



# **CprE 281: Digital Logic**

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**<http://www.ece.iastate.edu/~alexs/classes/>**

# Examples of Solved Problems

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*Iowa State University, Ames, IA*  
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# Administrative Stuff

- **HW4 is due today**

# **Administrative Stuff**

- **HW5 is out**
- **It is due on Monday Sep 27 @ 4pm.**
- **It is posted on the class web page**
- **I also sent you an e-mail with the link.**
- **You can use this as a preparation for the exam.**

# **Administrative Stuff**

- **Midterm Exam #1**
- **When: Friday Sep 24.**
- **What: Chapter 1 and Chapter 2 plus number systems**
- **The exam will be closed book but open notes (you can bring up to 3 pages of handwritten notes).**
- **It will be in this room.**
- **Review session: This Wednesday during lecture**

# Topics for the Midterm Exam

- **Binary Numbers**
- **Octal Numbers**
- **Hexadecimal Numbers**
- **Conversion between the different number systems**
- **Truth Tables**
- **Boolean Algebra**
- **Logic Gates**
- **Circuit Synthesis with AND, OR, NOT**
- **Circuit Synthesis with NAND, NOR**
- **Converting an AND/OR/NOT circuit to NAND circuit**
- **Converting an AND/OR/NOT circuit to NOR circuit**
- **SOP and POS expressions**

# Topics for the Midterm Exam

- **Mapping a Circuit to Verilog code**
- **Mapping Verilog code to a circuit**
- **Multiplexers**
- **Venn Diagrams**
- **K-maps for 2, 3, and 4 variables**
- **Minimization of Boolean expressions using theorems**
- **Minimization of Boolean expressions with K-maps**
- **Incompletely specified functions (with don't cares)**
- **Functions with multiple outputs**
- **Something from Star Wars**

**All possible Boolean functions  
with two input variables**





# There are 16 possible Boolean functions with two input variables

x	y	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	F <sub>5</sub>	F <sub>6</sub>	F <sub>7</sub>	F <sub>8</sub>	F <sub>9</sub>	F <sub>10</sub>	F <sub>11</sub>	F <sub>12</sub>	F <sub>13</sub>	F <sub>14</sub>	F <sub>15</sub>
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1



constant 0




constant 1

# There are 16 possible Boolean functions with two input variables


x	y	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	F <sub>5</sub>	F <sub>6</sub>	F <sub>7</sub>	F <sub>8</sub>	F <sub>9</sub>	F <sub>10</sub>	F <sub>11</sub>	F <sub>12</sub>	F <sub>13</sub>	F <sub>14</sub>	F <sub>15</sub>
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1


  
NOR(x, y)

  
OR(x, y)

# There are 16 possible Boolean functions with two input variables

x	y	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	F <sub>5</sub>	F <sub>6</sub>	F <sub>7</sub>	F <sub>8</sub>	F <sub>9</sub>	F <sub>10</sub>	F <sub>11</sub>	F <sub>12</sub>	F <sub>13</sub>	F <sub>14</sub>	F <sub>15</sub>
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1


$$m_1 = \bar{x} y$$


$$M_1 = x + \bar{y}$$

# There are 16 possible Boolean functions with two input variables


x	y	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	F <sub>5</sub>	F <sub>6</sub>	F <sub>7</sub>	F <sub>8</sub>	F <sub>9</sub>	F <sub>10</sub>	F <sub>11</sub>	F <sub>12</sub>	F <sub>13</sub>	F <sub>14</sub>	F <sub>15</sub>
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1


↑  
NOT(x)

↑  
x

# There are 16 possible Boolean functions with two input variables

x	y	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	F <sub>5</sub>	F <sub>6</sub>	F <sub>7</sub>	F <sub>8</sub>	F <sub>9</sub>	F <sub>10</sub>	F <sub>11</sub>	F <sub>12</sub>	F <sub>13</sub>	F <sub>14</sub>	F <sub>15</sub>
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

  
 $m_2 = x \bar{y}$

  
 $M_2 = \bar{x} + y$

# There are 16 possible Boolean functions with two input variables

x	y	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	F <sub>5</sub>	F <sub>6</sub>	F <sub>7</sub>	F <sub>8</sub>	F <sub>9</sub>	F <sub>10</sub>	F <sub>11</sub>	F <sub>12</sub>	F <sub>13</sub>	F <sub>14</sub>	F <sub>15</sub>
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

↑  
NOT(y)

↑  
y

# There are 16 possible Boolean functions with two input variables

x	y	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	F <sub>5</sub>	F <sub>6</sub>	F <sub>7</sub>	F <sub>8</sub>	F <sub>9</sub>	F <sub>10</sub>	F <sub>11</sub>	F <sub>12</sub>	F <sub>13</sub>	F <sub>14</sub>	F <sub>15</sub>
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1



XOR(x, y)



XNOR(x, y)



# There are 16 possible Boolean functions with two input variables

x	y	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	F <sub>5</sub>	F <sub>6</sub>	F <sub>7</sub>	F <sub>8</sub>	F <sub>9</sub>	F <sub>10</sub>	F <sub>11</sub>	F <sub>12</sub>	F <sub>13</sub>	F <sub>14</sub>	F <sub>15</sub>
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1



NAND(x, y)    AND(x, y)

# There are 16 possible Boolean functions with two input variables

x	y	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	F <sub>5</sub>	F <sub>6</sub>	F <sub>7</sub>	F <sub>8</sub>	F <sub>9</sub>	F <sub>10</sub>	F <sub>11</sub>	F <sub>12</sub>	F <sub>13</sub>	F <sub>14</sub>	F <sub>15</sub>
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

0  
 NOR  
 $m_1$   
 $\bar{x}$   
 $m_2$   
 $\bar{y}$   
 XOR  
 NAND  
 AND  
 XNOR  
 $y$   
 $M_2$   
 $x$   
 $M_1$   
 OR  
 1

# There are 16 possible Boolean functions with two input variables

x	y	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	F <sub>5</sub>	F <sub>6</sub>	F <sub>7</sub>	F <sub>8</sub>	F <sub>9</sub>	F <sub>10</sub>	F <sub>11</sub>	F <sub>12</sub>	F <sub>13</sub>	F <sub>14</sub>	F <sub>15</sub>
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

0    m<sub>0</sub>    m<sub>1</sub>     $\bar{x}$     m<sub>2</sub>     $\bar{y}$     XOR    M<sub>3</sub>    m<sub>3</sub>    XNOR    y    M<sub>2</sub>    x    M<sub>1</sub>    M<sub>0</sub>    1

# There are 16 possible Boolean functions with two input variables

x	y	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	F <sub>5</sub>	F <sub>6</sub>	F <sub>7</sub>	F <sub>8</sub>	F <sub>9</sub>	F <sub>10</sub>	F <sub>11</sub>	F <sub>12</sub>	F <sub>13</sub>	F <sub>14</sub>	F <sub>15</sub>
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

0    m<sub>0</sub>    m<sub>1</sub>     $\bar{x}$     m<sub>2</sub>     $\bar{y}$     m<sub>1</sub> + m<sub>2</sub>    M<sub>3</sub>    m<sub>3</sub>    M<sub>1</sub> • M<sub>2</sub>    y    M<sub>2</sub>    x    M<sub>1</sub>    M<sub>0</sub>    1

# There are 16 possible Boolean functions with two input variables

x	y	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	F <sub>5</sub>	F <sub>6</sub>	F <sub>7</sub>	F <sub>8</sub>	F <sub>9</sub>	F <sub>10</sub>	F <sub>11</sub>	F <sub>12</sub>	F <sub>13</sub>	F <sub>14</sub>	F <sub>15</sub>
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

$0$      $m_0$      $m_1$      $m_0 + m_1$      $m_2$      $m_0 + m_2$      $m_1 + m_2$      $M_3$      $m_3$      $M_1 \cdot M_2$      $M_0 \cdot M_2$      $M_2$      $M_0 \cdot M_1$      $M_1$      $M_0$      $1$

# These are all valid groupings

		x	
	y		
		0	1
0		1	0
1		0	0

$$\bar{x} \bar{y}$$

		x	
	y		
		0	1
0		0	0
1		1	0

$$\bar{x} y$$

		x	
	y		
		0	1
0		0	1
1		0	0

$$x \bar{y}$$

		x	
	y		
		0	1
0		0	0
1		0	1

$$x y$$

# These are all valid groupings

	x	
y	0	1
0	1	0
1	1	0

$\bar{x}$

	x	
y	0	1
0	0	1
1	0	1

$x$

	x	
y	0	1
0	1	1
1	0	0

$\bar{y}$

	x	
y	0	1
0	0	0
1	1	1

$y$

# These are all valid groupings

	x	
y	0	1
0	1	1
1	1	0

$$\bar{x} + \bar{y}$$

	x	
y	0	1
0	1	0
1	1	1

$$\bar{x} + y$$

	x	
y	0	1
0	1	1
1	0	1

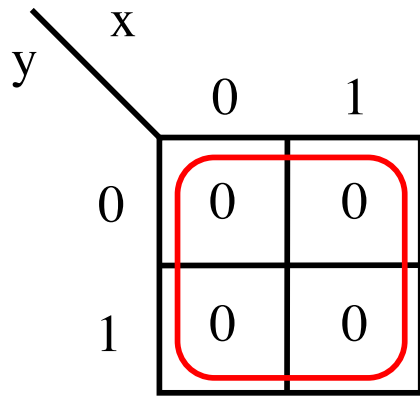
$$x + \bar{y}$$

	x	
y	0	1
0	0	1
1	1	1

$$x + y$$



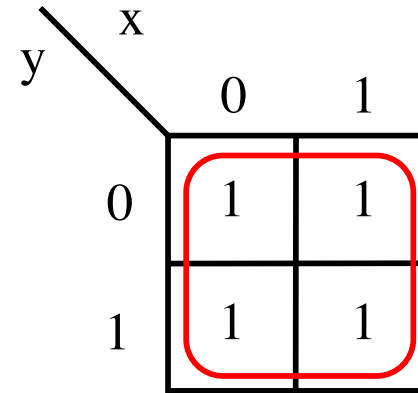
# These are valid too



A Karnaugh map for two variables, x and y. The x-axis is labeled with 0 and 1, and the y-axis is labeled with 0 and 1. The map is a 2x2 grid. All four cells contain the value 0. A red rounded square highlights the entire 2x2 grid.

	x	0	1
y	0	0	0
1	0	0	0

constant 0



A Karnaugh map for two variables, x and y. The x-axis is labeled with 0 and 1, and the y-axis is labeled with 0 and 1. The map is a 2x2 grid. All four cells contain the value 1. A red rounded square highlights the entire 2x2 grid.

	x	0	1
y	0	1	1
1	1	1	1

constant 1

**These are not valid groupings,  
but they correspond to XOR and XNOR**

		x	
		0	1
y	0	0	1
	1	1	0

		x	
		0	1
y	0	1	0
	1	0	1

**These are not valid groupings,  
but they correspond to XOR and XNOR**

		x	
	y		
		0	1
0		0	1
1		1	0

**XOR(x, y)**

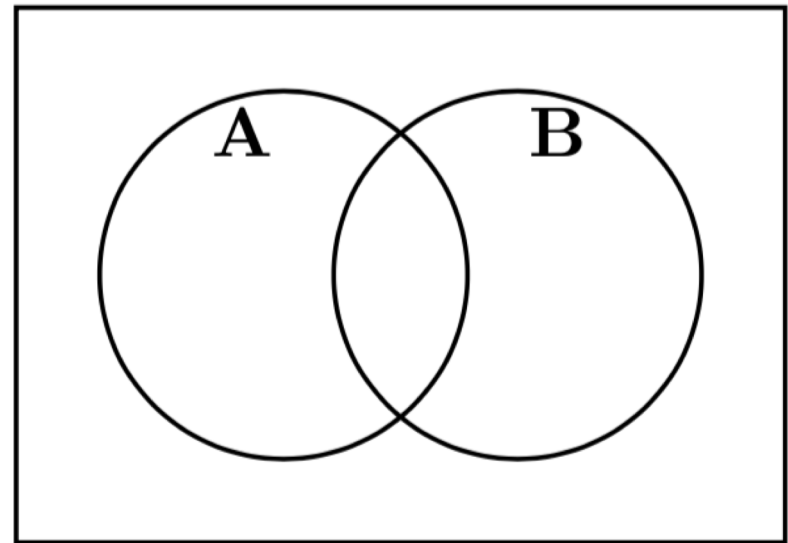
		x	
	y		
		0	1
0		1	0
1		0	1

**XNOR(x, y)**

# **The Link Between Truth Tables and Venn Diagrams**

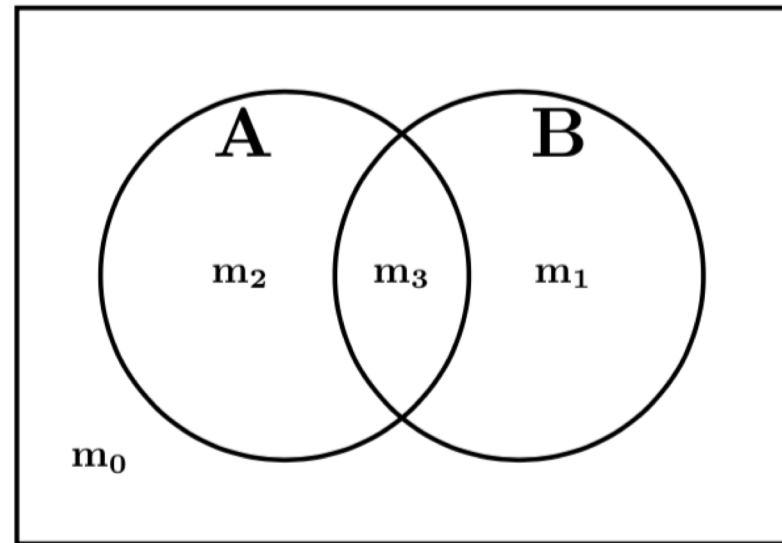
# Place the minterms on the Venn diagram

<b>A</b>	<b>B</b>	
0	0	$m_0$
0	1	$m_1$
1	0	$m_2$
1	1	$m_3$



# Place the minterms on the Venn diagram

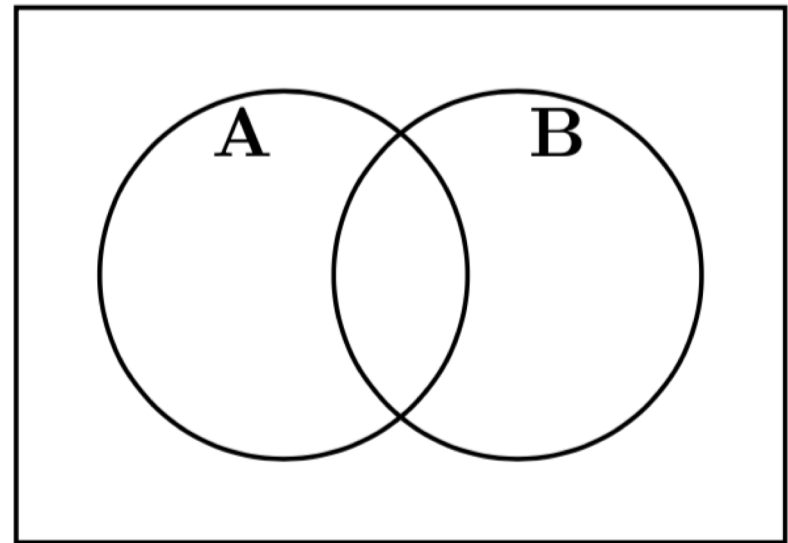
<b>A</b>	<b>B</b>	
0	0	$m_0$
0	1	$m_1$
1	0	$m_2$
1	1	$m_3$





# Color the Venn diagram for XOR

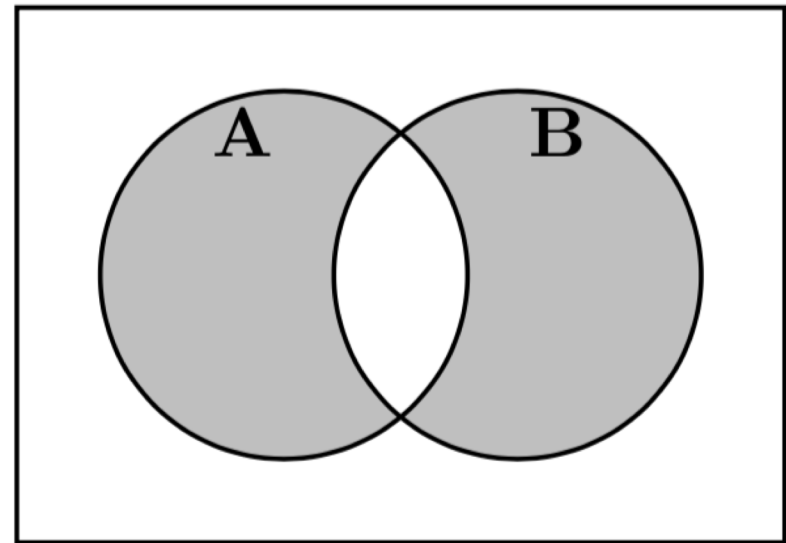
A	B	F
0	0	0
0	1	1
1	0	1
1	1	0





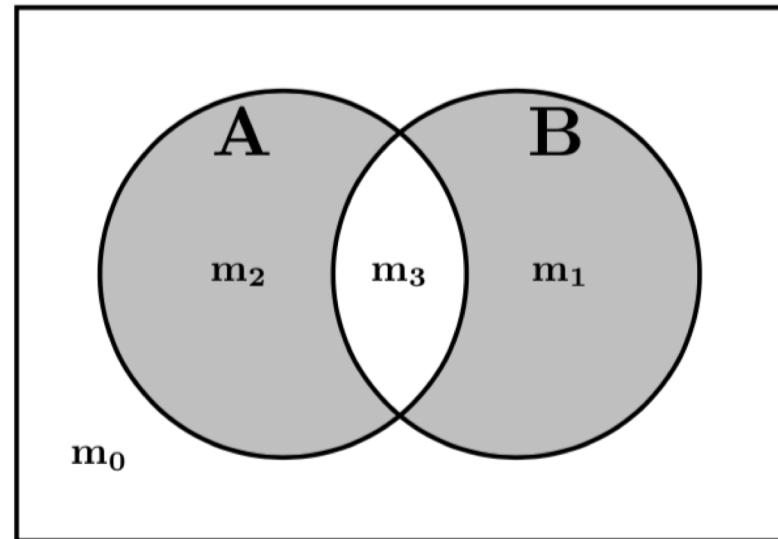
# Color the Venn diagram for XOR

A	B	F
0	0	0
0	1	1
1	0	1
1	1	0



# Color the Venn diagram for XOR

	<b>A</b>	<b>B</b>	<b>F</b>
$m_0$	0	0	0
$m_1$	0	1	1
$m_2$	1	0	1
$m_3$	1	1	0

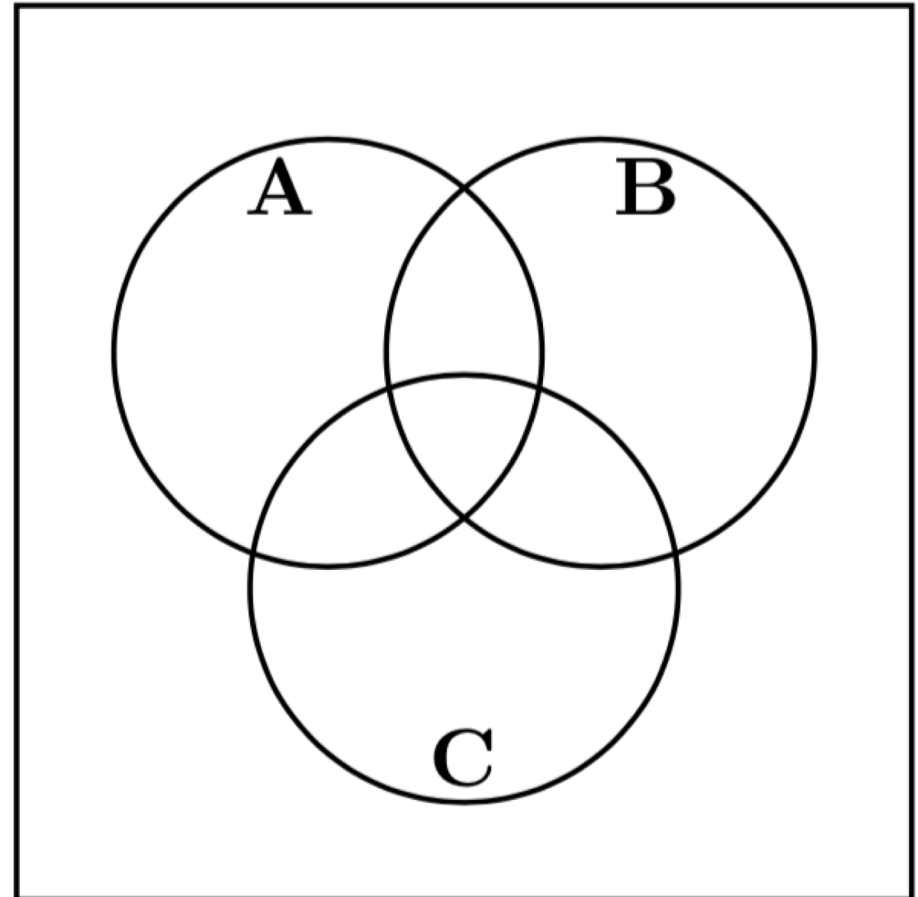


$$F = \bar{A}B + A\bar{B}$$



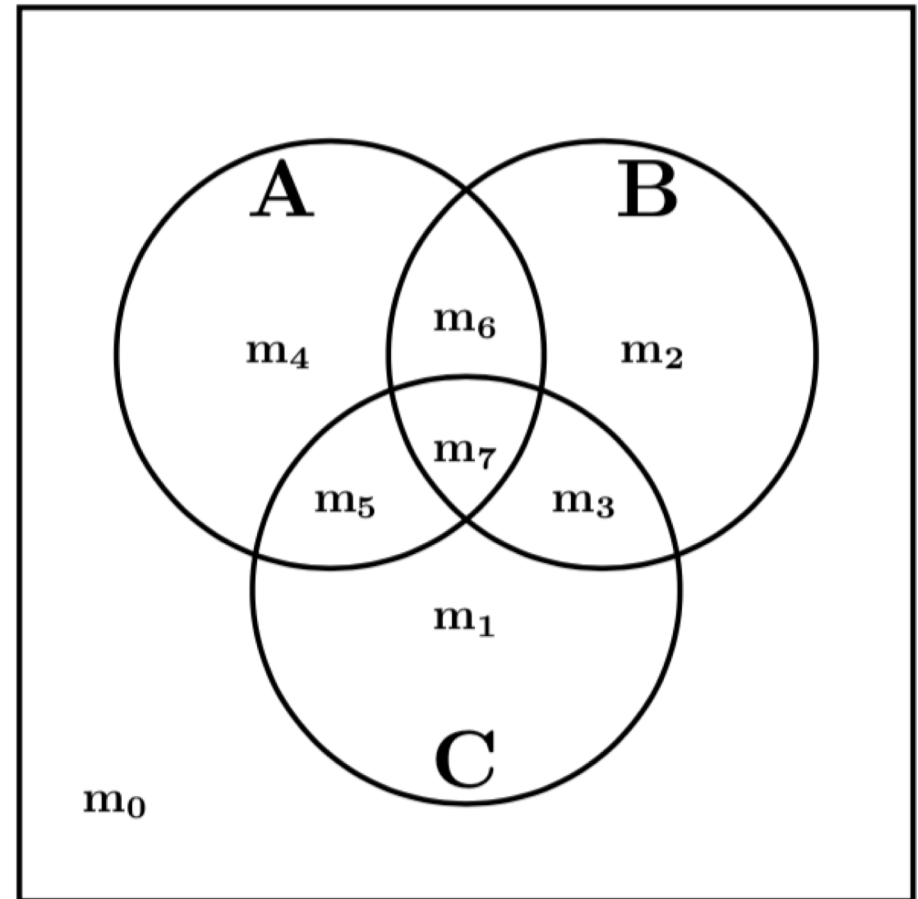
# Place the minterms on the Venn diagram

A	B	C	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$



# Place the minterms on the Venn diagram

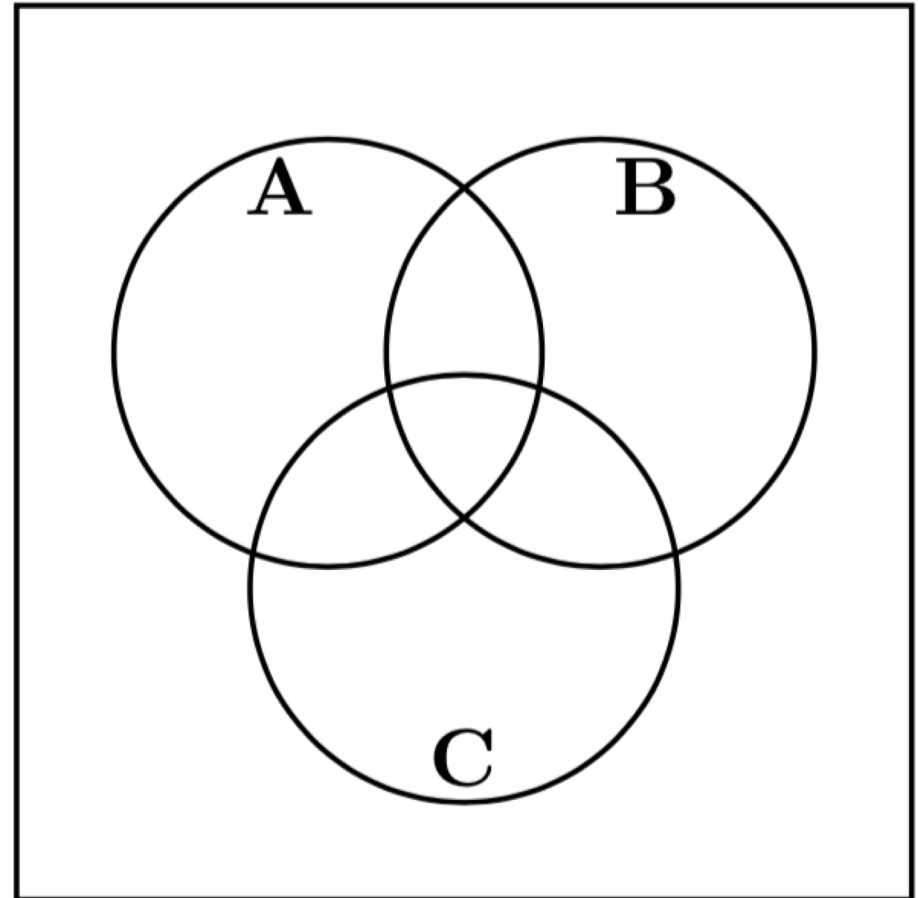
A	B	C	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$





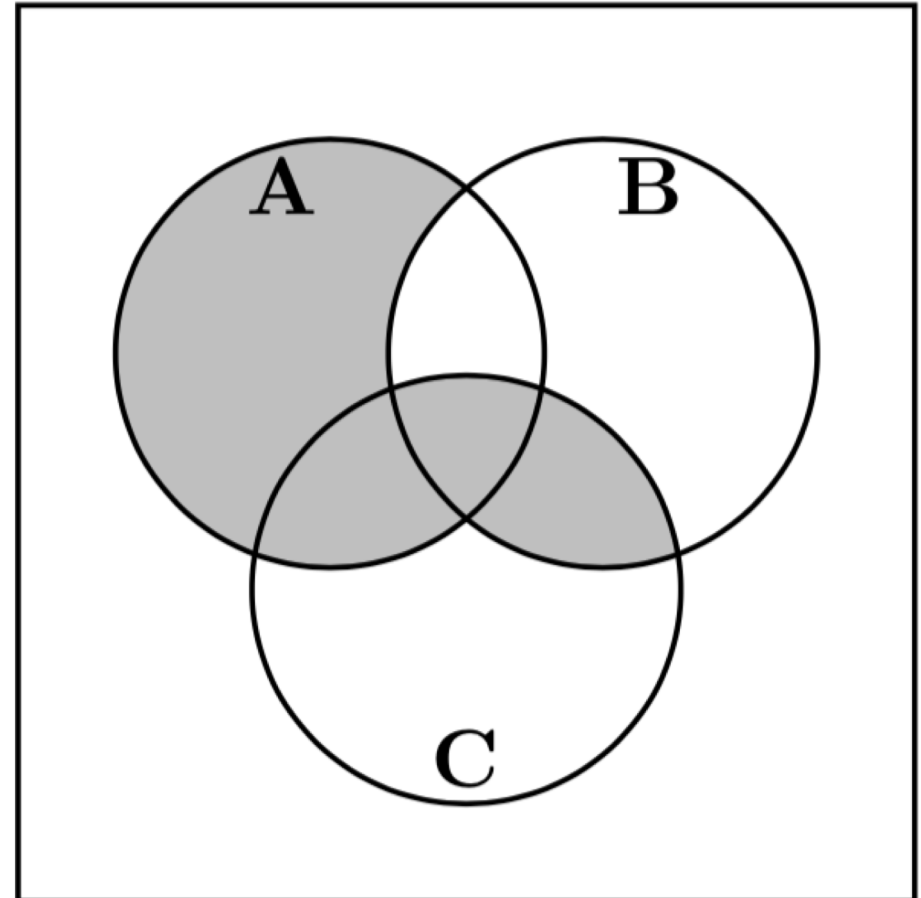
# Color the Venn diagram for this function

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1



# Color the Venn diagram for this function

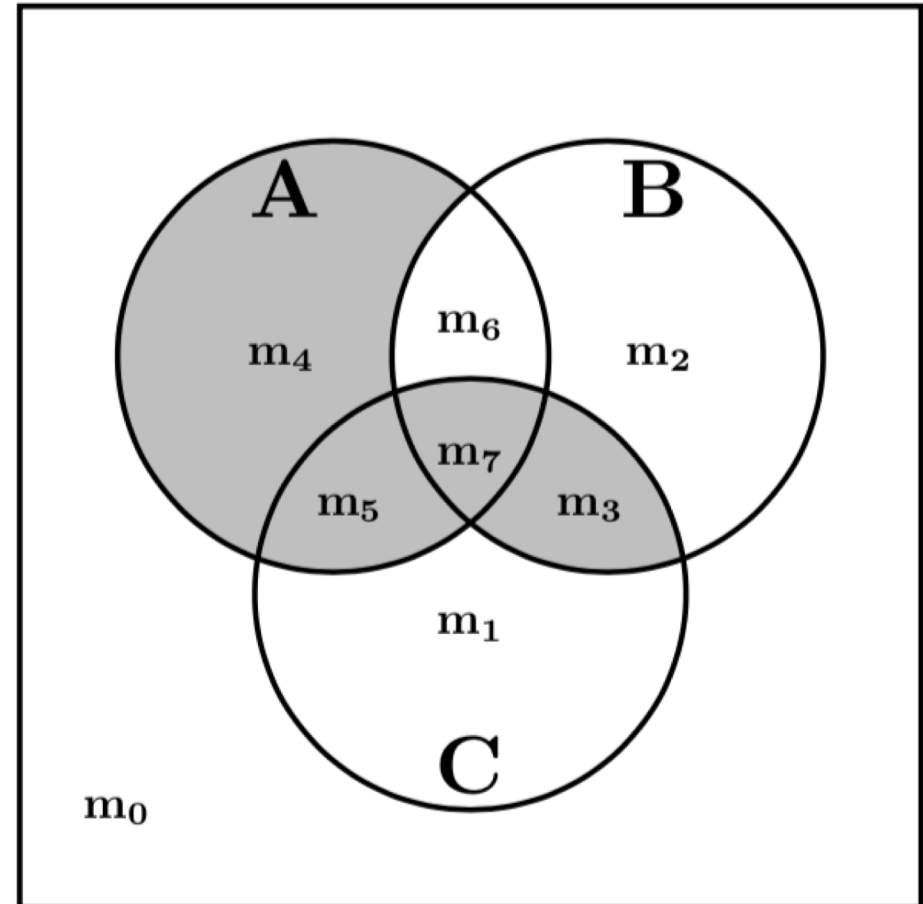
A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1





# Place the minterms on the Venn diagram

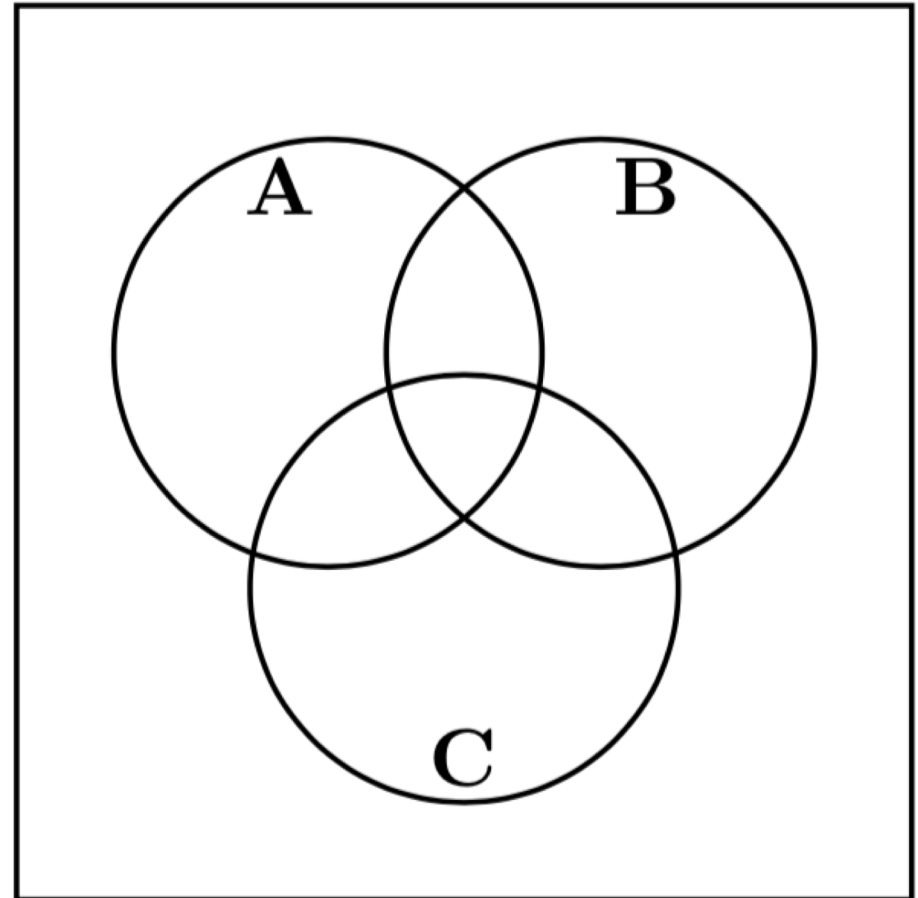
	<b>A</b>	<b>B</b>	<b>C</b>	<b>F</b>
$m_0$	0	0	0	0
$m_1$	0	0	1	0
$m_2$	0	1	0	0
$m_3$	0	1	1	1
$m_4$	1	0	0	1
$m_5$	1	0	1	1
$m_6$	1	1	0	0
$m_7$	1	1	1	1





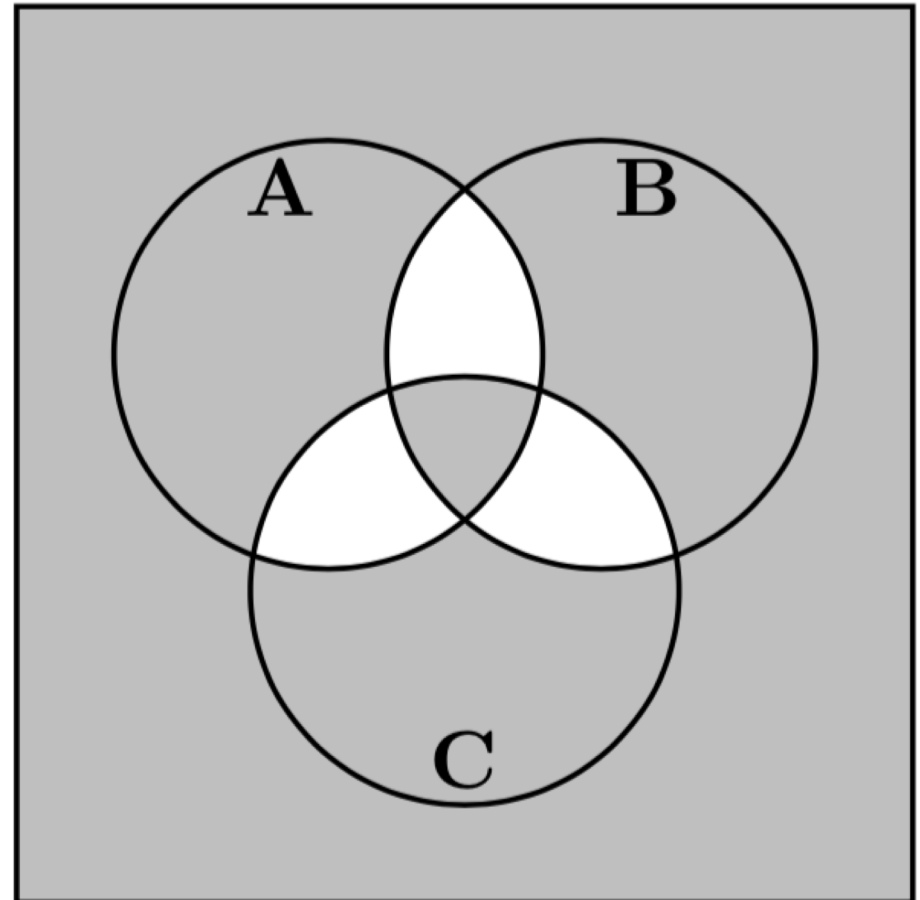
# Color the Venn diagram for this function

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1



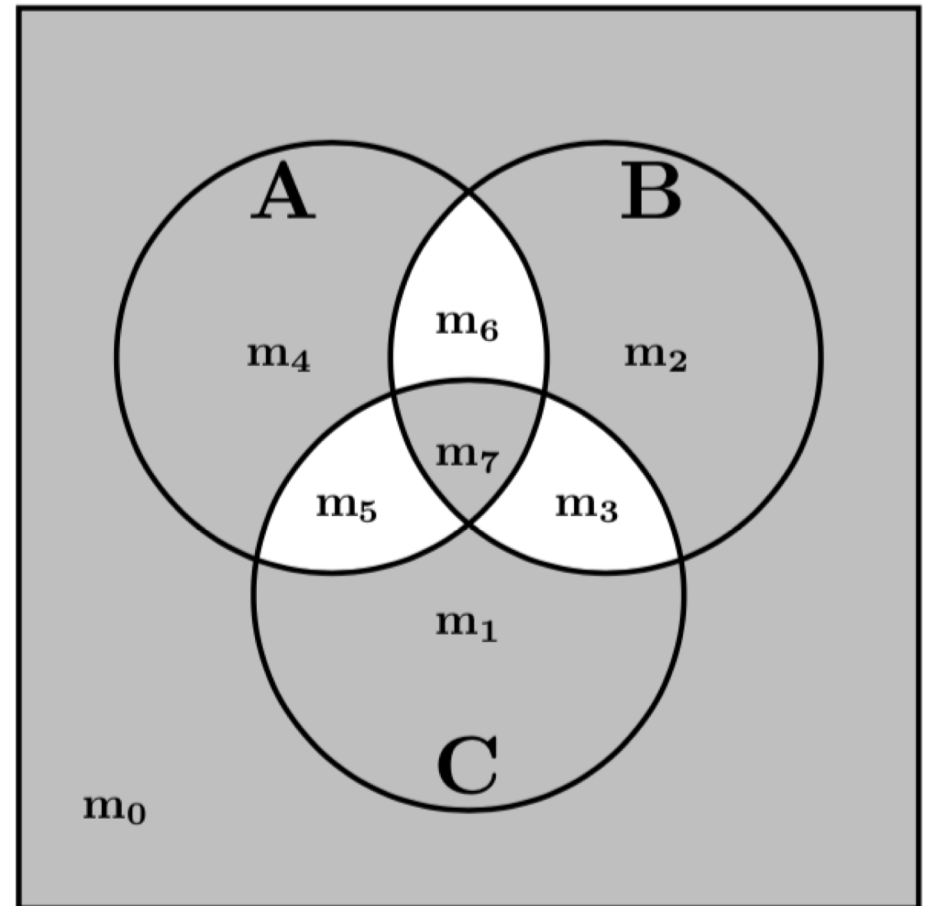
# Color the Venn diagram for this function

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1



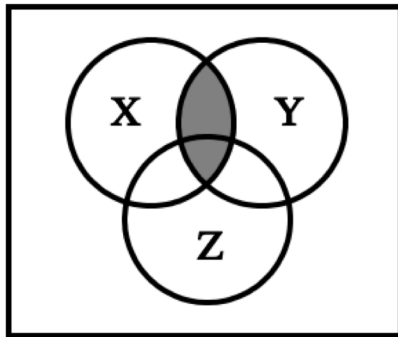
# Place the minterms on the Venn diagram

	<b>A</b>	<b>B</b>	<b>C</b>	<b>F</b>
$m_0$	0	0	0	1
$m_1$	0	0	1	1
$m_2$	0	1	0	1
$m_3$	0	1	1	0
$m_4$	1	0	0	1
$m_5$	1	0	1	0
$m_6$	1	1	0	0
$m_7$	1	1	1	1

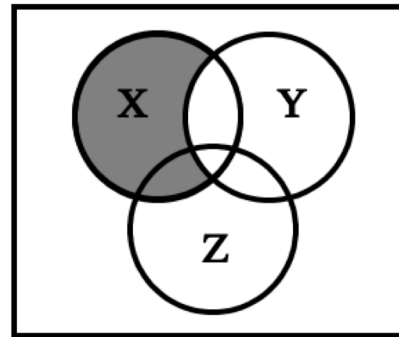




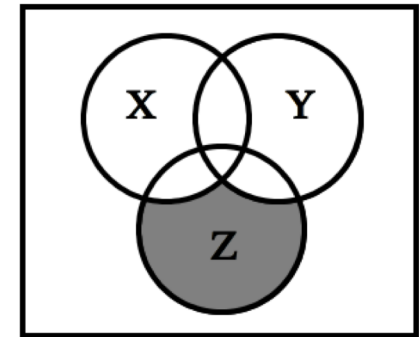
**Write the expression that is represented by each of the three Venn diagrams:**



**(A)**

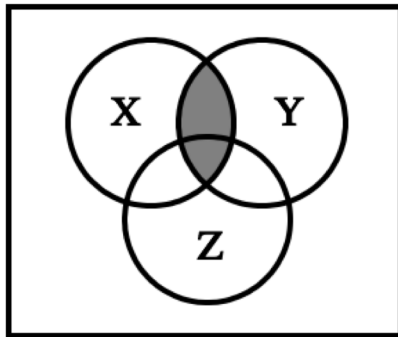


**(B)**



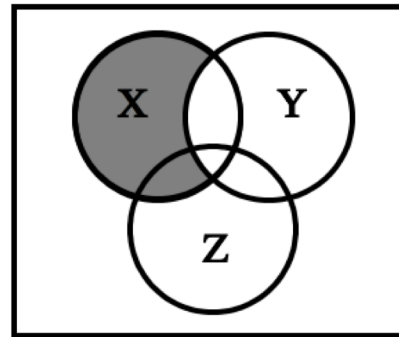
**(C)**

**Write the expression that is represented by each of the three Venn diagrams:**



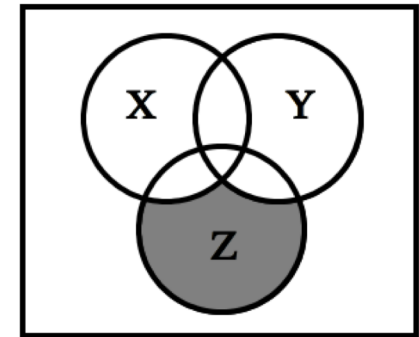
**(A)**

$$XY$$



**(B)**

$$X\bar{Y}$$



**(C)**

$$\bar{X}\bar{Y}Z$$



# Example 1

Determine if the following equation is valid

$$\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2 = \bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3$$

$$\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2 \stackrel{?}{=} \bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3$$

$$\underbrace{\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2}_{\text{LHS}} \stackrel{?}{=} \underbrace{\bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3}_{\text{RHS}}$$

LHS

RHS

# Left-Hand Side (LHS)

Row number	$x_1$	$x_2$	$x_3$	$\overline{x_1 x_3}$	$x_2 x_3$	$x_1 \overline{x_2}$	$f$
0	0	0	0				
1	0	0	1				
2	0	1	0				
3	0	1	1				
4	1	0	0				
5	1	0	1				
6	1	1	0				
7	1	1	1				

# Left-Hand Side (LHS)

Row number	$x_1$	$x_2$	$x_3$	$\overline{x_1 x_3}$	$x_2 x_3$	$x_1 \overline{x_2}$	$f$
0	0	0	0	1	0	0	
1	0	0	1	0	0	0	
2	0	1	0	1	0	0	
3	0	1	1	0	1	0	
4	1	0	0	0	0	1	
5	1	0	1	0	0	1	
6	1	1	0	0	0	0	
7	1	1	1	0	1	0	

# Left-Hand Side (LHS)

Row number	$x_1$	$x_2$	$x_3$	$\overline{x_1 x_3}$	$x_2 x_3$	$x_1 \overline{x_2}$	$f$
0	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
2	0	1	0	1	0	0	1
3	0	1	1	0	1	0	1
4	1	0	0	0	0	1	1
5	1	0	1	0	0	1	1
6	1	1	0	0	0	0	0
7	1	1	1	0	1	0	1

# Right-Hand Side (RHS)

Row number	$x_1$	$x_2$	$x_3$	$\overline{x_1 x_2}$	$x_1 x_3$	$\overline{x_2} \overline{x_3}$	$f$
0	0	0	0				
1	0	0	1				
2	0	1	0				
3	0	1	1				
4	1	0	0				
5	1	0	1				
6	1	1	0				
7	1	1	1				

# Right-Hand Side (RHS)

Row number	$x_1$	$x_2$	$x_3$	$\overline{x_1 x_2}$	$x_1 x_3$	$\overline{x_2} \overline{x_3}$	$f$
0	0	0	0	0	0	1	
1	0	0	1	0	0	0	
2	0	1	0	1	0	0	
3	0	1	1	1	0	0	
4	1	0	0	0	0	1	
5	1	0	1	0	1	0	
6	1	1	0	0	0	0	
7	1	1	1	0	1	0	



# Right-Hand Side (RHS)

Row number	$x_1$	$x_2$	$x_3$	$\overline{x_1 x_2}$	$x_1 x_3$	$\overline{x_2} \overline{x_3}$	$f$
0	0	0	0	0	0	1	1
1	0	0	1	0	0	0	0
2	0	1	0	1	0	0	1
3	0	1	1	1	0	0	1
4	1	0	0	0	0	1	1
5	1	0	1	0	1	0	1
6	1	1	0	0	0	0	0
7	1	1	1	0	1	0	1

$$\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2 \stackrel{?}{=} \bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3$$

LHS
RHS

$f$

---

1  
0  
1  
1  
1  
1  
1  
0  
1

$f$

---

1  
0  
1  
1  
1  
1  
1  
0  
1

They are equal.

## Example 2

**Design the minimum-cost product-of-sums expression for the function**

$$f(x_1, x_2, x_3) = \sum m(0, 2, 4, 5, 6, 7)$$

# Minterms and Maxterms (with three variables)

Row number	$x_1$	$x_2$	$x_3$	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

# Minterms and Maxterms (with three variables)

Row number	$x_1$	$x_2$	$x_3$	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

**The function is  
1 for these rows**

# Minterms and Maxterms (with three variables)

Row number	$x_1$	$x_2$	$x_3$	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

**The function is  
1 for these rows**

**The function is  
0 for these rows**

# Two different ways to specify the same function $f$ of three variables

$$f(x_1, x_2, x_3) = \sum m(0, 2, 4, 5, 6, 7)$$

$$f(x_1, x_2, x_3) = \prod M(1, 3)$$

# Two different ways to specify the same function $f$ of three variables

$$f(x_1, x_2, x_3) = \sum m(0, 2, 4, 5, 6, 7)$$

$$f(x_1, x_2, x_3) = \prod M(1, 3)$$

$$= M_1 \cdot M_3$$

$$= (x_1 + x_2 + \bar{x}_3) \cdot (x_1 + \bar{x}_2 + \bar{x}_3)$$



# The Minimum POS Expression

$$\begin{aligned}f(x_1, x_2, x_3) &= (x_1 + x_2 + \bar{x}_3) \cdot (x_1 + \bar{x}_2 + \bar{x}_3) \\ &= (x_1 + \bar{x}_3 + x_2) \cdot (x_1 + \bar{x}_3 + \bar{x}_2) \\ &= (x_1 + \bar{x}_3)\end{aligned}$$

**Hint: Use the following Boolean Algebra theorem**

$$**14b. \quad (x + y) \cdot (x + \bar{y}) = x**$$

# Alternative Solution Using K-Maps

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

		$x_1 x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map

# Alternative Solution Using K-Maps

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

$x_3$	$x_1 x_2$	00	01	11	10
0		$m_0$	$m_2$	$m_6$	$m_4$
1		$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map

# Alternative Solution Using K-Maps

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

		$x_1 x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map

# Alternative Solution Using K-Maps

		$x_1x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

# Alternative Solution Using K-Maps

		$x_1x_2$			
		00	01	11	10
$x_3$	0	1	1	1	1
	1	0	0	1	1

# Alternative Solution Using K-Maps

$x_3 \backslash x_1 x_2$	00	01	11	10
0	1	1	1	1
1	0	0	1	1

$$(x_1 + \bar{x}_3)$$

# Example 3

**Problem:** A circuit that controls a given digital system has three inputs:  $x_1$ ,  $x_2$ , and  $x_3$ . It has to recognize three different conditions:

- Condition  $A$  is true if  $x_3$  is true and either  $x_1$  is true or  $x_2$  is false
- Condition  $B$  is true if  $x_1$  is true and either  $x_2$  or  $x_3$  is false
- Condition  $C$  is true if  $x_2$  is true and either  $x_1$  is true or  $x_3$  is false

The control circuit must produce an output of 1 if at least two of the conditions  $A$ ,  $B$ , and  $C$  are true. Design the simplest circuit that can be used for this purpose.



# Condition A

Condition  $A$  is true if  $x_3$  is true and either  $x_1$  is true or  $x_2$  is false

# Condition A

Condition  $A$  is true if  $x_3$  is true and either  $x_1$  is true or  $x_2$  is false

$$A = x_3(x_1 + \bar{x}_2) = x_3x_1 + x_3\bar{x}_2$$

# Condition B

Condition  $B$  is true if  $x_1$  is true and either  $x_2$  or  $x_3$  is false

# Condition B

Condition  $B$  is true if  $x_1$  is true and either  $x_2$  or  $x_3$  is false

$$B = x_1(\bar{x}_2 + \bar{x}_3) = x_1\bar{x}_2 + x_1\bar{x}_3$$

# Condition C

Condition  $C$  is true if  $x_2$  is true and either  $x_1$  is true or  $x_3$  is false

# Condition C

Condition  $C$  is true if  $x_2$  is true and either  $x_1$  is true or  $x_3$  is false

$$C = x_2(x_1 + \bar{x}_3) = x_2x_1 + x_2\bar{x}_3$$

The output of the circuit can be expressed as

$$f = \mathbf{AB} + AC + BC$$

$$AB = (x_3x_1 + x_3\bar{x}_2)(x_1\bar{x}_2 + x_1\bar{x}_3)$$

$$= x_3x_1x_1\bar{x}_2 + x_3x_1x_1\bar{x}_3 + x_3\bar{x}_2x_1\bar{x}_2 + x_3\bar{x}_2x_1\bar{x}_3$$

$$= x_3x_1\bar{x}_2 + 0 + x_3\bar{x}_2x_1 + 0$$

$$= x_1\bar{x}_2x_3$$

The output of the circuit can be expressed as

$$f = AB + AC + BC$$

$$AC = (x_3x_1 + x_3\bar{x}_2)(x_2x_1 + x_2\bar{x}_3)$$

$$= x_3x_1x_2x_1 + x_3x_1x_2\bar{x}_3 + x_3\bar{x}_2x_2x_1 + x_3\bar{x}_2x_2\bar{x}_3$$

$$= x_3x_1x_2 + 0 + 0 + 0$$

$$= x_1x_2x_3$$



The output of the circuit can be expressed as

$$f = AB + AC + BC$$

$$BC = (x_1\bar{x}_2 + x_1\bar{x}_3)(x_2x_1 + x_2\bar{x}_3)$$

$$= x_1\bar{x}_2x_2x_1 + x_1\bar{x}_2x_2\bar{x}_3 + x_1\bar{x}_3x_2x_1 + x_1\bar{x}_3x_2\bar{x}_3$$

$$= 0 + 0 + x_1\bar{x}_3x_2 + x_1\bar{x}_3x_2$$

$$= x_1x_2\bar{x}_3$$

**Finally, we get**

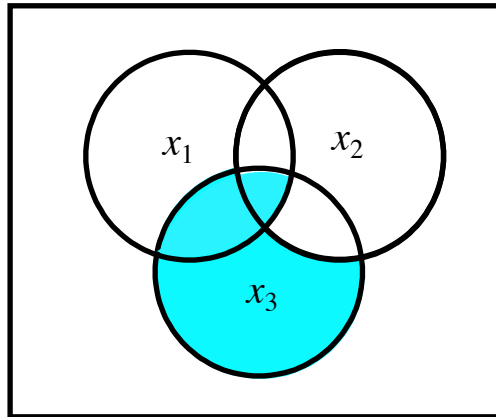
$$\begin{aligned} f &= x_1 \bar{x}_2 x_3 + x_1 x_2 x_3 + x_1 x_2 \bar{x}_3 \\ &= x_1 (\bar{x}_2 + x_2) x_3 + x_1 x_2 (x_3 + \bar{x}_3) \\ &= x_1 x_3 + x_1 x_2 \\ &= x_1 (x_3 + x_2) \end{aligned}$$

## **Example 4**

**Solve the previous problem using Venn diagrams.**

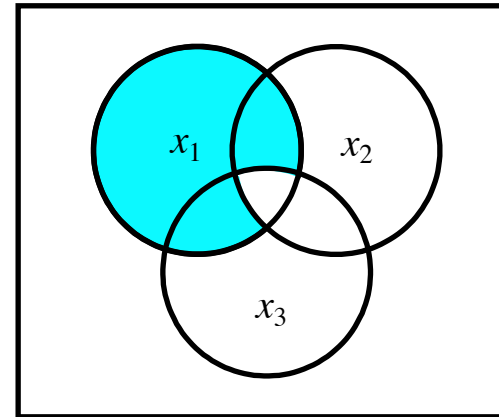
# Venn Diagrams

(find the areas that are shaded at least two times)



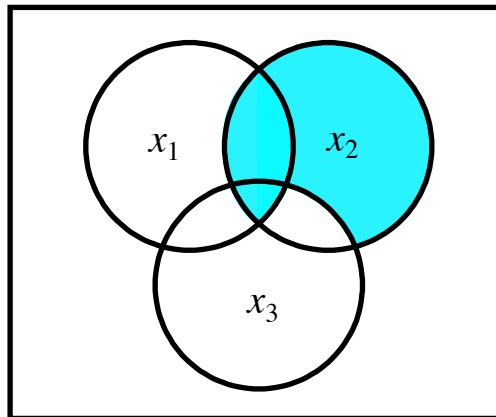
(a) Function A:

$$x_3x_1 + x_3\bar{x}_2$$



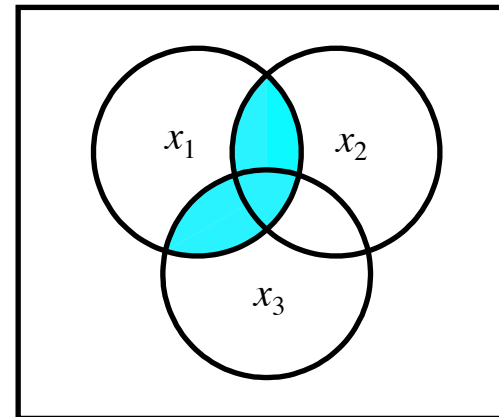
(b) Function B

$$x_1\bar{x}_2 + x_1\bar{x}_3$$



(c) Function C

$$x_2x_1 + x_2\bar{x}_3$$



(d) Function f

$$x_1(x_3 + x_2)$$

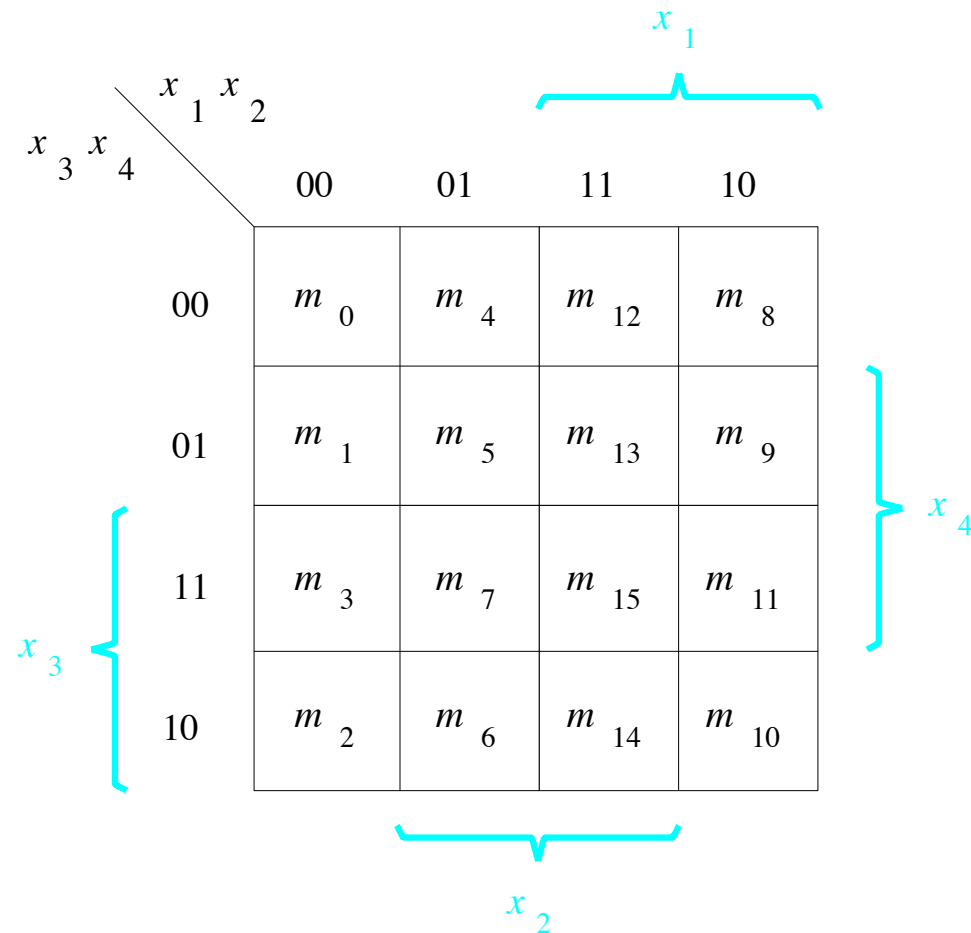
# Example 5

**Design the minimum-cost SOP and POS expression for the function**

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

# Let's Use a K-Map

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$



# Let's Use a K-Map

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00	0	1	1	1
01	0	d	0	d
11	d	d	1	1
10	0	1	0	1

The Karnaugh map is annotated with cyan brackets indicating groupings for variables  $x_1$ ,  $x_2$ , and  $x_4$ . A horizontal bracket above the columns 11 and 10 is labeled  $x_1$ . A vertical bracket to the right of the rows 01 and 11 is labeled  $x_4$ . A horizontal bracket below the columns 00 and 01 is labeled  $x_2$ . A vertical bracket to the left of the rows 11 and 10 is labeled  $x_3$ .

# Let's Use a K-Map

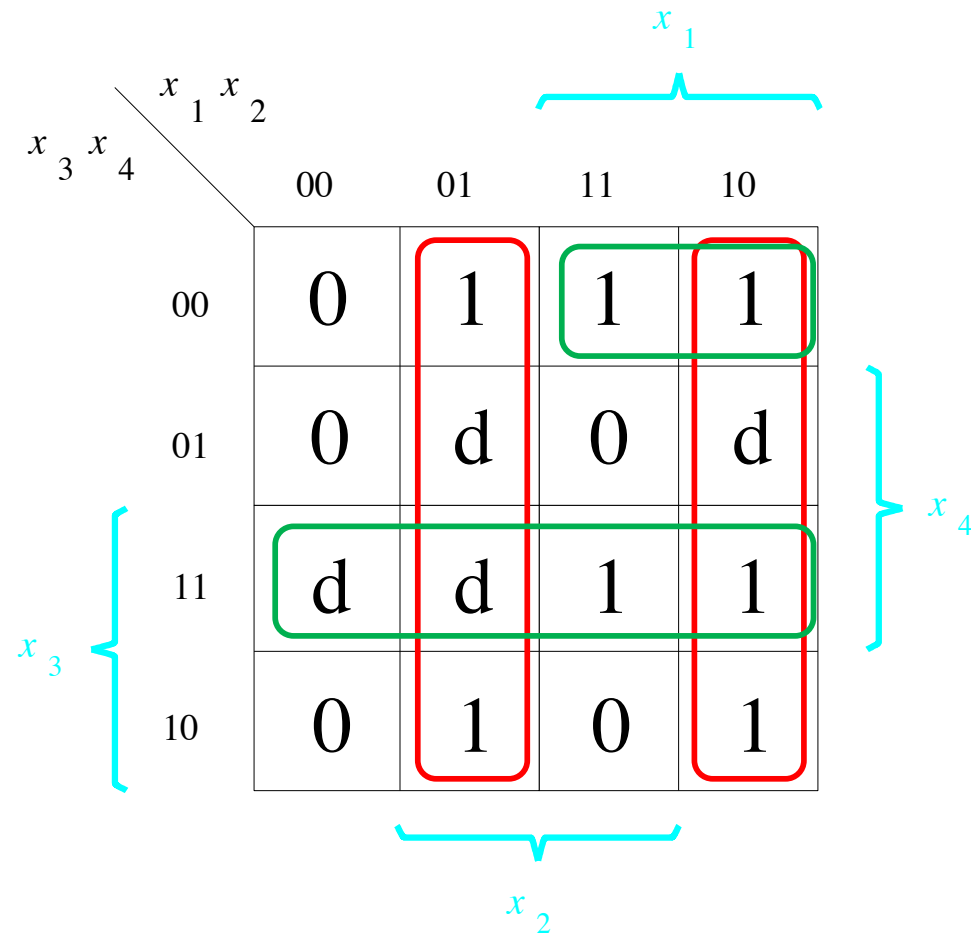
$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00	0	1	1	1
01	0	d	0	d
11	d	d	1	1
10	0	1	0	1



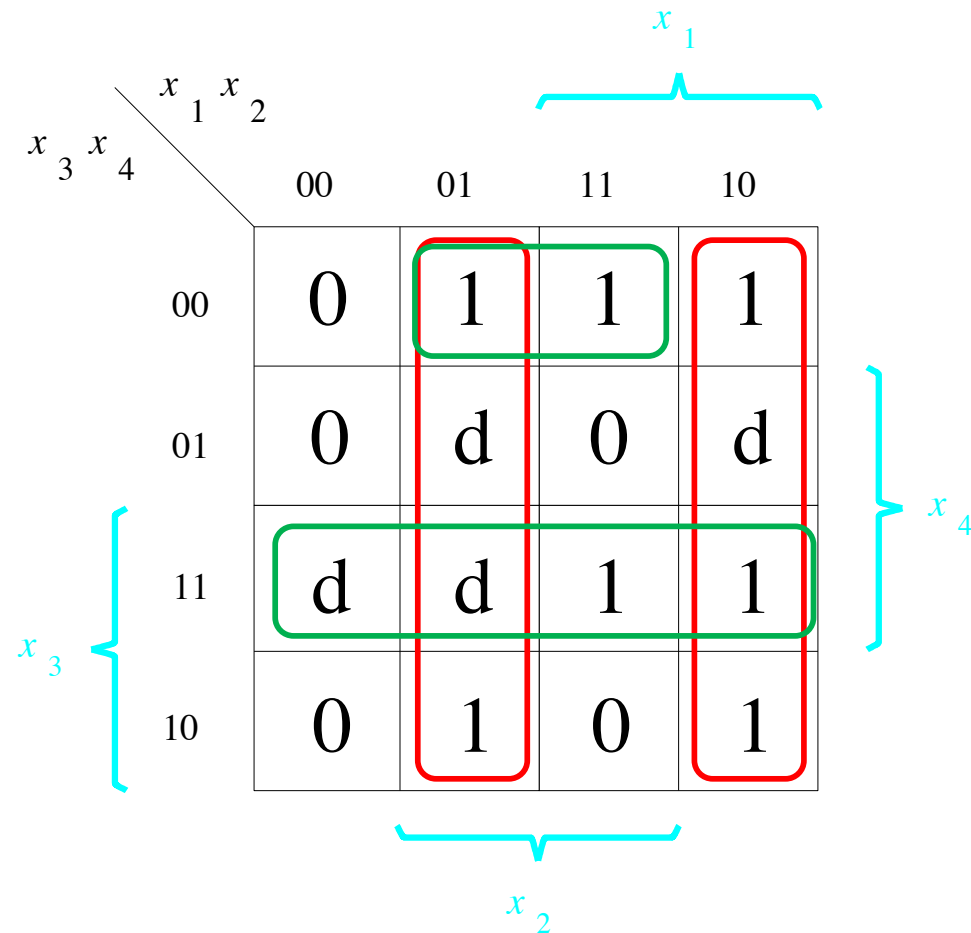
# Let's Use a K-Map

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$



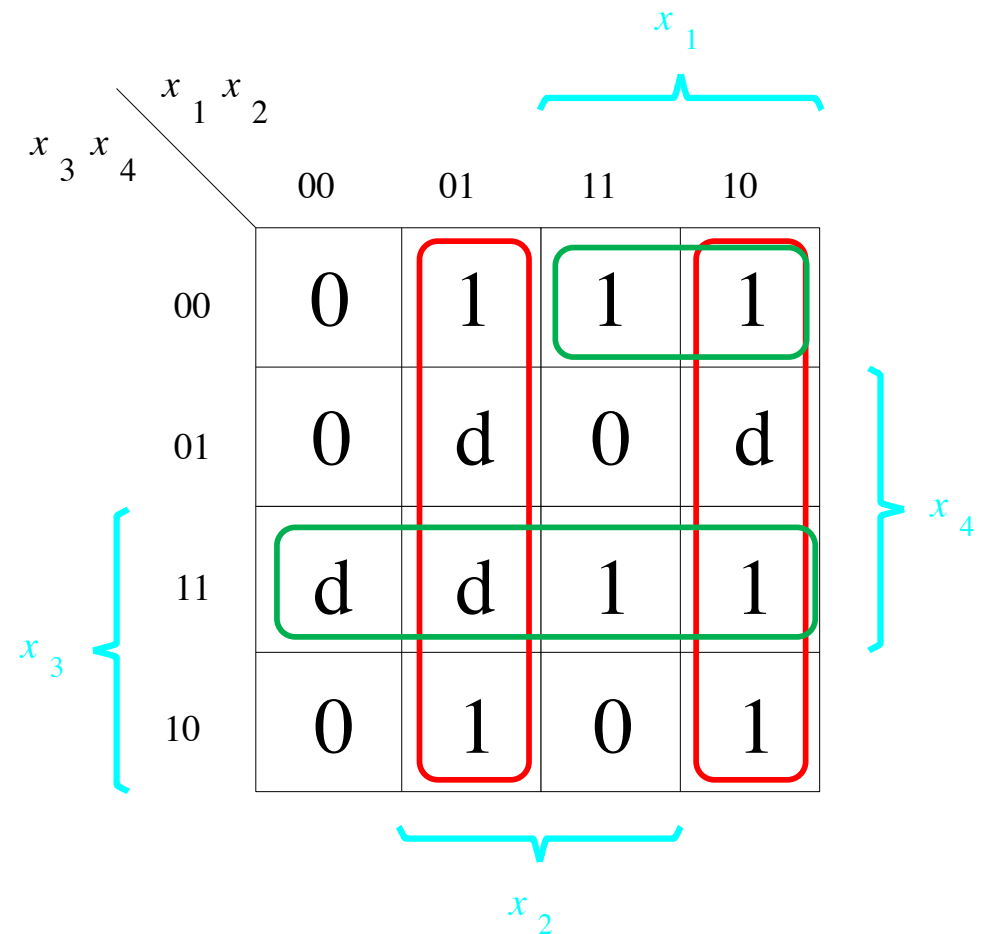
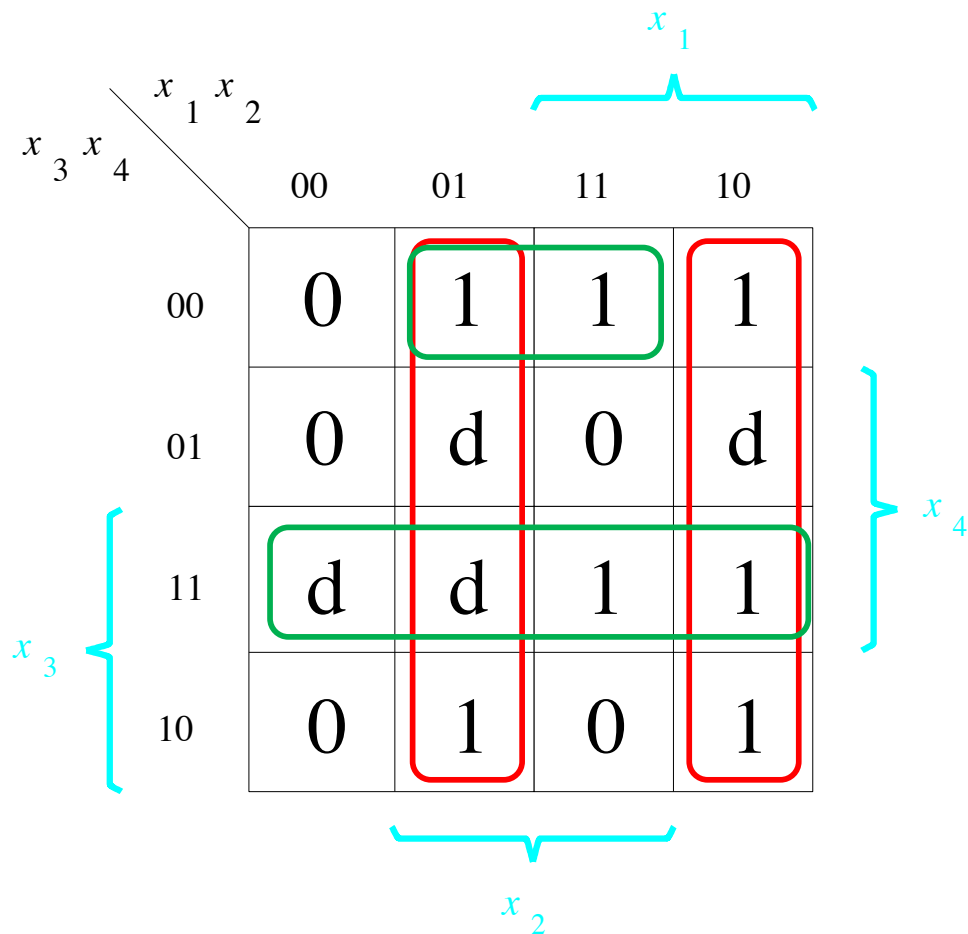
# Let's Use a K-Map

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

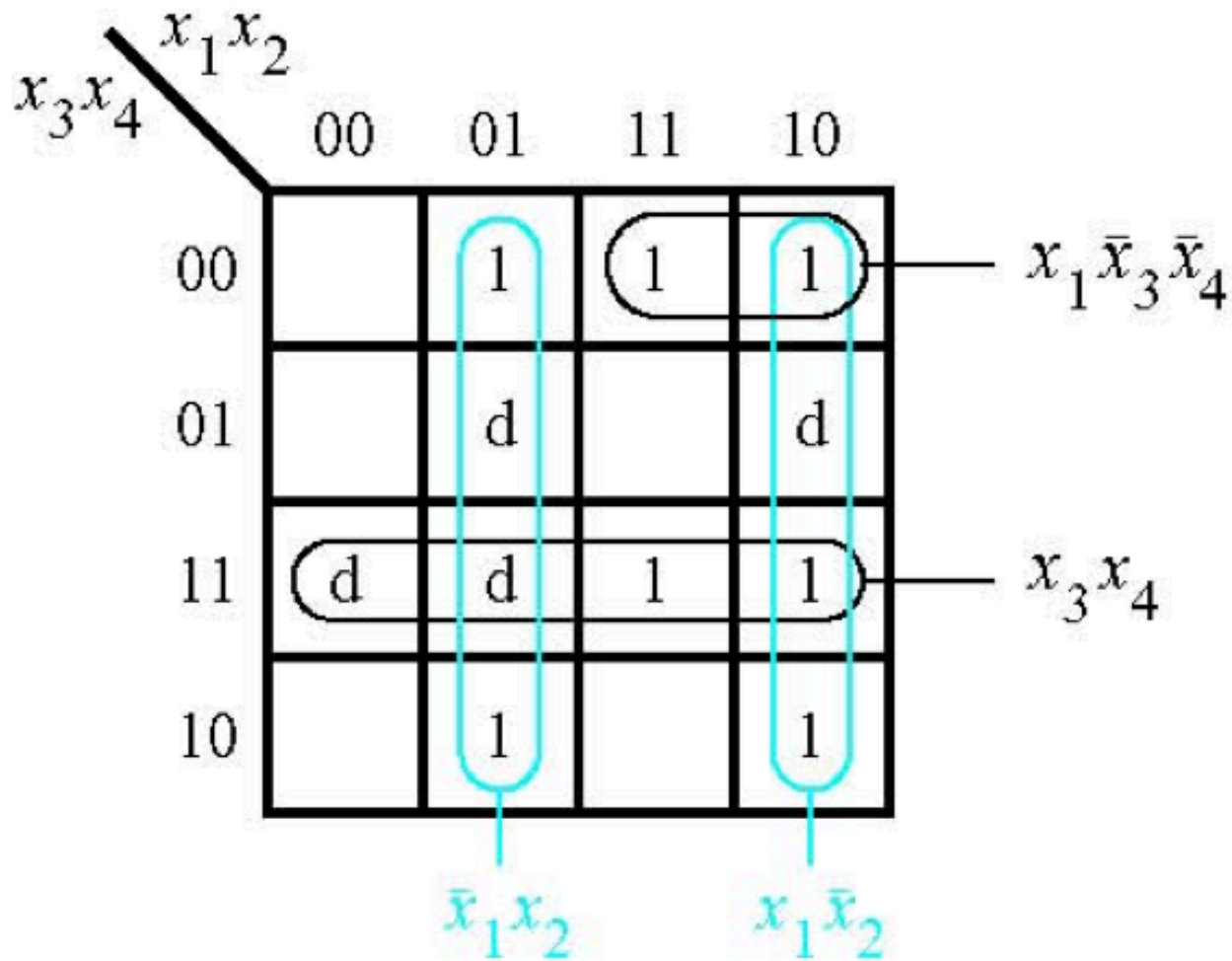


# Two Alternative Solutions

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$



# The SOP Expression



$$f = \bar{x}_1 x_2 + x_1 \bar{x}_2 + x_1 \bar{x}_3 \bar{x}_4 + x_3 x_4$$

# What about the POS Expression?

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

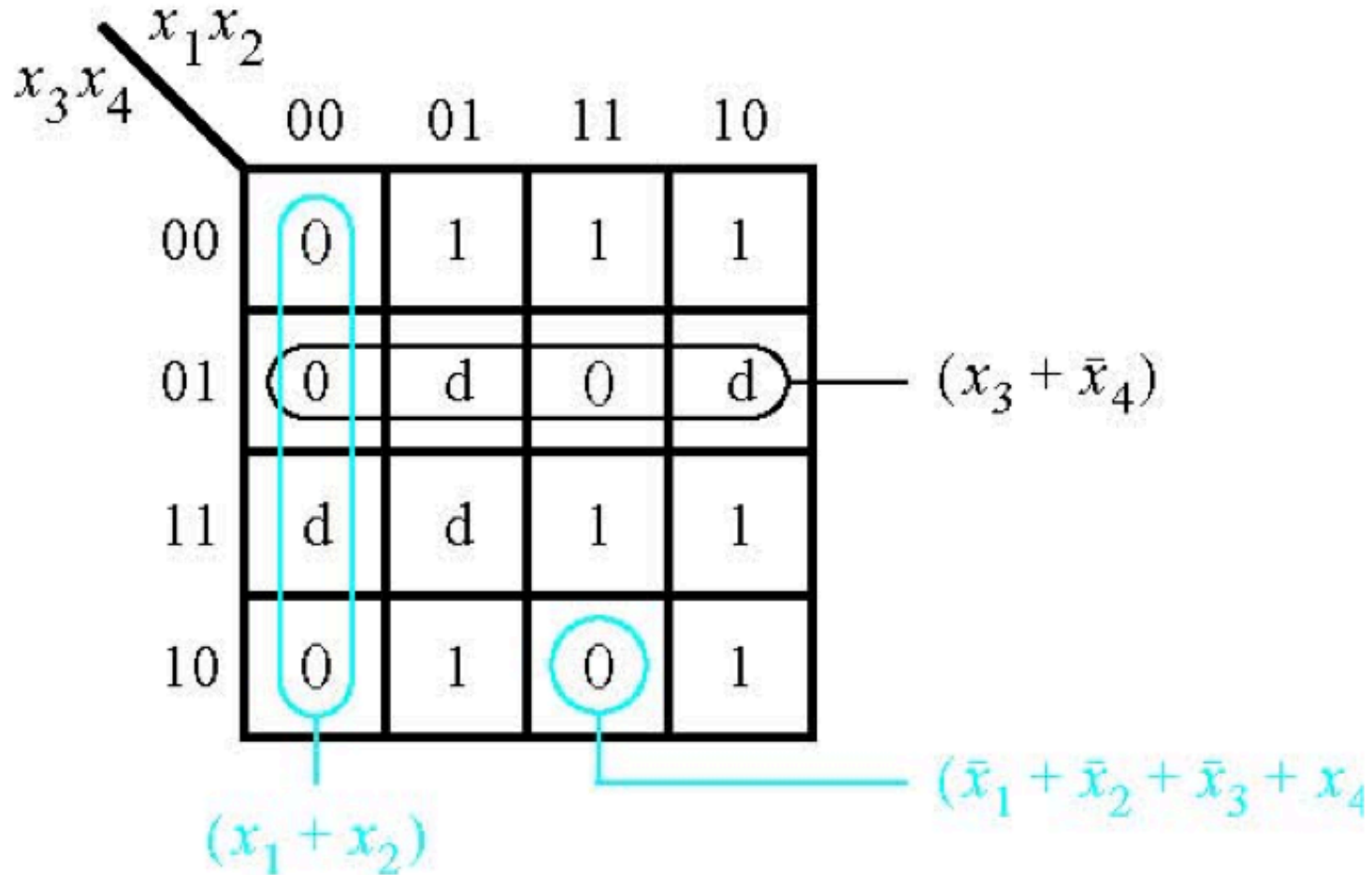
		$x_1$		$x_2$	
		00	01	11	10
$x_3$	00	0	1	1	1
	01	0	d	0	d
$x_3$	11	d	d	1	1
	10	0	1	0	1

# What about the POS Expression?

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

$x_3 \ x_4$		$x_1 \ x_2$					
		00	01	11	10	$x_1$	
00	00	0	1	1	1	$x_4$	
	01	0	d	0	d		
11	d	d	1	1			
10	0	1	0	1	$x_2$		

# The POS Expression



$$f = (x_1 + x_2) \cdot (x_3 + \bar{x}_4) \cdot (\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + x_4)$$

## Example 6

**Use K-maps to find the minimum-cost SOP and POS expression for the function**

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

assuming that there are also don't-cares defined as  $D = \sum(9, 12, 14)$ .



# Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

$$D = \sum(9, 12, 14).$$

$x_3 \ x_4$	$x_1 \ x_2$	00	01	11	10
00	$m_0$	$m_4$	$m_{12}$	$m_8$	
01	$m_1$	$m_5$	$m_{13}$	$m_9$	
11	$m_3$	$m_7$	$m_{15}$	$m_{11}$	
10	$m_2$	$m_6$	$m_{14}$	$m_{10}$	

# Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

$$D = \sum(9, 12, 14).$$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00	$m_0$	$m_4$	d	$m_8$
01	$m_1$	$m_5$	$m_{13}$	d
11	$m_3$	$m_7$	$m_{15}$	$m_{11}$
10	$m_2$	$m_6$	d	$m_{10}$

# Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00	$m_0$	$m_4$	d	$m_8$
01	$m_1$	$m_5$	$m_{13}$	d
11	$m_3$	$m_7$	$m_{15}$	$m_{11}$
10	$m_2$	$m_6$	d	$m_{10}$

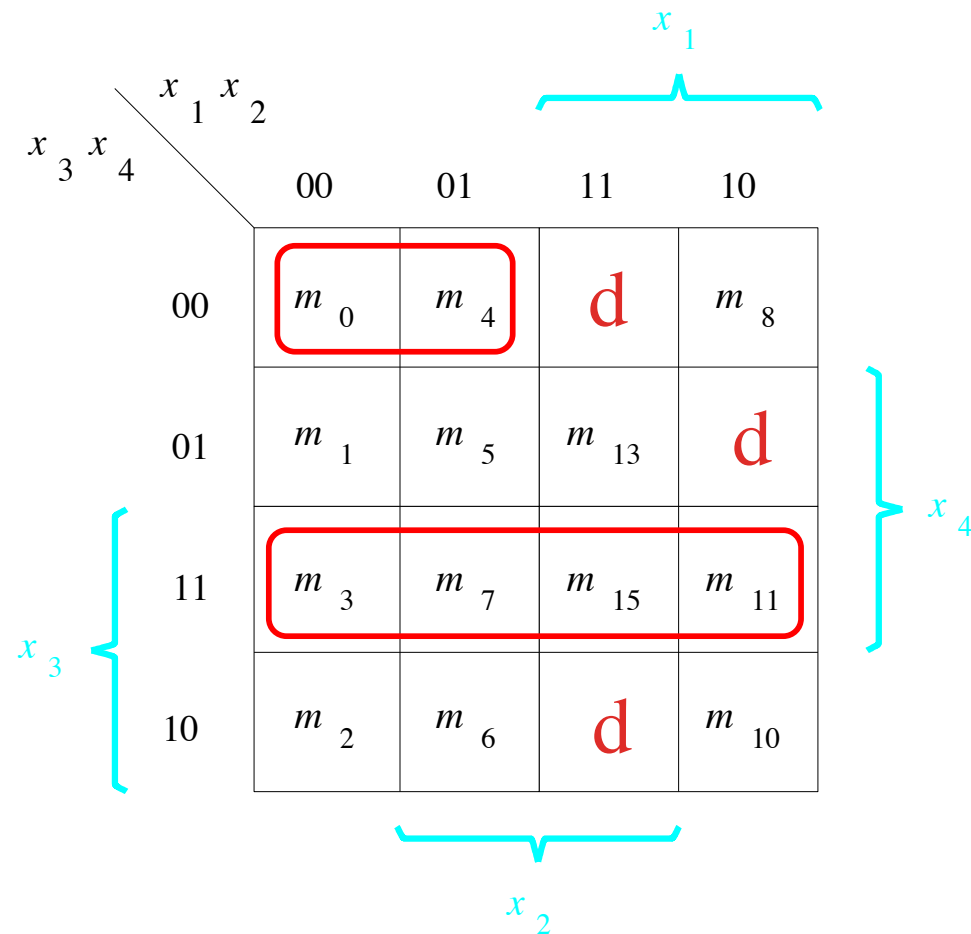
# Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00	$m_0$	$m_4$	d	$m_8$
01	$m_1$	$m_5$	$m_{13}$	d
11	$m_3$	$m_7$	$m_{15}$	$m_{11}$
10	$m_2$	$m_6$	d	$m_{10}$

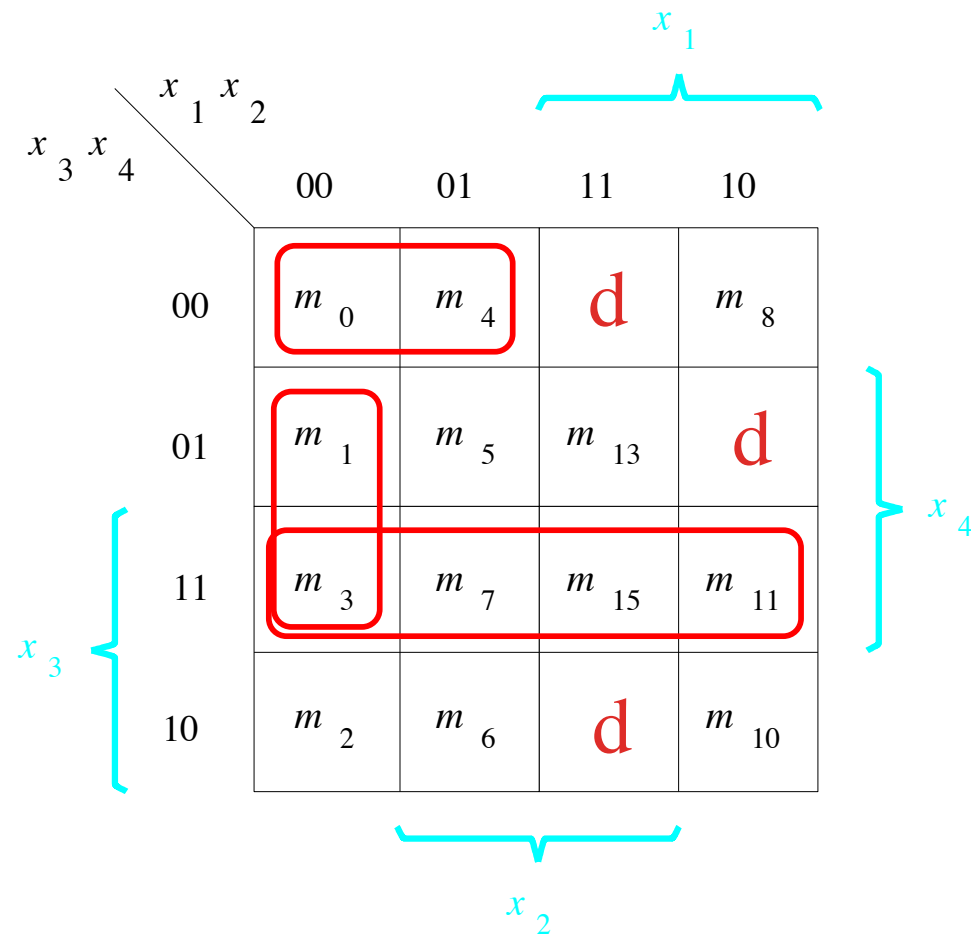
# Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$



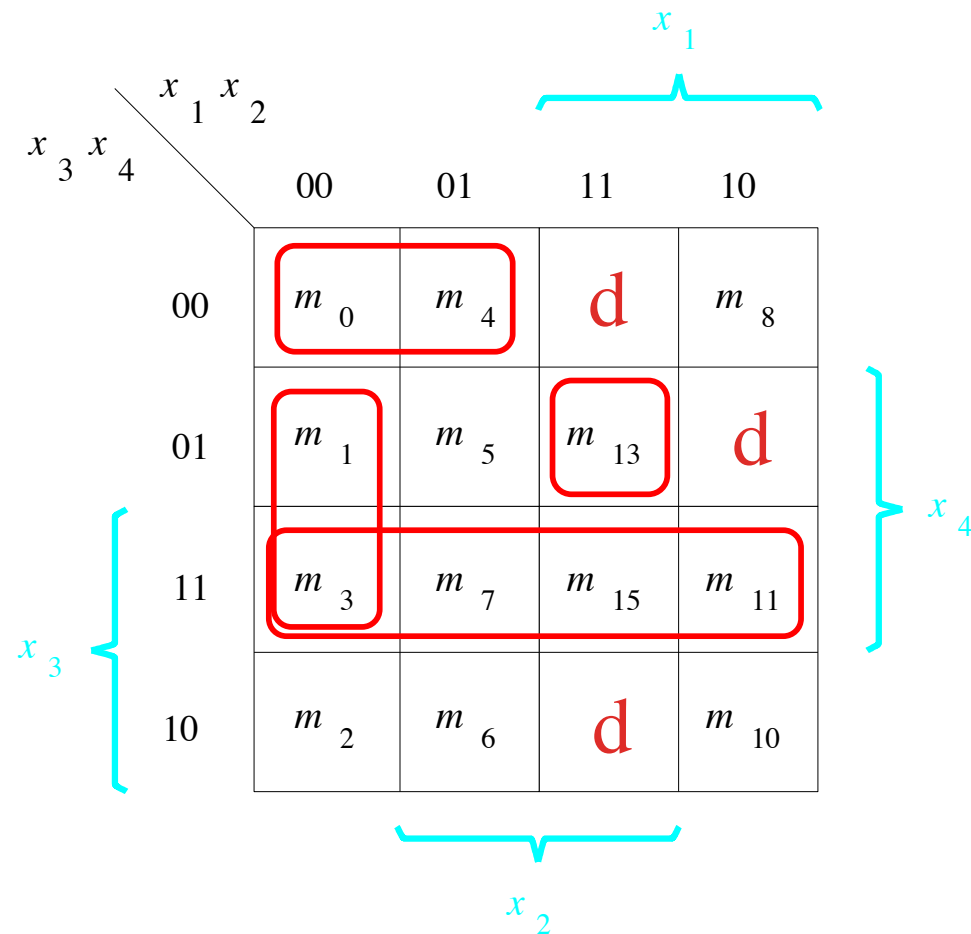
# Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \boxed{\bar{x}_1\bar{x}_2x_4} + x_1x_2\bar{x}_3x_4$$



# Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$



# Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00	1	1	d	0
01	1	0	1	d
11	1	1	1	1
10	0	0	d	0



# Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

A 4x4 Karnaugh map for the function  $f(x_1, x_2, x_3, x_4)$ . The columns are labeled  $x_1 x_2$  with values 00, 01, 11, 10. The rows are labeled  $x_3 x_4$  with values 00, 01, 11, 10. The map contains the following values:

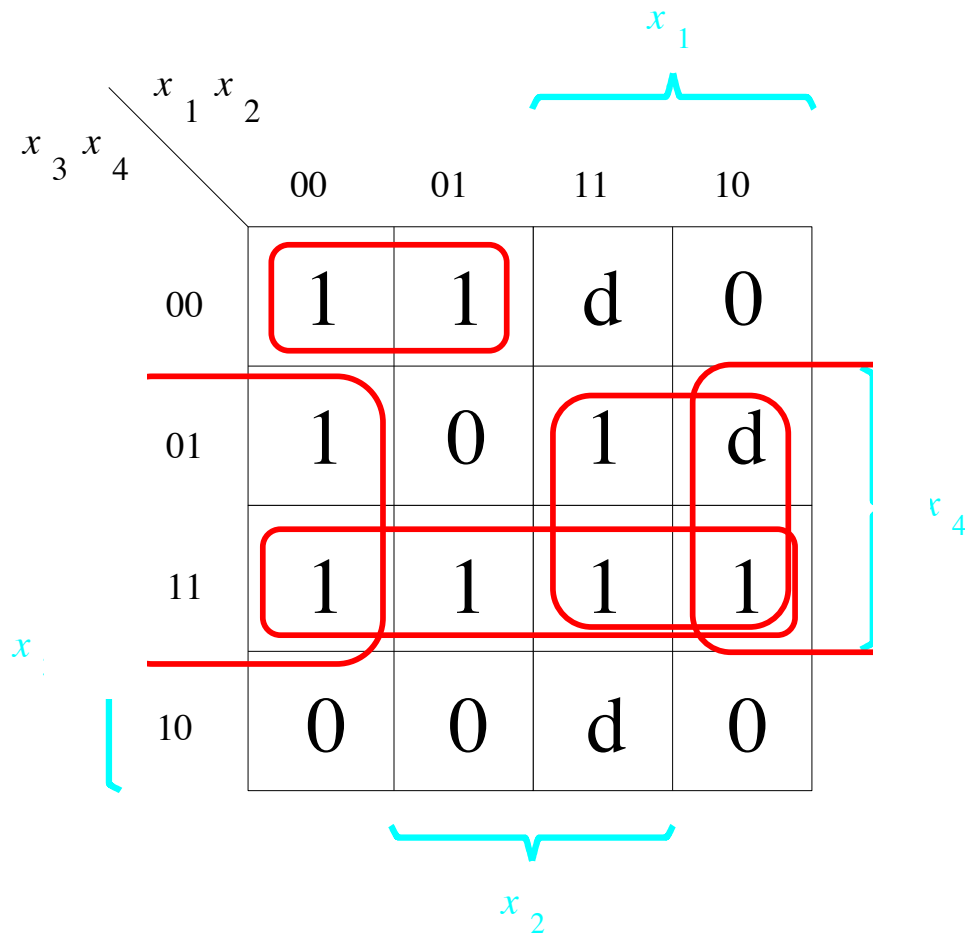
$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00	1	1	d	0
01	1	0	1	d
11	1	1	1	1
10	0	0	d	0

Annotations in cyan:

- A bracket above the columns labeled  $x_1$  spans the 11 and 10 columns.
- A bracket to the right of the rows labeled  $x_4$  spans the 01 and 11 rows.
- A bracket to the left of the rows labeled  $x_3$  spans the 11 and 10 rows.
- A bracket below the columns labeled  $x_2$  spans the 00 and 01 columns.

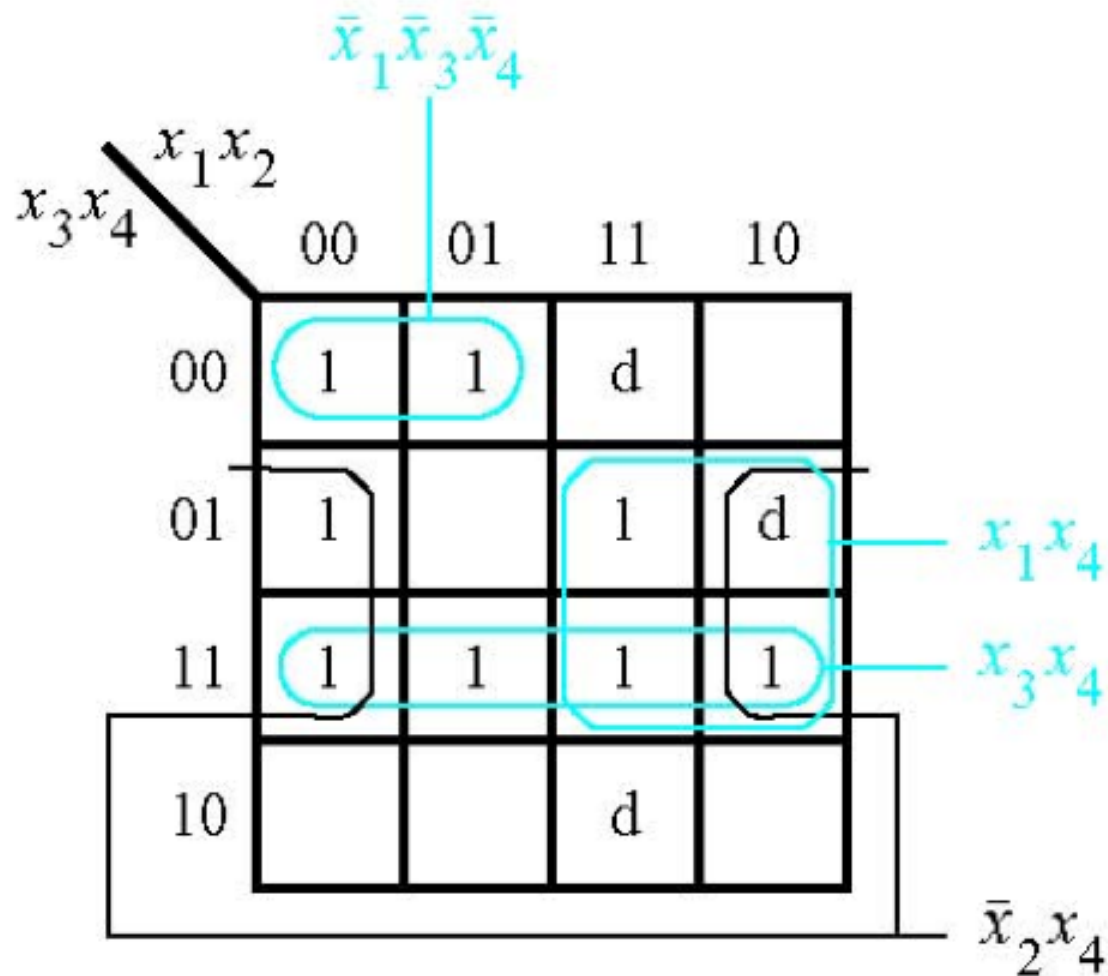
# Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$



# The SOP Expression

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$



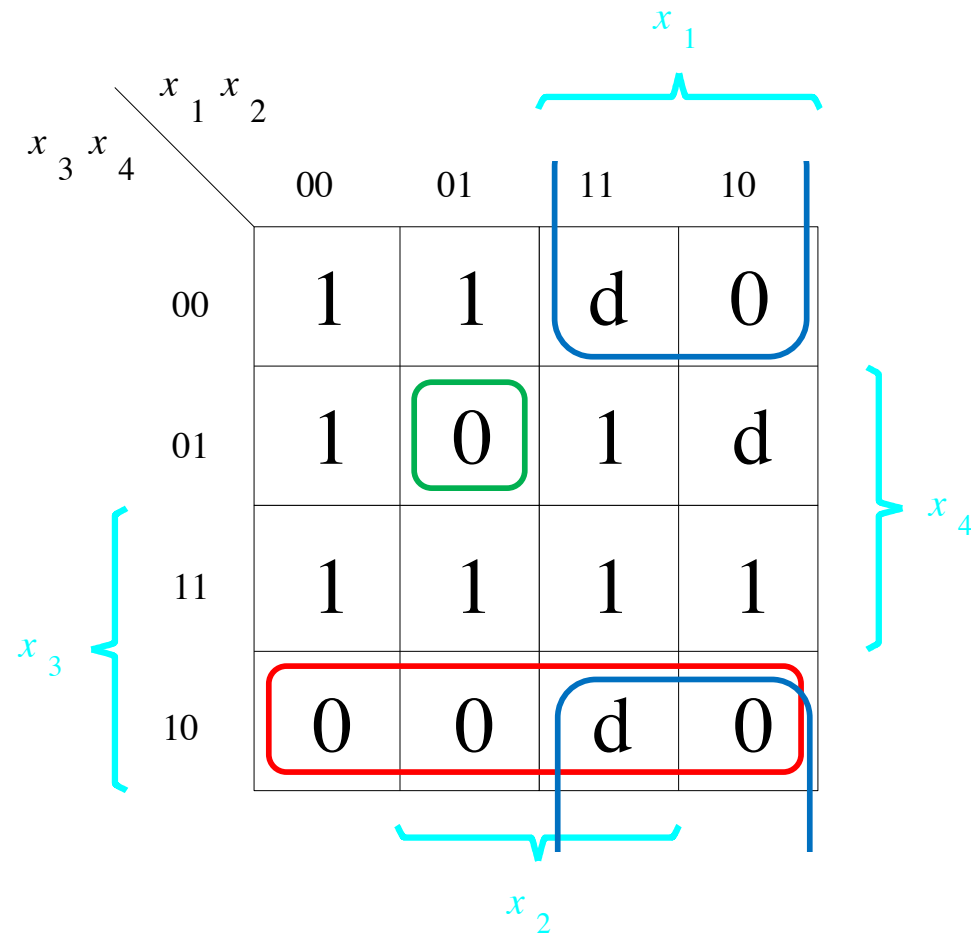
$$f = x_3x_4 + \bar{x}_1\bar{x}_3\bar{x}_4 + \bar{x}_2x_4 + x_1x_4$$

[ Figure 2.68a from the textbook ]

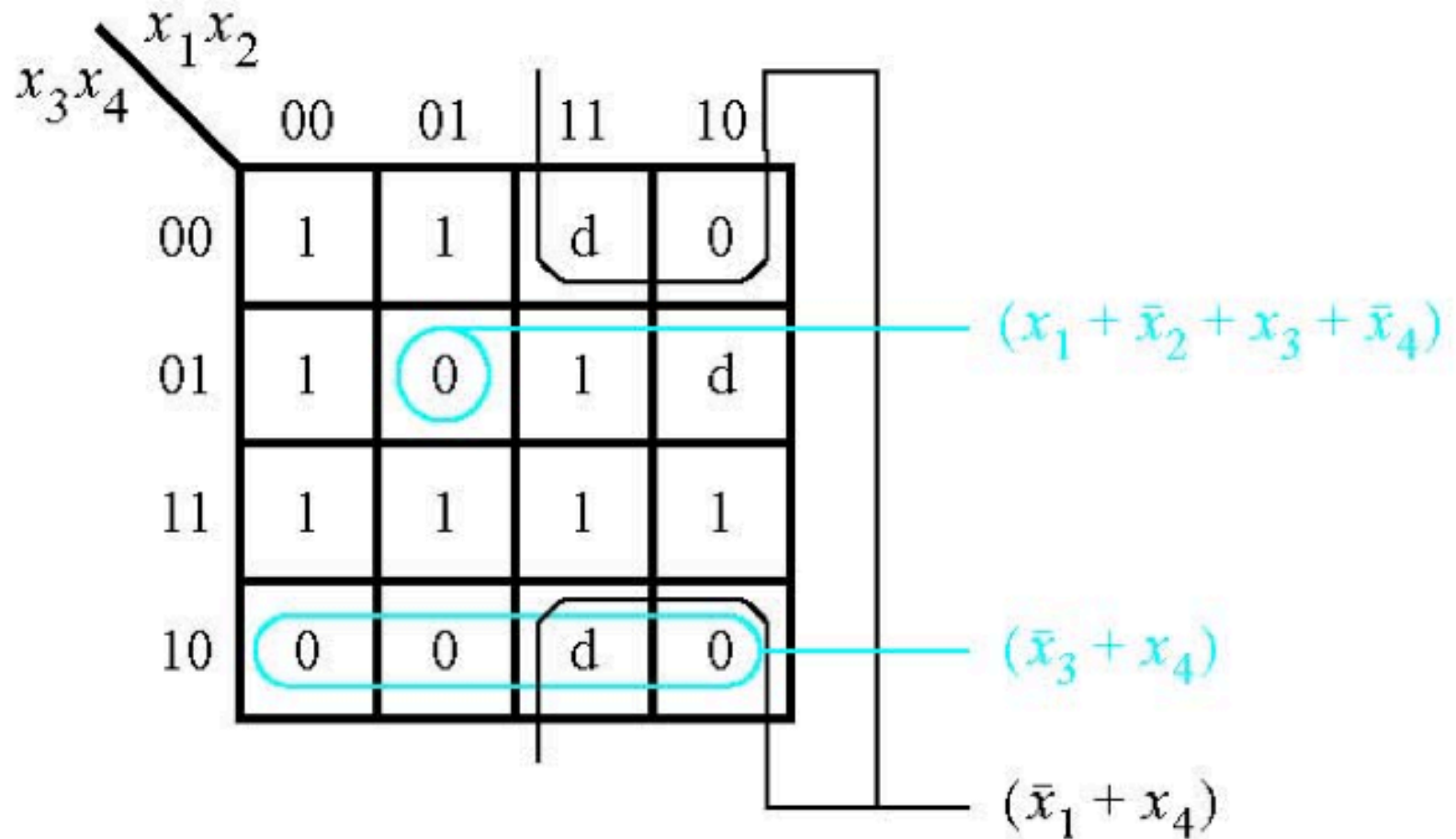
# What about the POS Expression?

		$x_1$		$x_2$	
		$x_3$	$x_4$	$x_3$	$x_4$
$x_3$	$x_4$	00	01	11	10
00	1	1	d	0	
01	1	0	1	d	
11	1	1	1	1	
10	0	0	d	0	

# What about the POS Expression?



# The POS Expression



$$f = (\bar{x}_3 + x_4)(\bar{x}_1 + x_4)(x_1 + \bar{x}_2 + x_3 + \bar{x}_4)$$

## Example 7

Derive the minimum-cost SOP expression for

$$f = s_3(\bar{s}_1 + \bar{s}_2) + s_1s_2$$

**First, expand the expression  
using property 12a**

$$f = s_3(\bar{s}_1 + \bar{s}_2) + s_1s_2$$

$$f = \bar{s}_1s_3 + \bar{s}_2s_3 + s_1s_2$$



# Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

$s_1$	$s_2$	$s_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

		$s_1 s_2$			
		00	01	11	10
$s_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map

# Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

$s_1$	$s_2$	$s_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

		$s_1 s_2$			
		00	01	11	10
$s_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map

# Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

$s_1$	$s_2$	$s_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

		$s_1 s_2$			
		00	01	11	10
$s_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map

# Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

$s_1$	$s_2$	$s_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

		$s_1 s_2$			
		00	01	11	10
$s_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map

# Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

$s_1$	$s_2$	$s_3$	
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(a) Truth table

		$s_1 s_2$			
		00	01	11	10
$s_3$	0	0	0	1	0
	1	1	1	1	1

(b) Karnaugh map

# Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

$s_1$	$s_2$	$s_3$	
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(a) Truth table

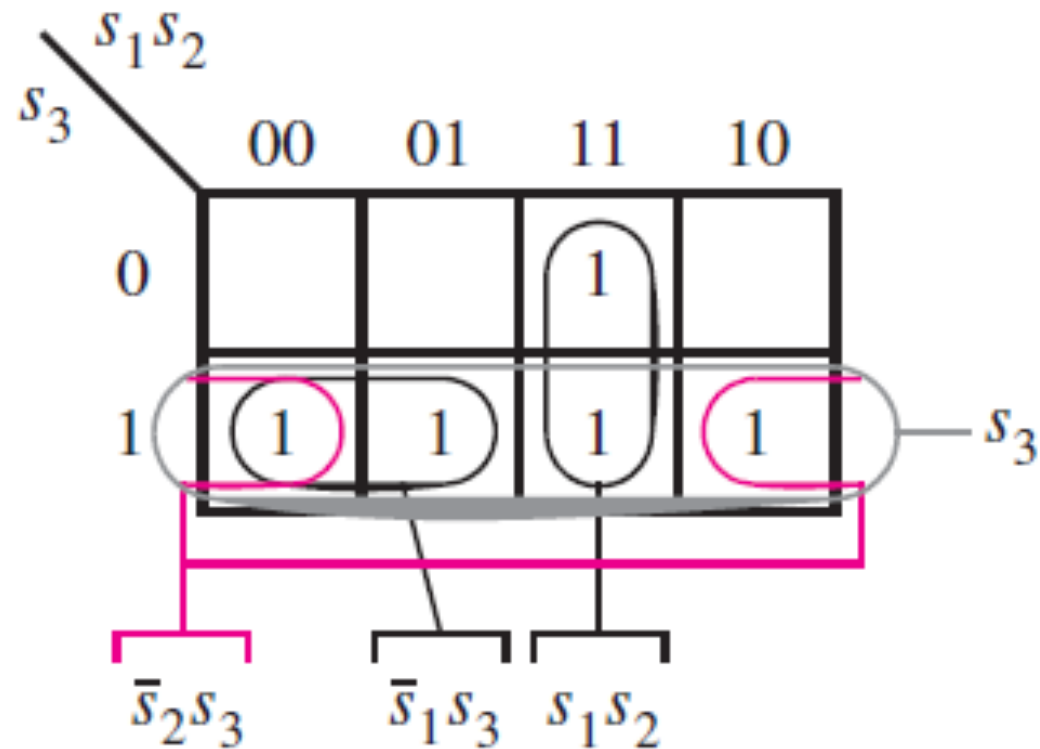
		$s_1 s_2$			
		00	01	11	10
$s_3$	0	0	0	1	0
	1	1	1	1	1

(b) Karnaugh map

**Simplified Expression:  $f = s_3 + s_1 s_2$**

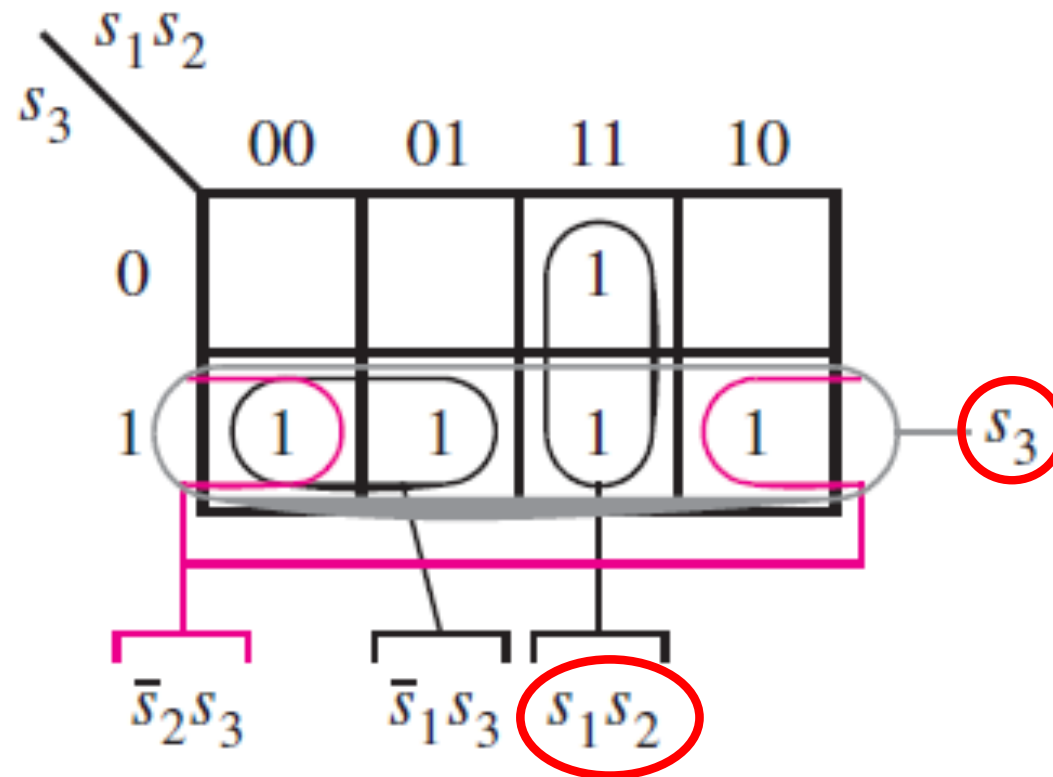
# Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$



# Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$



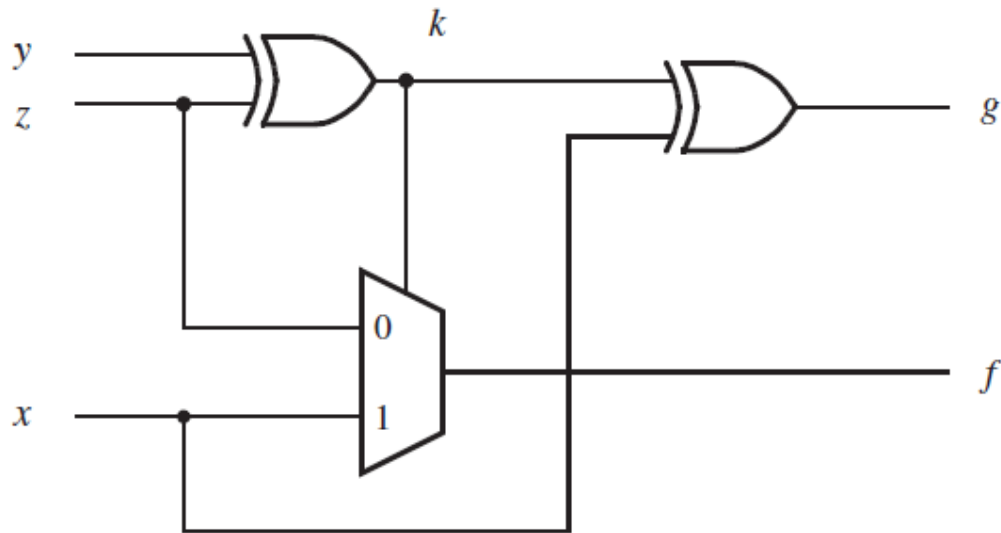
**Simplified Expression:  $f = s_3 + s_1 s_2$**



# Example 8

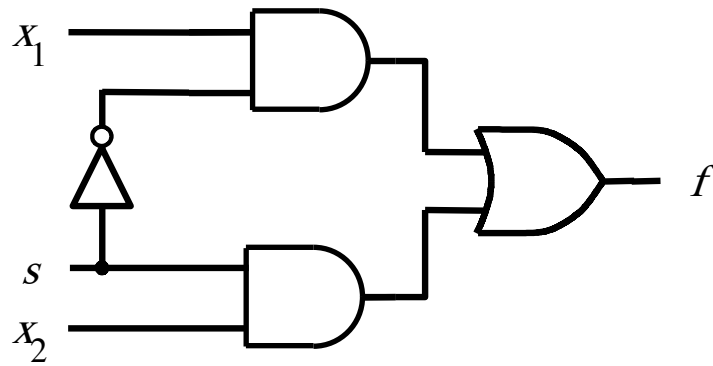
**Write the Verilog code for the following circuit ...**

# Logic Circuit

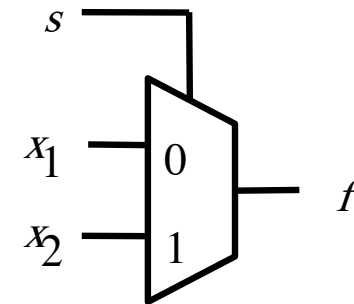


[ Figure 2.70 from the textbook ]

# Circuit for 2-1 Multiplexer



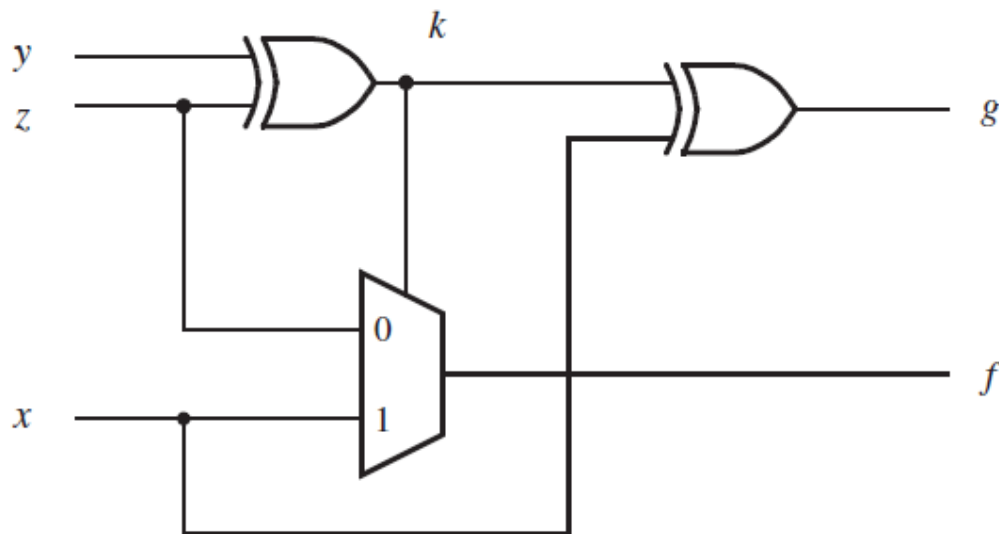
(b) Circuit



(c) Graphical symbol

$$f(s, x_1, x_2) = \bar{s} x_1 + s x_2$$

# Logic Circuit vs Verilog Code

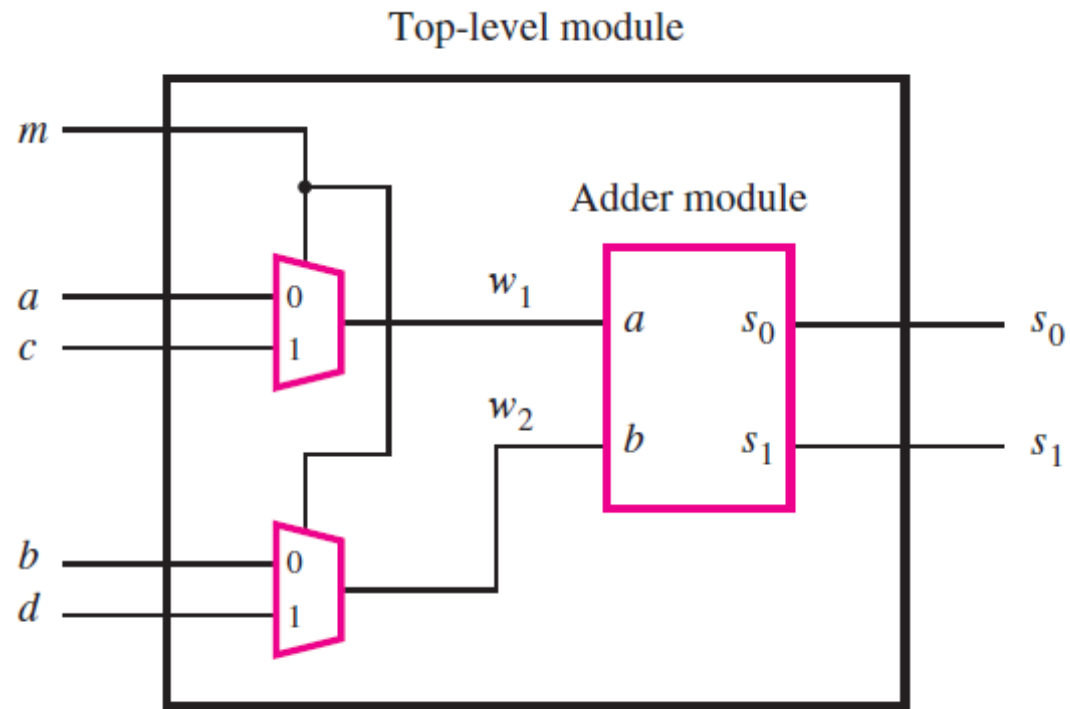


```
module f_g (x, y, z, f, g);  
  input x, y, z;  
  output f, g;  
  wire k;  
  
  assign k = y ^ z;  
  assign g = k ^ x;  
  assign f = (~k & z) | (k & x);  
  
endmodule
```

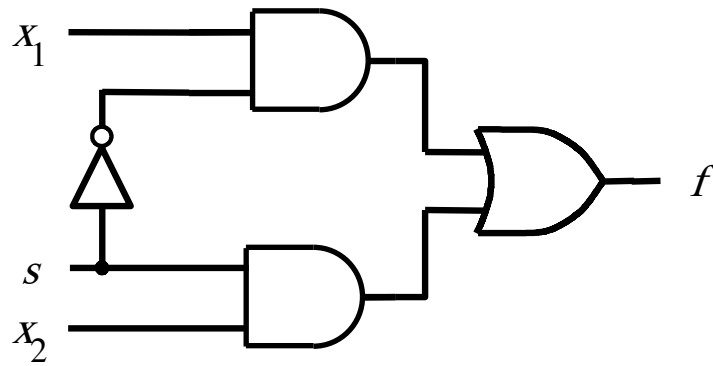
# Example 9

**Write the Verilog code for the following circuit ...**

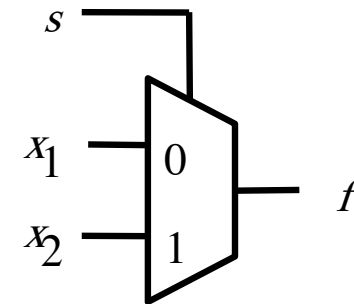
# The Logic Circuit for this Example



# Circuit for 2-1 Multiplexer



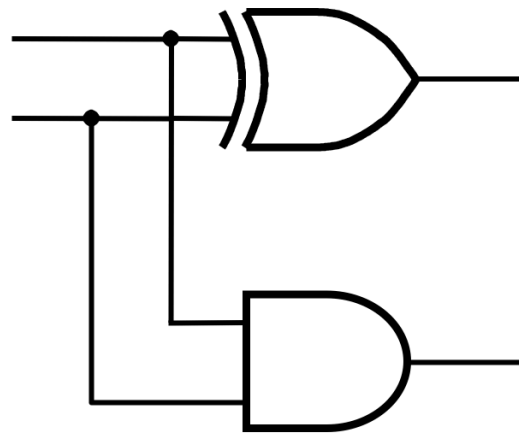
(b) Circuit



(c) Graphical symbol

$$f(s, x_1, x_2) = \bar{s} x_1 + s x_2$$

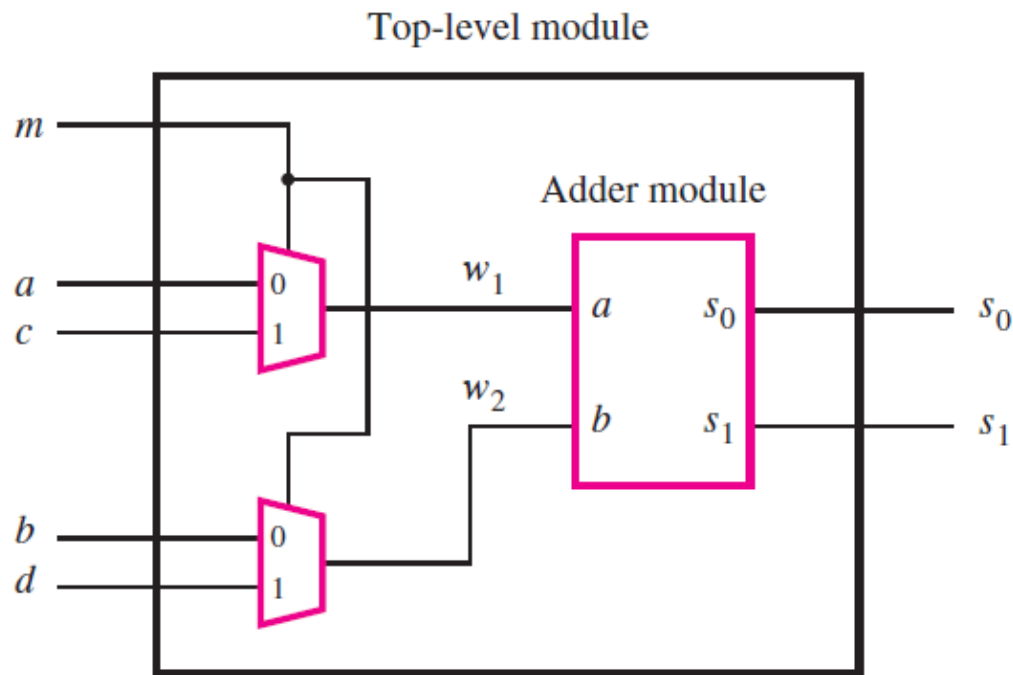
# Addition of Binary Numbers



$a$	$b$	$s_1$	$s_0$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0



# Logic Circuit vs Verilog Code



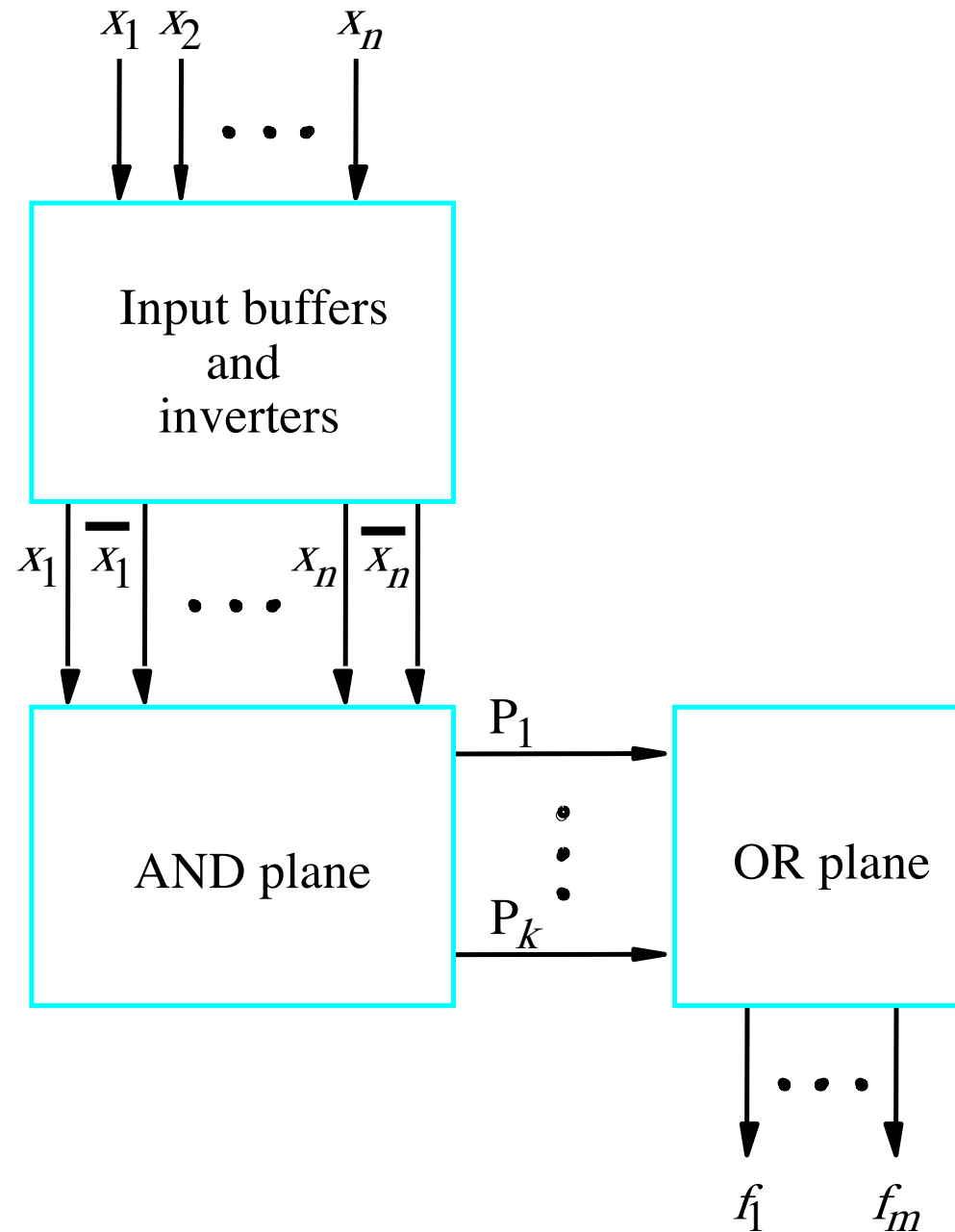
```
module shared (a, b, c, d, m, s1, s0);  
  input a, b, c, d, m;  
  output s1, s0;  
  wire w1, w2;  
  mux2to1 U1 (a, c, m, w1);  
  mux2to1 U2 (b, d, m, w2);  
  adder U3 (w1, w2, s1, s0);  
endmodule
```

```
module mux2to1 (x1, x2, s, f);  
  input x1, x2, s;  
  output f;  
  assign f = (~s & x1) | (s & x2);  
endmodule
```

```
module adder (a, b, s1, s0);  
  input a, b;  
  output s1, s0;  
  assign s1 = a & b;  
  assign s0 = a ^ b;  
endmodule
```

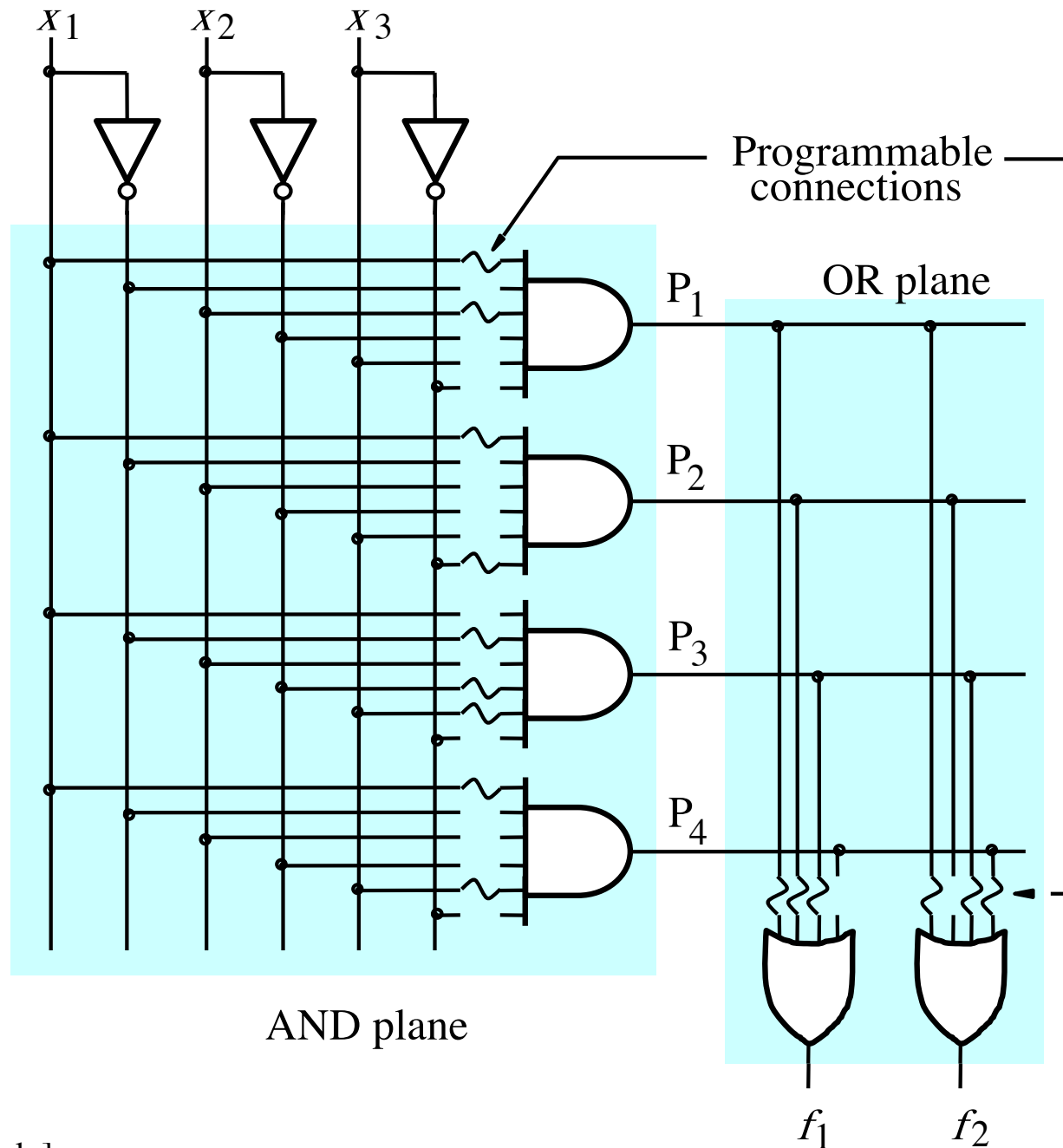
**Some material from Appendix B**

# Programmable Logic Array (PLA)



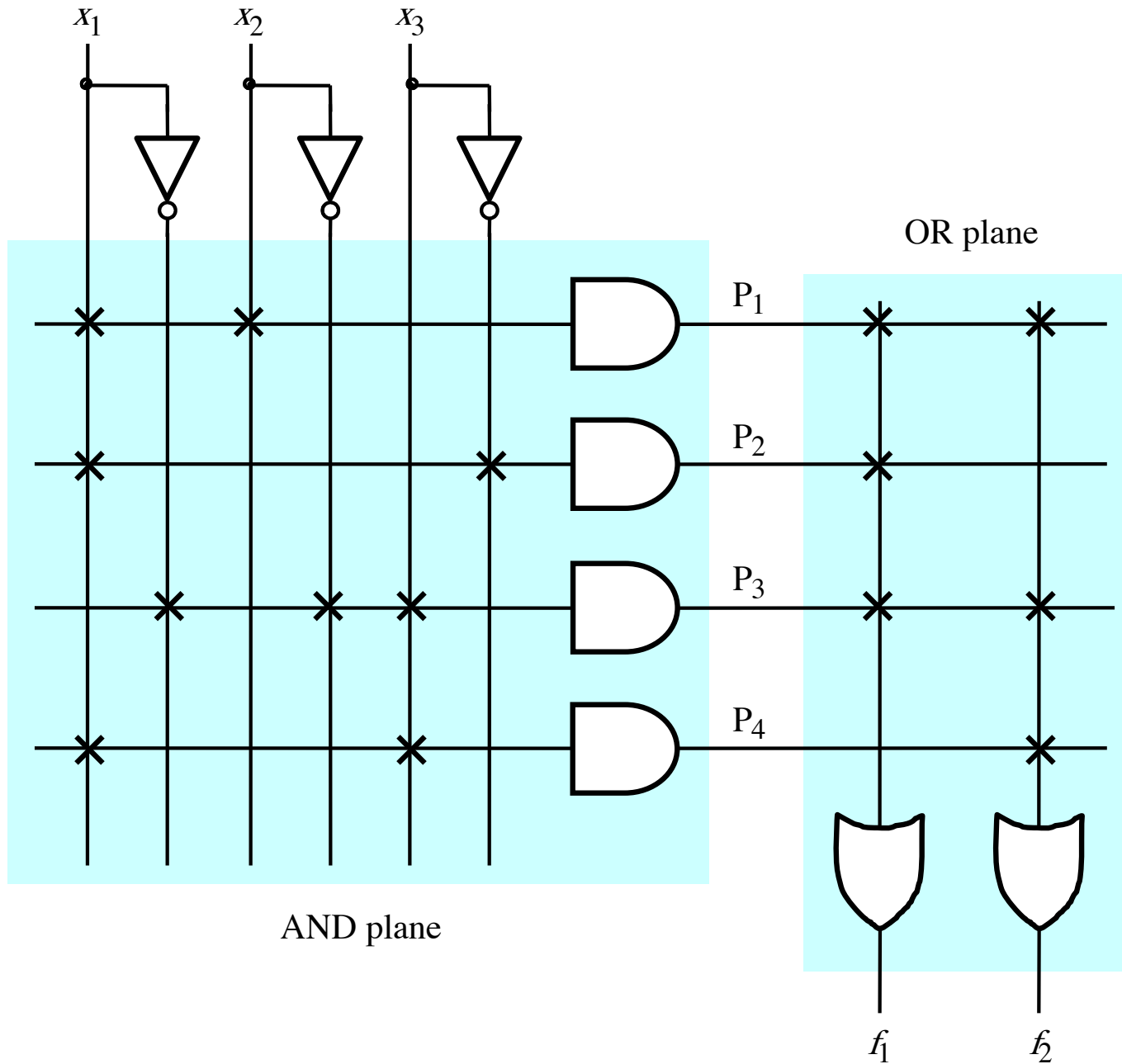
[ Figure B.25 from textbook ]

# Gate-Level Diagram of a PLA



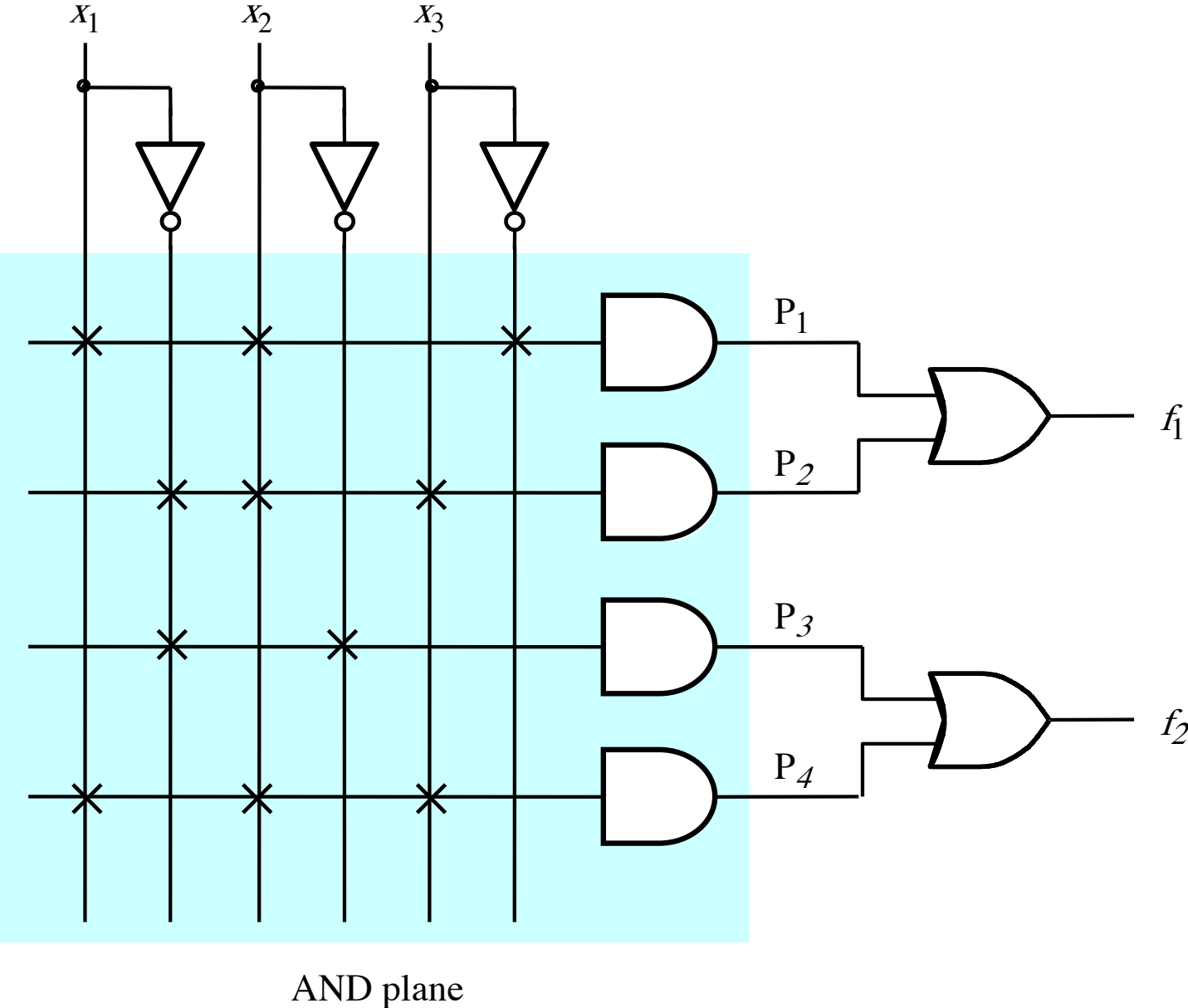
[ Figure B.26 from textbook ]

# Customary Schematic for PLA



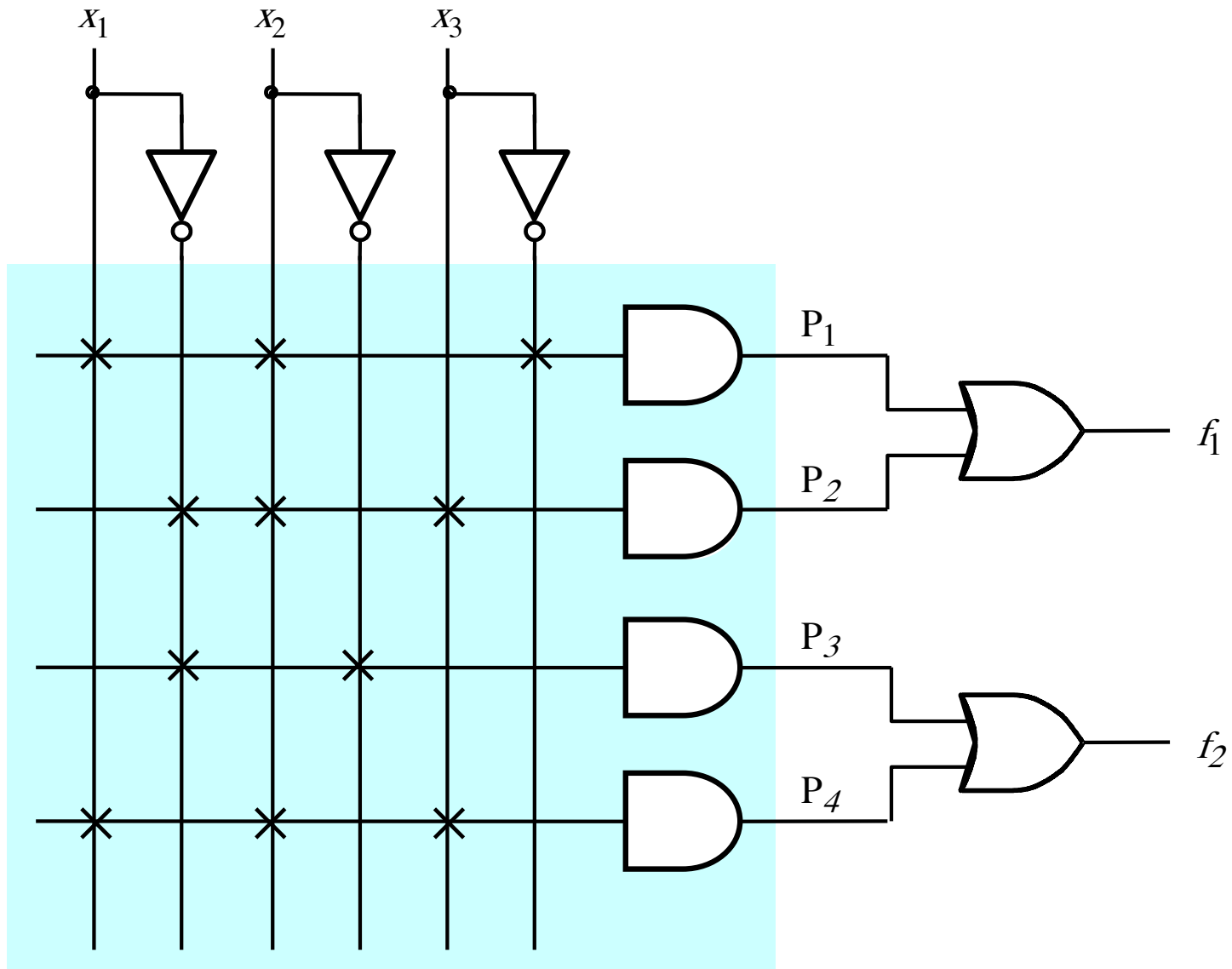
[ Figure B.27 from textbook ]

# Programmable Array Logic (PAL)



[ Figure B.28 from textbook ]

# Programmable Array Logic (PAL)



AND plane

Only the AND plane is programmable.  
The OR plane is fixed.

**Questions?**



**THE END**