



CprE 281: Digital Logic

Instructor: Alexander Stoytchev

<http://www.ece.iastate.edu/~alexs/classes/>

Binary Numbers

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Iowa State University, Ames, IA
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Administrative Stuff

This is the official class web page:

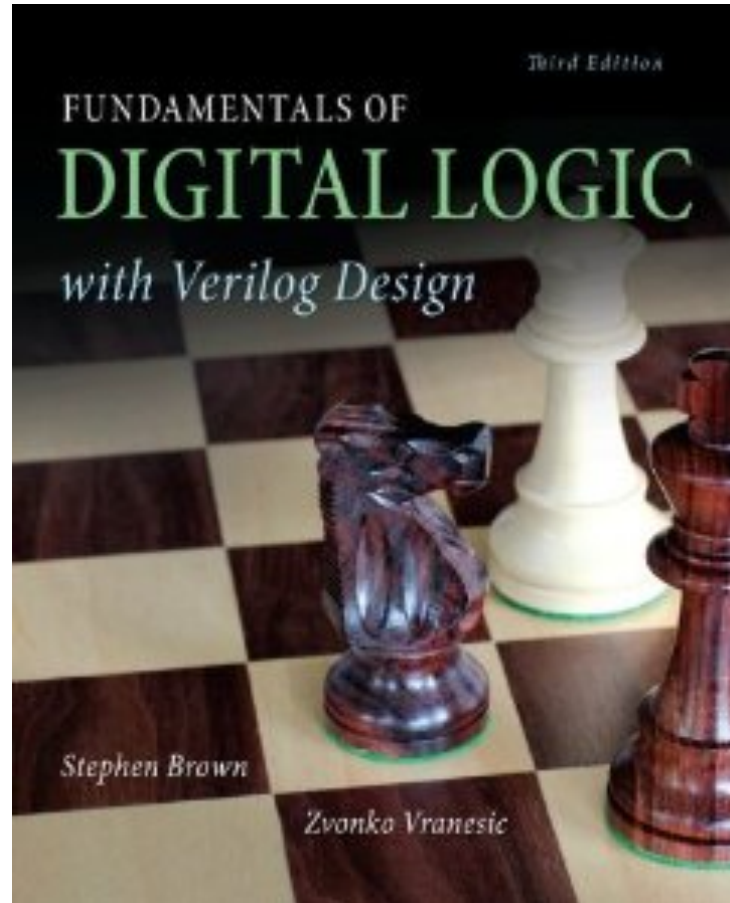
http://www.ece.iastate.edu/~alexs/classes/2022_Fall_281/

If you missed the first lecture, the syllabus and other class materials are posted there.

Administrative Stuff

- **HW1 is out**
- **It is due on Monday Aug 29 @ 10 pm.**
- **Submit it on Canvas before the start of the lecture**

Did you get the textbook?



Administrative Stuff

The labs and recitations start next week:

- **Section 9: Tuesday 2:10 PM - 5:00 PM (Coover Hall, room 2042)**
- **Section 6: Wednesday 7:45 AM - 10:35 AM (Coover Hall, room 2042)**
- **Section 13: Wednesday 11:00 AM - 1:50 PM (Coover Hall, room 2042)**
- **Section 12: Wednesday 6:10 PM - 9:00 PM (Coover Hall, room 2042)**
- **Section 15: Thursday 8:00 AM - 10:50 AM (Coover Hall, room 2042)**
- **Section 11: Thursday 11:00 AM - 1:50 PM (Coover Hall, room 2042)**
- **Section 8: Thursday 2:10 PM - 5:00 PM (Coover Hall, room 2042)**
- **Section 7: Thursday 5:10 PM - 8:00 PM (Coover Hall, room 2042)**

- **The lab schedule is also posted on the class web page**

The Labs Start Next Week

- Please download and read the lab assignment for next week before you go to your lab section.
- You must print the answer sheet for each lab ahead of time.
- Then answer the pre-lab questions before the start of the lab.
- The TAs will check your answers at the beginning of the lab.

The Labs Start Next Week

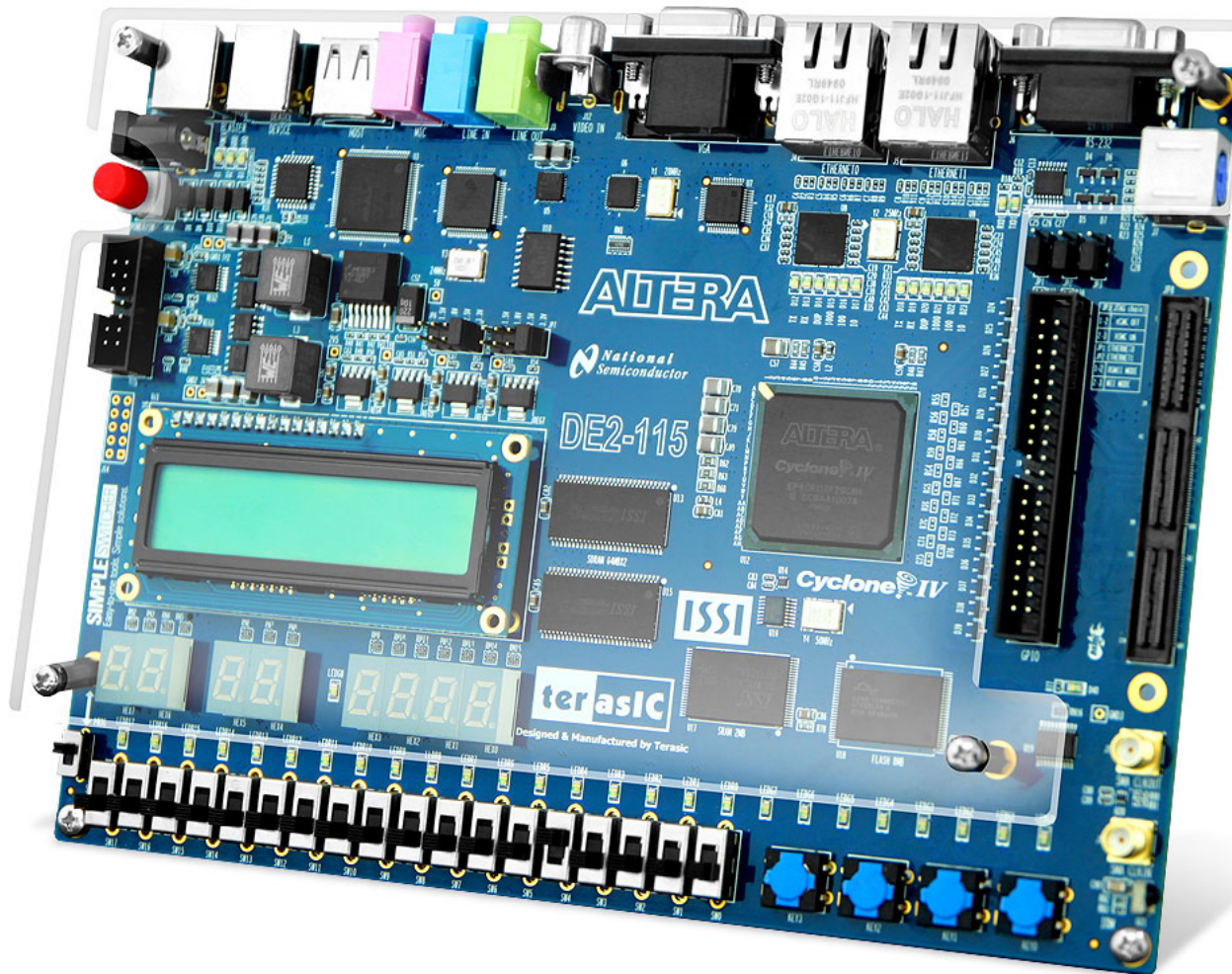
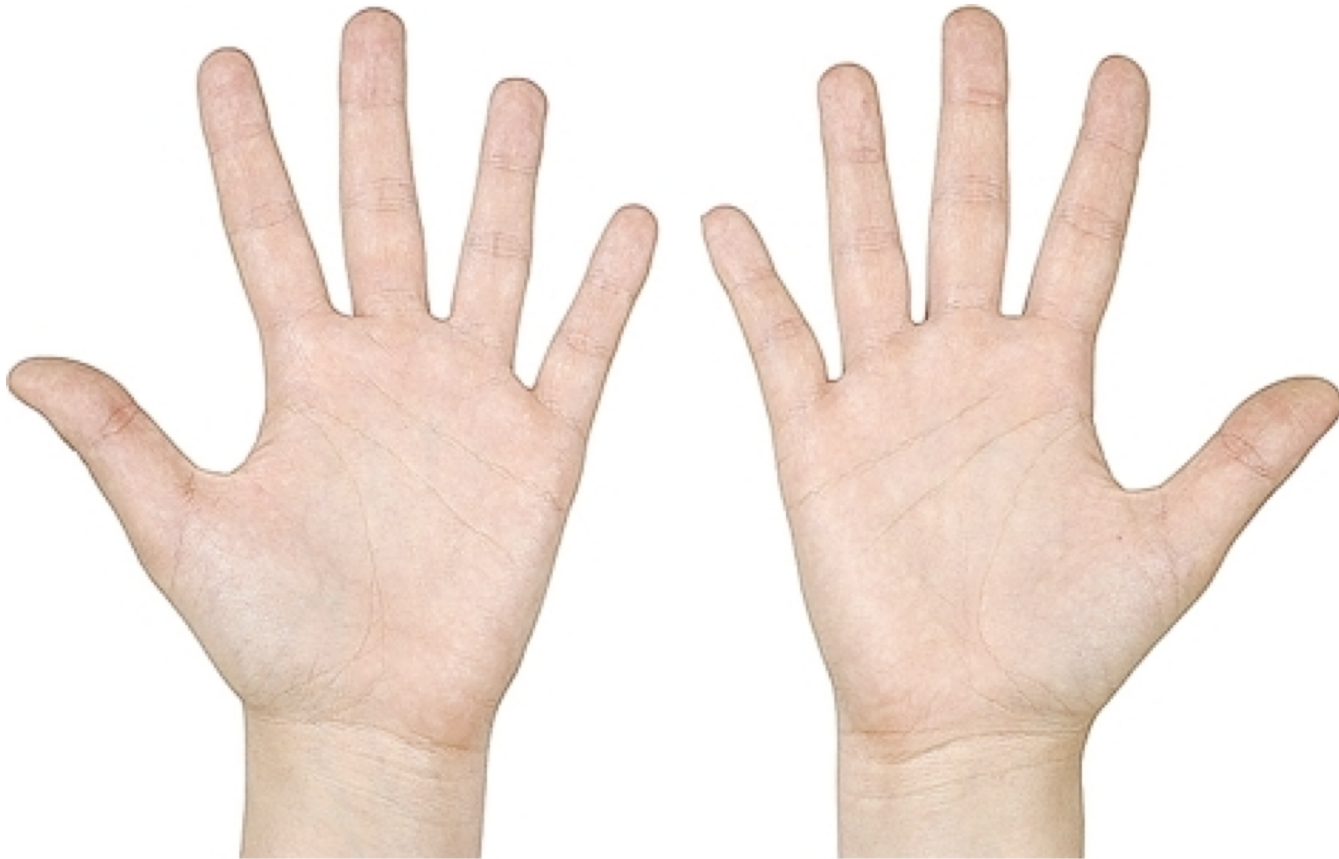
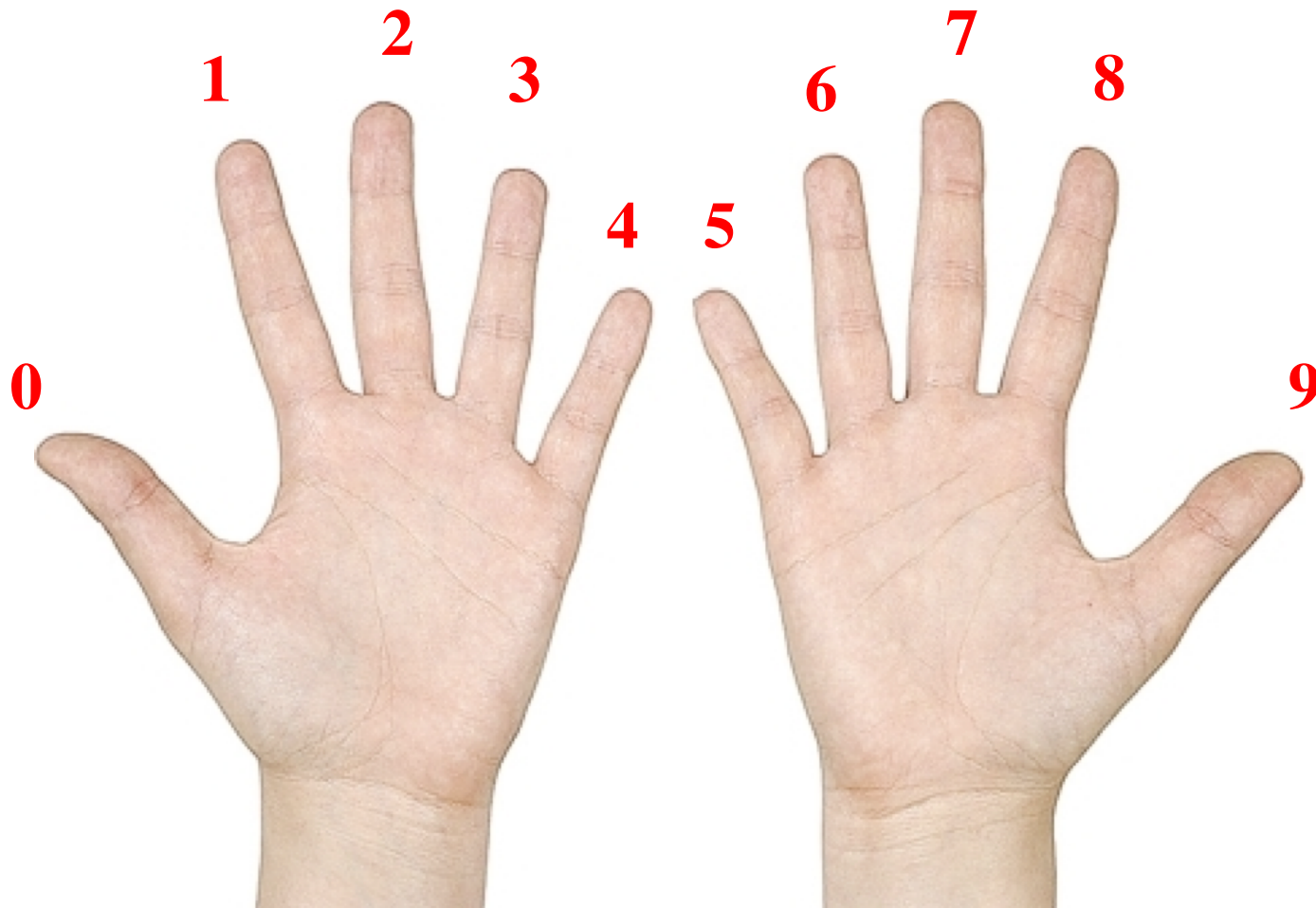


Figure 1.5 in the textbook: An FPGA board.

The Decimal System



The Decimal System



What number system is this one?



The Binary System



The Binary System



Number Systems

$$N = d_n B^n + d_{n-1} B^{n-1} + \cdots + d_1 B^1 + d_0 B^0$$


Number Systems

$$N = d_n B^n + d_{n-1} B^{n-1} + \dots + d_1 B^1 + d_0 B^0$$

n-th digit
(most significant)



0-th digit
(least significant)



Number Systems

base

power

$$N = d_n B^n + d_{n-1} B^{n-1} + \dots + d_1 B^1 + d_0 B^0$$

n-th digit
(most significant)

0-th digit
(least significant)

The Decimal System

$$524_{10} = 5 \times 10^2 + 2 \times 10^1 + 4 \times 10^0$$

The Decimal System

$$\begin{aligned}524_{10} &= 5 \times 10^2 + 2 \times 10^1 + 4 \times 10^0 \\ &= 5 \times 100 + 2 \times 10 + 4 \times 1 \\ &= 500 + 20 + 4 \\ &= 524_{10}\end{aligned}$$

Another Way to Look at This

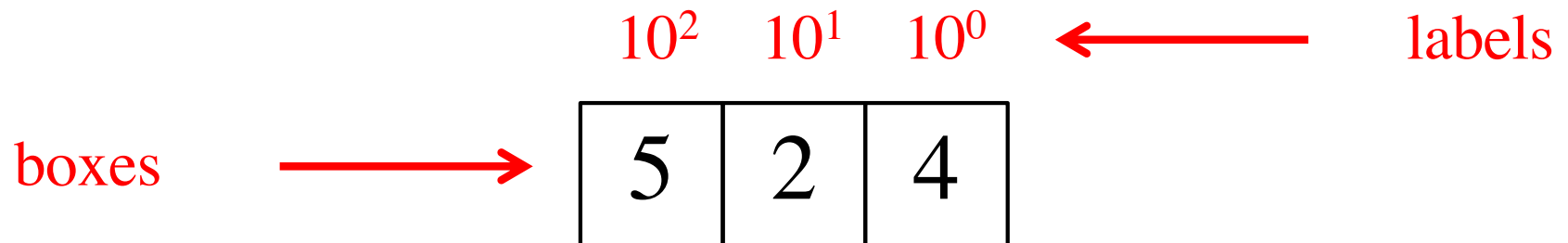
5	2	4
---	---	---

Another Way to Look at This

10^2 10^1 10^0

5	2	4
---	---	---

Another Way to Look at This



Each box can contain only one digit and has only one label. From right to left, the labels are increasing powers of the base, starting from 0.

Base 7

$$524_7 = 5 \times 7^2 + 2 \times 7^1 + 4 \times 7^0$$

Base 7

base

power

$$524_7 = 5 \times 7^2 + 2 \times 7^1 + 4 \times 7^0$$

Base 7

base

power

$$524_7 = 5 \times 7^2 + 2 \times 7^1 + 4 \times 7^0$$

most significant digit

least significant digit

Base 7

$$\begin{aligned}524_7 &= 5 \times 7^2 + 2 \times 7^1 + 4 \times 7^0 \\ &= 5 \times 49 + 2 \times 7 + 4 \times 1 \\ &= 245 + 14 + 4 \\ &= 263_{10}\end{aligned}$$

Another Way to Look at This

$$\begin{array}{|c|c|c|} \hline 7^2 & 7^1 & 7^0 \\ \hline 5 & 2 & 4 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 10^2 & 10^1 & 10^0 \\ \hline 2 & 6 & 3 \\ \hline \end{array}$$

Binary Numbers (Base 2)

$$1001_2 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

Binary Numbers (Base 2)

base power

$$1001_2 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

most significant bit least significant bit

Binary Numbers (Base 2)

$$\begin{aligned} 1001_2 &= 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = \\ &= 1 \times 8 + 0 \times 4 + 0 \times 2 + 1 \times 1 = \\ &= 8 + 0 + 0 + 1 = \\ &= 9_{10} \end{aligned}$$

Another Example

$$\begin{aligned} 11101_2 &= 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = \\ &= 1 \times 16 + 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 = \\ &= 16 + 8 + 4 + 0 + 1 = 29_{10} \end{aligned}$$

Powers of 2

$$2^{10} = 1024$$

$$2^9 = 512$$

$$2^8 = 256$$

$$2^7 = 128$$

$$2^6 = 64$$

$$2^5 = 32$$

$$2^4 = 16$$

$$2^3 = 8$$

$$2^2 = 4$$

$$2^1 = 2$$

$$2^0 = 1$$

What is the value of this binary number?

- **0 0 1 0 1 1 0 0**

- **0 0 1 0 1 1 0 0**

- **$0*2^7 + 0*2^6 + 1*2^5 + 0*2^4 + 1*2^3 + 1*2^2 + 0*2^1 + 0*2^0$**

- **$0*128 + 0*64 + 1*32 + 0*16 + 1*8 + 1*4 + 0*2 + 0*1$**

- **$0*128 + 0*64 + 1*32 + 0*16 + 1*8 + 1*4 + 0*2 + 0*1$**

- **$32 + 8 + 4 = 44$ (in decimal)**

Another Way to Look at This

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	0	1	0	1	1	0	0

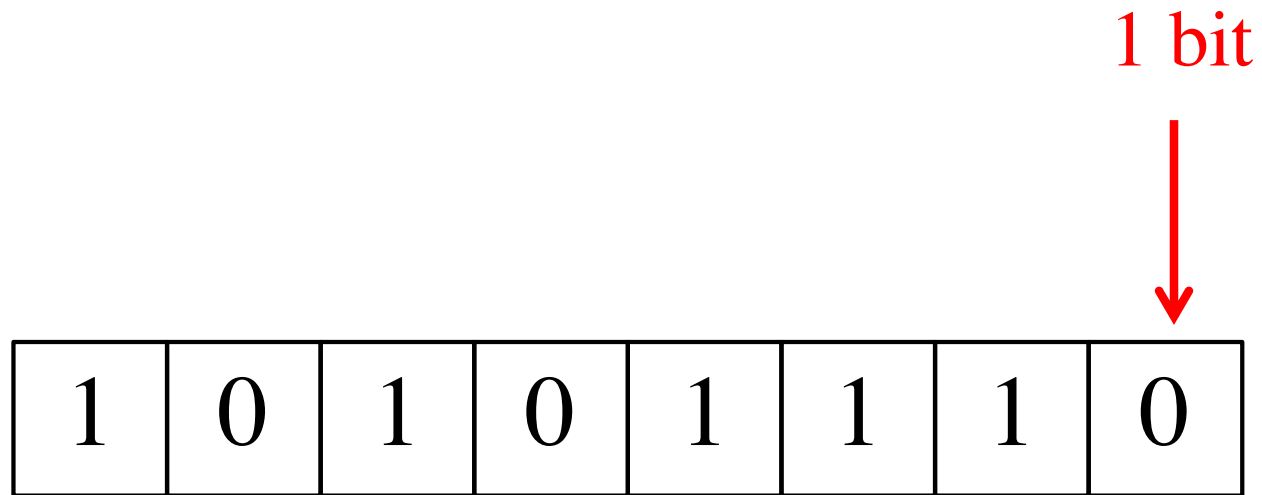
Some Terminology

- **A binary digit is called a *bit***
- **A group of eight bits is called a byte**
- **One bit can represent only two possible states, which are denoted with 1 and 0**

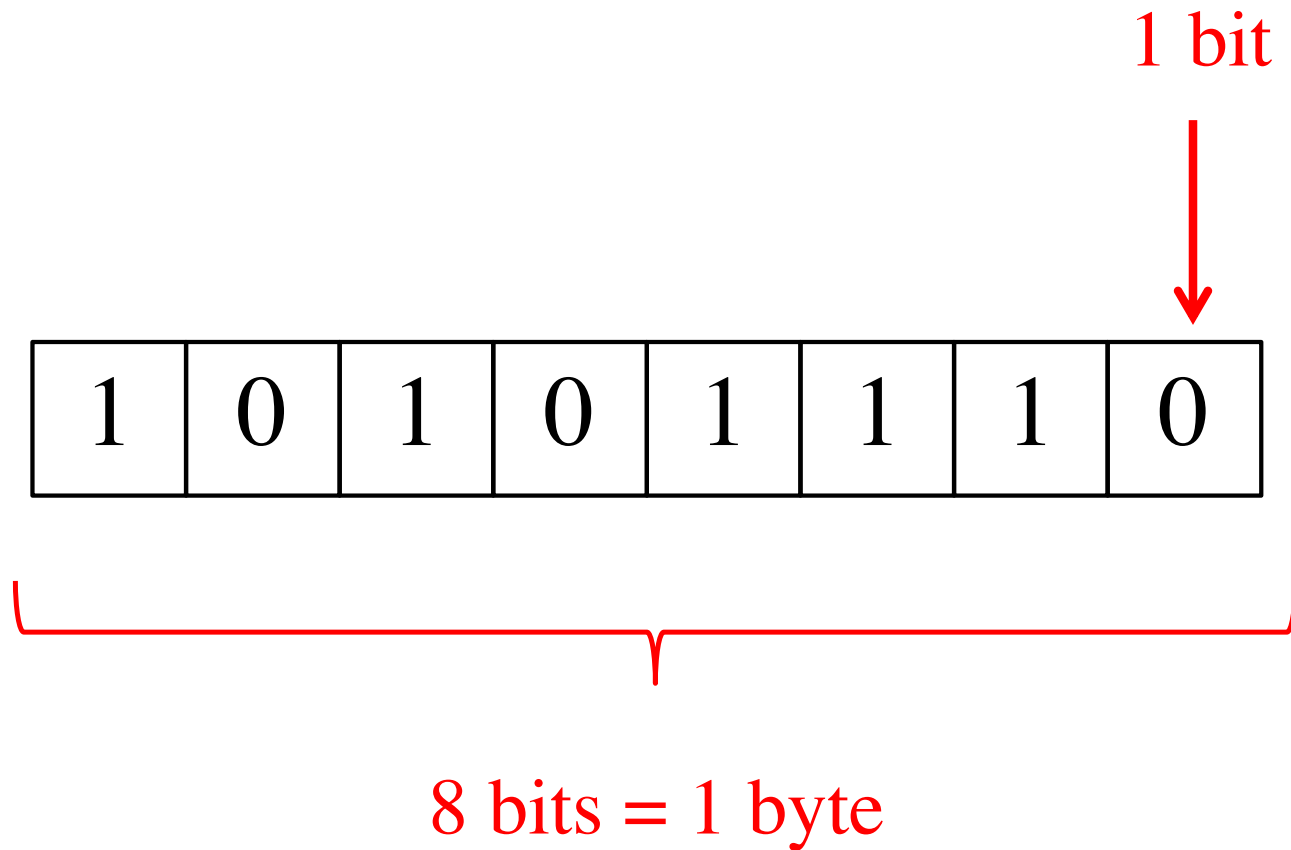
Relationship Between a Byte and a Bit

1	0	1	0	1	1	1	0
---	---	---	---	---	---	---	---

Relationship Between a Byte and a Bit



Relationship Between a Byte and a Bit



Bit Permutations


<u>1 bit</u>	<u>2 bits</u>	<u>3 bits</u>	<u>4 bits</u>	
0	00	000	0000	1000
1	01	001	0001	1001
	10	010	0010	1010
	11	011	0011	1011
		100	0100	1100
		101	0101	1101
		110	0110	1110
		111	0111	1111

Each additional bit doubles the number of possible permutations

Bit Permutations

- Each permutation can represent a particular item
- There are 2^N permutations of N bits
- Therefore, N bits are needed to represent 2^N unique items

How many
items can be
represented by



1 bit ?	$2^1 = 2$ items
2 bits ?	$2^2 = 4$ items
3 bits ?	$2^3 = 8$ items
4 bits ?	$2^4 = 16$ items
5 bits ?	$2^5 = 32$ items

What is the maximum number that can be stored in one byte (8 bits)?

What is the maximum number that can be stored in one byte (8 bits)?

- 1 1 1 1 1 1 1 1

- 1 1 1 1 1 1 1 1

- $1*2^7 + 1*2^6 + 1*2^5 + 1*2^4 + 1*2^3 + 1*2^2 + 1*2^1 + 1*2^0$

- $1*128 + 1*64 + 1*32 + 1*16 + 1*8 + 1*4 + 1*2 + 1*1$

- $128 + 64 + 32 + 16 + 8 + 4 + 2 + 1 = 255$ (in decimal)

- Another way is: $1*2^8 - 1 = 256 - 1 = 255$

Analogy with car odometers



Analogy with car odometers



Decimal to Binary Conversion (Using Guessing)

$$17 = 16 + 1 \rightarrow 10001_2$$

$$2^7 = 128$$

$$2^6 = 64$$

$$2^5 = 32$$

$$2^4 = 16 \quad \checkmark$$

$$2^3 = 8$$

$$2^2 = 4$$

$$2^1 = 2$$

$$2^0 = 1 \quad \checkmark$$

Decimal to Binary Conversion (Using Guessing)


$$212 = 128 + 64 + 16 + 4 \rightarrow 11010100_2$$

2^7	=	128	✓
2^6	=	64	✓
2^5	=	32	
2^4	=	16	✓
2^3	=	8	
2^2	=	4	✓
2^1	=	2	
2^0	=	1	

Converting from Decimal to Binary

				<i>result</i>	<i>remainder</i>
235	/	2	=	117	1
117	/	2	=	58	1
58	/	2	=	29	0
29	/	2	=	14	1
14	/	2	=	7	0
7	/	2	=	3	1
3	/	2	=	1	1
1	/	2	=	0	1

Converting from Decimal to Binary

				<i>result</i>	<i>remainder</i>	
235	/	2	=	117	1	
117	/	2	=	58	1	
58	/	2	=	29	0	
29	/	2	=	14	1	
14	/	2	=	7	0	
7	/	2	=	3	1	
3	/	2	=	1	1	
1	/	2	=	0	1	

$$235_{10} = 11101011_2$$

Convert $(857)_{10}$

			Remainder	
$857 \div 2$	$=$	428	1	LSB
$428 \div 2$	$=$	214	0	
$214 \div 2$	$=$	107	0	
$107 \div 2$	$=$	53	1	
$53 \div 2$	$=$	26	1	
$26 \div 2$	$=$	13	0	
$13 \div 2$	$=$	6	1	
$6 \div 2$	$=$	3	0	
$3 \div 2$	$=$	1	1	
$1 \div 2$	$=$	0	1	MSB

Result is $(1101011001)_2$

Octal System (Base 8)

0	1	2	3	4	5	6	7
10	11	12	13	14	15	16	17
20	21	22	23	24	25	26	27
30	31	32	33	34	35	36	37
40	41	42	43	44	45	46	47
50	51	52	53	54	55	56	57
60	61	62	63	64	65	66	67
70	71	72	73	74	75	76	77

Binary to Octal Conversion

000 → 0

001 → 1

010 → 2

011 → 3

100 → 4

101 → 5

110 → 6

111 → 7

Binary to Octal Conversion

$$101110010111_2 = ?_8$$

Binary to Octal Conversion

$$101110010111_2 = ?_8$$

101 110 010 111

Binary to Octal Conversion

$$101110010111_2 = ?_8$$

$$\begin{array}{cccc} 101 & 110 & 010 & 111 \\ \underbrace{\hspace{1em}} & \underbrace{\hspace{1em}} & \underbrace{\hspace{1em}} & \underbrace{\hspace{1em}} \\ 5 & 6 & 2 & 7 \end{array}$$

Binary to Octal Conversion

$$101110010111_2 = ?_8$$

$$\begin{array}{cccc} 101 & 110 & 010 & 111 \\ \underbrace{\hspace{1em}} & \underbrace{\hspace{1em}} & \underbrace{\hspace{1em}} & \underbrace{\hspace{1em}} \\ 5 & 6 & 2 & 7 \end{array}$$

$$\text{Thus, } 101110010111_2 = 5627_8$$

Hexadecimal System (Base 16)

$$52_{16} = 5 \times 16^1 + 2 \times 16^0 =$$

$$5 \times 16 + 2 \times 1 =$$

$$80 + 2 = 82_{10}$$

The 16 Hexadecimal Digits

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

The 16 Hexadecimal Digits

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

10, 11, 12, 13, 14, 15

Hexadecimal to Decimal Conversion

$$C3_{16} = C \times 16^1 + 3 \times 16^0$$

$$= 12 \times 16 + 3 \times 1$$

$$= 192 + 3$$

$$= 195_{10}$$

Hexadecimal to Decimal Conversion

$$BEEF_{16} = ?_{10}$$

Hexadecimal to Decimal Conversion

$$\begin{aligned} BEEF_{16} &= B_{16} \times 16^3 + E_{16} \times 16^2 + E_{16} \times 16^1 + F_{16} \times 16^0 \\ &= 11 \times 16^3 + 14 \times 16^2 + 14 \times 16^1 + 15 \times 16^0 \\ &= 11 \times 4096 + 14 \times 256 + 14 \times 16 + 15 \times 1 \\ &= 45056 + 3584 + 224 + 15 \\ &= 48879_{10} \end{aligned}$$

Binary to Hexadecimal Conversion

0000	→	0
0001	→	1
0010	→	2
0011	→	3
0100	→	4
0101	→	5
0110	→	6
0111	→	7
1000	→	8
1001	→	9
1010	→	<i>A</i>
1011	→	<i>B</i>
1100	→	<i>C</i>
1101	→	<i>D</i>
1110	→	<i>E</i>
1111	→	<i>F</i>

Binary to Hexadecimal Conversion

0000	→	0	→	0
0001	→	1	→	1
0010	→	2	→	2
0011	→	3	→	3
0100	→	4	→	4
0101	→	5	→	5
0110	→	6	→	6
0111	→	7	→	7
1000	→	8	→	8
1001	→	9	→	9
1010	→	10	→	<i>A</i>
1011	→	11	→	<i>B</i>
1100	→	12	→	<i>C</i>
1101	→	13	→	<i>D</i>
1110	→	14	→	<i>E</i>
1111	→	15	→	<i>F</i>

Binary to Hexadecimal Conversion

$$101110010111_2 = ?_{16}$$

Binary to Hexadecimal Conversion

$$101110010111_2 = ?_{16}$$

1011 1001 0111

Binary to Hexadecimal Conversion

$$101110010111_2 = ?_{16}$$

1011 1001 0111



B

9

7

Binary to Hexadecimal Conversion

$$101110010111_2 = ?_{16}$$

1011 1001 0111




B 9 7

Thus, $101110010111_2 = \text{B97}_{16}$


Decimal to Hexadecimal Conversion

$$1396_{10} = 574_{16}$$

				<i>result</i>	<i>remainder</i>	
1396	/	16	=	87	4	
87	/	16	=	5	7	
5	/	16	=	0	5	

Decimal to Hexadecimal Conversion

$$502_{10} = 1F6_{16}$$

				<i>result</i>	<i>remainder</i>	
502	/	16	=	31	6	
31	/	16	=	1	15	
1	/	16	=	0	1	

Signed integers are more complicated

We will talk more about them when we start with Chapter 3 in a couple of weeks.

On-line IEEE 754 Converter

- <https://www.h-schmidt.net/FloatConverter/IEEE754.html>
- **More about floating point numbers in Chapter 3.**

Storing Characters

- **This requires some convention that maps binary numbers to characters.**
- **ASCII table**
- **Unicode**

ASCII Table

Dec	Hx	Oct	Char	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr
0	0	000	NUL (null)	32	20	040	 	Space	64	40	100	@	@	96	60	140	`	`
1	1	001	SOH (start of heading)	33	21	041	!	!	65	41	101	A	A	97	61	141	a	a
2	2	002	STX (start of text)	34	22	042	"	"	66	42	102	B	B	98	62	142	b	b
3	3	003	ETX (end of text)	35	23	043	#	#	67	43	103	C	C	99	63	143	c	c
4	4	004	EOT (end of transmission)	36	24	044	$	\$	68	44	104	D	D	100	64	144	d	d
5	5	005	ENQ (enquiry)	37	25	045	%	%	69	45	105	E	E	101	65	145	e	e
6	6	006	ACK (acknowledge)	38	26	046	&	&	70	46	106	F	F	102	66	146	f	f
7	7	007	BEL (bell)	39	27	047	'	'	71	47	107	G	G	103	67	147	g	g
8	8	010	BS (backspace)	40	28	050	((72	48	110	H	H	104	68	150	h	h
9	9	011	TAB (horizontal tab)	41	29	051))	73	49	111	I	I	105	69	151	i	i
10	A	012	LF (NL line feed, new line)	42	2A	052	*	*	74	4A	112	J	J	106	6A	152	j	j
11	B	013	VT (vertical tab)	43	2B	053	+	+	75	4B	113	K	K	107	6B	153	k	k
12	C	014	FF (NP form feed, new page)	44	2C	054	,	,	76	4C	114	L	L	108	6C	154	l	l
13	D	015	CR (carriage return)	45	2D	055	-	-	77	4D	115	M	M	109	6D	155	m	m
14	E	016	SO (shift out)	46	2E	056	.	.	78	4E	116	N	N	110	6E	156	n	n
15	F	017	SI (shift in)	47	2F	057	/	/	79	4F	117	O	O	111	6F	157	o	o
16	10	020	DLE (data link escape)	48	30	060	0	0	80	50	120	P	P	112	70	160	p	p
17	11	021	DC1 (device control 1)	49	31	061	1	1	81	51	121	Q	Q	113	71	161	q	q
18	12	022	DC2 (device control 2)	50	32	062	2	2	82	52	122	R	R	114	72	162	r	r
19	13	023	DC3 (device control 3)	51	33	063	3	3	83	53	123	S	S	115	73	163	s	s
20	14	024	DC4 (device control 4)	52	34	064	4	4	84	54	124	T	T	116	74	164	t	t
21	15	025	NAK (negative acknowledge)	53	35	065	5	5	85	55	125	U	U	117	75	165	u	u
22	16	026	SYN (synchronous idle)	54	36	066	6	6	86	56	126	V	V	118	76	166	v	v
23	17	027	ETB (end of trans. block)	55	37	067	7	7	87	57	127	W	W	119	77	167	w	w
24	18	030	CAN (cancel)	56	38	070	8	8	88	58	130	X	X	120	78	170	x	x
25	19	031	EM (end of medium)	57	39	071	9	9	89	59	131	Y	Y	121	79	171	y	y
26	1A	032	SUB (substitute)	58	3A	072	:	:	90	5A	132	Z	Z	122	7A	172	z	z
27	1B	033	ESC (escape)	59	3B	073	;	:	91	5B	133	[[123	7B	173	{	{
28	1C	034	FS (file separator)	60	3C	074	<	<	92	5C	134	\	\	124	7C	174	|	
29	1D	035	GS (group separator)	61	3D	075	=	=	93	5D	135]]	125	7D	175	}	}
30	1E	036	RS (record separator)	62	3E	076	>	>	94	5E	136	^	^	126	7E	176	~	~
31	1F	037	US (unit separator)	63	3F	077	?	?	95	5F	137	_	_	127	7F	177		DEL

Extended ASCII Codes

128	Ç	144	É	161	í	177	⋮	193	⊥	209	ƒ	225	β	241	±
129	ü	145	æ	162	ó	178	■	194	⊤	210	π	226	Γ	242	≥
130	é	146	Æ	163	ú	179		195	⊢	211	ℓ	227	π	243	≤
131	â	147	ô	164	ñ	180	†	196	—	212	ℓ	228	Σ	244	∫
132	ä	148	ö	165	Ñ	181	‡	197	‡	213	ƒ	229	σ	245	∫
133	à	149	ò	166	ª	182	‡	198	‡	214	ƒ	230	μ	246	+
134	å	150	û	167	º	183	π	199	‡	215	‡	231	τ	247	±
135	ç	151	ù	168	¿	184	‡	200	ℓ	216	‡	232	Φ	248	°
136	ê	152	—	169	—	185	‡	201	ƒ	217	∫	233	⊙	249	.
137	ë	153	Ö	170	¬	186		202	≡	218	∫	234	Ω	250	.
138	è	154	Û	171	½	187	‡	203	ƒ	219	■	235	δ	251	√
139	ì	156	£	172	¼	188	∫	204	‡	220	■	236	∞	252	—
140	î	157	₣	173	¡	189	∫	205	=	221	■	237	φ	253	z
141	ï	158	—	174	«	190	∫	206	‡	222	■	238	ε	254	■
142	Ä	159	f	175	»	191	∫	207	≡	223	■	239	∩	255	
143	Å	160	á	176	⋮	192	L	208	≡	224	α	240	≡		

Source: www.LookupTables.com












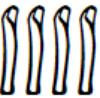

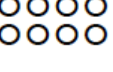











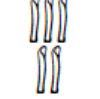

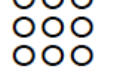











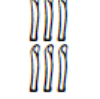














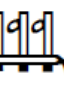















The Unicode Character Code

- <http://www.unicode.org/charts/>

Egyptian Hieroglyphs

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2	13002	13012	13022	13032	13042	13052	13062	13072	13082	13092	130A2	130B2	130C2	130D2
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5	13005	13015	13025	13035	13045	13055	13065	13075	13085	13095	130A5	130B5	130C5	130D5
6	13006	13016	13026	13036	13046	13056	13066	13076	13086	13096	130A6	130B6	130C6	130D6
7	13007	13017	13027	13037	13047	13057	13067	13077	13087	13097	130A7	130B7	130C7	130D7
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A	1300A	1301A	1302A	1303A	1304A	1305A	1306A	1307A	1308A	1309A	130AA	130EA	130CA	130DA
B	1300B	1301B	1302B	1303B	1304B	1305B	1306B	1307B	1308B	1309B	130AB	130BB	130CB	130DB
C	1300C	1301C	1302C	1303C	1304C	1305C	1306C	1307C	1308C	1309C	130AC	130BC	130CC	130DC
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E	1300E	1301E	1302E	1303E	1304E	1305E	1306E	1307E	1308E	1309E	130AE	130BE	130CE	130DE
F	1300F	1301F	1302F	1303F	1304F	1305F	1306F	1307F	1308F	1309F	130AF	130BF	130CF	130DF

Close up

	1300	1301	1302	1303	1304	1305	1306	1307	1308	1309	130A	130B	130C	130D
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2	 13002	 13012	 13022	 13032	 13042	 13052	 13062	 13072	 13082	 13092	 130A2	 130B2	 130C2	 130D2
3	 13003	 13013	 13023	 13033	 13043	 13053	 13063	 13073	 13083	 13093	 130A3	 130B3	 130C3	 130D3
4	 13004	 13014	 13024	 13034	 13044	 13054	 13064	 13074	 13084	 13094	 130A4	 130B4	 130C4	 130D4

Questions?

THE END