

# **CprE 281: Digital Logic**

**Instructor: Alexander Stoytchev**

**<http://www.ece.iastate.edu/~alexs/classes/>**

# Karnaugh Maps

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Iowa State University, Ames, IA  
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# **Administrative Stuff**

- **HW3 is due today**

# **Administrative Stuff**

- **HW4 is out**
- **It is due on Monday Sep 23 @ 10 pm**
- **It is posted on the class web page**
- **I also sent you an e-mail with the link.**

# TA Office Hours

- **Hanif Lashari: Mondays 2:10 – 3:10pm**
- **Le Zhang: Wednesdays 11 am – 1pm**
- **Himani Kohli: Thursdays 9 – 10 am**

**Go to the Transformative Learning Area (TLA) on the first floor in Coover Hall. Look for a sign that says “CprE 2810 TA.”**

# **Administrative Stuff**

- **Midterm Exam #1**
- **When: Friday Sep 27.**
- **What: Chapter 1 and Chapter 2 plus number systems**
- **The exam will be closed book but open notes (you can bring up to 3 pages of handwritten notes).**
- **Sample exams are posted on the class web page.**
- **More details to follow.**

# **Quick Review**

# Do You Still Remember This Boolean Algebra Theorem?

14a.  $x \cdot y + x \cdot \bar{y} = x$

Combining

14b.  $(x + y) \cdot (x + \bar{y}) = x$



Let's prove 14.a

$x$	$y$	$\mathbf{x \cdot y + x \cdot \bar{y} = x}$
0	0	
0	1	
1	0	
1	1	

Let's prove 14.a

$x$	$y$	$\mathbf{x \cdot y + x \cdot \bar{y} = x}$
0	0	0
0	1	0
1	0	0
1	1	1

Let's prove 14.a

$x$	$y$	$\mathbf{x} \cdot \mathbf{y} + \mathbf{x} \cdot \overline{\mathbf{y}} = \mathbf{x}$
0	0	0
0	1	0
1	0	1
1	1	0

Let's prove 14.a

$x$	$y$	$\mathbf{x} \cdot \mathbf{y} + \mathbf{x} \cdot \overline{\mathbf{y}} = \mathbf{x}$
0	0	0 0 0
0	1	0 0 0
1	0	0 1 1
1	1	1 1 0

Let's prove 14.a

$x$	$y$	$\mathbf{x} \cdot \mathbf{y} + \mathbf{x} \cdot \overline{\mathbf{y}} = \mathbf{x}$
0	0	0 0 0 0
0	1	0 0 0 0
1	0	0 1 1 1
1	1	1 1 0 1

# Let's prove 14.a

$x$	$y$	$\mathbf{x \cdot y + x \cdot \bar{y} = x}$
0	0	0 0 0 0
0	1	0 0 0 0
1	0	0 1 1 1
1	1	1 1 0 1

They are equal.



# Motivation

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

**An approach for simplifying logic expressions.**

**How do we guarantee that we have reached the minimum SOP/POS representation?**



# **This method was described in 1953**

**M. Karnaugh, “The map method for synthesis of combinational logic circuits”**

**Transactions of the American Institute of Electrical Engineers, Part I: Communication and Electronics (pages 593 – 599, Volume: 72, Issue: 5, Nov. 1953)**

**<https://ieeexplore.ieee.org/document/6371932>**

# **Two-Variable K-Map**

# Karnaugh Map (K-map)

- A visual representation of the function
- Same information as the truth table
- Easier to group minterms

$x_1$	$x_2$	$f$
0	0	0
0	1	1
1	0	0
1	1	1

(a) Truth table

$x_2$	$x_1$	0	1
0		0	0
1		1	1

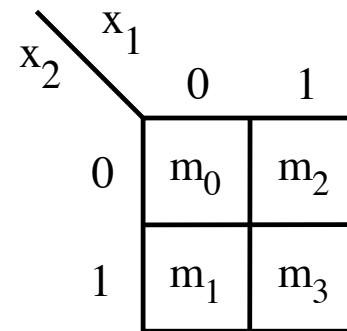
(b) Karnaugh map

# Karnaugh Map (K-map)

- A visual representation of the function
- Same information as the truth table
- Easier to group minterms

$x_1$	$x_2$	$f$
0	0	$m_0$
0	1	$m_1$
1	0	$m_2$
1	1	$m_3$

(a) Truth table



(b) Karnaugh map

# Minterms

$x_1$	$x_2$	f
0	0	$m_0$
0	1	$m_1$
1	0	$m_2$
1	1	$m_3$

$x_1$	$x_2$	$m_0$	$m_1$	$m_2$	$m_3$
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

# Minterm Addition Example

$x_1$	$x_2$	f
0	0	0
0	1	1
1	0	0
1	1	1

$x_1$	$x_2$	$m_0$	$m_1$	$m_2$	$m_3$	$m_1 + m_3$
0	0	1	0	0	0	0
0	1	0	1	0	0	1
1	0	0	0	1	0	0
1	1	0	0	0	1	1

# Minterm Addition Example

$x_1$	$x_2$	f
0	0	0
0	1	1
1	0	0
1	1	1

$x_1$	$x_2$	$m_0$	$m_1$	$m_2$	$m_3$	$m_1 + m_3$
0	0	1	0	0	0	0
0	1	0	1	0	0	1
1	0	0	0	1	0	0
1	1	0	0	0	1	1

$$\bar{x}_1x_2 + x_1x_2 = x_2$$

# Minterm Addition Example

$x_1$	$x_2$	$f$
0	0	0
0	1	1
1	0	0
1	1	1

$x_2 \backslash x_1$	0	1
0	0	0
1	1	1

$$\bar{x}_1x_2 + x_1x_2 = x_2$$



# Minterm Addition Example

$x_1$	$x_2$	$f$
0	0	0
0	1	1
1	0	0
1	1	1

$x_2 \backslash x_1$	0	1
0	0	0
1	1	1

$x_2$

$$\bar{x}_1x_2 + x_1x_2 = x_2$$

# Another Grouping Example

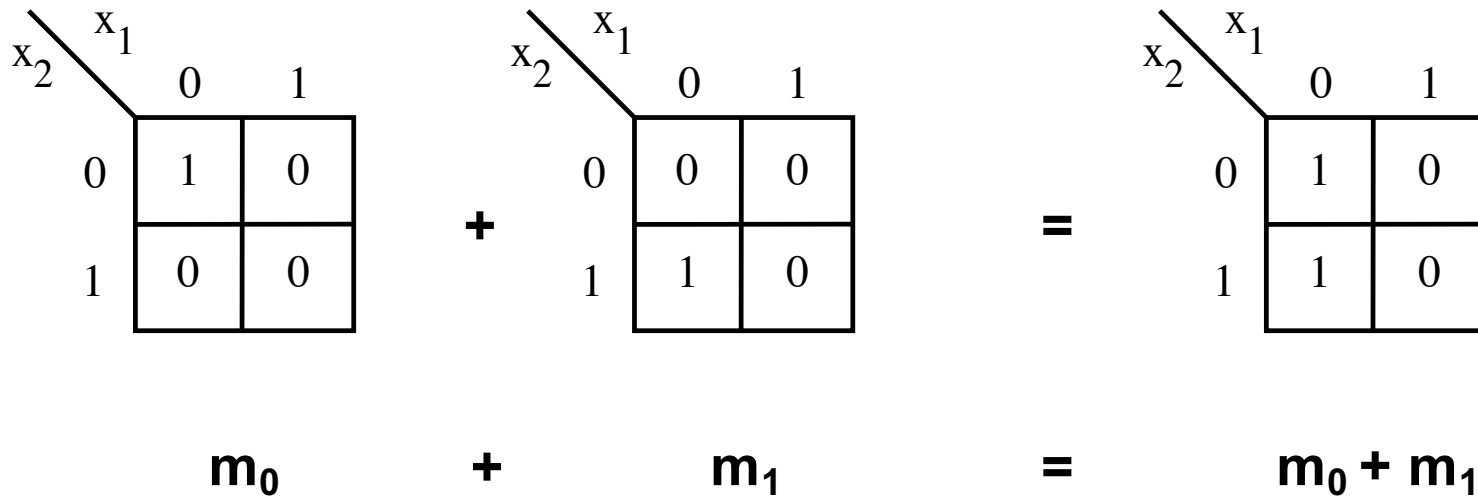
	$x_1$	0	1
$x_2$			
0		1	0
1		0	0

$m_0$

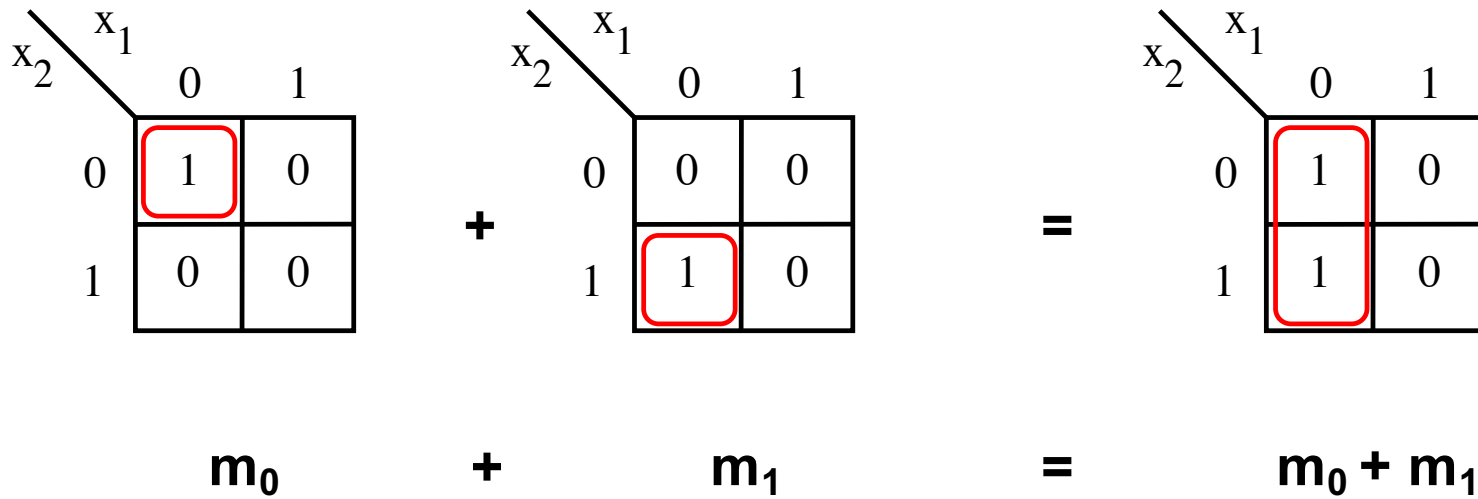
	$x_1$	0	1
$x_2$			
0		0	0
1		1	0

$m_1$

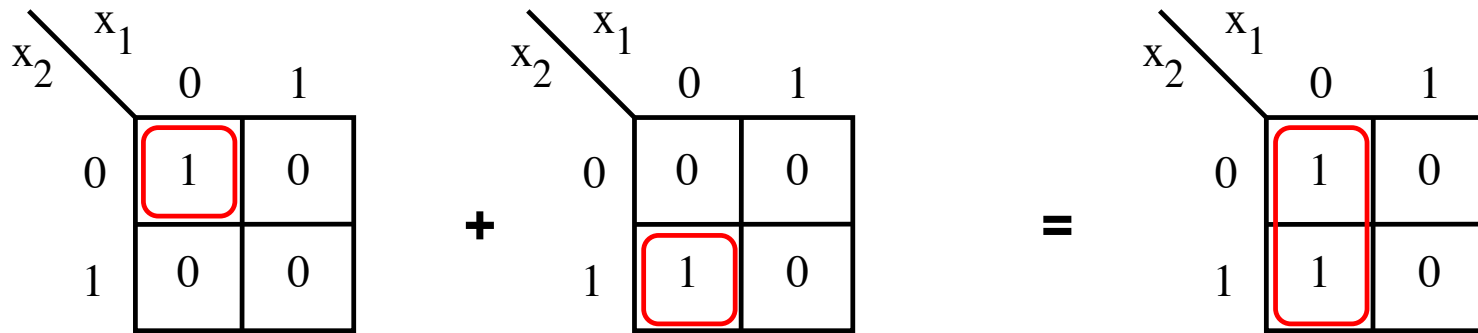
# Another Grouping Example



# Another Grouping Example



# Another Grouping Example



$m_0$

+

$m_1$

=

$m_0 + m_1$

$\bar{x}_1\bar{x}_2$

+

$\bar{x}_1x_2$

=

$\bar{x}_1$

Property 14a (Combining)

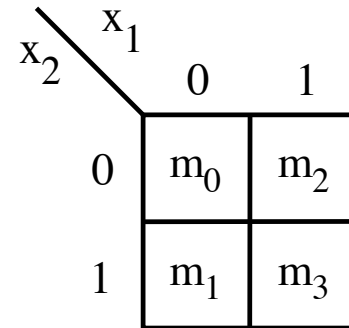
# Grouping Rules

- **Group “1”s with rectangles**
- **Both sides must be a power of 2:**
  - 1x1, 1x2, 2x1, 2x2, 1x4, 4x1, 2x4, 4x2, 4x4
- **Can use/cover the same minterm more than once**
- **Can wrap around the edges of the map**
- **Some rules in selecting groups:**
  - Try to use as few groups as possible to cover all “1”s.
  - For each group, try to make it as large as you can (i.e., if you can use a 2x2, don't use a 2x1 even if that is enough).

# Two-Variable K-map

$x_1$	$x_2$	$f$
0	0	$m_0$
0	1	$m_1$
1	0	$m_2$
1	1	$m_3$

(a) Truth table



(b) Karnaugh map

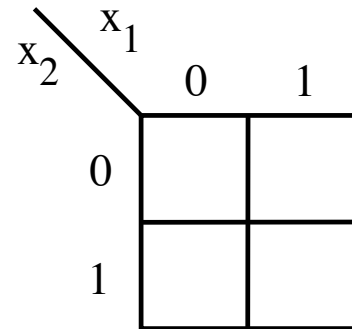
# Step-By-Step Example

$x_1$	$x_2$	$f$
0	0	1
0	1	1
1	0	0
1	1	1



# 1. Draw The Map

$x_1$	$x_2$	$f$
0	0	1
0	1	1
1	0	0
1	1	1



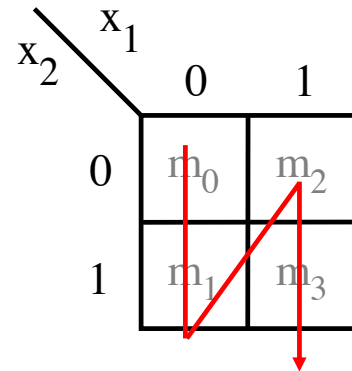
## 2. Fill The Map

	$x_1$	$x_2$	$f$
$m_0$	0	0	1
$m_1$	0	1	1
$m_2$	1	0	0
$m_3$	1	1	1

$x_2$	$x_1$	0	1
0		$m_0$	$m_2$
1		$m_1$	$m_3$

## 2. Fill The Map

	$x_1$	$x_2$	$f$
$m_0$	0	0	1
$m_1$	0	1	1
$m_2$	1	0	0
$m_3$	1	1	1



## 2. Fill The Map

	$x_1$	$x_2$	$f$
$m_0$	0	0	1
$m_1$	0	1	1
$m_2$	1	0	0
$m_3$	1	1	1

$x_2$	$x_1$	0	1
0		1	0
1		1	1

# 3. Group

	$x_1$	$x_2$	$f$
$m_0$	0	0	1
$m_1$	0	1	1
$m_2$	1	0	0
$m_3$	1	1	1

$x_2 \backslash x_1$	0	1
0	1	0
1	1	1

# 3. Group

	$x_1$	$x_2$	$f$
$m_0$	0	0	1
$m_1$	0	1	1
$m_2$	1	0	0
$m_3$	1	1	1

$x_2 \backslash x_1$	0	1
0	1	0
1	1	1

# 3. Group

	$x_1$	$x_2$	$f$
$m_0$	0	0	1
$m_1$	0	1	1
$m_2$	1	0	0
$m_3$	1	1	1

A Karnaugh map for the function  $f(x_1, x_2)$ . The horizontal axis is labeled  $x_1$  with values 0 and 1. The vertical axis is labeled  $x_2$  with values 0 and 1. The map contains the following values:

$x_2 \backslash x_1$	0	1
0	1	0
1	1	1

The prime implicants are highlighted with colored boxes:

- A red box highlights the cells (0,0) and (0,1), representing the prime implicant  $\bar{x}_1$ .
- A green box highlights the cells (1,0) and (1,1), representing the prime implicant  $x_1$ .

## 4. Write The Expression

$x_1$	$x_2$	$f$
0	0	1
0	1	1
1	0	0
1	1	1

$x_2 \backslash x_1$	0	1
0	1	0
1	1	1



## 4. Write The Expression

$x_1$	$x_2$	$f$
0	0	1
0	1	1
1	0	0
1	1	1

$x_2 \backslash x_1$	0	1
0	1	0
1	1	1

$$\bar{x}_1 + x_2$$

# Writing The Expression

- Find which variable is constant

		$x_1$	
		0	1
$x_2$	0	1	0
	1	1	0

$$\bar{x}_1$$

# Writing The Expression

- Find which variable is constant

		$x_1$	
	$x_2$		
		0	1
0		0	1
1		0	1

$x_1$

# Writing The Expression

- Find which variable is constant

		$x_1$	
	$x_2$		
		0	1
0		1	1
1		0	0

$\overline{x_2}$

# Writing The Expression

- Find which variable is constant

		$x_1$	
	$x_2$		
		0	1
0		0	0
1		1	1

$x_2$



# These are all valid groupings

	$x_1$	0	1
$x_2$	0	1	0
	1	0	0

$$\bar{x}_1 \bar{x}_2$$

	$x_1$	0	1
$x_2$	0	0	0
	1	1	0

$$\bar{x}_1 x_2$$

	$x_1$	0	1
$x_2$	0	0	1
	1	0	0

$$x_1 \bar{x}_2$$

	$x_1$	0	1
$x_2$	0	0	0
	1	0	1

$$x_1 x_2$$

# These are all valid groupings

	$x_1$	0	1
$x_2$	0	1	0
	1	1	0

$\bar{x}_1$

	$x_1$	0	1
$x_2$	0	0	1
	1	0	1

$x_1$

	$x_1$	0	1
$x_2$	0	1	1
	1	0	0

$\bar{x}_2$

	$x_1$	0	1
$x_2$	0	0	0
	1	1	1

$x_2$



**This one is valid too**

	$x_1$	0	1
$x_2$	0	1	1
1	1	1	1

In this case the result is the constant function 1.

# Invalid Groupings

	$x_1$	0	1
$x_2$	0	1	0
	1	0	1

A Karnaugh map for two variables,  $x_1$  and  $x_2$ . The map is a 2x2 grid. The columns are labeled 0 and 1, and the rows are labeled 0 and 1. The cells contain the following values: (0,0) is 1, (0,1) is 0, (1,0) is 0, and (1,1) is 1. A red line is drawn around the four cells, forming a square that is rotated 45 degrees. This grouping is invalid because it does not consist of a power of two adjacent cells.

	$x_1$	0	1
$x_2$	0	0	1
	1	1	0

A Karnaugh map for two variables,  $x_1$  and  $x_2$ . The map is a 2x2 grid. The columns are labeled 0 and 1, and the rows are labeled 0 and 1. The cells contain the following values: (0,0) is 0, (0,1) is 1, (1,0) is 1, and (1,1) is 0. A red line is drawn around the four cells, forming a square that is rotated 45 degrees. This grouping is invalid because it does not consist of a power of two adjacent cells.

# Can't group diagonally. Why?

$x_2 \backslash x_1$	0	1
0	1	0
1	0	0

$m_0$

$x_2 \backslash x_1$	0	1
0	0	0
1	0	1

$m_3$

# Can't group diagonally. Why?

$x_2$	$x_1$		
		0	1
0		1	0
1		0	0

$m_0$

+

$x_2$	$x_1$		
		0	1
0		0	0
1		0	1

$m_3$

=

$x_2$	$x_1$		
		0	1
0		1	0
1		0	1

$m_0 + m_3$

# Can't group diagonally. Why?

The diagram illustrates the addition of two minterms,  $m_0$  and  $m_3$ , in a 2x2 Karnaugh map. The variables  $x_1$  and  $x_2$  are shown as axes. The minterms are represented by 1s in the cells where they occur, and the 1s are circled in red. The addition is shown as  $m_0 + m_3 = m_0 + m_3$ .

$x_2 \backslash x_1$	0	1
0	1	0
1	0	0

$m_0$

+

$x_2 \backslash x_1$	0	1
0	0	0
1	0	1

$m_3$

=

$x_2 \backslash x_1$	0	1
0	1	0
1	0	1

$m_0 + m_3$

# Can't group diagonally. Why?

<table style="border-collapse: collapse;"> <tr> <td style="border: none;"></td> <td style="border: none; padding: 5px;"><math>x_1</math></td> <td style="border: none;"></td> <td style="border: none;"></td> </tr> <tr> <td style="border: none; padding: 5px;"><math>x_2</math></td> <td style="border: none; padding: 5px;">/</td> <td style="border: none; padding: 5px;"></td> <td style="border: none; padding: 5px;"></td> </tr> <tr> <td style="border: none;"></td> <td style="border: none; padding: 5px;"></td> <td style="border: none; padding: 5px;">0</td> <td style="border: none; padding: 5px;">1</td> </tr> <tr> <td style="border: none; padding: 5px;">0</td> <td style="border: 1px solid black; padding: 5px;">1</td> <td style="border: 1px solid black; padding: 5px;">0</td> <td style="border: none;"></td> </tr> <tr> <td style="border: none; padding: 5px;">1</td> <td style="border: 1px solid black; padding: 5px;">0</td> <td style="border: 1px solid black; padding: 5px;">0</td> <td style="border: none;"></td> </tr> </table>		$x_1$			$x_2$	/					0	1	0	1	0		1	0	0		+	<table style="border-collapse: collapse;"> <tr> <td style="border: none;"></td> <td style="border: none; padding: 5px;"><math>x_1</math></td> <td style="border: none;"></td> <td style="border: none;"></td> </tr> <tr> <td style="border: none; padding: 5px;"><math>x_2</math></td> <td style="border: none; padding: 5px;">/</td> <td style="border: none; padding: 5px;"></td> <td style="border: none; padding: 5px;"></td> </tr> <tr> <td style="border: none;"></td> <td style="border: none; padding: 5px;"></td> <td style="border: none; padding: 5px;">0</td> <td style="border: none; padding: 5px;">1</td> </tr> <tr> <td style="border: none; padding: 5px;">0</td> <td style="border: 1px solid black; padding: 5px;">0</td> <td style="border: 1px solid black; padding: 5px;">0</td> <td style="border: none;"></td> </tr> <tr> <td style="border: none; padding: 5px;">1</td> <td style="border: 1px solid black; padding: 5px;">0</td> <td style="border: 1px solid black; padding: 5px;">1</td> <td style="border: none;"></td> </tr> </table>		$x_1$			$x_2$	/					0	1	0	0	0		1	0	1		=	<table style="border-collapse: collapse;"> <tr> <td style="border: none;"></td> <td style="border: none; padding: 5px;"><math>x_1</math></td> <td style="border: none;"></td> <td style="border: none;"></td> </tr> <tr> <td style="border: none; padding: 5px;"><math>x_2</math></td> <td style="border: none; padding: 5px;">/</td> <td style="border: none; padding: 5px;"></td> <td style="border: none; padding: 5px;"></td> </tr> <tr> <td style="border: none;"></td> <td style="border: none; padding: 5px;"></td> <td style="border: none; padding: 5px;">0</td> <td style="border: none; padding: 5px;">1</td> </tr> <tr> <td style="border: none; padding: 5px;">0</td> <td style="border: 1px solid black; padding: 5px;">1</td> <td style="border: 1px solid black; padding: 5px;">0</td> <td style="border: none;"></td> </tr> <tr> <td style="border: none; padding: 5px;">1</td> <td style="border: 1px solid black; padding: 5px;">0</td> <td style="border: 1px solid black; padding: 5px;">1</td> <td style="border: none;"></td> </tr> </table>		$x_1$			$x_2$	/					0	1	0	1	0		1	0	1	
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$\bar{x}_1\bar{x}_2$	+	$x_1x_2$	=	$\bar{x}_1\bar{x}_2 + x_1x_2$																																																												

We can't use Property 14a here. This can't be simplified.

# Three-Variable K-Map

# Location of three-variable minterms

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

		$x_1 x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map



# Location of three-variable minterms

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

		$x_1 x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map

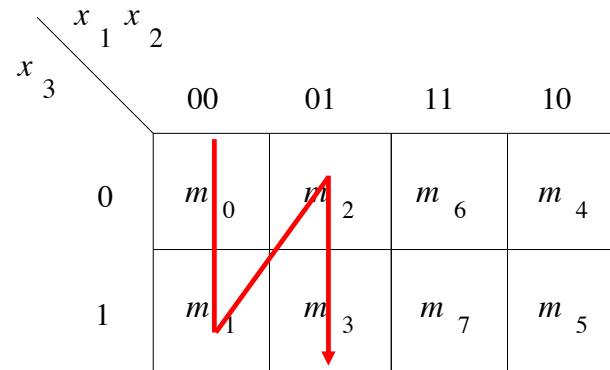
**Notice the placement of**

- **Variables**
- **Binary pair values**
- **Minterms**

# Location of three-variable minterms

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
<hr/>			
0	1	0	$m_2$
0	1	1	$m_3$
<hr/>			
1	0	0	$m_4$
1	0	1	$m_5$
<hr/>			
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table



(b) Karnaugh map

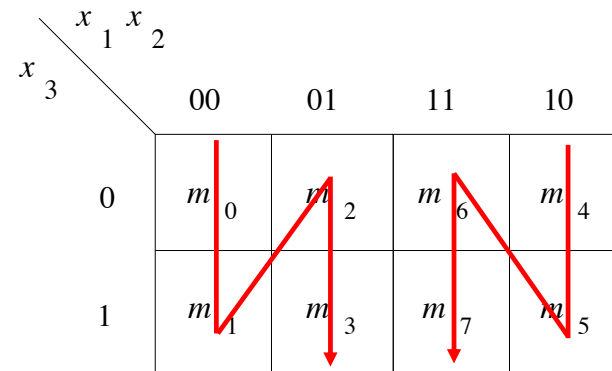
**Notice the placement of**

- **Variables**
- **Binary pair values**
- **Minterms**

# Location of three-variable minterms

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
<hr/>			
0	1	0	$m_2$
0	1	1	$m_3$
<hr/>			
1	0	0	$m_4$
1	0	1	$m_5$
<hr/>			
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table



(b) Karnaugh map

**Notice the placement of**

- **Variables**
- **Binary pair values**
- **Minterms**

# Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

00

01

11

10

# Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

# Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000  
001  
011  
010  
110  
111  
101  
100

# Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000  
001  
011  
010  
110  
111  
101  
100

# Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100



# Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

000

001

011

010

110

111

101

100

# Gray Code

- Sequence of binary codes
- Two neighboring lines vary by only 1 bit

000

001

011

010

110

111

101

100

# Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

# Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

$x_2$ \ $s x_1$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

$x_2 \backslash s x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

		$s \ x_1$			
		00	01	11	10
$x_2$	0	000	010	110	100
	1	001	011	111	101

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

$x_2 \backslash s x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101



# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

$x_2 \backslash s x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

$x_2 \backslash s x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

$x_2 \backslash s x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These two neighbors  
differ only in the LAST bit

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

$x_2 \backslash s x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These two neighbors  
differ only in the LAST bit

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

$x_2 \backslash s x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These two neighbors  
differ only in the **FIRST** bit

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

$x_2 \backslash s x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101


These four neighbors  
differ in the FIRST and LAST bit

They are similar in their MIDDLE bit

# Adjacency Rules

		$x_1x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

adjacent  
columns



# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

$x_2 \backslash s x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These two neighbors  
differ only in the **FIRST** bit



# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

$x_2 \backslash s x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

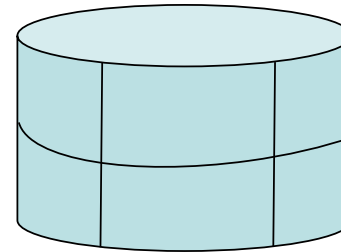
These four neighbors  
differ in the FIRST and LAST bit

They are similar in their MIDDLE bit

# Adjacency Rules

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

adjacent  
columns

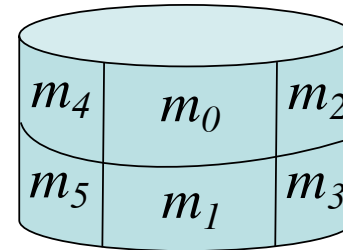


As if the K-map were  
drawn on a cylinder

# Adjacency Rules

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

adjacent  
columns

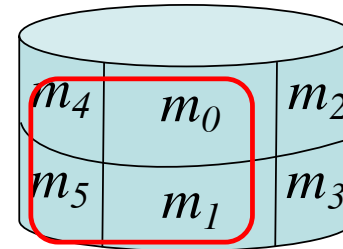


As if the K-map were  
drawn on a cylinder

# Adjacency Rules

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

adjacent  
columns

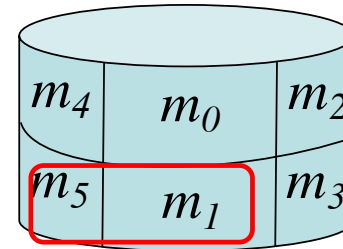


As if the K-map were  
drawn on a cylinder

# Adjacency Rules

$x_3 \backslash x_1 x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

adjacent  
columns



As if the K-map were  
drawn on a cylinder

# **Examples of Valid Groupings**

# Groupings and Expressions

		$x_1x_2$			
		00	01	11	10
$x_3$	0	1	0	0	0
	1	0	0	0	0

$\overline{x_1} \overline{x_2} \overline{x_3}$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	1	0	0
	1	0	0	0	0

$\overline{x_1} x_2 \overline{x_3}$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	1	0
	1	0	0	0	0

$\overline{x_1} \overline{x_2} x_3$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	0	1
	1	0	0	0	0

$\overline{x_1} \overline{x_2} x_3$

# Groupings and Expressions

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	0	0
	1	1	0	0	0

$$\overline{x_1} \overline{x_2} x_3$$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	0	0
	1	0	1	0	0

$$\overline{x_1} x_2 \overline{x_3}$$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	0	0
	1	0	0	1	0

$$x_1 x_2 x_3$$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	0	0
	1	0	0	0	1

$$\overline{x_1} \overline{x_2} x_3$$



# Groupings and Expressions

		$x_1x_2$			
		00	01	11	10
$x_3$	0	1	0	0	0
	1	1	0	0	0

$\overline{x_1} \overline{x_2}$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	1	0	0
	1	0	1	0	0

$\overline{x_1} x_2$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	1	0
	1	0	0	1	0

$x_1 x_2$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	0	1
	1	0	0	0	1

$x_1 \overline{x_2}$

# Groupings and Expressions

		$x_1x_2$			
		00	01	11	10
$x_3$	0	1	1	0	0
	1	0	0	0	0

$$\overline{x_1} \overline{x_3}$$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	1	1	0
	1	0	0	0	0

$$x_2 \overline{x_3}$$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	1	1
	1	0	0	0	0

$$x_1 \overline{x_3}$$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	1	0	0	1
	1	0	0	0	0

$$\overline{x_2} \overline{x_3}$$

# Groupings and Expressions

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	0	0
	1	1	1	0	0

$$\overline{x_1} x_3$$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	0	0
	1	0	1	1	0

$$x_2 x_3$$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	0	0
	1	0	0	1	1

$$x_1 x_3$$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	0	0
	1	1	0	0	1

$$\overline{x_2} x_3$$

# Groupings and Expressions

		$x_1x_2$			
		00	01	11	10
$x_3$	0	1	1	0	0
	1	1	1	0	0

$\overline{x_1}$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	1	1	0
	1	0	1	1	0

$x_2$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	1	1
	1	0	0	1	1

$x_1$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	1	0	0	1
	1	1	0	0	1

$\overline{x_2}$

# Groupings and Expressions

		$x_1x_2$			
		00	01	11	10
$x_3$	0	1	1	1	1
	1	0	0	0	0

$\overline{x_3}$

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	0	0
	1	1	1	1	1

$x_3$

# Groupings and Expressions

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	0	0
	1	0	0	0	0

0

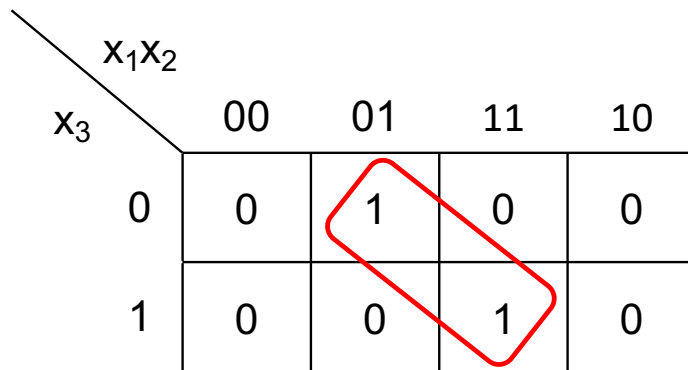
		$x_1x_2$			
		00	01	11	10
$x_3$	0	1	1	1	1
	1	1	1	1	1

1

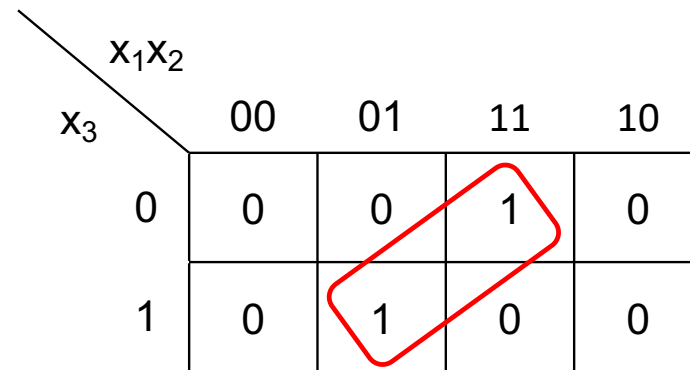
# Examples of **Invalid** Groupings

# Some **Invalid** Groupings

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	1	0	0
	1	0	0	1	0

A Karnaugh map for three variables x1, x2, and x3. The columns are labeled x1x2 with values 00, 01, 11, 10. The rows are labeled x3 with values 0 and 1. The map contains 1s at (0,01) and (1,11). A red rounded rectangle is drawn diagonally, enclosing the two 1s. This represents an invalid grouping because the two 1s do not share a common variable.

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	1	0
	1	0	1	0	0

A Karnaugh map for three variables x1, x2, and x3. The columns are labeled x1x2 with values 00, 01, 11, 10. The rows are labeled x3 with values 0 and 1. The map contains 1s at (0,11) and (1,01). A red rounded rectangle is drawn diagonally, enclosing the two 1s. This represents an invalid grouping because the two 1s do not share a common variable.

Can't group diagonally.



# Some **Invalid** Groupings

		$x_1x_2$			
		00	01	11	10
$x_3$	0	1	1	1	0
	1	0	0	0	0

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	0	0
	1	0	1	1	1

Can't group three in a row.  
Each side must be a power of 2.

# Some **Invalid** Groupings

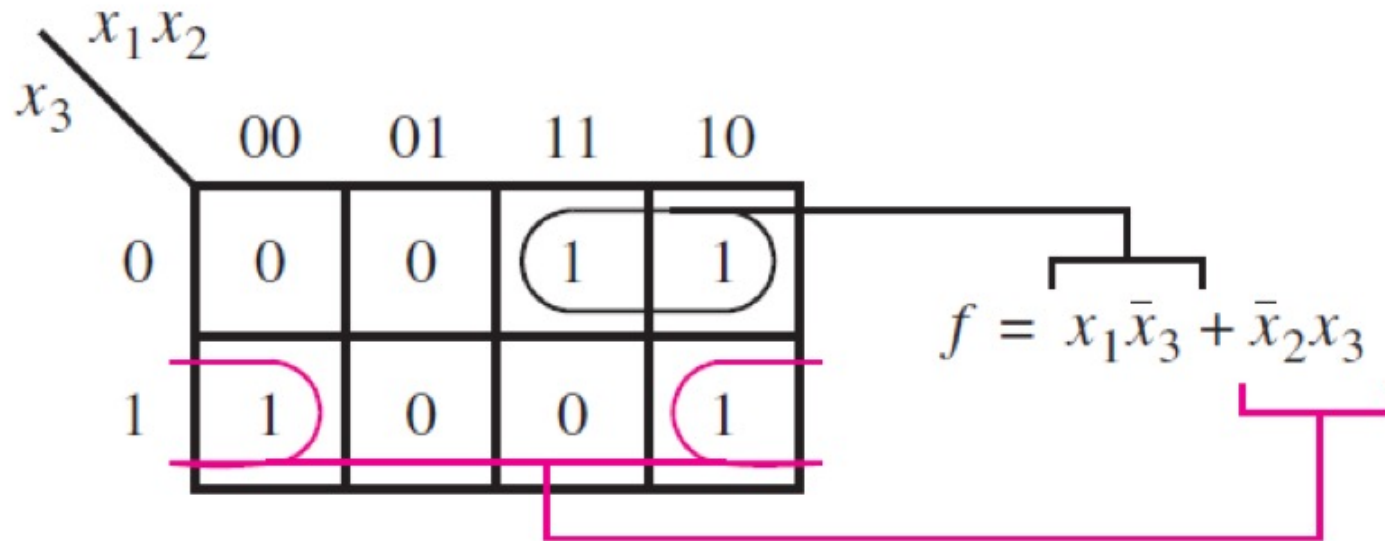
		$x_1x_2$			
		00	01	11	10
$x_3$	0	1	0	1	1
	1	0	0	0	0

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	1	0
	1	0	1	1	0

Can't group zeros and ones together.

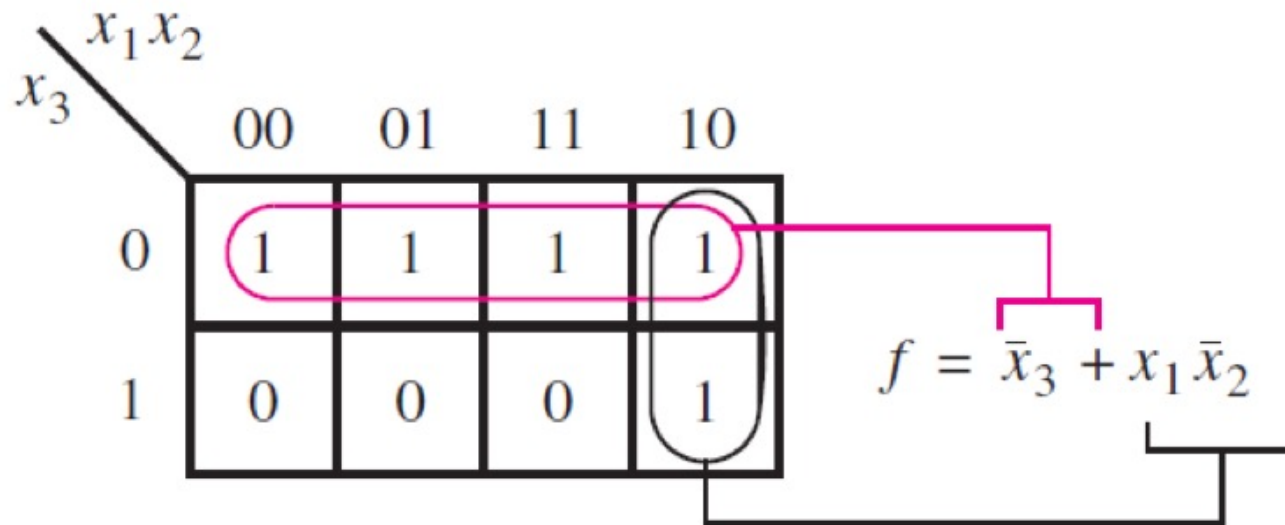
# **Minimization Examples with 3-variable K-Maps**

# Examples of three-variable Karnaugh maps



[ Figure 2.52a from the textbook ]

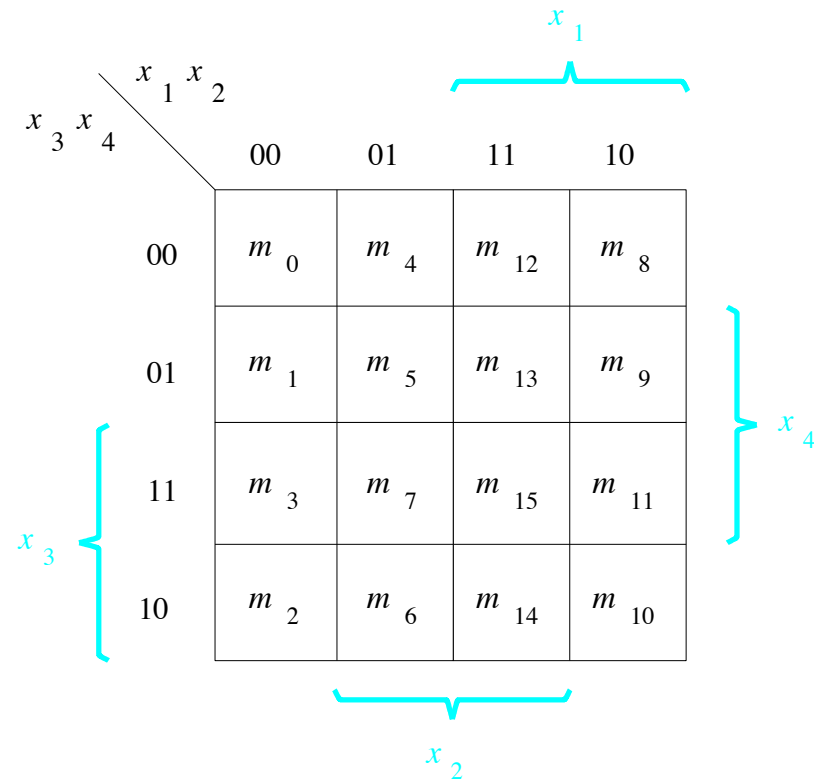
# Examples of three-variable Karnaugh maps



[ Figure 2.52b from the textbook ]

# Four-Variable K-Map

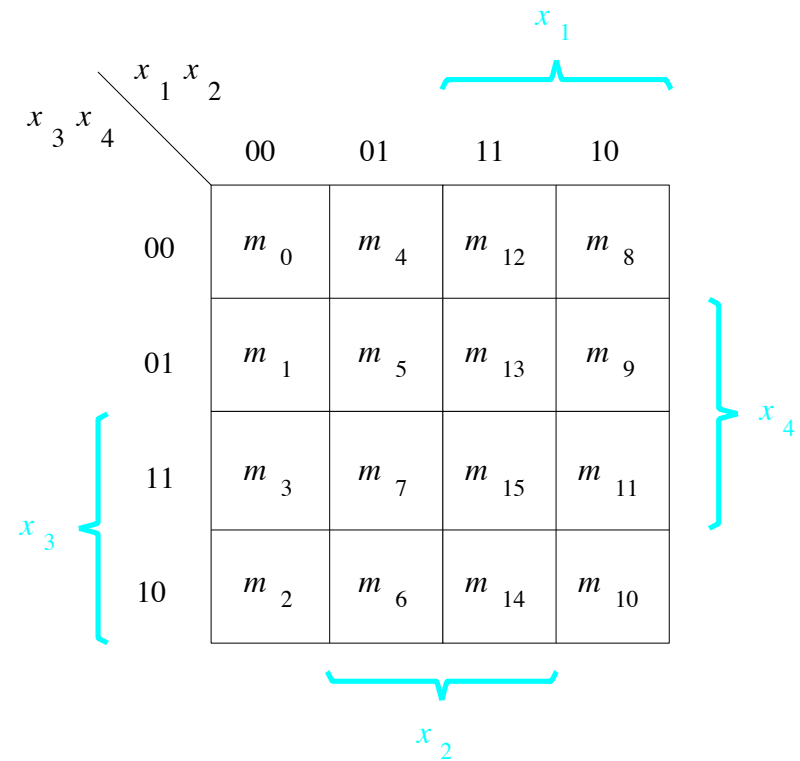
# A four-variable Karnaugh map



[ Figure 2.53 from the textbook ]

# A four-variable Karnaugh map

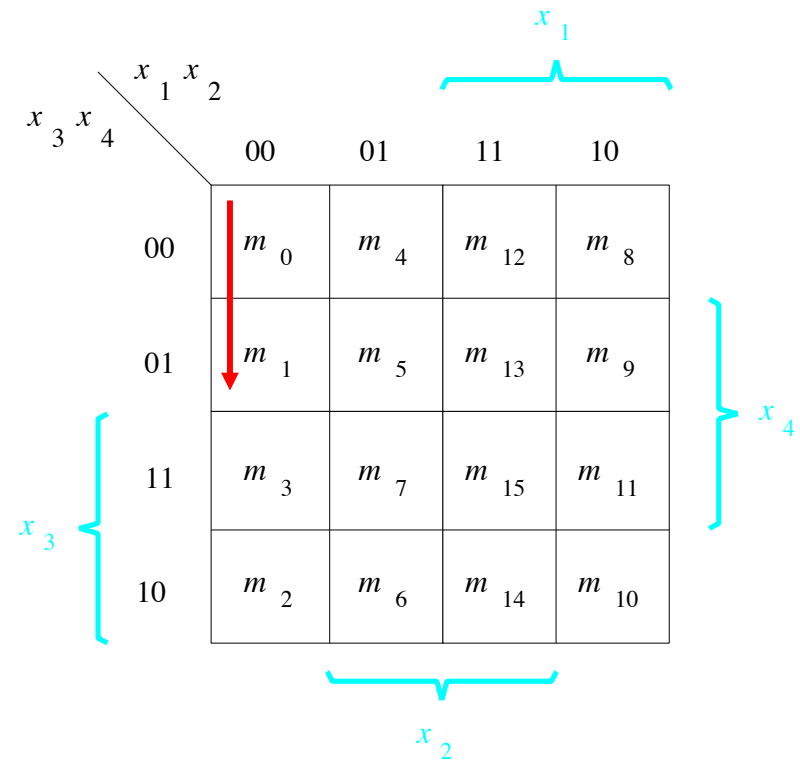
x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15





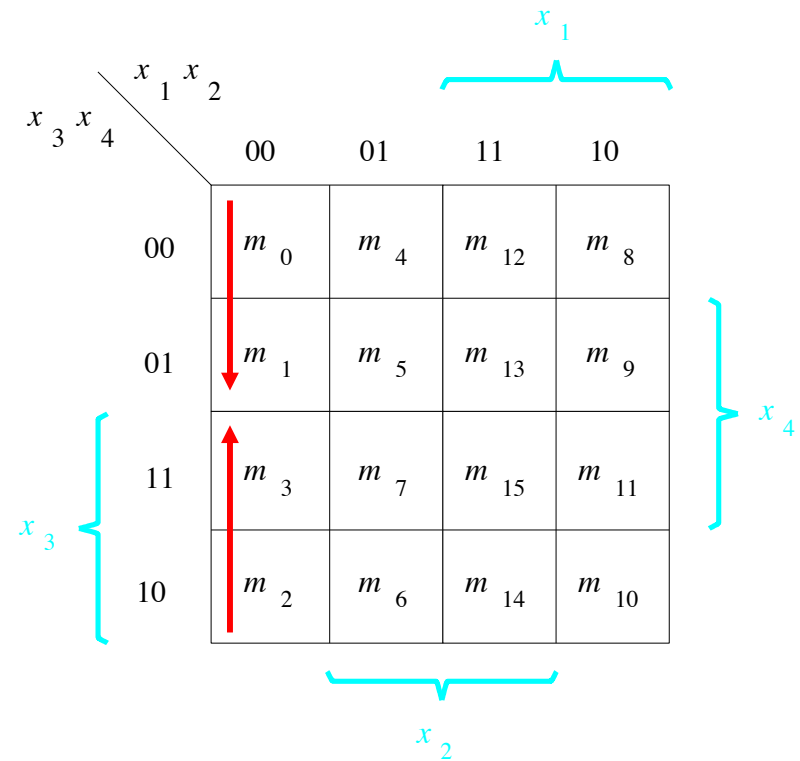
# A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



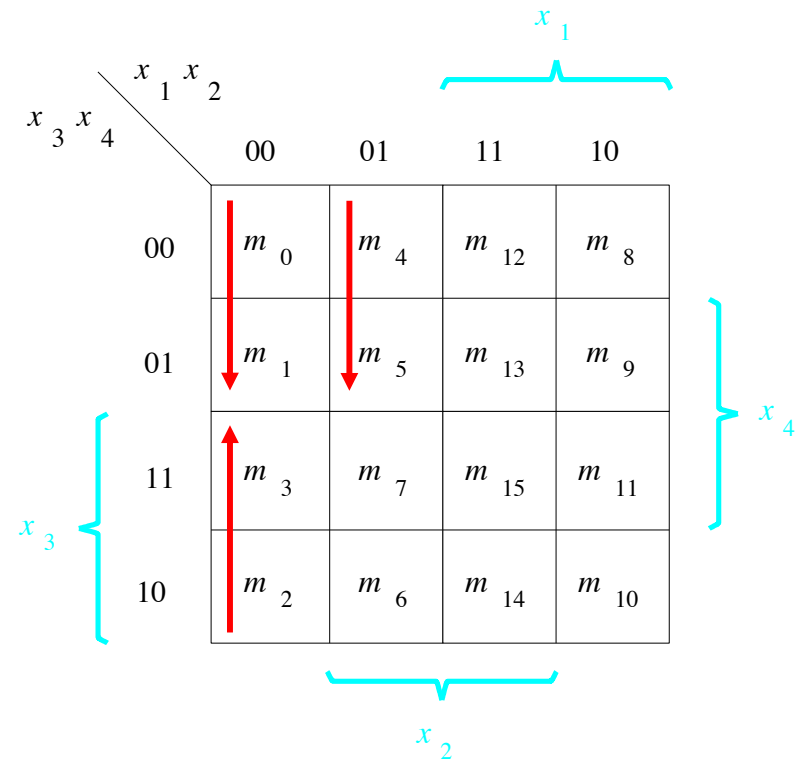
# A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



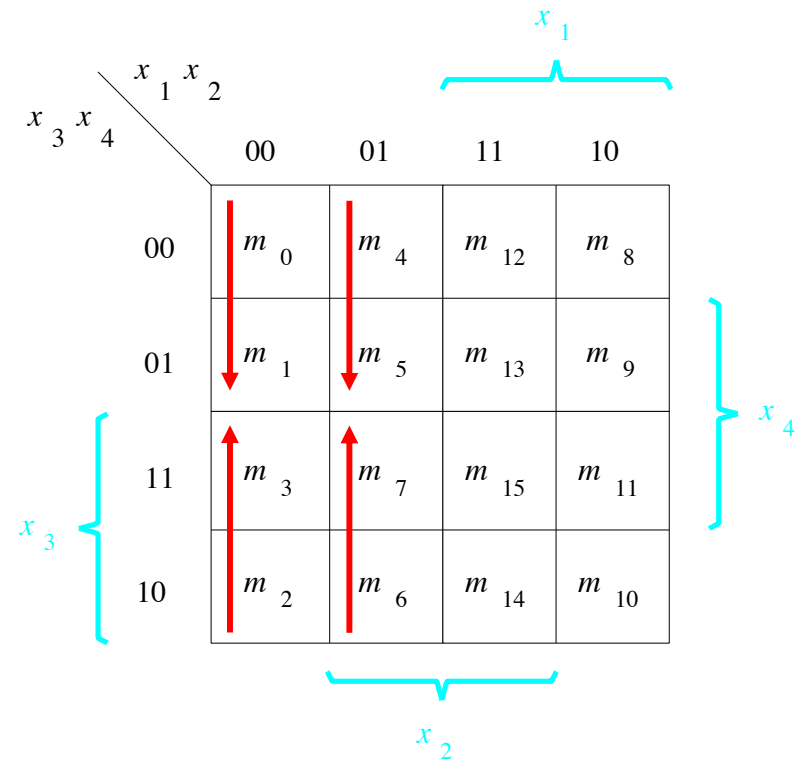
# A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



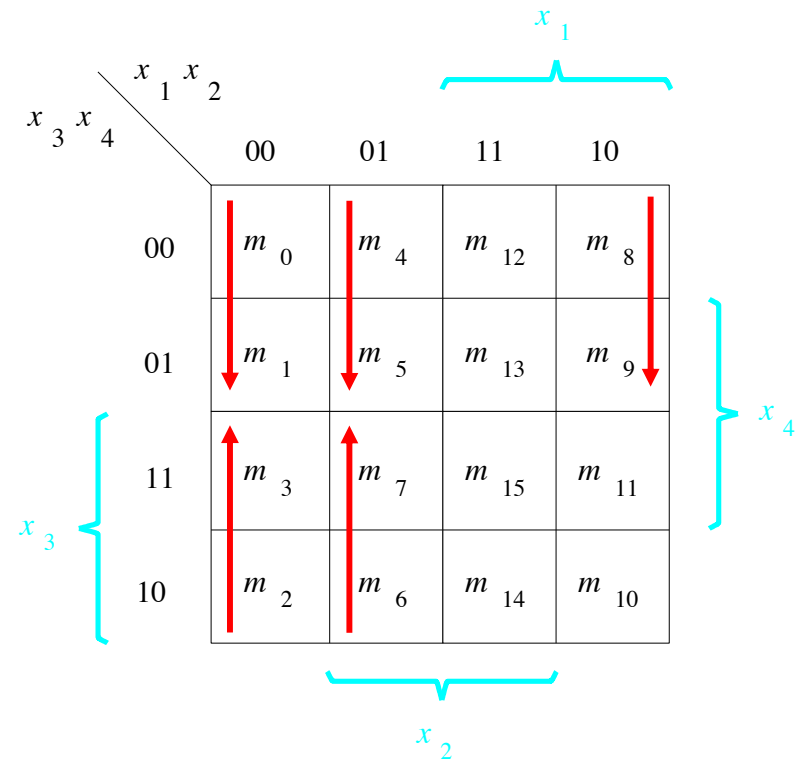
# A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



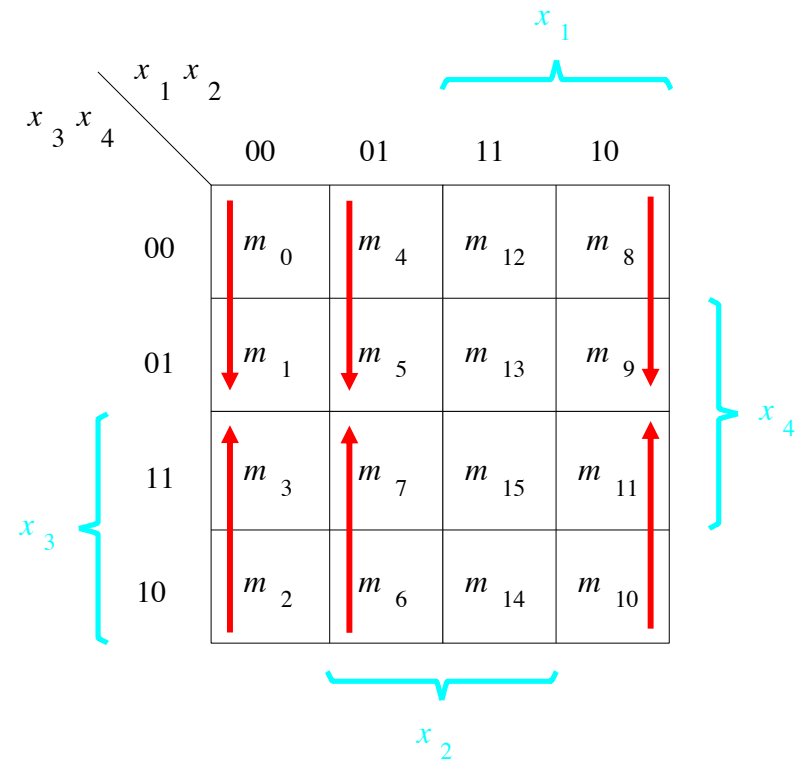
# A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



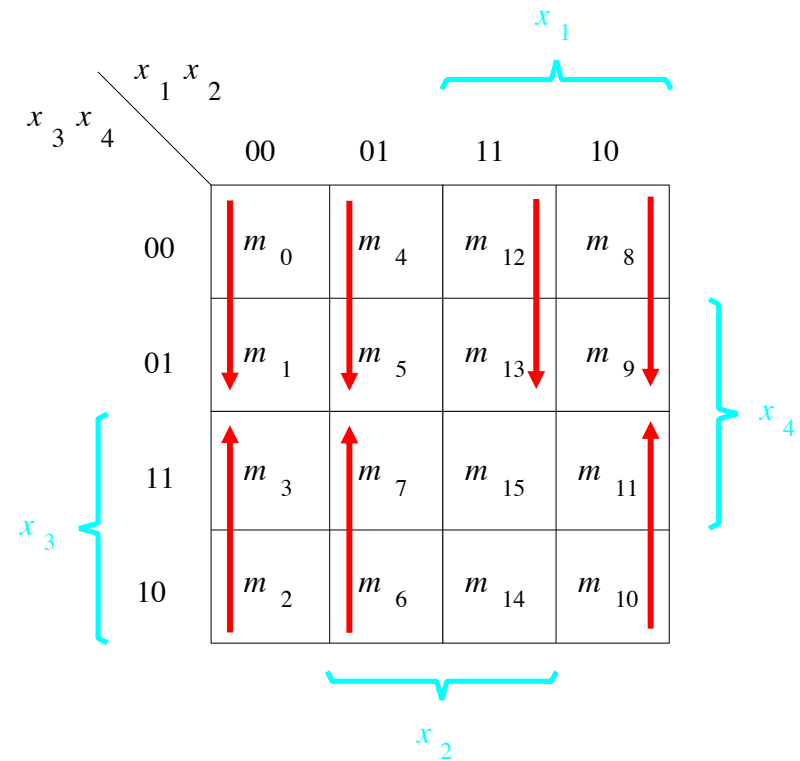
# A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



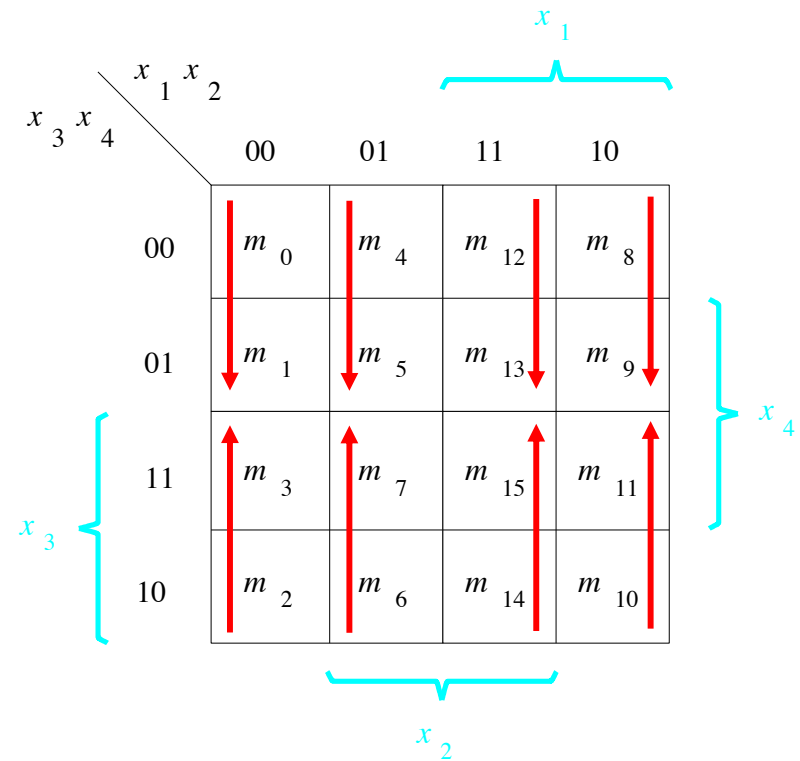
# A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



# A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15





# Adjacency Rules

		$x_1x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

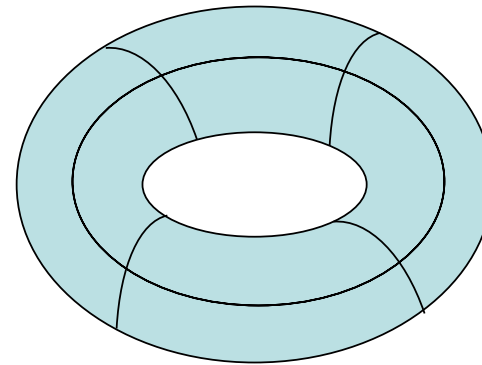
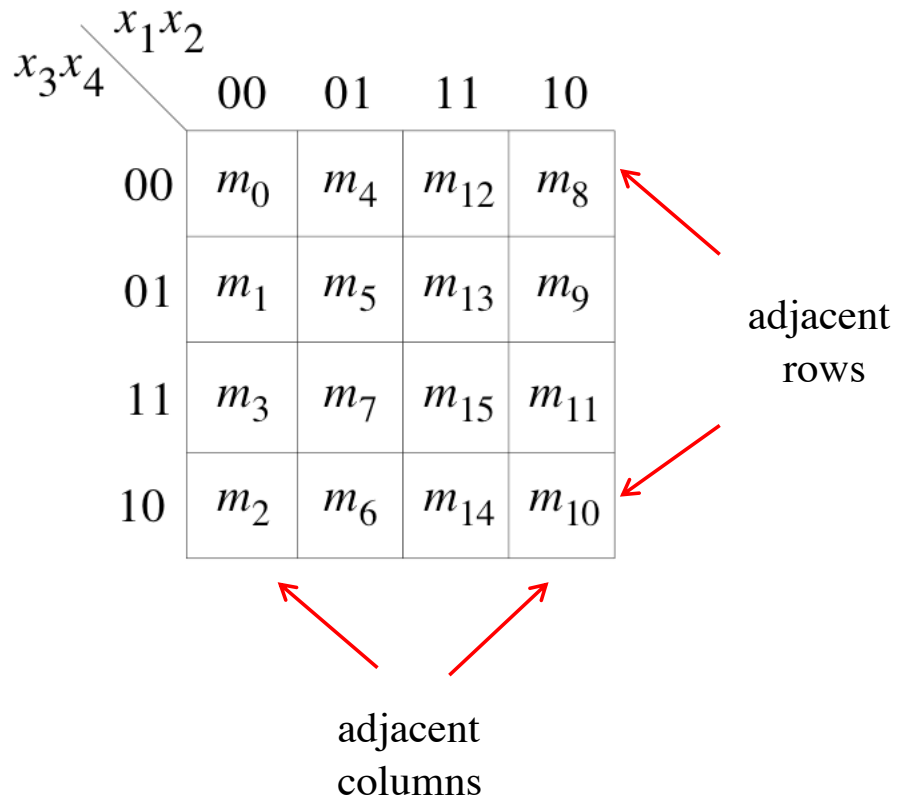
adjacent  
columns

		$x_1x_2$			
		00	01	11	10
$x_3x_4$	00	$m_0$	$m_4$	$m_{12}$	$m_8$
	01	$m_1$	$m_5$	$m_{13}$	$m_9$
	11	$m_3$	$m_7$	$m_{15}$	$m_{11}$
	10	$m_2$	$m_6$	$m_{14}$	$m_{10}$

adjacent  
rows

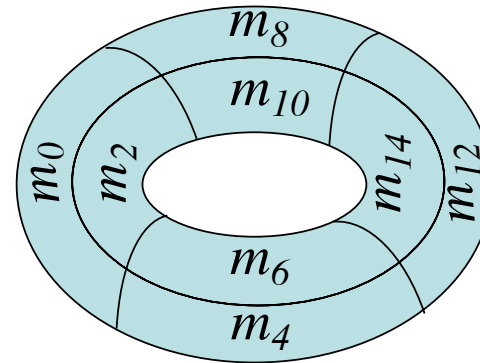
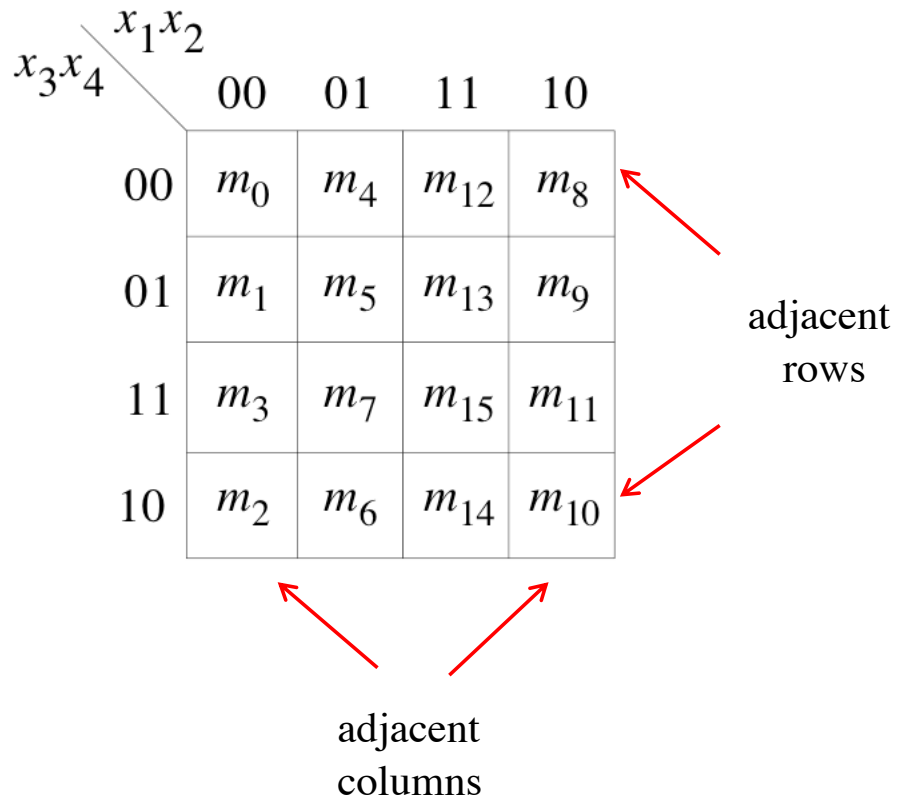
adjacent  
columns

# Adjacency Rules



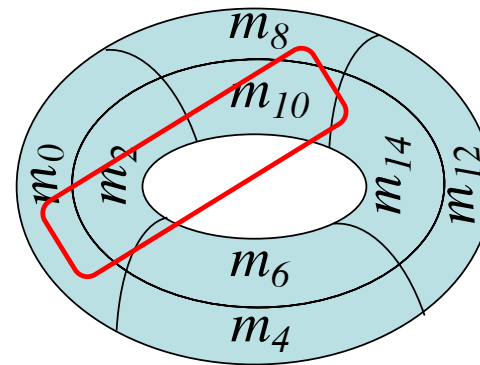
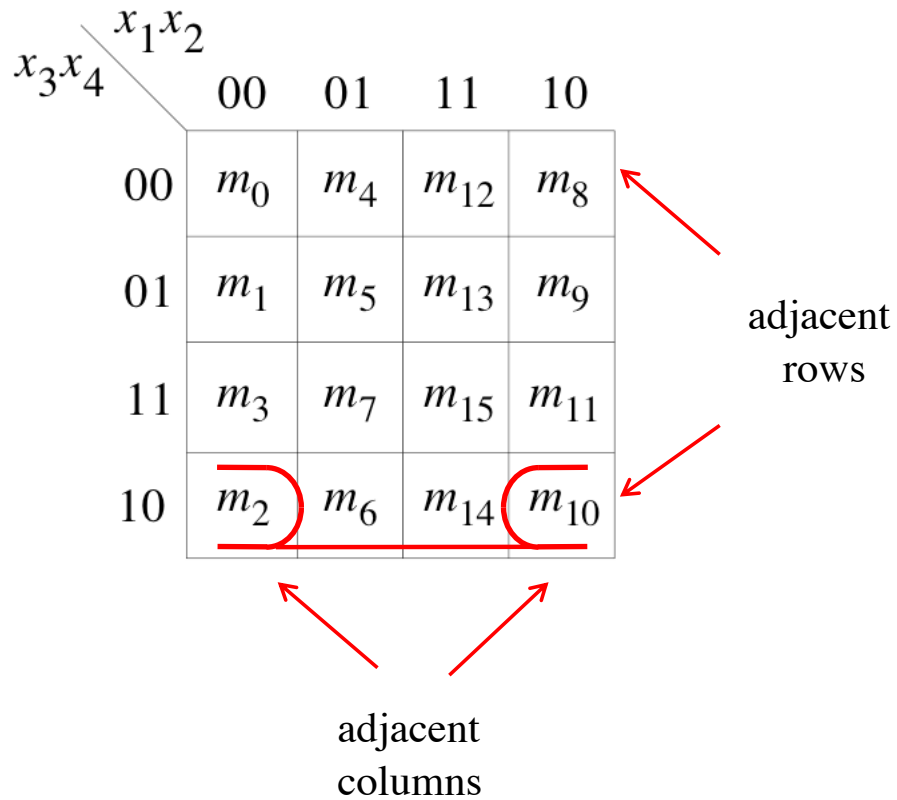
As if the K-map were drawn on a torus

# Adjacency Rules



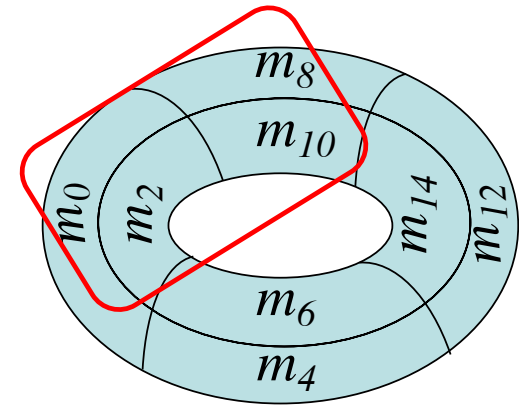
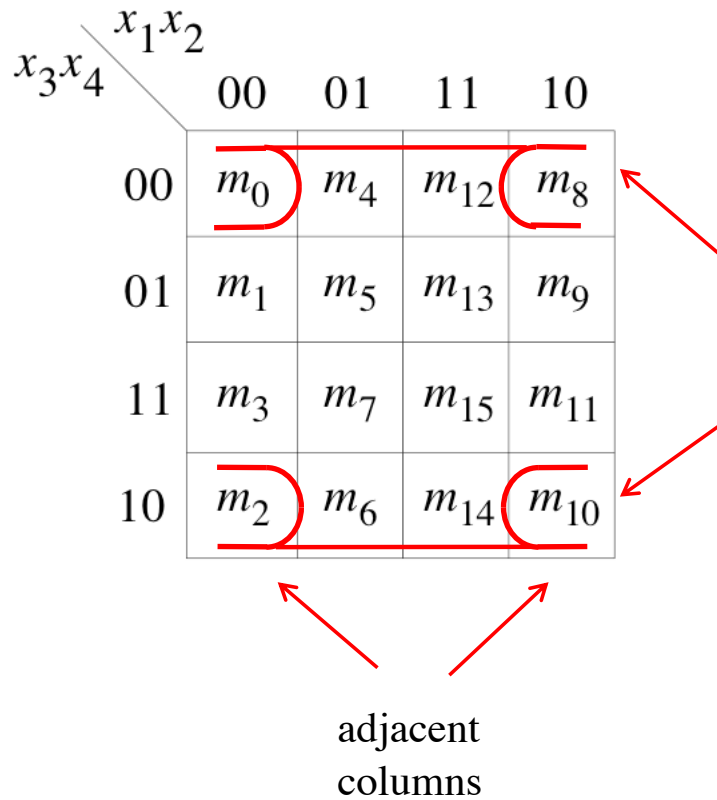
As if the K-map were drawn on a torus

# Adjacency Rules



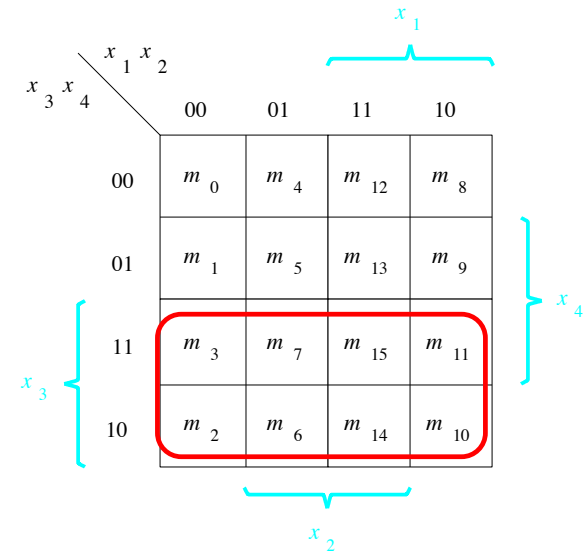
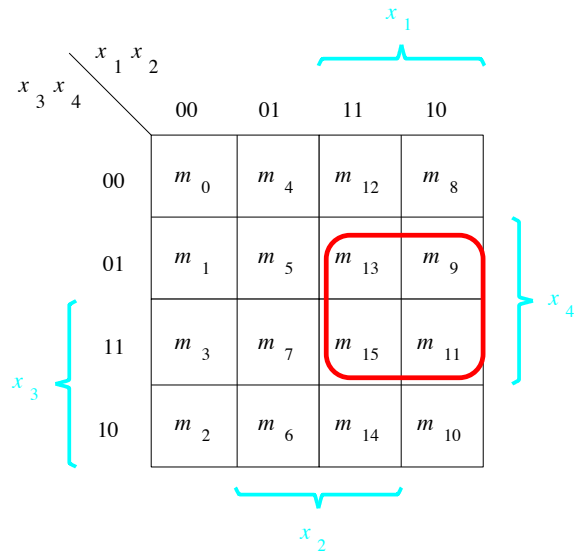
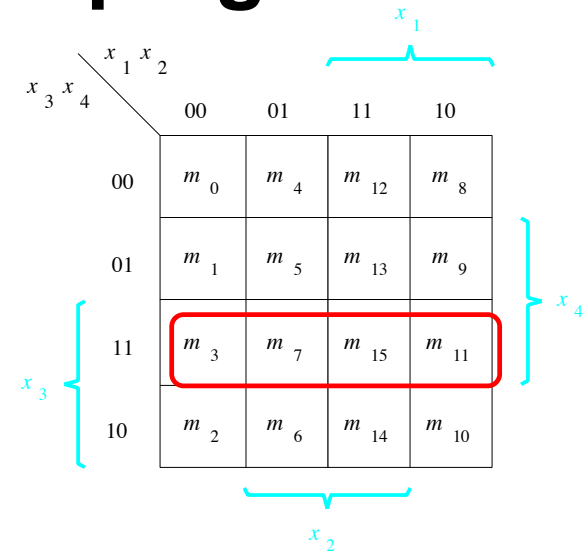
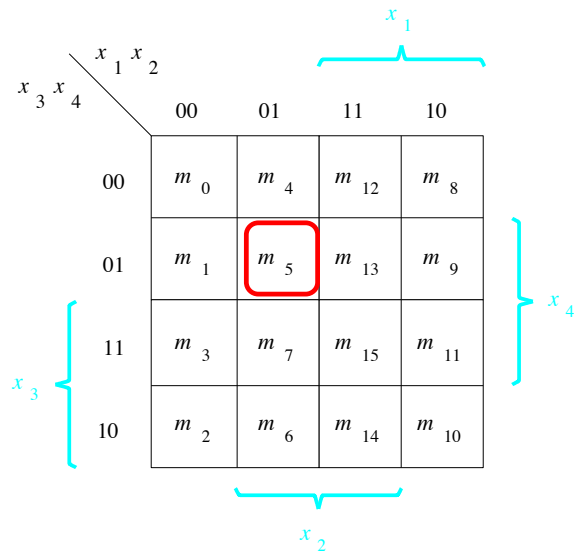
As if the K-map were drawn on a torus

# Adjacency Rules

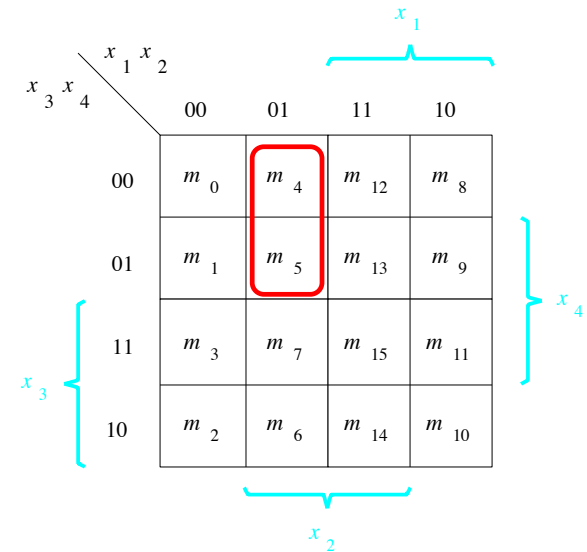
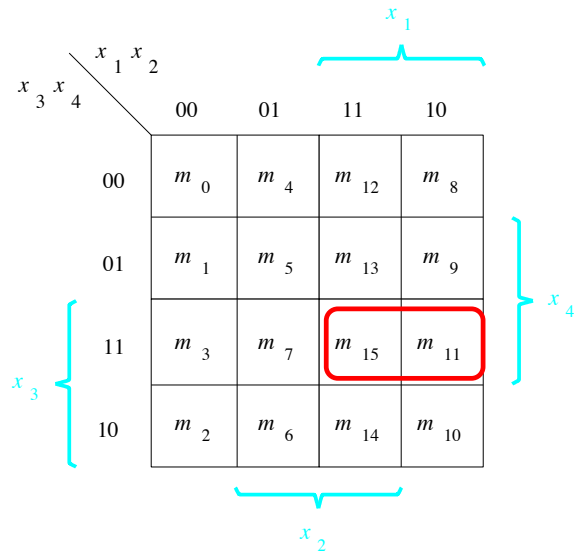
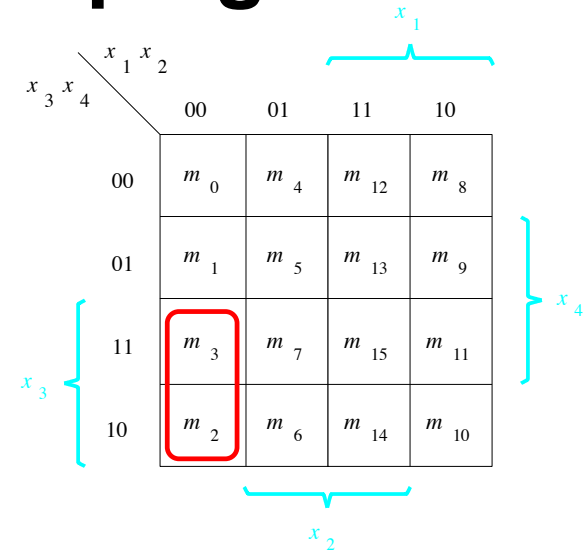
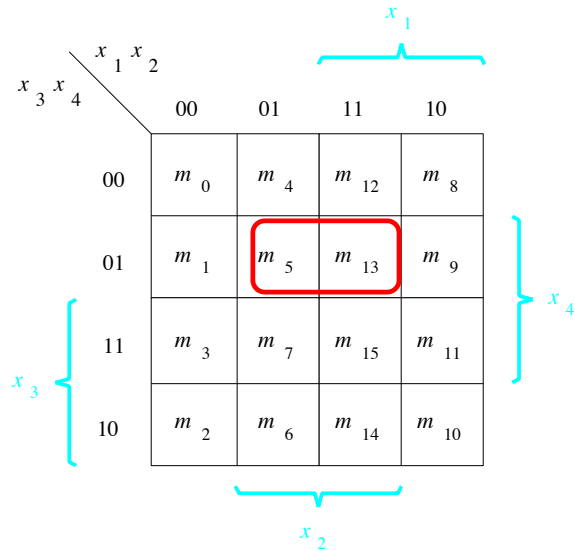


As if the K-map were drawn on a torus

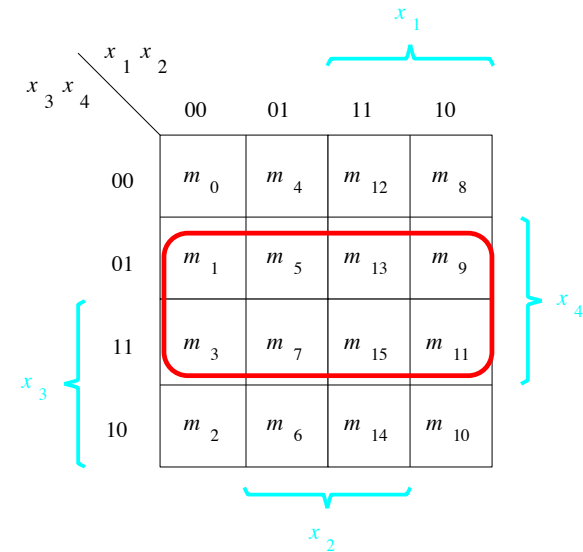
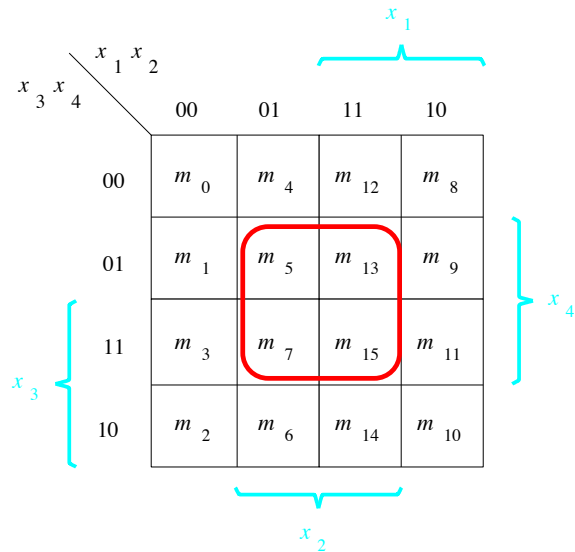
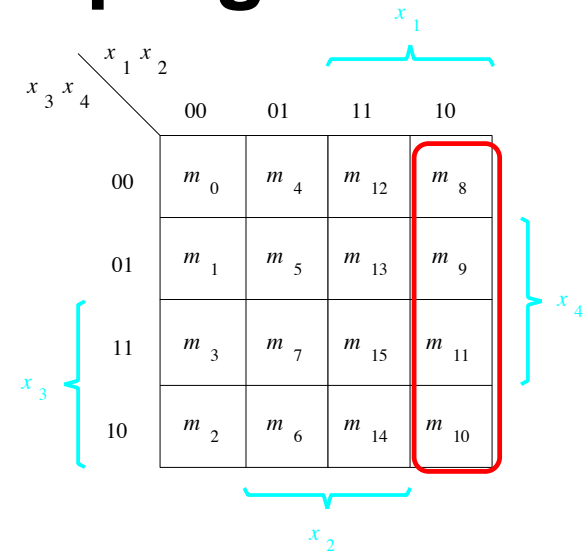
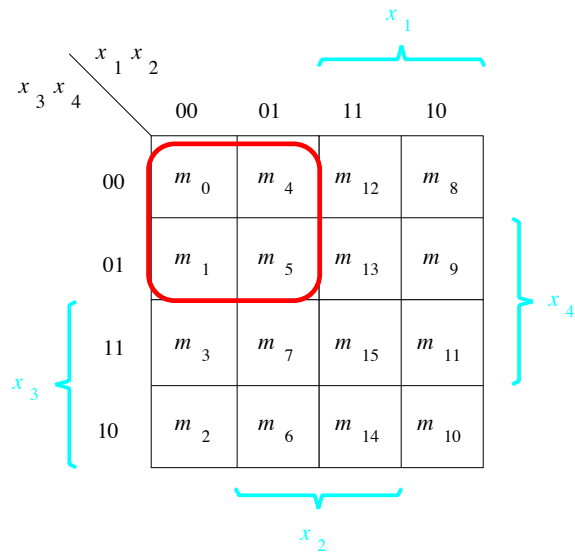
# Some Valid Groupings



# Some Valid Groupings

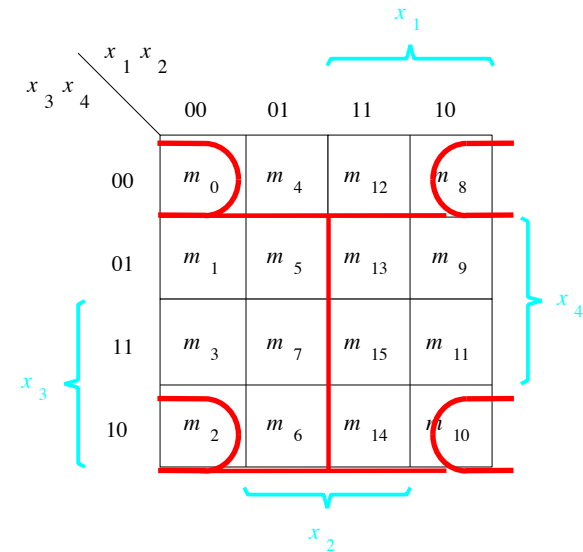
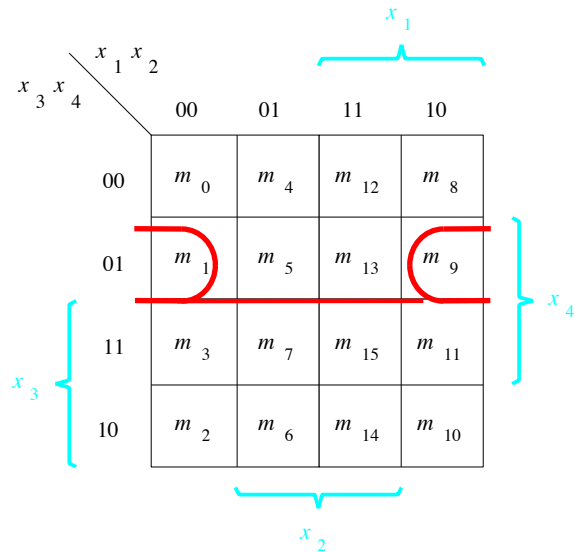
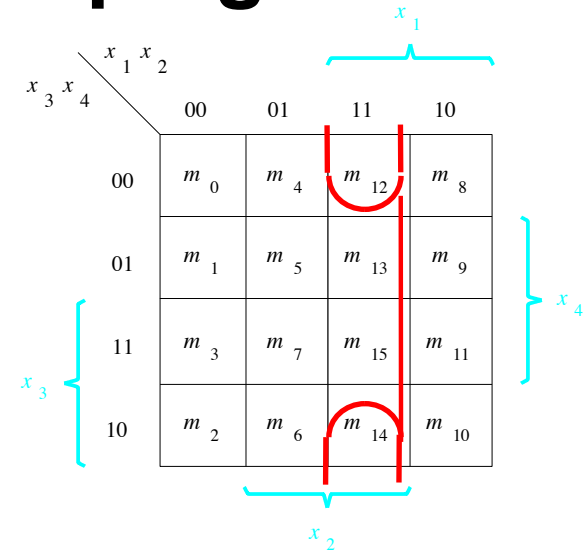
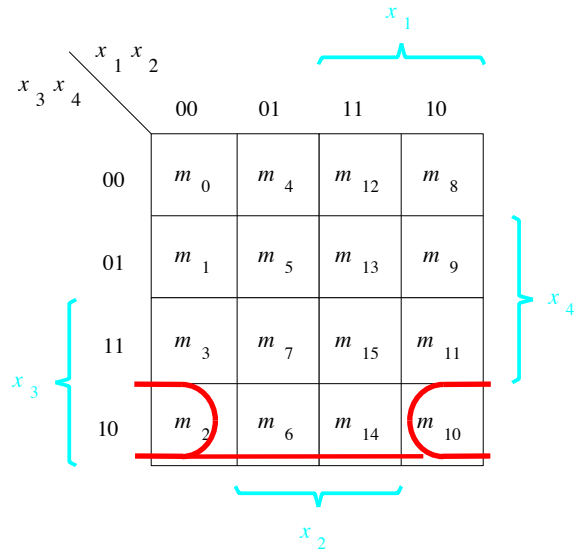


# Some Valid Groupings

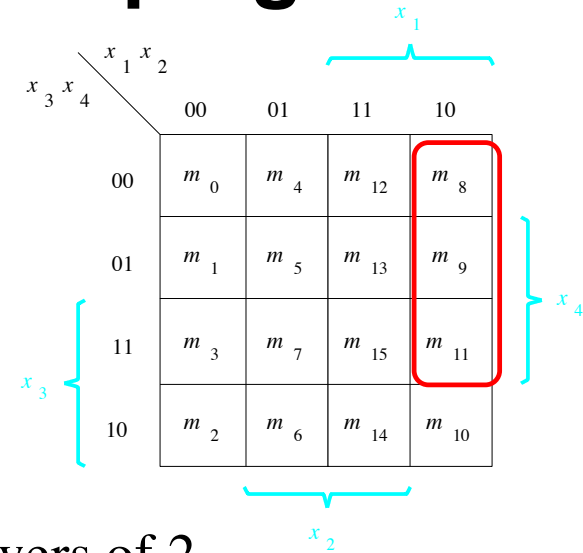
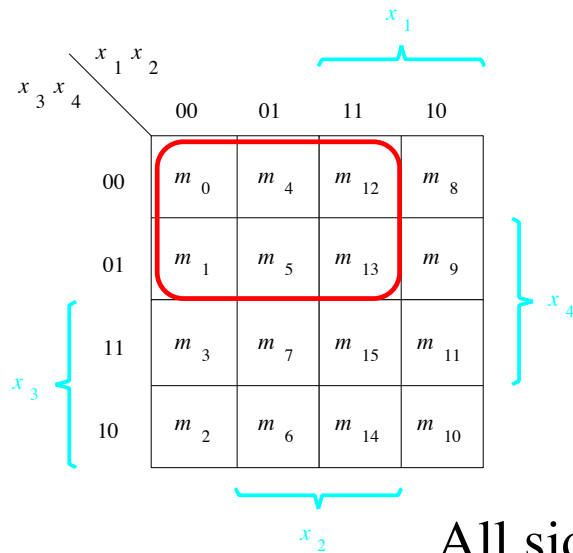




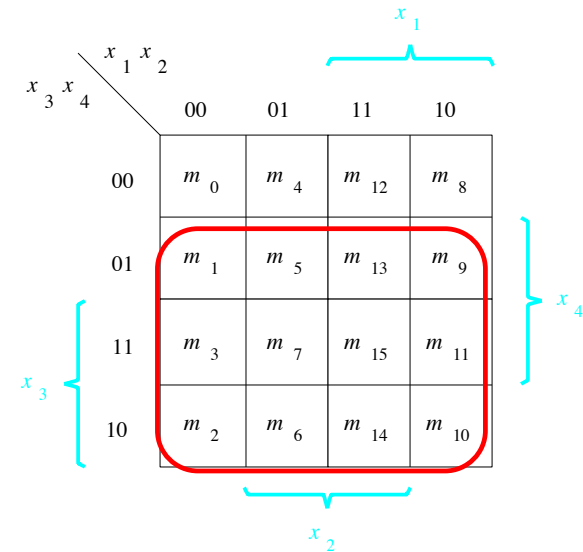
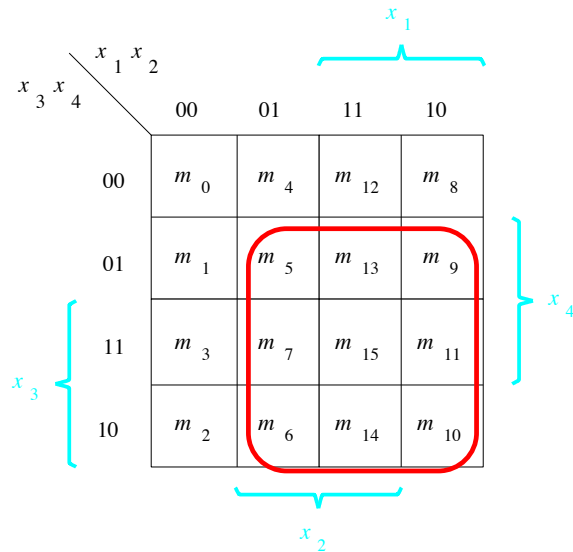
# Some Valid Groupings



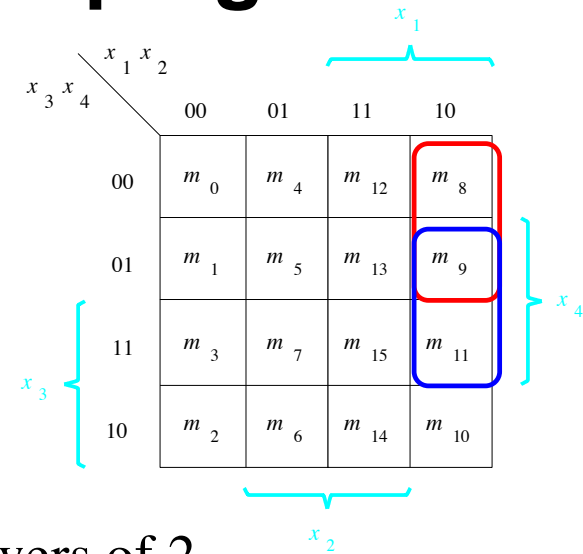
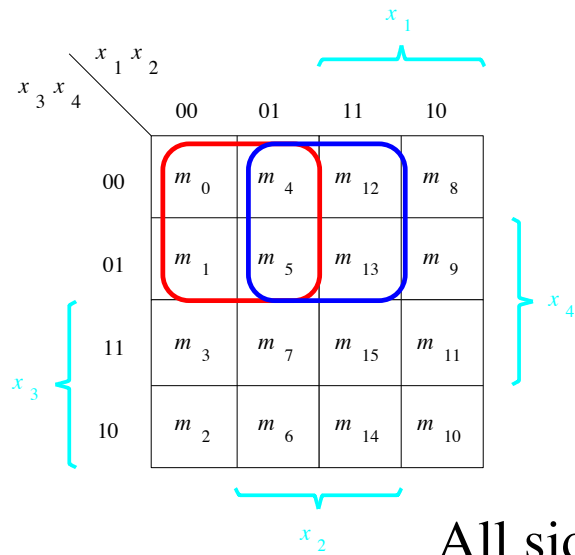
# Some Invalid Groupings



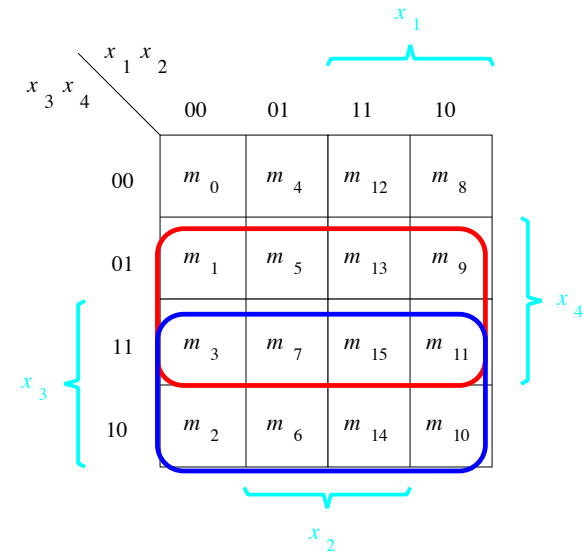
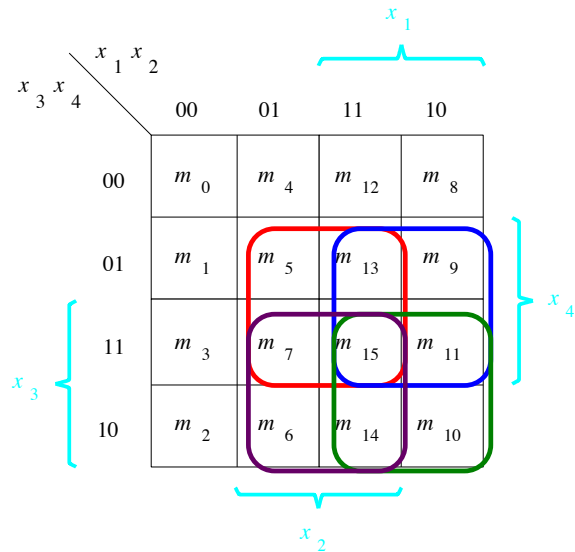
All sides must be powers of 2.



# Some **valid** Groupings

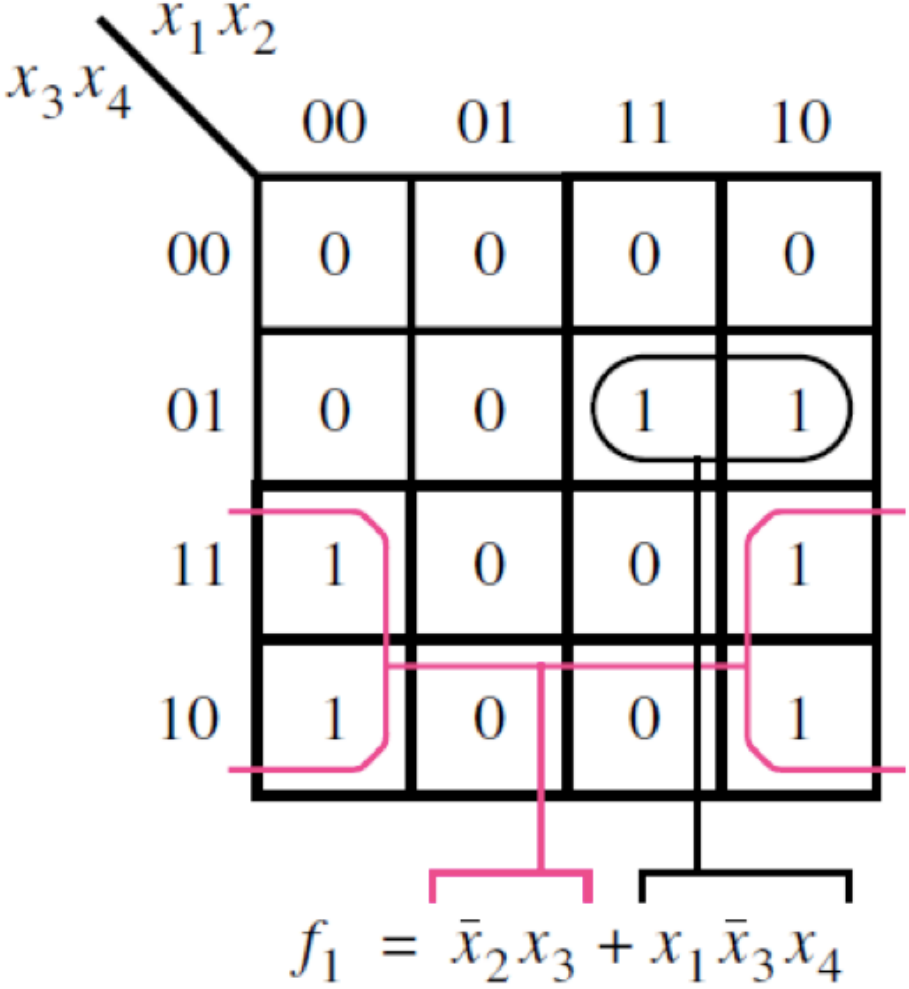


All sides must be powers of 2.



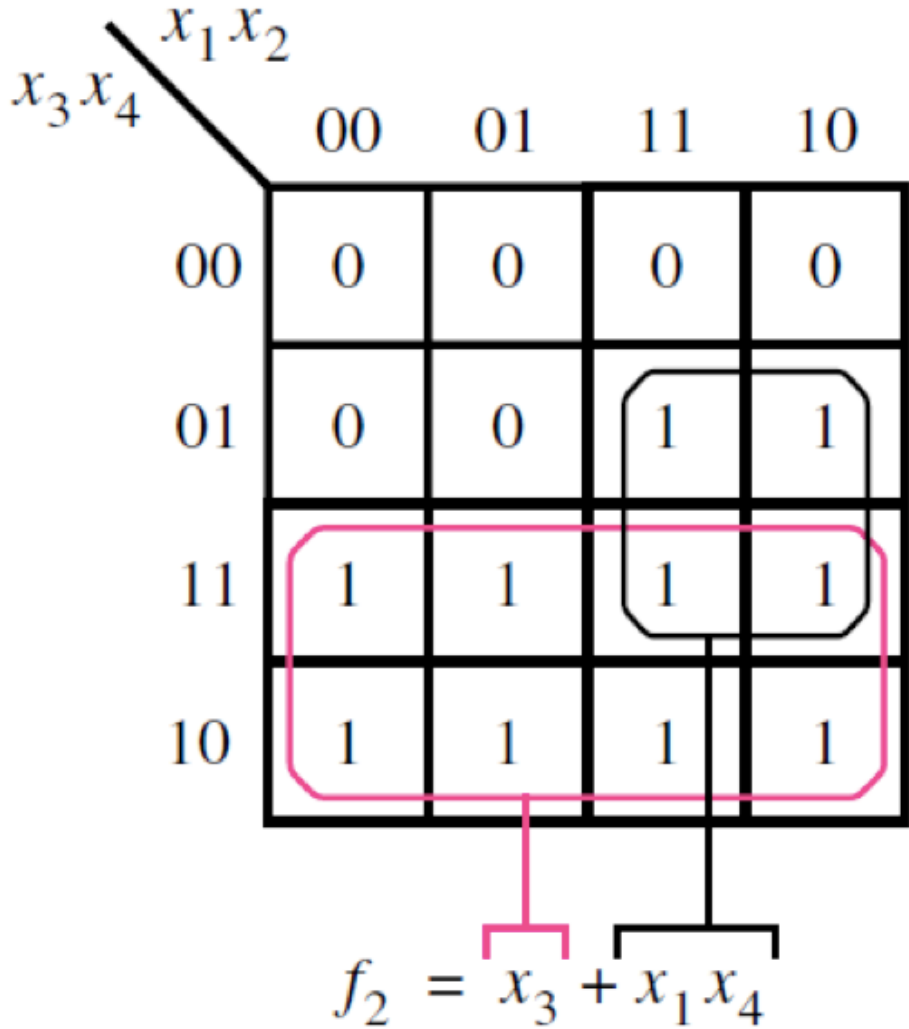
# **Minimization Examples with 4-variable K-Maps**

# Example of a four-variable Karnaugh map



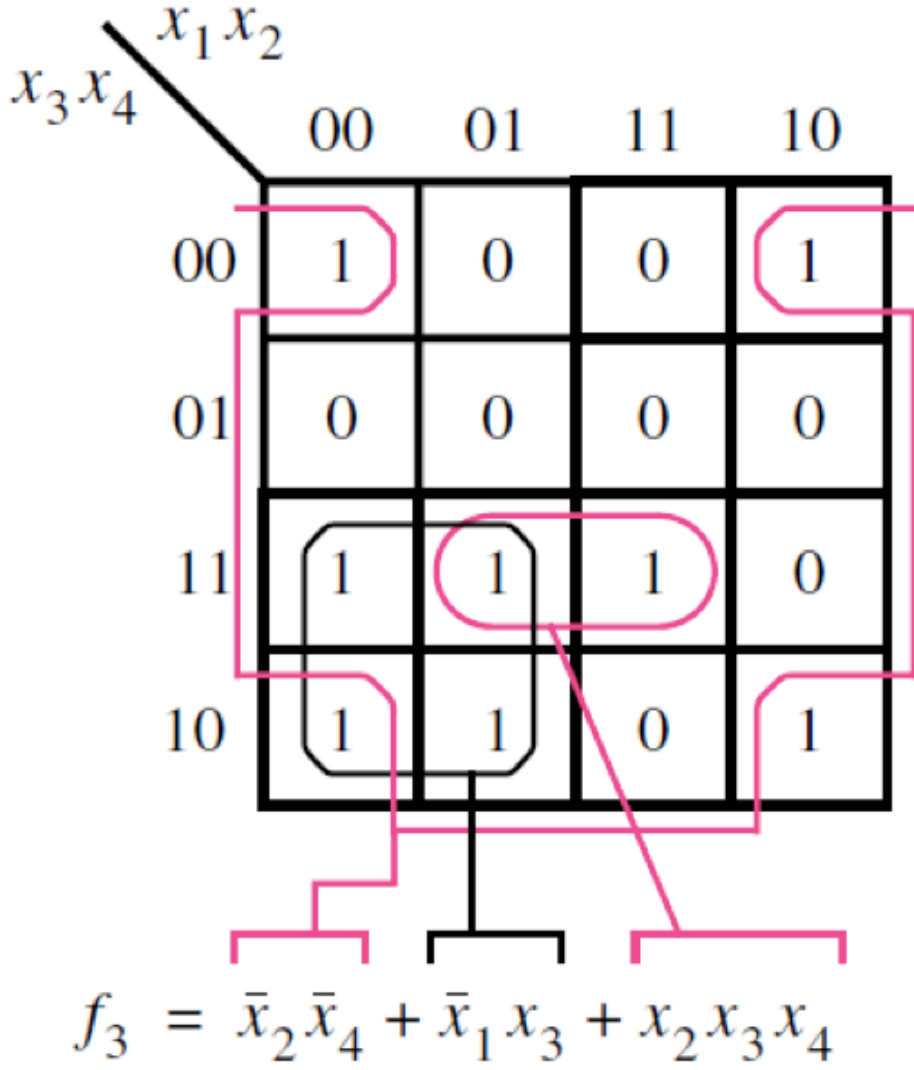
[ Figure 2.54 from the textbook ]

# Example of a four-variable Karnaugh map



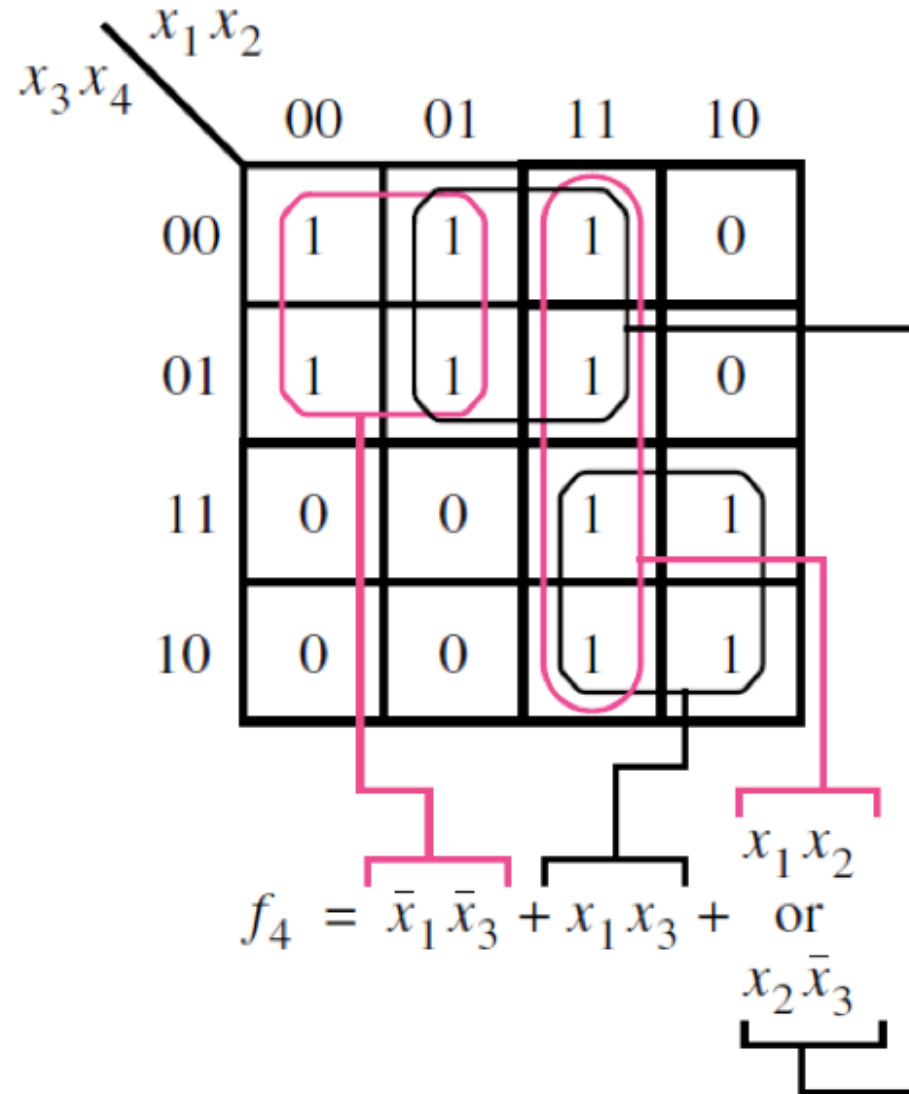
[ Figure 2.54 from the textbook ]

# Example of a four-variable Karnaugh map



[ Figure 2.54 from the textbook ]

# Example of a four-variable Karnaugh map

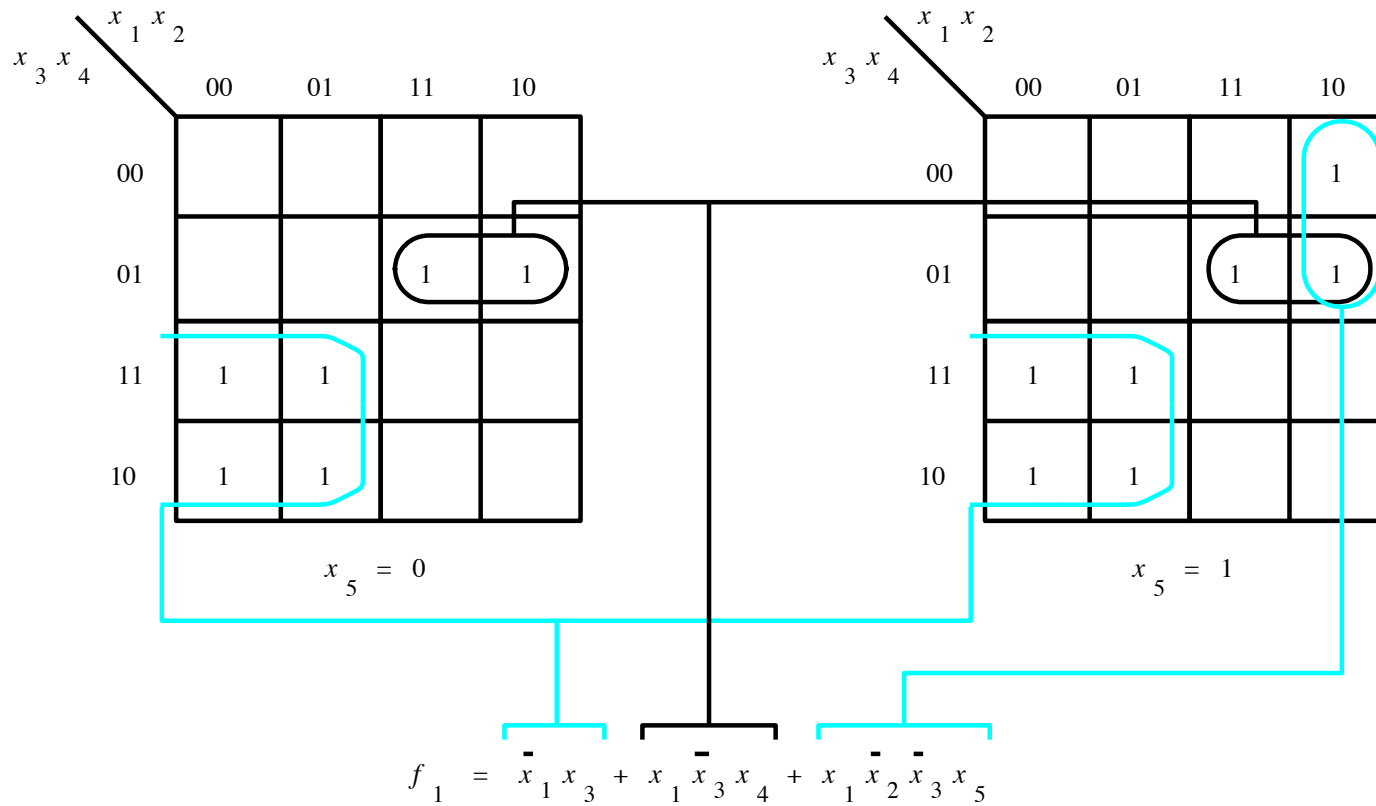


[ Figure 2.54 from the textbook ]



# Five-Variable K-Map

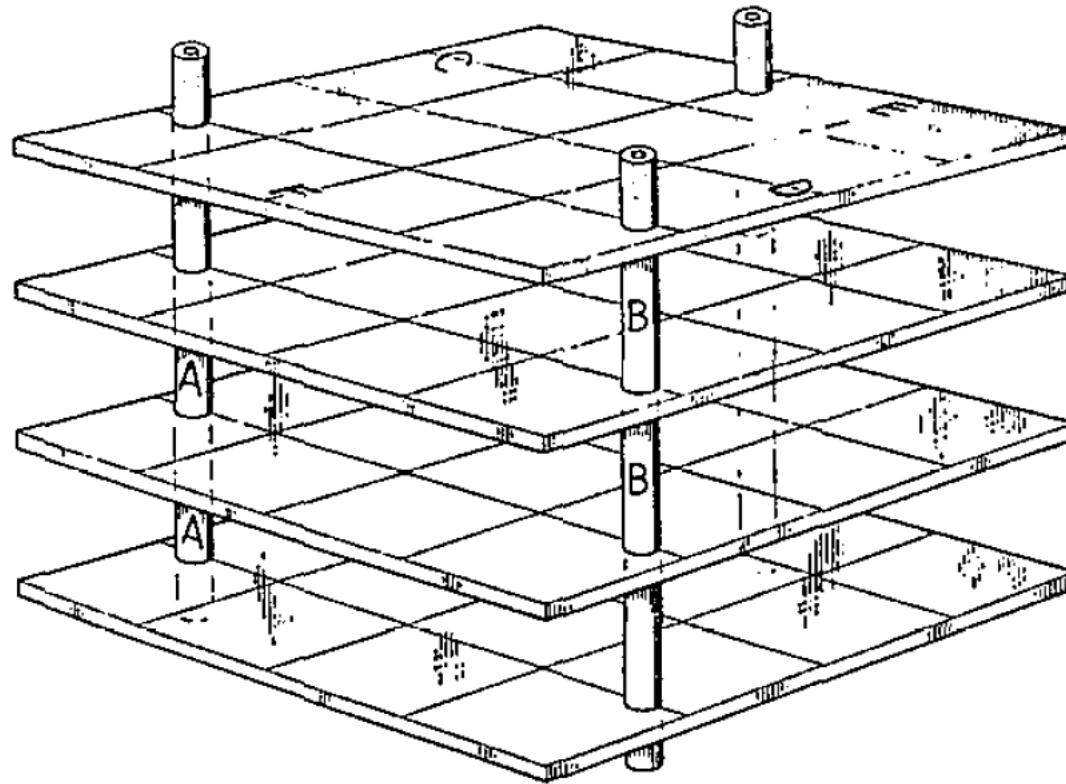
# A five-variable Karnaugh map



[ Figure 2.55 from the textbook ]

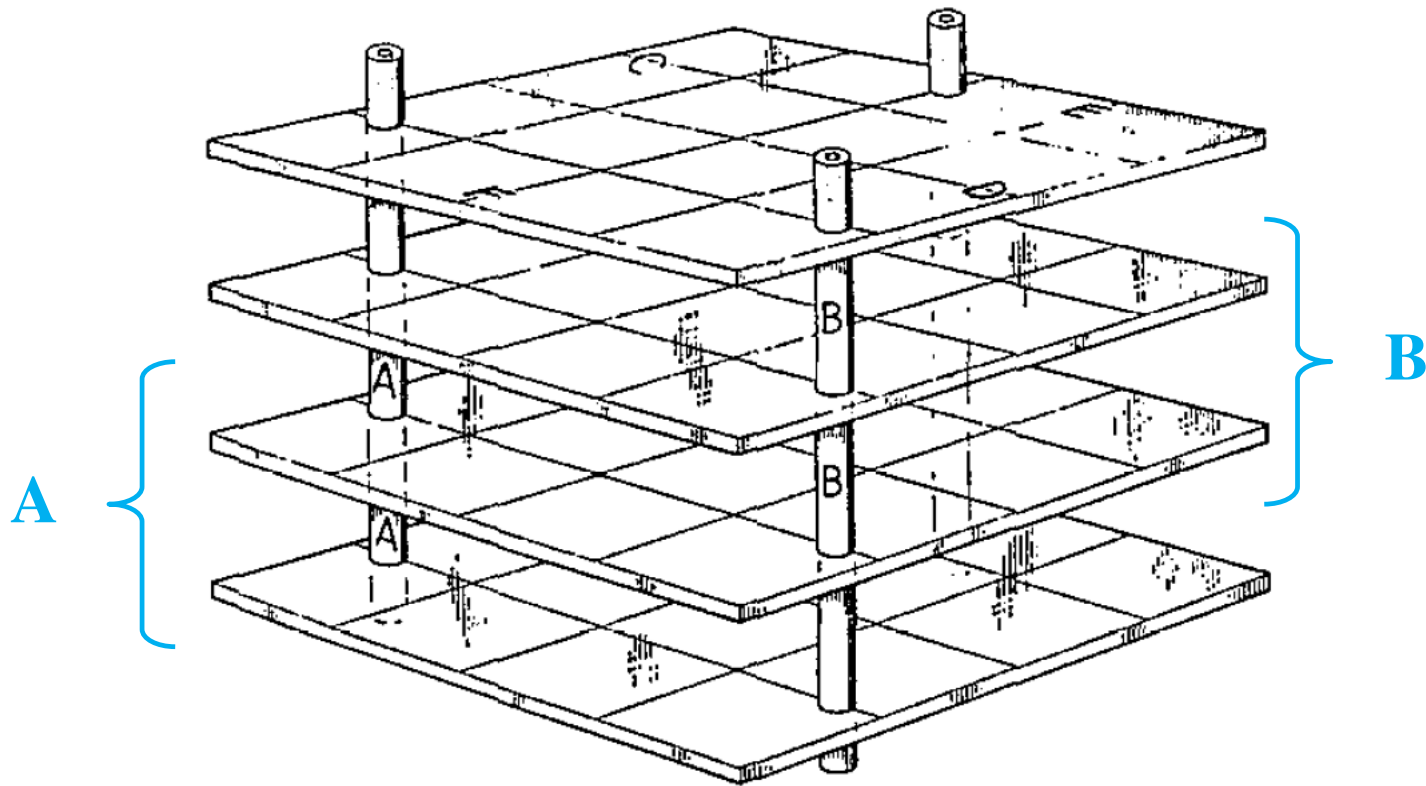
# **Six-Variable K-Map**

# A six-variable Karnaugh map



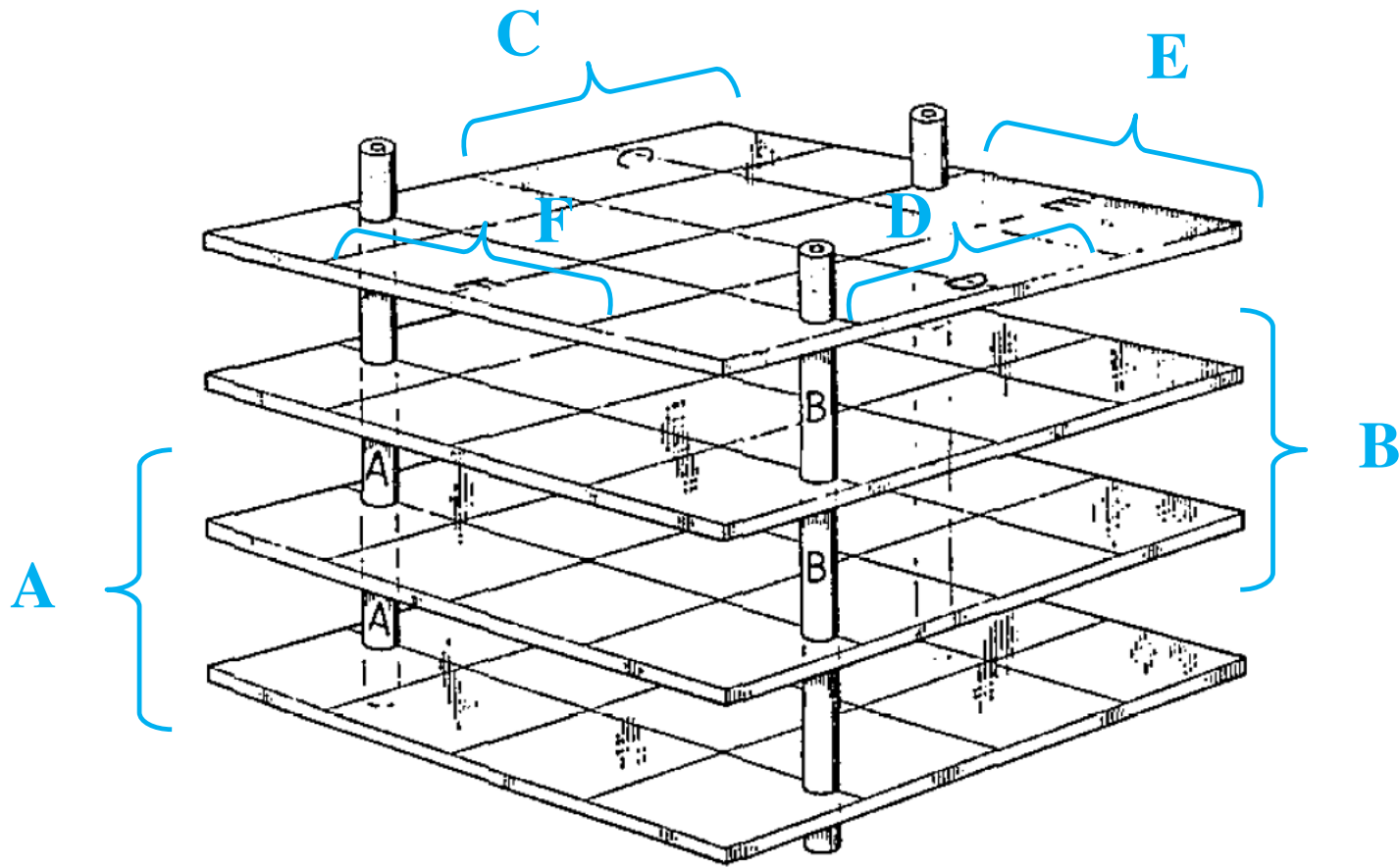
[ Figure 16, in Karnaugh 1953]

# A six-variable Karnaugh map



[ Figure 16, in Karnaugh 1953]

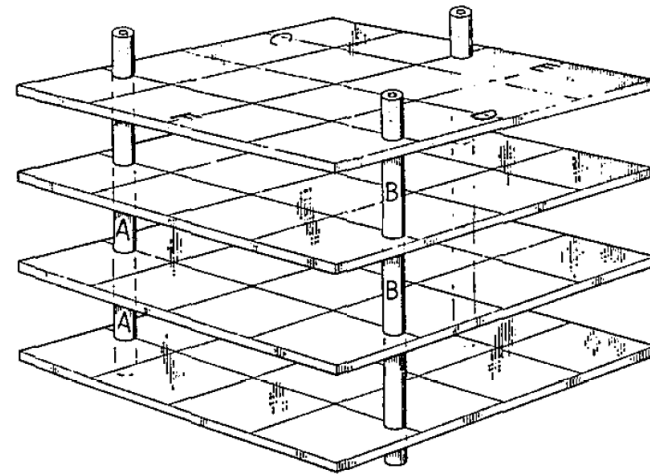
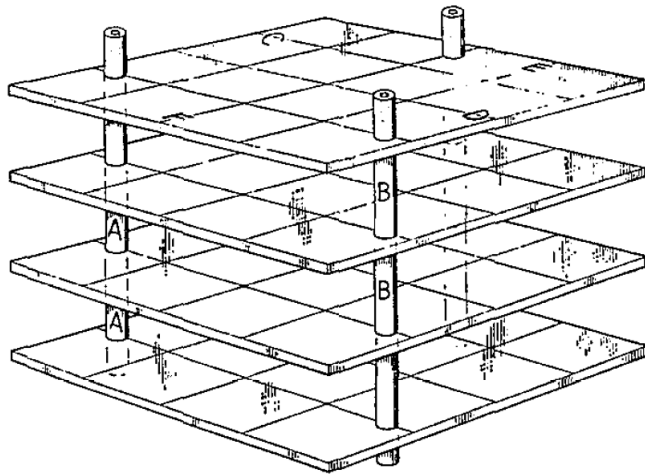
# A six-variable Karnaugh map



[ Figure 16, in Karnaugh 1953]

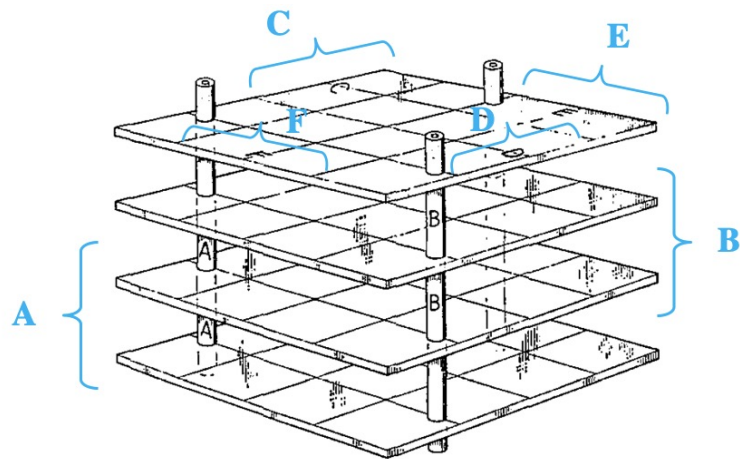
# Seven-Variable K-Map

# A seven-variable Karnaugh map

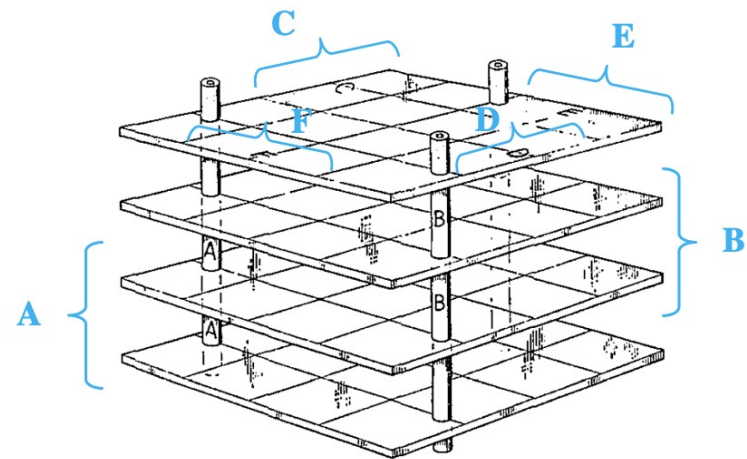




# A seven-variable Karnaugh map



**G = 0**



**G = 1**

[Suggested in Karnaugh 1953]

**Questions?**

**THE END**