



CprE 281: Digital Logic

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<http://www.ece.iastate.edu/~alexs/classes/>

Karnaugh Maps

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Iowa State University, Ames, IA
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Administrative Stuff

- **HW3 is due today**

Administrative Stuff

- **HW4 is out**
- **It is due on Monday Sep 18 @ 10 pm**
- **It is posted on the class web page**
- **I also sent you an e-mail with the link.**

Administrative Stuff

- **Midterm Exam #1**
- **When: Friday Sep 22.**
- **What: Chapter 1 and Chapter 2 plus number systems**
- **The exam will be closed book but open notes (you can bring up to 3 pages of handwritten notes).**
- **Sample exams are posted on the class web page.**
- **More details to follow.**

Quick Review

Do You Still Remember This Boolean Algebra Theorem?

14a. $x \cdot y + x \cdot \bar{y} = x$

Combining

14b. $(x + y) \cdot (x + \bar{y}) = x$

Let's prove 14.a

x	y	$\mathbf{x \cdot y + x \cdot \bar{y} = x}$
0	0	
0	1	
1	0	
1	1	

Let's prove 14.a

x	y	$\mathbf{x \cdot y + x \cdot \bar{y} = x}$
0	0	0
0	1	0
1	0	0
1	1	1

Let's prove 14.a

x	y	$\mathbf{x \cdot y + x \cdot \bar{y} = x}$
0	0	0
0	1	0
1	0	1
1	1	1

Let's prove 14.a

x	y	$\mathbf{x \cdot y + x \cdot \bar{y} = x}$
0	0	0 0 0
0	1	0 0 0
1	0	0 1 1
1	1	1 1 0

Let's prove 14.a

x	y	$\mathbf{x \cdot y + x \cdot \bar{y} = x}$
0	0	0
0	1	0
1	0	1
1	1	1

Let's prove 14.a

x	y	$\mathbf{x} \cdot \mathbf{y} + \mathbf{x} \cdot \bar{\mathbf{y}} = \mathbf{x}$				
0	0	0	0	0	0	0
0	1	0	0	0	0	0
1	0	0	1	1	1	1
1	1	1	1	1	0	1

They are equal.

Motivation

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

An approach for simplifying logic expressions.

How do we guarantee that we have reached the minimum SOP/POS representation?

This method was described in 1953

M. Karnaugh, “The map method for synthesis of combinational logic circuits”

Transactions of the American Institute of Electrical Engineers, Part I: Communication and Electronics (pages 593 – 599, Volume: 72, Issue: 5, Nov. 1953)

<https://ieeexplore.ieee.org/document/6371932>

Two-Variable K-Map

Karnaugh Map (K-map)

- A visual representation of the function
- Same information as the truth table
- Easier to group minterms

x_1	x_2	f
0	0	0
0	1	1
1	0	0
1	1	1

(a) Truth table

x_2 \ x_1	0	1
0	0	0
1	1	1

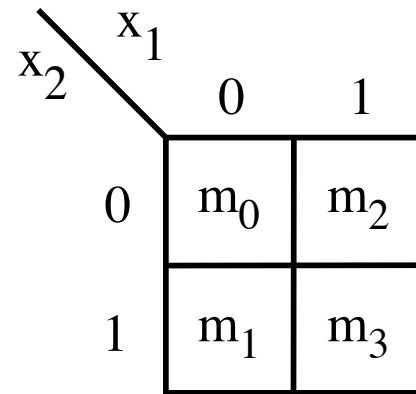
(b) Karnaugh map

Karnaugh Map (K-map)

- A visual representation of the function
- Same information as the truth table
- Easier to group minterms

x_1	x_2	f
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

(a) Truth table



(b) Karnaugh map

Minterms

x_1	x_2	f
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

x_1	x_2	m_0	m_1	m_2	m_3
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

Minterm Addition Example

x_1	x_2	f
0	0	0
0	1	1
1	0	0
1	1	1

x_1	x_2	m_0	m_1	m_2	m_3	$m_1 + m_3$
0	0	1	0	0	0	0
0	1	0	1	0	0	1
1	0	0	0	1	0	0
1	1	0	0	0	1	1

Minterm Addition Example

x_1	x_2	f
0	0	0
0	1	1
1	0	0
1	1	1

x_1	x_2	m_0	m_1	m_2	m_3	$m_1 + m_3$
0	0	1	0	0	0	0
0	1	0	1	0	0	1
1	0	0	0	1	0	0
1	1	0	0	0	1	1

$$\bar{x}_1x_2 + x_1x_2 = x_2$$

Minterm Addition Example

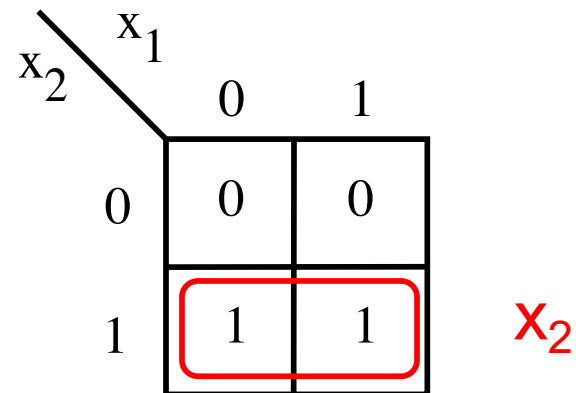
x_1	x_2	f
0	0	0
0	1	1
1	0	0
1	1	1

x_2	x_1	0	1
0	0	0	0
1	0	1	1

$$\bar{x}_1x_2 + x_1x_2 = x_2$$

Minterm Addition Example

x_1	x_2	f
0	0	0
0	1	1
1	0	0
1	1	1



$$\bar{x}_1x_2 + x_1x_2 = x_2$$

Another Grouping Example

	x_1	0	1
x_2	0	1	0
	1	0	0

m_0

	x_1	0	1
x_2	0	0	0
	1	1	0

m_1

Another Grouping Example

	x_1	0	1
x_2			
0		1	0
1		0	0

m_0

+

	x_1	0	1
x_2			
0		0	0
1		1	0

m_1

=

	x_1	0	1
x_2			
0		1	0
1		1	0

$m_0 + m_1$

Another Grouping Example

	x_1	0	1
x_2	0	1	0
	1	0	0

m_0

+

	x_1	0	1
x_2	0	0	0
	1	1	0

m_1

=

	x_1	0	1
x_2	0	1	0
	1	1	0

$m_0 + m_1$

Another Grouping Example

	x_1	0	1
x_2	0	1	0
	1	0	0

m_0

$\overline{x_1}\overline{x_2}$

+

	x_1	0	1
x_2	0	0	0
	1	1	0

m_1

$\overline{x_1}x_2$

=

	x_1	0	1
x_2	0	1	0
	1	1	0

$m_0 + m_1$

$\overline{x_1}$

Property 14a (Combining)

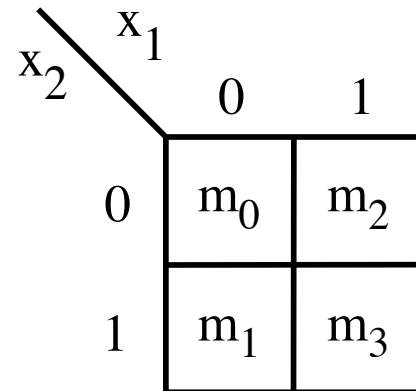
Grouping Rules

- **Group “1”s with rectangles**
- **Both sides must be a power of 2:**
 - **1x1, 1x2, 2x1, 2x2, 1x4, 4x1, 2x4, 4x2, 4x4**
- **Can use/cover the same minterm more than once**
- **Can wrap around the edges of the map**
- **Some rules in selecting groups:**
 - **Try to use as few groups as possible to cover all “1”s.**
 - **For each group, try to make it as large as you can (i.e., if you can use a 2x2, don’t use a 2x1 even if that is enough).**

Two-Variable K-map

x_1	x_2	f
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

(a) Truth table



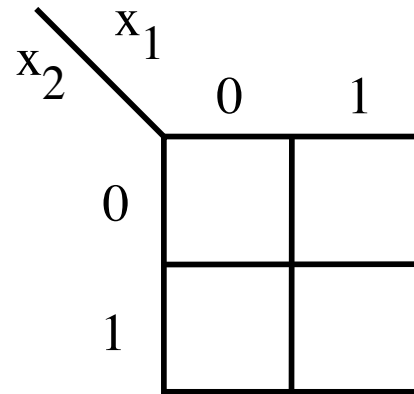
(b) Karnaugh map

Step-By-Step Example

x_1	x_2	f
0	0	1
0	1	1
1	0	0
1	1	1

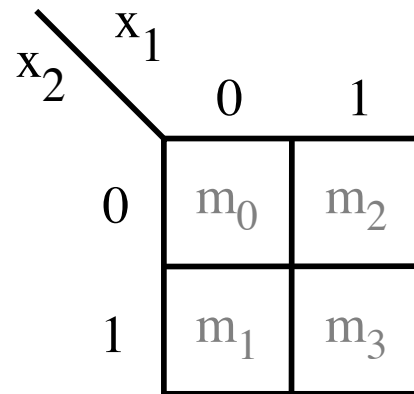
1. Draw The Map

x_1	x_2	f
0	0	1
0	1	1
1	0	0
1	1	1



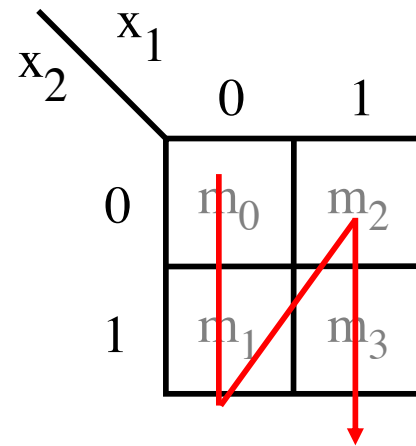
2. Fill The Map

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1



2. Fill The Map

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1



2. Fill The Map

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

x_2	x_1	0	1
0		1	0
1		1	1

3. Group

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

x_2	x_1	0	1
0		1	0
1		1	1

3. Group

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

	x_1	0	1
x_2	0	1	0
	1	1	1

3. Group

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

A Karnaugh map for the function $f(x_1, x_2)$. The horizontal axis is labeled x_1 with values 0 and 1. The vertical axis is labeled x_2 with values 0 and 1. The map contains the following values:

$x_2 \backslash x_1$	0	1
0	1	0
1	1	1

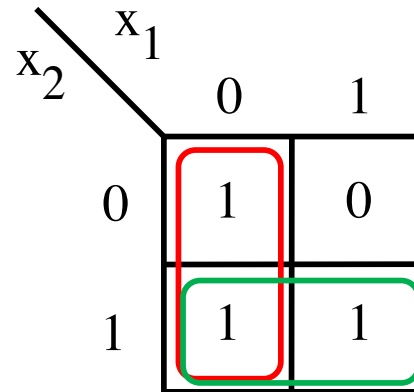
The prime implicants are highlighted with colored boxes:

- A red box highlights the prime implicant \bar{x}_2 , which covers the cells (0,0) and (1,0).
- A green box highlights the prime implicant x_2 , which covers the cells (1,0) and (1,1).

4. Write The Expression

x_1	x_2	f
0	0	1
0	1	1
1	0	0
1	1	1

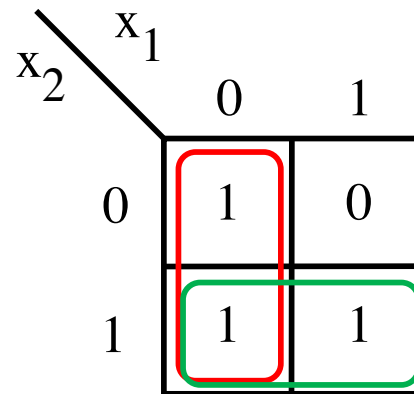
$x_2 \backslash x_1$	0	1
0	1	0
1	1	1



The Karnaugh map shows the function $f(x_1, x_2)$ with prime implicants highlighted. A red box encloses the cells (0,0) and (1,0), representing the prime implicant \bar{x}_2 . A green box encloses the cells (0,1) and (1,1), representing the prime implicant x_2 . The function is the sum of these two prime implicants: $f(x_1, x_2) = \bar{x}_2 + x_2$.

4. Write The Expression

x_1	x_2	f
0	0	1
0	1	1
1	0	0
1	1	1



$$\bar{x}_1 + x_2$$

Writing The Expression

- Find which variable is constant

		x_1	
		0	1
x_2	0	1	0
	1	1	0

$$\overline{x_1}$$

Writing The Expression

- Find which variable is constant

		x_1	
		0	1
x_2	0	0	1
	1	0	1

x_1

Writing The Expression

- Find which variable is constant

		x_1	
		0	1
x_2	0	1	1
	1	0	0

$$\overline{x_2}$$

Writing The Expression

- Find which variable is constant

		x_1	
		0	1
x_2	0	0	0
	1	1	1

x_2

These are all valid groupings

	x_1	0	1
x_2	0	1	0
	1	0	0

$$\overline{x_1} \overline{x_2}$$

	x_1	0	1
x_2	0	0	0
	1	1	0

$$\overline{x_1} x_2$$

	x_1	0	1
x_2	0	0	1
	1	0	0

$$x_1 \overline{x_2}$$

	x_1	0	1
x_2	0	0	0
	1	0	1

$$x_1 x_2$$

These are all valid groupings

	x_1	0	1
x_2	0	1	0
	1	1	0

\bar{x}_1

	x_1	0	1
x_2	0	0	1
	1	0	1

x_1

	x_1	0	1
x_2	0	1	1
	1	0	0

\bar{x}_2

	x_1	0	1
x_2	0	0	0
	1	1	1

x_2

This one is valid too

$x_2 \backslash x_1$	0	1
0	1	1
1	1	1

In this case the result is the constant function 1.

Invalid Groupings

	x_1	0	1
x_2	0	1	0
	1	0	1

	x_1	0	1
x_2	0	0	1
	1	1	0

Can't group diagonally. Why?

	x_1		
x_2		0	1
0		1	0
1		0	0

m_0

	x_1		
x_2		0	1
0		0	0
1		0	1

m_3

Can't group diagonally. Why?

	x_1		
x_2		0	1
0		1	0
1		0	0

m_0

+

	x_1		
x_2		0	1
0		0	0
1		0	1

m_3

=

	x_1		
x_2		0	1
0		1	0
1		0	1

$m_0 + m_3$

Can't group diagonally. Why?

	x_1	0	1
x_2			
0		1	0
1		0	0

m_0

+

	x_1	0	1
x_2			
0		0	0
1		0	1

m_3

=

	x_1	0	1
x_2			
0		1	0
1		0	1

$m_0 + m_3$

Can't group diagonally. Why?

<table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="border: none;"></td> <td style="border: none;">x_1</td> <td style="border: none;">0</td> <td style="border: none;">1</td> </tr> <tr> <td style="border: none;">x_2</td> <td style="border: none;"></td> <td style="border: 1px solid red;">1</td> <td>0</td> </tr> <tr> <td style="border: none;">0</td> <td style="border: none;"></td> <td style="border: 1px solid red;">1</td> <td>0</td> </tr> <tr> <td style="border: none;">1</td> <td style="border: none;"></td> <td>0</td> <td>0</td> </tr> </table>		x_1	0	1	x_2		1	0	0		1	0	1		0	0	+	<table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="border: none;"></td> <td style="border: none;">x_1</td> <td style="border: none;">0</td> <td style="border: none;">1</td> </tr> <tr> <td style="border: none;">x_2</td> <td style="border: none;"></td> <td>0</td> <td>0</td> </tr> <tr> <td style="border: none;">0</td> <td style="border: none;"></td> <td>0</td> <td>0</td> </tr> <tr> <td style="border: none;">1</td> <td style="border: none;"></td> <td>0</td> <td style="border: 1px solid red;">1</td> </tr> </table>		x_1	0	1	x_2		0	0	0		0	0	1		0	1	=	<table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="border: none;"></td> <td style="border: none;">x_1</td> <td style="border: none;">0</td> <td style="border: none;">1</td> </tr> <tr> <td style="border: none;">x_2</td> <td style="border: none;"></td> <td style="border: 1px solid red;">1</td> <td>0</td> </tr> <tr> <td style="border: none;">0</td> <td style="border: none;"></td> <td style="border: 1px solid red;">1</td> <td>0</td> </tr> <tr> <td style="border: none;">1</td> <td style="border: none;"></td> <td>0</td> <td style="border: 1px solid red;">1</td> </tr> </table>		x_1	0	1	x_2		1	0	0		1	0	1		0	1
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0		1	0																																																	
1		0	1																																																	
m_0	+	m_3	=	$m_0 + m_3$																																																
$\overline{x_1}\overline{x_2}$	+	x_1x_2	=	$\overline{\overline{x_1}\overline{x_2}} + x_1x_2$																																																

We can't use Property 14a here. This can't be simplified.

Three-Variable K-Map

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3	$x_1 x_2$	00	01	11	10
0		m_0	m_2	m_6	m_4
1		m_1	m_3	m_7	m_5

(b) Karnaugh map

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3	$x_1 x_2$	00	01	11	10
0		m_0	m_2	m_6	m_4
1		m_1	m_3	m_7	m_5

(b) Karnaugh map

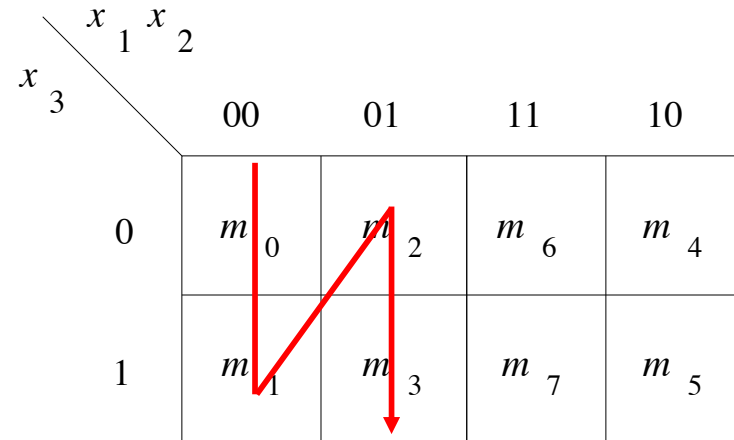
Notice the placement of

- **Variables**
- **Binary pair values**
- **Minterms**

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
<hr/>			
0	1	0	m_2
0	1	1	m_3
<hr/>			
1	0	0	m_4
1	0	1	m_5
<hr/>			
1	1	0	m_6
1	1	1	m_7

(a) Truth table



(b) Karnaugh map

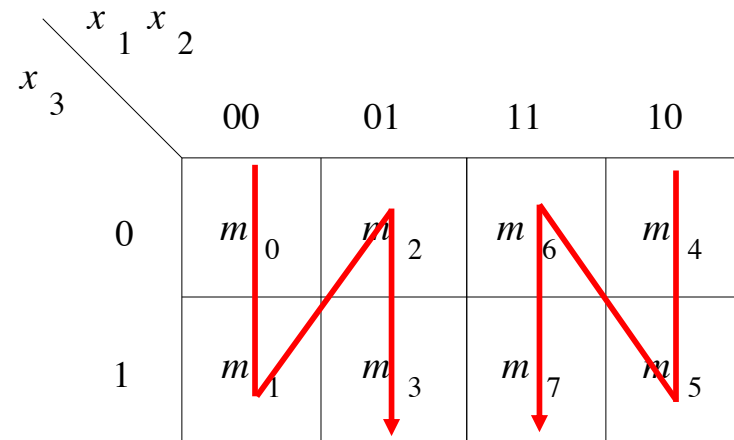
Notice the placement of

- **Variables**
- **Binary pair values**
- **Minterms**

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
<hr/>			
0	1	0	m_2
0	1	1	m_3
<hr/>			
1	0	0	m_4
1	0	1	m_5
<hr/>			
1	1	0	m_6
1	1	1	m_7

(a) Truth table



(b) Karnaugh map

Notice the placement of

- **Variables**
- **Binary pair values**
- **Minterms**

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

00

01

11

10

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000
001
011
010
110
111
101
100

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s x_1$			
		00	01	11	10
x_2	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

x_2	$s x_1$	00	01	11	10
0	000	010	110	100	
1	001	011	111	101	

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

x_2	s	x_1				
			00	01	11	10
0	000	010	110	100		
1	001	011	111	101		

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the LAST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the LAST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the FIRST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These four neighbors
differ in the FIRST and LAST bit

They are similar in their MIDDLE bit

Adjacency Rules

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

adjacent
columns



Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

x_2	s	x_1				
			00	01	11	10
0			000	010	110	100
1			001	011	111	101

These two neighbors
differ only in the FIRST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

x_2	$s x_1$			
	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These four neighbors
differ in the FIRST and LAST bit

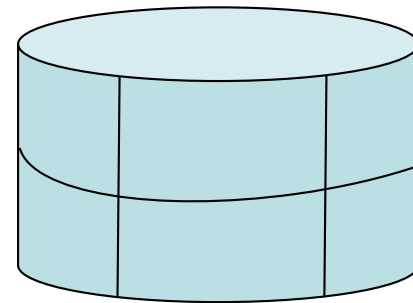
They are similar in their MIDDLE bit

Adjacency Rules

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5



adjacent
columns



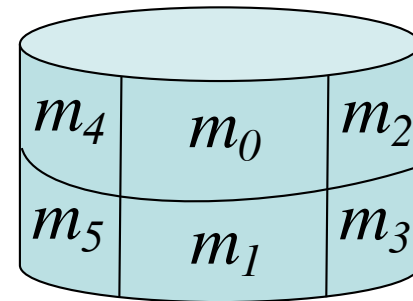
As if the K-map were
drawn on a cylinder

Adjacency Rules

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

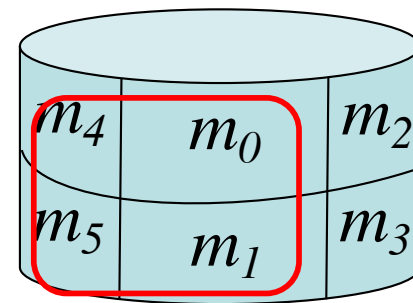
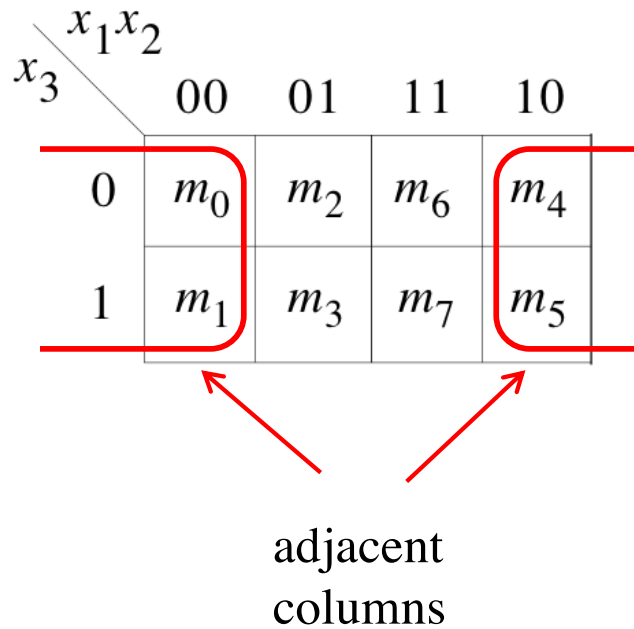


adjacent
columns



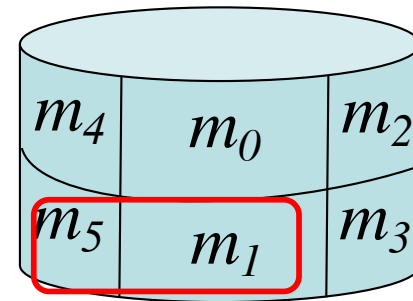
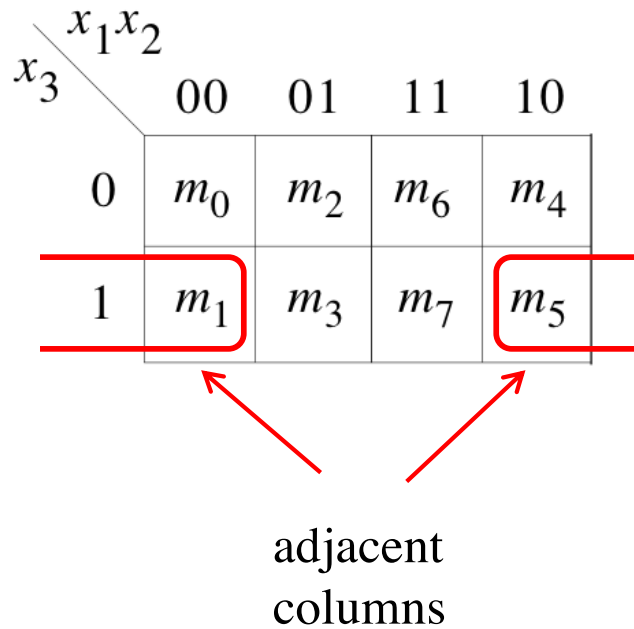
As if the K-map were
drawn on a cylinder

Adjacency Rules



As if the K-map were drawn on a cylinder

Adjacency Rules



As if the K-map were drawn on a cylinder

Examples of Valid Groupings

Groupings and Expressions

		x_1x_2			
		00	01	11	10
x_3	0	1	0	0	0
	1	0	0	0	0

$$\overline{x_1} \overline{x_2} \overline{x_3}$$

		x_1x_2			
		00	01	11	10
x_3	0	0	1	0	0
	1	0	0	0	0

$$\overline{x_1} x_2 \overline{x_3}$$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	1	0
	1	0	0	0	0

$$x_1 x_2 \overline{x_3}$$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	1
	1	0	0	0	0

$$x_1 \overline{x_2} \overline{x_3}$$

Groupings and Expressions

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	1	0	0	0

$$\overline{\overline{x_1}} \overline{\overline{x_2}} x_3$$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	0	1	0	0

$$\overline{\overline{x_1}} x_2 \overline{\overline{x_3}}$$

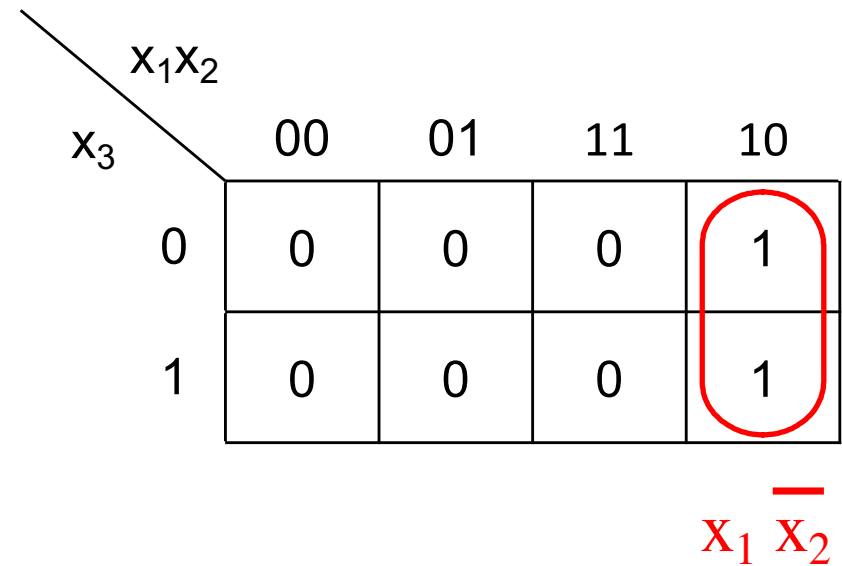
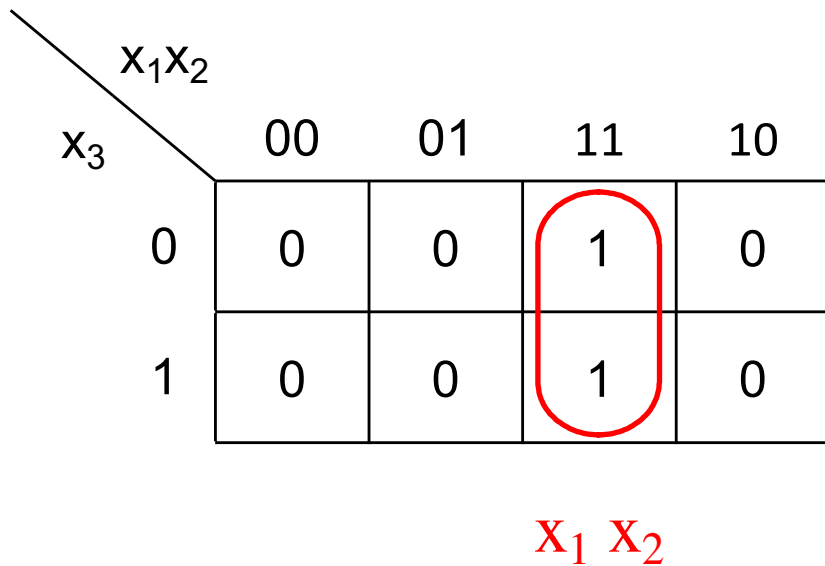
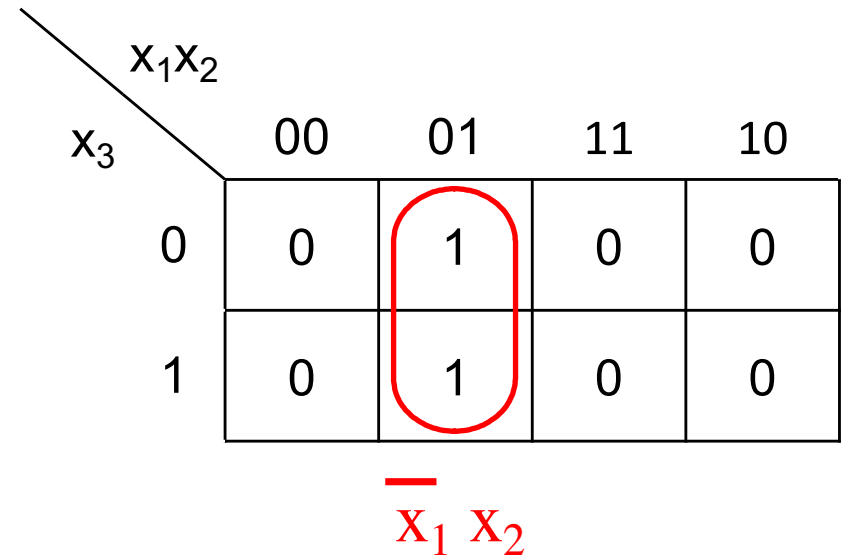
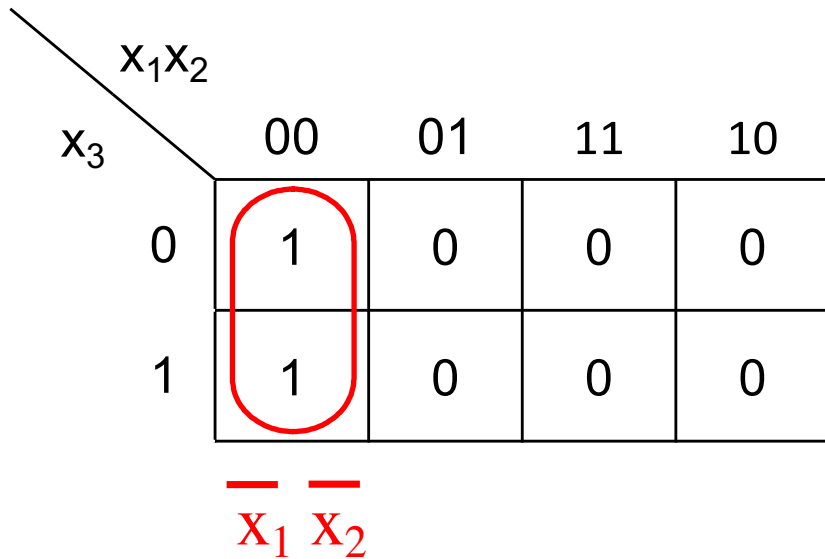
		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	0	0	1	0

$$x_1 x_2 x_3$$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	0	0	0	1

$$x_1 \overline{\overline{x_2}} \overline{\overline{x_3}}$$

Groupings and Expressions



Groupings and Expressions

		x_1x_2			
		00	01	11	10
x_3	0	1	1	0	0
	1	0	0	0	0

$$\overline{x_1} \overline{x_3}$$

		x_1x_2			
		00	01	11	10
x_3	0	0	1	1	0
	1	0	0	0	0

$$\overline{x_2} \overline{x_3}$$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	1	1
	1	0	0	0	0

$$\overline{x_1} \overline{x_3}$$

		x_1x_2			
		00	01	11	10
x_3	0	1	0	0	1
	1	0	0	0	0

$$\overline{x_2} \overline{x_3}$$

Groupings and Expressions

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	1	1	0	0

$$\overline{x_1} x_3$$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	0	1	1	0

$$x_2 x_3$$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	0	0	1	1

$$x_1 x_3$$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	1	0	0	1

$$\overline{x_2} x_3$$

Groupings and Expressions

		x_1x_2			
		00	01	11	10
x_3	0	1	1	0	0
	1	1	1	0	0

$\overline{x_1}$

		x_1x_2			
		00	01	11	10
x_3	0	0	1	1	0
	1	0	1	1	0

x_2

		x_1x_2			
		00	01	11	10
x_3	0	0	0	1	1
	1	0	0	1	1

x_1

		x_1x_2			
		00	01	11	10
x_3	0	1	0	0	1
	1	1	0	0	1

$\overline{x_2}$

Groupings and Expressions

		x_1x_2			
		00	01	11	10
x_3	0	1	1	1	1
	1	0	0	0	0

$\overline{x_3}$

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	1	1	1	1

x_3

Groupings and Expressions

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	0	0	0	0

0

		x_1x_2			
		00	01	11	10
x_3	0	1	1	1	1
	1	1	1	1	1

1

Examples of **Invalid** Groupings

Some **Invalid** Groupings

		x_1x_2			
		00	01	11	10
x_3	0	0	1	0	0
	1	0	0	1	0

		x_1x_2			
		00	01	11	10
x_3	0	0	0	1	0
	1	0	1	0	0

Can't group diagonally.

Some **Invalid** Groupings

		x_1x_2			
		00	01	11	10
x_3	0	1	1	1	0
	1	0	0	0	0

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	0	1	1	1

Can't group three in a row.
Each side must be a power of 2.

Some **Invalid** Groupings

		x_1x_2			
		00	01	11	10
x_3	0	1	0	1	1
	1	0	0	0	0

A 2x4 Karnaugh map for variables x_1, x_2, x_3 . The columns are labeled x_1x_2 with values 00, 01, 11, 10. The rows are labeled x_3 with values 0 and 1. A red rounded rectangle groups the four cells in the $x_3=0$ row. The cell at $(x_3=0, x_1x_2=01)$ contains a red **0**, while the other three cells contain 1.

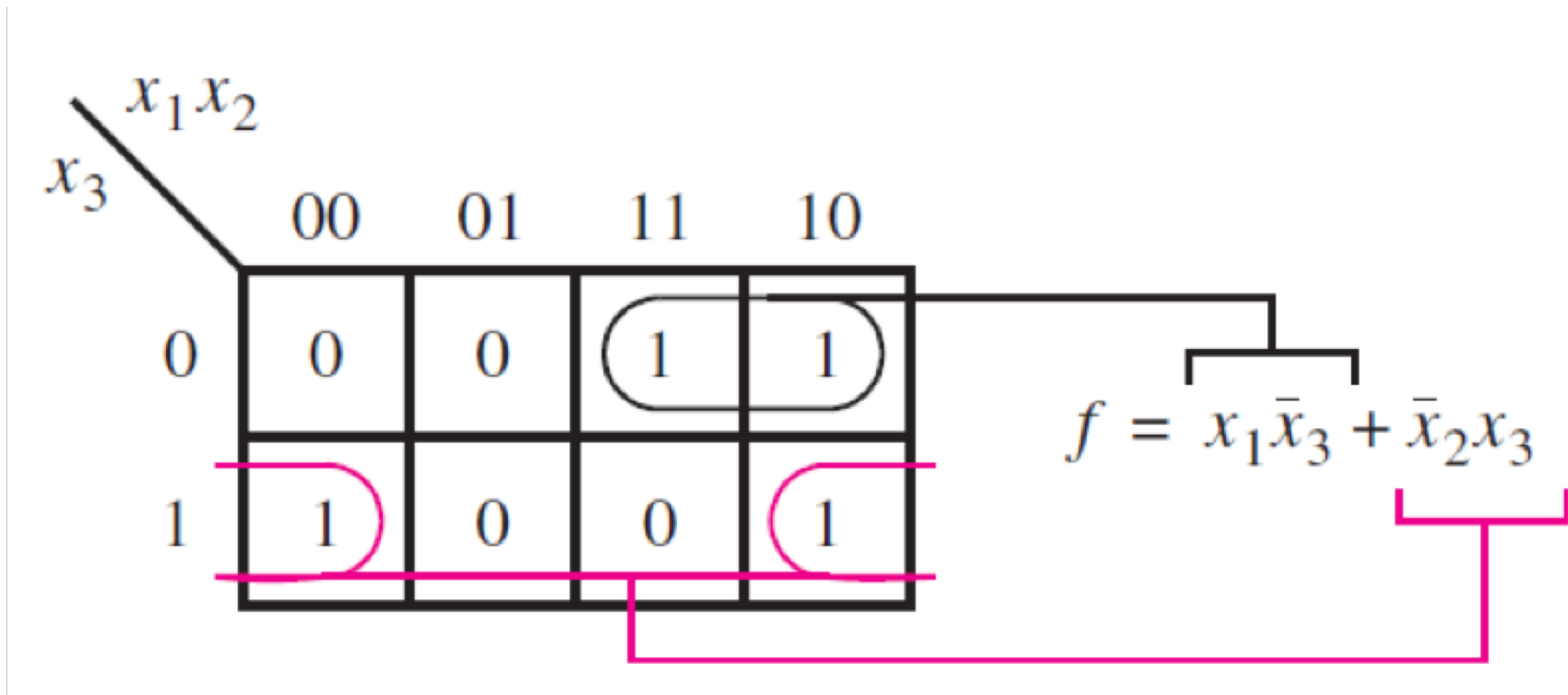
		x_1x_2			
		00	01	11	10
x_3	0	0	0	1	0
	1	0	1	1	0

A 2x4 Karnaugh map for variables x_1, x_2, x_3 . The columns are labeled x_1x_2 with values 00, 01, 11, 10. The rows are labeled x_3 with values 0 and 1. A red rounded rectangle groups the four cells in the $x_1x_2 \in \{01, 11\}$ columns. The cell at $(x_3=0, x_1x_2=01)$ contains a red **0**, while the other three cells contain 1.

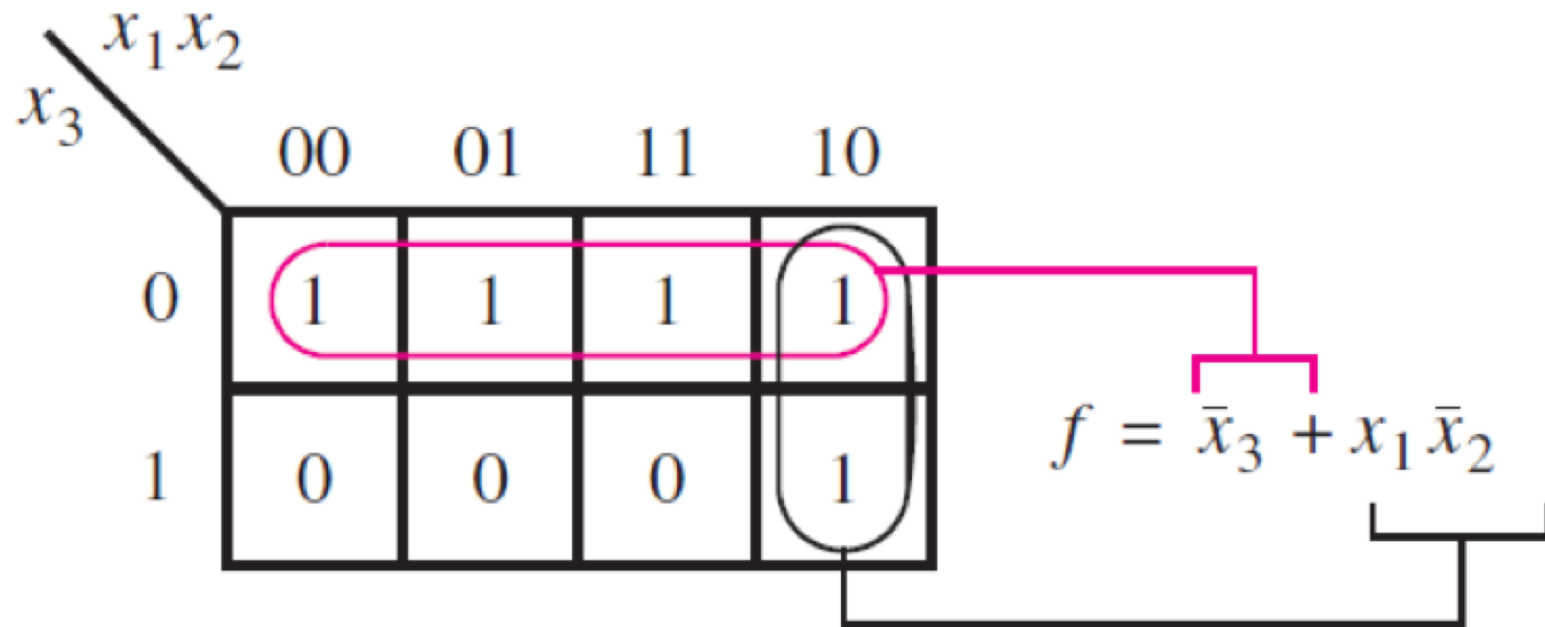
Can't group zeros and ones together.

Minimization Examples with 3-variable K-Maps

Examples of three-variable Karnaugh maps

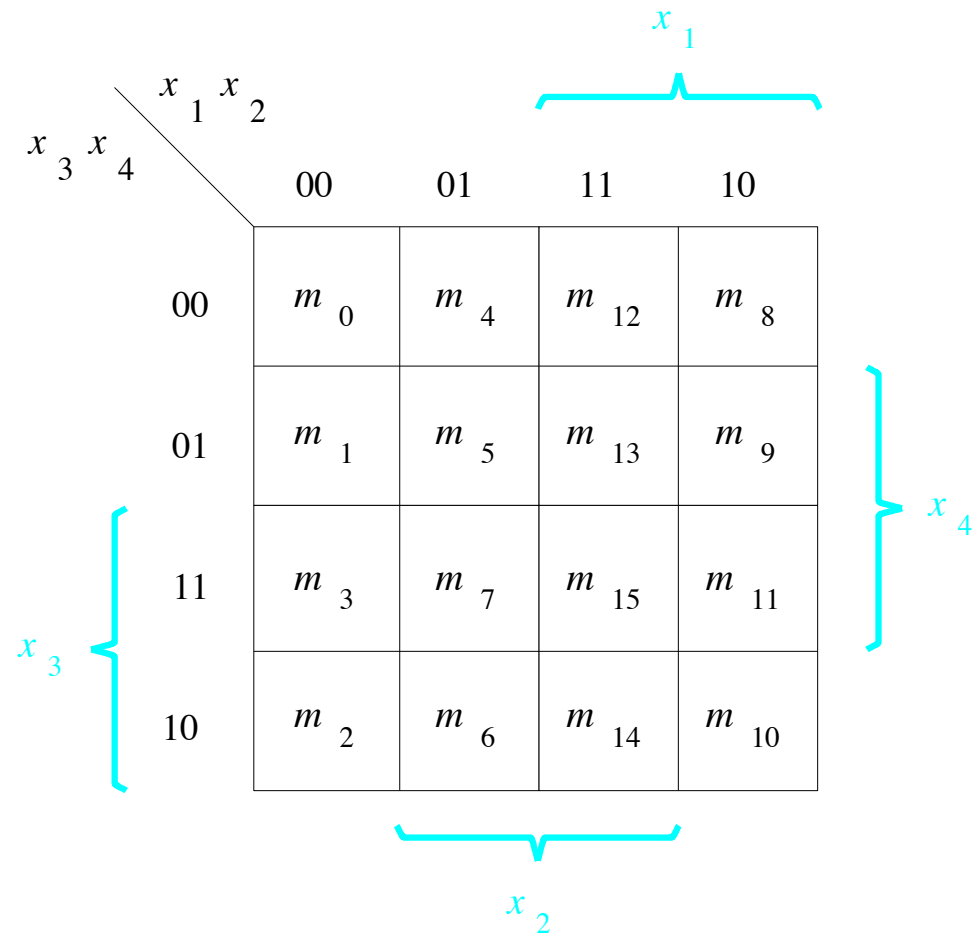


Examples of three-variable Karnaugh maps



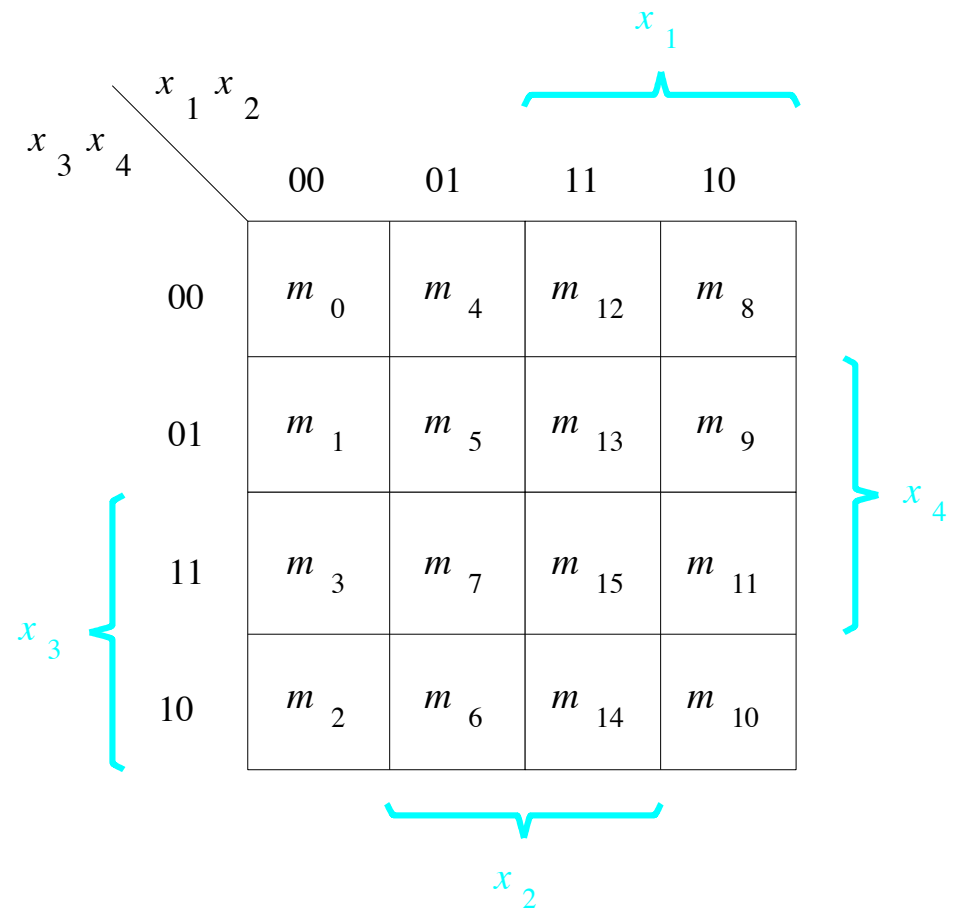
Four-Variable K-Map

A four-variable Karnaugh map



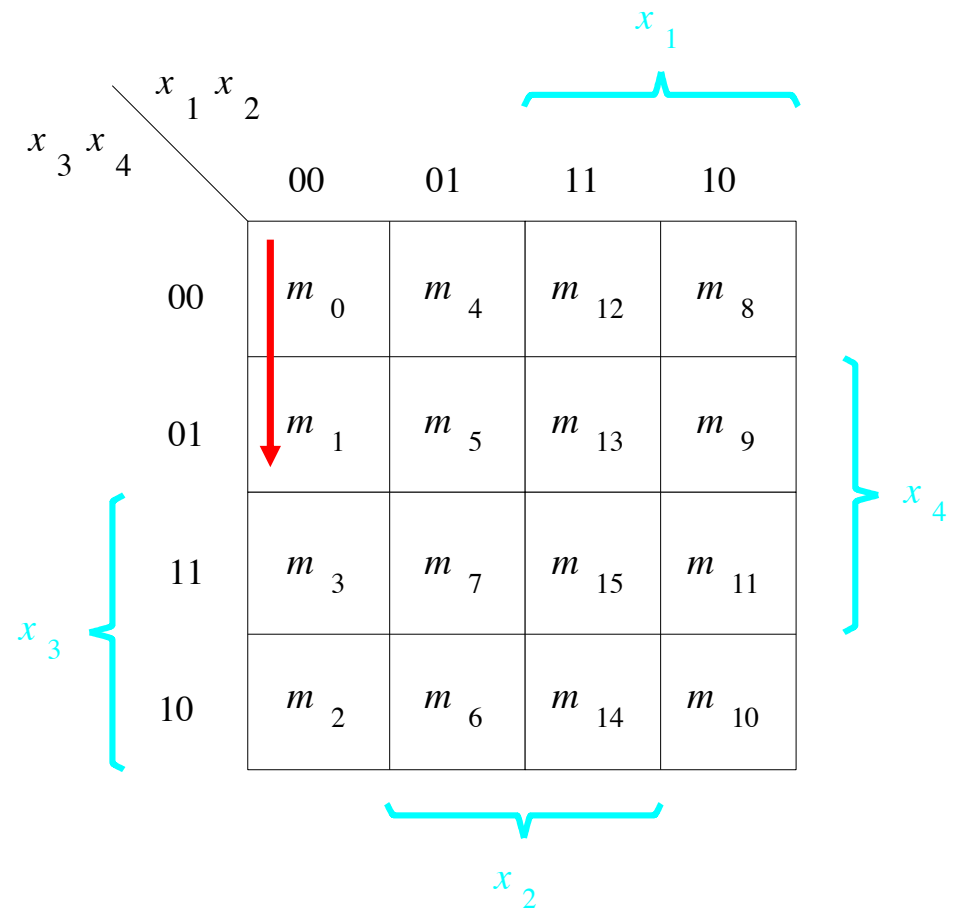
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



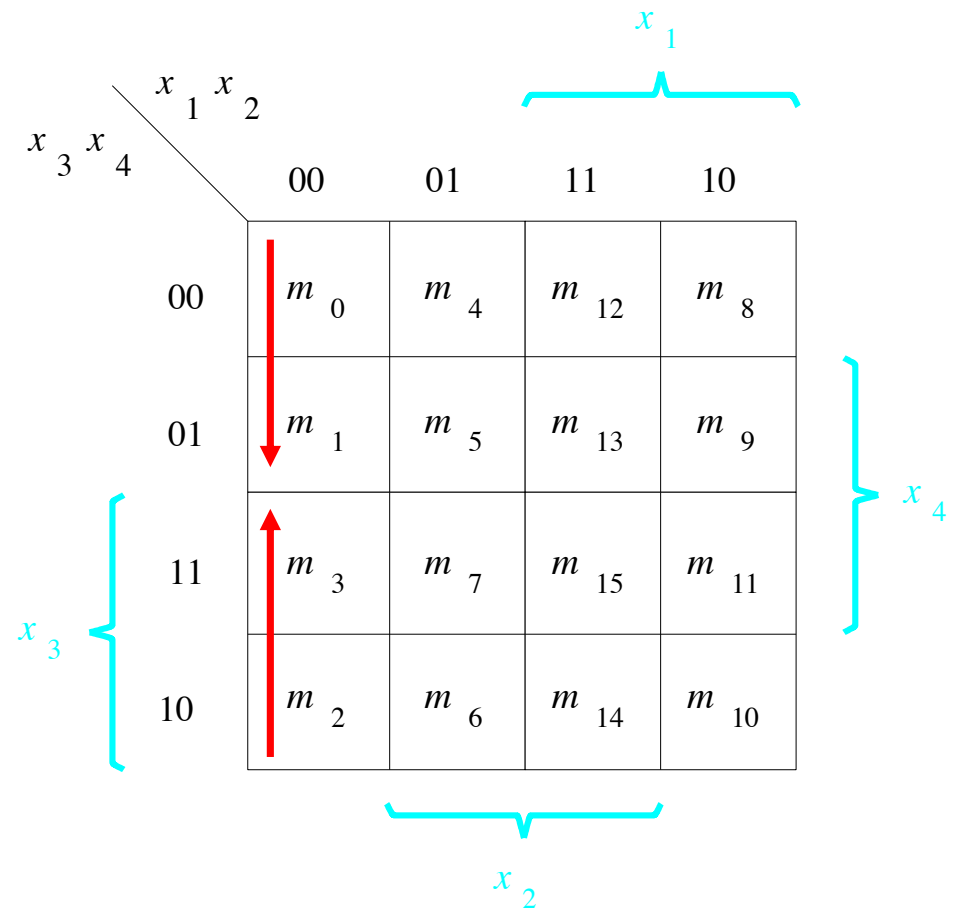
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



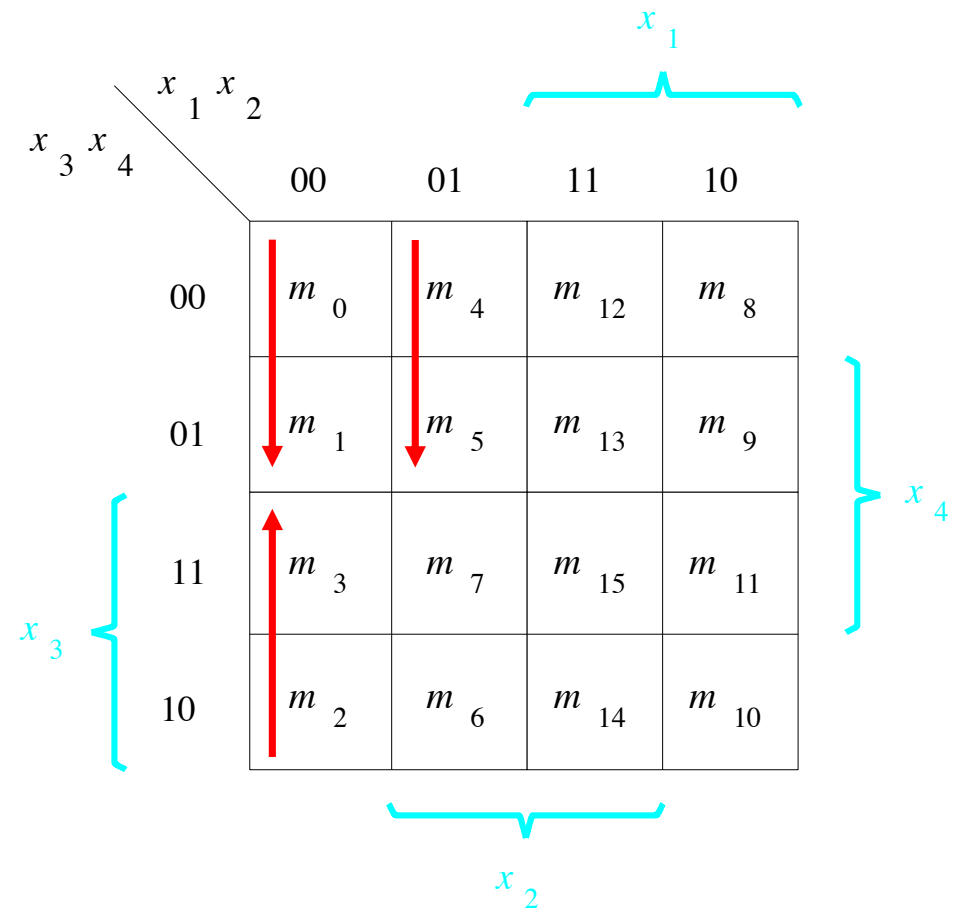
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



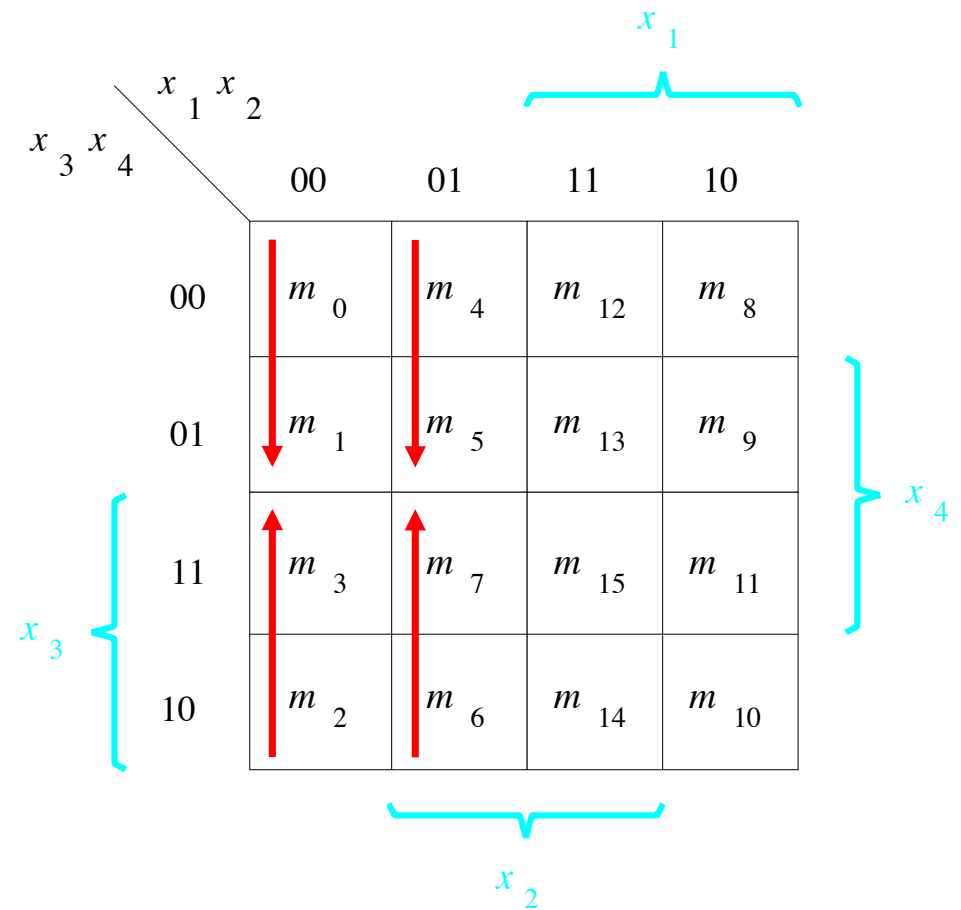
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



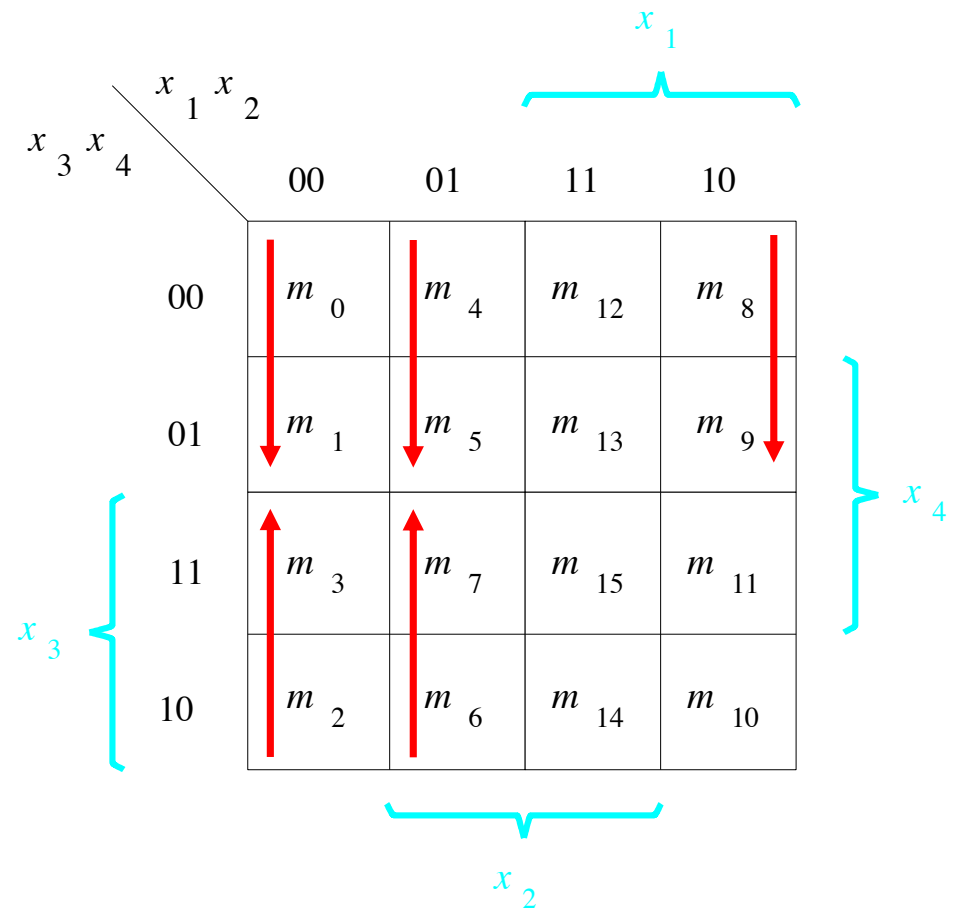
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



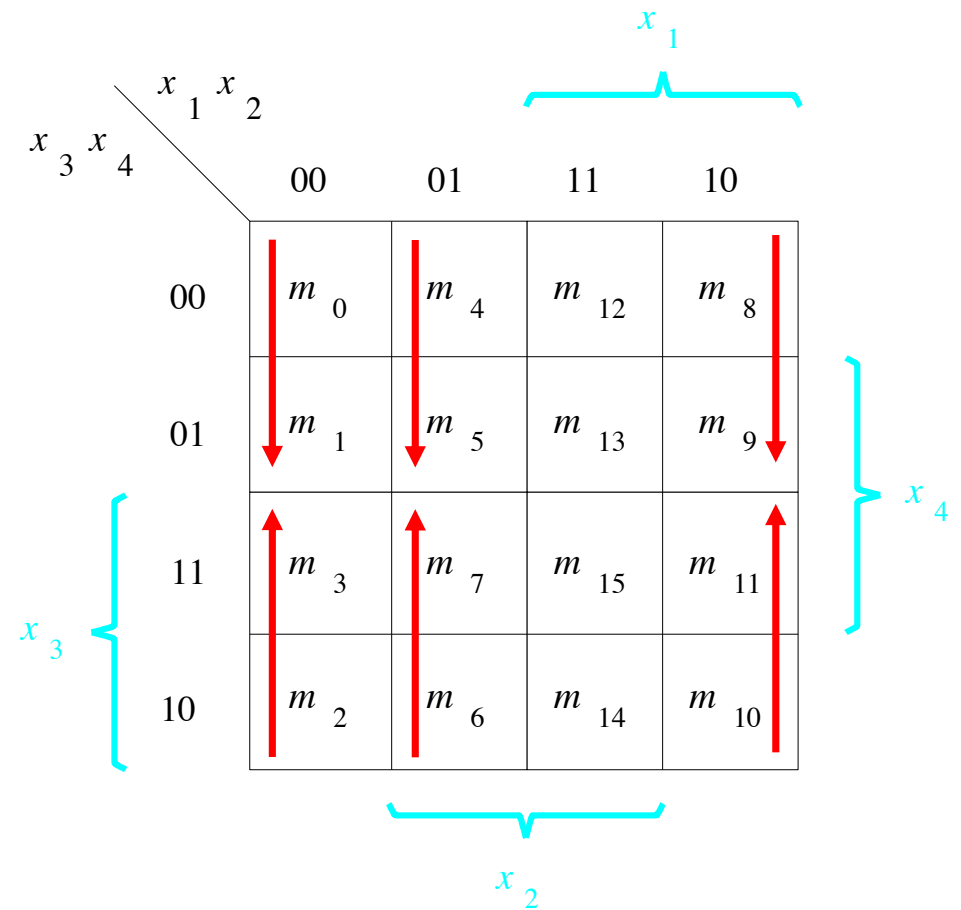
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



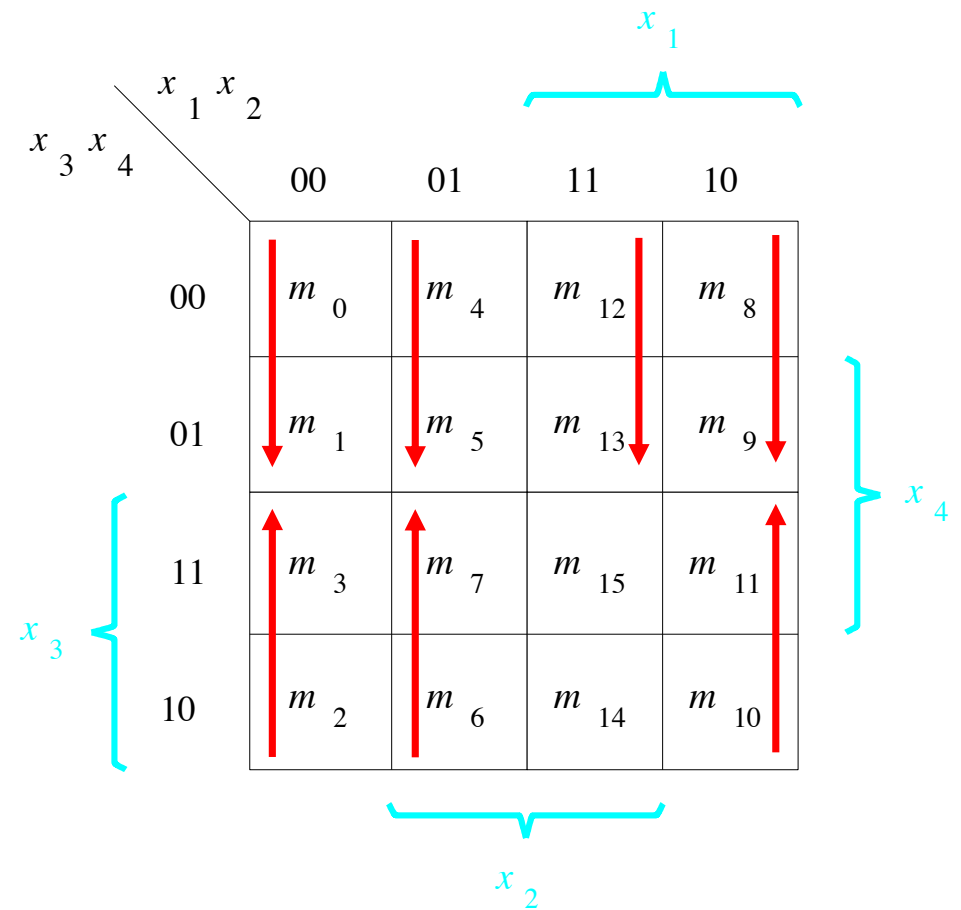
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



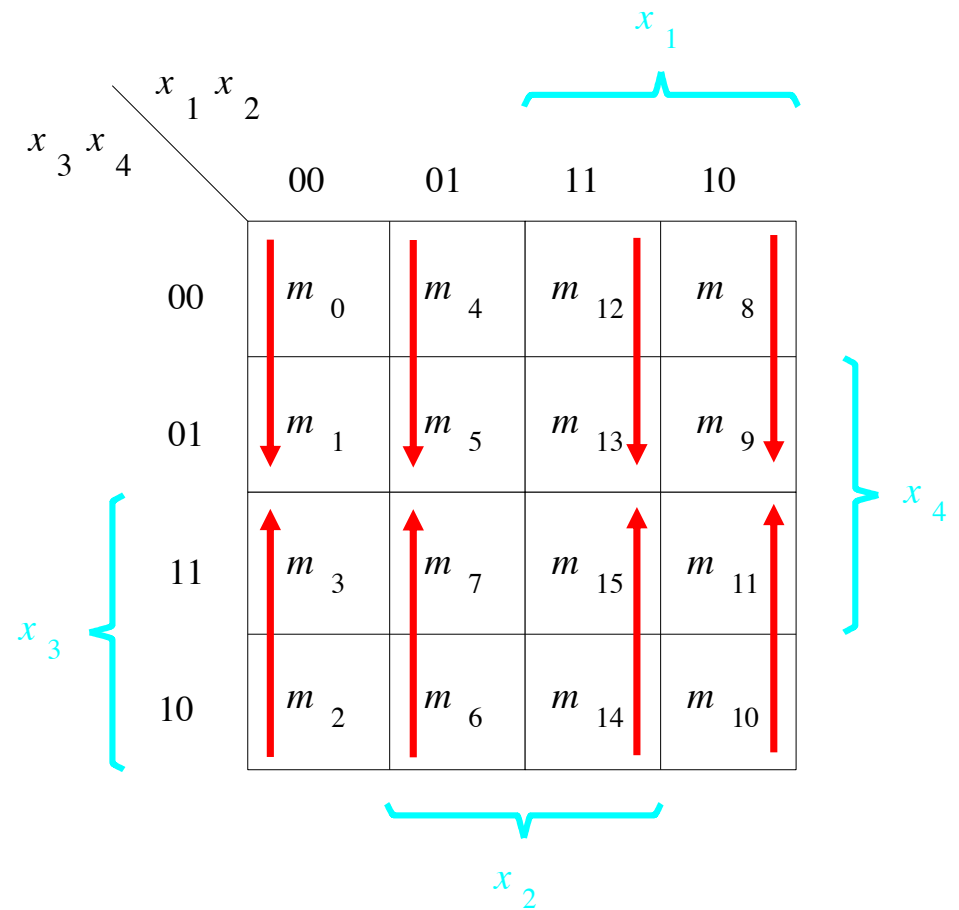
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



Adjacency Rules

x_3	x_1x_2	00	01	11	10
0		m_0	m_2	m_6	m_4
1		m_1	m_3	m_7	m_5

adjacent
columns

x_3x_4	x_1x_2	00	01	11	10
00		m_0	m_4	m_{12}	m_8
01		m_1	m_5	m_{13}	m_9
11		m_3	m_7	m_{15}	m_{11}
10		m_2	m_6	m_{14}	m_{10}

adjacent
columns

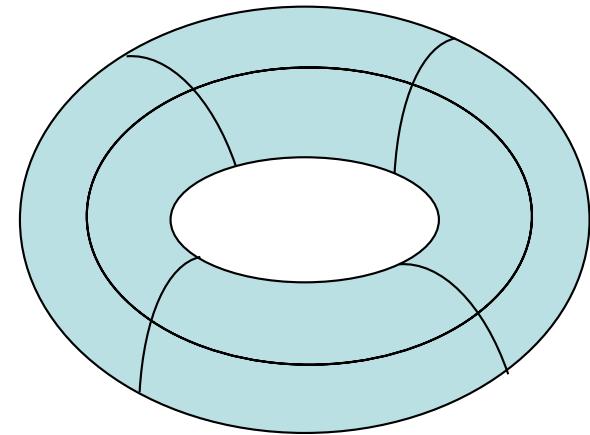
adjacent
rows

Adjacency Rules

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

adjacent
rows

adjacent
columns



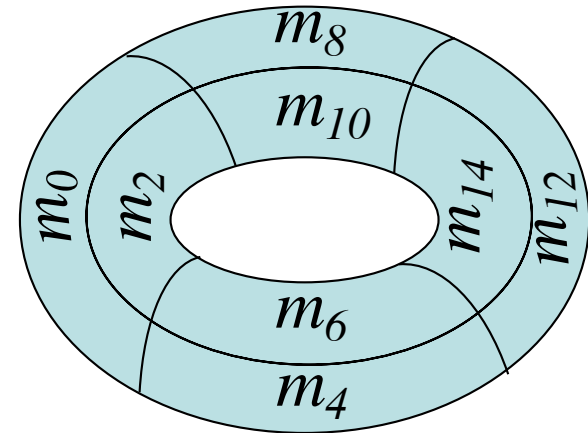
As if the K-map were
drawn on a torus

Adjacency Rules

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	m_0	m_4	m_{12}	m_8
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10	m_2	m_6	m_{14}	m_{10}

adjacent
rows

adjacent
columns



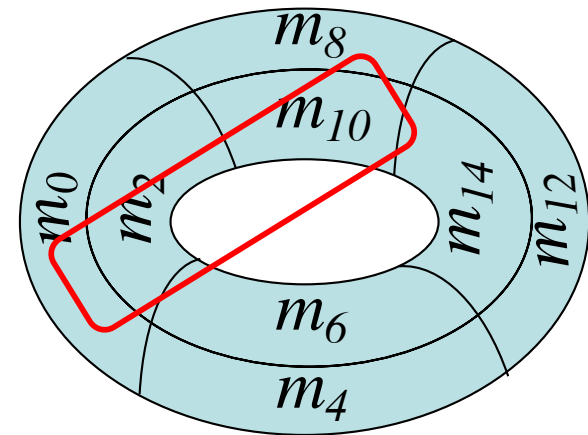
As if the K-map were
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Adjacency Rules

$x_3x_4 \backslash x_1x_2$	00	01	11	10
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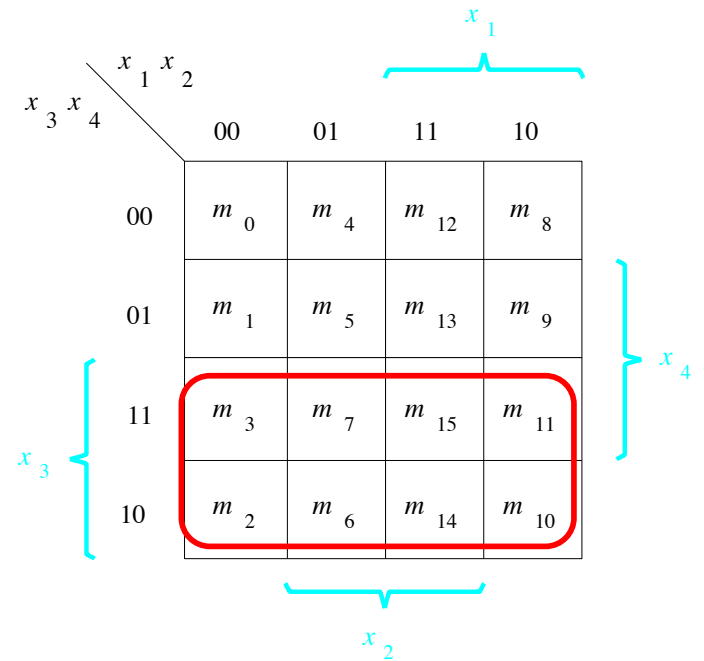
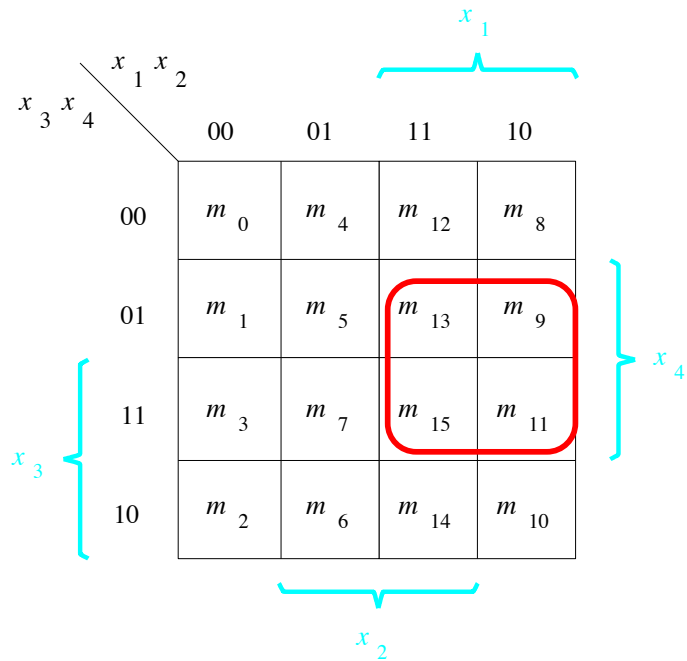
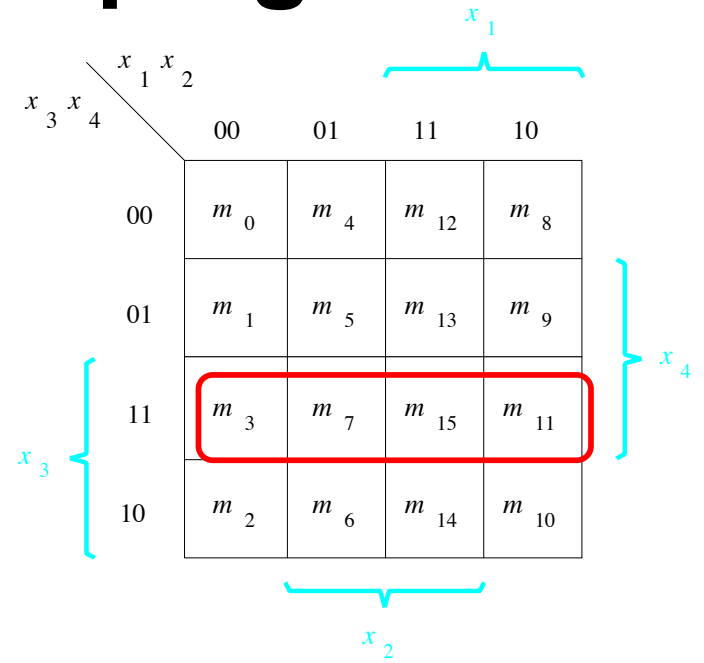
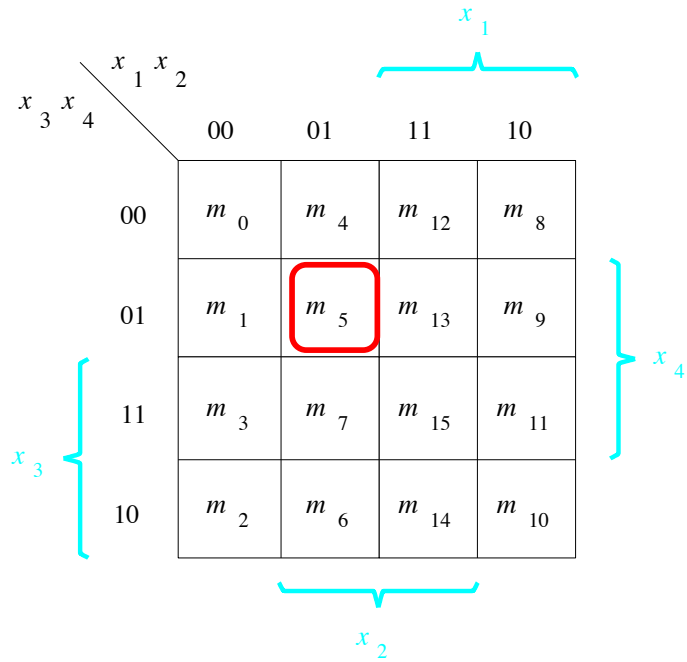
adjacent
rows

adjacent
columns

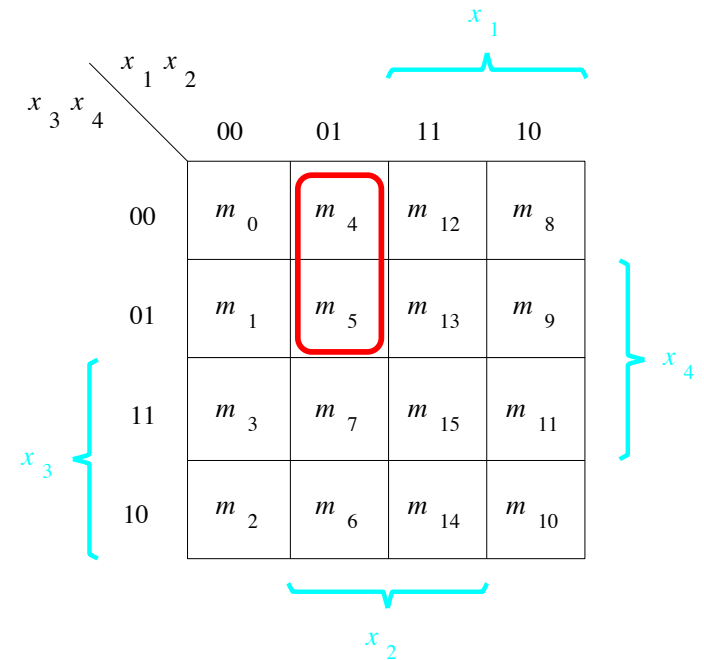
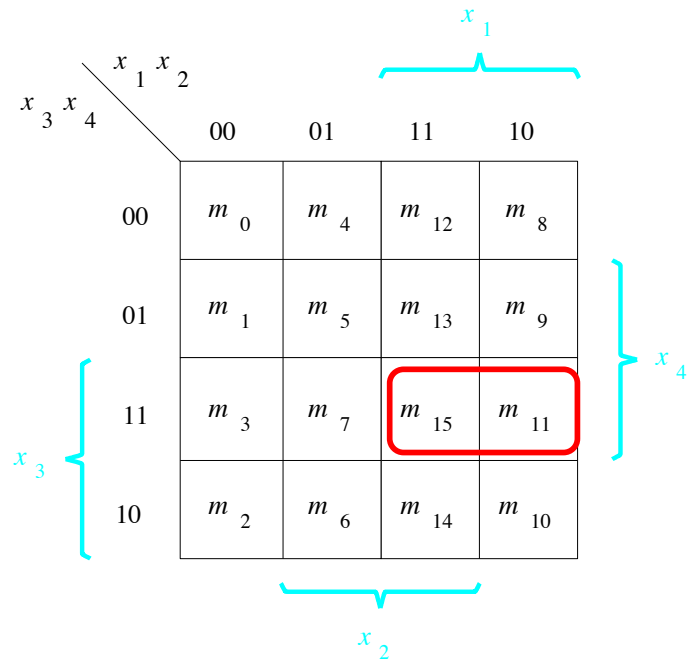
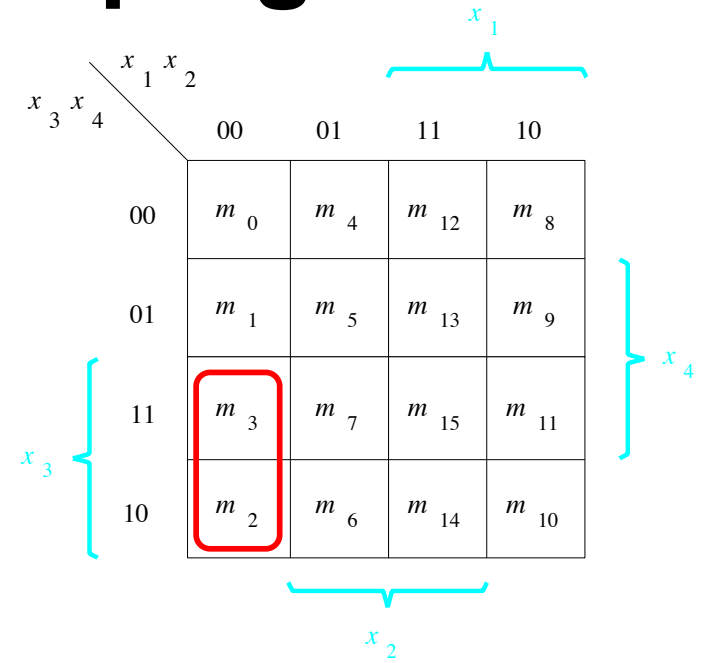
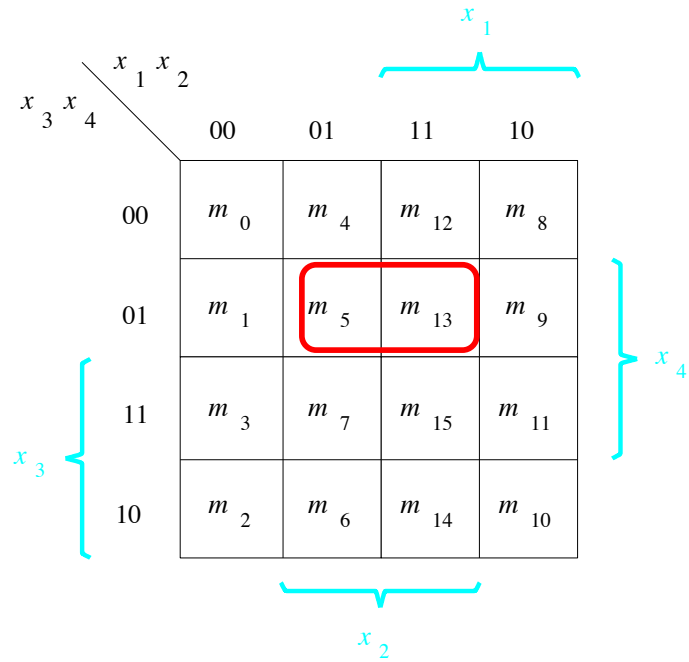


As if the K-map were
drawn on a torus

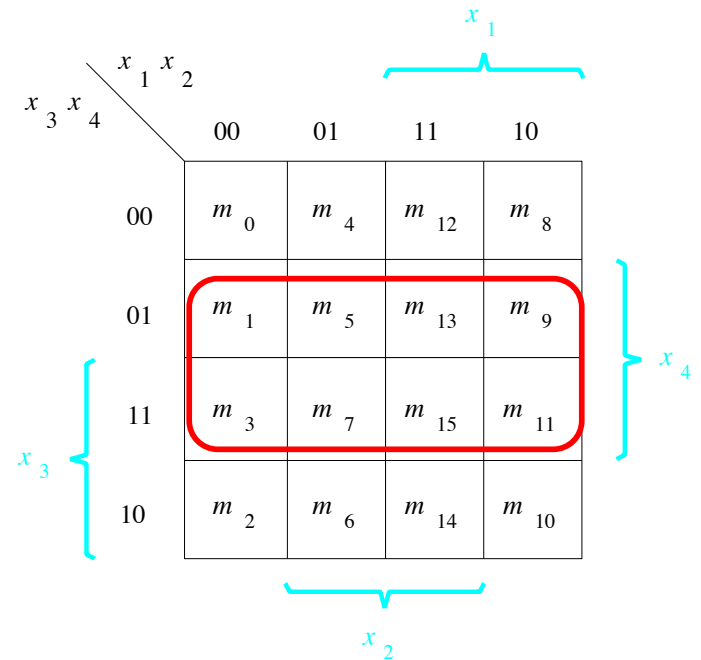
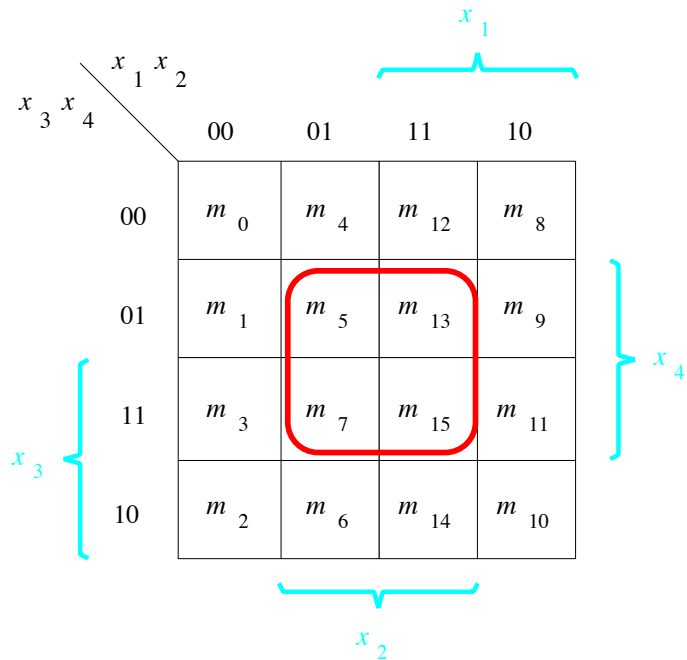
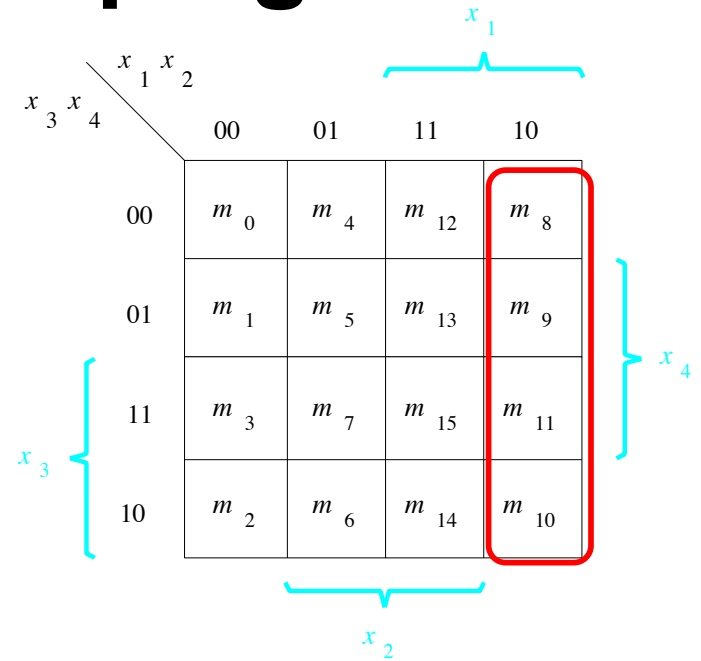
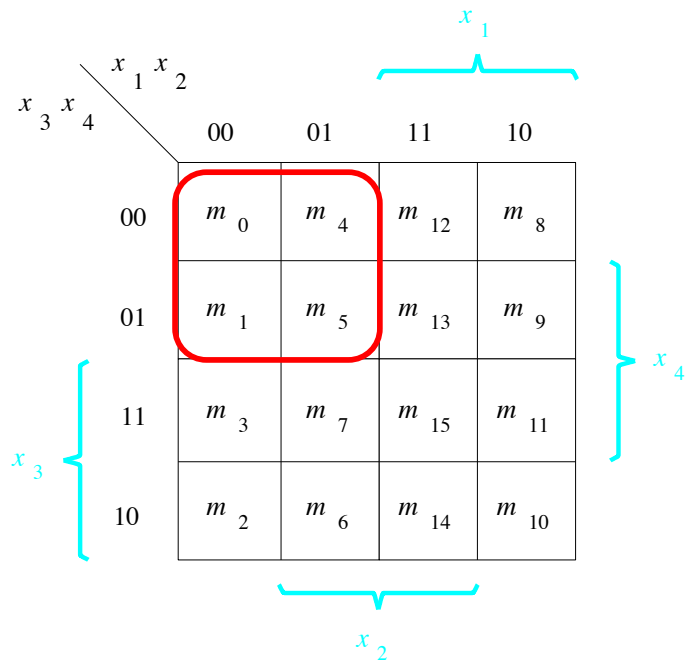
Some Valid Groupings



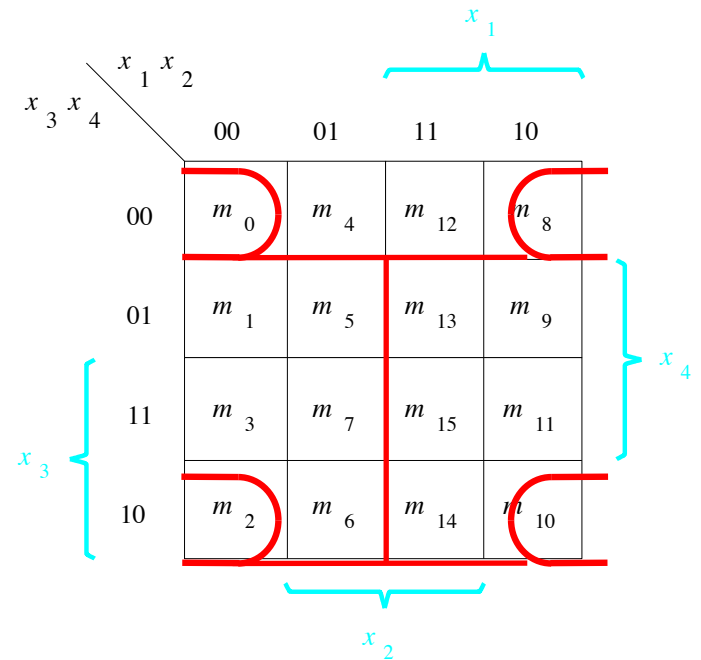
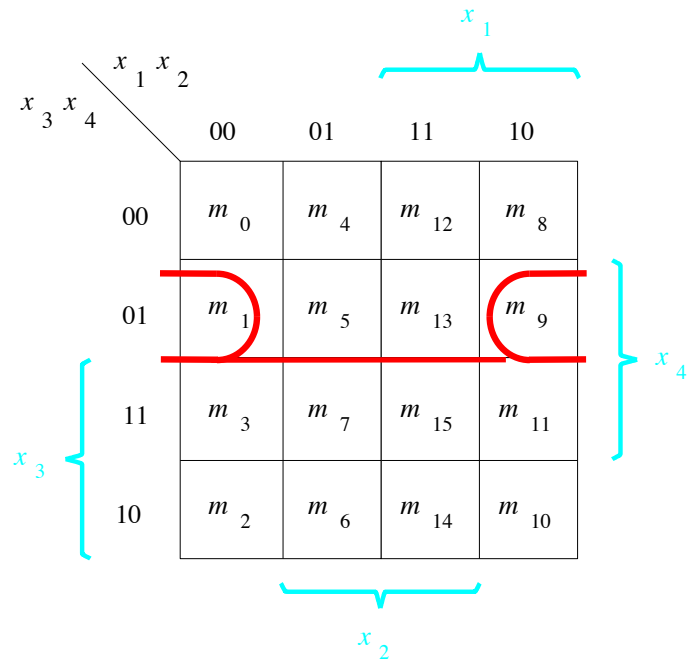
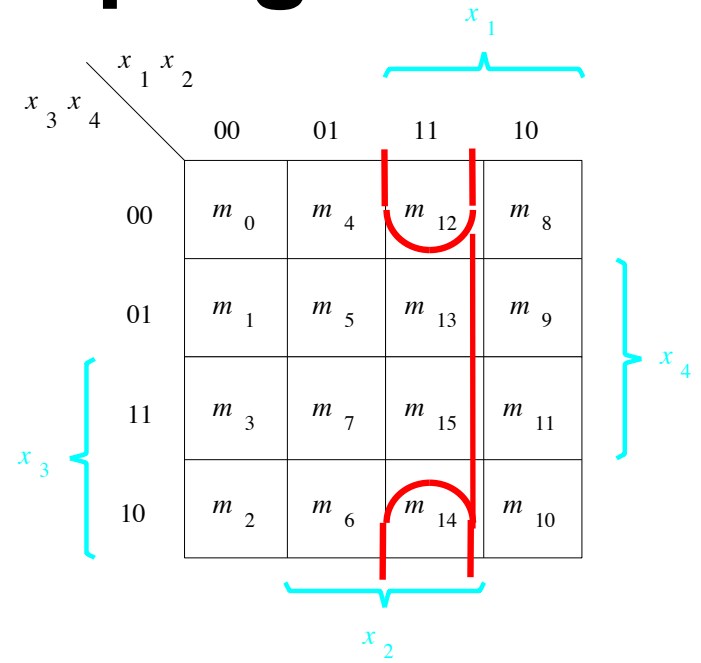
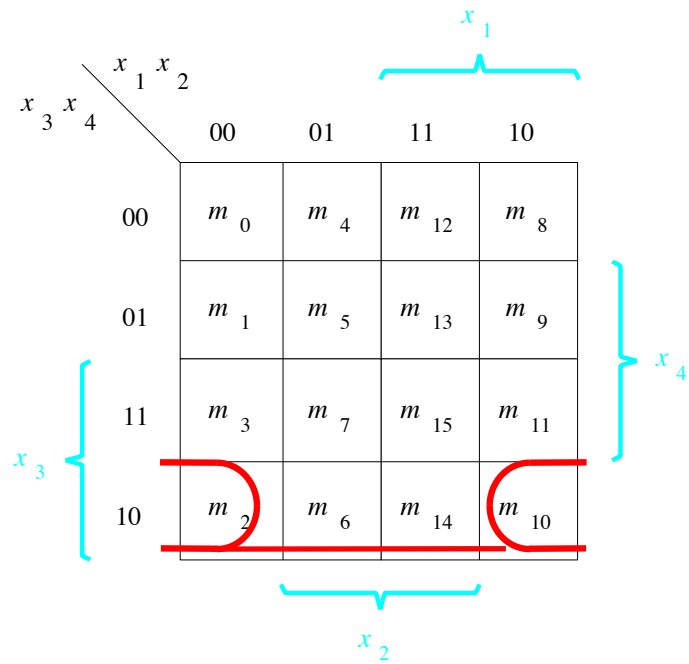
Some Valid Groupings



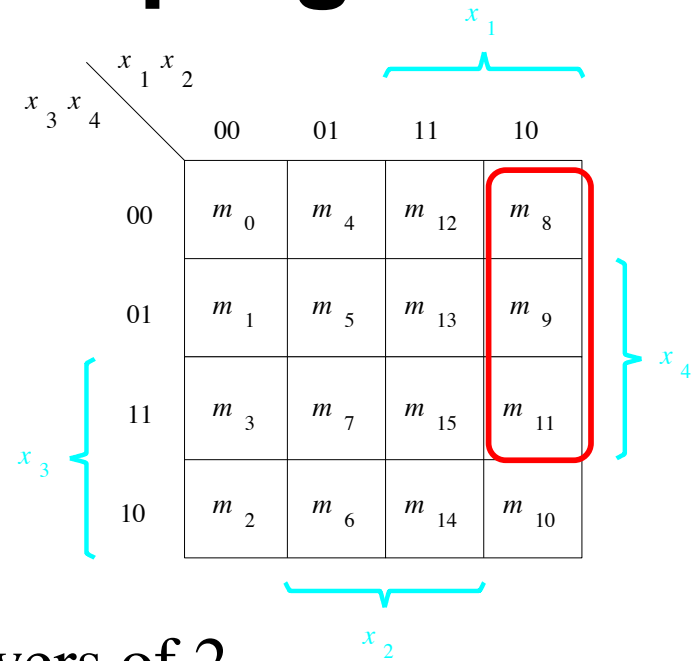
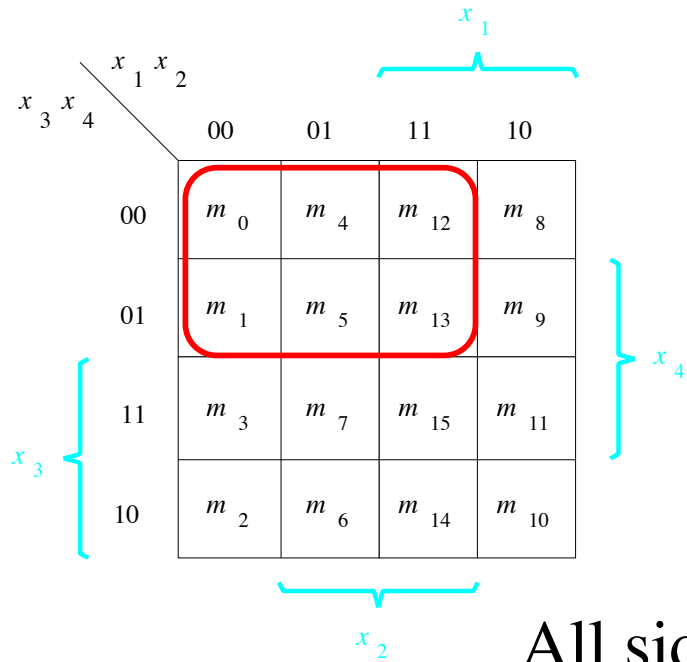
Some Valid Groupings



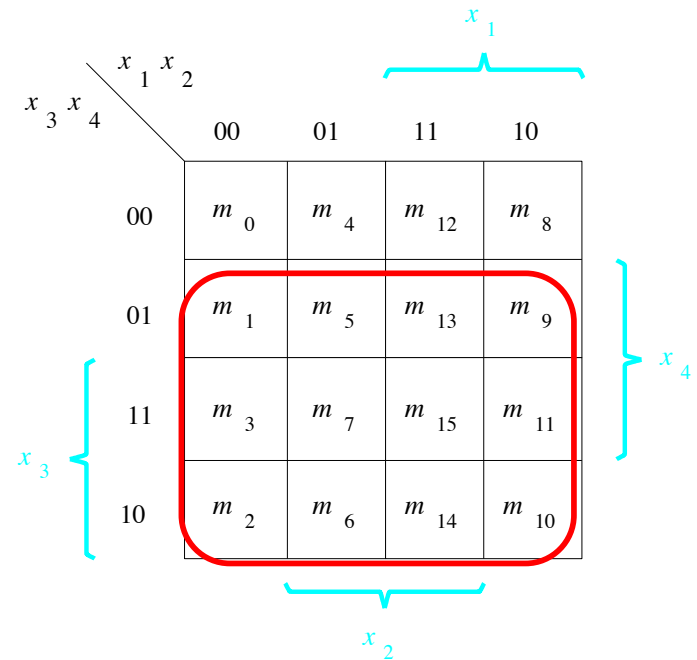
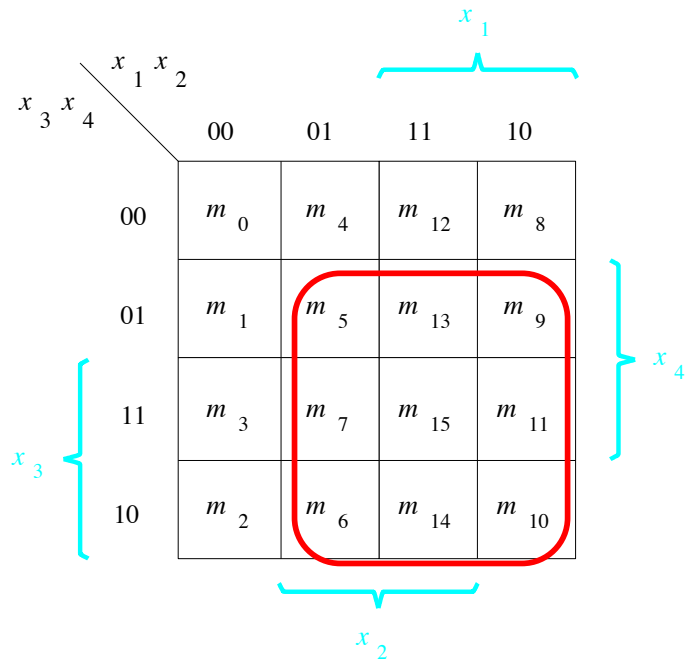
Some Valid Groupings



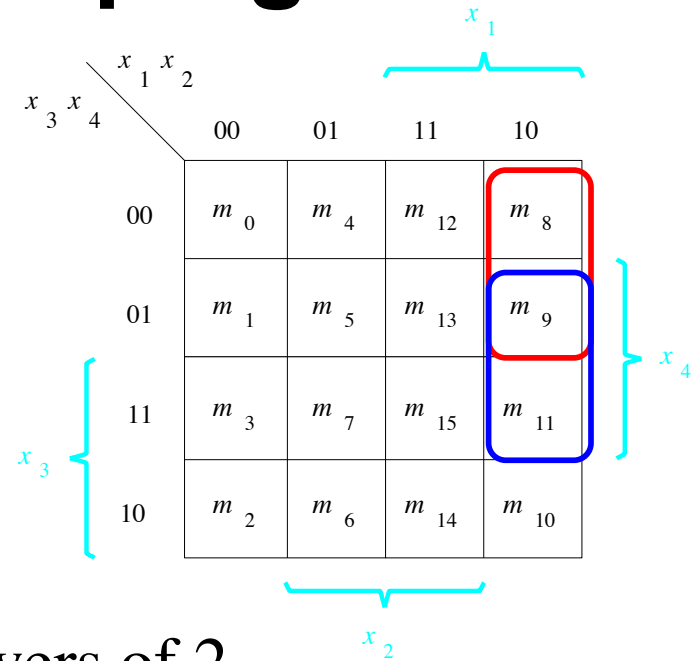
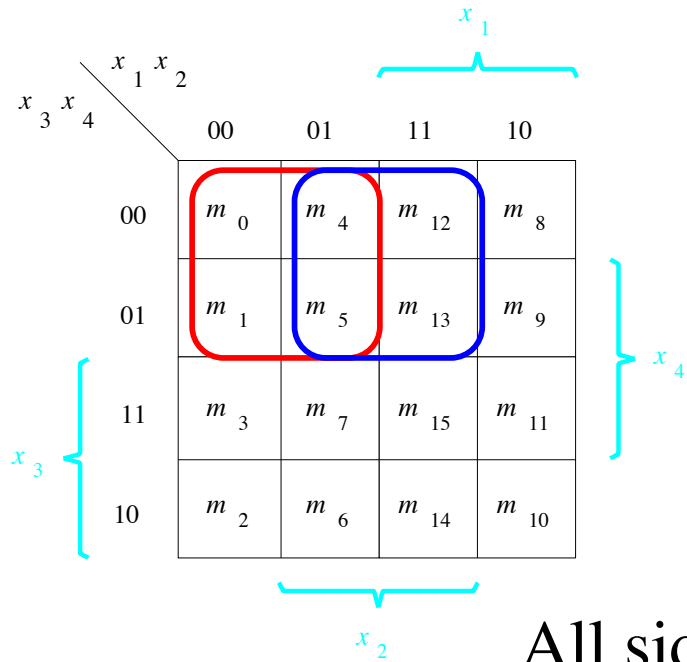
Some Invalid Groupings



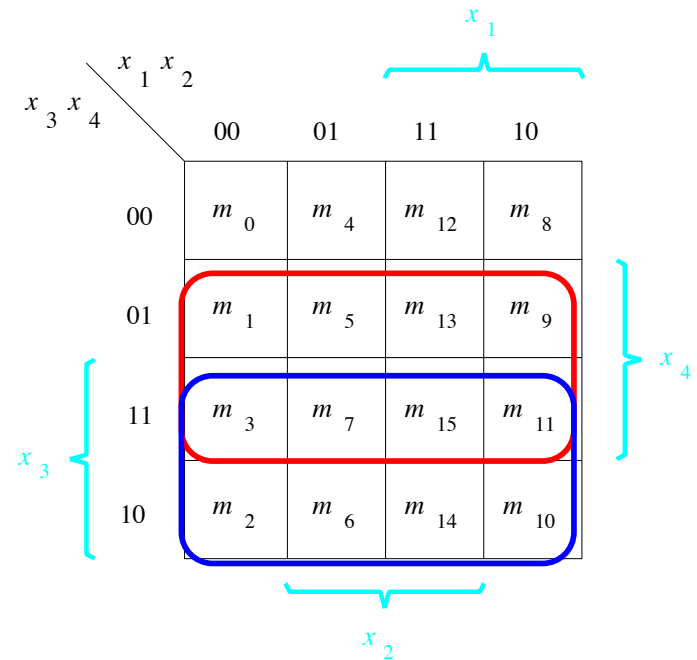
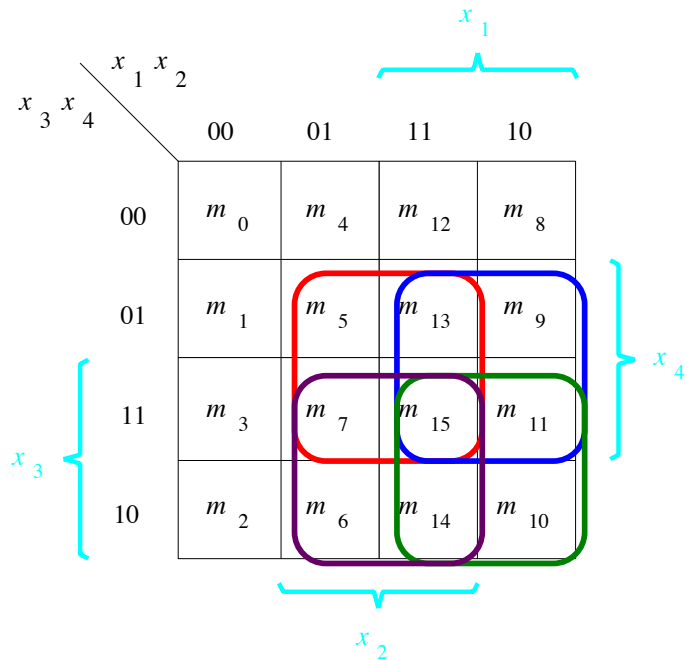
All sides must be powers of 2.



Some **valid** Groupings

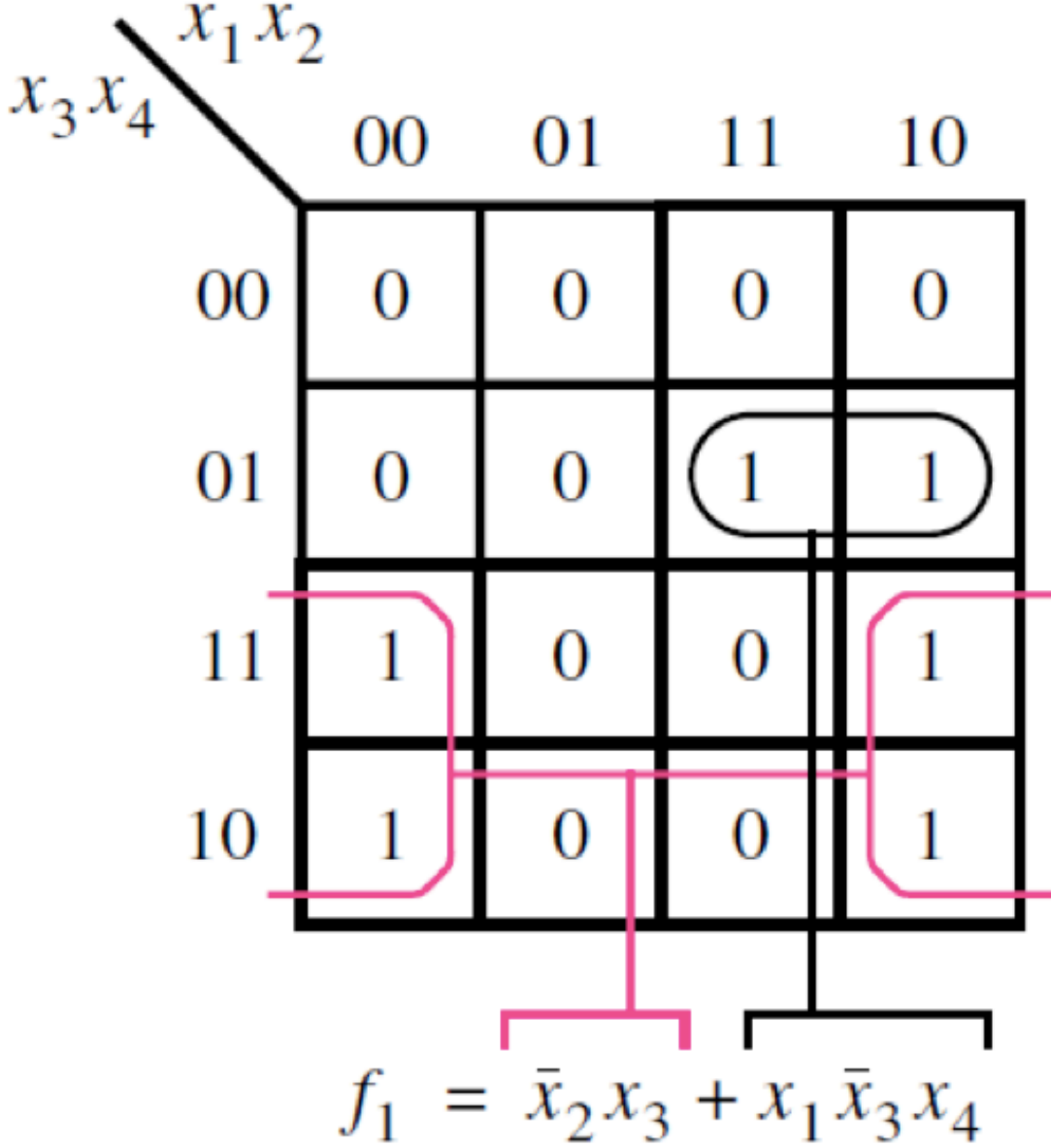


All sides must be powers of 2.



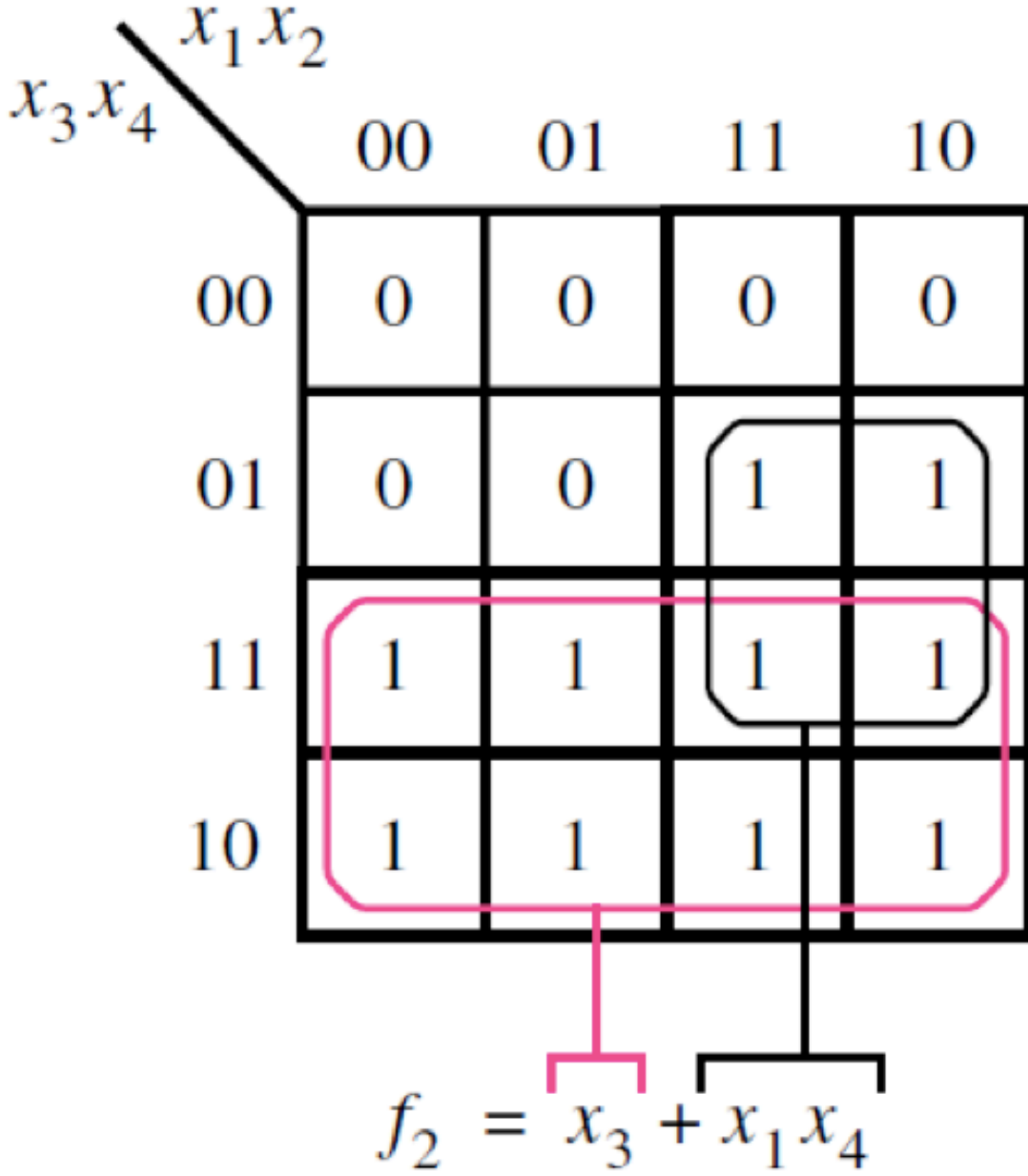
Minimization Examples with 4-variable K-Maps

Example of a four-variable Karnaugh map



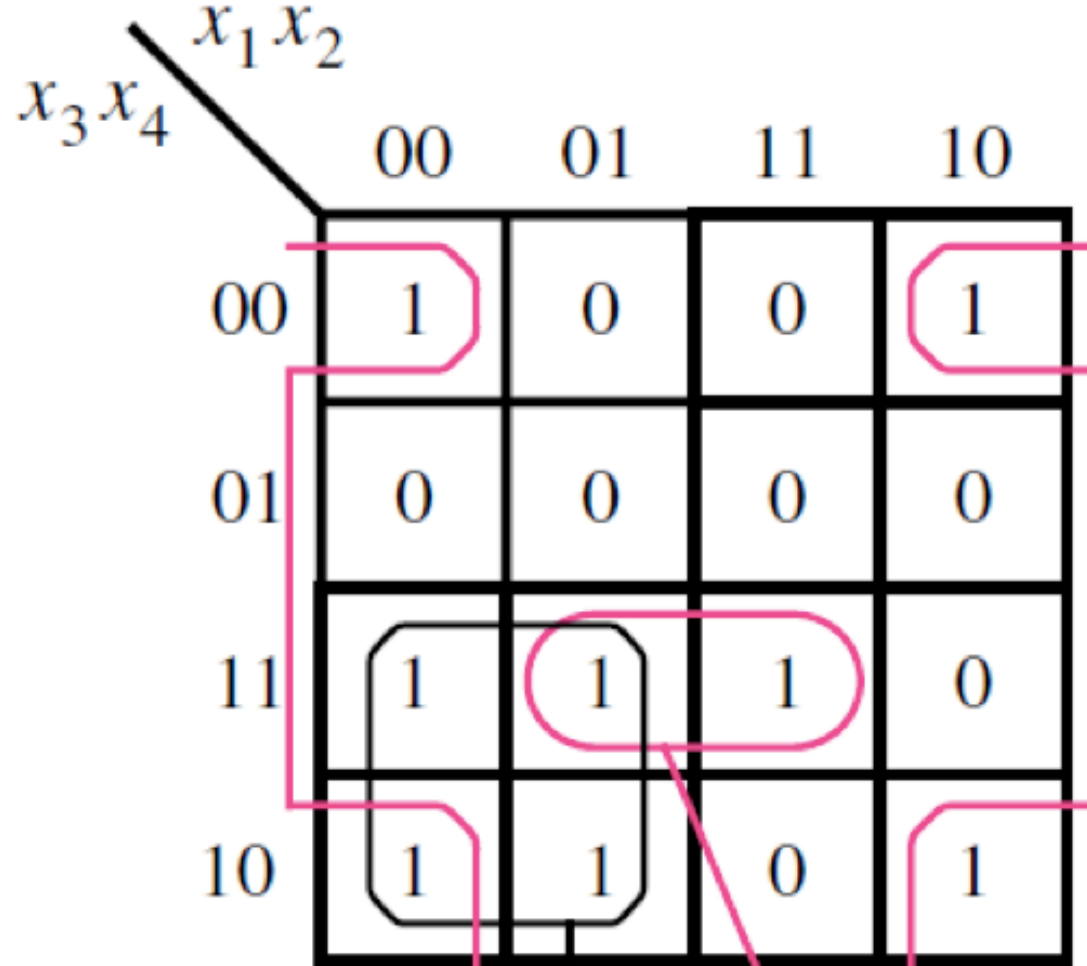
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



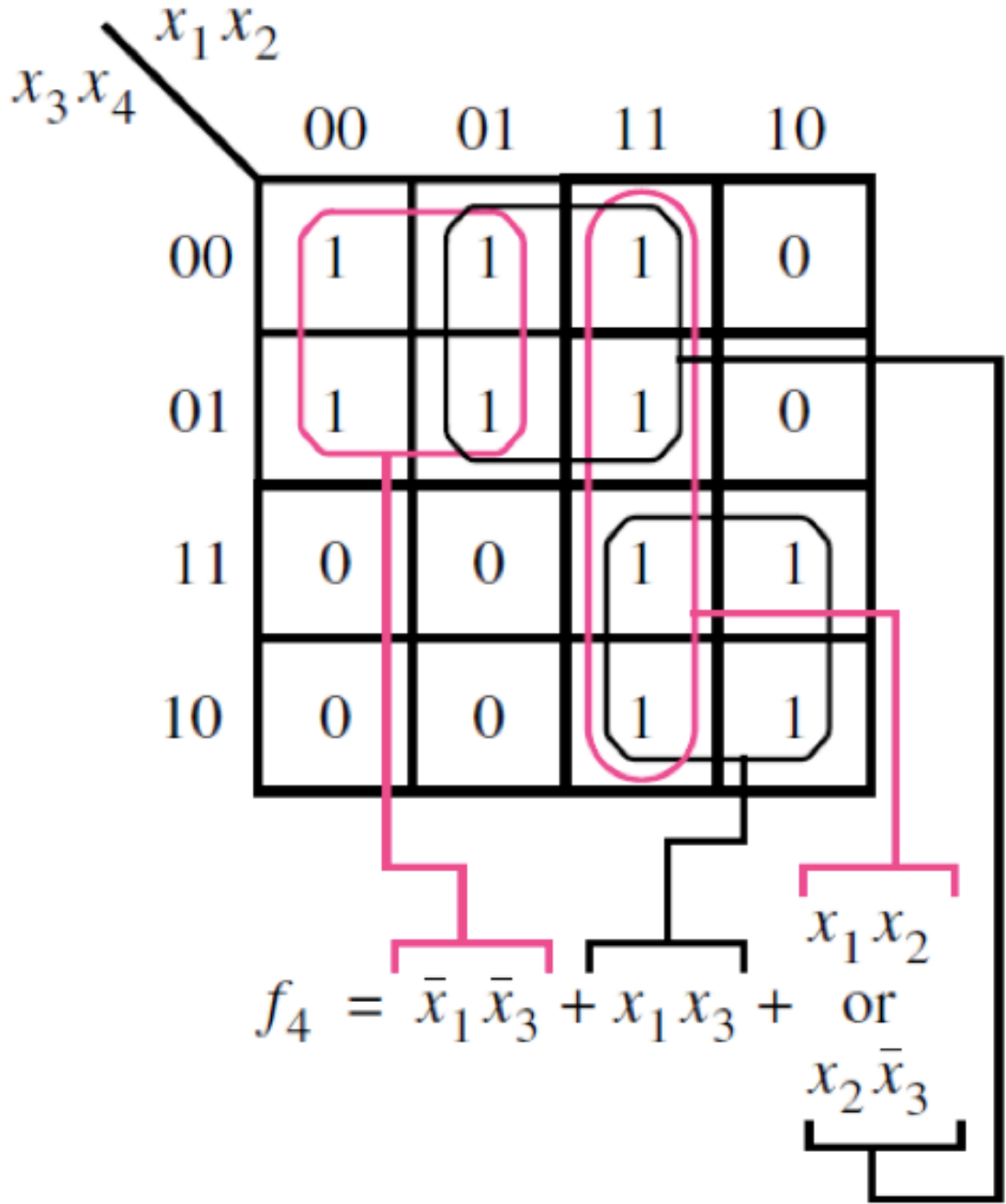
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



$$f_3 = \bar{x}_2\bar{x}_4 + \bar{x}_1x_3 + x_2x_3x_4$$

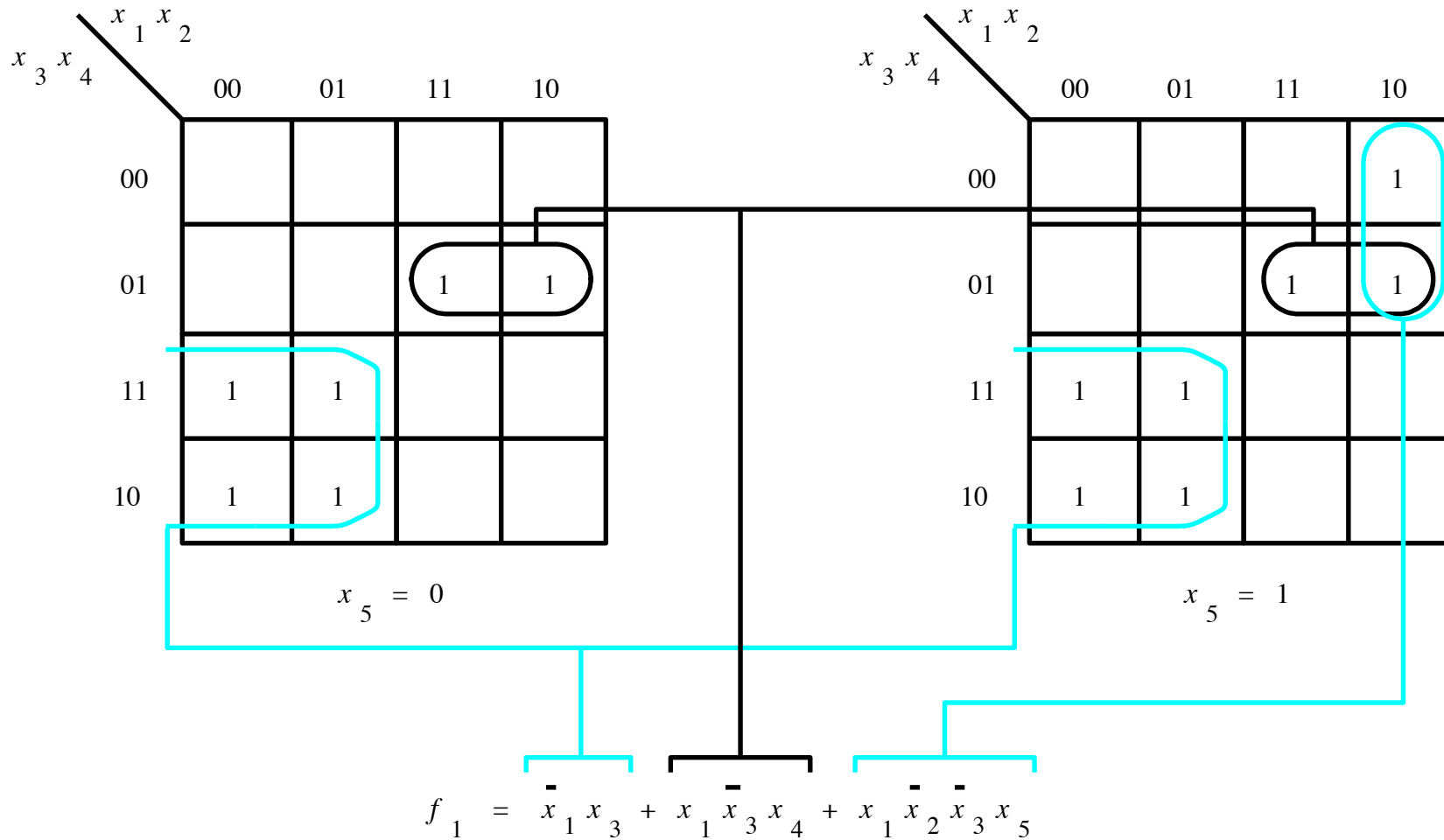
Example of a four-variable Karnaugh map



[Figure 2.54 from the textbook]

Five-Variable K-Map

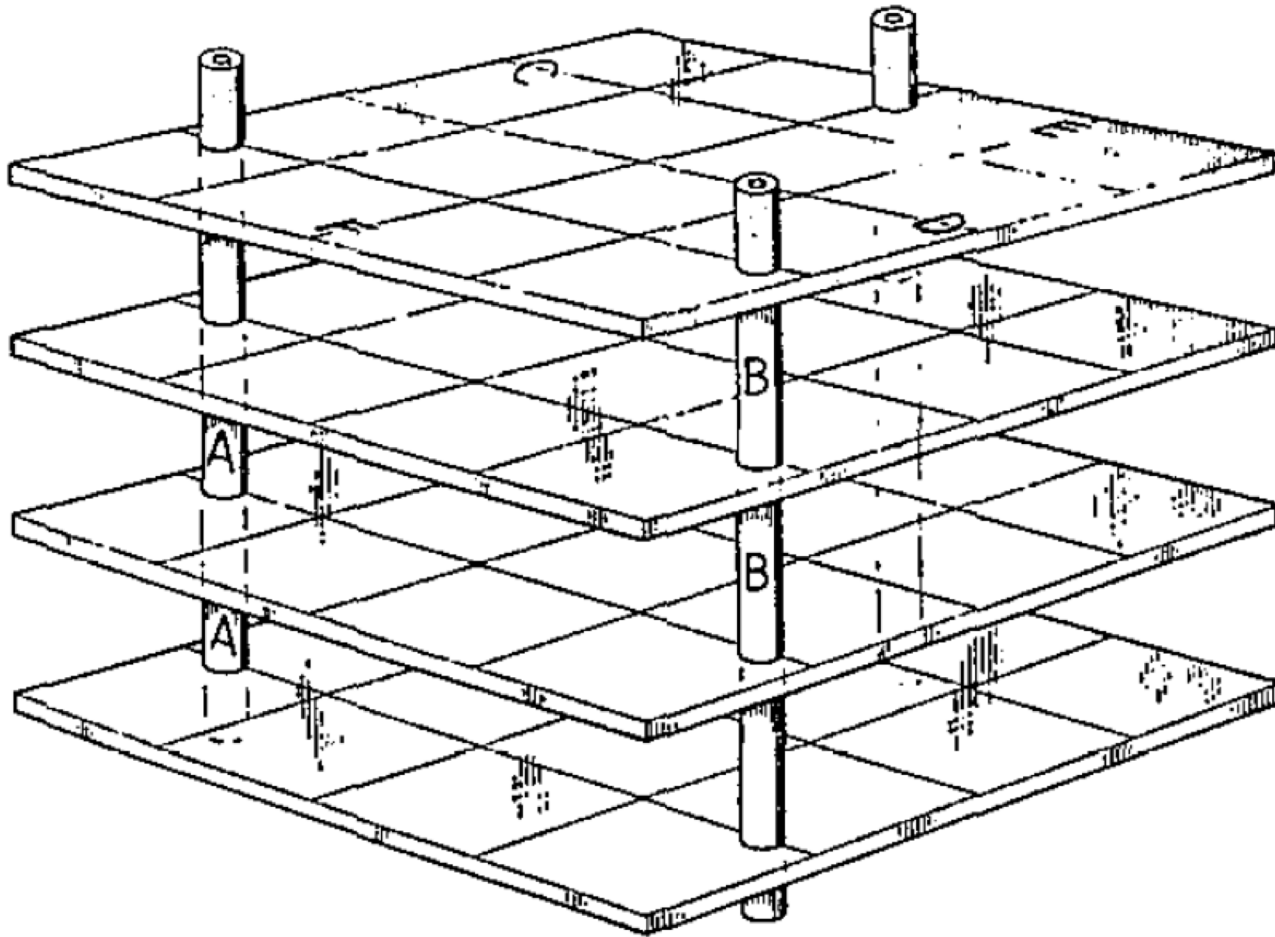
A five-variable Karnaugh map



[Figure 2.55 from the textbook]

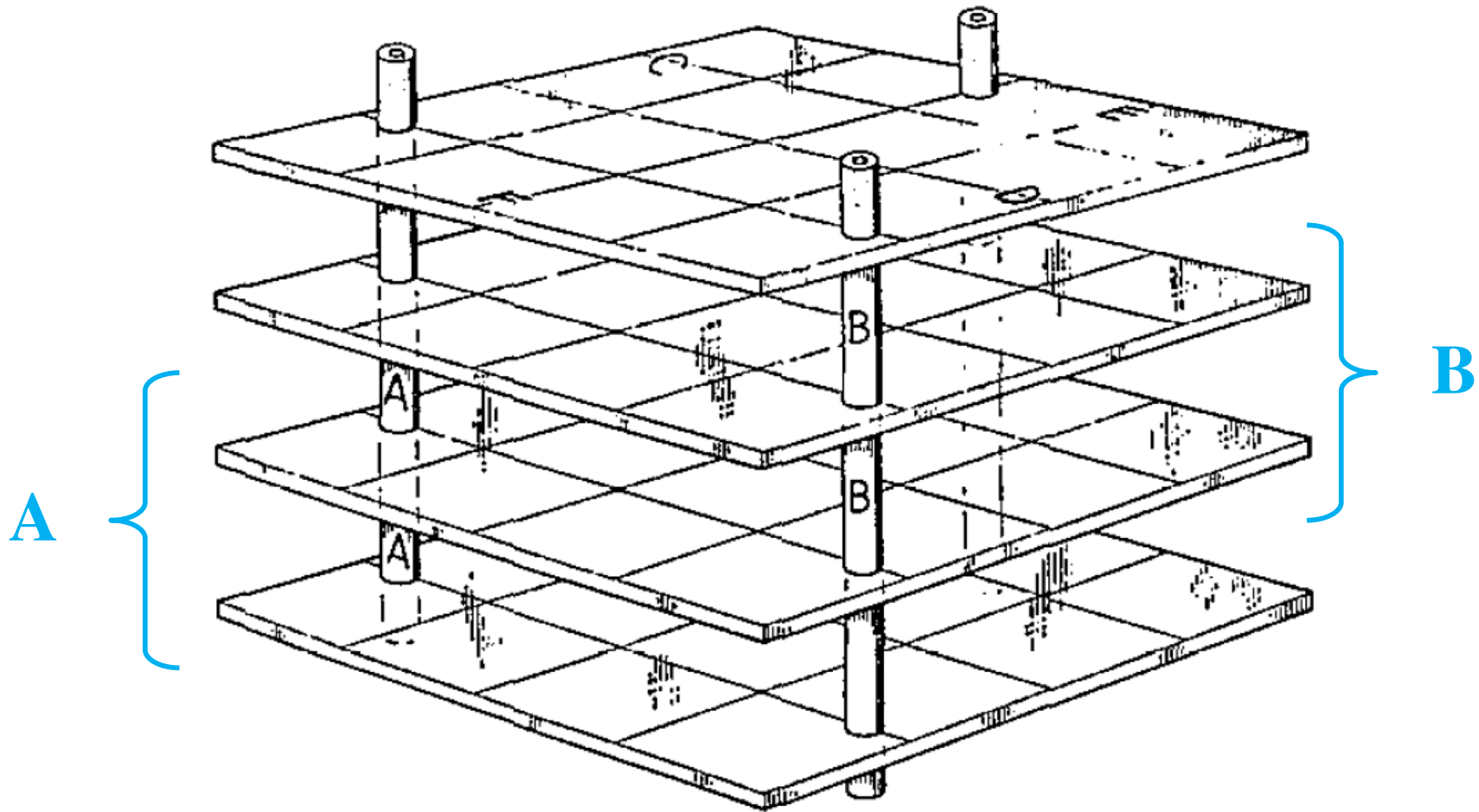
Six-Variable K-Map

A six-variable Karnaugh map

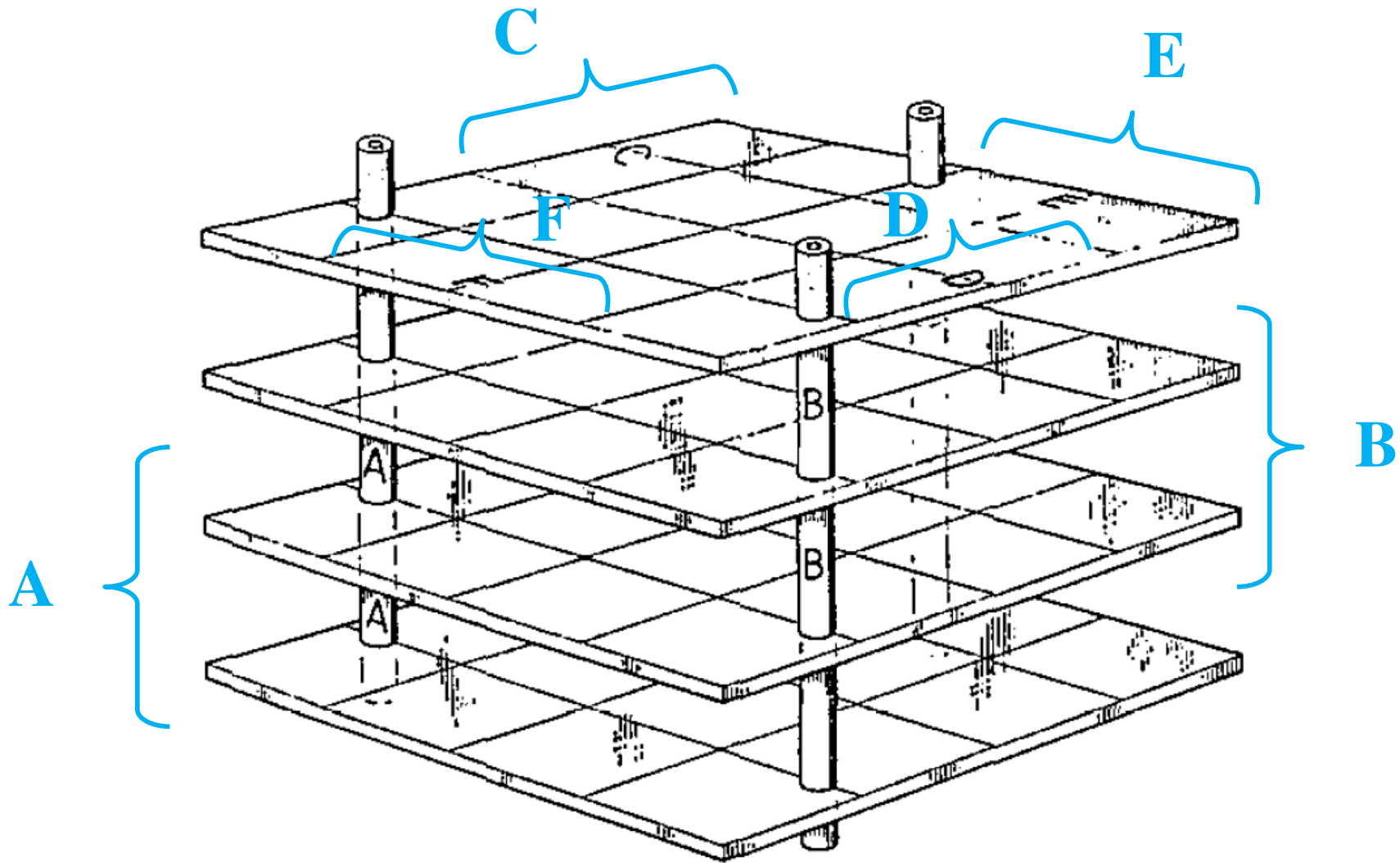


[Figure 16, in Karnaugh 1953]

A six-variable Karnaugh map

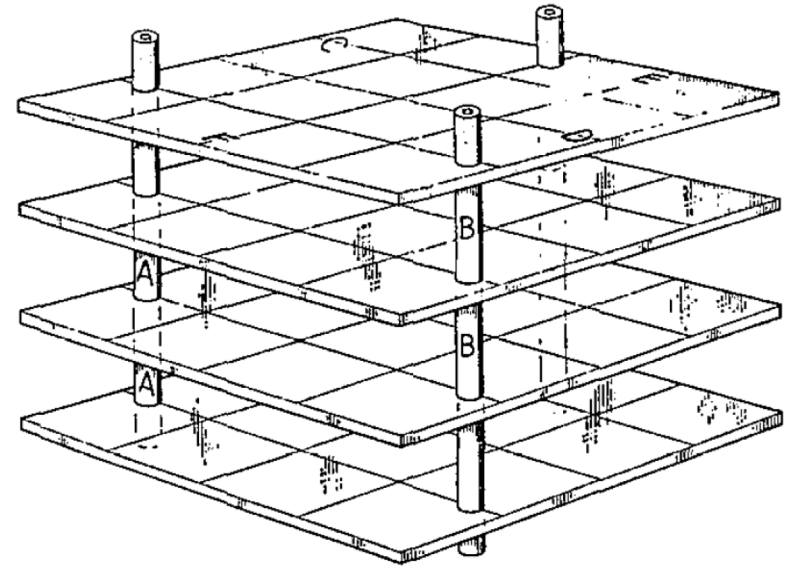
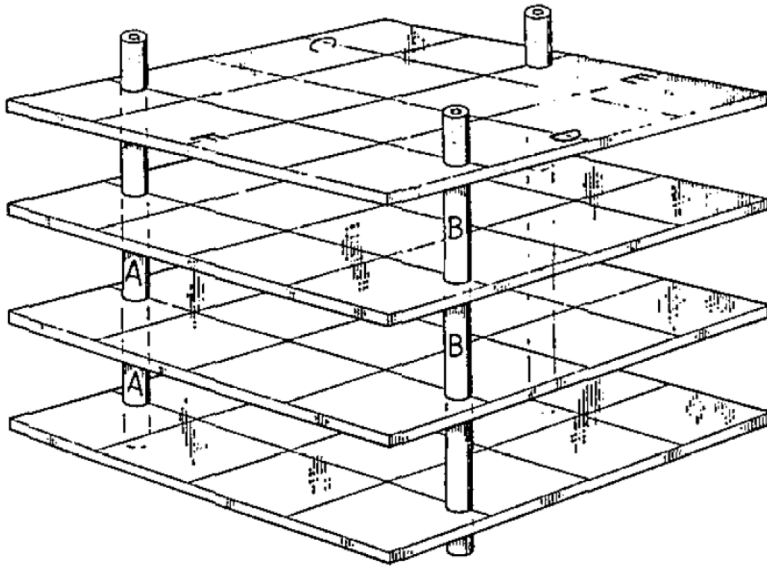


A six-variable Karnaugh map

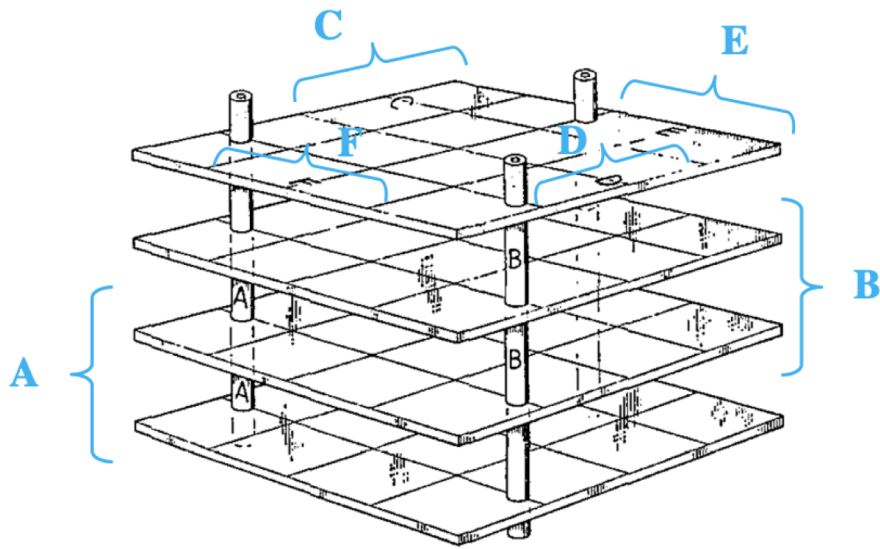


Seven-Variable K-Map

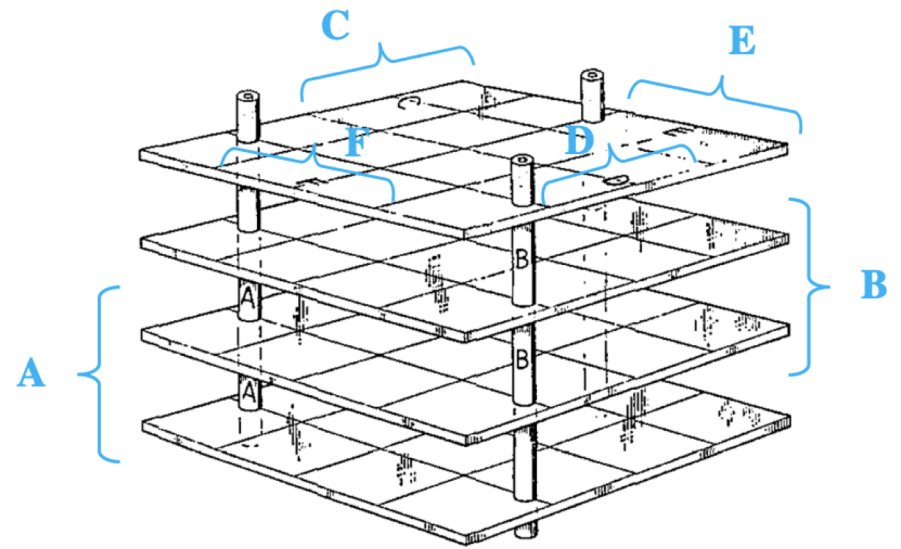
A seven-variable Karnaugh map



A seven-variable Karnaugh map



$G = 0$



$G = 1$

Questions?

THE END