---|---|---|---|---|---|---|
| 0 | + | 0 | = | 0 | 0 | $= 0_{10}$ |
| 0 | + | 1 | = | 0 | 1 | $= 1_{10}$ |
| 1 | + | 0 | = | 0 | 1 | $= 1_{10}$ |
| 1 | + | 1 | = | 1 | 0 | $= 2_{10}$ |

# Addition of two 1-bit numbers

?

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

AND

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

?

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

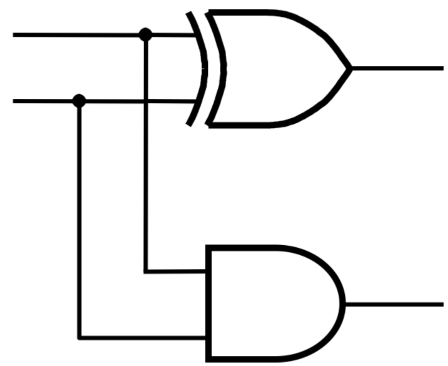
# Addition of two 1-bit numbers

| $x$ | $y$ | $c$ | $s$ (XOR) |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers

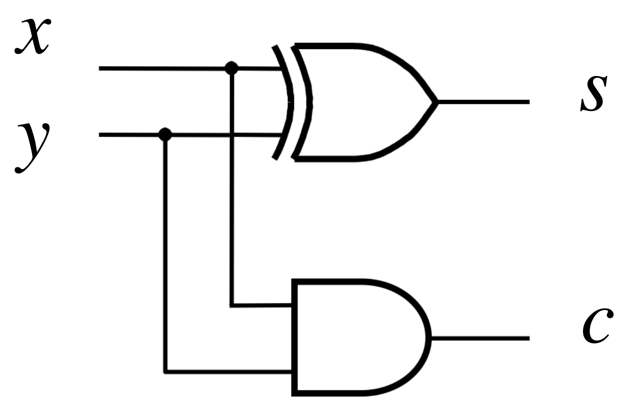
| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers



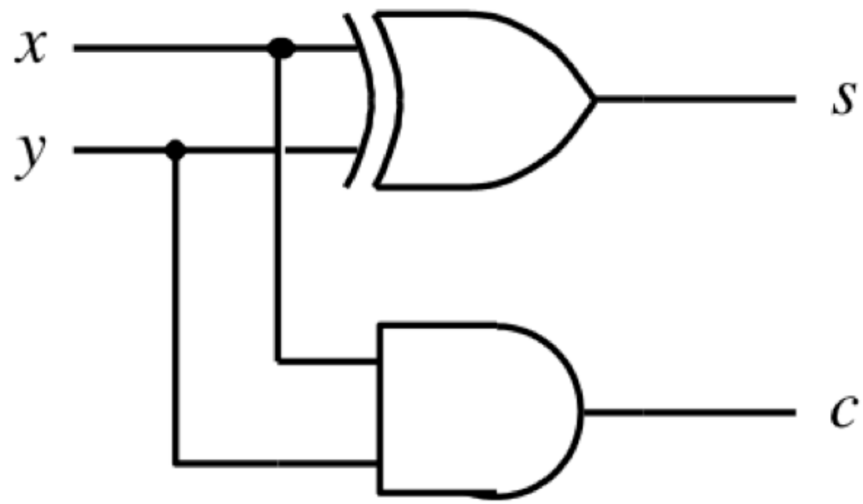
| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers



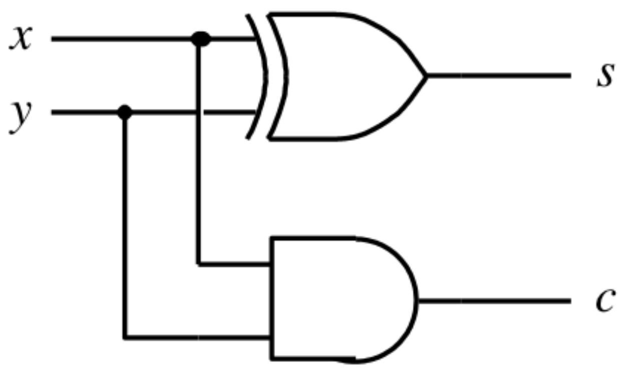

| $x$ | $y$ | $c$ | $s$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Addition of two 1-bit numbers (the logic circuit)

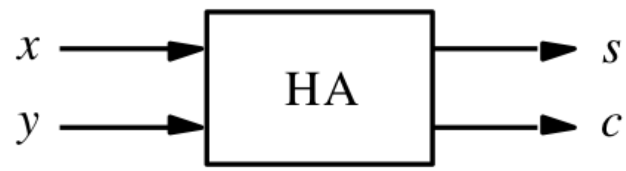



[ Figure 3.1c from the textbook ]

# The Half-Adder




(c) Circuit




(d) Graphical symbol

[ Figure 3.1c-d from the textbook ]

# Addition of Multibit Unsigned Numbers

# Analogy with addition in base 10

$$
\begin{array}{r}
x_2\ x_1\ x_0 \\
+\quad y_2\ y_1\ y_0 \\
\hline
s_2\ s_1\ s_0
\end{array}
$$

# Analogy with addition in base 10

$$
\begin{array}{rccc}
 & 3 & 8 & 9 \\
+ & 1 & 5 & 7 \\
\hline
 & 5 & 4 & 6
\end{array}
$$

# Analogy with addition in base 10

| | | | | |
|---|---|---|---|---|
| carry | 0 | 1 | 1 | 0 |
| | | 3 | 8 | 9 |
| + | | 1 | 5 | 7 |
| | | 5 | 4 | 6 |

# Analogy with addition in base 10

$$
\begin{array}{rcccc}
 & c_3 & c_2 & c_1 & c_0 \\
 & & x_2 & x_1 & x_0 \\
+ & & y_2 & y_1 & y_0 \\
\hline
 & & s_2 & s_1 & s_0
\end{array}
$$

# Analogy with addition in base 10

$$
\begin{array}{ccccc}
 & c_3 & c_2 & c_1 & c_0 \\
 & & x_2 & x_1 & x_0 \\
+ & & y_2 & y_1 & y_0 \\
\hline
 & & s_2 & s_1 & s_0
\end{array}
$$

given these
3 inputs

# Analogy with addition in base 10

$$\begin{array}{ccccc} & c_3 & c_2 & \boxed{c_1} & \boxed{c_0} \\ + & & x_2 & x_1 & \boxed{x_0} \\ & & y_2 & y_1 & \boxed{y_0} \\ \hline & & s_2 & s_1 & \boxed{s_0} \end{array}$$

given these
3 inputs

compute these
2 outputs

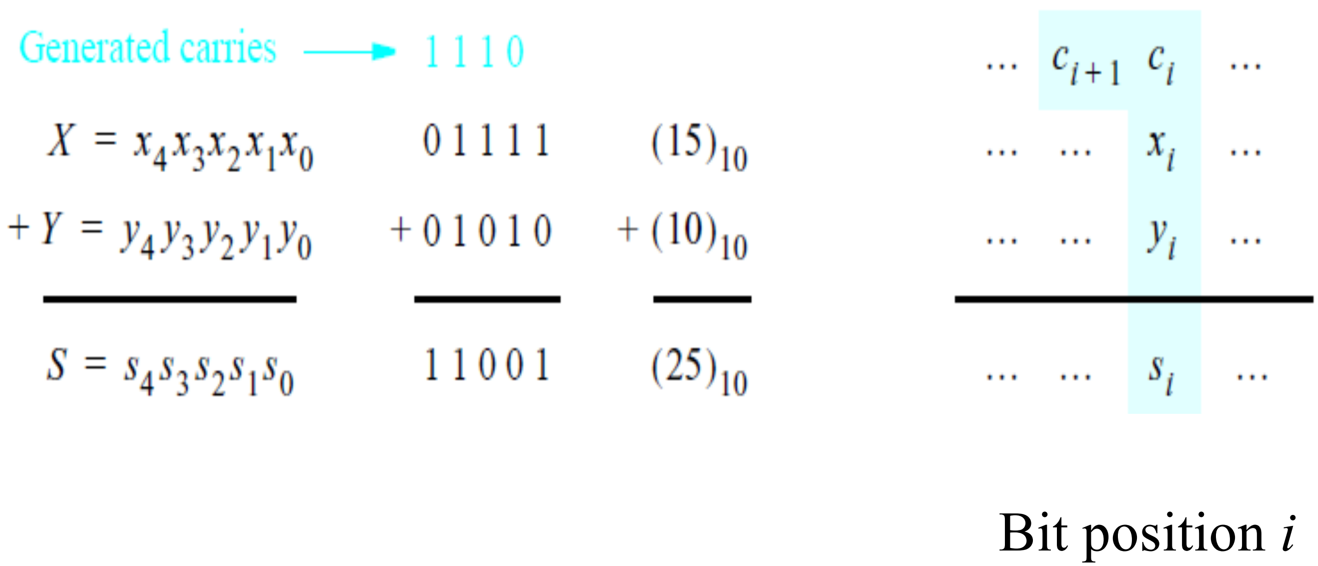
# Analogy with addition in base 10

| | | | | |
|---|---|---|---|---|
| | $c_3$ | $c_2$ | $c_1$ | $c_0$ |
| | | $x_2$ | $x_1$ | $x_0$ |
| $+$ | | $y_2$ | $y_1$ | $y_0$ |
| | | $s_2$ | $s_1$ | $s_0$ |

# Analogy with addition in base 10

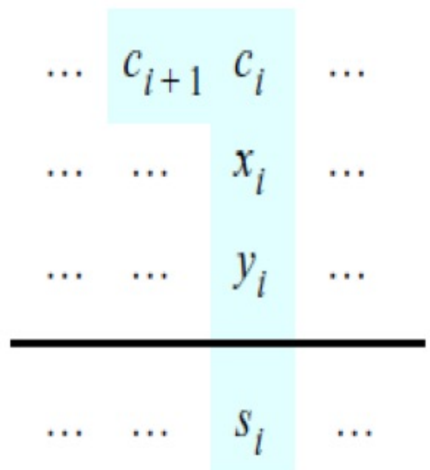
$$
\begin{array}{rcccc}
 & c_3 & c_2 & c_1 & c_0 \\
 & & x_2 & x_1 & x_0 \\
+ & & y_2 & y_1 & y_0 \\
\hline
 & & s_2 & s_1 & s_0
\end{array}
$$

# Addition of multibit numbers




[ Figure 3.2 from the textbook ]

# Problem Statement and Truth Table

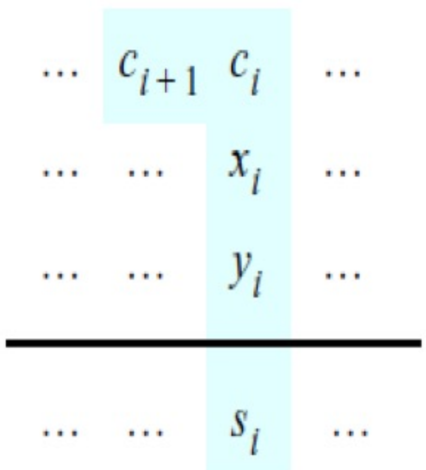



[ Figure 3.2b from the textbook ]

| $c_i$ | $x_i$ | $y_i$ | $c_{i+1}$ | $s_i$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

[ Figure 3.3a from the textbook ]

# Problem Statement and Truth Table



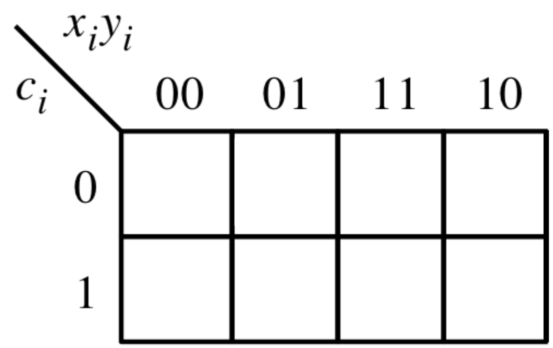

[ Figure 3.2b from the textbook ]

| $c_i$ $x_i$ $y_i$ | | $c_{i+1}$ | $s_i$ | |
|---|---|---|---|---|
| 0 + 0 + 0 | = | 0 | 0 | $= 0_{10}$ |
| 0 + 0 + 1 | = | 0 | 1 | $= 1_{10}$ |
| 0 + 1 + 0 | = | 0 | 1 | $= 1_{10}$ |
| 0 + 1 + 1 | = | 1 | 0 | $= 2_{10}$ |
| 1 + 0 + 0 | = | 0 | 1 | $= 1_{10}$ |
| 1 + 0 + 1 | = | 1 | 0 | $= 2_{10}$ |
| 1 + 1 + 0 | = | 1 | 0 | $= 2_{10}$ |
| 1 + 1 + 1 | = | 1 | 1 | $= 3_{10}$ |

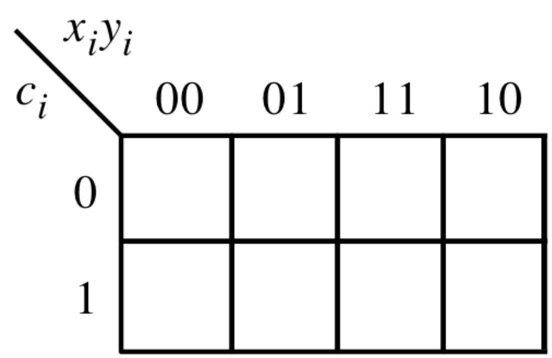
[ Figure 3.3a from the textbook ]

# Let's fill-in the two K-maps

| $c_i$ | $x_i$ | $y_i$ | $c_{i+1}$ | $s_i$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |




$s_i =$



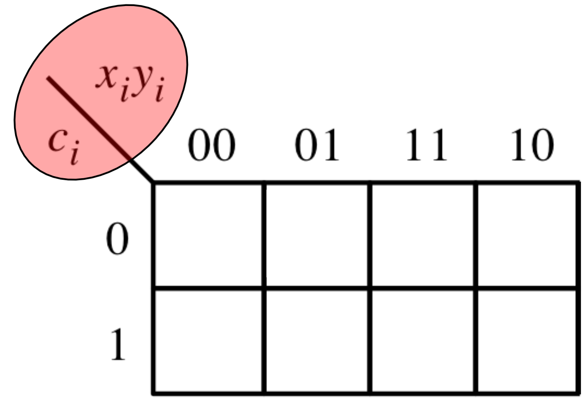

$c_{i+1} =$

[ Figure 3.3a-b from the textbook ]

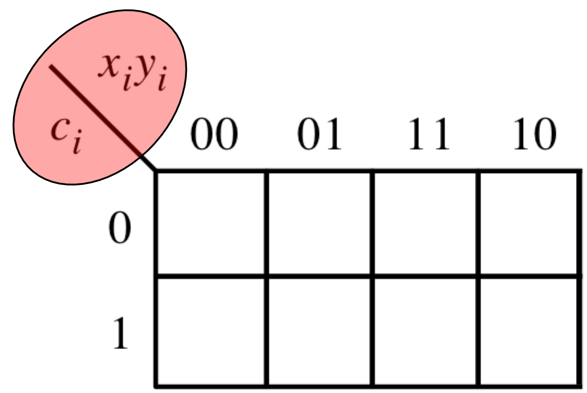
# Let’s fill-in the two K-maps

Note that the textbook switched to the other way to draw a K-Map

| $c_i$ | $x_i$ | $y_i$ | $c_{i+1}$ | $s_i$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |




$s_i =$



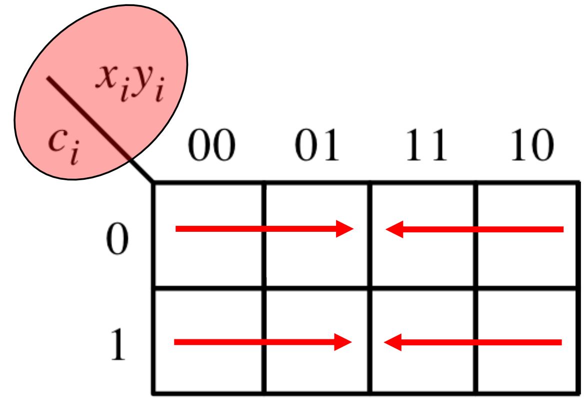

$c_{i+1} =$

[ Figure 3.3a-b from the textbook ]

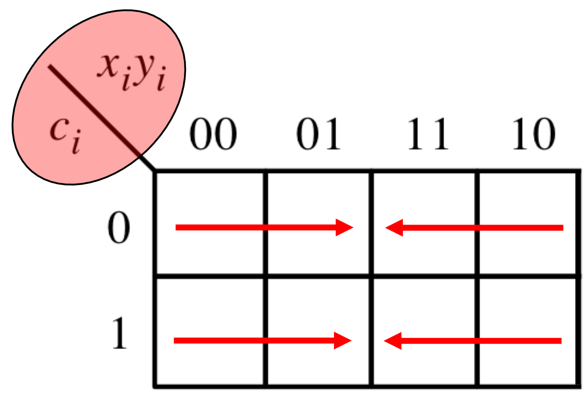
# Let's fill-in the two K-maps

Note that the textbook switched to the other way to draw a K-Map

| $c_i$ | $x_i$ | $y_i$ | $c_{i+1}$ | $s_i$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |




$s_i =$




$c_{i+1} =$

[ Figure 3.3a-b from the textbook ]

# Let's fill-in the two K-maps

| $c_i$ | $x_i$ | $y_i$ | $c_{i+1}$ | $s_i$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

| $c_i$ \ $x_i y_i$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | 1 | | 1 |
| 1 | 1 | | 1 | |

$$s_i = x_i \oplus y_i \oplus c_i$$

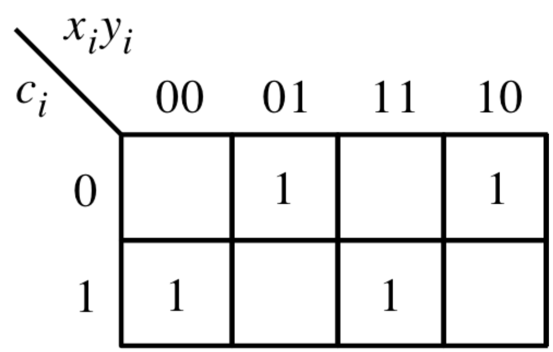
| $c_i$ \ $x_i y_i$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | | 1 | |
| 1 | | 1 | 1 | 1 |

$$c_{i+1} = x_i y_i + x_i c_i + y_i c_i$$

[ Figure 3.3a-b from the textbook ]

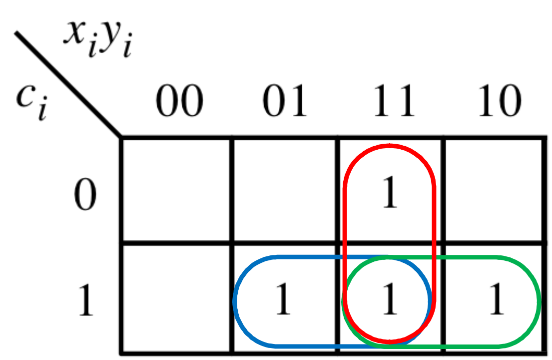
# Let's fill-in the two K-maps

| $c_i$ | $x_i$ | $y_i$ | $c_{i+1}$ | $s_i$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |




3-input XOR

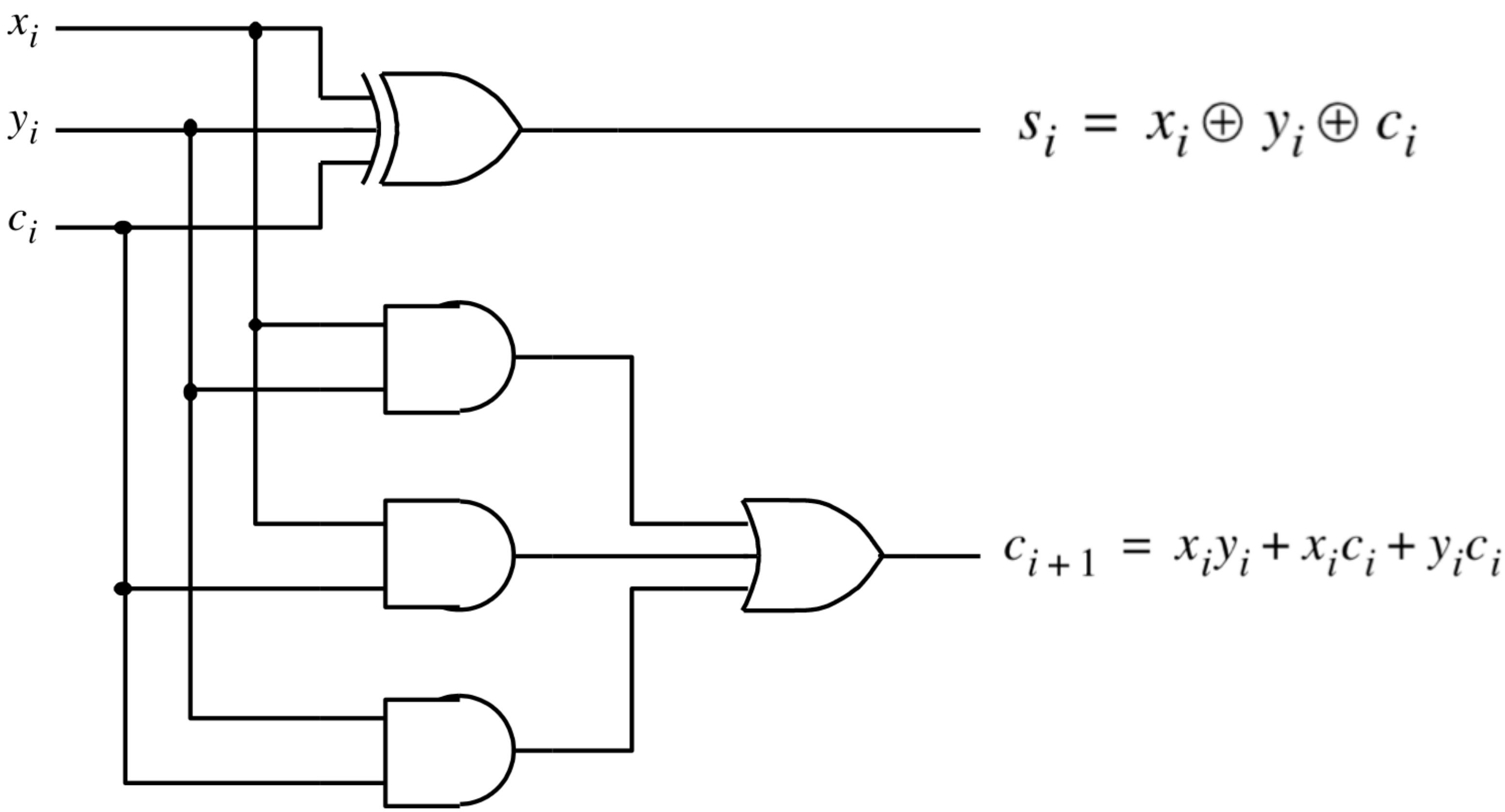
$$s_i = x_i \oplus y_i \oplus c_i$$




$$c_{i+1} = x_i y_i + x_i c_i + y_i c_i$$

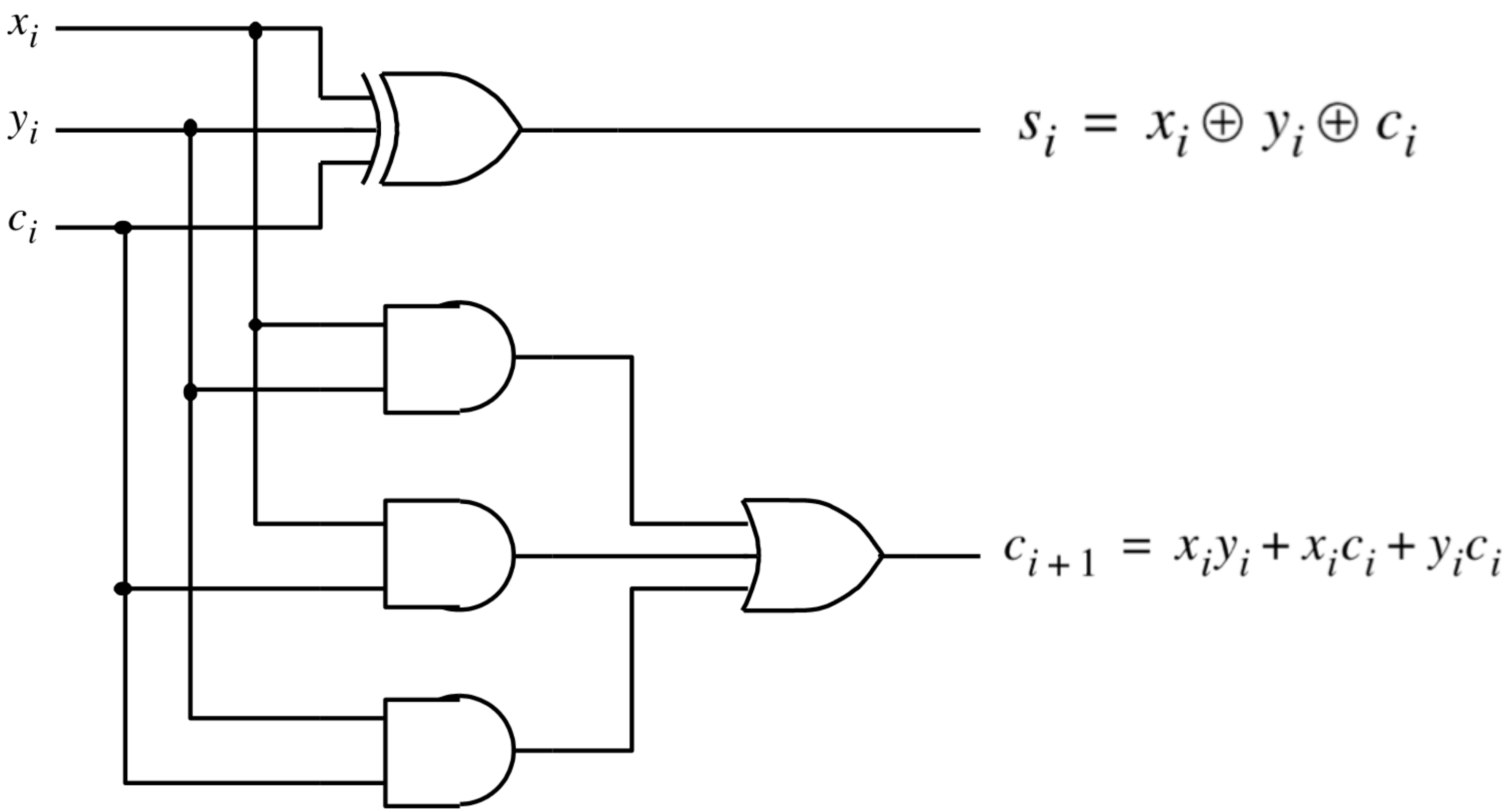
[ Figure 3.3a-b from the textbook ]

# The circuit for the two expressions



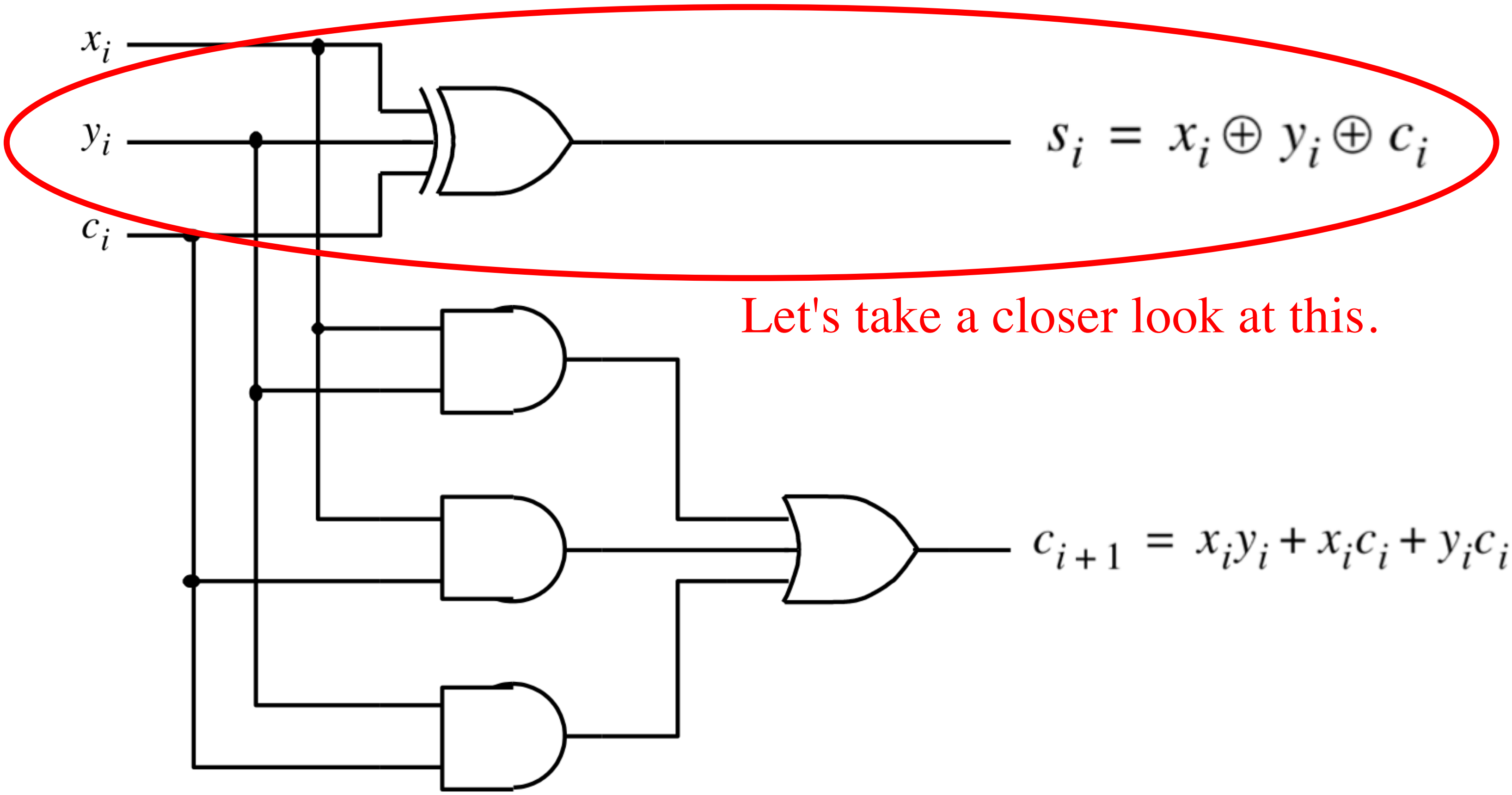

[ Figure 3.3c from the textbook ]

# This is called the Full-Adder




[ Figure 3.3c from the textbook ]

# This is called the Full-Adder




[ Figure 3.3c from the textbook ]

# XOR Magic

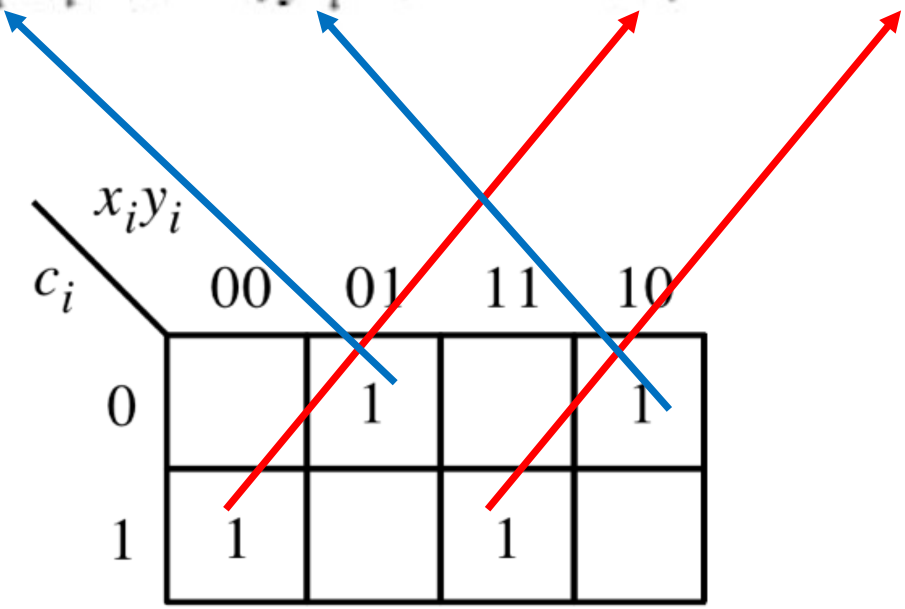
$$s_i = \bar{x}_i y_i \bar{c}_i + x_i \bar{y}_i \bar{c}_i + \bar{x}_i \bar{y}_i c_i + x_i y_i c_i$$

| $c_i$ \ $x_i y_i$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | 1 | | 1 |
| 1 | 1 | | 1 | |

$$s_i = x_i \oplus y_i \oplus c_i$$

# XOR Magic

$$s_i = \bar{x}_i y_i \bar{c}_i + x_i \bar{y}_i \bar{c}_i + \bar{x}_i \bar{y}_i c_i + x_i y_i c_i$$




$$s_i = x_i \oplus y_i \oplus c_i$$

# XOR Magic

$$s_i = \bar{x}_i y_i \bar{c}_i + x_i \bar{y}_i \bar{c}_i + \bar{x}_i \bar{y}_i c_i + x_i y_i c_i$$

$$\begin{aligned} s_i &= (\bar{x}_i y_i + x_i \bar{y}_i)\bar{c}_i + (\bar{x}_i \bar{y}_i + x_i y_i)c_i \\ &= (x_i \oplus y_i)\bar{c}_i + \overline{(x_i \oplus y_i)}c_i \\ &= (x_i \oplus y_i) \oplus c_i \end{aligned}$$

# XOR Magic

$$s_i = \bar{x}_i y_i \bar{c}_i + x_i \bar{y}_i \bar{c}_i + \bar{x}_i \bar{y}_i c_i + x_i y_i c_i$$

Can you prove this?

$$\begin{aligned} s_i &= (\bar{x}_i y_i + x_i \bar{y}_i)\bar{c}_i + (\bar{x}_i \bar{y}_i + x_i y_i) c_i \\ &= (x_i \oplus y_i)\bar{c}_i + \overline{(x_i \oplus y_i)} c_i \\ &= (x_i \oplus y_i) \oplus c_i \end{aligned}$$

# XOR Magic

$$\overline{x_i}\,\overline{y_i} + x_i\, y_i = \overline{x_i \oplus y_i}$$

# XOR Magic

$$\overline{x_i}\,\overline{y_i} + x_i y_i = \overline{x_i \oplus y_i}$$

XNOR

# XOR Magic

$$\overline{x_i}\,\overline{y_i} + x_i y_i = \overline{x_i \oplus y_i}$$

XOR

# XOR Magic

$$\overline{x_i}\,\overline{y_i} + x_i\, y_i = \overline{x_i \oplus y_i}$$

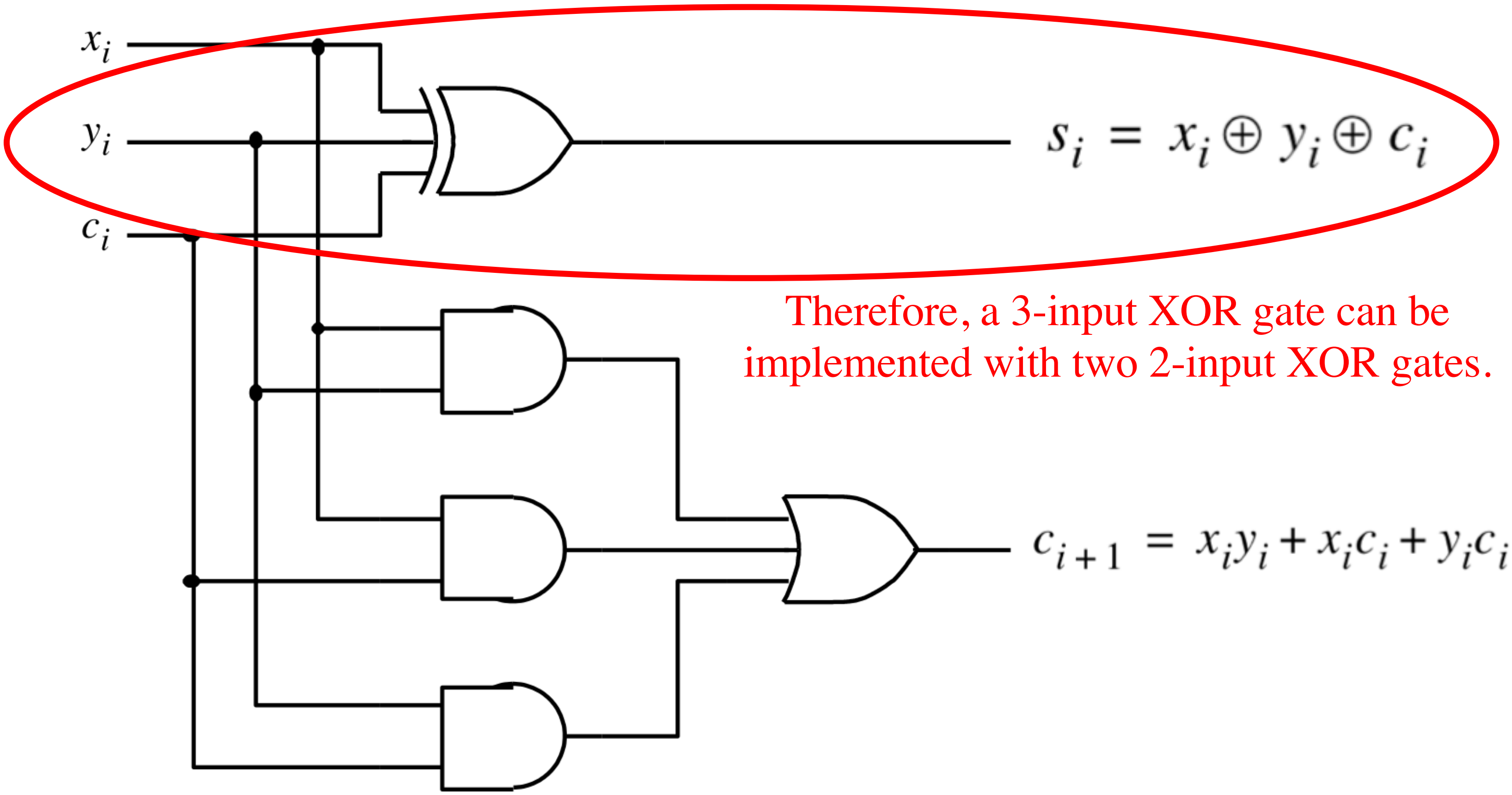
XNOR

# XOR Magic

$$\overline{x_i}\,\overline{y_i} + x_i y_i = \overline{x_i \oplus y_i}$$

You can also prove this using the theorems of Boolean algebra.
Try that at home.

# The Full-Adder Circuit

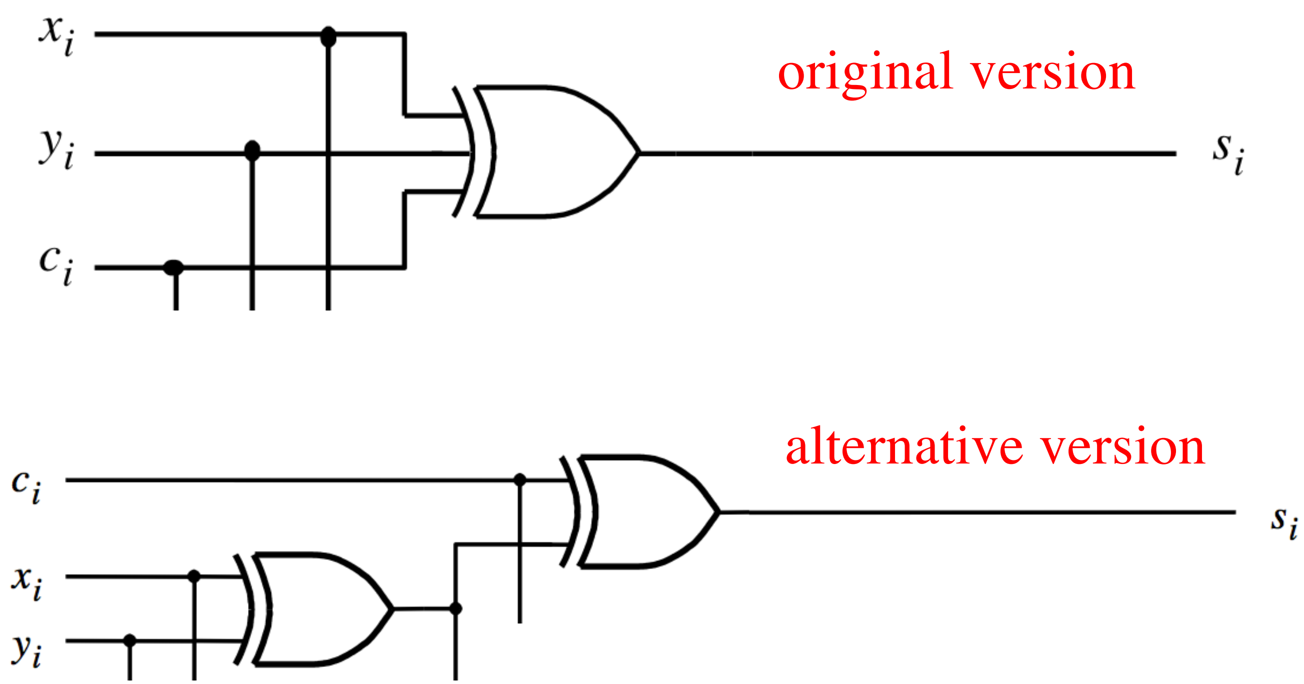



Therefore, a 3-input XOR gate can be implemented with two 2-input XOR gates.

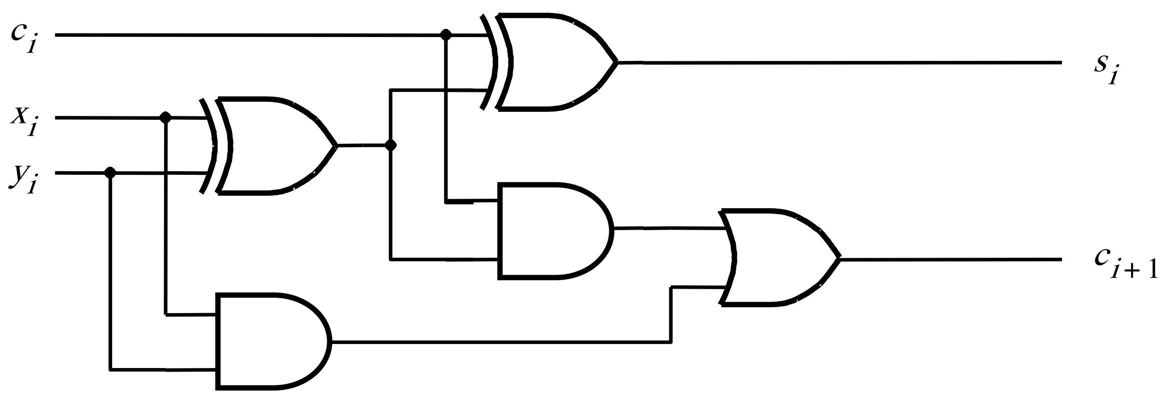
[ Figure 3.3c from the textbook ]

# $s_i$ can be implemented in two different ways

$$s_i = x_i \oplus y_i \oplus c_i$$

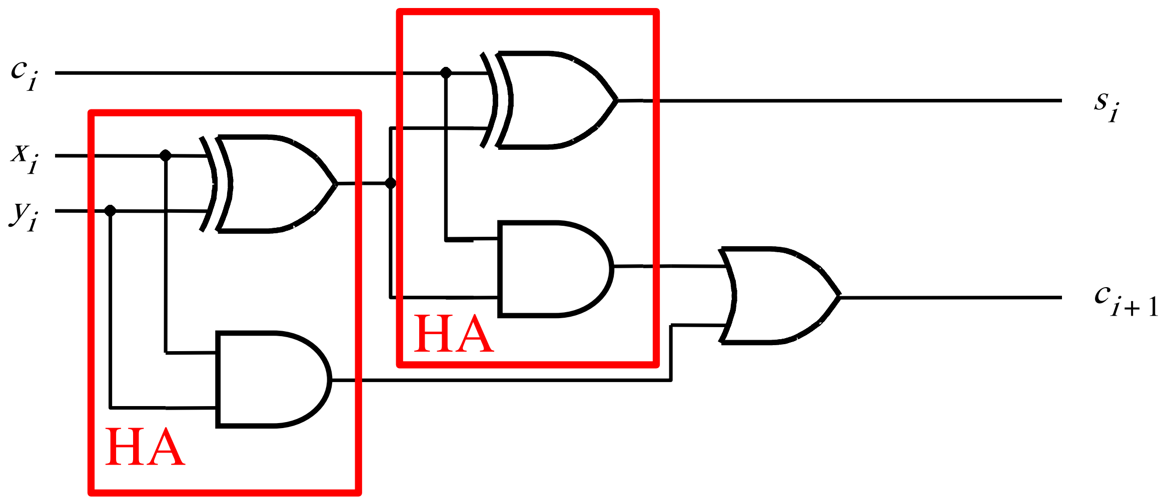


# The Full-Adder Circuit (alternative drawing)



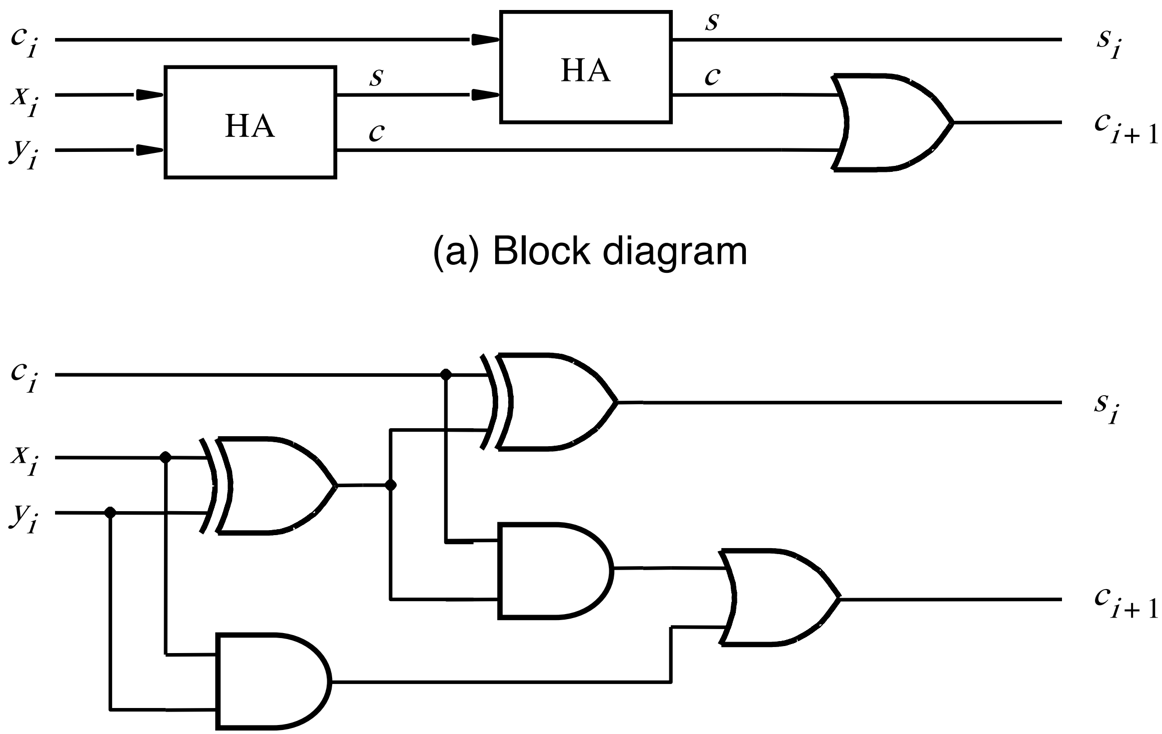

[ Figure 3.4b from the textbook ]

# The Full-Adder Circuit (alternative drawing)




[ Figure 3.4b from the textbook ]

# The Full-Adder Circuit (alternative drawing)

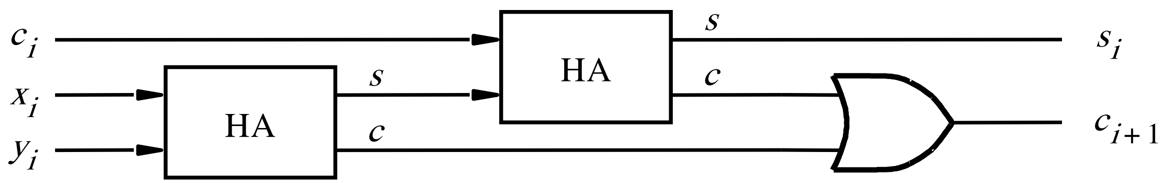



(a) Block diagram

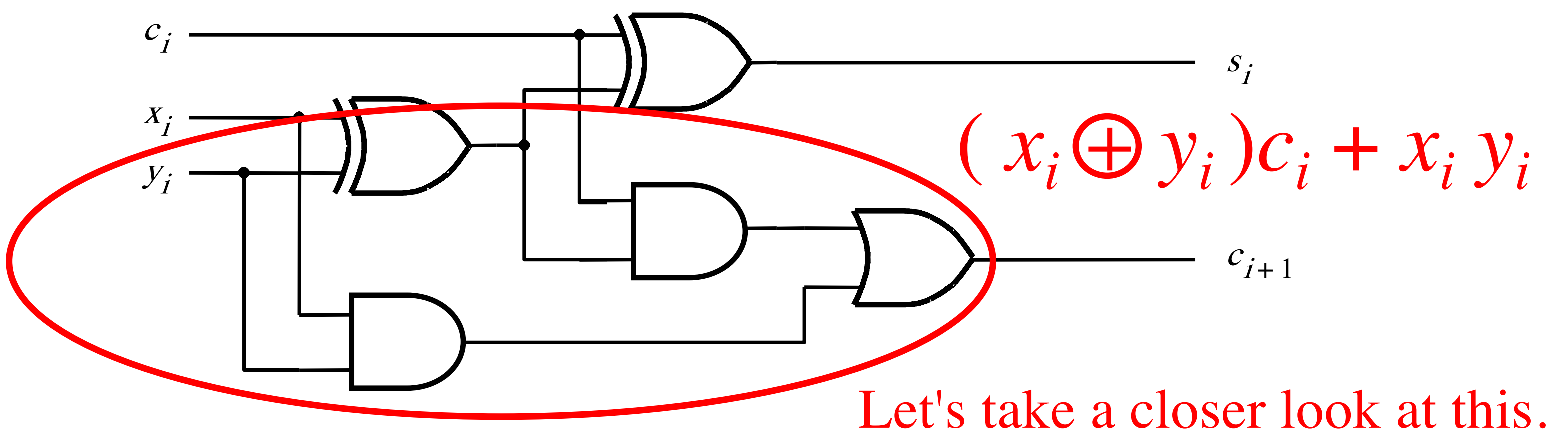
(b) Detailed diagram

[ Figure 3.4 from the textbook ]

# The Full-Adder Circuit (alternative drawing)



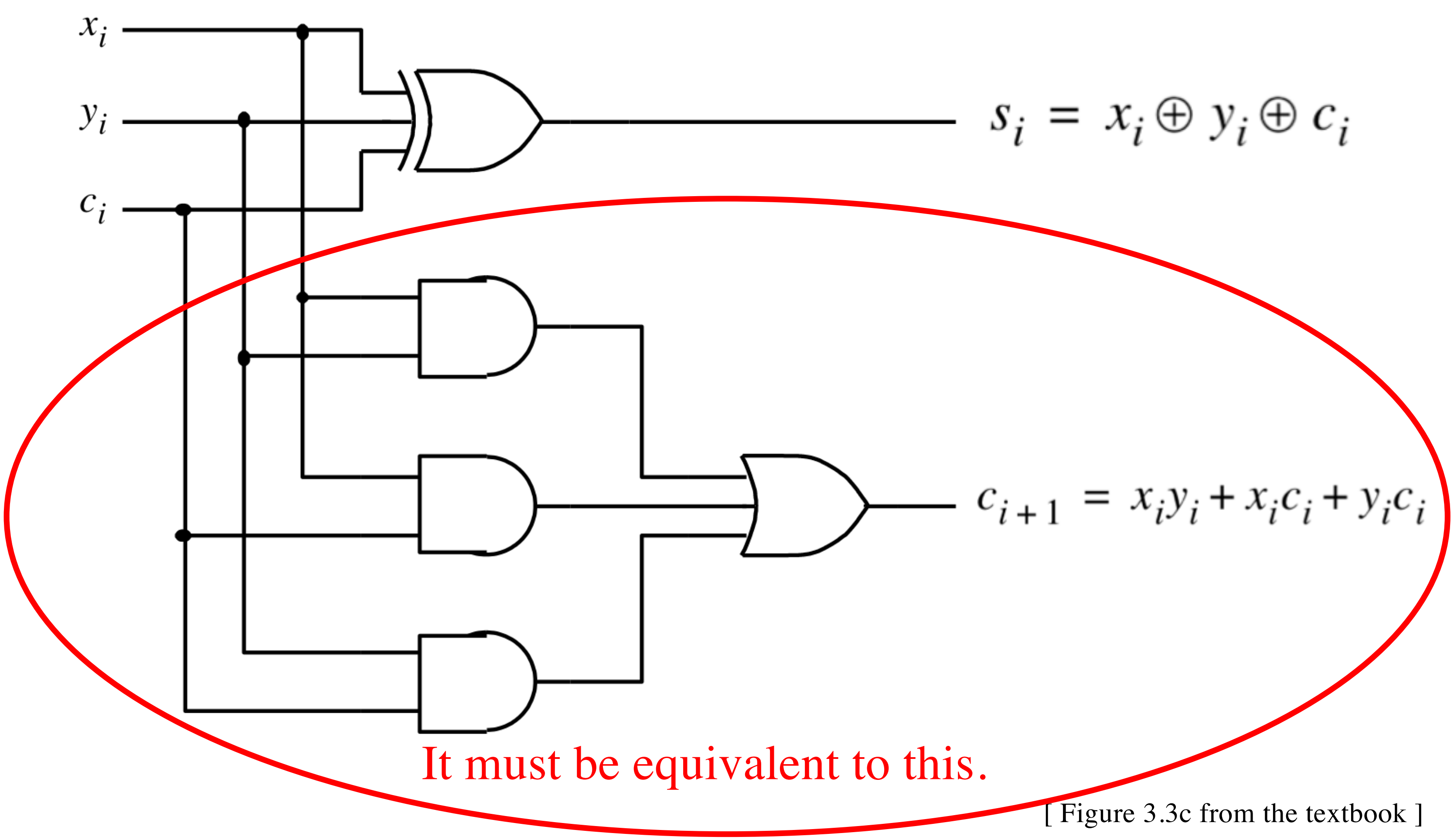

(a) Block diagram




(b) Detailed diagram

[ Figure 3.4 from the textbook ]

# The Full-Adder Circuit




[ Figure 3.3c from the textbook ]

# Let’s Prove This

$$(x_i \oplus y_i)c_i + x_i y_i \stackrel{?}{=} x_i y_i + x_i c_i + c_i y_i$$

# Let's Prove This

$$(x_i \oplus y_i)c_i + x_i y_i =$$

# Let's Prove This

$$(x_i \oplus y_i)c_i + x_i y_i = (\overline{x_i}\, y_i + x_i \overline{y_i})c_i + x_i y_i$$

# Let's Prove This

$$(x_i \oplus y_i)c_i + x_i y_i = (\overline{x_i} y_i + x_i \overline{y_i})c_i + x_i y_i$$

$$= \overline{x_i} y_i c_i + x_i \overline{y_i} c_i + x_i y_i + x_i y_i$$

double
this term

# Let's Prove This

$$(x_i \oplus y_i)c_i + x_i y_i = (\overline{x_i} y_i + x_i \overline{y_i})c_i + x_i y_i$$

$$= \overline{x_i} y_i c_i + x_i \overline{y_i} c_i + x_i y_i + x_i y_i$$

# Let's Prove This

$$(x_i \oplus y_i)c_i + x_i y_i = (\overline{x_i} y_i + x_i \overline{y_i})c_i + x_i y_i$$

$$= \overline{x_i} y_i c_i + x_i \overline{y_i} c_i + x_i y_i + x_i y_i$$

$$= (\overline{x_i} c_i + x_i) y_i + x_i (\overline{y_i} c_i + y_i)$$

# Let's Prove This

$$(x_i \oplus y_i)c_i + x_i y_i = (\overline{x_i} y_i + x_i \overline{y_i})c_i + x_i y_i$$

$$= \overline{x_i} y_i c_i + x_i \overline{y_i} c_i + x_i y_i + x_i y_i$$

$$= (\overline{x_i} c_i + x_i) y_i + x_i (\overline{y_i} c_i + y_i)$$

use Theorem 16a twice

$$= (c_i + x_i) y_i + x_i (c_i + y_i)$$

# Let's Prove This

$$(x_i \oplus y_i)c_i + x_i y_i = (\overline{x_i} y_i + x_i \overline{y_i})c_i + x_i y_i$$

$$= \overline{x_i} y_i c_i + x_i \overline{y_i} c_i + x_i y_i + x_i y_i$$

$$= (\overline{x_i} c_i + x_i) y_i + x_i (\overline{y_i} c_i + y_i)$$

$$= (c_i + x_i) y_i + x_i (c_i + y_i)$$

$$= c_i y_i + x_i y_i + x_i c_i + x_i y_i$$

# Let's Prove This

$$(x_i \oplus y_i)c_i + x_i y_i = (\overline{x_i} y_i + x_i \overline{y_i})c_i + x_i y_i$$

$$= \overline{x_i} y_i c_i + x_i \overline{y_i} c_i + x_i y_i + x_i y_i$$

$$= (\overline{x_i} c_i + x_i) y_i + x_i (\overline{y_i} c_i + y_i)$$

$$= (c_i + x_i) y_i + x_i (c_i + y_i)$$

$$= c_i y_i + x_i y_i + x_i c_i + x_i y_i$$

remove one copy of this doubled term

# Let's Prove This

$$(x_i \oplus y_i)c_i + x_i y_i = (\overline{x_i} y_i + x_i \overline{y_i})c_i + x_i y_i$$

$$= \overline{x_i} y_i c_i + x_i \overline{y_i} c_i + x_i y_i + x_i y_i$$

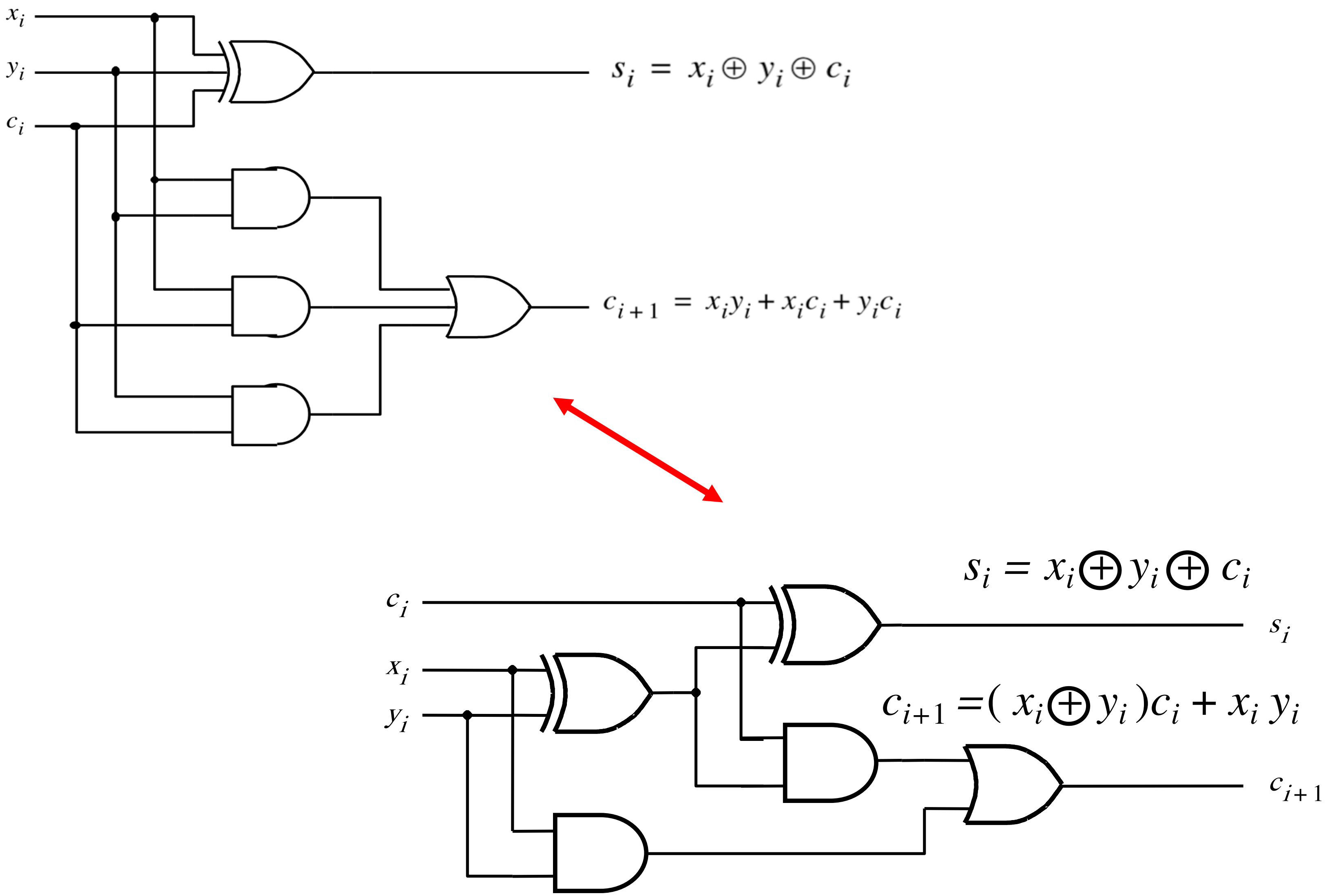
$$= (\overline{x_i} c_i + x_i) y_i + x_i (\overline{y_i} c_i + y_i)$$

$$= (c_i + x_i) y_i + x_i (c_i + y_i)$$

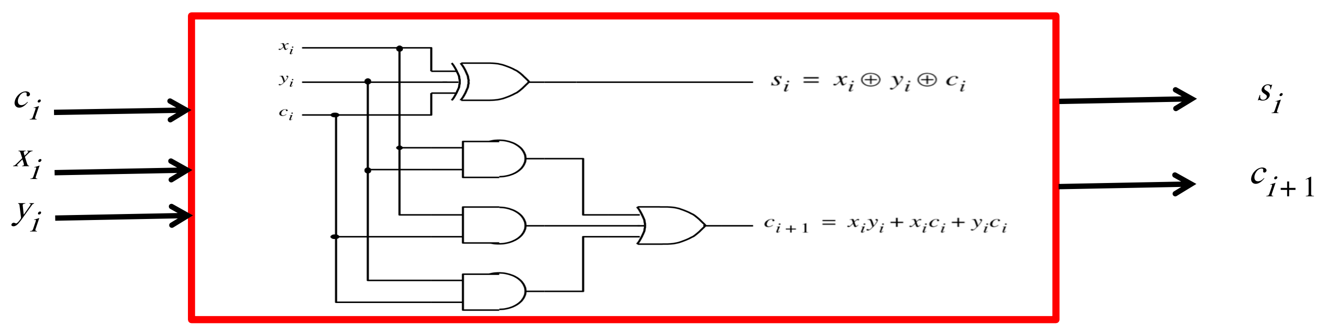
$$= c_i y_i + x_i y_i + x_i c_i + x_i y_i$$

$$= c_i y_i + x_i y_i + x_i c_i$$

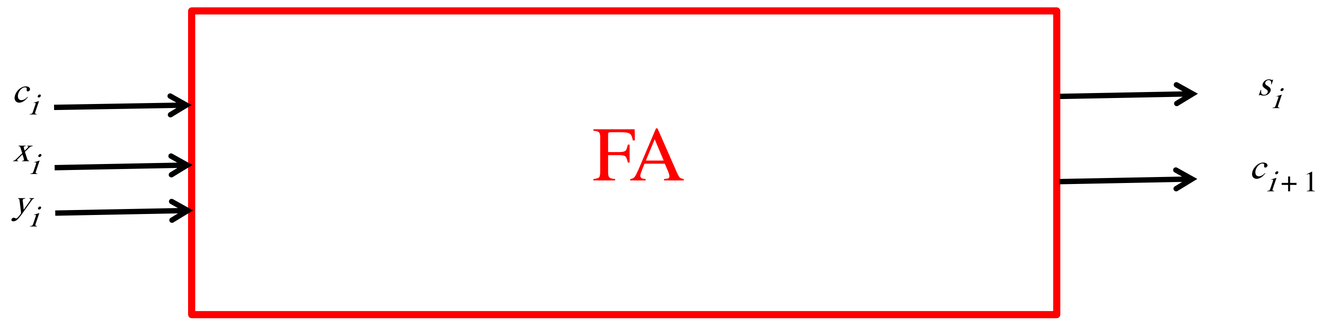
# Therefore, these circuits are equivalent



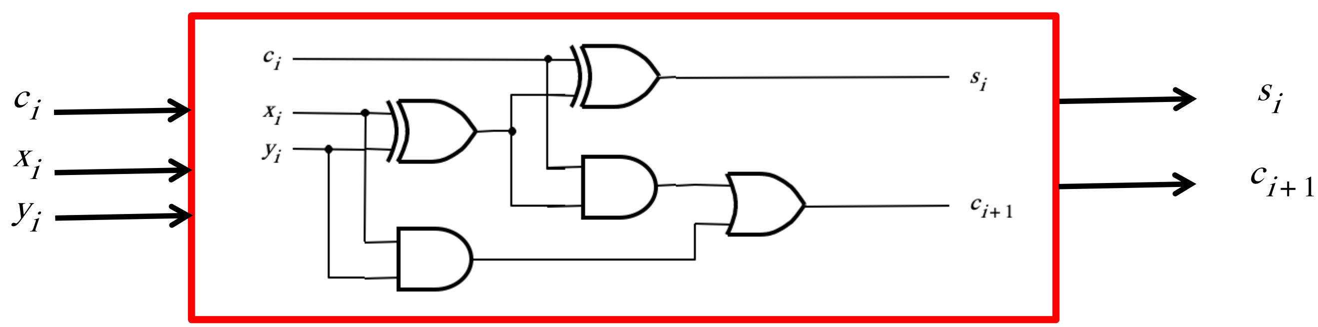
# The Full-Adder Abstraction



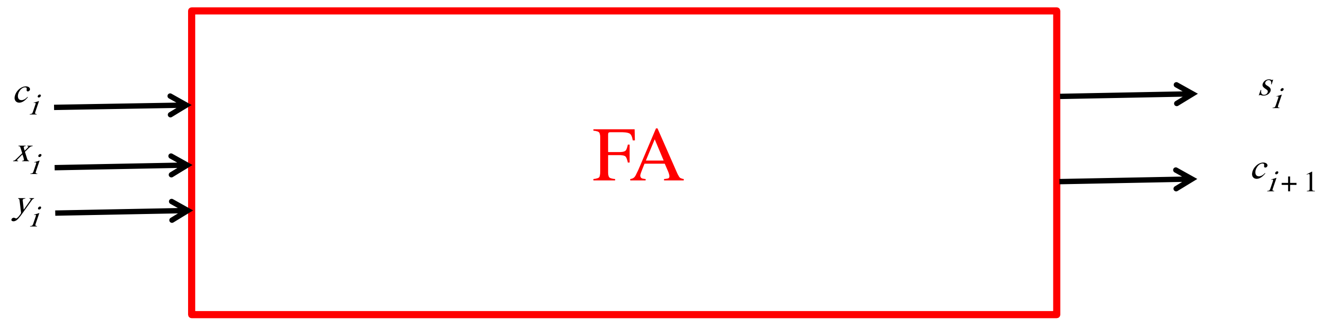
# The Full-Adder Abstraction



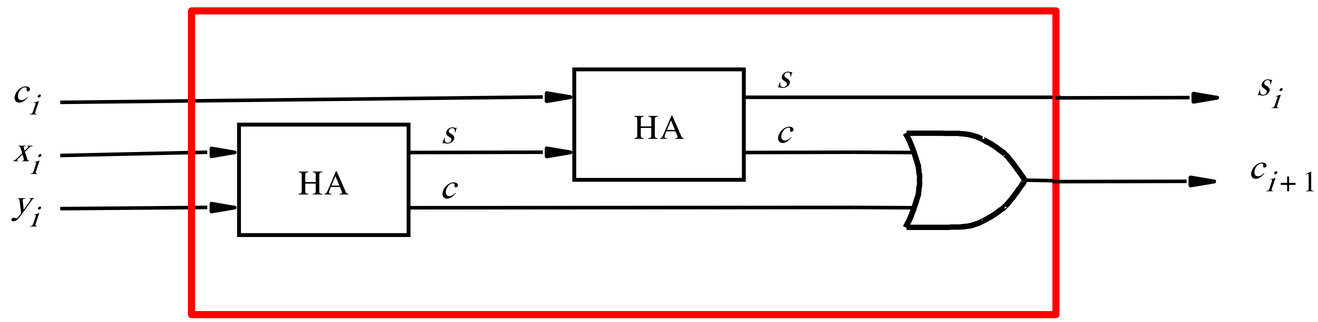
# The Full-Adder Abstraction



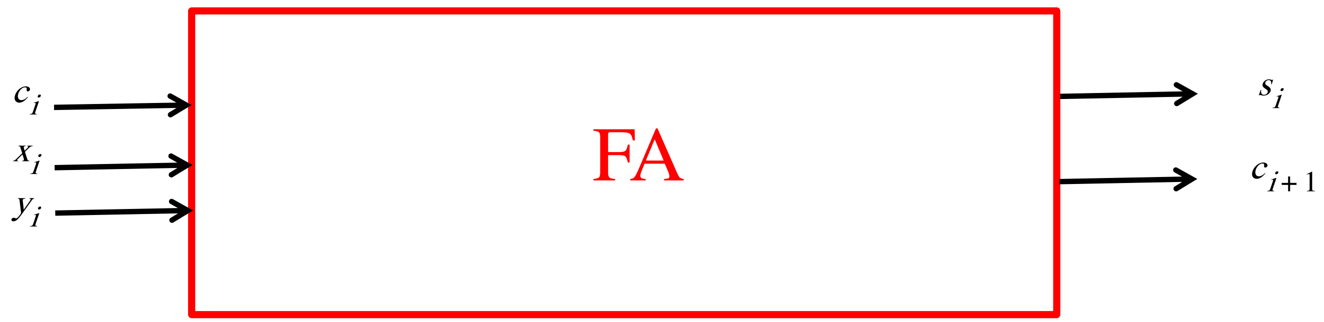
# The Full-Adder Abstraction



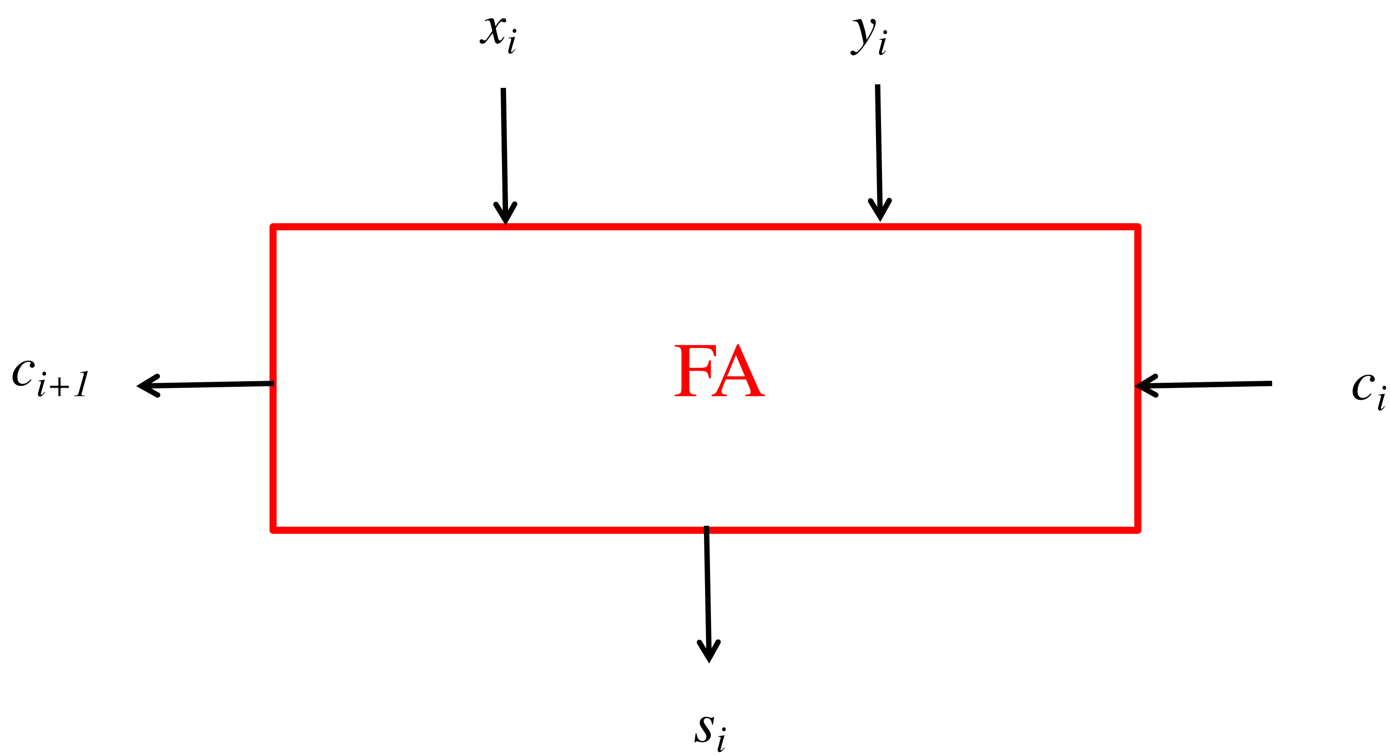
# The Full-Adder Abstraction



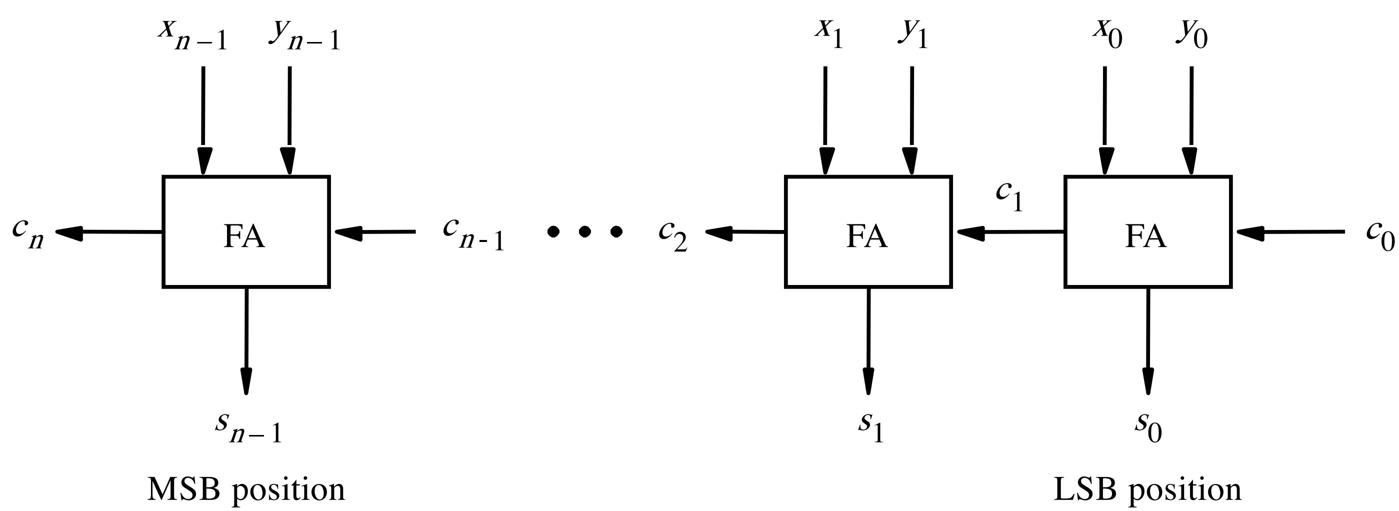
# The Full-Adder Abstraction



# We can place the arrows anywhere

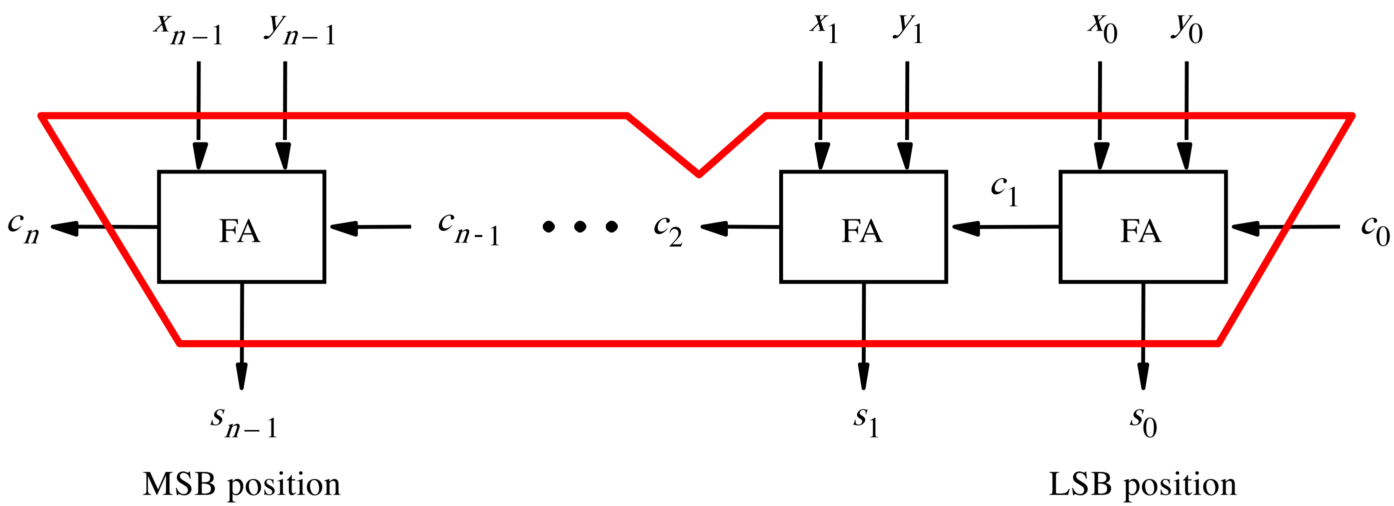


# *n*-bit ripple-carry adder

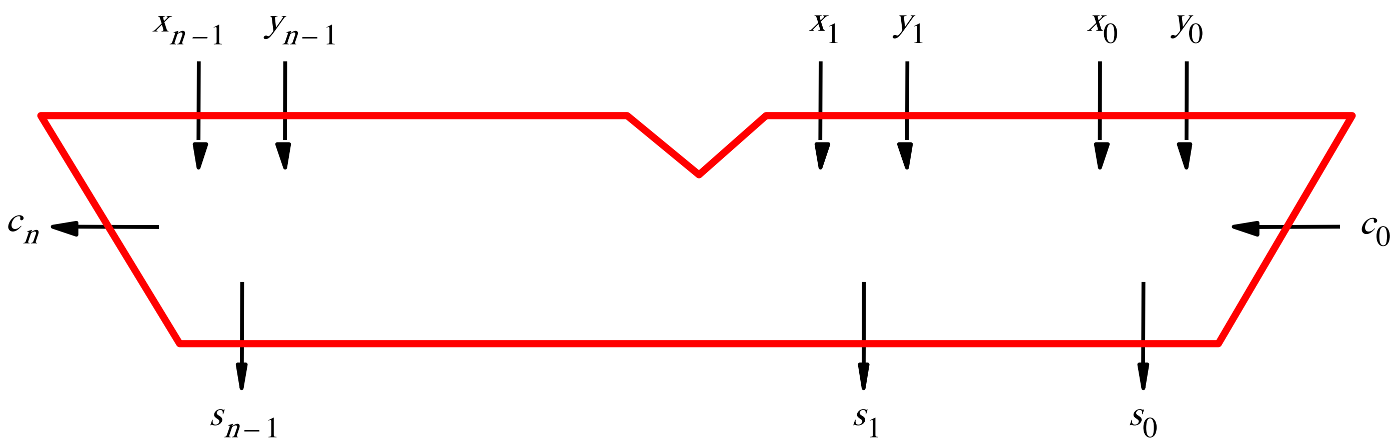



[ Figure 3.5 from the textbook ]

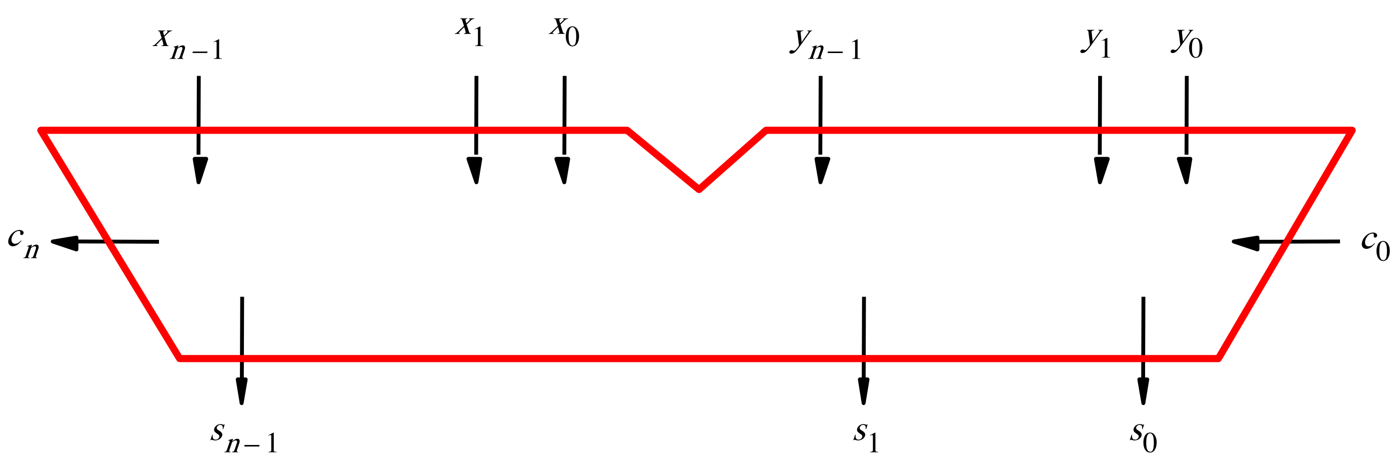
# *n*-bit ripple-carry adder abstraction



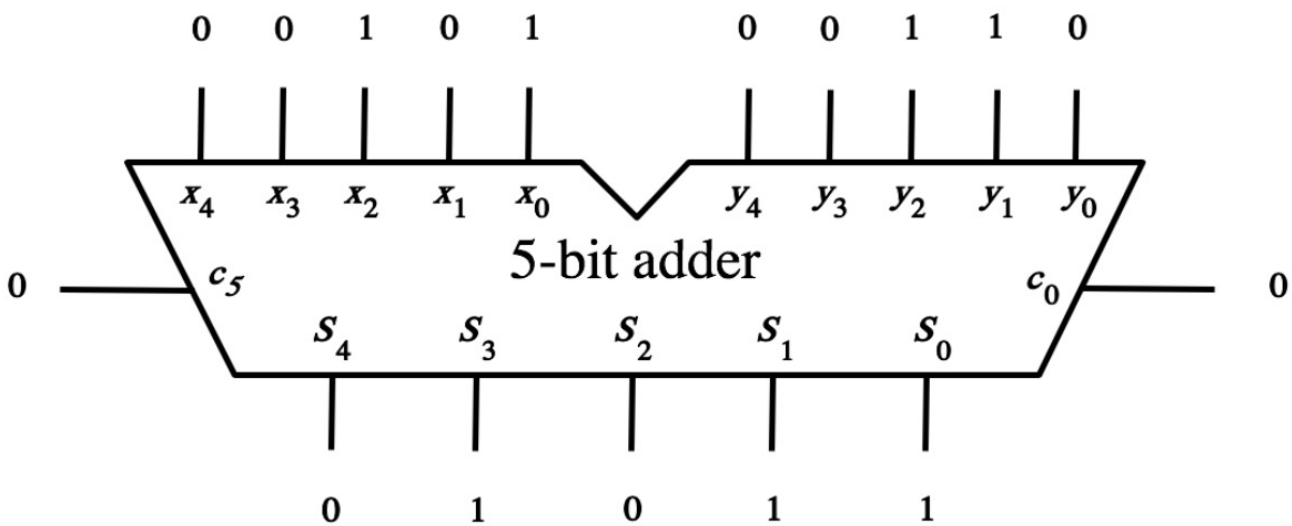
# *n*-bit ripple-carry adder abstraction



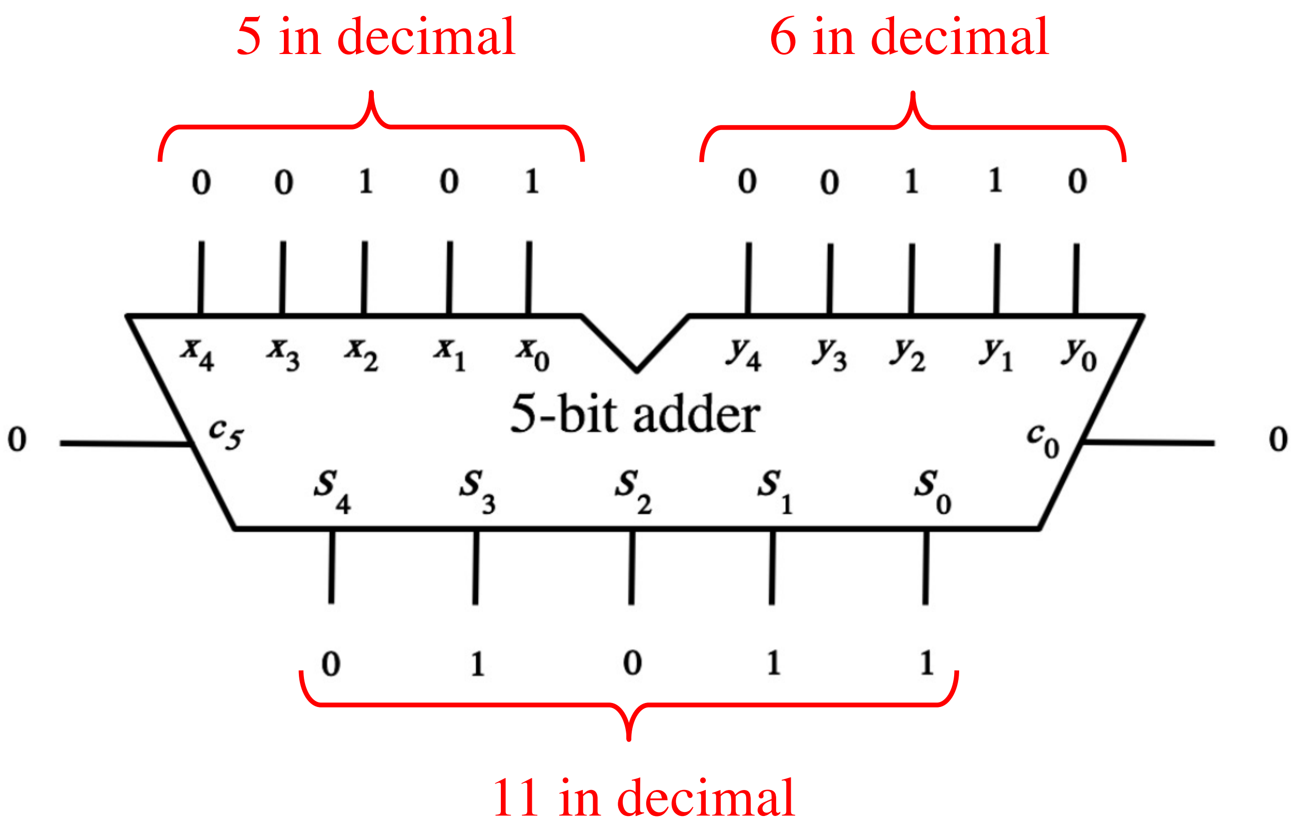
## The x and y lines are typically grouped together for better visualization, but the underlying logic remains the same



# Example:
# Computing 5+6 using a 5-bit adder



# Example:
# Computing 5+6 using a 5-bit adder

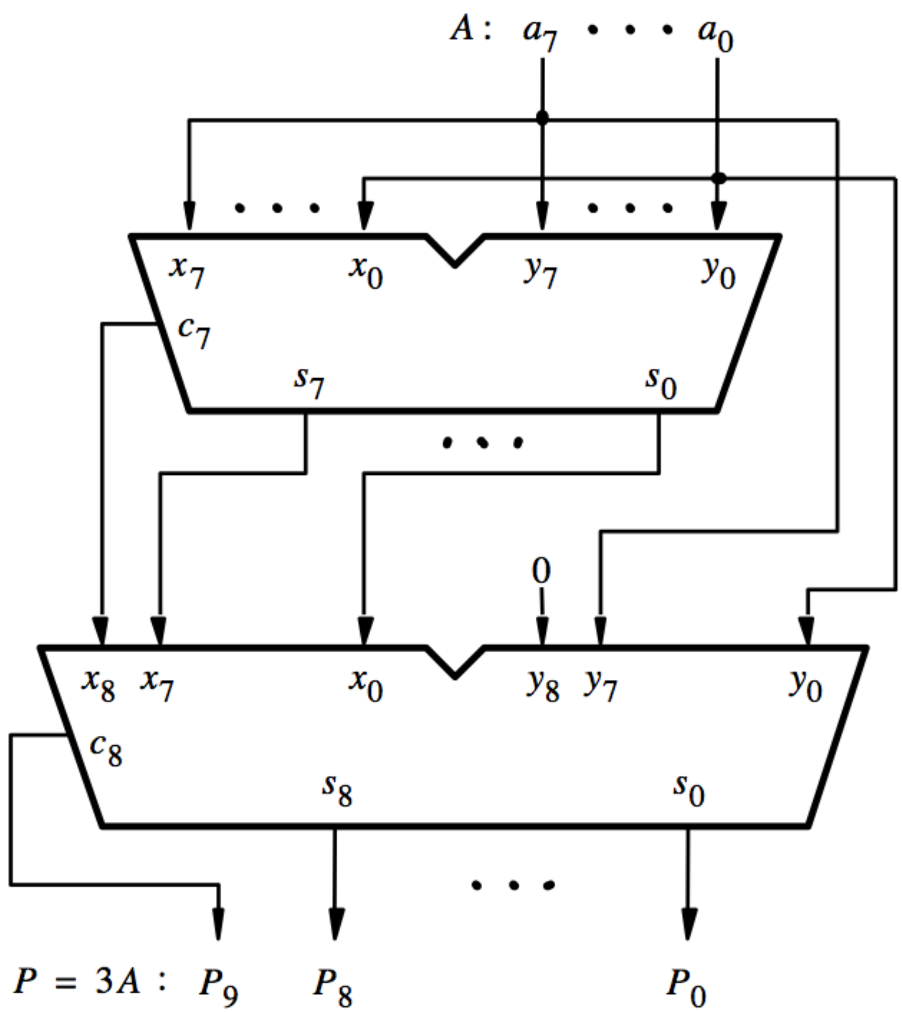


# Design Example:

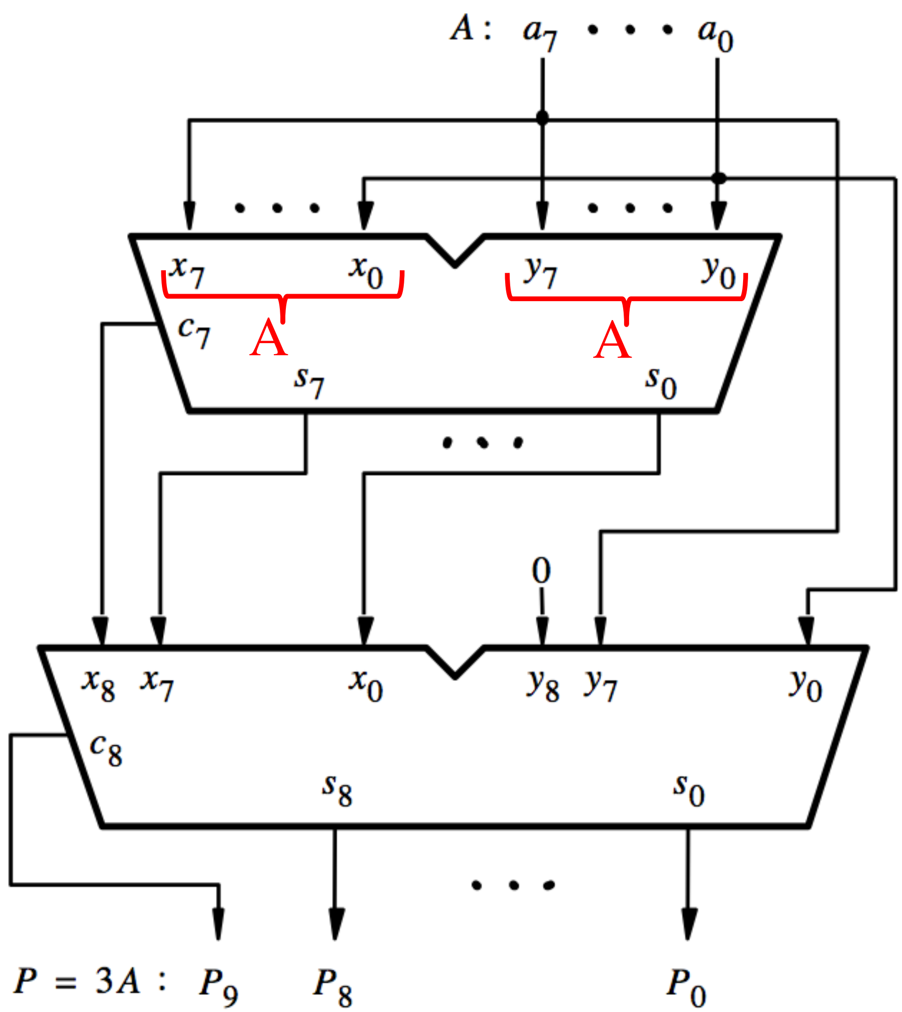
## Create a circuit that multiplies a number by 3

# How to Get 3A from A?

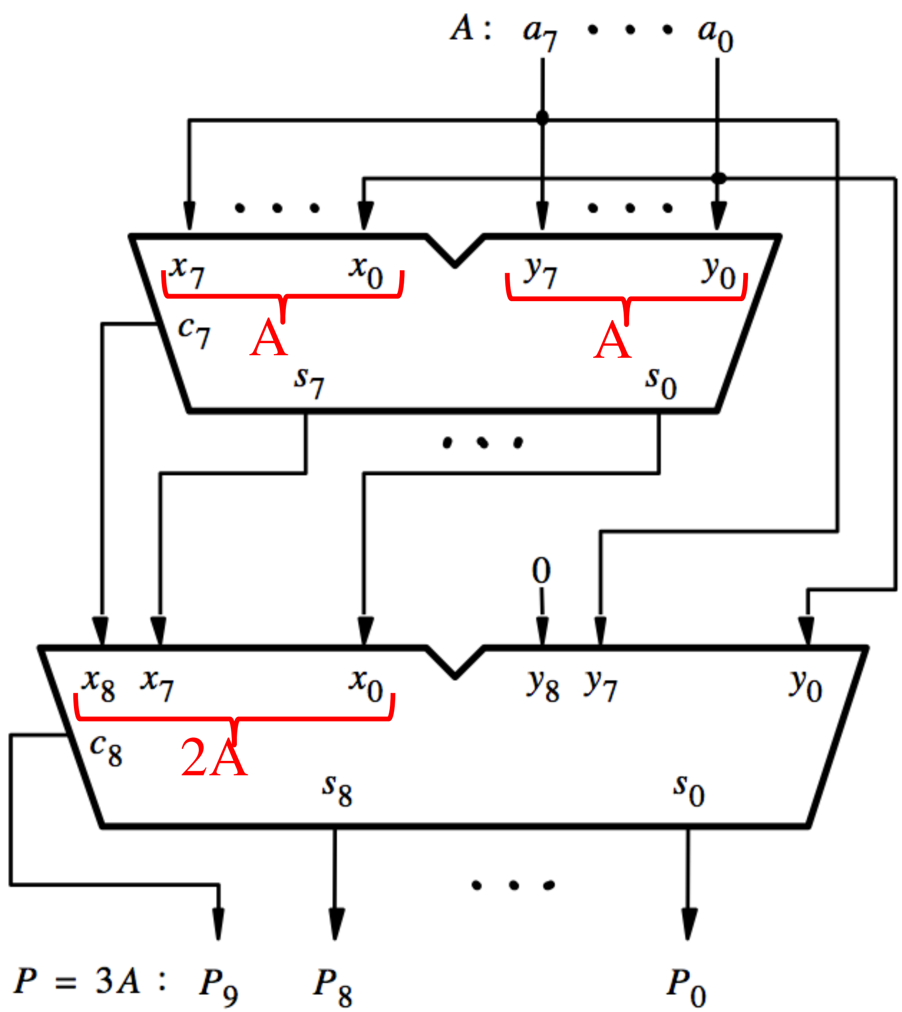
- $3A = A + A + A$
- $3A = (A+A) + A$
- $3A = 2A + A$



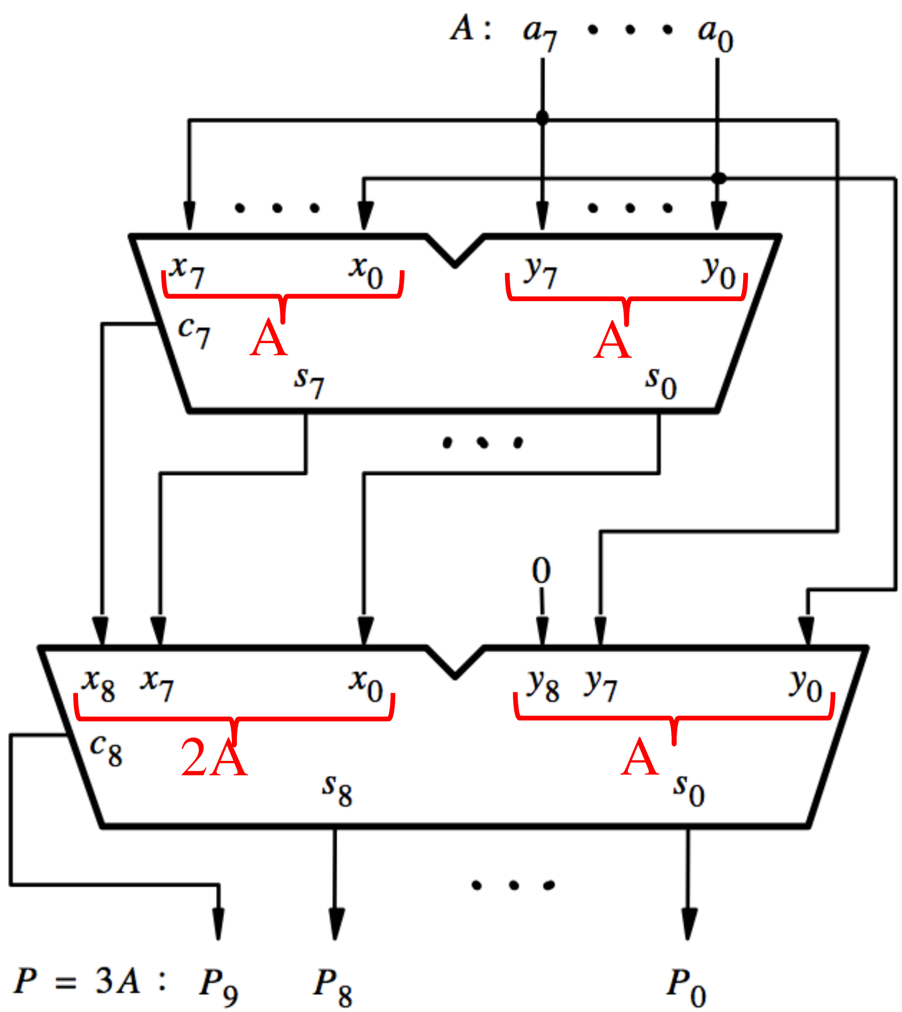

[ Figure 3.6a from the textbook ]



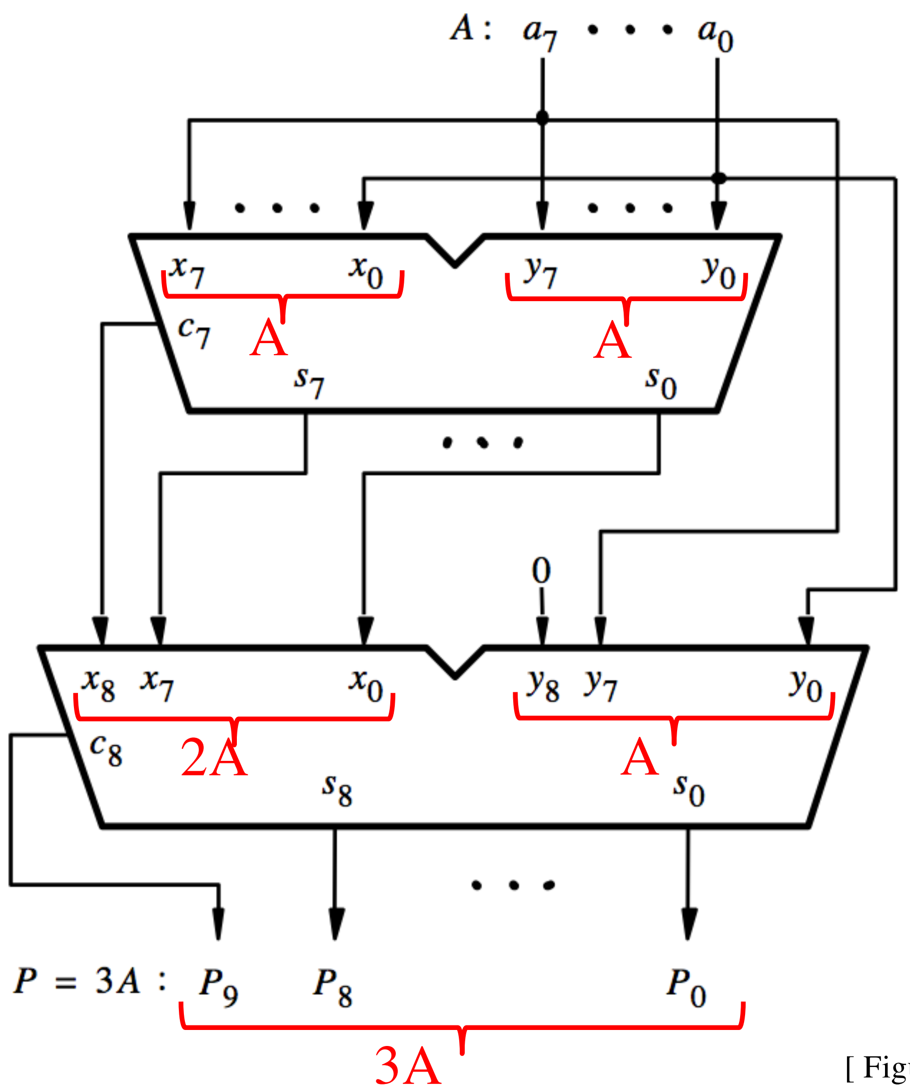

[ Figure 3.6a from the textbook ]




[ Figure 3.6a from the textbook ]




[ Figure 3.6a from the textbook ]




[ Figure 3.6a from the textbook ]

# Decimal Multiplication by 10

**What happens when we multiply a number by 10?**

**4 x 10 = ?**

**542 x 10 = ?**

**1245 x 10 = ?**

# Decimal Multiplication by 10

**What happens when we multiply a number by 10?**

**4 x 10 = 40**

**542 x 10 = 5420**

**1245 x 10 = 12450**

# Decimal Multiplication by 10

**What happens when we multiply a number by 10?**

**4 x 10 = 40**

**542 x 10 = 5420**

**1245 x 10 = 12450**

**You simply add a zero as the rightmost number**

# Binary Multiplication by 2

What happens when we multiply a number by 2?

011 times 2 = ?

101 times 2 = ?

110011 times 2 = ?

# Binary Multiplication by 2

**What happens when we multiply a number by 2?**

**011 times 2 = 0110**

**101 times 2 = 1010**

**110011 times 2 = 1100110**

# Binary Multiplication by 2

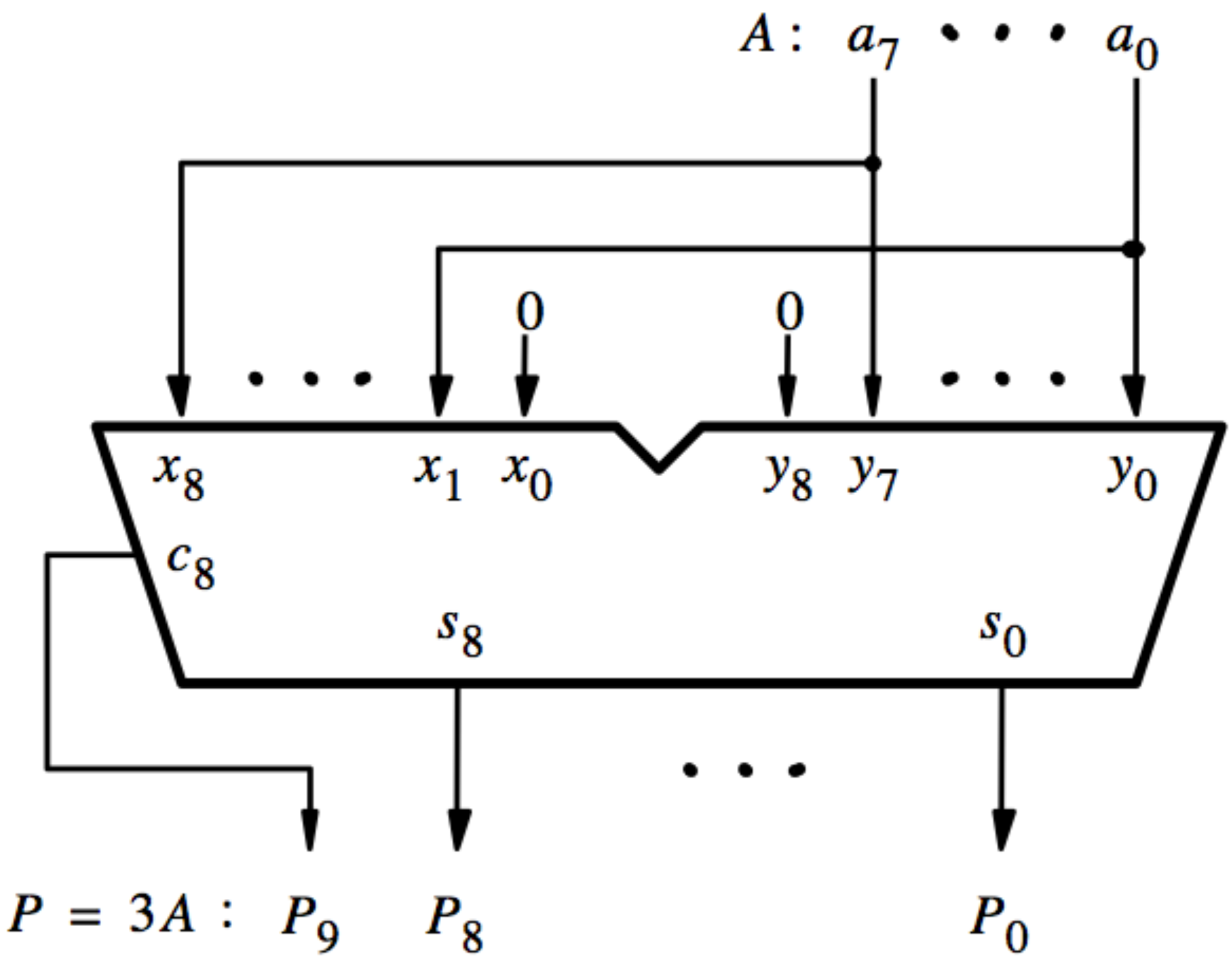
**What happens when we multiply a number by 2?**

**011 times 2 = 0110**

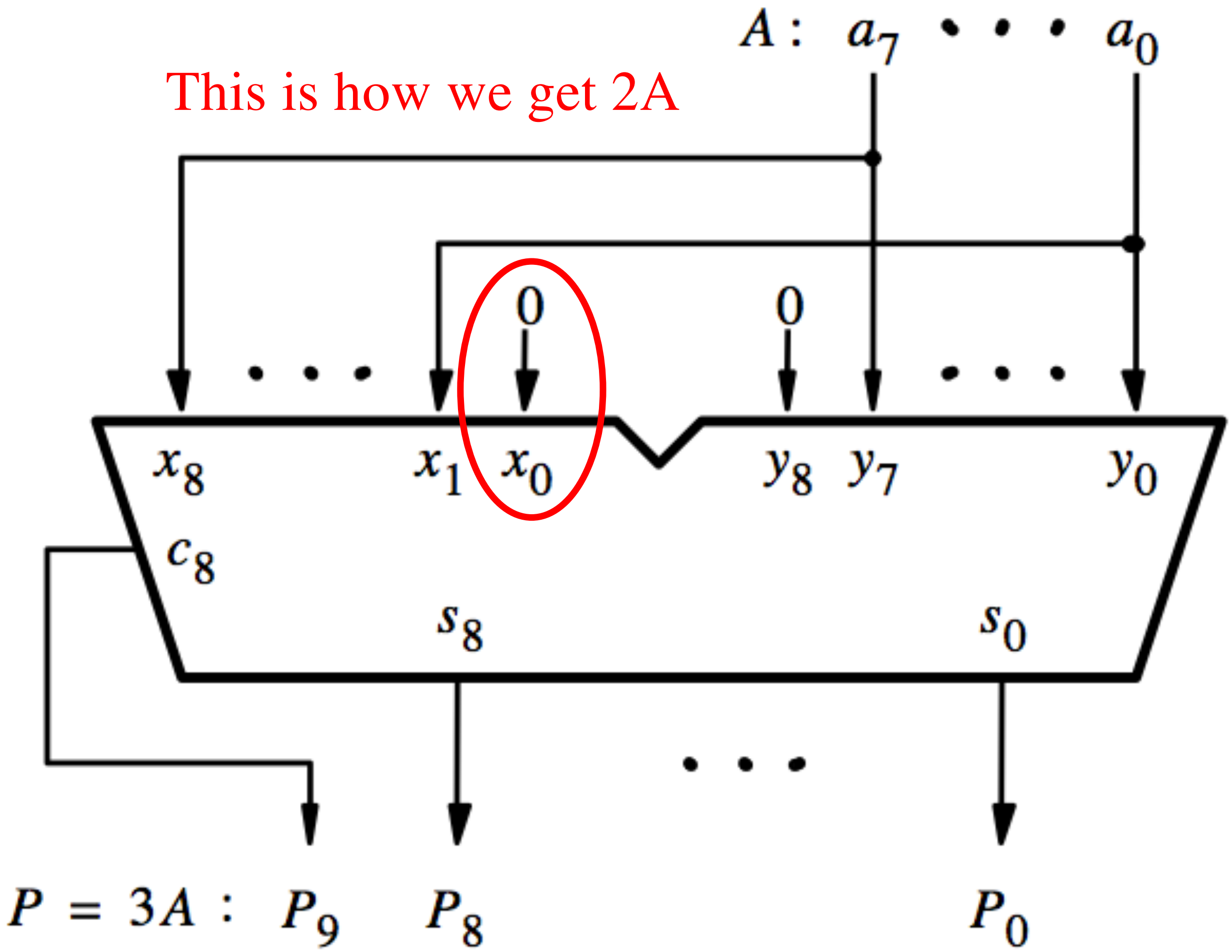
**101 times 2 = 1010**

**110011 times 2 = 1100110**

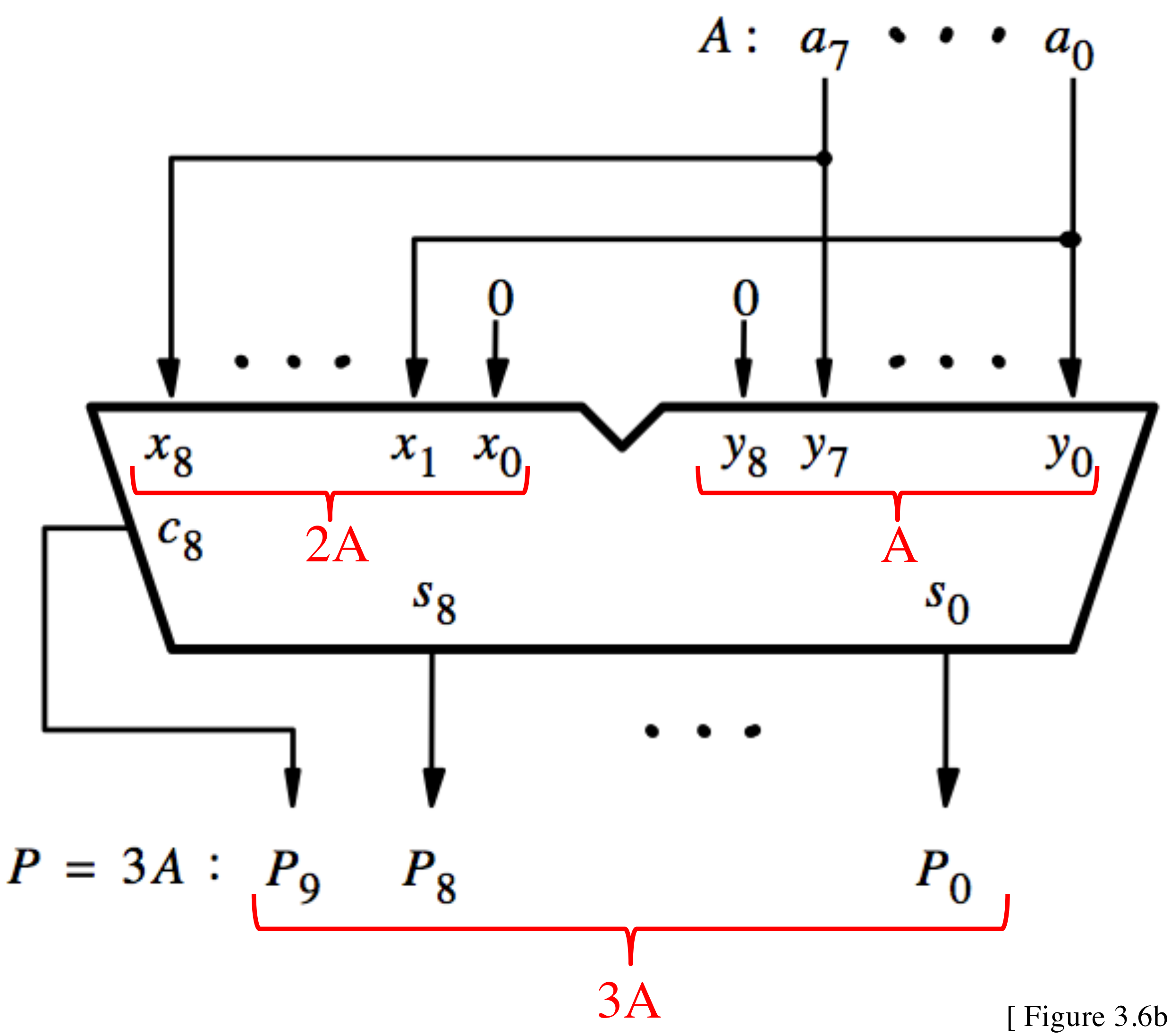
**You simply add a zero as the rightmost number**




[ Figure 3.6b from the textbook ]




[ Figure 3.6b from the textbook ]




[ Figure 3.6b from the textbook ]

# Questions?

**THE END**