



CprE 281: Digital Logic

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<http://www.ece.iastate.edu/~alexs/classes/>

Design Examples

CprE 281: Digital Logic
Iowa State University, Ames, IA
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Administrative Stuff

- **HW2 is due today @ 10pm**
- **Please write clearly on the first page the following three things:**
 - **Your First and Last Name**
 - **Your Student ID Number**
 - **Your Lab Section Letter**
- **Submit on Canvas as *one* PDF file.**
- **Please orient your pages such that the text can be read without the need to rotate the page.**

Administrative Stuff

- **HW3 is due on Monday Sep 11 @ 10pm**
- **Please write clearly on the first page the following three things:**
 - **Your First and Last Name**
 - **Your Student ID Number**
 - **Your Lab Section Letter**
- **Submit on Canvas as *one* PDF file.**
- **Please orient your pages such that the text can be read without the need to rotate the page.**

Quick Review

Axioms of Boolean Algebra

1a. $0 \cdot 0 = 0$

1b. $1 + 1 = 1$

2a. $1 \cdot 1 = 1$

2b. $0 + 0 = 0$

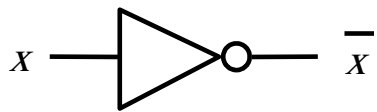
3a. $0 \cdot 1 = 1 \cdot 0 = 0$

3b. $1 + 0 = 0 + 1 = 1$

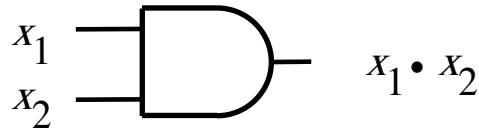
4a. If $x=0$, then $\bar{x} = 1$

4b. If $x=1$, then $\bar{x} = 0$

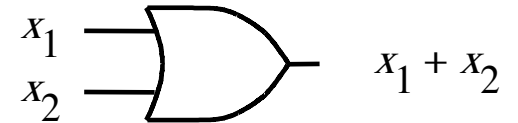
The Three Basic Logic Gates



NOT gate



AND gate



OR gate

Single-Variable Theorems

$$5a. \quad \mathbf{x \cdot 0 = 0}$$

$$5b. \quad \mathbf{x + 1 = 1}$$

$$6a. \quad \mathbf{x \cdot 1 = x}$$

$$6b. \quad \mathbf{x + 0 = x}$$

$$7a. \quad \mathbf{x \cdot x = x}$$

$$7b. \quad \mathbf{x + x = x}$$

$$8a. \quad \mathbf{x \cdot \overline{x} = 0}$$

$$8b. \quad \mathbf{x + \overline{x} = 1}$$

$$9. \quad \mathbf{\overline{\overline{x}} = x}$$

Two- and Three-Variable Properties

10a. $x \cdot y = y \cdot x$

Commutative

10b. $x + y = y + x$

11a. $x \cdot (y \cdot z) = (x \cdot y) \cdot z$

Associative

11b. $x + (y + z) = (x + y) + z$

12a. $x \cdot (y + z) = x \cdot y + x \cdot z$

Distributive

12b. $x + y \cdot z = (x + y) \cdot (x + z)$

13a. $x + x \cdot y = x$

Absorption

13b. $x \cdot (x + y) = x$

Two- and Three-Variable Properties

$$14a. \quad x \cdot y + x \cdot \bar{y} = x$$

Combining

$$14b. \quad (x + y) \cdot (x + \bar{y}) = x$$

$$15a. \quad \overline{x \cdot y} = \bar{x} + \bar{y}$$

DeMorgan's
theorem

$$15b. \quad \overline{x + y} = \bar{x} \cdot \bar{y}$$

$$16a. \quad x + \bar{x} \cdot y = x + y$$

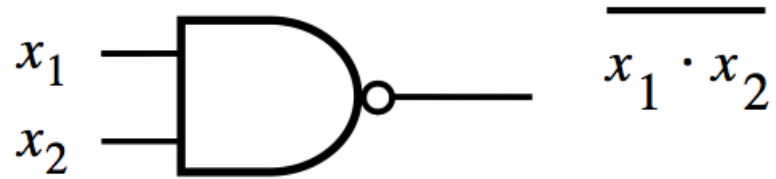
$$16b. \quad x \cdot (\bar{x} + y) = x \cdot y$$

$$17a. \quad x \cdot y + y \cdot z + \bar{x} \cdot z = x \cdot y + \bar{x} \cdot z$$

Consensus

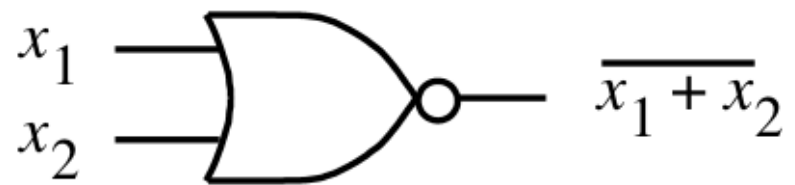
$$17b. \quad (x+y) \cdot (y+z) \cdot (\bar{x}+z) = (x+y) \cdot (\bar{x}+z)$$

NAND Gate



x_1	x_2	f
0	0	1
0	1	1
1	0	1
1	1	0

NOR Gate



x_1	x_2	f
0	0	1
0	1	0
1	0	0
1	1	0

Why do we need two more gates?

They can be implemented with fewer transistors.

Each of the new gates can be used to implement the three basic logic gates: NOT, AND, OR.

Implications

**Any Boolean function can be implemented
with only NAND gates!**

Implications

**Any Boolean function can be implemented
with only NAND gates!**

The same is also true for NOR gates!

minterms
(for two variables)

The Four minterms

x	y	m_0
0	0	1
0	1	0
1	0	0
1	1	0

$m_0(x, y)$

x	y	m_1
0	0	0
0	1	1
1	0	0
1	1	0

$m_1(x, y)$

x	y	m_2
0	0	0
0	1	0
1	0	1
1	1	0

$m_2(x, y)$

x	y	m_3
0	0	0
0	1	0
1	0	0
1	1	1

$m_3(x, y)$

The Four minterms

x	y	m_0
0	0	1
0	1	0
1	0	0
1	1	0

$m_0(x, y)$

x	y	m_1
0	0	0
0	1	1
1	0	0
1	1	0

$m_1(x, y)$

x	y	m_2
0	0	0
0	1	0
1	0	1
1	1	0

$m_2(x, y)$

x	y	m_3
0	0	0
0	1	0
1	0	0
1	1	1

$m_3(x, y)$

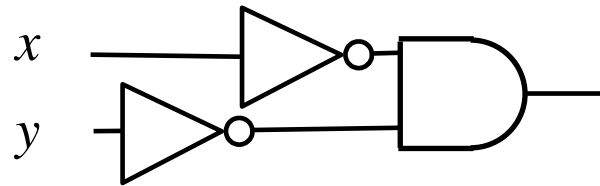
The Four minterms

x	y	$m_0(x, y)$	$m_1(x, y)$	$m_2(x, y)$	$m_3(x, y)$
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

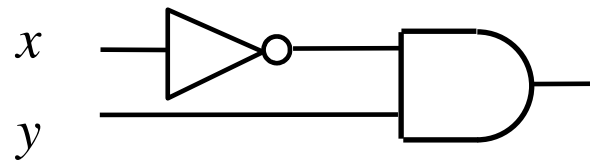
The Four minterms

x	y	$\bar{x}\bar{y}$	$\bar{x}y$	$x\bar{y}$	xy
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

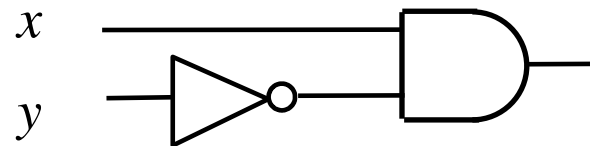
Circuits for the four minterms



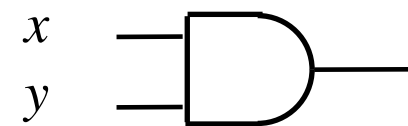
$$m_0(x, y) = \bar{x} \bar{y}$$



$$m_1(x, y) = \bar{x} y$$



$$m_2(x, y) = x \bar{y}$$



$$m_3(x, y) = x y$$

Maxterms (for two variables)

The Four Maxterms

x	y	M_0
0	0	0
0	1	1
1	0	1
1	1	1

$M_0(x, y)$

x	y	M_1
0	0	1
0	1	0
1	0	1
1	1	1

$M_1(x, y)$

x	y	M_2
0	0	1
0	1	1
1	0	0
1	1	1

$M_2(x, y)$

x	y	M_3
0	0	1
0	1	1
1	0	1
1	1	0

$M_3(x, y)$

The Four Maxterms

x	y	M_0
0	0	0
0	1	1
1	0	1
1	1	1

$M_0(x, y)$

x	y	M_1
0	0	1
0	1	0
1	0	1
1	1	1

$M_1(x, y)$

x	y	M_2
0	0	1
0	1	1
1	0	0
1	1	1

$M_2(x, y)$

x	y	M_3
0	0	1
0	1	1
1	0	1
1	1	0

$M_3(x, y)$

The Four Maxterms

x	y	$M_0(x, y)$	$M_1(x, y)$	$M_2(x, y)$	$M_3(x, y)$
0	0	0	1	1	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	0

The Four Maxterms

x	y	$x + y$	$x + \bar{y}$	$\bar{x} + y$	$\bar{x} + \bar{y}$
0	0	0	1	1	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	0

minterms and Maxterms (for two variables)

minterms and Maxterms

x	y	m_0
0	0	1
0	1	0
1	0	0
1	1	0

x	y	m_1
0	0	0
0	1	1
1	0	0
1	1	0

x	y	m_2
0	0	0
0	1	0
1	0	1
1	1	0

x	y	m_3
0	0	0
0	1	0
1	0	0
1	1	1

x	y	M_0
0	0	0
0	1	1
1	0	1
1	1	1

x	y	M_1
0	0	1
0	1	0
1	0	1
1	1	1

x	y	M_2
0	0	1
0	1	1
1	0	0
1	1	1

x	y	M_3
0	0	1
0	1	1
1	0	1
1	1	0

minterms and Maxterms

$$m_0(x, y) = \bar{x} \bar{y}$$

$$M_0(x, y) = x + y$$

$$m_1(x, y) = \bar{x} y$$

$$M_1(x, y) = x + \bar{y}$$

$$m_2(x, y) = x \bar{y}$$

$$M_2(x, y) = \bar{x} + y$$

$$m_3(x, y) = x y$$

$$M_3(x, y) = \bar{x} + \bar{y}$$

minterms
(for three variables)

Expressions for the minterms

$$m_0 = \bar{x} \bar{y} \bar{z}$$

$$m_1 = \bar{x} \bar{y} z$$

$$m_2 = \bar{x} y \bar{z}$$

$$m_3 = \bar{x} y z$$

$$m_4 = x \bar{y} \bar{z}$$

$$m_5 = x \bar{y} z$$

$$m_6 = x y \bar{z}$$

$$m_7 = x y z$$

Expressions for the minterms

$$0 \ 0 \ 0 \quad m_0 = \bar{x} \bar{y} \bar{z}$$

$$0 \ 0 \ 1 \quad m_1 = \bar{x} \bar{y} z$$

$$0 \ 1 \ 0 \quad m_2 = \bar{x} y \bar{z}$$

$$0 \ 1 \ 1 \quad m_3 = \bar{x} y z$$

$$1 \ 0 \ 0 \quad m_4 = x \bar{y} \bar{z}$$

$$1 \ 0 \ 1 \quad m_5 = x \bar{y} z$$

$$1 \ 1 \ 0 \quad m_6 = x y \bar{z}$$

$$1 \ 1 \ 1 \quad m_7 = x y z$$

The bars coincide
with the 0's
in the binary expansion
of the minterm sub-index

Maxterms (for three variables)

Expressions for the Maxterms

$$M_0 = x + y + z$$

$$M_1 = x + y + \bar{z}$$

$$M_2 = x + \bar{y} + z$$

$$M_3 = x + \bar{y} + \bar{z}$$

$$M_4 = \bar{x} + y + z$$

$$M_5 = \bar{x} + y + \bar{z}$$

$$M_6 = \bar{x} + \bar{y} + z$$

$$M_7 = \bar{x} + \bar{y} + \bar{z}$$

Expressions for the Maxterms

$$0 \ 0 \ 0 \quad M_0 = x + y + z$$

$$0 \ 0 \ 1 \quad M_1 = x + y + \bar{z}$$

$$0 \ 1 \ 0 \quad M_2 = x + \bar{y} + z$$

$$0 \ 1 \ 1 \quad M_3 = x + \bar{y} + \bar{z}$$

$$1 \ 0 \ 0 \quad M_4 = \bar{x} + y + z$$

$$1 \ 0 \ 1 \quad M_5 = \bar{x} + y + \bar{z}$$

$$1 \ 1 \ 0 \quad M_6 = \bar{x} + \bar{y} + z$$

$$1 \ 1 \ 1 \quad M_7 = \bar{x} + \bar{y} + \bar{z}$$

The bars coincide
with the 1's
in the binary expansion
of the maxterm sub-index

minterms and Maxterms (for three variables)

minterms and Maxterms

$$m_0 = \bar{x} \bar{y} \bar{z}$$

$$m_1 = \bar{x} \bar{y} z$$

$$m_2 = \bar{x} y \bar{z}$$

$$m_3 = \bar{x} y z$$

$$m_4 = x \bar{y} \bar{z}$$

$$m_5 = x \bar{y} z$$

$$m_6 = x y \bar{z}$$

$$m_7 = x y z$$

$$M_0 = x + y + z$$

$$M_1 = x + y + \bar{z}$$

$$M_2 = x + \bar{y} + z$$

$$M_3 = x + \bar{y} + \bar{z}$$

$$M_4 = \bar{x} + y + z$$

$$M_5 = \bar{x} + y + \bar{z}$$

$$M_6 = \bar{x} + \bar{y} + z$$

$$M_7 = \bar{x} + \bar{y} + \bar{z}$$

Synthesis Example

Truth table for a three-way light control

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Let's Derive the SOP form

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

Let's Derive the SOP form

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Let's Derive the SOP form

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$x_1 x_2 x_3$

$x_1 x_2 x_3$

$x_1 x_2 x_3$

$x_1 x_2 x_3$

Let's Derive the SOP form

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$$\overline{x}_1 \overline{x}_2 x_3$$

$$\overline{x}_1 x_2 \overline{x}_3$$

$$x_1 \overline{x}_2 \overline{x}_3$$

$$x_1 x_2 x_3$$

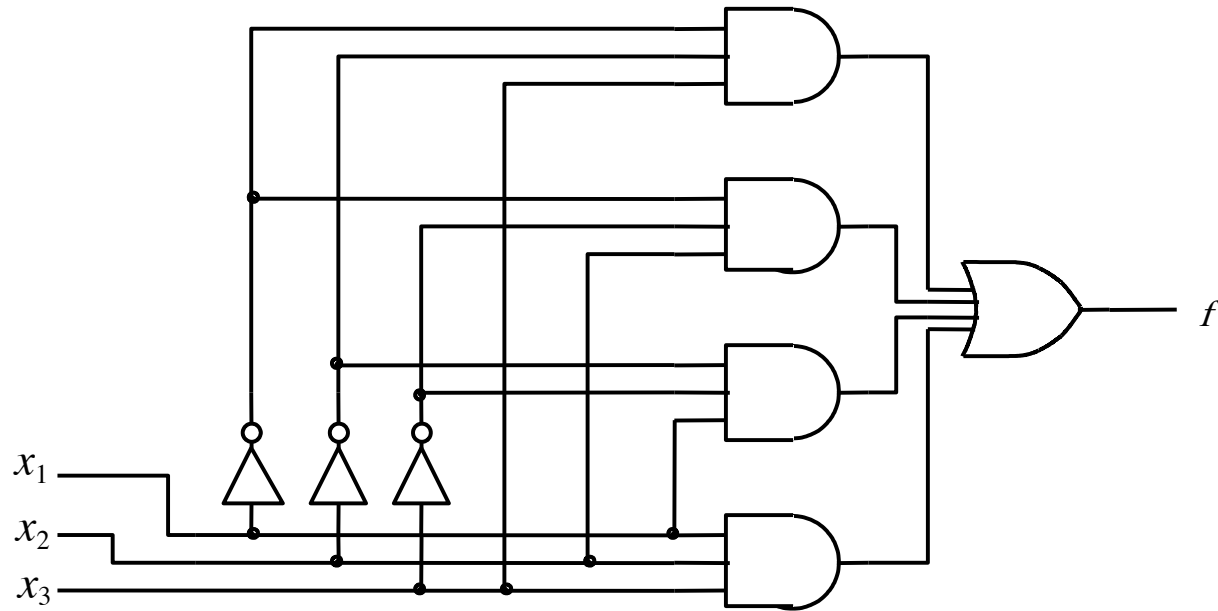
Let's Derive the SOP form

x_1	x_2	x_3	f	
0	0	0	0	
0	0	1	1	$\bar{x}_1 \bar{x}_2 x_3$
0	1	0	1	$\bar{x}_1 x_2 \bar{x}_3$
0	1	1	0	
1	0	0	1	$x_1 \bar{x}_2 \bar{x}_3$
1	0	1	0	
1	1	0	0	
1	1	1	1	$x_1 x_2 x_3$

$$f = m_1 + m_2 + m_4 + m_7$$

$$= \bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 x_3$$

Sum-of-products realization



[Figure 2.32a from the textbook]

Let's Derive the POS form

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Let's Derive the POS form

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

Let's Derive the POS form

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Let's Derive the POS form

x_1	x_2	x_3	f	
0	0	0	0	$(x_1 + x_2 + x_3)$
0	0	1	1	
0	1	0	1	
0	1	1	0	$(x_1 + x_2 + x_3)$
1	0	0	1	
1	0	1	0	$(x_1 + x_2 + x_3)$
1	1	0	0	$(x_1 + x_2 + x_3)$
1	1	1	1	

Let's Derive the POS form

x_1	x_2	x_3	f	
0	0	0	0	$(x_1 + x_2 + x_3)$
0	0	1	1	
0	1	0	1	
0	1	1	0	$(x_1 + \bar{x}_2 + \bar{x}_3)$
1	0	0	1	
1	0	1	0	$(\bar{x}_1 + x_2 + \bar{x}_3)$
1	1	0	0	$(\bar{x}_1 + \bar{x}_2 + x_3)$
1	1	1	1	

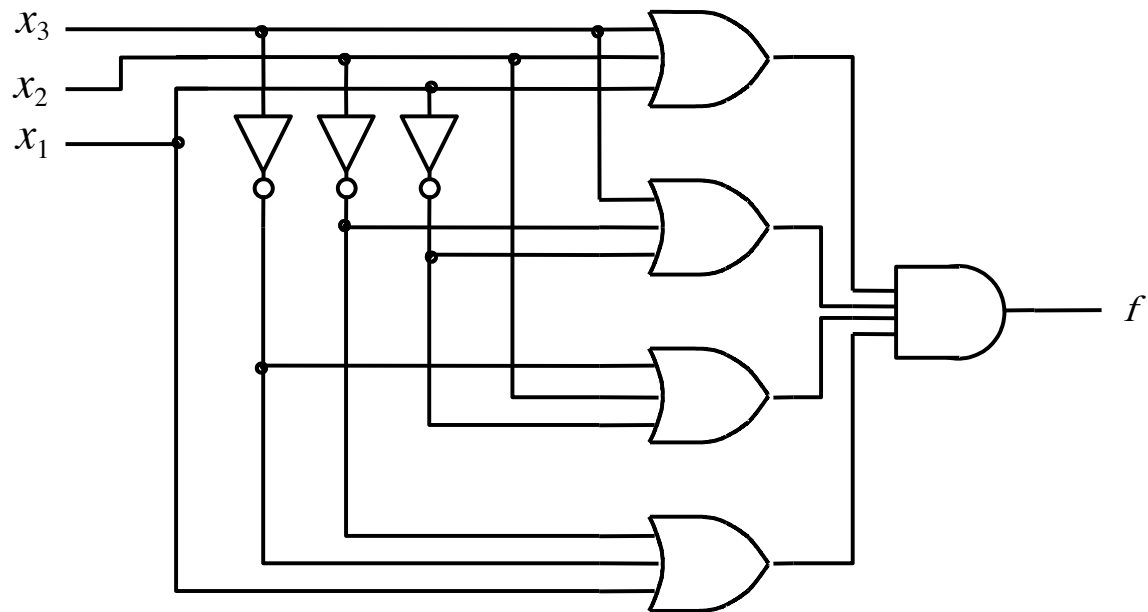
Let's Derive the POS form

x_1	x_2	x_3	f	
0	0	0	0	$(x_1 + x_2 + x_3)$
0	0	1	1	
0	1	0	1	
0	1	1	0	$(x_1 + \bar{x}_2 + \bar{x}_3)$
1	0	0	1	
1	0	1	0	$(\bar{x}_1 + x_2 + \bar{x}_3)$
1	1	0	0	$(\bar{x}_1 + \bar{x}_2 + x_3)$
1	1	1	1	

$$f = M_0 \cdot M_3 \cdot M_5 \cdot M_6$$

$$= (x_1 + x_2 + x_3)(x_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + x_2 + \bar{x}_3)(\bar{x}_1 + \bar{x}_2 + x_3)$$

Product-of-sums realization



Function Synthesis

Example 2.10

Implement the function $f(x_1, x_2, x_3) = \sum m(2, 3, 4, 6, 7)$

Minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

$$f(x_1, x_2, x_3) = \sum m(2, 3, 4, 6, 7)$$

- **The SOP expression is:**

$$\begin{aligned} f &= m_2 + m_3 + m_4 + m_6 + m_7 \\ &= \bar{x}_1 x_2 \bar{x}_3 + \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 \bar{x}_3 + x_1 x_2 x_3 \end{aligned}$$

- **This could be simplified as follows:**

$$\begin{aligned} f &= \bar{x}_1 x_2 (\bar{x}_3 + x_3) + x_1 (\bar{x}_2 + x_2) \bar{x}_3 + x_1 x_2 (\bar{x}_3 + x_3) \\ &= \bar{x}_1 x_2 + x_1 \bar{x}_3 + x_1 x_2 \\ &= (\bar{x}_1 + x_1) x_2 + x_1 \bar{x}_3 \\ &= x_2 + x_1 \bar{x}_3 \end{aligned}$$

Recall Property 14a

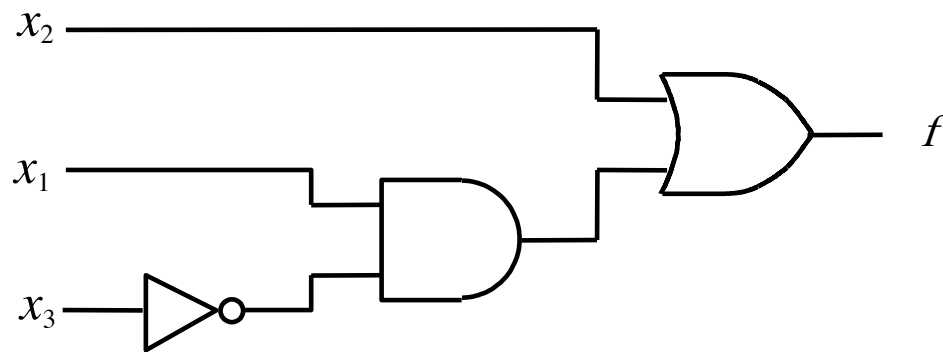
$$14a. \quad x \cdot y + x \cdot \bar{y} = x$$

Combining

$$14b. \quad (x + y) \cdot (x + \bar{y}) = x$$

SOP realization of the function

The SOP expression is: $f = x_2 + x_1\bar{x}_3$



Example 2.12

Implement the function $f(x_1, x_2, x_3) = \prod M(0, 1, 5)$,

which is equivalent to $f(x_1, x_2, x_3) = \sum m(2, 3, 4, 6, 7)$

minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

$$f(x_1, x_2, x_3) = \Pi M(0, 1, 5)$$

- **The POS expression is:**

$$\begin{aligned} f &= M_0 \cdot M_1 \cdot M_5 \\ &= (x_1 + x_2 + x_3)(x_1 + x_2 + \bar{x}_3)(\bar{x}_1 + x_2 + \bar{x}_3) \end{aligned}$$

- **This could be simplified as follows:**

$$\begin{aligned} f &= (x_1 + x_2 + x_3)(x_1 + x_2 + \bar{x}_3)(x_1 + x_2 + \bar{x}_3)(\bar{x}_1 + x_2 + \bar{x}_3) \\ &= ((x_1 + x_2) + x_3)((x_1 + x_2) + \bar{x}_3)(x_1 + (x_2 + \bar{x}_3))(\bar{x}_1 + (x_2 + \bar{x}_3)) \\ &= ((x_1 + x_2) + x_3\bar{x}_3)(x_1\bar{x}_1 + (x_2 + \bar{x}_3)) \\ &= (x_1 + x_2)(x_2 + \bar{x}_3) \end{aligned}$$

Recall Property 14b

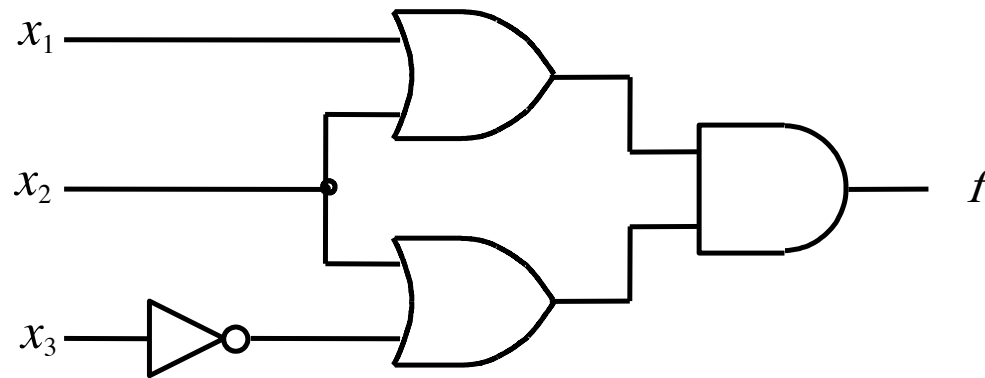
$$14a. \quad x \cdot y + x \cdot \bar{y} = x$$

Combining

$$14b. \quad (x + y) \cdot (x + \bar{y}) = x$$

POS realization of the function

The POS expression is: $f = (x_1 + x_2)(x_2 + \bar{x}_3)$



More Examples

Example 2.14

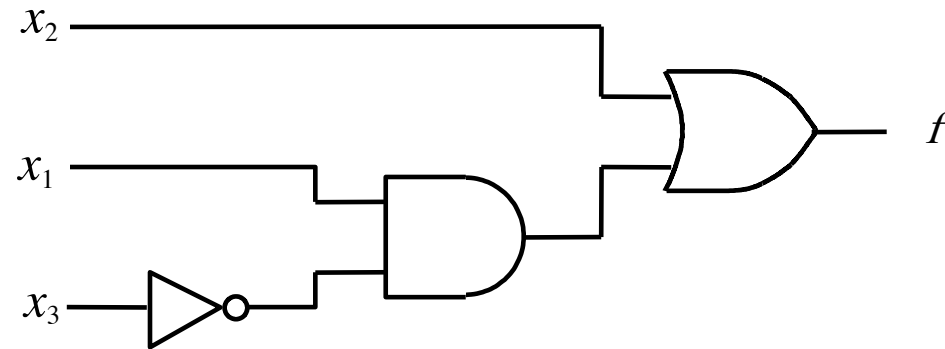
Implement the function $f(x_1, x_2, x_3) = \sum m(2, 3, 4, 6, 7)$ using only NAND gates.

Example 2.14

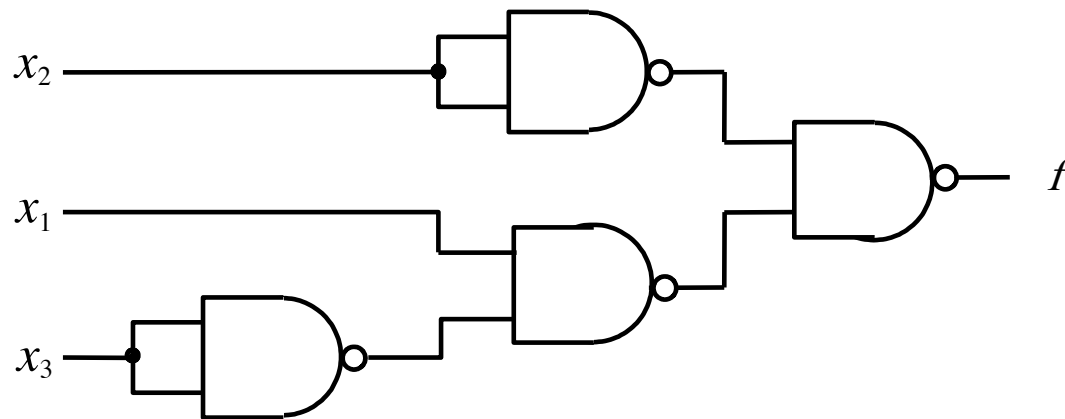
Implement the function $f(x_1, x_2, x_3) = \sum m(2, 3, 4, 6, 7)$ using only NAND gates.

The SOP expression is: $f = x_2 + x_1\bar{x}_3$

NAND-gate realization of the function



(a) SOP implementation



(b) NAND implementation

Example 2.13

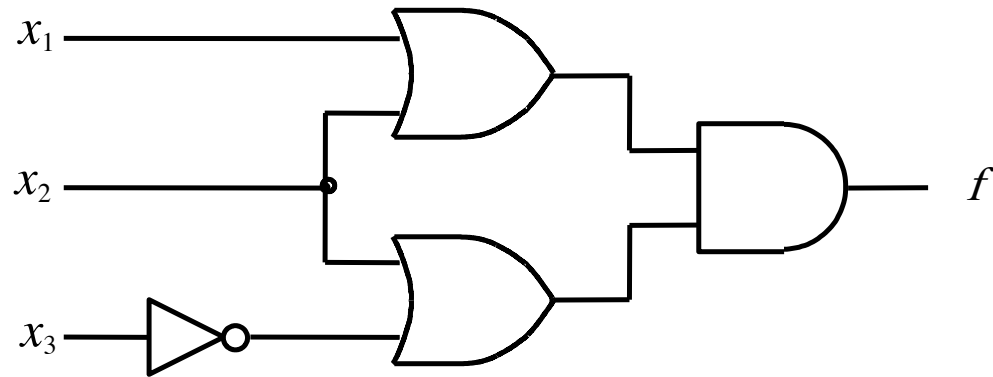
Implement the function $f(x_1, x_2, x_3) = \sum m(2, 3, 4, 6, 7)$ using only NOR gates.

Example 2.13

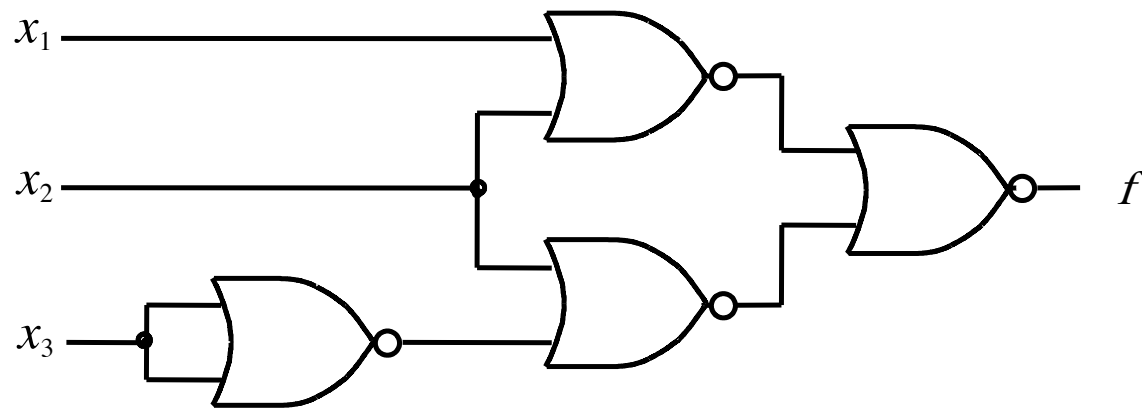
Implement the function $f(x_1, x_2, x_3) = \sum m(2, 3, 4, 6, 7)$ using only NOR gates.

The POS expression is: $f = (x_1 + x_2) (x_2 + \bar{x}_3)$

NOR-gate realization of the function

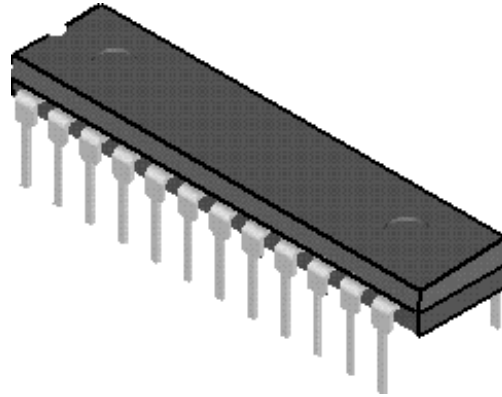


(a) POS implementation

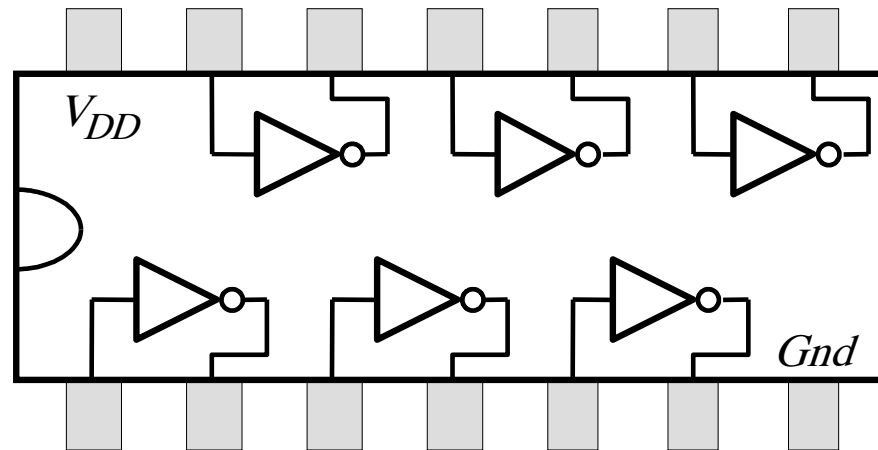


(b) NOR implementation

Implementation with Chips



(a) Dual-inline package



(b) Structure of 7404 chip

Figure B.21. A 7400-series chip.

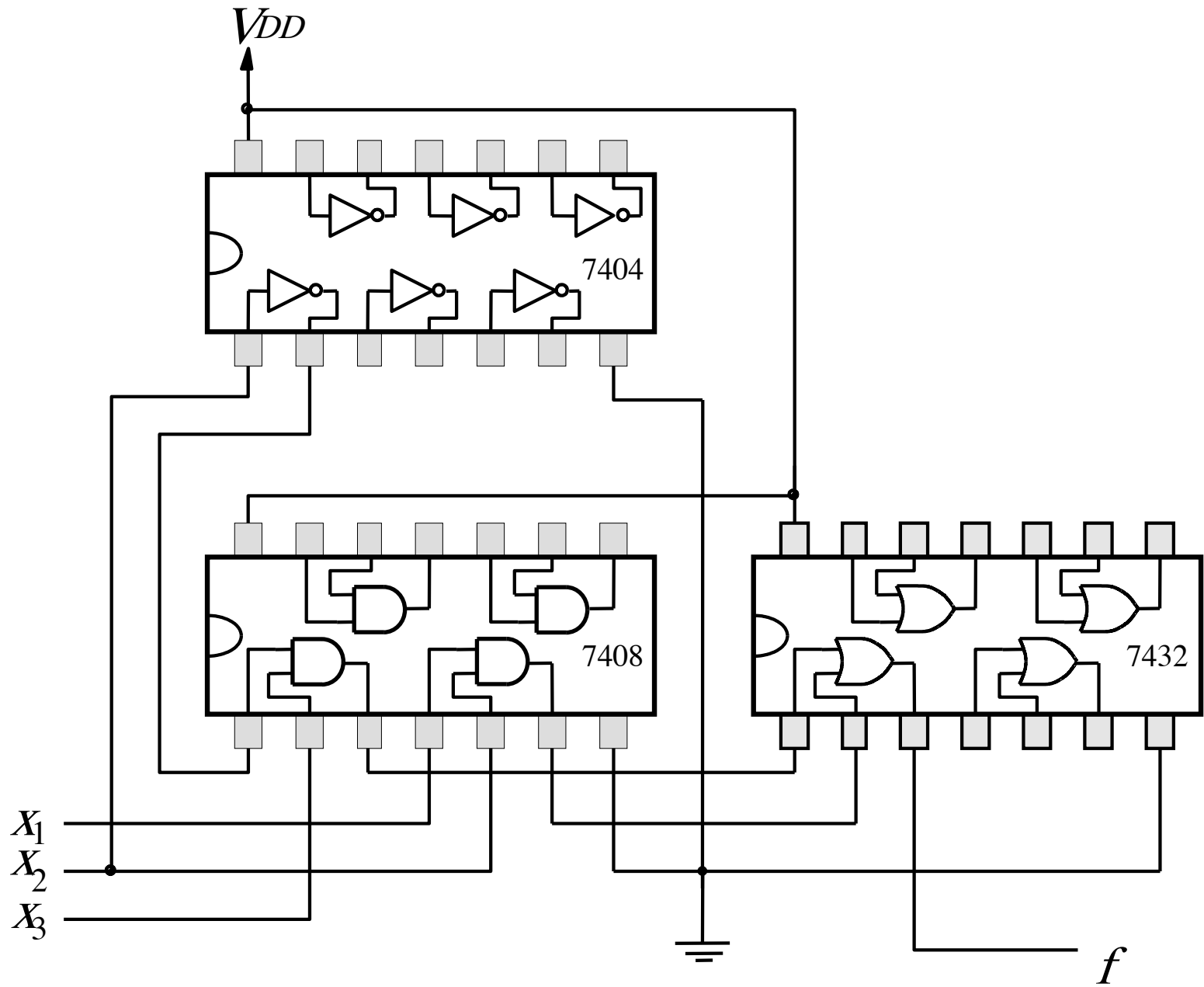


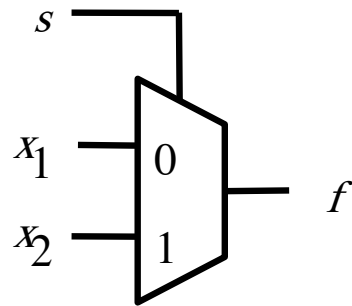
Figure B.22. An implementation of $f = x_1x_2 + \overline{x_2}x_3$.

Multiplexers

2-to-1 Multiplexer (Definition)

- Has two inputs: x_1 and x_2
- Also has another input line s
- If $s=0$, then the output is equal to x_1
- If $s=1$, then the output is equal to x_2

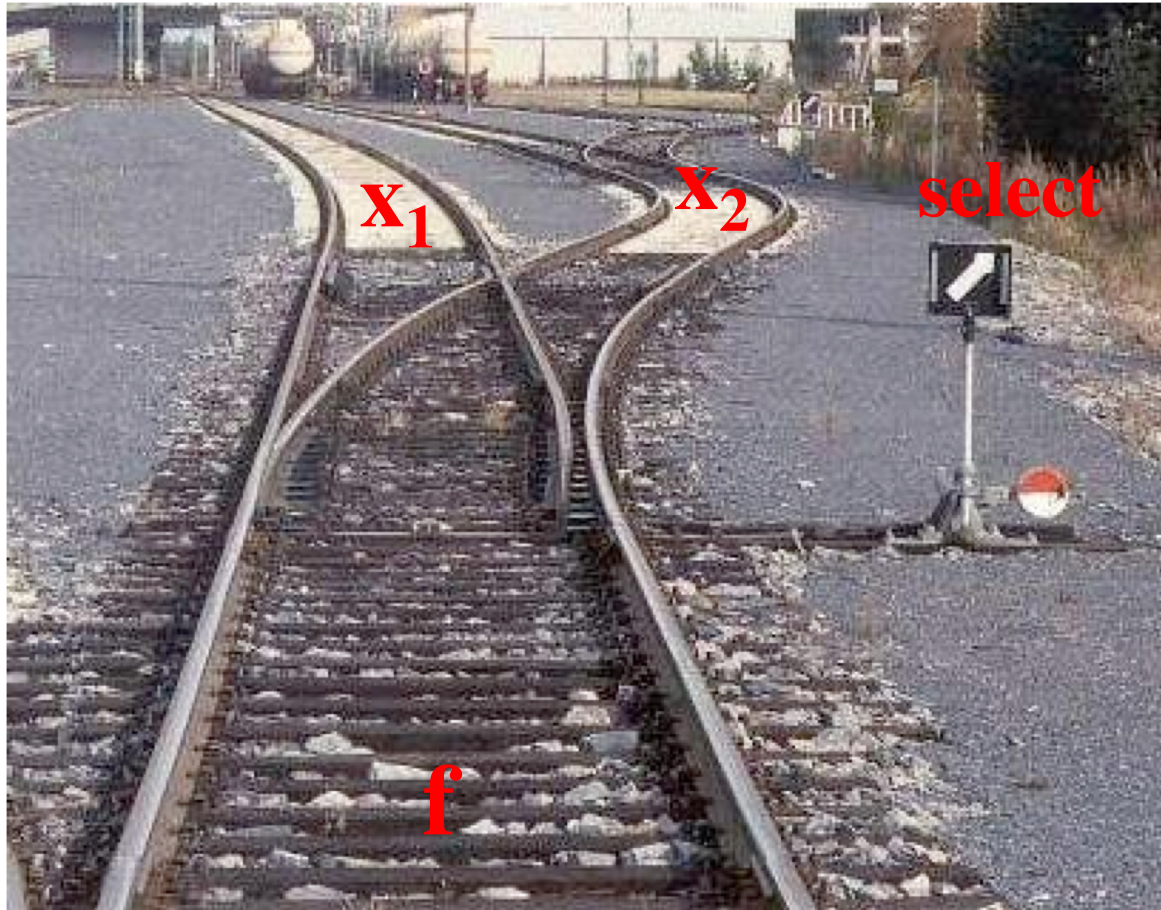
Graphical Symbol for a 2-to-1 Multiplexer



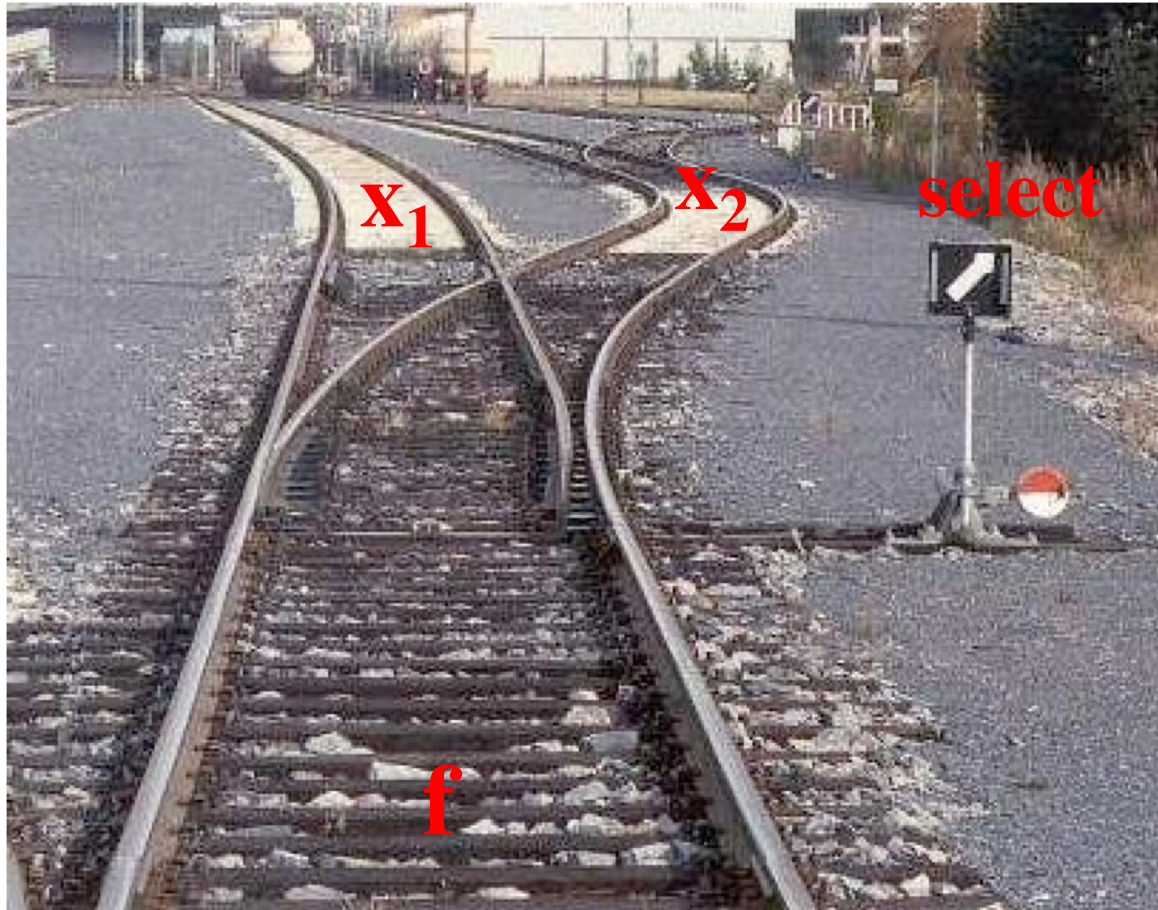
Analogy: Railroad Switch



Analogy: Railroad Switch



Analogy: Railroad Switch



This is not a perfect analogy because the trains can go in either direction, while the multiplexer would only allow them to go from top to bottom.

Truth Table for a 2-to-1 Multiplexer

s x_1 x_2	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

Let's Derive the SOP form

$s \ x_1 \ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

Let's Derive the SOP form

$s \ x_1 \ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

Let's Derive the SOP form

$s \ x_1 \ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

Where should we
put the negation signs?

$s \ x_1 \ x_2$

$s \ x_1 \ x_2$

$s \ x_1 \ x_2$

$s \ x_1 \ x_2$

Let's Derive the SOP form

$s \ x_1 \ x_2$	$f(s, x_1, x_2)$	
0 0 0	0	
0 0 1	0	
0 1 0	1	$\bar{s} \ x_1 \ \bar{x}_2$
0 1 1	1	$\bar{s} \ x_1 \ x_2$
1 0 0	0	
1 0 1	1	$s \ \bar{x}_1 \ x_2$
1 1 0	0	
1 1 1	1	$s \ x_1 \ x_2$

Let's Derive the SOP form

$s \ x_1 \ x_2$	$f(s, x_1, x_2)$	
0 0 0	0	
0 0 1	0	
0 1 0	1	$\bar{s} x_1 \bar{x}_2$
0 1 1	1	$\bar{s} x_1 x_2$
1 0 0	0	
1 0 1	1	$s \bar{x}_1 x_2$
1 1 0	0	
1 1 1	1	$s x_1 x_2$

$$f(s, x_1, x_2) = \bar{s} x_1 \bar{x}_2 + \bar{s} x_1 x_2 + s \bar{x}_1 x_2 + s x_1 x_2$$

Let's simplify this expression

$$f(s, x_1, x_2) = \bar{s} x_1 \bar{x}_2 + \bar{s} x_1 x_2 + s \bar{x}_1 x_2 + s x_1 x_2$$

Let's simplify this expression

$$f(s, x_1, x_2) = \bar{s} x_1 \bar{x}_2 + \bar{s} x_1 x_2 + s \bar{x}_1 x_2 + s x_1 x_2$$

$$f(s, x_1, x_2) = \bar{s} x_1 (\bar{x}_2 + x_2) + s (\bar{x}_1 + x_1) x_2$$

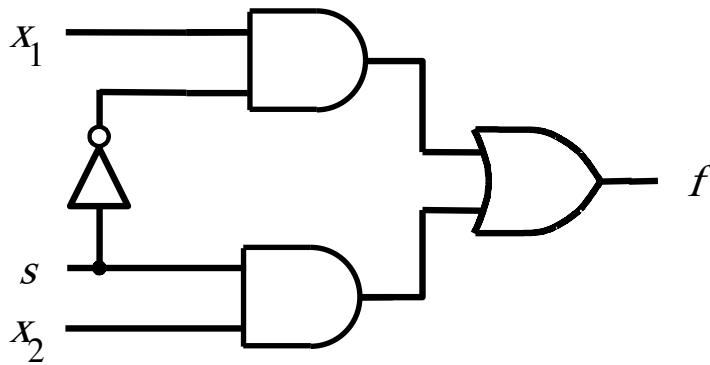
Let's simplify this expression

$$f(s, x_1, x_2) = \bar{s} x_1 \bar{x}_2 + \bar{s} x_1 x_2 + s \bar{x}_1 x_2 + s x_1 x_2$$

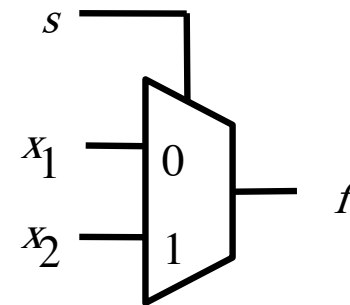
$$f(s, x_1, x_2) = \bar{s} x_1 (\bar{x}_2 + x_2) + s (\bar{x}_1 + x_1) x_2$$

$$f(s, x_1, x_2) = \bar{s} x_1 + s x_2$$

Circuit for 2-to-1 Multiplexer



(b) Circuit



(c) Graphical symbol

$$f(s, x_1, x_2) = \bar{s}x_1 + sx_2$$

More Compact Truth-Table Representation

$s \ x_1 \ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

(a) Truth table

s	$f(s, x_1, x_2)$
0	x_1
1	x_2

4-to-1 Multiplexer (Definition)

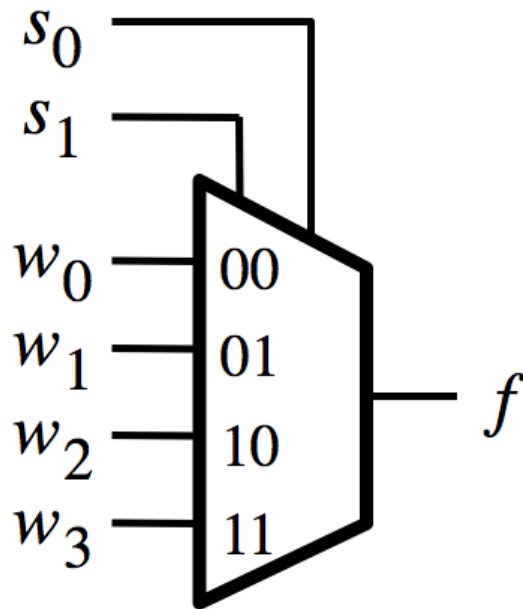
- Has four inputs: w_0 , w_1 , w_2 , w_3
- Also has two select lines: s_1 and s_0
- If $s_1=0$ and $s_0=0$, then the output f is equal to w_0
- If $s_1=0$ and $s_0=1$, then the output f is equal to w_1
- If $s_1=1$ and $s_0=0$, then the output f is equal to w_2
- If $s_1=1$ and $s_0=1$, then the output f is equal to w_3

4-to-1 Multiplexer (Definition)

- Has four inputs: w_0 , w_1 , w_2 , w_3
- Also has two select lines: s_1 and s_0
- If $s_1=0$ and $s_0=0$, then the output f is equal to w_0
- If $s_1=0$ and $s_0=1$, then the output f is equal to w_1
- If $s_1=1$ and $s_0=0$, then the output f is equal to w_2
- If $s_1=1$ and $s_0=1$, then the output f is equal to w_3

We'll talk more about this when we get to chapter 4, but here is a quick preview.

Graphical Symbol and Truth Table



(a) Graphic symbol

s_1	s_0	f
0	0	w_0
0	1	w_1
1	0	w_2
1	1	w_3

(b) Truth table

The long-form truth table

The long-form truth table

$S_1 S_0$	$I_3 I_2 I_1 I_0$	F	$S_1 S_0$	$I_3 I_2 I_1 I_0$	F	$S_1 S_0$	$I_3 I_2 I_1 I_0$	F	$S_1 S_0$	$I_3 I_2 I_1 I_0$	F
0 0	0 0 0 0	0	0 1	0 0 0 0	0	1 0	0 0 0 0	0	1 1	0 0 0 0	0
	0 0 0 1	1		0 0 0 1	0		0 0 0 1	0		0 0 0 1	0
	0 0 1 0	0		0 0 1 0	1		0 0 1 0	0		0 0 1 0	0
	0 0 1 1	1		0 0 1 1	1		0 0 1 1	0		0 0 1 1	0
	0 1 0 0	0		0 1 0 0	0		0 1 0 0	1		0 1 0 0	0
	0 1 0 1	1		0 1 0 1	0		0 1 0 1	1		0 1 0 1	0
	0 1 1 0	0		0 1 1 0	1		0 1 1 0	1		0 1 1 0	0
	0 1 1 1	1		0 1 1 1	1		0 1 1 1	1		0 1 1 1	0
	1 0 0 0	0		1 0 0 0	0		1 0 0 0	0		1 0 0 0	1
	1 0 0 1	1		1 0 0 1	0		1 0 0 1	0		1 0 0 1	1
	1 0 1 0	0		1 0 1 0	1		1 0 1 0	0		1 0 1 0	1
	1 0 1 1	1		1 0 1 1	1		1 0 1 1	0		1 0 1 1	1
	1 1 0 0	0		1 1 0 0	0		1 1 0 0	1		1 1 0 0	1
	1 1 0 1	1		1 1 0 1	0		1 1 0 1	1		1 1 0 1	1
	1 1 1 0	0		1 1 1 0	1		1 1 1 0	1		1 1 1 0	1
	1 1 1 1	1		1 1 1 1	1		1 1 1 1	1		1 1 1 1	1

The long-form truth table

$S_1 S_0$	I_3	I_2	I_1	I_0	F	$S_1 S_0$	I_3	I_2	I_1	I_0	F	$S_1 S_0$	I_3	I_2	I_1	I_0	F	$S_1 S_0$	I_3	I_2	I_1	I_0	F
0 0	0	0	0	0	0	0 1	0	0	0	0	0	1 0	0	0	0	0	0	1 1	0	0	0	0	0
0 0	0	0	0	1	1	0 1	0	0	0	1	0	1 0	0	0	0	1	0	1 1	0	0	0	1	0
0 0	0	0	1	0	0	0 1	0	0	1	0	1	1 0	0	0	1	0	0	1 1	0	0	1	0	0
0 0	0	0	1	1	1	0 1	0	0	1	1	1	1 0	0	0	1	1	0	1 1	0	0	1	1	0
0 0	0	1	0	0	0	0 1	0	1	0	0	0	1 0	0	1	0	0	0	1 1	0	1	0	0	0
0 0	0	1	0	1	1	0 1	0	1	0	1	0	1 0	0	1	0	1	0	1 1	0	1	0	1	0
0 0	0	1	1	0	0	0 1	0	1	1	0	1	1 0	0	1	1	0	0	1 1	0	1	1	0	0
0 0	0	1	1	1	1	0 1	0	1	1	1	1	1 0	0	1	1	1	0	1 1	0	1	1	1	0
0 1	0	0	0	0	0	0 1	1	0	0	0	0	1 0	1	0	0	0	0	1 1	1	0	0	0	1
0 1	0	0	0	1	1	0 1	1	0	0	1	0	1 0	1	0	0	1	0	1 1	1	0	0	1	1
0 1	0	0	1	0	0	0 1	1	0	1	0	1	1 0	1	0	1	0	0	1 1	1	0	1	0	1
0 1	0	0	1	1	1	0 1	1	0	1	1	1	1 0	1	0	1	1	0	1 1	1	0	1	1	1
0 1	0	1	0	0	0	0 1	1	1	0	0	0	1 0	1	1	0	0	0	1 1	1	1	0	0	1
0 1	0	1	0	1	1	0 1	1	1	0	1	0	1 0	1	1	0	1	0	1 1	1	1	0	1	1
0 1	0	1	1	0	0	0 1	1	1	0	0	0	1 0	1	1	0	0	0	1 1	1	1	0	0	1
0 1	0	1	1	1	1	0 1	1	1	0	1	0	1 0	1	1	0	1	0	1 1	1	1	0	1	1
0 1	0	1	1	1	1	0 1	1	1	1	1	1	1 0	1	1	1	1	0	1 1	1	1	1	0	1
0 1	1	0	0	0	0	0 1	1	1	1	1	1	1 0	1	1	1	1	0	1 1	1	1	1	1	1
0 1	1	0	0	1	1	0 1	1	1	1	1	0	1 0	1	1	1	1	0	1 1	1	1	1	1	1
0 1	1	0	1	0	0	0 1	1	1	1	1	1	1 0	1	1	1	1	0	1 1	1	1	1	1	1
0 1	1	0	1	1	1	0 1	1	1	1	1	1	1 0	1	1	1	1	0	1 1	1	1	1	1	1
0 1	1	1	0	0	0	0 1	1	1	1	1	0	1 0	1	1	1	1	0	1 1	1	1	1	1	1
0 1	1	1	0	1	1	0 1	1	1	1	1	0	1 0	1	1	1	1	0	1 1	1	1	1	1	1
0 1	1	1	1	0	0	0 1	1	1	1	1	1	1 0	1	1	1	1	0	1 1	1	1	1	1	1
0 1	1	1	1	1	1	0 1	1	1	1	1	1	1 0	1	1	1	1	0	1 1	1	1	1	1	1

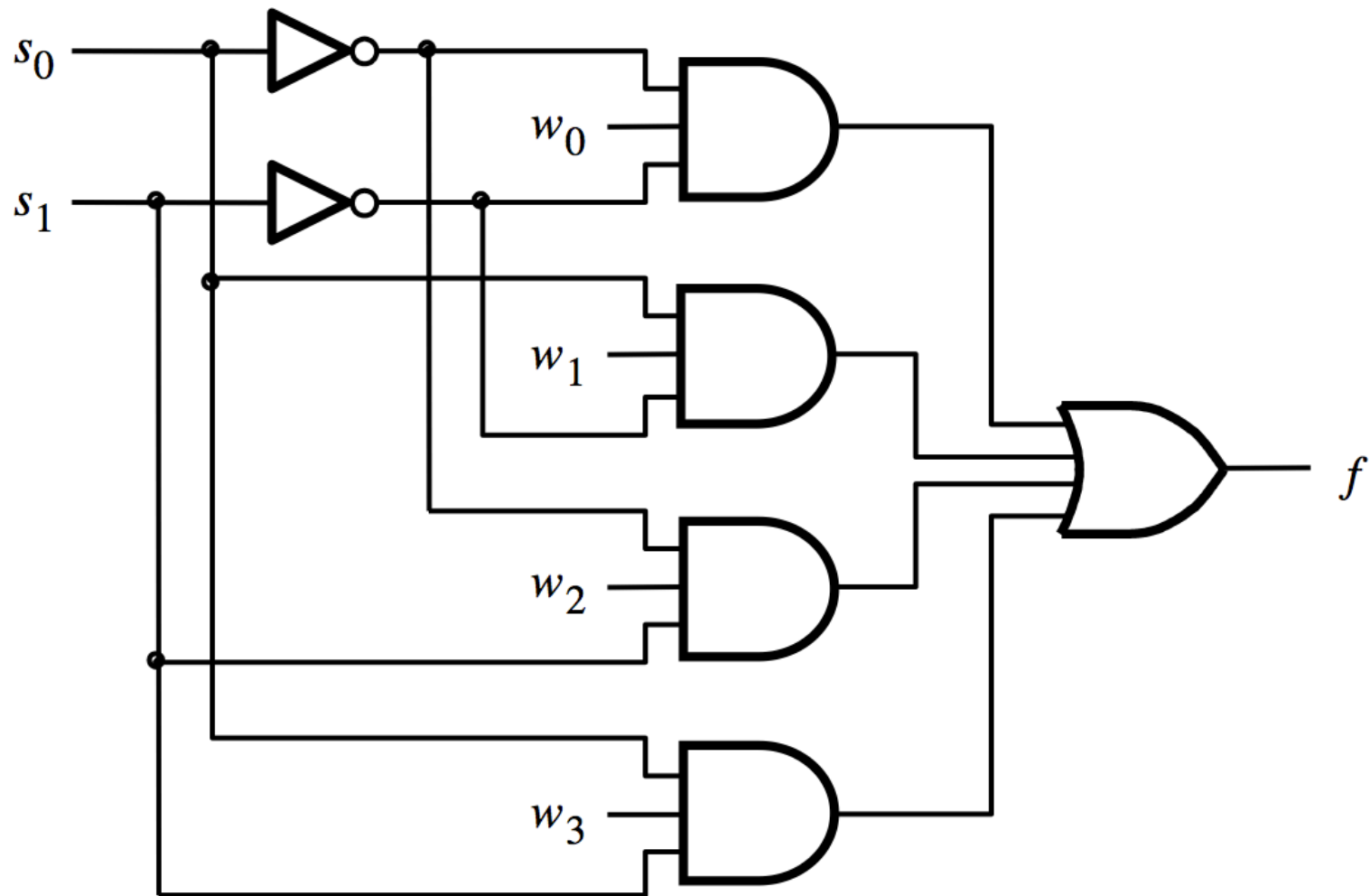
The long-form truth table

$S_1 S_0$	I_3	I_2	I_1	I_0	F	$S_1 S_0$	I_3	I_2	I_1	I_0	F	$S_1 S_0$	I_3	I_2	I_1	I_0	F	$S_1 S_0$	I_3	I_2	I_1	I_0	F	
0 0	0	0	0	0	0	0 1	0	0	0	0	0	1 0	0	0	0	0	0	0	1 1	0	0	0	0	0
	0	0	0	1	1		0	0	0	1	0		0	0	0	1	0		0	0	0	1	0	
	0	0	1	0	0		0	0	1	0	1		0	0	1	0	0		0	0	1	0	0	
	0	0	1	1	1		0	0	1	1	1		0	0	1	1	0		0	0	1	1	0	
	0	1	0	0	0		0	1	0	0	0		0	1	0	0	1		0	1	0	0	0	
	0	1	0	1	1		0	1	0	1	0		0	1	0	1	1		0	1	0	1	0	
	0	1	1	0	0		0	1	1	0	1		0	1	1	0	1		0	1	1	0	0	
	0	1	1	1	1		0	1	1	1	1		0	1	1	1	1		0	1	1	1	0	
	1	0	0	0	0		1	0	0	0	0		1	0	0	0	0		1	0	0	0	1	
	1	0	0	1	1		1	0	0	1	0		1	0	0	1	0		1	0	0	1	1	
	1	0	1	0	0		1	0	1	0	1		1	0	1	0	0		1	0	1	0	1	
	1	0	1	1	1		1	0	1	1	1		1	0	1	1	0		1	0	1	1	1	
	1	1	0	0	0		1	1	0	0	0		1	1	0	0	1		1	1	0	0	1	
	1	1	0	1	1		1	1	0	1	0		1	1	0	1	1		1	1	0	1	1	
	1	1	1	0	0		1	1	1	0	1		1	1	1	0	1		1	1	1	0	1	
	1	1	1	1	1		1	1	1	1	1		1	1	1	1	1		1	1	1	1	1	

The long-form truth table

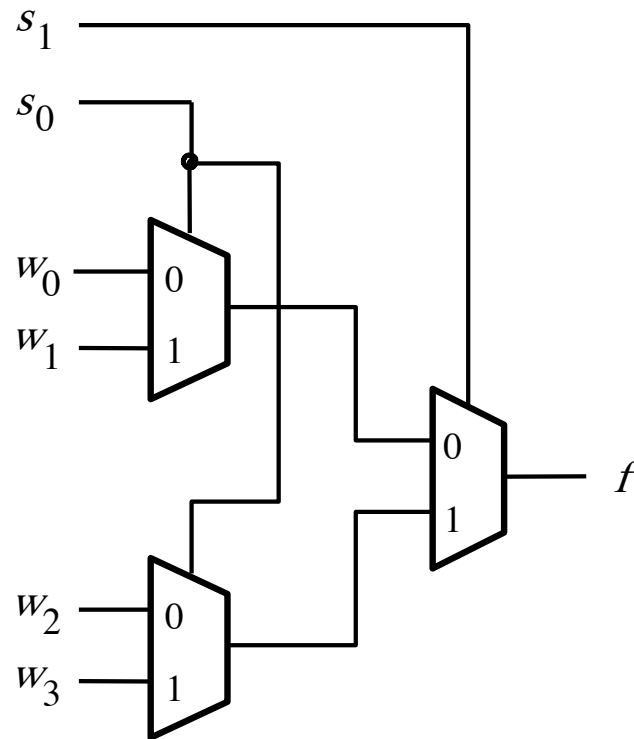
$S_1 S_0$	$I_3 I_2 I_1 I_0$	F	$S_1 S_0$	$I_3 I_2 I_1 I_0$	F	$S_1 S_0$	$I_3 I_2 I_1 I_0$	F	$S_1 S_0$	$I_3 I_2 I_1 I_0$	F
0 0	0 0 0 0	0	0 1	0 0 0 0	0	1 0	0 0 0 0	0	1 1	0 0 0 0	0
	0 0 0 1	1		0 0 0 1	0		0 0 0 1	0		0 0 0 1	0
	0 0 1 0	0		0 0 1 0	1		0 0 1 0	0		0 0 1 0	0
	0 0 1 1	1		0 0 1 1	1		0 0 1 1	0		0 0 1 1	0
	0 1 0 0	0		0 1 0 0	0		0 1 0 0	1		0 1 0 0	0
	0 1 0 1	1		0 1 0 1	0		0 1 0 1	1		0 1 0 1	0
	0 1 1 0	0		0 1 1 0	1		0 1 1 0	1		0 1 1 0	0
	0 1 1 1	1		0 1 1 1	1		0 1 1 1	1		0 1 1 1	0
	1 0 0 0	0		1 0 0 0	0		1 0 0 0	0		1 0 0 0	1
	1 0 0 1	1		1 0 0 1	0		1 0 0 1	0		1 0 0 1	1
	1 0 1 0	0		1 0 1 0	1		1 0 1 0	0		1 0 1 0	1
	1 0 1 1	1		1 0 1 1	1		1 0 1 1	0		1 0 1 1	1
	1 1 0 0	0		1 1 0 0	0		1 1 0 0	1		1 1 0 0	1
	1 1 0 1	1		1 1 0 1	0		1 1 0 1	1		1 1 0 1	1
	1 1 1 0	0		1 1 1 0	1		1 1 1 0	1		1 1 1 0	1
	1 1 1 1	1		1 1 1 1	1		1 1 1 1	1		1 1 1 1	1

4-to-1 Multiplexer (SOP circuit)



[Figure 4.2c from the textbook]

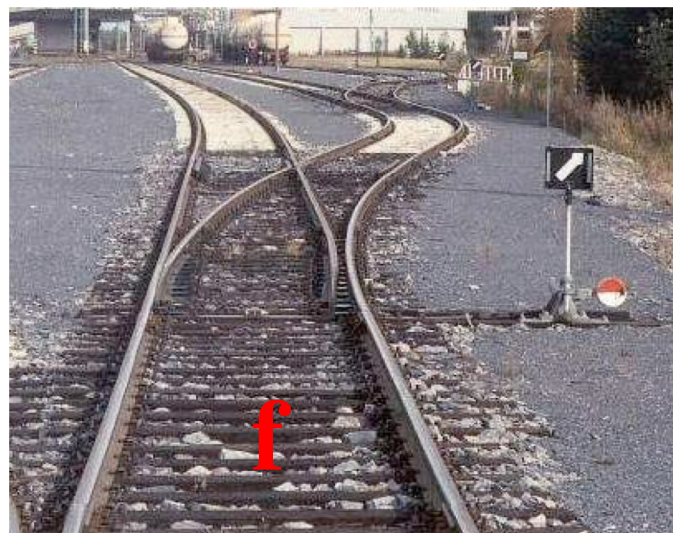
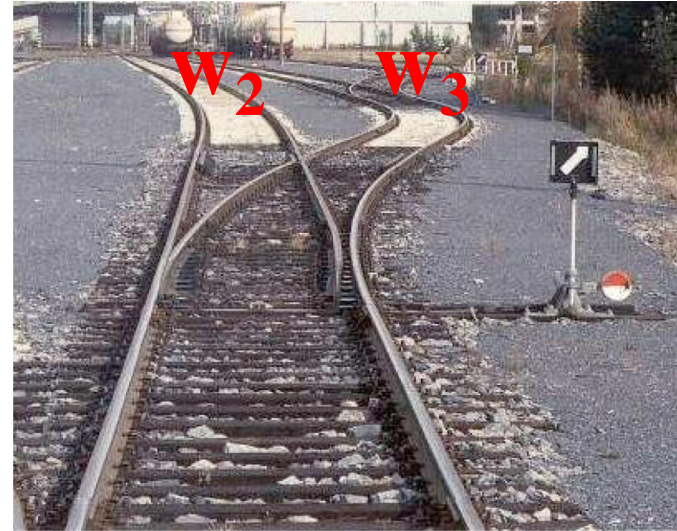
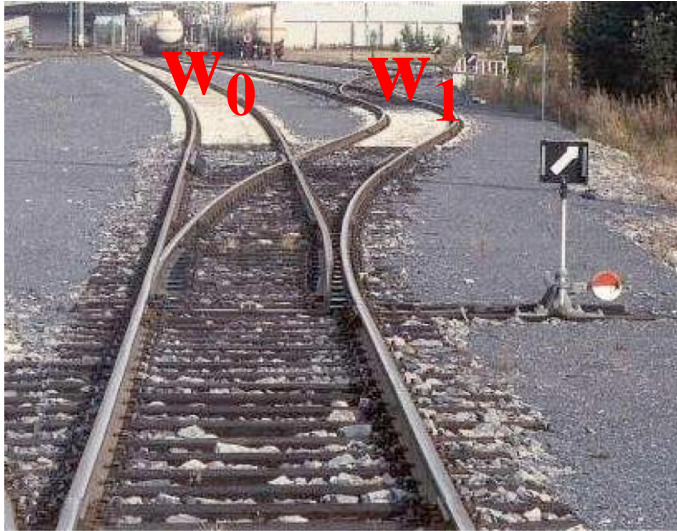
Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



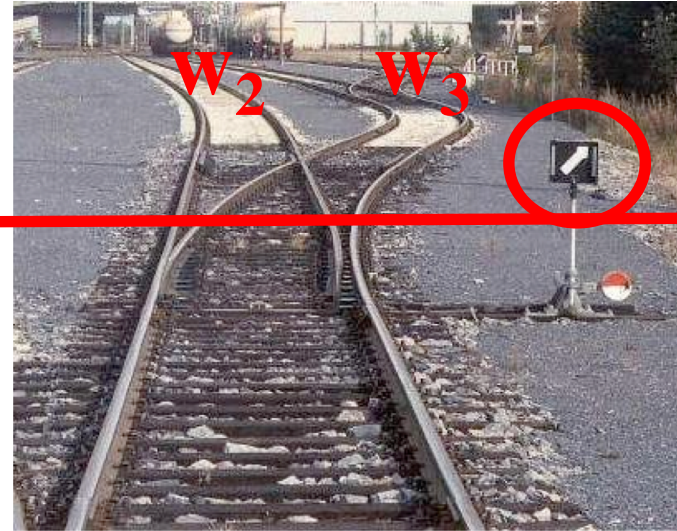
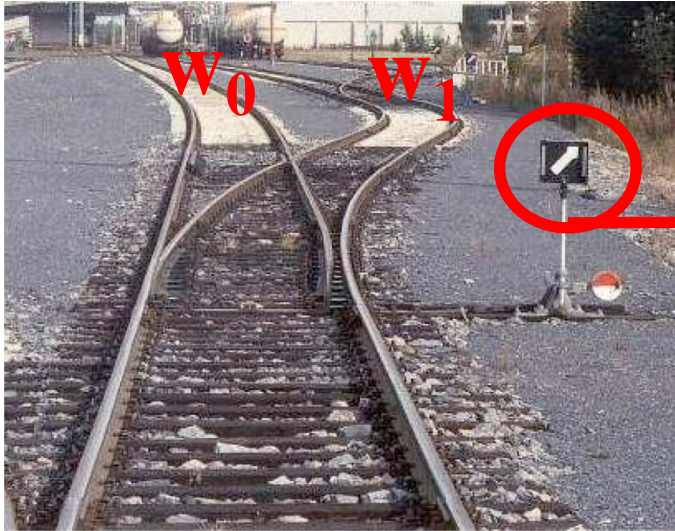
Analogy: Railroad Switches



Analogy: Railroad Switches

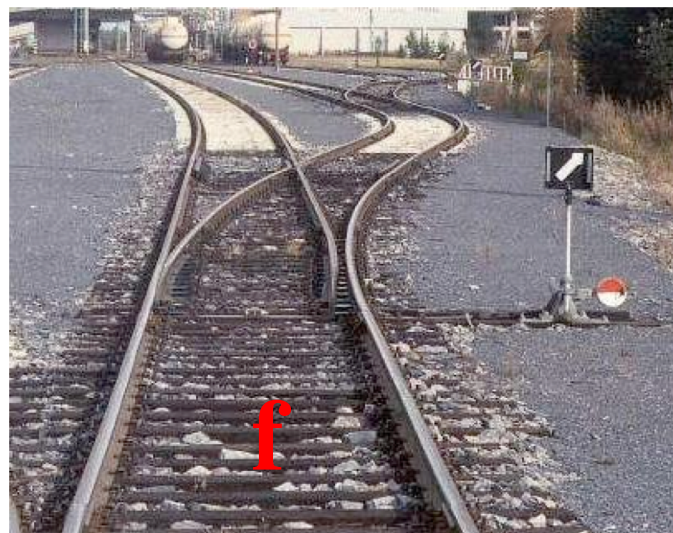


Analogy: Railroad Switches



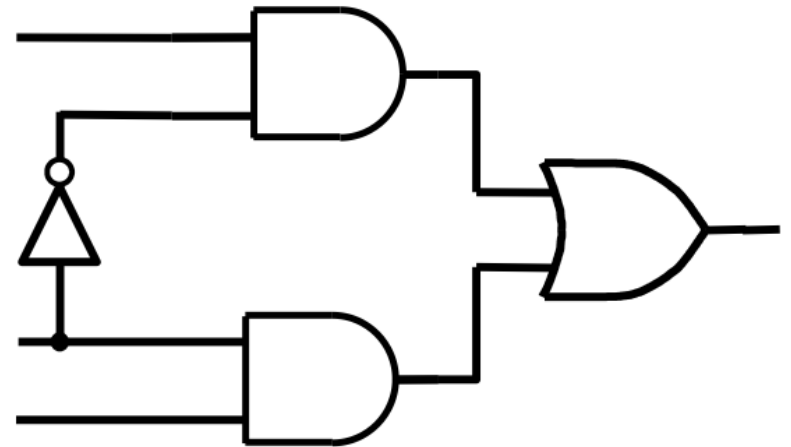
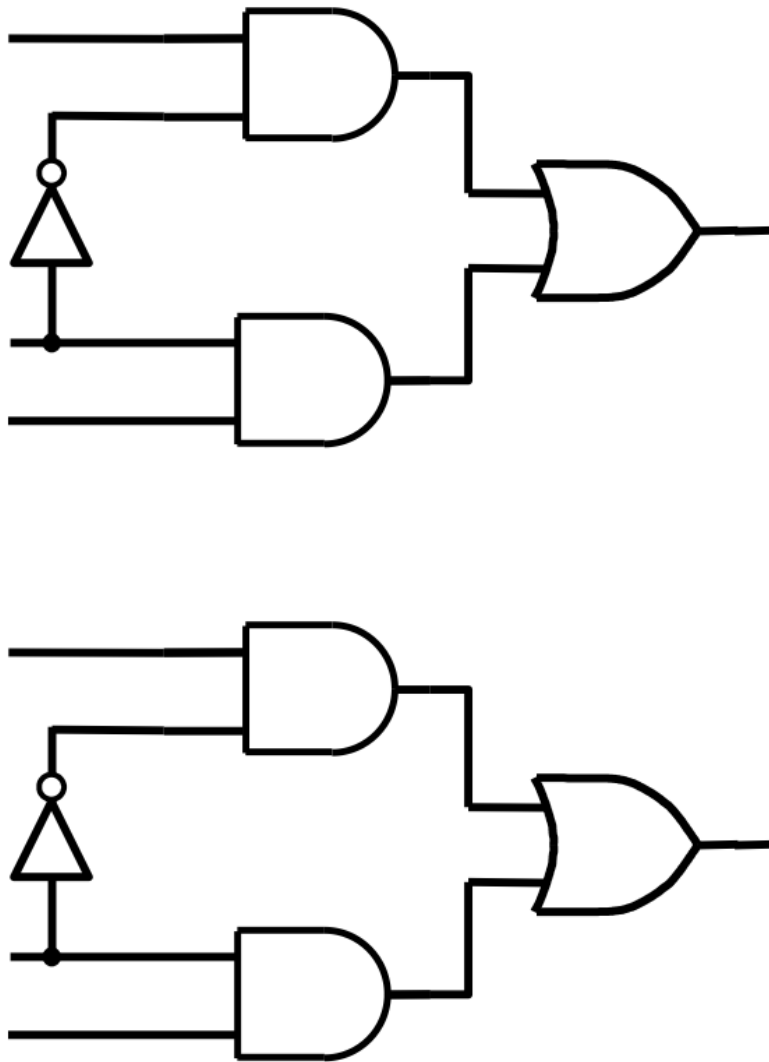
S_0

these two
switches are
controlled
together

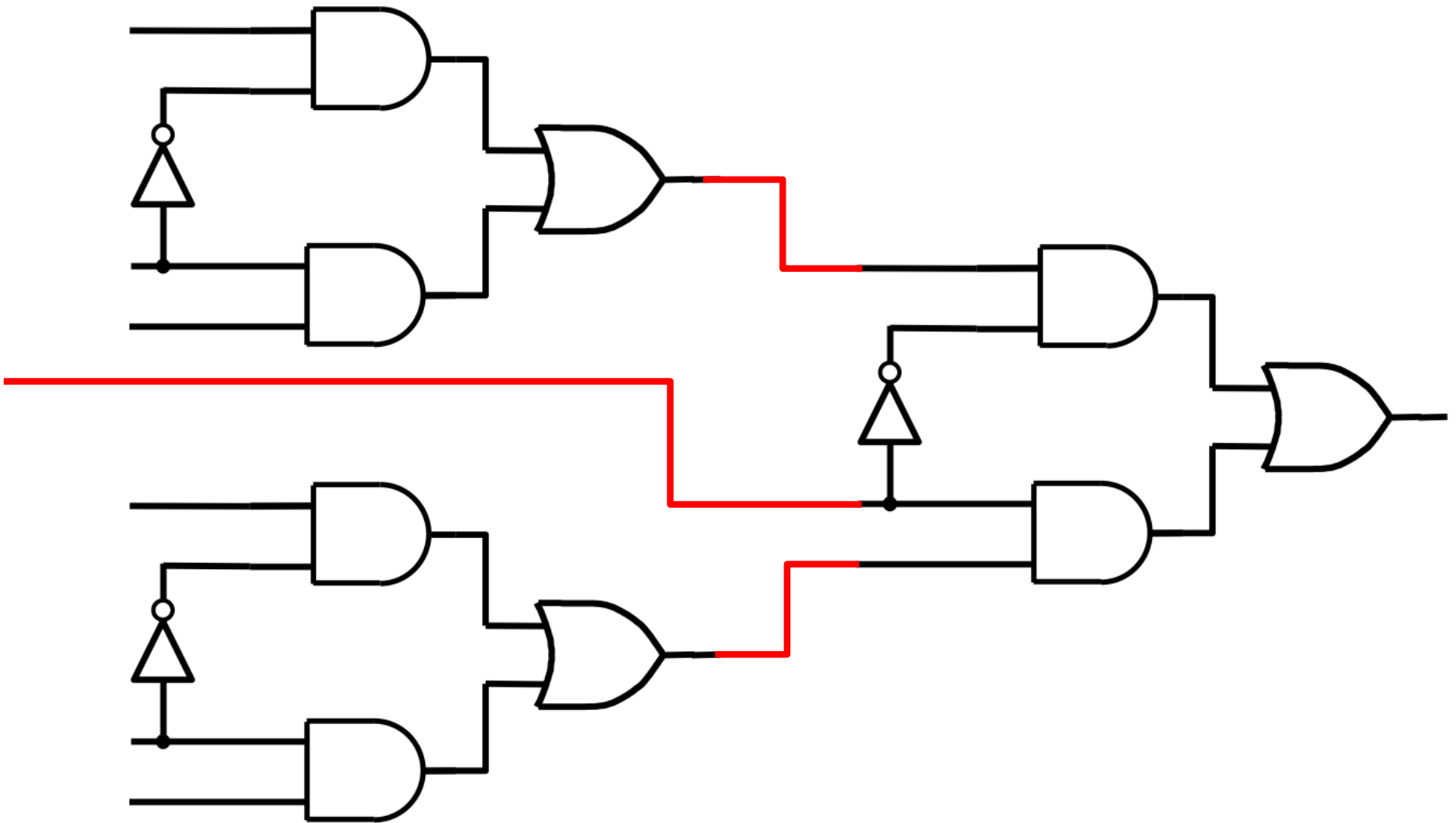


S_1

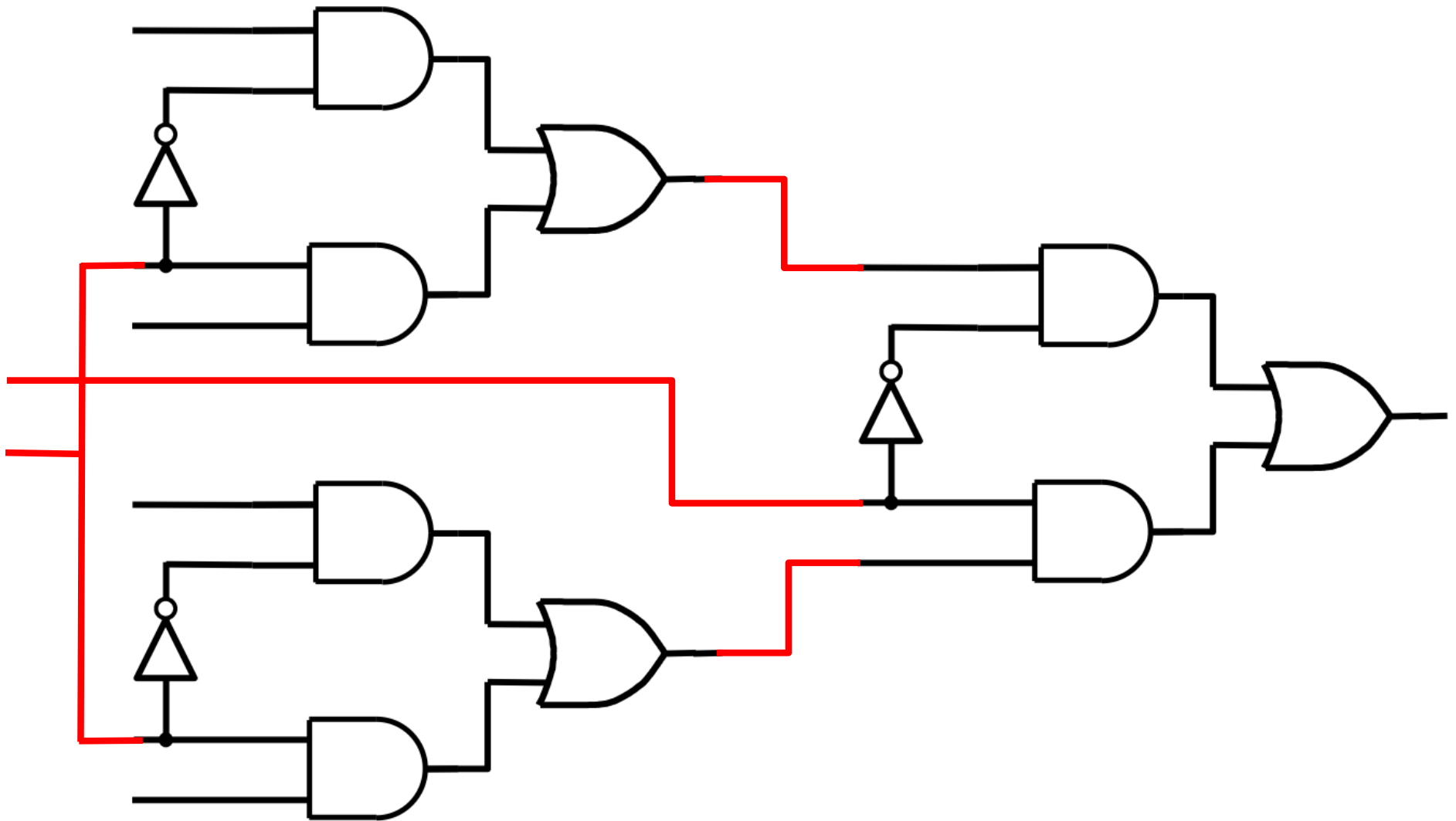
Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



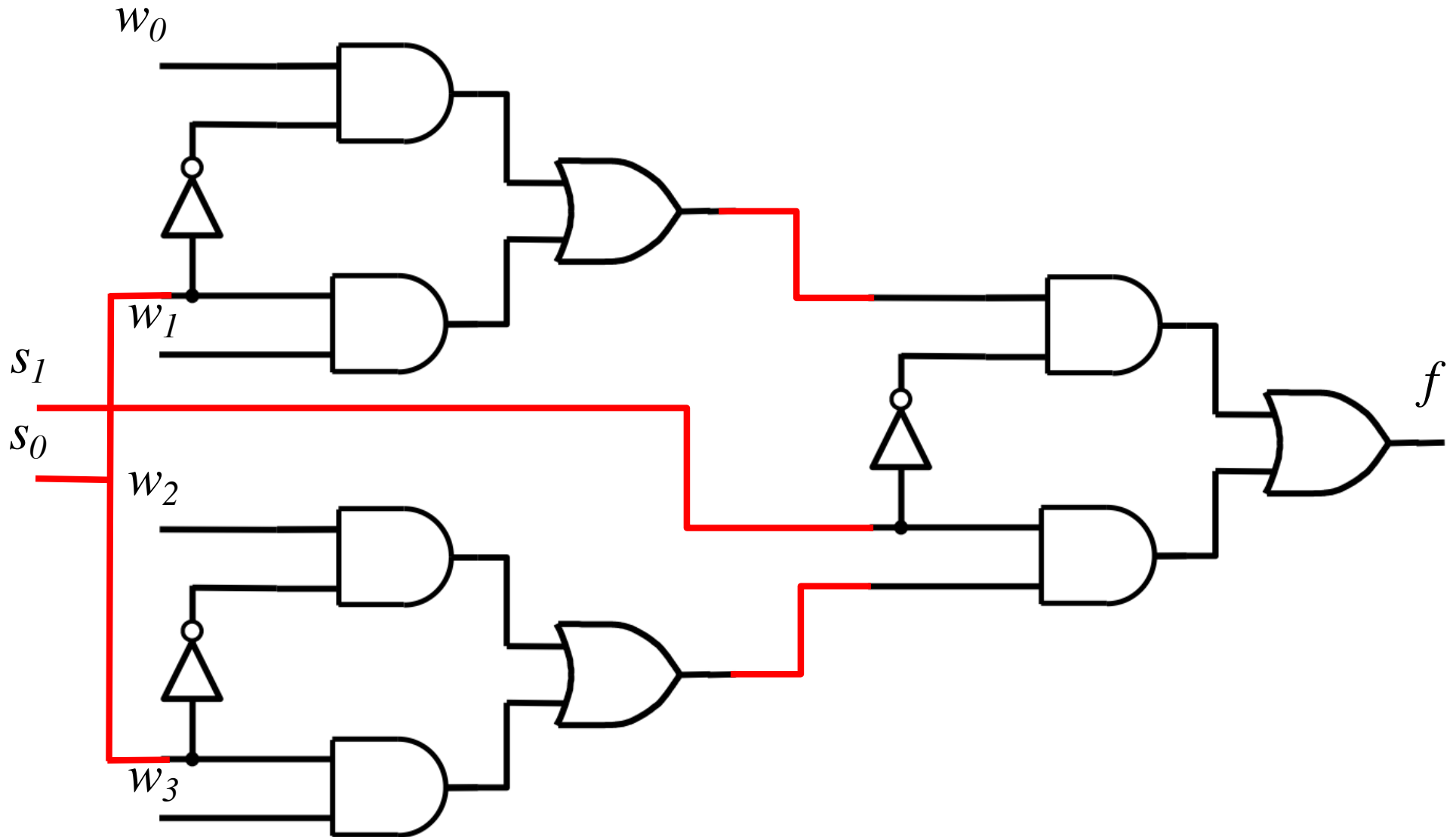
Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



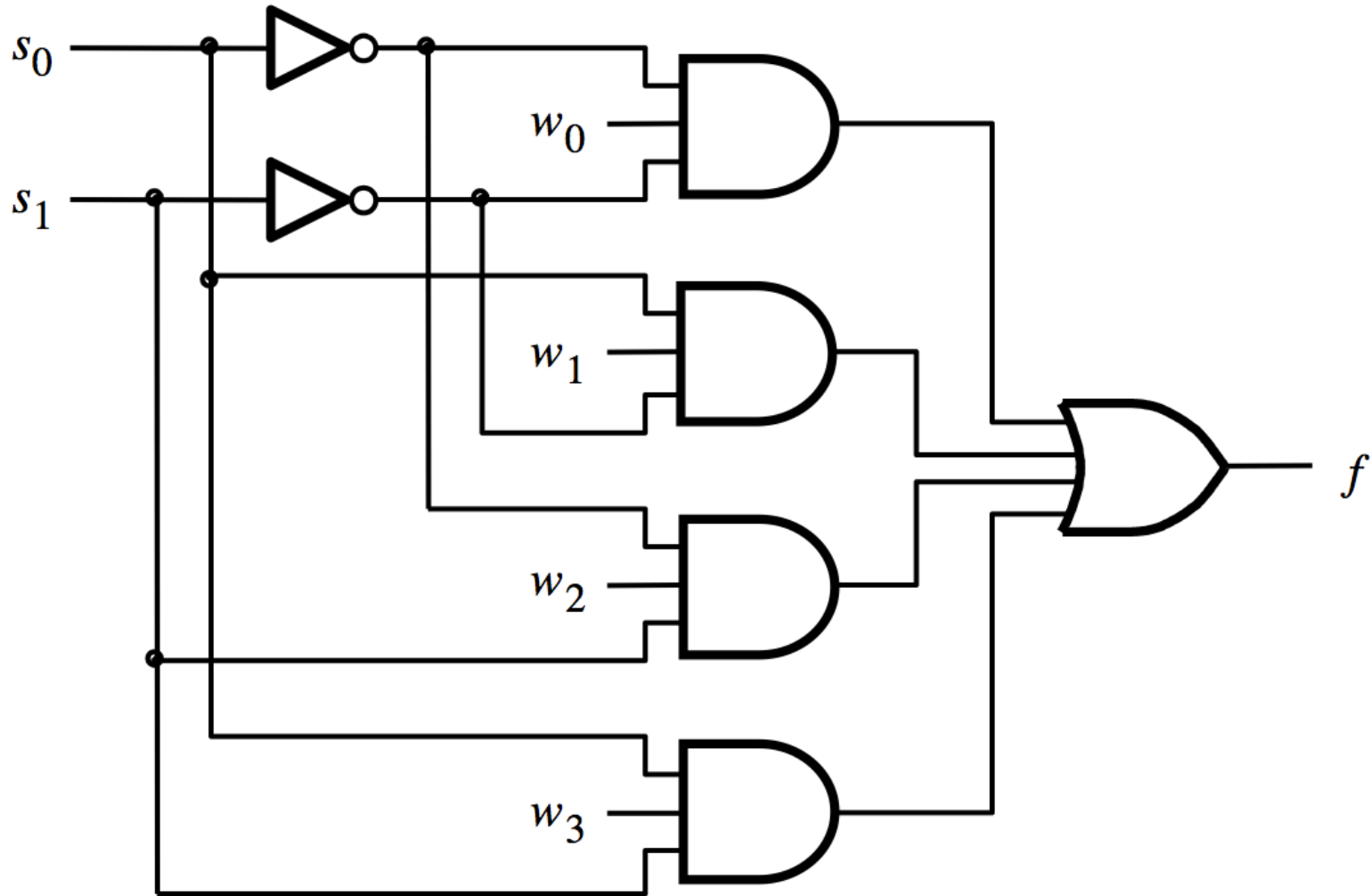
Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



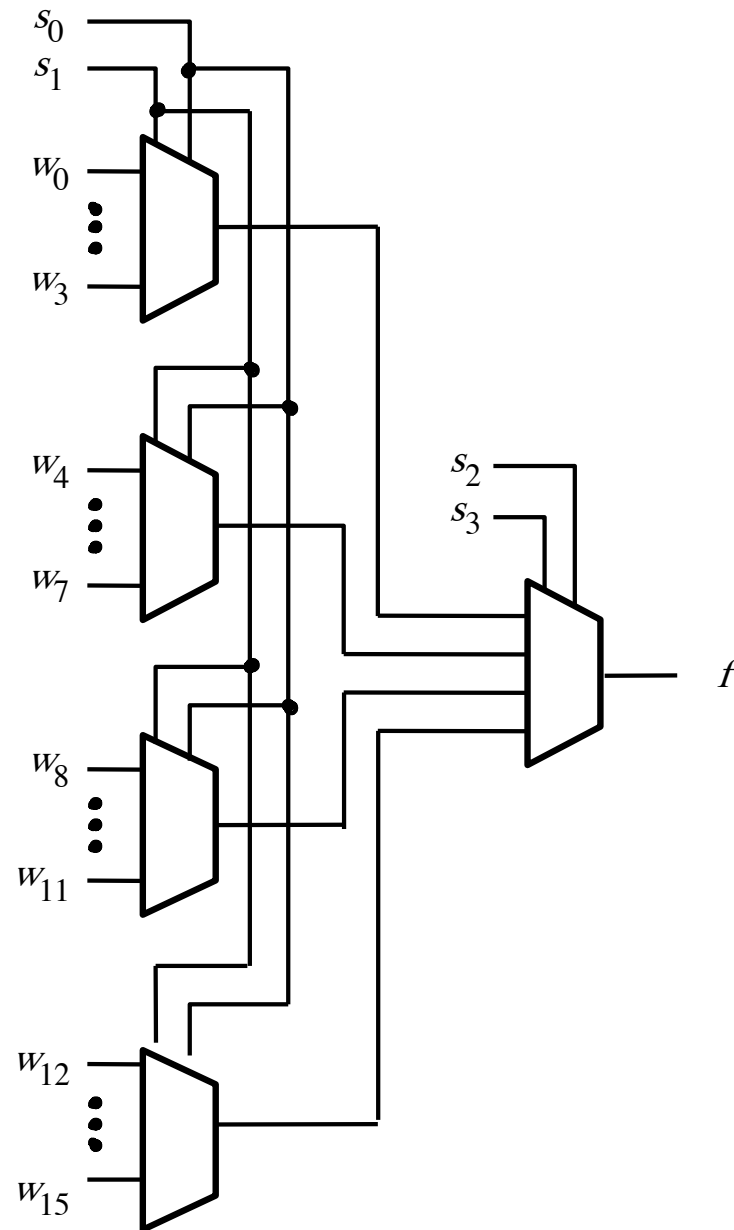
Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



That is different from the SOP form of the 4-to-1 multiplexer shown below, which uses fewer gates



16-to-1 Multiplexer



[Figure 4.4 from the textbook]



[<http://upload.wikimedia.org/wikipedia/commons/2/26/SunsetTracksCrop.JPG>]

Questions?

THE END