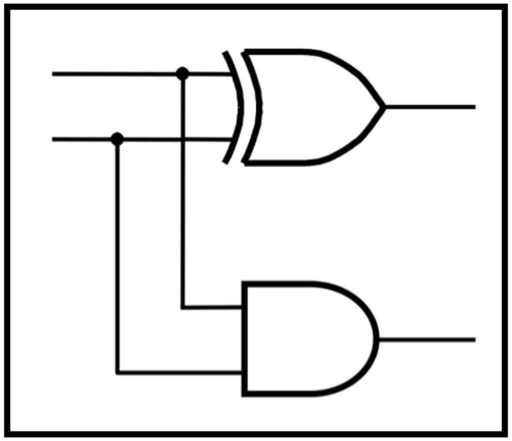


# CprE 281:
# Digital Logic

**Instructor: Alexander Stoytchev**

**http://www.ece.iastate.edu/~alexs/classes/**

# Synthesis
# Using AND, OR, and NOT Gates

*CprE 281: Digital Logic*
*Iowa State University, Ames, IA*

# Administrative Stuff

- HW2 is due on Wednesday Sep 6 @ 10pm

- Please write clearly on the first page the following three things:

  - Your First and Last Name
  - Your Student ID Number
  - Your Lab Section Letter

- Submit on Canvas as *one* PDF file.

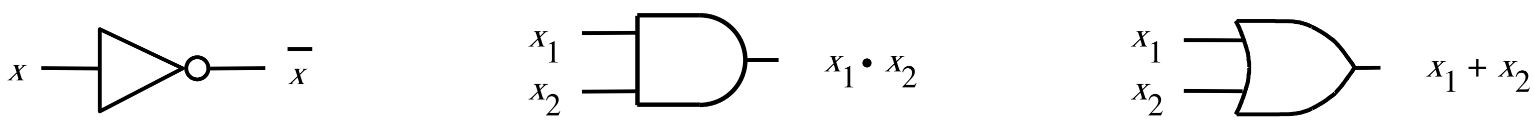
- Please orient your pages such that the text can be read without the need to rotate the page.

# Administrative Stuff

- Next week we will start with Lab2
- Read the lab assignment and do the prelab at home.
- Complete the prelab on paper before you go to the lab. Otherwise you'll lose 20% of your grade for that lab.

# Quick Review

# The Three Basic Logic Gates



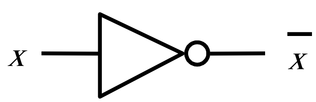

NOT gate

AND gate

OR gate

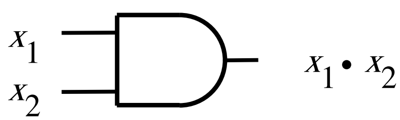
[ Figure 2.8 from the textbook ]

# Truth Table for NOT



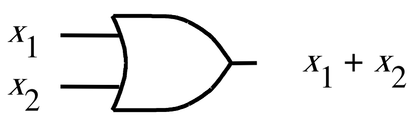

| $x$ | $\overline{x}$ |
|---|---|
| 0 | 1 |
| 1 | 0 |

# Truth Table for AND




| $x_1$ | $x_2$ | $x_1 \cdot x_2$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

# Truth Table for OR



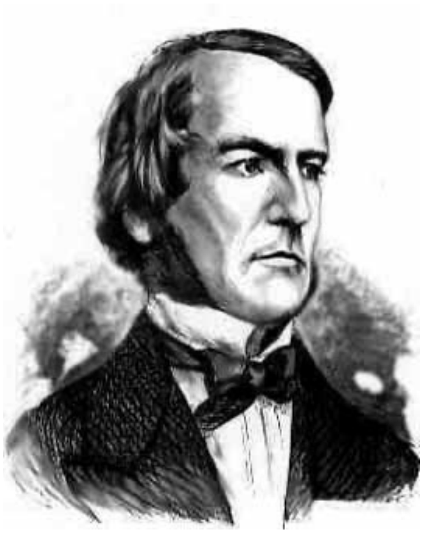

| $x_1$ | $x_2$ | $x_1 + x_2$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

# Truth Tables for AND and OR

| $x_1$ | $x_2$ | $x_1 \cdot x_2$ | $x_1 + x_2$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |
| | | AND | OR |

[ Figure 2.6b from the textbook ]

# Boolean Algebra



George Boole
1815-1864

- **An algebraic structure consists of**
  - **a set of elements {0, 1}**
  - **binary operators {+, •}**
  - **and a unary operator { ' } or { ¯ } or { ~ }**
- **Introduced by George Boole in 1854**
- **An effective means of describing circuits built with switches**
- **A powerful tool that can be used for designing and analyzing logic circuits**

# Different Notations for Negation

- **All three of these mean "negate x"**
  - $x'$
  - $\overline{x}$
  - $\sim x$

# Operator Precedence

- In regular arithmetic and algebra, multiplication takes precedence over addition.
- This is also true in Boolean algebra.
- For example, $x + y \bullet z$ means

  multiply y by z and add the product to x .
- In other words, $x + y \bullet z$ is equal to $x + (y \bullet z)$ ,

  not $(x + y) \bullet z$ .

# The multiplication dot is optional

- In regular algebra, the multiplication operator is often omitted to shorten the equations.
- This is also true in Boolean algebra.
- Both of these mean the same thing:

  $xy$ is equal to $x \cdot y$

# Operator Precedence
# (three different ways to write the same)

$$x_1 \cdot x_2 + \overline{x}_1 \cdot \overline{x}_2$$

$$(x_1 \cdot x_2) + ((\overline{x}_1) \cdot (\overline{x}_2))$$

$$x_1 x_2 + \overline{x}_1 \overline{x}_2$$

# Operator Precedence

- **Negation of a single variable takes precedence over multiplication of that variable with another variable.**

- **For example,**

  **$\overline{A}$ B means negate A first and then multiply $\overline{A}$ by B**

# Operator Precedence

- However, a horizontal bar over a product of two variables means that the negation is performed after the product is computed.

- For example,

  $\overline{A\,B}$ means multiply A and B and then negate

# Operator Precedence

- Note that these two expressions are different:

$\overline{A\,B}$ is not equal to $\overline{A}\,\overline{B}$

$\overline{A\,B}$ means multiply A and B and then negate

$\overline{A}\,\overline{B}$ means negate A and B separately and then multiply

# Operator Precedence

- **Note that these two expressions are different:**

$$\overline{A\,B} \text{ is not equal to } \overline{A}\ \overline{B}$$

| A | B | $\overline{AB}$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

| A | B | $\overline{A}\,\overline{B}$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

# DeMorgan's Theorem

$$\text{15a.} \quad \overline{x \bullet y} = \overline{x} + \overline{y}$$

$$\text{15b.} \quad \overline{x + y} = \overline{x} \bullet \overline{y}$$

# Proof of DeMorgan's theorem

15a. $\overline{x \cdot y} = \overline{x} + \overline{y}$

| $x$ | $y$ | $x \cdot y$ | $\overline{x \cdot y}$ | $\bar{x}$ | $\bar{y}$ | $\bar{x} + \bar{y}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |

LHS RHS

# Proof of DeMorgan's theorem

**15a.** $\overline{x \cdot y} = \overline{x} + \overline{y}$

| $x$ | $y$ | $x \cdot y$ | $\overline{x \cdot y}$ | $\overline{x}$ | $\overline{y}$ | $\overline{x} + \overline{y}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |

LHS RHS

These two columns are equal. Therefore, the theorem is true.

# How to remember DeMorgan's theorem

$$\overline{x \bullet y}$$

start with the
left-hand side

# How to remember DeMorgan's theorem

$$\overline{x \cdot y}$$

divide the bar
into 3 equal parts

# How to remember DeMorgan's theorem

$$\overline{x} \bullet \overline{y}$$

erase the
middle segment

# How to remember DeMorgan's theorem

$$\overline{x} + \overline{y}$$

change the
product to a sum

# How to remember DeMorgan's theorem

$$\overline{x} + \overline{y}$$

this is the
right-hand side

# How to remember DeMorgan's theorem

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

# Proof of the other DeMorgan's theorem

**15b.** $\overline{x + y} = \overline{x} \bullet \overline{y}$

| $x$ | $y$ | $x + y$ | $\overline{x + y}$ | $\overline{x}$ | $\overline{y}$ | $\overline{x} \bullet \overline{y}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |

LHS RHS

# Proof of the other DeMorgan's theorem

**15b.** $\overline{x + y} = \overline{x} \bullet \overline{y}$

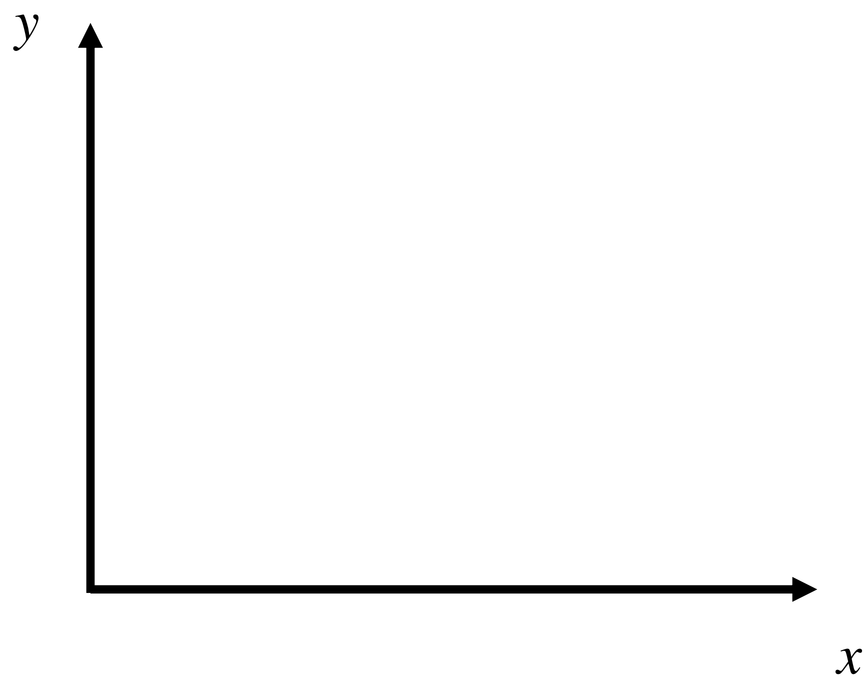
| $x$ | $y$ | $x+y$ | $\overline{x+y}$ | $\bar{x}$ | $\bar{y}$ | $\bar{x} \bullet \bar{y}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |

LHS RHS

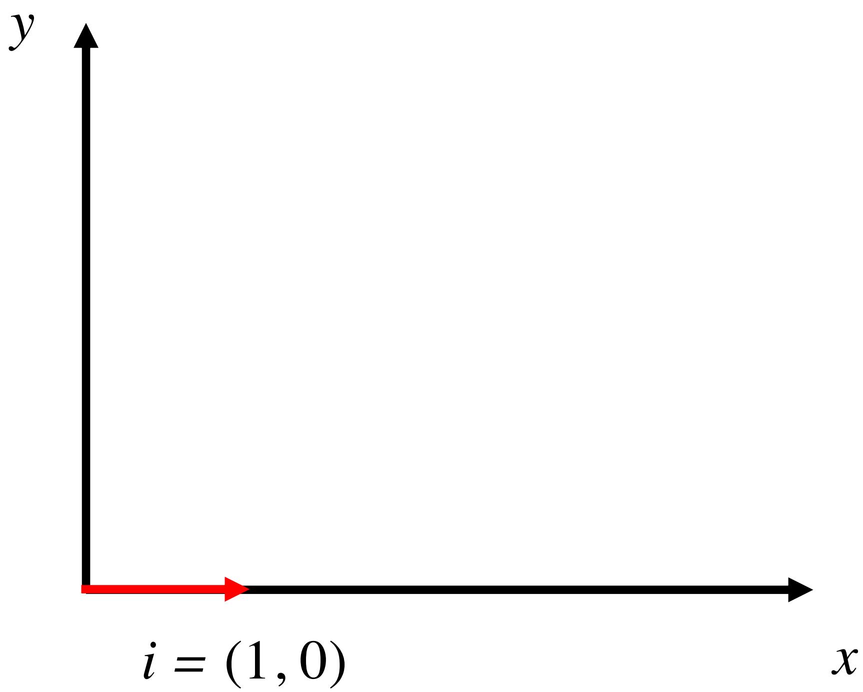
These two columns are equal. Therefore, the theorem is true.

# A Short Digression

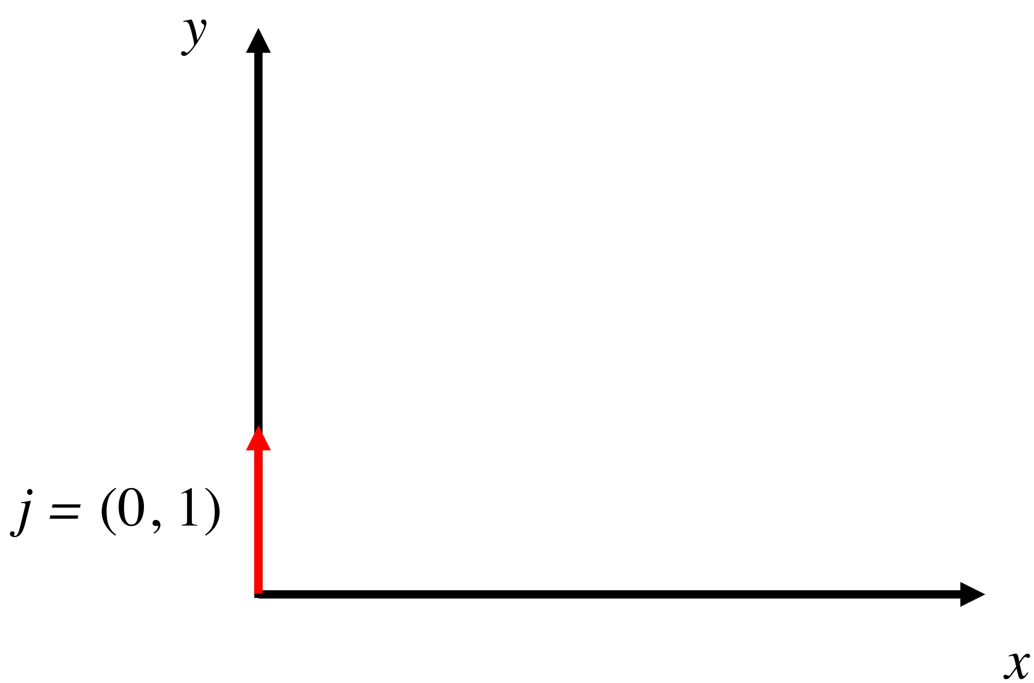
# The 2D Plane



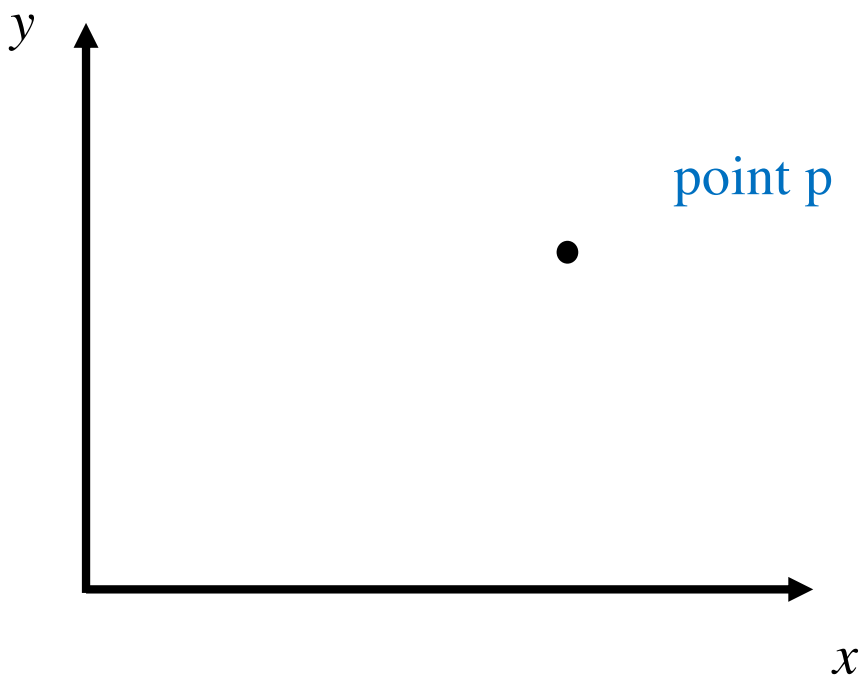
# The 2D Plane



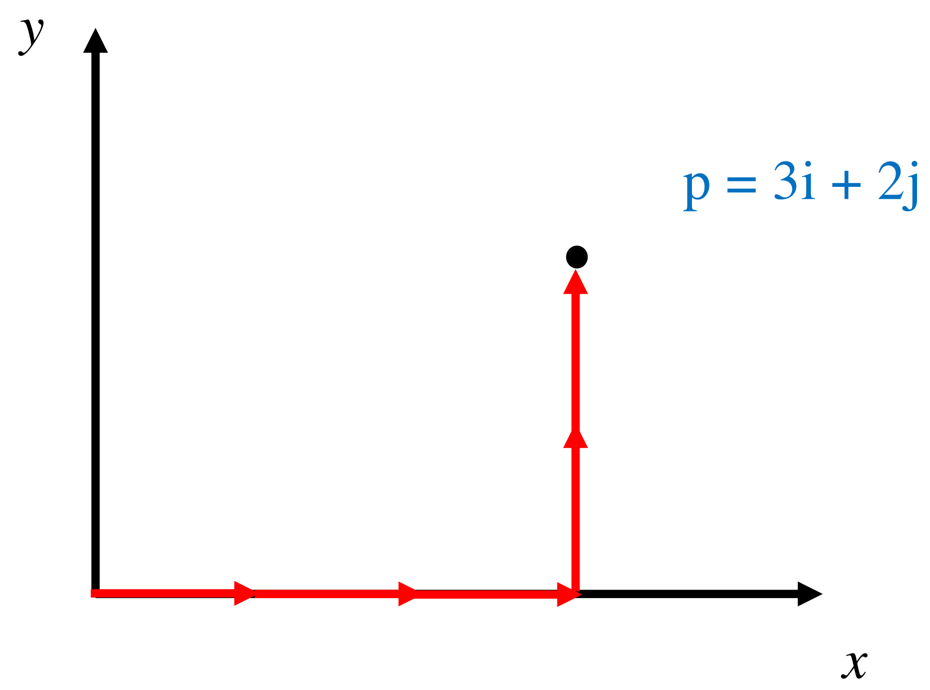
# The 2D Plane



# The 2D Plane



# The 2D Plane



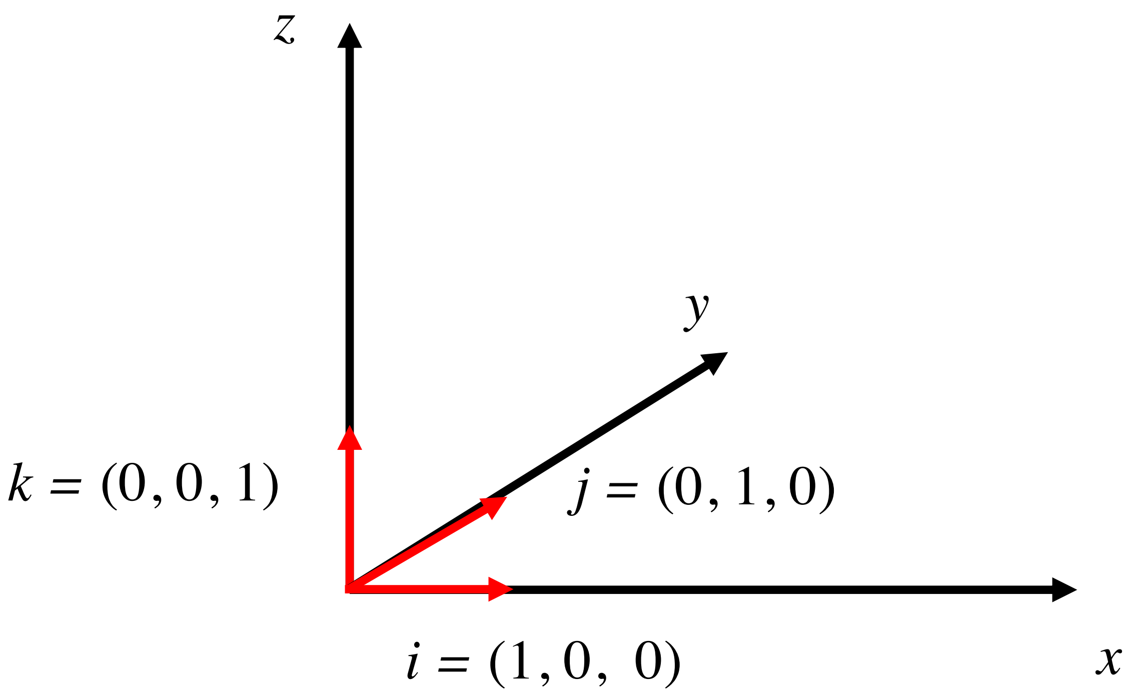
# The unit vectors `i` and `j` form a basis

- **Any point in the 2D plane can be represented as a linear combination of these two vectors.**

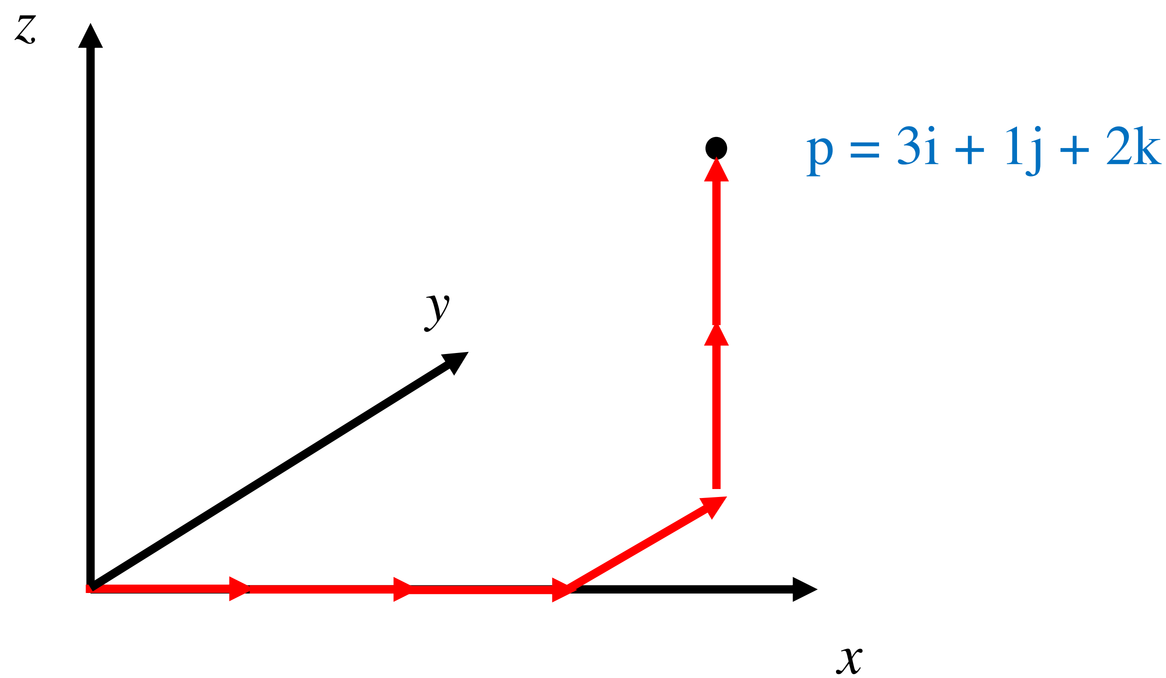
```
i=(1, 0)
j=(0, 1)
```

**Note that there is only one 1 in each.**

# 3D Space



# 3D Space



# The 3D Basis

- **In 3D we have `i, j,` and `k`**

```
i=(1, 0, 0)
j=(0, 1, 0)
k=(0, 0, 1)
```

**Note that there is only one 1 in each.**

**Any point in the 3D space can be represented as a linear combination of these three basis vectors.**

# The 4D Basis

- In 4D we have `four vectors`

$$x^1 = (1, 0, 0, 0)$$
$$x^2 = (0, 1, 0, 0)$$
$$x^3 = (0, 0, 1, 0)$$
$$x^4 = (0, 0, 0, 1)$$

Note that there is only one 1 in each.

Any point in this 4D space can be represented as a linear combination of these four basis vectors.

# Basis Functions
# (for two variables)

# The Four Basis Functions

| x | y | $f_{00}$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

$f_{00}(x, y)$

| x | y | $f_{01}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

$f_{01}(x, y)$

| x | y | $f_{10}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

$f_{10}(x, y)$

| x | y | $f_{11}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$f_{11}(x, y)$

# The Four Basis Functions

| x | y | $f_{00}$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

$f_{00}(x, y)$

| x | y | $f_{01}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

$f_{01}(x, y)$

| x | y | $f_{10}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

$f_{10}(x, y)$

| x | y | $f_{11}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$f_{11}(x, y)$

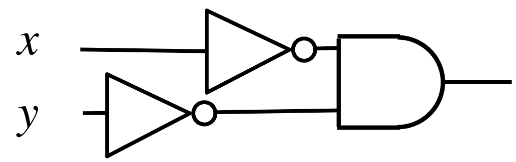
# The Four Basis Functions

| x | y | | $f_{00}(x, y)$ | $f_{01}(x, y)$ | $f_{10}(x, y)$ | $f_{11}(x, y)$ |
|---|---|---|---|---|---|---|
| 0 | 0 | | 1 | 0 | 0 | 0 |
| 0 | 1 | | 0 | 1 | 0 | 0 |
| 1 | 0 | | 0 | 0 | 1 | 0 |
| 1 | 1 | | 0 | 0 | 0 | 1 |

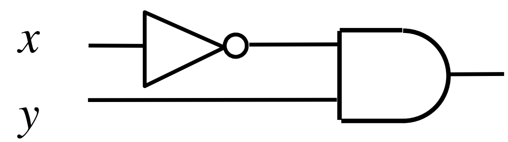
# The Four Basis Functions

| x | y | | $\overline{x}\,\overline{y}$ | $\overline{x}\,y$ | $x\,\overline{y}$ | $x\,y$ |
|---|---|---|---|---|---|---|
| 0 | 0 | | 1 | 0 | 0 | 0 |
| 0 | 1 | | 0 | 1 | 0 | 0 |
| 1 | 0 | | 0 | 0 | 1 | 0 |
| 1 | 1 | | 0 | 0 | 0 | 1 |

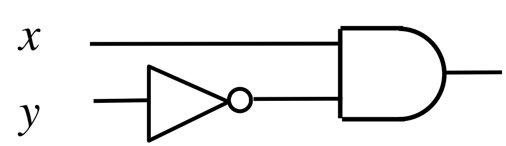
# Circuits for the four basis functions



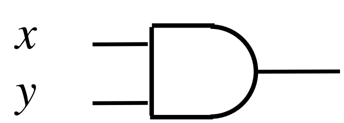

$$f_{00}(x, y) = \overline{x}\ \overline{y}$$




$$f_{01}(x, y) = \overline{x}\ y$$




$$f_{10}(x, y) = x\ \overline{y}$$




$$f_{11}(x, y) = x\ y$$

# The Four Basis Functions

| x | y | $f_{00}$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

| x | y | $f_{01}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

| x | y | $f_{10}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

| x | y | $f_{11}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$f_{00}(x, y) = \overline{x}\ \overline{y}$ $\quad f_{01}(x, y) = \overline{x}\ y$ $\quad f_{10}(x, y) = x\ \overline{y}$ $\quad f_{11}(x, y) = x\ y$

# The Four Basis Functions

| x | y | $f_{00}$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

| x | y | $f_{01}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

| x | y | $f_{10}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

| x | y | $f_{11}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$f_{00}(x, y) = \bar{x}\,\bar{y}$ $\quad f_{01}(x, y) = \bar{x}\,y$ $\quad f_{10}(x, y) = x\,\bar{y}$ $\quad f_{11}(x, y) = x\,y$

Negate the variable if the corresponding subscript of f is 0.

# The Four Basis Functions

| x | y | $f_{00}$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

| x | y | $f_{01}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

| x | y | $f_{10}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

| x | y | $f_{11}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$f_{00}(x, y) = \overline{x}\ \overline{y}$ $\quad f_{01}(x, y) = \overline{x}\ y$ $\quad f_{10}(x, y) = x\ \overline{y}$ $\quad f_{11}(x, y) = x\ y$

Negate the variable if the corresponding subscript of f is 0.

# The Four Basis Functions

| x | y | $f_{00}$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

| x | y | $f_{01}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

| x | y | $f_{10}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

| x | y | $f_{11}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$f_{00}(x, y) = \overline{x}\ \overline{y}$ $f_{01}(x, y) = \overline{x}\ y$ $f_{10}(x, y) = x\ \overline{y}$ $f_{11}(x, y) = x\ y$

Negate the variable if the corresponding subscript of f is 0.

**minterms**
**(an alternative name for**
**the set of basis functions)**

# The Four Basis Functions

| x | y | $f_{00}$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

| x | y | $f_{01}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

| x | y | $f_{10}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

| x | y | $f_{11}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$f_{00}(x, y) = \overline{x}\,\overline{y}$ $f_{01}(x, y) = \overline{x}\,y$ $f_{10}(x, y) = x\,\overline{y}$ $f_{11}(x, y) = x\,y$

# The Four Basis Functions (alternative names)

| x | y | $f_{00}$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

| x | y | $f_{01}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

| x | y | $f_{10}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

| x | y | $f_{11}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$f_{00}(x, y) = \overline{x}\,\overline{y}$ $f_{01}(x, y) = \overline{x}\,y$ $f_{10}(x, y) = x\,\overline{y}$ $f_{11}(x, y) = x\,y$

$m_0$ $m_1$ $m_2$ $m_3$

# The Four Basis Functions
# ( minterms )

| x | y | $m_0$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

$f_{00}(x, y) = \overline{x}\,\overline{y}$

$m_0$

| x | y | $m_1$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

$f_{01}(x, y) = \overline{x}\,y$

$m_1$

| x | y | $m_2$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

$f_{10}(x, y) = x\,\overline{y}$

$m_2$

| x | y | $m_3$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$f_{11}(x, y) = x\,y$

$m_3$

# The Four Basis Functions
# ( minterms )

| x | y | | $\bar{x}\,\bar{y}$ | $\bar{x}\,y$ | $x\,\bar{y}$ | $x\,y$ |
|---|---|---|---|---|---|---|
| 0 | 0 | | 1 | 0 | 0 | 0 |
| 0 | 1 | | 0 | 1 | 0 | 0 |
| 1 | 0 | | 0 | 0 | 1 | 0 |
| 1 | 1 | | 0 | 0 | 0 | 1 |

# Expressions for the minterms

$m_0 = \overline{x}\ \overline{y}$

$m_1 = \overline{x}\ y$

$m_2 = x\ \overline{y}$

$m_3 = x\ y$

# Expressions for the minterms

| | | |
|---|---|---|
| 0 0 | $m_0 = \bar{x}\ \bar{y}$ | |
| 0 1 | $m_1 = \bar{x}\ y$ | |
| 1 0 | $m_2 = x\ \bar{y}$ | |
| 1 1 | $m_3 = x\ y$ | |

The bars coincide
with the 0's
in the binary expansion
of the minterm sub-index

# Expressions for the minterms

| | | | |
|---|---|---|---|
| 0 | 0 | | $m_0 = \overline{x}\ \overline{y}$ |
| 0 | 1 | | $m_1 = \overline{x}\ y$ |
| 1 | 0 | | $m_2 = x\ \overline{y}$ |
| 1 | 1 | | $m_3 = x\ y$ |

The bars coincide
with the 0's
in the binary expansion
of the minterm sub-index

# Function Synthesis Example
# (with two variables)

# Synthesize the Following Function

| $x_1$ | $x_2$ | $f(x_1, x_2)$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

# 1) Split the function into a sum of 4 functions

| $x_1$ | $x_2$ | $f(x_1, x_2)$ | | $f_{00}(x_1, x_2)$ | $f_{01}(x_1, x_2)$ | $f_{10}(x_1, x_2)$ | $f_{11}(x_1, x_2)$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | | 0 | 0 | 0 | 1 |

# 1) Split the function into a sum of 4 functions

| $x_1$ | $x_2$ | $f(x_1, x_2)$ | | $f_{00}(x_1, x_2)$ | $f_{01}(x_1, x_2)$ | $f_{10}(x_1, x_2)$ | $f_{11}(x_1, x_2)$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | | 0 | 0 | 0 | 1 |

$$f(x_1, x_2) = 1 \bullet f_{00} \quad + \quad 1 \bullet f_{01} \quad + \quad 0 \bullet f_{10} \quad + \quad 1 \bullet f_{11}$$

# 2) Write the expressions for all four

| $x_1$ | $x_2$ | $f(x_1, x_2)$ | | $f_{00}(x_1, x_2)$ | $f_{01}(x_1, x_2)$ | $f_{10}(x_1, x_2)$ | $f_{11}(x_1, x_2)$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | | $\boxed{1}$ | 0 | 0 | 0 |
| 0 | 1 | 1 | | 0 | $\boxed{1}$ | 0 | 0 |
| 1 | 0 | 0 | | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | | 0 | 0 | 0 | $\boxed{1}$ |

$$f(x_1, x_2) = \underbrace{1 \bullet f_{00}} + \underbrace{1 \bullet f_{01}} + \underbrace{0 \bullet f_{10}} + \underbrace{1 \bullet f_{11}}$$

# 2) Write the expressions for all four

| $x_1$ | $x_2$ | $f(x_1, x_2)$ | | $f_{00}(x_1, x_2)$ | $f_{01}(x_1, x_2)$ | $f_{10}(x_1, x_2)$ | $f_{11}(x_1, x_2)$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | | 0 | 0 | 0 | 1 |

$$f(x_1, x_2) = \underbrace{1 \bullet f_{00}}_{\bar{x}_1\bar{x}_2} + \underbrace{1 \bullet f_{01}}_{\bar{x}_1 x_2} + \underbrace{0 \bullet f_{10}}_{0} + \underbrace{1 \bullet f_{11}}_{x_1 x_2}$$

# 3) Then just add them together

| $x_1$ | $x_2$ | $f(x_1, x_2)$ | $f_{00}(x_1, x_2)$ | $f_{01}(x_1, x_2)$ | $f_{10}(x_1, x_2)$ | $f_{11}(x_1, x_2)$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 |

$$f(x_1, x_2) = 1 \bullet f_{00} + 1 \bullet f_{01} + 0 \bullet f_{10} + 1 \bullet f_{11}$$

$$f(x_1, x_2) = \bar{x}_1\bar{x}_2 + \bar{x}_1 x_2 + 0 + x_1 x_2$$

# 3) Then just add them together

| $x_1$ | $x_2$ | $f(x_1, x_2)$ | | $f_{00}(x_1, x_2)$ | $f_{01}(x_1, x_2)$ | $f_{10}(x_1, x_2)$ | $f_{11}(x_1, x_2)$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | | 0 | 0 | 0 | 1 |

$$f(x_1, x_2) = \bar{x}_1\bar{x}_2 + \bar{x}_1 x_2 + 0 + x_1 x_2$$

# A function to be synthesized

| $x_1$ | $x_2$ | $f(x_1, x_2)$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

[ Figure 2.19 from the textbook ]

**Let's look at it row by row.**
**How can we express the last row?**

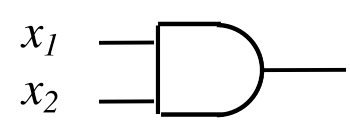
| $x_1$ | $x_2$ | $f(x_1, x_2)$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

**Let's look at it row by row.**
**How can we express the last row?**

| $x_1$ | $x_2$ | $f(x_1, x_2)$ | |
|---|---|---|---|
| 0 | 0 | 1 | |
| 0 | 1 | 1 | |
| 1 | 0 | 0 | |
| 1 | 1 | $\boxed{1}$ | $x_1x_2$ |

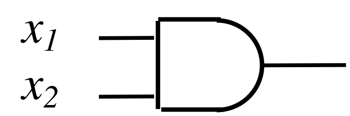
# Let's look at it row by row.
# How can we express the last row?

| $x_1$ | $x_2$ | $f(x_1, x_2)$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |



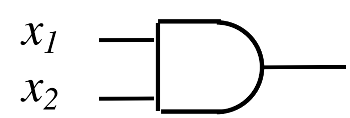
# What about this row?

| $x_1$ | $x_2$ | $f(x_1, x_2)$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |



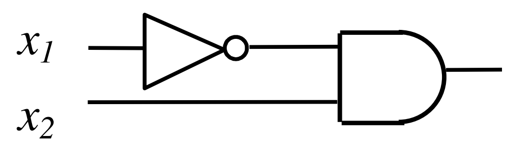
# What about this row?

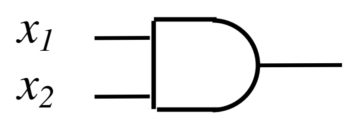
| $x_1$ | $x_2$ | $f(x_1, x_2)$ | |
|---|---|---|---|
| 0 | 0 | 1 | |
| 0 | 1 | $\boxed{1}$ | $\bar{x}_1 x_2$ |
| 1 | 0 | 0 | |
| 1 | 1 | 1 | |



# What about this row?

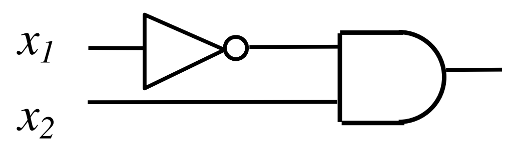
| $x_1$ | $x_2$ | $f(x_1, x_2)$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

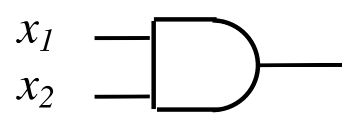


# What about the first row?

| $x_1$ | $x_2$ | $f(x_1, x_2)$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

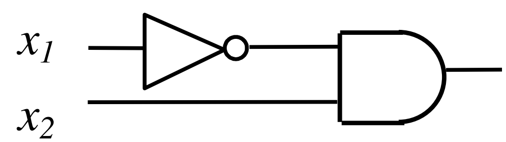


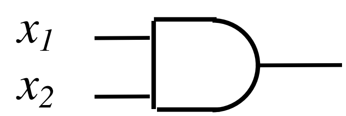



# What about the first row?

| $x_1$ | $x_2$ | $f(x_1, x_2)$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

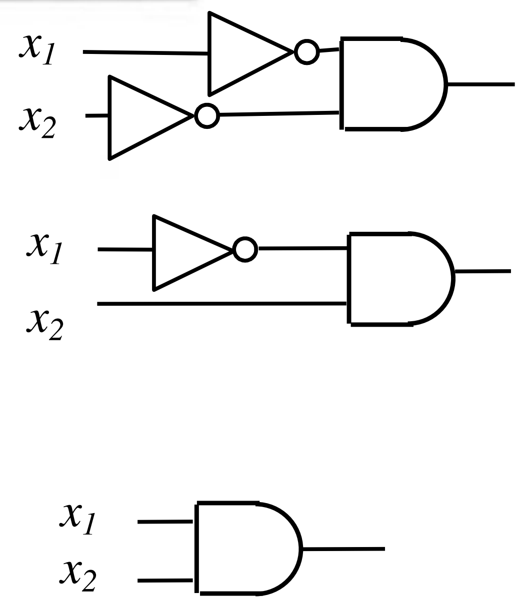
$\overline{x}_1\overline{x}_2$



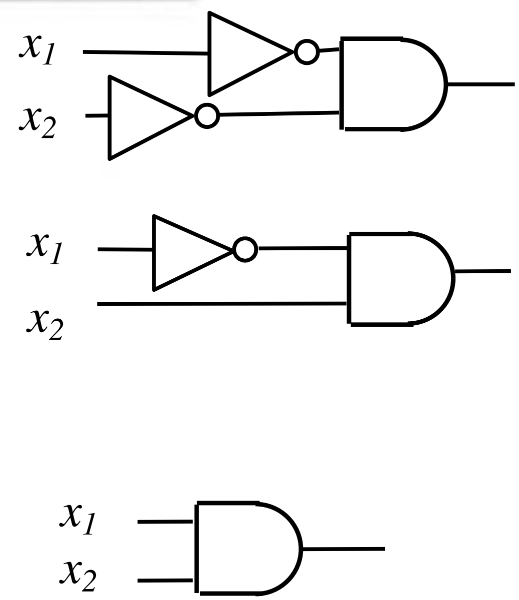
# What about the first row?

| $x_1$ | $x_2$ | $f(x_1, x_2)$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

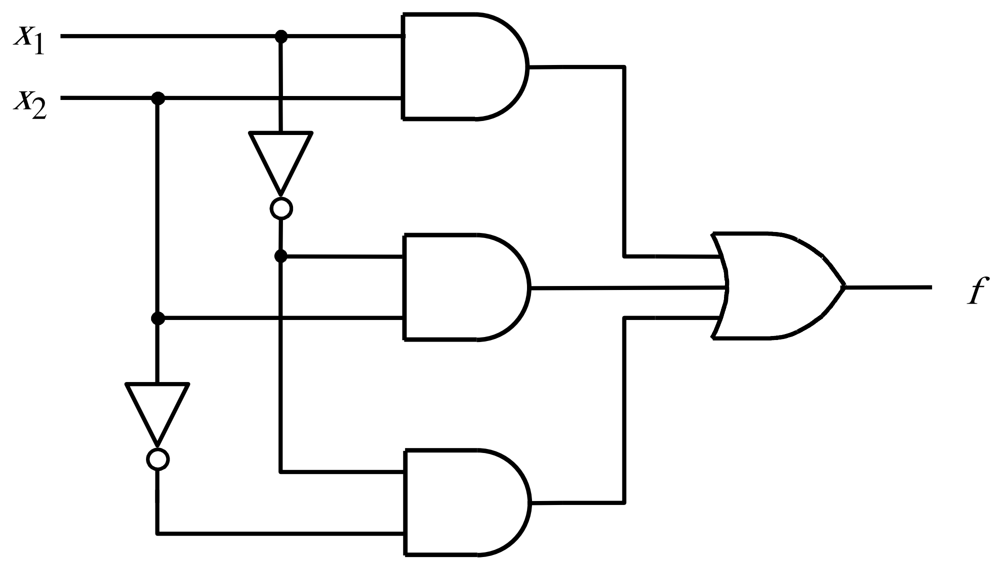


# Finally, what about the zero?

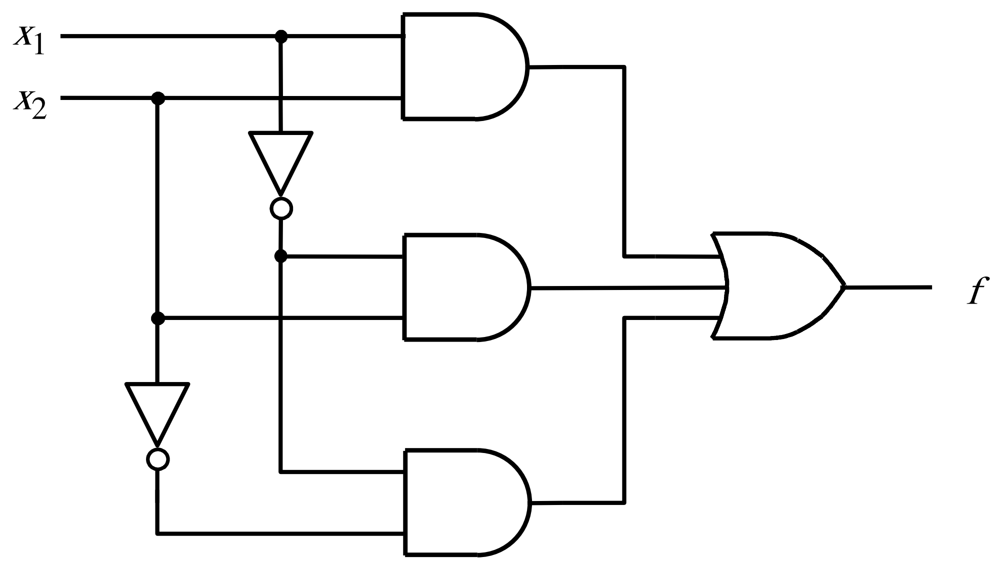
| $x_1$ | $x_2$ | $f(x_1, x_2)$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |



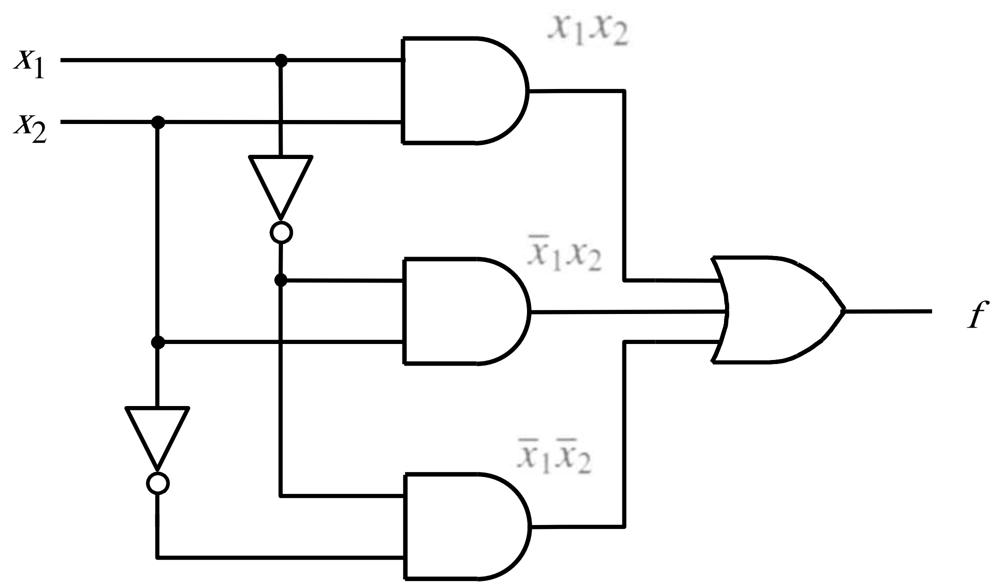
# Putting it all together



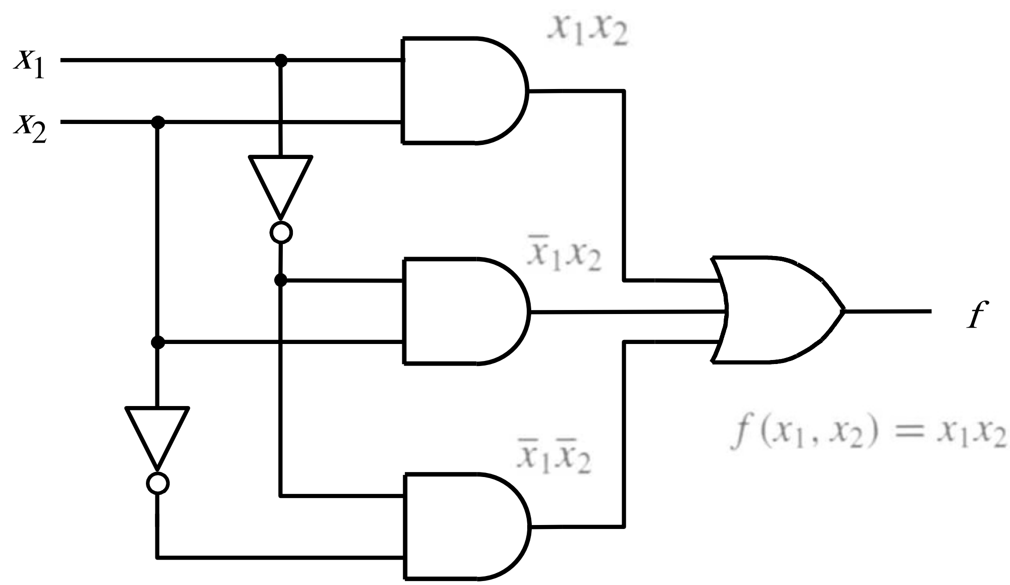
# Let's verify that this circuit implements correctly the target truth table



# Let’s verify that this circuit implements correctly the target truth table

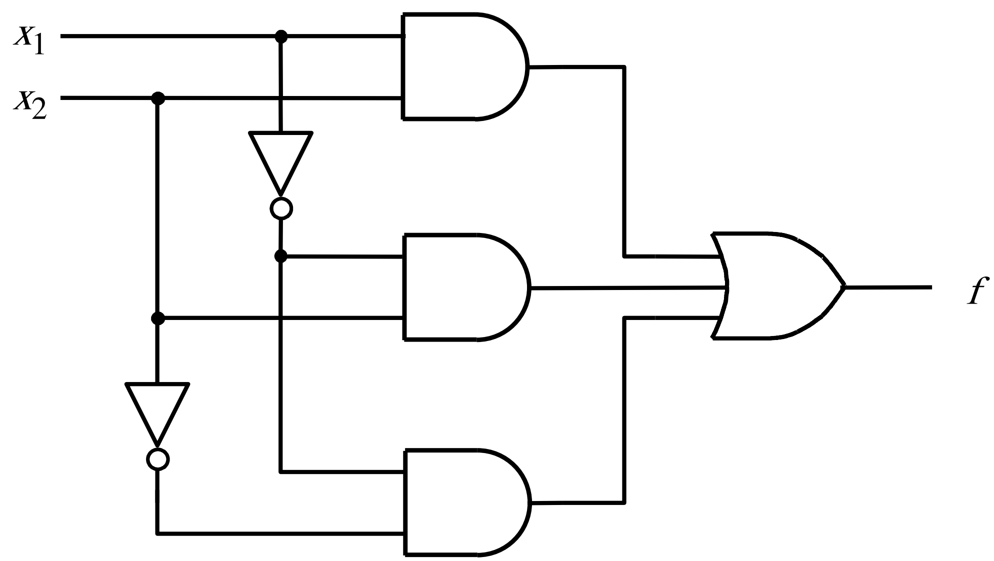


# Putting it all together




$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$

# Canonical Sum-Of-Products (SOP)




$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

[ Figure 2.20a from the textbook ]

# Summary of This Procedure

- **Get the truth table of the function**
- **Form a product term (AND gate) for each row of the table for which the function is 1**
- **Each product term contains all input variables**
- **In each row, if $x_i$=1 enter it as $x_i$, otherwise use $\overline{x_i}$**
- **Sum all of these products (OR gate) to get the function**

# Simplification Steps

$$f(x_1, x_2) = x_1 x_2 + \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2$$

# Simplification Steps

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \boxed{\bar{x}_1x_2}$$

replicate
this term

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2 + \bar{x}_1x_2$$

# Simplification Steps

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

group
these terms

$$f(x_1, x_2) = \boxed{x_1x_2} + \boxed{\bar{x}_1\bar{x}_2} + \boxed{\bar{x}_1x_2} + \boxed{\bar{x}_1x_2}$$

$$f(x_1, x_2) = (x_1 + \bar{x}_1)x_2 + \bar{x}_1(\bar{x}_2 + x_2)$$

# Simplification Steps

$$f(x_1, x_2) = x_1 x_2 + \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2$$

$$f(x_1, x_2) = x_1 x_2 + \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2 + \bar{x}_1 x_2$$

These two terms are trivially equal to 1

$$f(x_1, x_2) = (\boxed{x_1 + \bar{x}_1})x_2 + \bar{x}_1(\boxed{\bar{x}_2 + x_2})$$

$$f(x_1, x_2) = 1 \cdot x_2 + \bar{x}_1 \cdot 1$$

# Simplification Steps

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2 + \bar{x}_1x_2$$

$$f(x_1, x_2) = (x_1 + \bar{x}_1)x_2 + \bar{x}_1(\bar{x}_2 + x_2)$$

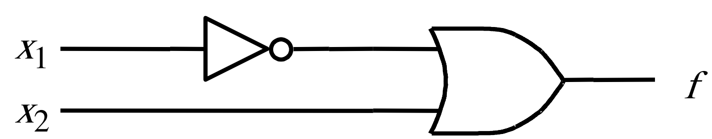
Drop the 1's

$$f(x_1, x_2) = \boxed{1} \cdot x_2 + \bar{x}_1 \cdot \boxed{1}$$

$$f(x_1, x_2) = x_2 + \bar{x}_1$$

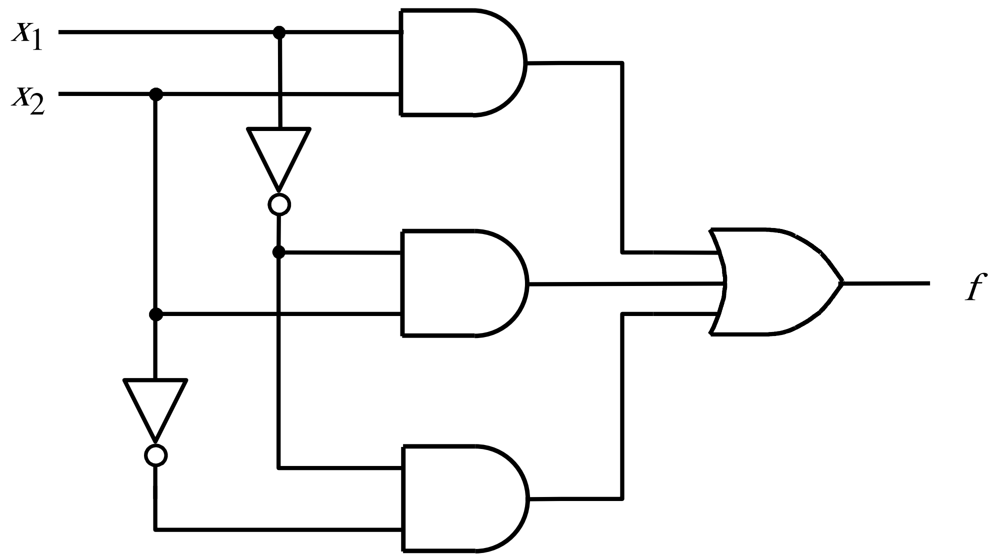
# Minimal-cost realization

$$f(x_1, x_2) = x_2 + \bar{x}_1$$

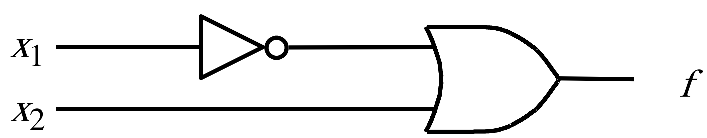



[ Figure 2.20b from the textbook ]

# Two implementations for the same function




(a) Canonical sum-of-products




(b) Minimal-cost realization

[ Figure 2.20 from the textbook ]

# Basis Functions / minterms
# (for three variables)

# The Eight Basis Functions

| x | y | z | $f_{000}$ | $f_{001}$ | $f_{010}$ | $f_{011}$ | $f_{100}$ | $f_{101}$ | $f_{110}$ | $f_{111}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

# The Eight Basis Functions

| x | y | z | $f_{000}$ | $f_{001}$ | $f_{010}$ | $f_{011}$ | $f_{100}$ | $f_{101}$ | $f_{110}$ | $f_{111}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

# The Eight minterms

| x | y | z | $m_0$ | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $m_5$ | $m_6$ | $m_7$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

# Expressions for the minterms

$$m_0 = \bar{x}\ \bar{y}\ \bar{z}$$

$$m_1 = \bar{x}\ \bar{y}\ z$$

$$m_2 = \bar{x}\ y\ \bar{z}$$

$$m_3 = \bar{x}\ y\ z$$

$$m_4 = x\ \bar{y}\ \bar{z}$$

$$m_5 = x\ \bar{y}\ z$$

$$m_6 = x\ y\ \bar{z}$$

$$m_7 = x\ y\ z$$

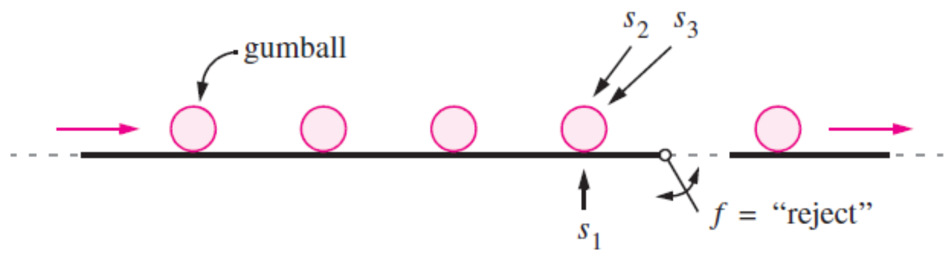
# Expressions for the minterms

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | $m_0 = \overline{x}\ \overline{y}\ \overline{z}$ |
| 0 | 0 | 1 | $m_1 = \overline{x}\ \overline{y}\ z$ |
| 0 | 1 | 0 | $m_2 = \overline{x}\ y\ \overline{z}$ |
| 0 | 1 | 1 | $m_3 = \overline{x}\ y\ z$ |
| 1 | 0 | 0 | $m_4 = x\ \overline{y}\ \overline{z}$ |
| 1 | 0 | 1 | $m_5 = x\ \overline{y}\ z$ |
| 1 | 1 | 0 | $m_6 = x\ y\ \overline{z}$ |
| 1 | 1 | 1 | $m_7 = x\ y\ z$ |

The bars coincide
with the 0's
in the binary expansion
of the minterm sub-index

# Function Synthesis Example
# (with three variables)

# Let’s look at another problem




(a) Conveyor and sensors

| $s_1$ | $s_2$ | $s_3$ | $f$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

(b) Truth table

[ Figure 2.21 from the textbook ]

# Let's look at another problem

| $s_1$ | $s_2$ | $s_3$ | $f$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

[ Figure 2.21b from the textbook ]

# Let's look at another problem

| $s_1$ | $s_2$ | $s_3$ | $f$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

# Let’s look at another problem

| $s_1$ | $s_2$ | $s_3$ | $f$ | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | |
| 0 | 0 | 1 | 1 | $\bar{s}_1\bar{s}_2 s_3$ |
| 0 | 1 | 0 | 0 | |
| 0 | 1 | 1 | 1 | $\bar{s}_1 s_2 s_3$ |
| 1 | 0 | 0 | 0 | |
| 1 | 0 | 1 | 1 | $s_1\bar{s}_2 s_3$ |
| 1 | 1 | 0 | 1 | $s_1 s_2\bar{s}_3$ |
| 1 | 1 | 1 | 1 | $s_1 s_2 s_3$ |

# Let's look at another problem

| $s_1$ | $s_2$ | $s_3$ | $f$ | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | |
| 0 | 0 | 1 | 1 | $\bar{s}_1\bar{s}_2 s_3$ |
| 0 | 1 | 0 | 0 | |
| 0 | 1 | 1 | 1 | $\bar{s}_1 s_2 s_3$ |
| 1 | 0 | 0 | 0 | |
| 1 | 0 | 1 | 1 | $s_1\bar{s}_2 s_3$ |
| 1 | 1 | 0 | 1 | $s_1 s_2\bar{s}_3$ |
| 1 | 1 | 1 | 1 | $s_1 s_2 s_3$ |

# Let’s look at another problem

| $s_1$ | $s_2$ | $s_3$ | $f$ | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | |
| 0 | 0 | 1 | 1 | $\bar{s}_1 \bar{s}_2 s_3$ |
| 0 | 1 | 0 | 0 | |
| 0 | 1 | 1 | 1 | $\bar{s}_1 s_2 s_3$ |
| 1 | 0 | 0 | 0 | |
| 1 | 0 | 1 | 1 | $s_1 \bar{s}_2 s_3$ |
| 1 | 1 | 0 | 1 | $s_1 s_2 \bar{s}_3$ |
| 1 | 1 | 1 | 1 | $s_1 s_2 s_3$ |

# Let's look at another problem

| $s_1$ | $s_2$ | $s_3$ | $f$ | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | |
| $\overline{0}$ | $\overline{0}$ | 1 | 1 | $\overline{s}_1\overline{s}_2 s_3$ |
| 0 | 1 | 0 | 0 | |
| $\overline{0}$ | 1 | 1 | 1 | $\overline{s}_1 s_2 s_3$ |
| 1 | 0 | 0 | 0 | |
| 1 | $\overline{0}$ | 1 | 1 | $s_1\overline{s}_2 s_3$ |
| 1 | 1 | 0 | 1 | $s_1 s_2\overline{s}_3$ |
| 1 | 1 | 1 | 1 | $s_1 s_2 s_3$ |

# Let’s look at another problem

| $s_1$ | $s_2$ | $s_3$ | $f$ | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | |
| $\overline{0}$ | $\overline{0}$ | 1 | 1 | $\overline{s}_1\overline{s}_2s_3$ |
| 0 | 1 | 0 | 0 | |
| $\overline{0}$ | 1 | 1 | 1 | $\overline{s}_1s_2s_3$ |
| 1 | 0 | 0 | 0 | |
| 1 | $\overline{0}$ | 1 | 1 | $s_1\overline{s}_2s_3$ |
| 1 | 1 | $\overline{0}$ | 1 | $s_1s_2\overline{s}_3$ |
| 1 | 1 | 1 | 1 | $s_1s_2s_3$ |

# Let's look at another problem

| $s_1$ | $s_2$ | $s_3$ | $f$ | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | |
| 0 | 0 | 1 | $\boxed{1}$ | $\bar{s}_1\bar{s}_2s_3$ |
| 0 | 1 | 0 | 0 | |
| 0 | 1 | 1 | $\boxed{1}$ | $\bar{s}_1s_2s_3$ |
| 1 | 0 | 0 | 0 | |
| 1 | 0 | 1 | $\boxed{1}$ | $s_1\bar{s}_2s_3$ |
| 1 | 1 | 0 | $\boxed{1}$ | $s_1s_2\bar{s}_3$ |
| 1 | 1 | 1 | $\boxed{1}$ | $s_1s_2s_3$ |

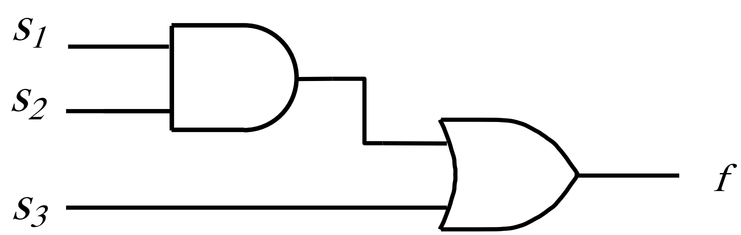
$$f = \bar{s}_1\bar{s}_2s_3 + \bar{s}_1s_2s_3 + s_1\bar{s}_2s_3 + s_1s_2\bar{s}_3 + s_1s_2s_3$$

# Let's look at another problem (minimization)

$$
\begin{aligned}
f &= \bar{s}_1\bar{s}_2 s_3 + \bar{s}_1 s_2 s_3 + s_1\bar{s}_2 s_3 + s_1 s_2 s_3 + s_1 s_2\bar{s}_3 + s_1 s_2 s_3 \\
&= \bar{s}_1 s_3(\bar{s}_2 + s_2) + s_1 s_3(\bar{s}_2 + s_2) + s_1 s_2(\bar{s}_3 + s_3) \\
&= \bar{s}_1 s_3 + s_1 s_3 + s_1 s_2 \\
&= s_3 + s_1 s_2
\end{aligned}
$$

# Let's look at another problem (minimization)

$$
\begin{aligned}
f &= \bar{s}_1\bar{s}_2 s_3 + \bar{s}_1 s_2 s_3 + s_1\bar{s}_2 s_3 + s_1 s_2 s_3 + s_1 s_2 \bar{s}_3 + s_1 s_2 s_3 \\
&= \bar{s}_1 s_3(\bar{s}_2 + s_2) + s_1 s_3(\bar{s}_2 + s_2) + s_1 s_2(\bar{s}_3 + s_3) \\
&= \bar{s}_1 s_3 + s_1 s_3 + s_1 s_2 \\
&= s_3 + s_1 s_2
\end{aligned}
$$



# Maxterms
# (an alternative set of basis functions)

# The Four Maxterms

| x | y | $M_0$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

$M_0(x, y)$

| x | y | $M_1$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

$M_1(x, y)$

| x | y | $M_2$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$M_2(x, y)$

| x | y | $M_3$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

$M_3(x, y)$

# The Four Maxterms

| x | y | $M_0$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

$M_0(x, y)$

| x | y | $M_1$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

$M_1(x, y)$

| x | y | $M_2$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$M_2(x, y)$

| x | y | $M_3$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

$M_3(x, y)$

# The Four Maxterms

| x | y | | $M_0(x, y)$ | $M_1(x, y)$ | $M_2(x, y)$ | $M_3(x, y)$ |
|---|---|---|---|---|---|---|
| 0 | 0 | | $\boxed{0}$ | 1 | 1 | 1 |
| 0 | 1 | | 1 | $\boxed{0}$ | 1 | 1 |
| 1 | 0 | | 1 | 1 | $\boxed{0}$ | 1 |
| 1 | 1 | | 1 | 1 | 1 | $\boxed{0}$ |

# The Four Maxterms

| x | y | | x + y | $x + \overline{y}$ | $\overline{x} + y$ | $\overline{x} + \overline{y}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | | 0 | 1 | 1 | 1 |
| 0 | 1 | | 1 | 0 | 1 | 1 |
| 1 | 0 | | 1 | 1 | 0 | 1 |
| 1 | 1 | | 1 | 1 | 1 | 0 |

# Expressions for the Maxterms

$$M_0 = x + y$$

$$M_1 = x + \overline{y}$$

$$M_2 = \overline{x} + y$$

$$M_3 = \overline{x} + \overline{y}$$

# Expressions for the Maxterms

$M_0 = x + y$

$M_1 = x + \overline{y}$

$M_2 = \overline{x} + y$

$M_3 = \overline{x} + \overline{y}$

Note that these are now sums, not products.

# Expressions for the Maxterms

| | | |
|---|---|---|
| 0 0 | $M_0 = x + y$ | |
| 0 1 | $M_1 = x + \overline{y}$ | |
| 1 0 | $M_2 = \overline{x} + y$ | |
| 1 1 | $M_3 = \overline{x} + \overline{y}$ | |

The bars coincide
with the 1's
in the binary expansion
of the maxterm sub-index

# Expressions for the Maxterms

| | | |
|---|---|---|
| 0 0 | $M_0 = x + y$ | |
| 0 1 | $M_1 = x + \overline{y}$ | |
| 1 0 | $M_2 = \overline{x} + y$ | |
| 1 1 | $M_3 = \overline{x} + \overline{y}$ | |

The bars coincide
with the 1's
in the binary expansion
of the maxterm sub-index

# Minterms and Maxterms

| Row number | $x_1$ | $x_2$ | Minterm | Maxterm |
|---|---|---|---|---|
| 0 | 0 | 0 | $m_0 = \overline{x}_1\overline{x}_2$ | $M_0 = x_1 + x_2$ |
| 1 | 0 | 1 | $m_1 = \overline{x}_1 x_2$ | $M_1 = x_1 + \overline{x}_2$ |
| 2 | 1 | 0 | $m_2 = x_1\overline{x}_2$ | $M_2 = \overline{x}_1 + x_2$ |
| 3 | 1 | 1 | $m_3 = x_1 x_2$ | $M_3 = \overline{x}_1 + \overline{x}_2$ |

# Minterms and Maxterms

| Row number | $x_1$ | $x_2$ | Minterm | Maxterm |
|---|---|---|---|---|
| 0 | 0 | 0 | $m_0 = \overline{x}_1\overline{x}_2$ | $M_0 = x_1 + x_2$ |
| 1 | 0 | 1 | $m_1 = \overline{x}_1 x_2$ | $M_1 = x_1 + \overline{x}_2$ |
| 2 | 1 | 0 | $m_2 = x_1\overline{x}_2$ | $M_2 = \overline{x}_1 + x_2$ |
| 3 | 1 | 1 | $m_3 = x_1 x_2$ | $M_3 = \overline{x}_1 + \overline{x}_2$ |

Use these for Sum-of-Products Minimization
(1's of the function)

Use these for Product-of-Sums Minimization
(0's of the function)

# Minterms and Maxterms

| Row number | $x_1$ | $x_2$ | Minterm | Maxterm |
|---|---|---|---|---|
| 0 | 0 | 0 | $m_0 = \overline{x}_1\overline{x}_2$ | $M_0 = x_1 + x_2$ |
| 1 | 0 | 1 | $m_1 = \overline{x}_1 x_2$ | $M_1 = x_1 + \overline{x}_2$ |
| 2 | 1 | 0 | $m_2 = x_1\overline{x}_2$ | $M_2 = \overline{x}_1 + x_2$ |
| 3 | 1 | 1 | $m_3 = x_1 x_2$ | $M_3 = \overline{x}_1 + \overline{x}_2$ |

Use these for
**Sum-of-Products**
Minimization
(1's of the function)

Use these for
**Product-of-Sums**
Minimization
(0's of the function)

# Sum-of-Products Form

**(uses the ones of the function)**

# Sum-of-Products Form

## (for the AND logic function)

| Row number | $x_1$ | $x_2$ | Minterm | $f(x_1, x_2)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | $m_0 = \overline{x}_1\overline{x}_2$ | 0 |
| 1 | 0 | 1 | $m_1 = \overline{x}_1 x_2$ | 0 |
| 2 | 1 | 0 | $m_2 = x_1\overline{x}_2$ | 0 |
| 3 | 1 | 1 | $m_3 = x_1 x_2$ | 1 |

# Sum-of-Products Form

## (for the AND logic function)

| Row number | $x_1$ | $x_2$ | Minterm | $f(x_1, x_2)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | $m_0 = \overline{x}_1\overline{x}_2$ | 0 |
| 1 | 0 | 1 | $m_1 = \overline{x}_1 x_2$ | 0 |
| 2 | 1 | 0 | $m_2 = x_1\overline{x}_2$ | 0 |
| 3 | 1 | 1 | $m_3 = x_1 x_2$ | 1 |

# Sum-of-Products Form
## (for the AND logic function)

| Row number | $x_1$ | $x_2$ | Minterm | $f(x_1, x_2)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | $m_0 = \overline{x}_1\overline{x}_2$ | 0 |
| 1 | 0 | 1 | $m_1 = \overline{x}_1 x_2$ | 0 |
| 2 | 1 | 0 | $m_2 = x_1\overline{x}_2$ | 0 |
| 3 | 1 | 1 | $m_3 = x_1 x_2$ | 1 |

$$f(x_1, x_2) = m_3 = x_1\, x_2$$

(In this case there is just one product and there is no need for a sum)

# Another Example

# Sum-of-Products Form

| Row number | $x_1$ | $x_2$ | Minterm | $f(x_1, x_2)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | $m_0 = \overline{x}_1\overline{x}_2$ | 1 |
| 1 | 0 | 1 | $m_1 = \overline{x}_1 x_2$ | 1 |
| 2 | 1 | 0 | $m_2 = x_1\overline{x}_2$ | 0 |
| 3 | 1 | 1 | $m_3 = x_1 x_2$ | 1 |

# Sum-of-Products Form

| Row number | $x_1$ | $x_2$ | Minterm | $f(x_1, x_2)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | $m_0 = \overline{x}_1\overline{x}_2$ | 1 |
| 1 | 0 | 1 | $m_1 = \overline{x}_1 x_2$ | 1 |
| 2 | 1 | 0 | $m_2 = x_1\overline{x}_2$ | 0 |
| 3 | 1 | 1 | $m_3 = x_1 x_2$ | 1 |

# Sum-of-Products Form

| Row number | $x_1$ | $x_2$ | Minterm | $f(x_1, x_2)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | $m_0 = \overline{x}_1\overline{x}_2$ | 1 |
| 1 | 0 | 1 | $m_1 = \overline{x}_1 x_2$ | 1 |
| 2 | 1 | 0 | $m_2 = x_1\overline{x}_2$ | 0 |
| 3 | 1 | 1 | $m_3 = x_1 x_2$ | 1 |

$$
\begin{aligned}
f &= m_0 \cdot 1 + m_1 \cdot 1 + m_2 \cdot 0 + m_3 \cdot 1 \\
&= m_0 + m_1 + m_3 \\
&= \bar{x}_1\bar{x}_2 + \bar{x}_1 x_2 + x_1 x_2
\end{aligned}
$$

# Product-of-Sums Form

**(uses the zeros of the function)**

# Product-of-Sums Form
## (for the OR logic function)

| Row number | $x_1$ | $x_2$ | Maxterm | $f(x_1, x_2)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | $M_0 = x_1 + x_2$ | 0 |
| 1 | 0 | 1 | $M_1 = x_1 + \overline{x}_2$ | 1 |
| 2 | 1 | 0 | $M_2 = \overline{x}_1 + x_2$ | 1 |
| 3 | 1 | 1 | $M_3 = \overline{x}_1 + \overline{x}_2$ | 1 |

# Product-of-Sums Form
## (for the OR logic function)

| Row number | $x_1$ | $x_2$ | Maxterm | $f(x_1, x_2)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | $M_0 = x_1 + x_2$ | $\boxed{0}$ |
| 1 | 0 | 1 | $M_1 = x_1 + \overline{x}_2$ | 1 |
| 2 | 1 | 0 | $M_2 = \overline{x}_1 + x_2$ | 1 |
| 3 | 1 | 1 | $M_3 = \overline{x}_1 + \overline{x}_2$ | 1 |

# Product-of-Sums Form
## (for the OR logic function)

| Row number | $x_1$ | $x_2$ | Maxterm | $f(x_1, x_2)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | $M_0 = x_1 + x_2$ | 0 |
| 1 | 0 | 1 | $M_1 = x_1 + \overline{x}_2$ | 1 |
| 2 | 1 | 0 | $M_2 = \overline{x}_1 + x_2$ | 1 |
| 3 | 1 | 1 | $M_3 = \overline{x}_1 + \overline{x}_2$ | 1 |

$$f(x_1, x_2) = M_0 = x_1 + x_2$$

(In this case there is just one sum and there is no need for a product)

# Another Example

# Product-of-Sums Form
## (for this logic function)

| Row number | $x_1$ | $x_2$ | Maxterm | $f(x_1, x_2)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | $M_0 = x_1 + x_2$ | 0 |
| 1 | 0 | 1 | $M_1 = x_1 + \overline{x}_2$ | 1 |
| 2 | 1 | 0 | $M_2 = \overline{x}_1 + x_2$ | 0 |
| 3 | 1 | 1 | $M_3 = \overline{x}_1 + \overline{x}_2$ | 1 |

# Product-of-Sums Form
## (for this logic function)

| Row number | $x_1$ | $x_2$ | Maxterm | $f(x_1, x_2)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | $M_0 = x_1 + x_2$ | 0 |
| 1 | 0 | 1 | $M_1 = x_1 + \overline{x}_2$ | 1 |
| 2 | 1 | 0 | $M_2 = \overline{x}_1 + x_2$ | 0 |
| 3 | 1 | 1 | $M_3 = \overline{x}_1 + \overline{x}_2$ | 1 |

# Product-of-Sums Form
## (for this logic function)

| Row number | $x_1$ | $x_2$ | Maxterm | $f(x_1, x_2)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | $M_0 = x_1 + x_2$ | 0 |
| 1 | 0 | 1 | $M_1 = x_1 + \overline{x}_2$ | 1 |
| 2 | 1 | 0 | $M_2 = \overline{x}_1 + x_2$ | 0 |
| 3 | 1 | 1 | $M_3 = \overline{x}_1 + \overline{x}_2$ | 1 |

$$f(x_1, x_2) = M_0 \bullet M_2 = (x_1 + x_2) \bullet (\overline{x_1} + x_2)$$

# Yet Another Example

# Product-of-Sums Form

| Row number | $x_1$ | $x_2$ | Maxterm | $f(x_1, x_2)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | $M_0 = x_1 + x_2$ | 1 |
| 1 | 0 | 1 | $M_1 = x_1 + \overline{x}_2$ | 1 |
| 2 | 1 | 0 | $M_2 = \overline{x}_1 + x_2$ | $\boxed{0}$ |
| 3 | 1 | 1 | $M_3 = \overline{x}_1 + \overline{x}_2$ | 1 |

We need to minimize using the zeros of the function f.
But let's first minimize the inverse of f, i.e., $\overline{\text{f}}$.

# Product-of-Sums Form

| Row number | $x_1$ | $x_2$ | Maxterm | $f(x_1, x_2)$ | $\overline{f}(x_1, x_2)$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | $M_0 = x_1 + x_2$ | 1 | 0 |
| 1 | 0 | 1 | $M_1 = x_1 + \overline{x}_2$ | 1 | 0 |
| 2 | 1 | 0 | $M_2 = \overline{x}_1 + x_2$ | 0 | 1 |
| 3 | 1 | 1 | $M_3 = \overline{x}_1 + \overline{x}_2$ | 1 | 0 |

# Product-of-Sums Form

| Row number | $x_1$ | $x_2$ | Maxterm | $f(x_1, x_2)$ | $\overline{f}(x_1, x_2)$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | $M_0 = x_1 + x_2$ | 1 | 0 |
| 1 | 0 | 1 | $M_1 = x_1 + \overline{x}_2$ | 1 | 0 |
| 2 | 1 | 0 | $M_2 = \overline{x}_1 + x_2$ | 0 | 1 |
| 3 | 1 | 1 | $M_3 = \overline{x}_1 + \overline{x}_2$ | 1 | 0 |

# Product-of-Sums Form

| Row number | $x_1$ | $x_2$ | Maxterm | $f(x_1, x_2)$ | $\overline{f}(x_1, x_2)$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | $M_0 = x_1 + x_2$ | 1 | 0 |
| 1 | 0 | 1 | $M_1 = x_1 + \overline{x}_2$ | 1 | 0 |
| 2 | 1 | 0 | $M_2 = \overline{x}_1 + x_2$ | 0 | $\boxed{1}$ |
| 3 | 1 | 1 | $M_3 = \overline{x}_1 + \overline{x}_2$ | 1 | 0 |

$$\begin{aligned}\overline{f}(x_1, x_2) &= m_2 \\ &= x_1\overline{x}_2\end{aligned}$$

# Product-of-Sums Form

| Row number | $x_1$ | $x_2$ | Maxterm | $f(x_1, x_2)$ | $\overline{f}(x_1, x_2)$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | $M_0 = x_1 + x_2$ | 1 | 0 |
| 1 | 0 | 1 | $M_1 = x_1 + \overline{x}_2$ | 1 | 0 |
| 2 | 1 | 0 | $M_2 = \overline{x}_1 + x_2$ | $\boxed{0}$ | 1 |
| 3 | 1 | 1 | $M_3 = \overline{x}_1 + \overline{x}_2$ | 1 | 0 |

$$\boxed{\begin{aligned}\overline{\overline{f}} = f &= \overline{x_1\overline{x}_2} \\ &= \overline{x}_1 + x_2\end{aligned}}$$

$$\begin{aligned}\overline{f}(x_1, x_2) &= m_2 \\ &= x_1\overline{x}_2\end{aligned}$$

# Product-of-Sums Form

| Row number | $x_1$ | $x_2$ | Maxterm | $f(x_1, x_2)$ | $\overline{f}(x_1, x_2)$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | $M_0 = x_1 + x_2$ | 1 | 0 |
| 1 | 0 | 1 | $M_1 = x_1 + \overline{x}_2$ | 1 | 0 |
| 2 | 1 | 0 | $M_2 = \overline{x}_1 + x_2$ | 0 | 1 |
| 3 | 1 | 1 | $M_3 = \overline{x}_1 + \overline{x}_2$ | 1 | 0 |

$$\overline{\overline{f}} = f = \overline{x_1 \overline{x}_2} = \overline{x}_1 + x_2$$

$$\overline{f}(x_1, x_2) = m_2 = x_1 \overline{x}_2$$

$$f = \overline{m}_2 = M_2$$

# minterms
# (for three variables)

# The Eight minterms

| x | y | z | $m_0$ | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $m_5$ | $m_6$ | $m_7$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

# The Eight minterms

| x | y | z | $m_0$ | $m_1$ | $m_2$ | $m_3$ | $m_4$ | $m_5$ | $m_6$ | $m_7$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

# Expressions for the minterms

$$m_0 = \bar{x}\ \bar{y}\ \bar{z}$$

$$m_1 = \bar{x}\ \bar{y}\ z$$

$$m_2 = \bar{x}\ y\ \bar{z}$$

$$m_3 = \bar{x}\ y\ z$$

$$m_4 = x\ \bar{y}\ \bar{z}$$

$$m_5 = x\ \bar{y}\ z$$

$$m_6 = x\ y\ \bar{z}$$

$$m_7 = x\ y\ z$$

# Expressions for the minterms

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | $m_0 = \bar{x}\ \bar{y}\ \bar{z}$ |
| 0 | 0 | 1 | $m_1 = \bar{x}\ \bar{y}\ z$ |
| 0 | 1 | 0 | $m_2 = \bar{x}\ y\ \bar{z}$ |
| 0 | 1 | 1 | $m_3 = \bar{x}\ y\ z$ |
| 1 | 0 | 0 | $m_4 = x\ \bar{y}\ \bar{z}$ |
| 1 | 0 | 1 | $m_5 = x\ \bar{y}\ z$ |
| 1 | 1 | 0 | $m_6 = x\ y\ \bar{z}$ |
| 1 | 1 | 1 | $m_7 = x\ y\ z$ |

The bars coincide
with the 0's
in the binary expansion
of the minterm sub-index

# Maxterms
# (for three variables)

# The Eight Maxterms

| x | y | z | $M_0$ | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ | $M_6$ | $M_7$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

# The Eight Maxterms

| x | y | z | $M_0$ | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ | $M_6$ | $M_7$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

# Expressions for the Maxterms

$$M_0 = x + y + z$$

$$M_1 = x + y + \overline{z}$$

$$M_2 = x + \overline{y} + z$$

$$M_3 = x + \overline{y} + \overline{z}$$

$$M_4 = \overline{x} + y + z$$

$$M_5 = \overline{x} + y + \overline{z}$$

$$M_6 = \overline{x} + \overline{y} + z$$

$$M_7 = \overline{x} + \overline{y} + \overline{z}$$

# Expressions for the Maxterms

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | $M_0 = x + y + z$ |
| 0 | 0 | 1 | $M_1 = x + y + \overline{z}$ |
| 0 | 1 | 0 | $M_2 = x + \overline{y} + z$ |
| 0 | 1 | 1 | $M_3 = x + \overline{y} + \overline{z}$ |
| 1 | 0 | 0 | $M_4 = \overline{x} + y + z$ |
| 1 | 0 | 1 | $M_5 = \overline{x} + y + \overline{z}$ |
| 1 | 1 | 0 | $M_6 = \overline{x} + \overline{y} + z$ |
| 1 | 1 | 1 | $M_7 = \overline{x} + \overline{y} + \overline{z}$ |

The bars coincide
with the 1's
in the binary expansion
of the maxterm sub-index

# minterms and Maxterms
# (for three variables)

# minterms and Maxterms

| | |
|---|---|
| $m_0 = \bar{x}\,\bar{y}\,\bar{z}$ | $M_0 = x + y + z$ |
| $m_1 = \bar{x}\,\bar{y}\,z$ | $M_1 = x + y + \bar{z}$ |
| $m_2 = \bar{x}\,y\,\bar{z}$ | $M_2 = x + \bar{y} + z$ |
| $m_3 = \bar{x}\,y\,z$ | $M_3 = x + \bar{y} + \bar{z}$ |
| $m_4 = x\,\bar{y}\,\bar{z}$ | $M_4 = \bar{x} + y + z$ |
| $m_5 = x\,\bar{y}\,z$ | $M_5 = \bar{x} + y + \bar{z}$ |
| $m_6 = x\,y\,\bar{z}$ | $M_6 = \bar{x} + \bar{y} + z$ |
| $m_7 = x\,y\,z$ | $M_7 = \bar{x} + \bar{y} + \bar{z}$ |

# minterms and Maxterms (with three variables)

| Row number | $x_1$ | $x_2$ | $x_3$ | Minterm | Maxterm |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$ | $M_0 = x_1 + x_2 + x_3$ |
| 1 | 0 | 0 | 1 | $m_1 = \overline{x}_1\overline{x}_2 x_3$ | $M_1 = x_1 + x_2 + \overline{x}_3$ |
| 2 | 0 | 1 | 0 | $m_2 = \overline{x}_1 x_2\overline{x}_3$ | $M_2 = x_1 + \overline{x}_2 + x_3$ |
| 3 | 0 | 1 | 1 | $m_3 = \overline{x}_1 x_2 x_3$ | $M_3 = x_1 + \overline{x}_2 + \overline{x}_3$ |
| 4 | 1 | 0 | 0 | $m_4 = x_1\overline{x}_2\overline{x}_3$ | $M_4 = \overline{x}_1 + x_2 + x_3$ |
| 5 | 1 | 0 | 1 | $m_5 = x_1\overline{x}_2 x_3$ | $M_5 = \overline{x}_1 + x_2 + \overline{x}_3$ |
| 6 | 1 | 1 | 0 | $m_6 = x_1 x_2\overline{x}_3$ | $M_6 = \overline{x}_1 + \overline{x}_2 + x_3$ |
| 7 | 1 | 1 | 1 | $m_7 = x_1 x_2 x_3$ | $M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$ |

[ Figure 2.22 from the textbook ]

# Examples with three-variable functions

# A three-variable function

| Row number | $x_1$ | $x_2$ | $x_3$ | $f(x_1, x_2, x_3)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 0 |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 1 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | 0 |

[ Figure 2.23 from the textbook ]

# Sum-of-Products (SOP)

| Row number | $x_1$ | $x_2$ | $x_3$ | $f(x_1, x_2, x_3)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | $\boxed{1}$ |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 0 |
| 4 | 1 | 0 | 0 | $\boxed{1}$ |
| 5 | 1 | 0 | 1 | $\boxed{1}$ |
| 6 | 1 | 1 | 0 | $\boxed{1}$ |
| 7 | 1 | 1 | 1 | 0 |

# Sum-of-Products (SOP)

| Row number | $x_1$ | $x_2$ | $x_3$ | $f(x_1, x_2, x_3)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 0 |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 1 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | 0 |

$$f(x_1, x_2, x_3) = \bar{x}_1\bar{x}_2x_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2x_3 + x_1x_2\bar{x}_3$$

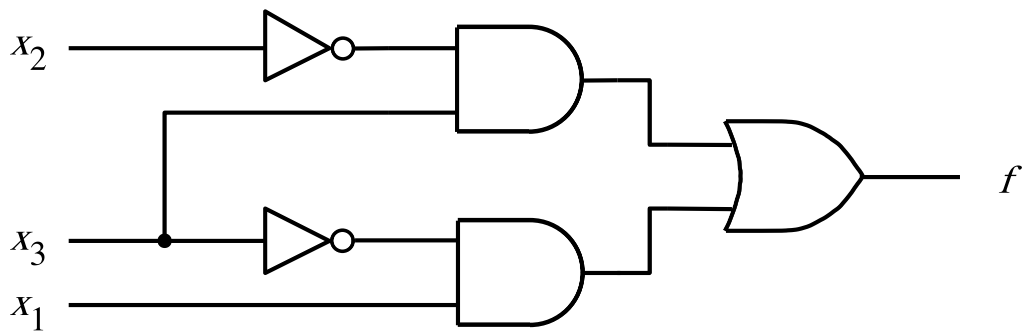
# Sum-of-Products (SOP)

| Row number | $x_1$ | $x_2$ | $x_3$ | $f(x_1, x_2, x_3)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 0 |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 1 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | 0 |

$$f(x_1, x_2, x_3) = \bar{x}_1\bar{x}_2x_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2x_3 + x_1x_2\bar{x}_3$$

$$\begin{aligned} f &= (\bar{x}_1 + x_1)\bar{x}_2x_3 + x_1(\bar{x}_2 + x_2)\bar{x}_3 \\ &= 1 \cdot \bar{x}_2x_3 + x_1 \cdot 1 \cdot \bar{x}_3 \\ &= \bar{x}_2x_3 + x_1\bar{x}_3 \end{aligned}$$

# Sum-of-products realization of this function




$$f = \overline{x_2}\, x_3 \;+\; x_1\, \overline{x_3}$$

[ Figure 2.24a from the textbook ]

# A three-variable function

| Row number | $x_1$ | $x_2$ | $x_3$ | $f(x_1, x_2, x_3)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 0 |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 1 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | 0 |

[ Figure 2.23 from the textbook ]

# Product-of-Sums (POS)

| Row number | $x_1$ | $x_2$ | $x_3$ | $f(x_1, x_2, x_3)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $\boxed{0}$ |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | $\boxed{0}$ |
| 3 | 0 | 1 | 1 | $\boxed{0}$ |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 1 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | $\boxed{0}$ |

# Product-of-Sums (POS)

| Row number | $x_1$ | $x_2$ | $x_3$ | $f(x_1, x_2, x_3)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $\boxed{0}$ |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | $\boxed{0}$ |
| 3 | 0 | 1 | 1 | $\boxed{0}$ |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 1 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | $\boxed{0}$ |

$$\begin{aligned} f &= \overline{m_0 + m_2 + m_3 + m_7} \\ &= \overline{m}_0 \cdot \overline{m}_2 \cdot \overline{m}_3 \cdot \overline{m}_7 \\ &= M_0 \cdot M_2 \cdot M_3 \cdot M_7 \\ &= (x_1 + x_2 + x_3)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + \overline{x}_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3) \end{aligned}$$

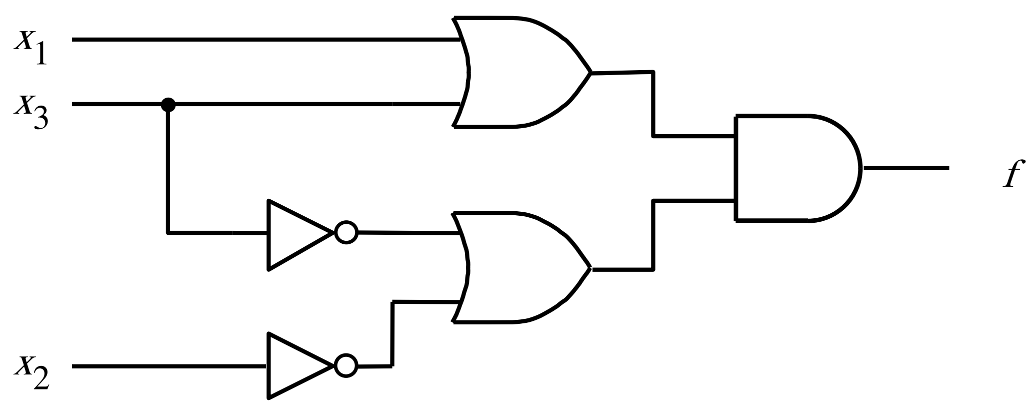
# Product-of-Sums (POS)

| Row number | $x_1$ | $x_2$ | $x_3$ | $f(x_1, x_2, x_3)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $\boxed{0}$ |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | $\boxed{0}$ |
| 3 | 0 | 1 | 1 | $\boxed{0}$ |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 1 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | $\boxed{0}$ |

$$f = ((x_1 + x_3) + x_2)((x_1 + x_3) + \bar{x}_2)(x_1 + (\bar{x}_2 + \bar{x}_3))(\bar{x}_1 + (\bar{x}_2 + \bar{x}_3))$$

$$f = (x_1 + x_3)(\bar{x}_2 + \bar{x}_3)$$

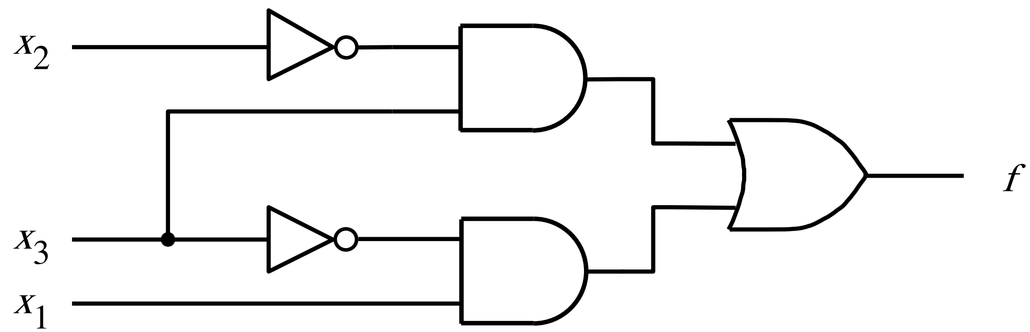
# Product-of-sums realization of this function



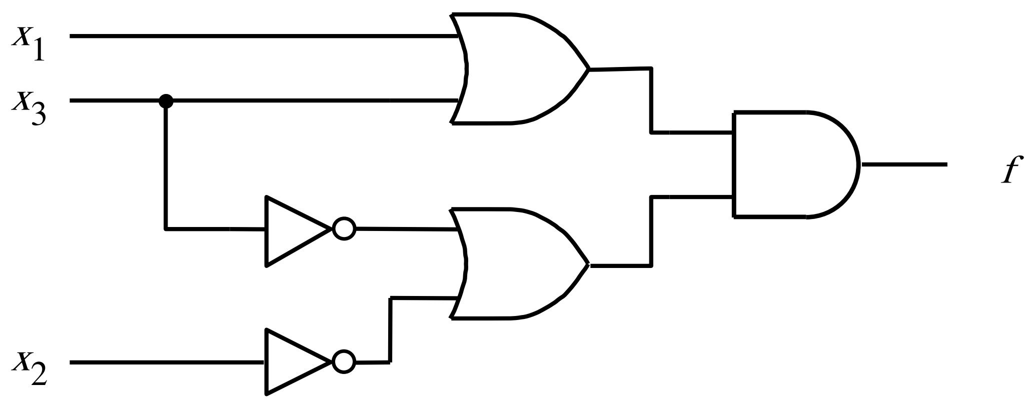

$$f = (x_1 + x_3) \bullet (\overline{x_2} + \overline{x_3})$$

[ Figure 2.24b from the textbook ]

# Two realizations of this function




(a) A minimal sum-of-products realization




(b) A minimal product-of-sums realization

[ Figure 2.24 from the textbook ]

# Shorthand Notation for SOP

| Row number | $x_1$ | $x_2$ | $x_3$ | $f(x_1, x_2, x_3)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 0 |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 1 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | 0 |

$$f(x_1, x_2, x_3) = \sum(m_1, m_4, m_5, m_6)$$

or

$$f(x_1, x_2, x_3) = \sum m(1, 4, 5, 6)$$

# Shorthand Notation for POS

| Row number | $x_1$ | $x_2$ | $x_3$ | $f(x_1, x_2, x_3)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 0 |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 1 | 1 | 0 | 1 |
| 7 | 1 | 1 | 1 | 0 |

$$f(x_1, x_2, x_3) = \Pi(M_0, M_2, M_3, M_7)$$

or

$$f(x_1, x_2, x_3) = \Pi M(0, 2, 3, 7)$$

# Shorthand Notation

- **Sum-of-Products (SOP)**

$$f(x_1, x_2, x_3) = \sum(m_1, m_4, m_5, m_6)$$

**or**

$$f(x_1, x_2, x_3) = \sum m(1, 4, 5, 6)$$

- **Product-of-Sums (POS)**

$$f(x_1, x_2, x_3) = \Pi(M_0, M_2, M_3, M_7)$$

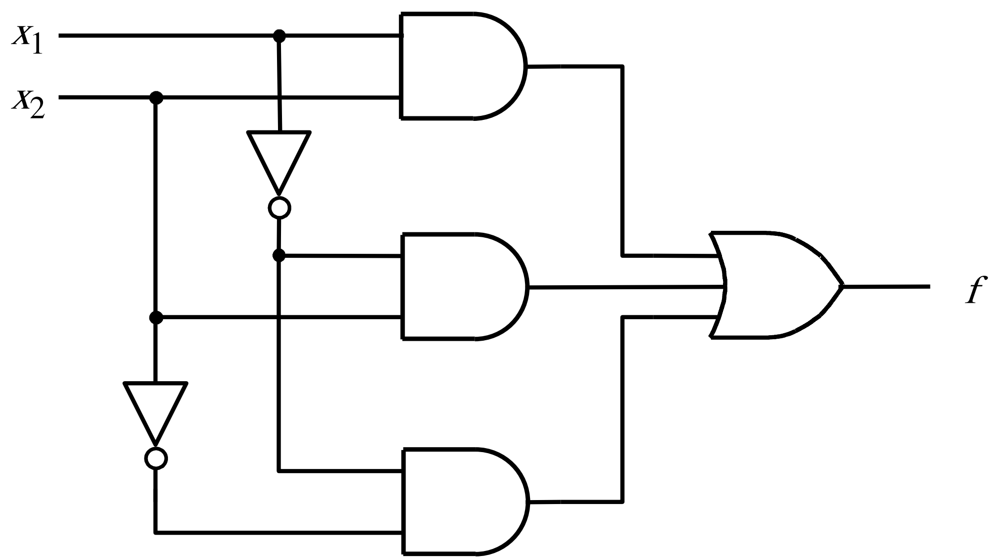
**or**

$$f(x_1, x_2, x_3) = \Pi M(0, 2, 3, 7)$$

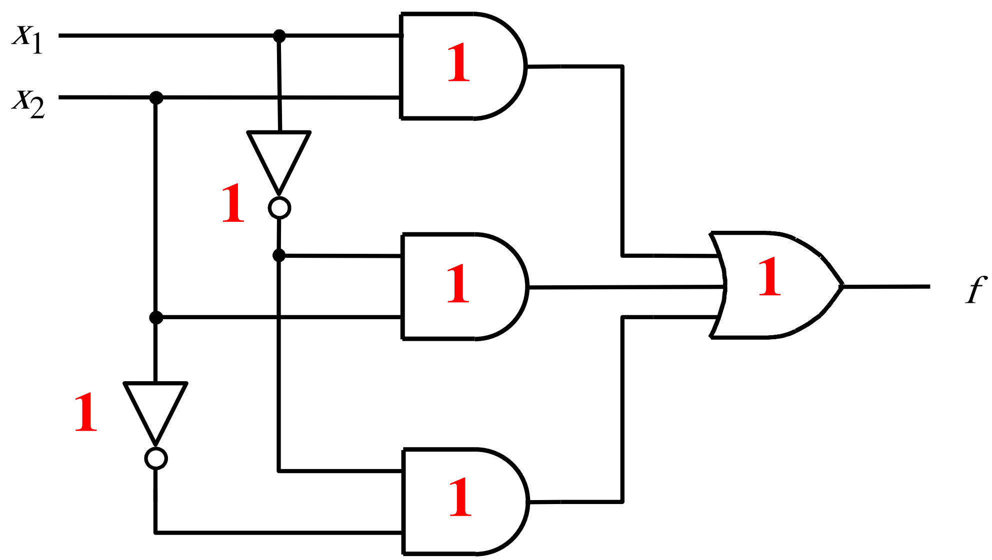
# The Cost of a Circuit

- Count all gates
- Count all inputs/wires to the gates
- Add the two partial counts. That is the cost.

# What is the cost of this circuit?

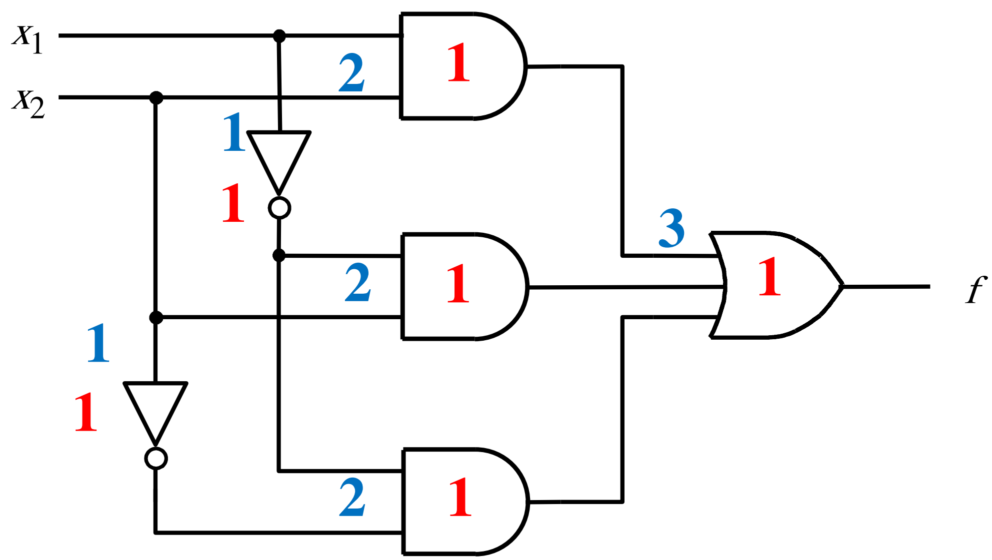


# What is the cost of this circuit?




**There are 6 gates.**

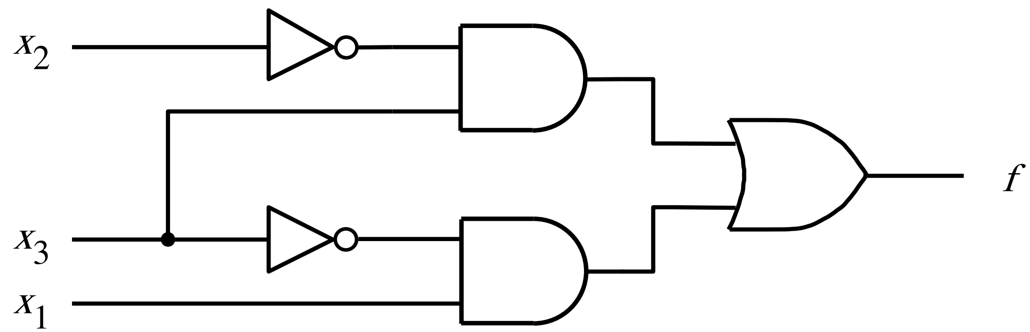
# What is the cost of this circuit?



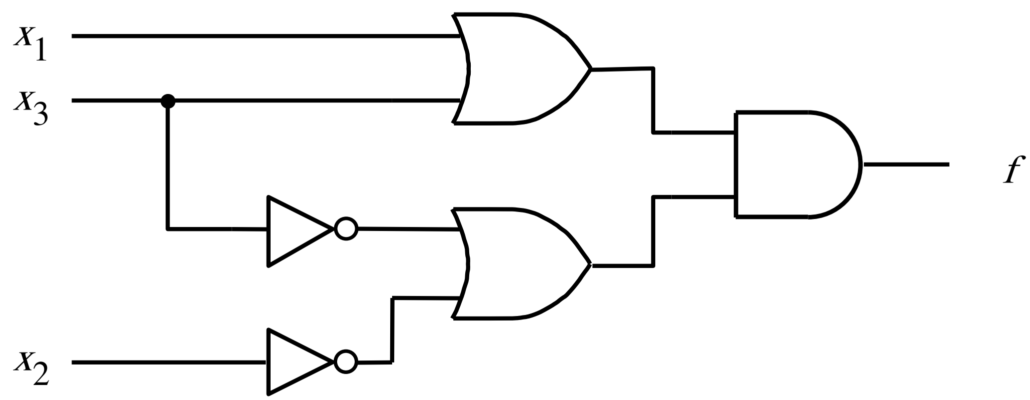

**There are 6 gates and 11 inputs.**

**The total cost is 17.**

# What is the cost of each circuit?




(a) A minimal sum-of-products realization




(b) A minimal product-of-sums realization

# Questions?

**THE END**