

CprE 281: Digital Logic

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<http://www.ece.iastate.edu/~alexs/classes/>

Karnaugh Maps

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Iowa State University, Ames, IA
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Administrative Stuff

- **HW3 is due on Monday Sep 7 @ 4pm**

Administrative Stuff

- **HW4 is out**
- **It is due on Monday Sep 14 @ 4 pm**
- **It is posted on the class web page**
- **I also sent you an e-mail with the link.**

Administrative Stuff

- **Midterm Exam #1**
- **When: Friday Sep 18.**
- **What: Chapter 1 and Chapter 2 plus number systems**
- **The exam will be closed book but open notes (you can bring up to 3 pages of handwritten notes).**
- **More details to follow.**

Quick Review

Do You Still Remember This Boolean Algebra Theorem?

14a. $x \cdot y + x \cdot \bar{y} = x$

Combining

14b. $(x + y) \cdot (x + \bar{y}) = x$

Let's prove 14.a

x	y	$\mathbf{x \cdot y + x \cdot \bar{y} = x}$
0	0	
0	1	
1	0	
1	1	

Let's prove 14.a

x	y	$\mathbf{x \cdot y + x \cdot \bar{y} = x}$
0	0	0
0	1	0
1	0	0
1	1	1

Let's prove 14.a

x	y	$\mathbf{x \cdot y + x \cdot \bar{y} = x}$
0	0	0
0	1	0
1	0	1
1	1	1

Let's prove 14.a

x	y	$\mathbf{x} \cdot \mathbf{y} + \mathbf{x} \cdot \bar{\mathbf{y}} = \mathbf{x}$
0	0	0 0 0
0	1	0 0 0
1	0	0 1 1
1	1	1 1 0

Let's prove 14.a

x	y	$\mathbf{x \cdot y + x \cdot \bar{y} = x}$
0	0	0
0	1	0
1	0	1
1	1	1

Let's prove 14.a

x	y	$\mathbf{x} \cdot \mathbf{y} + \mathbf{x} \cdot \bar{\mathbf{y}} = \mathbf{x}$				
0	0	0	0	0	0	
0	1	0	0	0	0	
1	0	0	1	1	1	
1	1	1	1	0	1	

They are equal.

Motivation

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

An approach for simplifying logic expressions.

How do we guarantee we have reached the minimum SOP/POS representation?

Two-Variable K-Map

Karnaugh Map (K-map)

- A visual representation of the function
- Same information as the truth table
- Easier to group minterms

x_1	x_2	f
0	0	0
0	1	1
1	0	0
1	1	1

(a) Truth table

$x_2 \backslash x_1$	0	1
0	0	0
1	1	1

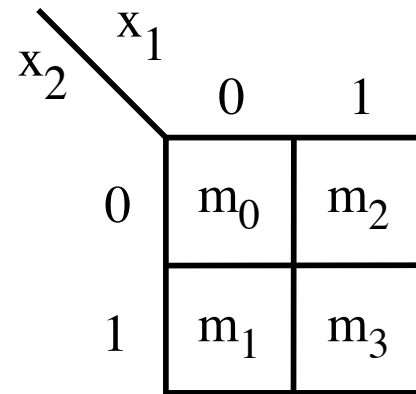
(b) Karnaugh map

Karnaugh Map (K-map)

- A visual representation of the function
- Same information as the truth table
- Easier to group minterms

x_1	x_2	f
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

(a) Truth table



(b) Karnaugh map

Minterms

x_1	x_2	f
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

x_1	x_2	m_0	m_1	m_2	m_3
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

Minterm Addition Example

x_1	x_2	f
0	0	0
0	1	1
1	0	0
1	1	1

x_1	x_2	m_0	m_1	m_2	m_3	$m_1 + m_3$
0	0	1	0	0	0	0
0	1	0	1	0	0	1
1	0	0	0	1	0	0
1	1	0	0	0	1	1

Minterm Addition Example

x_1	x_2	f
0	0	0
0	1	1
1	0	0
1	1	1

x_1	x_2	m_0	m_1	m_2	m_3	$m_1 + m_3$
0	0	1	0	0	0	0
0	1	0	1	0	0	1
1	0	0	0	1	0	0
1	1	0	0	0	1	1

$$\bar{x}_1x_2 + x_1x_2 = x_2$$

Minterm Addition Example

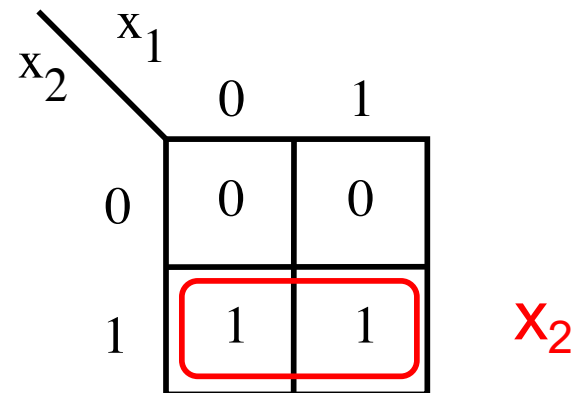
x_1	x_2	f
0	0	0
0	1	1
1	0	0
1	1	1

x_2	x_1	0	1
0	0	0	0
1	0	1	1

$$\bar{x}_1x_2 + x_1x_2 = x_2$$

Minterm Addition Example

x_1	x_2	f
0	0	0
0	1	1
1	0	0
1	1	1



$$\bar{x}_1x_2 + x_1x_2 = x_2$$

Another Grouping Example

	x_1	0	1
x_2	0	1	0
	1	0	0

m_0

	x_1	0	1
x_2	0	0	0
	1	1	0

m_1

Another Grouping Example

	x_1	0	1
x_2			
0		1	0
1		0	0

m_0

+

	x_1	0	1
x_2			
0		0	0
1		1	0

m_1

=

	x_1	0	1
x_2			
0		1	0
1		1	0

$m_0 + m_1$

Another Grouping Example

	x_1	0	1
x_2	0	1	0
	1	0	0

m_0

+

	x_1	0	1
x_2	0	0	0
	1	1	0

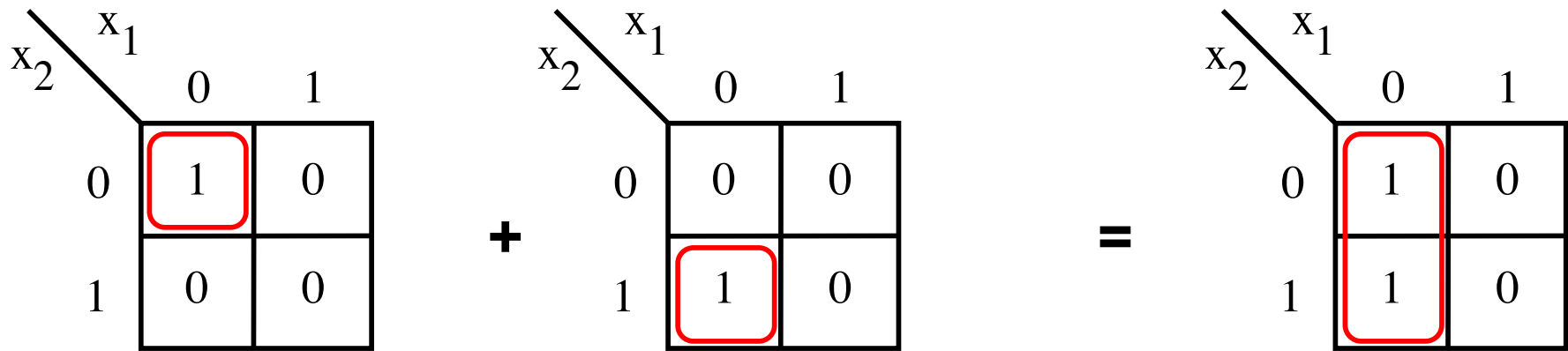
m_1

=

	x_1	0	1
x_2	0	1	0
	1	1	0

$m_0 + m_1$

Another Grouping Example



m_0

+

m_1

=

$m_0 + m_1$

$\overline{x_1}\overline{x_2}$

+

$\overline{x_1}x_2$

=

$\overline{x_1}$

Property 14a (Combining)

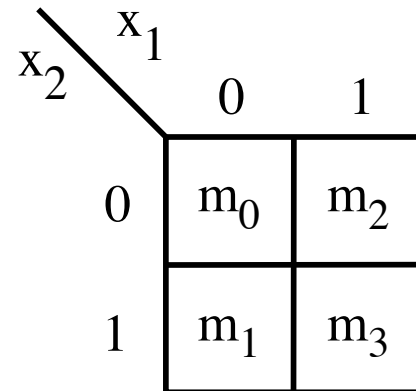
Grouping Rules

- **Group “1”s with rectangles**
- **Both sides a power of 2:**
 - **1x1, 1x2, 2x1, 2x2, 1x4, 4x1, 2x4, 4x2, 4x4**
- **Can use the same minterm more than once**
- **Can wrap around the edges of the map**
- **Some rules in selecting groups:**
 - **Try to use as few groups as possible to cover all “1”s.**
 - **For each group, try to make it as large as you can (i.e., if you can use a 2x2, don’t use a 2x1 even if that is enough).**

Two-Variable K-map

x_1	x_2	f
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

(a) Truth table



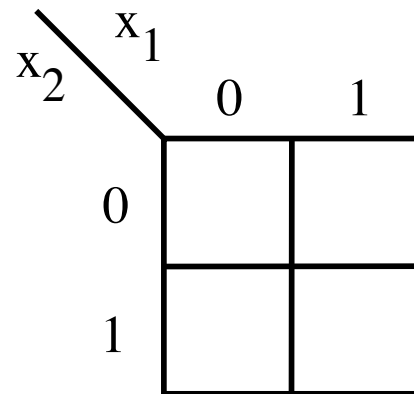
(b) Karnaugh map

Step-By-Step Example

x_1	x_2	f
0	0	1
0	1	1
1	0	0
1	1	1

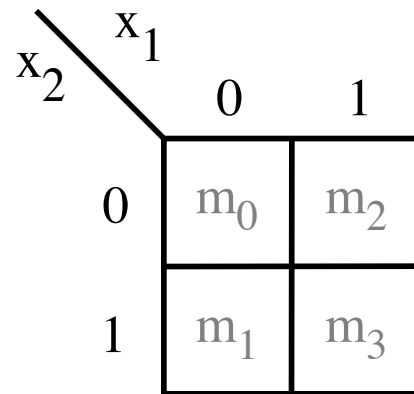
1. Draw The Map

x_1	x_2	f
0	0	1
0	1	1
1	0	0
1	1	1



2. Fill The Map

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1



2. Fill The Map

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

x_2	x_1	0	1
0		1	0
1		1	1

3. Group

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

x_2	x_1	0	1
0		1	0
1		1	1

3. Group

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

	x_1	0	1
x_2	0	1	0
	1	1	1

3. Group

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

A Karnaugh map for the function $f(x_1, x_2)$. The horizontal axis is labeled x_1 with values 0 and 1. The vertical axis is labeled x_2 with values 0 and 1. The map contains the following values:

$x_2 \backslash x_1$	0	1
0	1	0
1	1	1

The prime implicants are highlighted with colored boxes:

- A red box highlights the prime implicant \bar{x}_2 , which covers the cells (0,0) and (1,0).
- A green box highlights the prime implicant x_2 , which covers the cells (1,0) and (1,1).

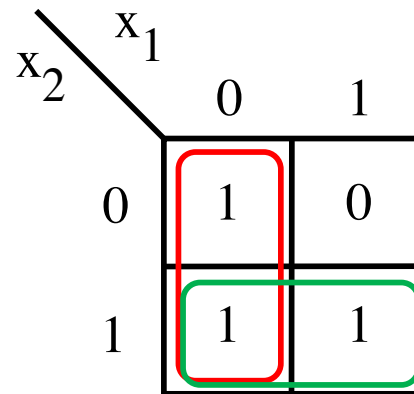
4. Write The Expression

x_1	x_2	f
0	0	1
0	1	1
1	0	0
1	1	1

$x_2 \backslash x_1$	0	1
0	1	0
1	1	1

4. Write The Expression

x_1	x_2	f
0	0	1
0	1	1
1	0	0
1	1	1



$$\bar{x}_1 + x_2$$

Writing The Expression

- Find which variable is constant

		x_1	
		0	1
x_2	0	1	0
	1	1	0

$\overline{x_1}$ is constant

Writing The Expression

- Find which variable is constant

		x_1	
		0	1
x_2	0	0	1
	1	0	1

x_1 is constant

Writing The Expression

- Find which variable is constant

		x_1	
		0	1
x_2	0	1	1
	1	0	0

$\overline{x_2}$ is constant

Writing The Expression

- Find which variable is constant

		x_1	
		0	1
x_2	0	0	0
	1	1	1

x_2 is constant

These are all valid groupings

	x_1	0	1
x_2	0	m_0	m_2
	1	m_1	m_3

	x_1	0	1
x_2	0	m_0	m_2
	1	m_1	m_3

	x_1	0	1
x_2	0	m_0	m_2
	1	m_1	m_3

	x_1	0	1
x_2	0	m_0	m_2
	1	m_1	m_3

These are also valid

	x_1		
x_2		0	1
0		m_0	m_2
1		m_1	m_3

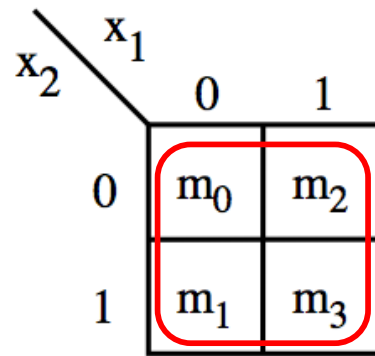
	x_1		
x_2		0	1
0		m_0	m_2
1		m_1	m_3

	x_1		
x_2		0	1
0		m_0	m_2
1		m_1	m_3

	x_1		
x_2		0	1
0		m_0	m_2
1		m_1	m_3

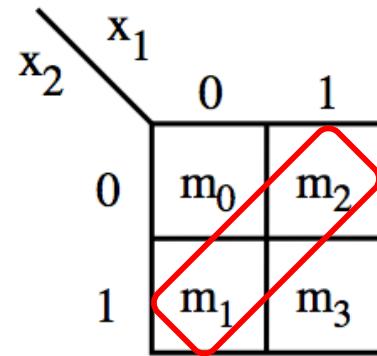
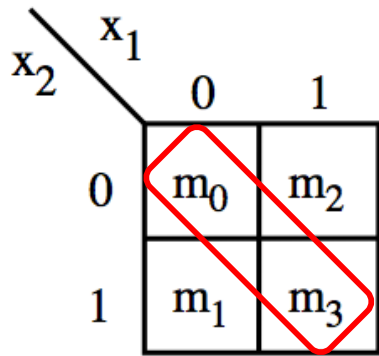
But try to use larger rectangles if possible.

This one is valid too



In this case the result is the constant function 1.

Why are these two not valid?



Let's Find Out

	x_1	0	1
x_2	0	1	0
	1	0	0

m_0

	x_1	0	1
x_2	0	0	0
	1	0	1

m_3

Let's Find Out

	x_1	0	1
x_2			
0		1	0
1		0	0

m_0

+

	x_1	0	1
x_2			
0		0	0
1		0	1

m_3

=

	x_1	0	1
x_2			
0		1	0
1		0	1

$m_0 + m_3$

Let's Find Out

	x_1	0	1
x_2	0	1	0
	1	0	0

m_0

+

	x_1	0	1
x_2	0	0	0
	1	0	1

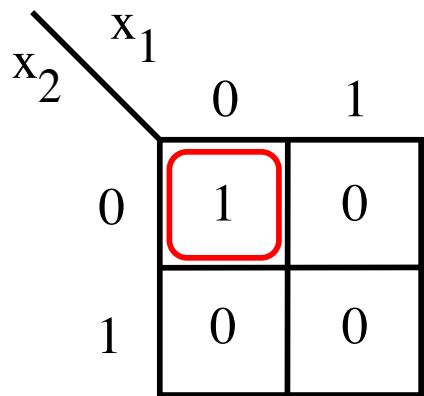
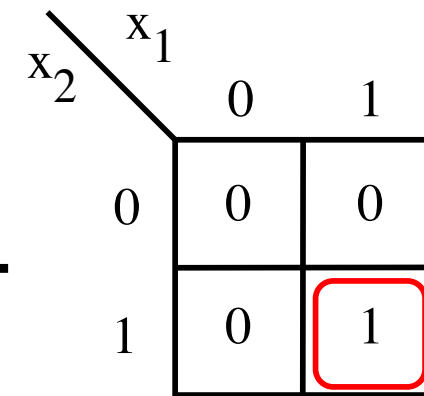
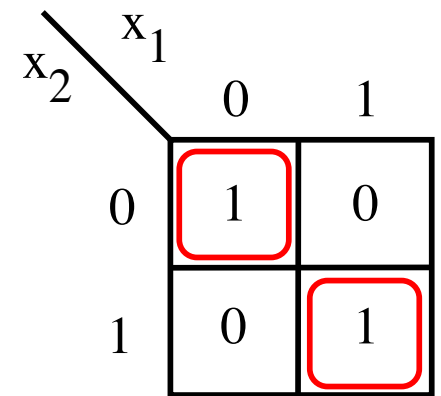
m_3

=

	x_1	0	1
x_2	0	1	0
	1	0	1

$m_0 + m_3$

Let's Find Out

	+		=	
m_0	+	m_3	=	$m_0 + m_3$
$\overline{x_1}\overline{x_2}$	+	x_1x_2	=	$\overline{x_1}\overline{x_2} + x_1x_2$

We can't use Property 14a here. This can't be simplified.

Three-Variable K-Map

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3	$x_1 x_2$	00	01	11	10
0		m_0	m_2	m_6	m_4
1		m_1	m_3	m_7	m_5

(b) Karnaugh map

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3	$x_1 x_2$	00	01	11	10
0		m_0	m_2	m_6	m_4
1		m_1	m_3	m_7	m_5

(b) Karnaugh map

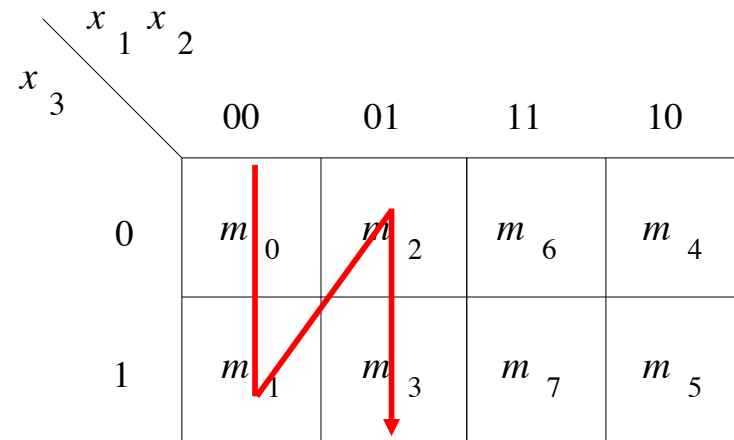
Notice the placement of

- **Variables**
- **Binary pair values**
- **Minterms**

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
<hr/>			
0	1	0	m_2
0	1	1	m_3
<hr/>			
1	0	0	m_4
1	0	1	m_5
<hr/>			
1	1	0	m_6
1	1	1	m_7

(a) Truth table



(b) Karnaugh map

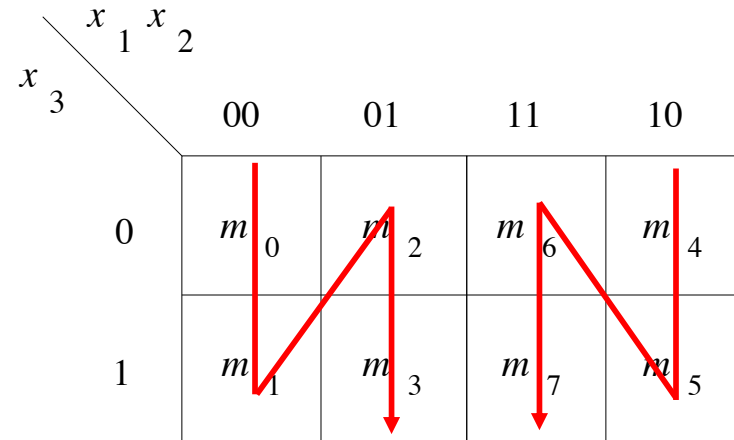
Notice the placement of

- **Variables**
- **Binary pair values**
- **Minterms**

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
<hr/>			
0	1	0	m_2
0	1	1	m_3
<hr/>			
1	0	0	m_4
1	0	1	m_5
<hr/>			
1	1	0	m_6
1	1	1	m_7

(a) Truth table



(b) Karnaugh map

Notice the placement of

- **Variables**
- **Binary pair values**
- **Minterms**

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

00

01

11

10

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000
001
011
010
110
111
101
100

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

000

001

011

010

110

111

101

100

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s x_1$			
		00	01	11	10
x_2	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

x_2	$s x_1$	00	01	11	10
0	000	010	110	100	
1	001	011	111	101	

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the LAST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the LAST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the FIRST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These four neighbors
differ in the FIRST and LAST bit

They are similar in their MIDDLE bit

Adjacency Rules

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5



adjacent
columns

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

x_2	s	x_1				
			00	01	11	10
0			000	010	110	100
1			001	011	111	101

These two neighbors
differ only in the FIRST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

x_2	$s x_1$			
	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These four neighbors
differ in the FIRST and LAST bit

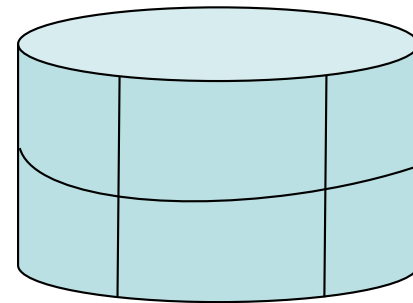
They are similar in their MIDDLE bit

Adjacency Rules

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5



adjacent
columns



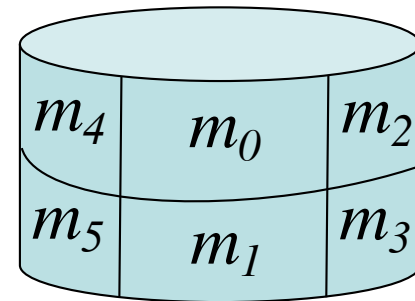
As if the K-map were
drawn on a cylinder

Adjacency Rules

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5



adjacent
columns



As if the K-map were
drawn on a cylinder

These are valid groupings

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

These are valid groupings

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

These are valid groupings

$x_3 \backslash x_1x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

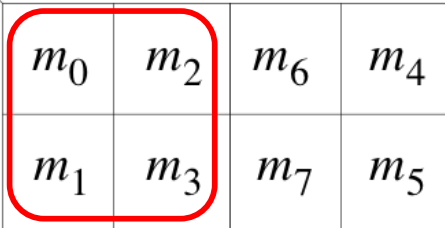
$x_3 \backslash x_1x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

$x_3 \backslash x_1x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

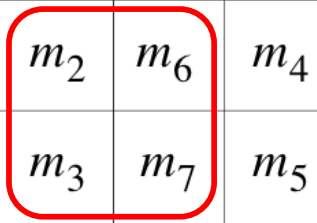
$x_3 \backslash x_1x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

These are valid groupings

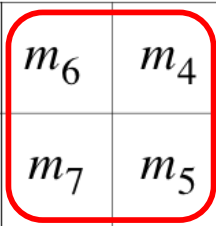
$x_3 \backslash x_1x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5



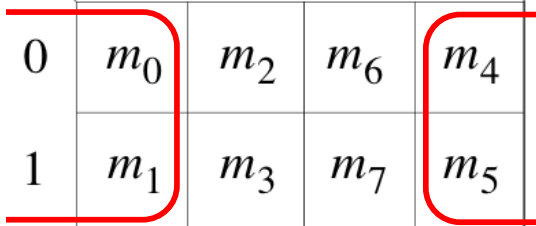
$x_3 \backslash x_1x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5



$x_3 \backslash x_1x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5



$x_3 \backslash x_1x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5



These are valid groupings

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

This is a valid grouping

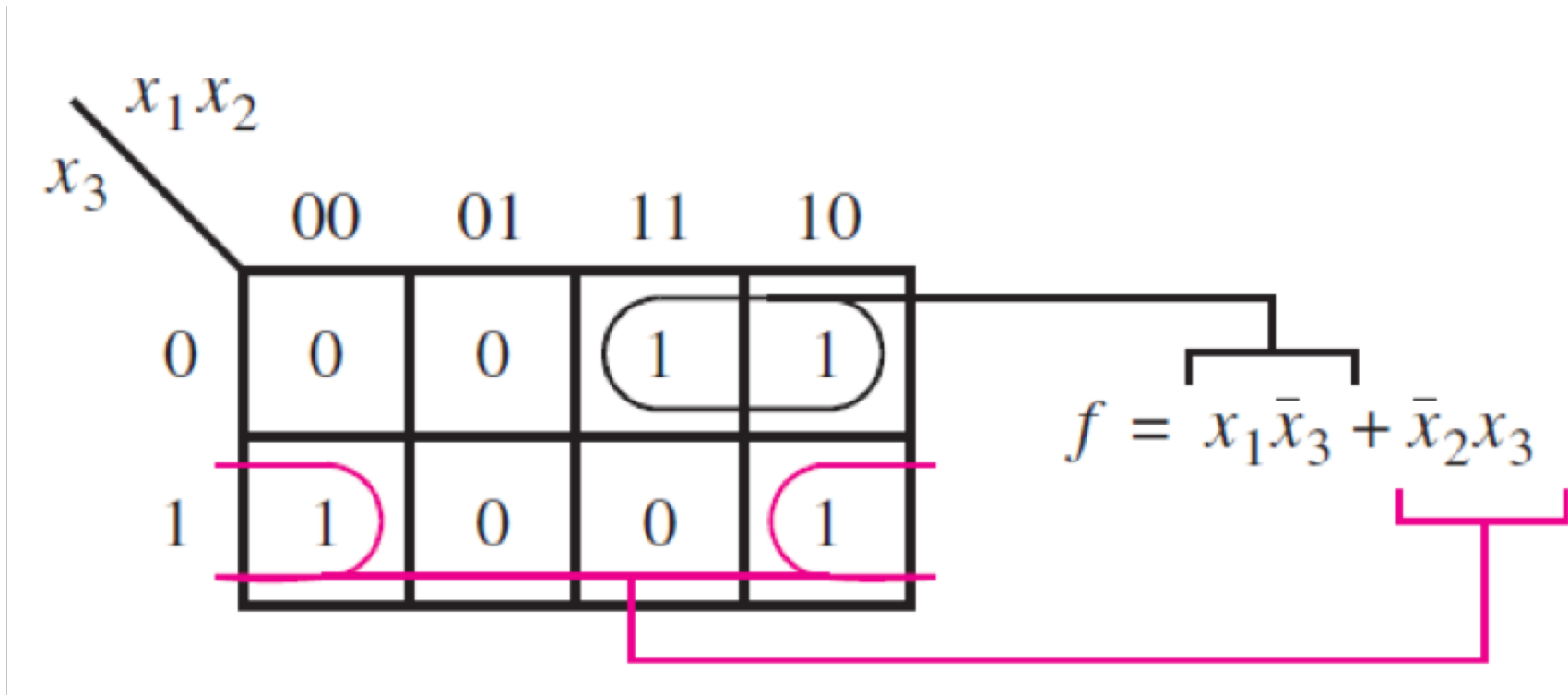
		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

Some invalid groupings

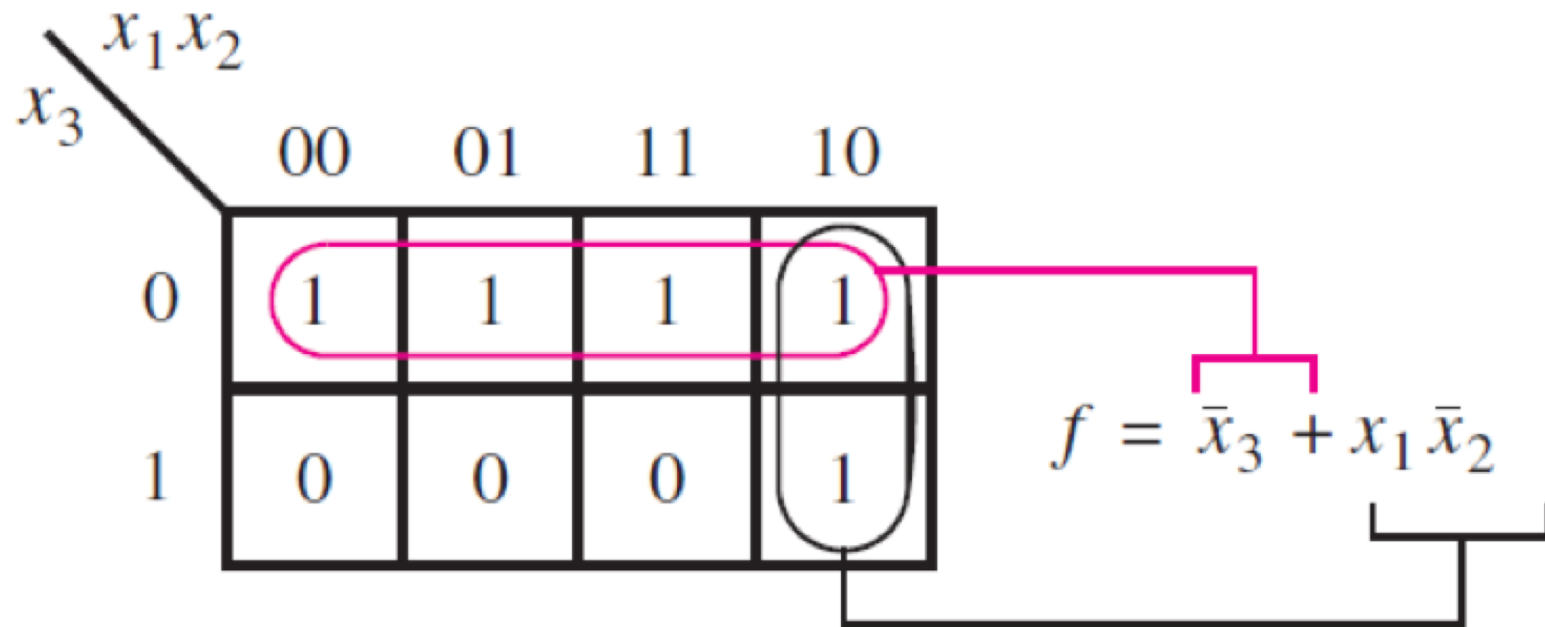
		x_1x_2			
x_3		00	01	11	10
0	m_0	m_2	m_6	m_4	
1	m_1	m_3	m_7	m_5	

		x_1x_2			
x_3		00	01	11	10
0	m_0	m_2	m_6	m_4	
1	m_1	m_3	m_7	m_5	

Examples of three-variable Karnaugh maps

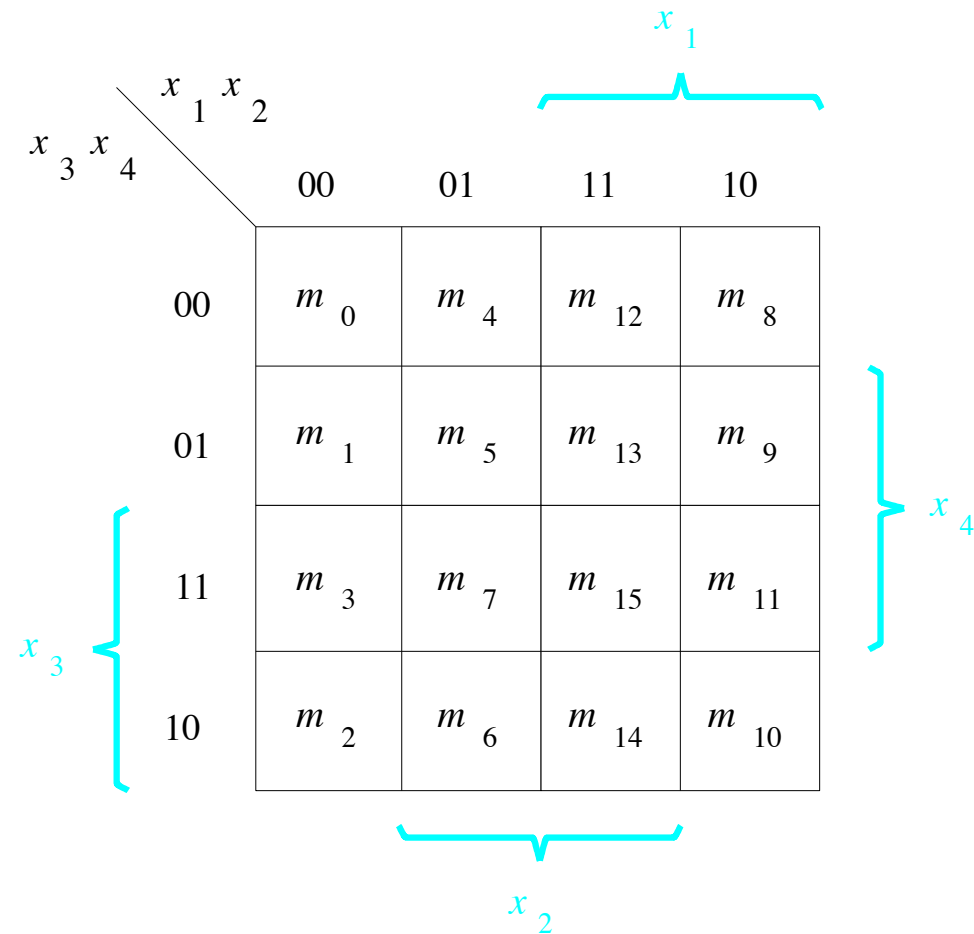


Examples of three-variable Karnaugh maps



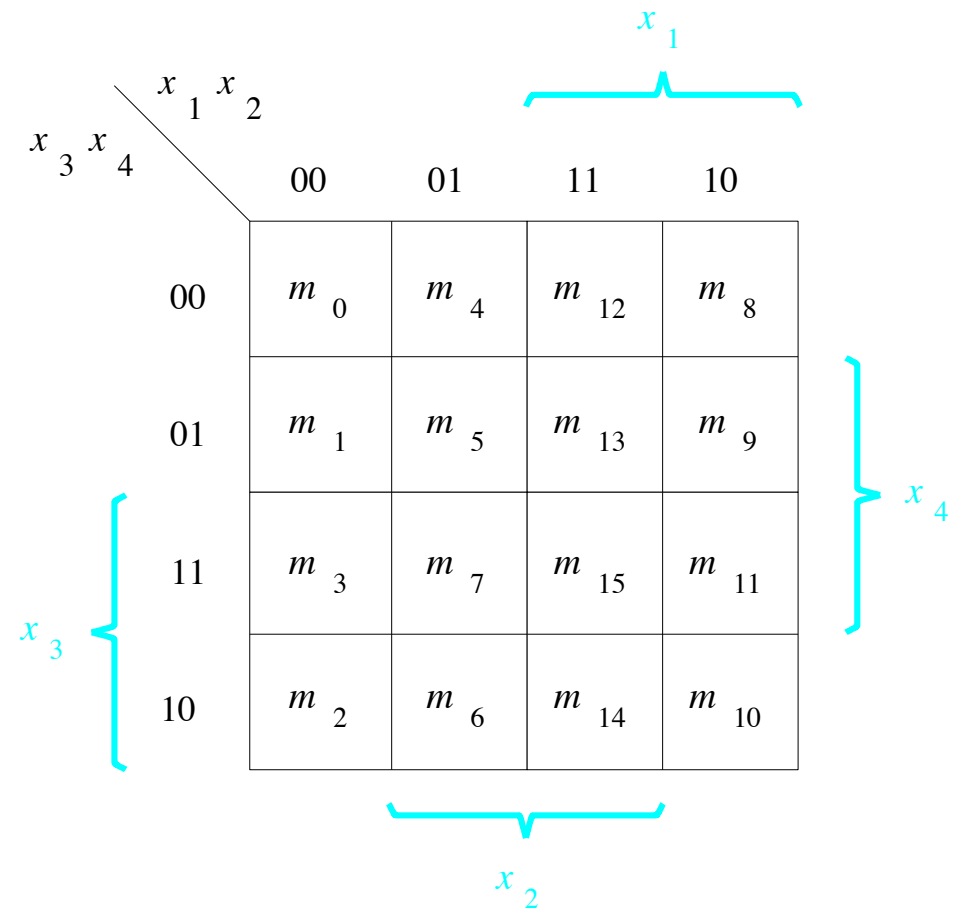
Four-Variable K-Map

A four-variable Karnaugh map



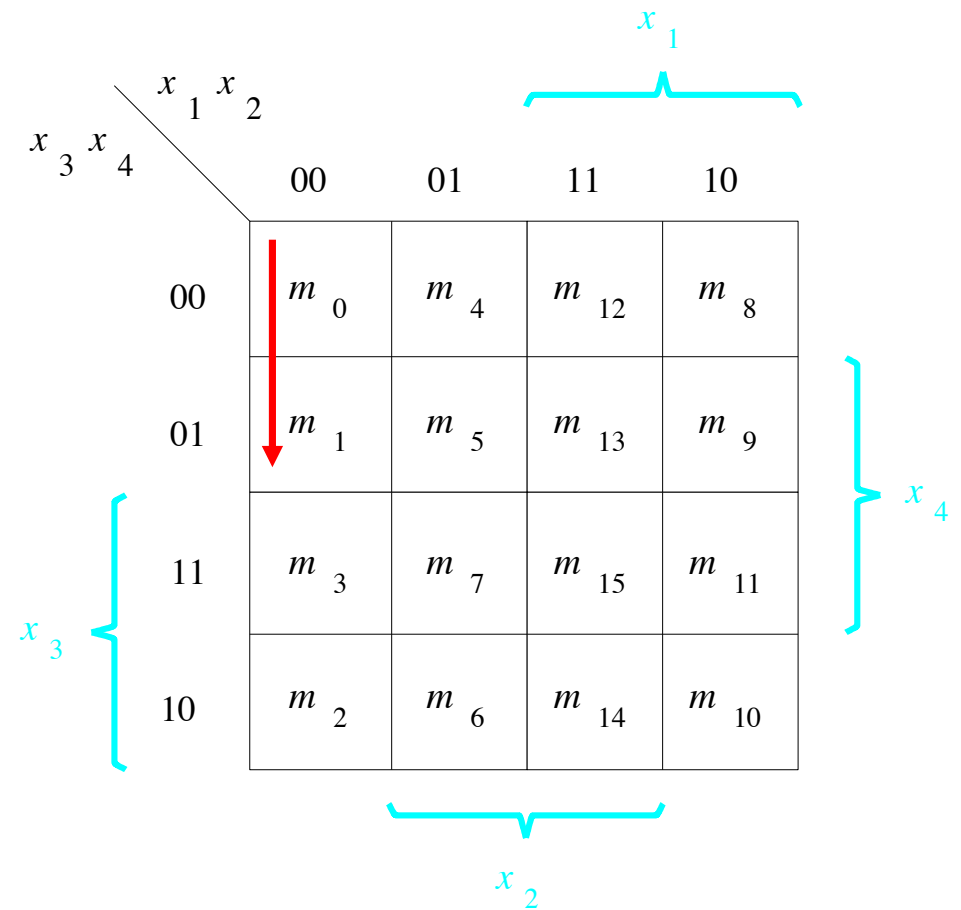
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



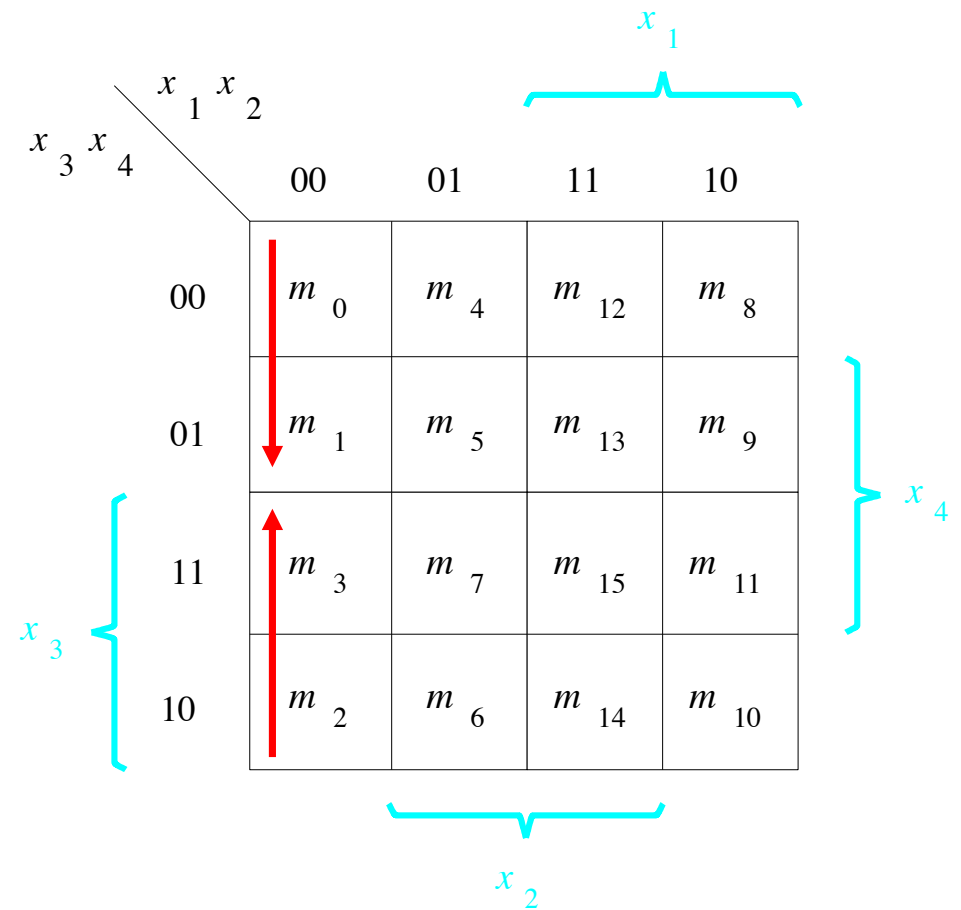
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



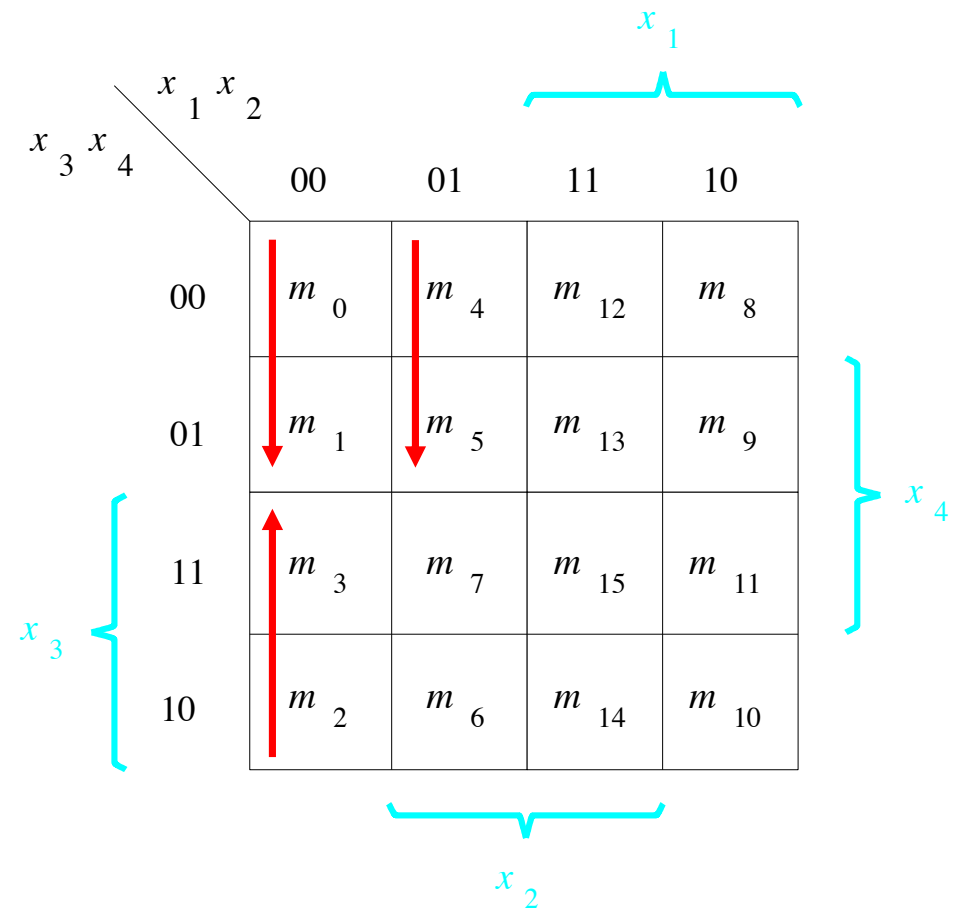
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



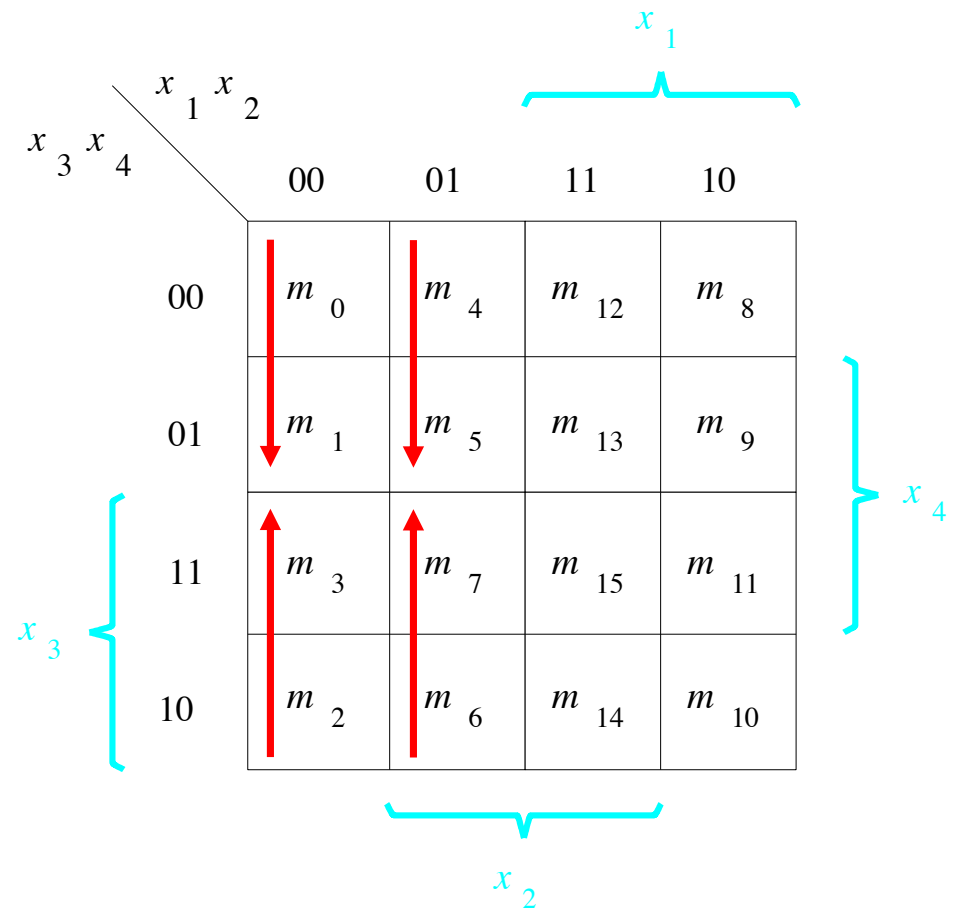
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



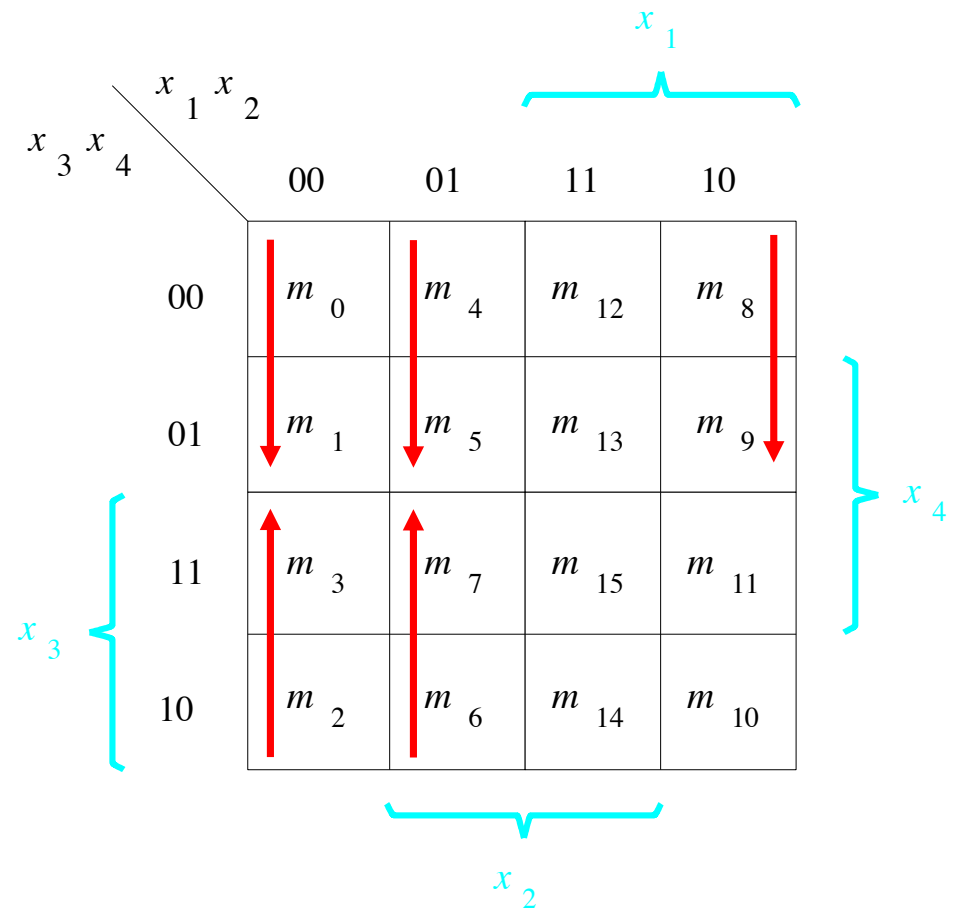
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



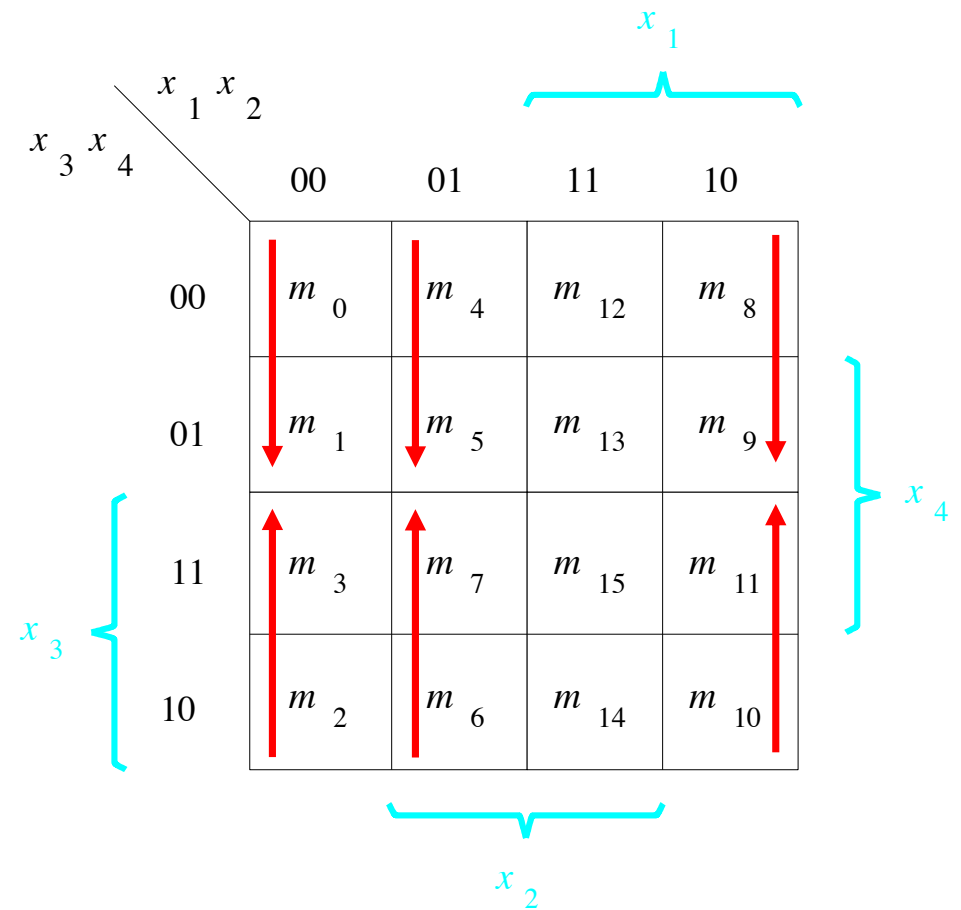
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



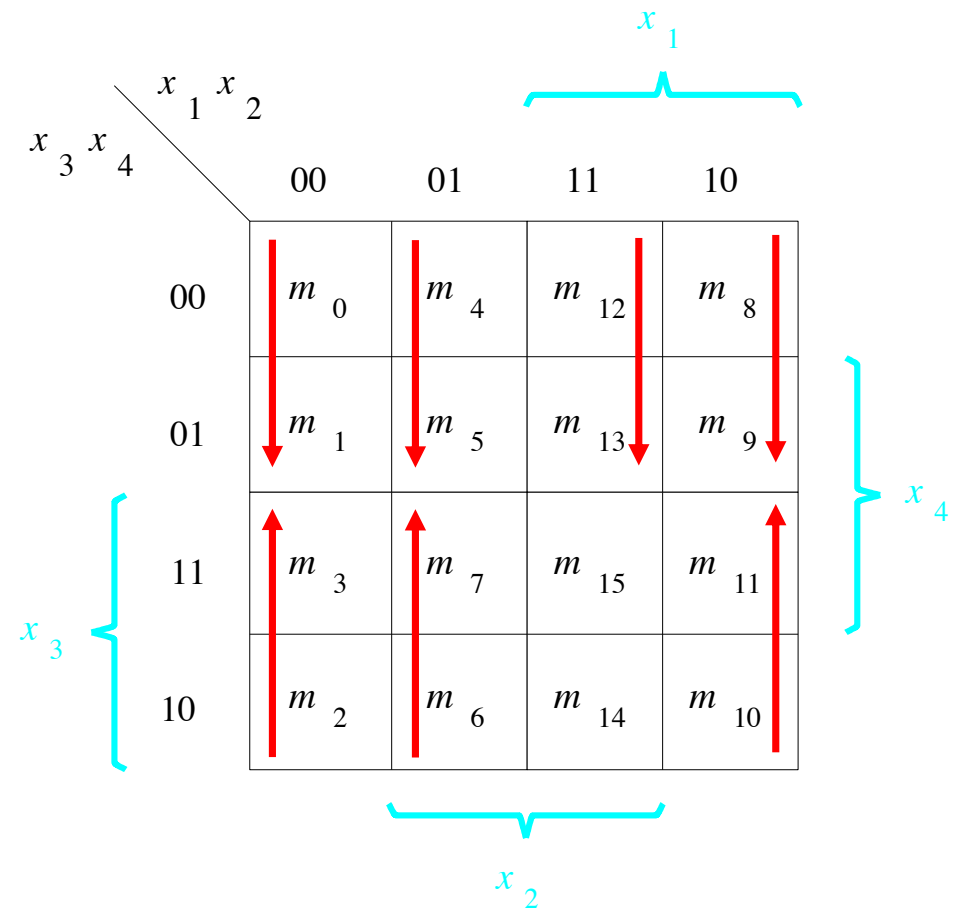
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



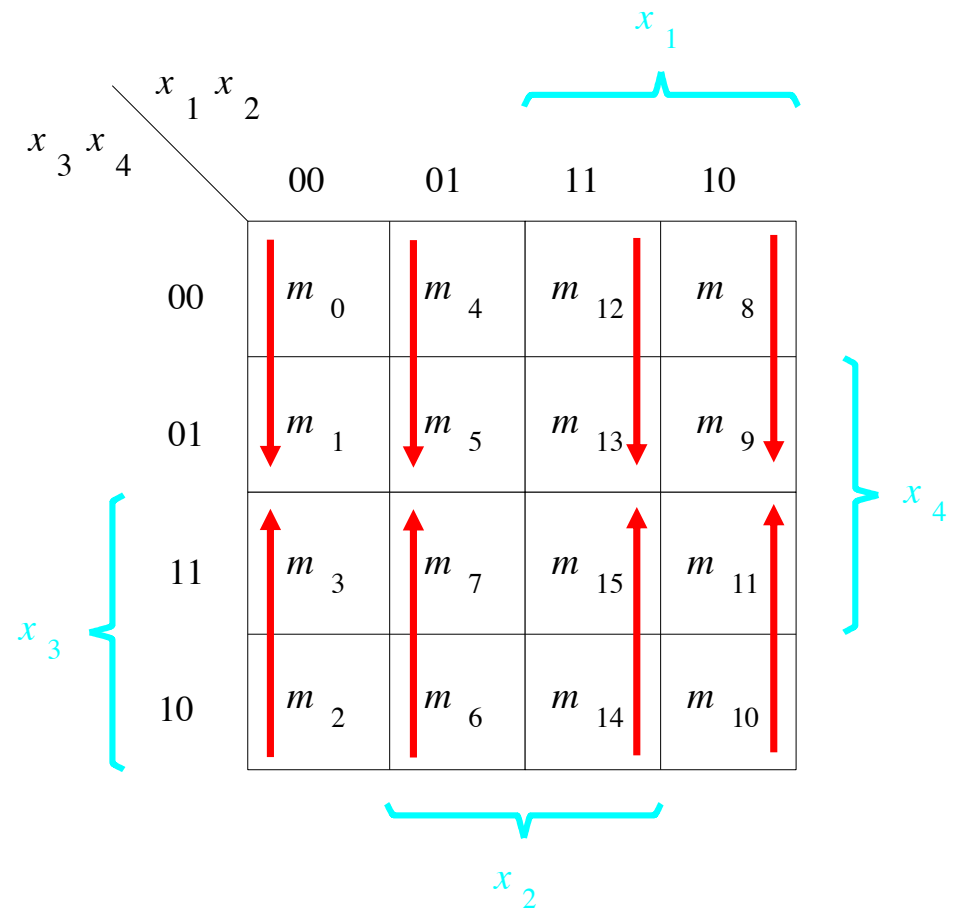
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



Adjacency Rules

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

adjacent
columns

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

adjacent
columns

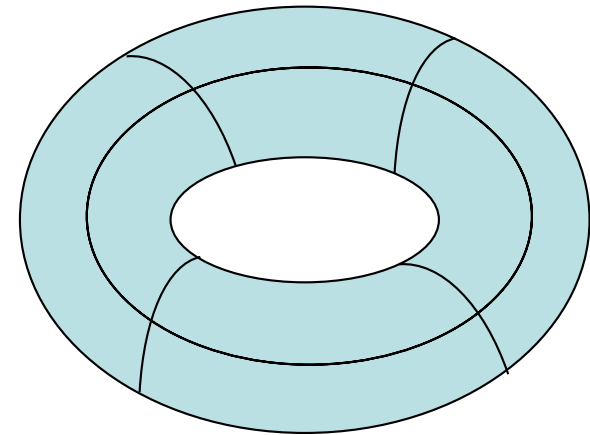
adjacent
rows

Adjacency Rules

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

adjacent
rows

adjacent
columns



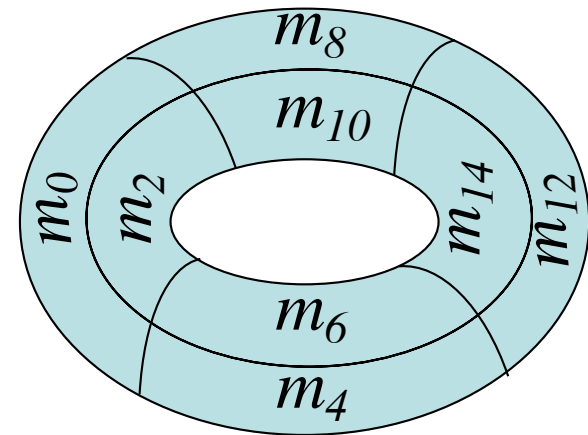
As if the K-map were
drawn on a torus

Adjacency Rules

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

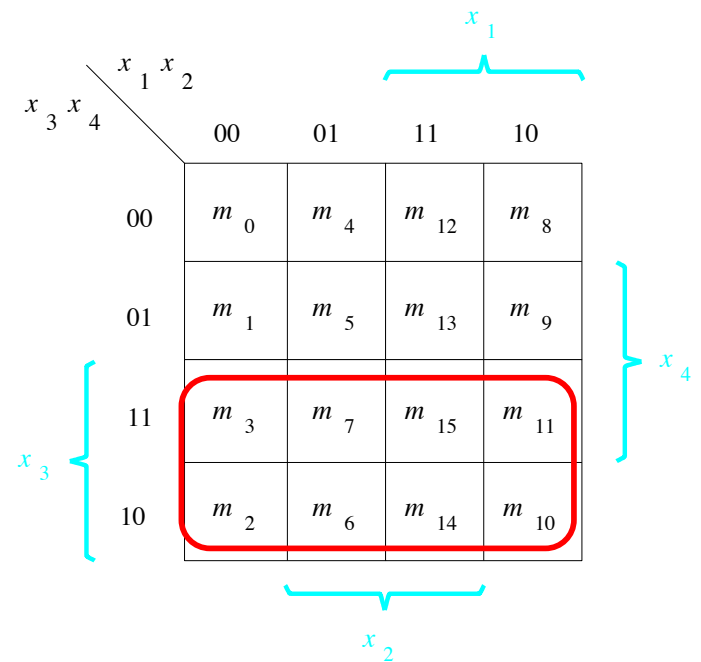
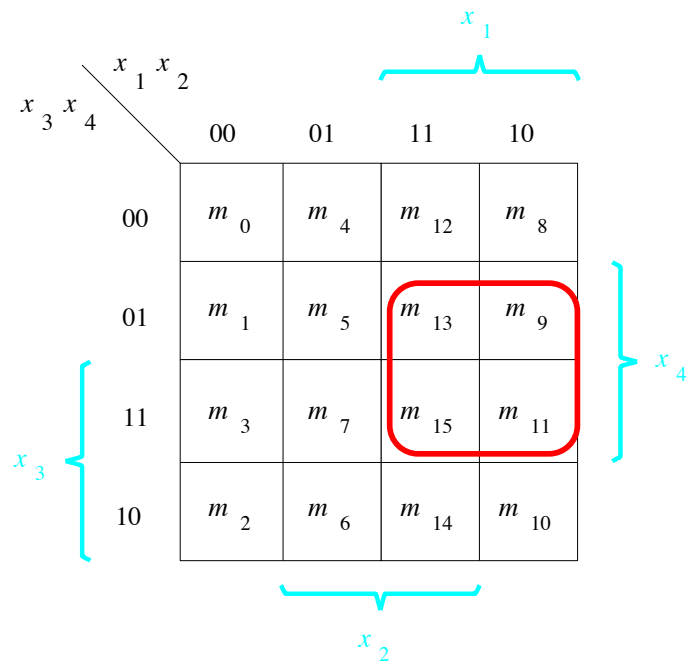
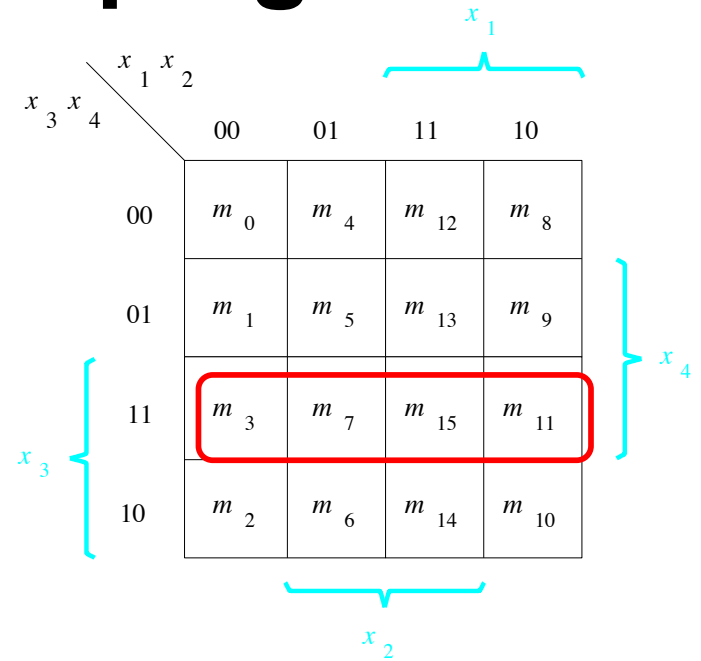
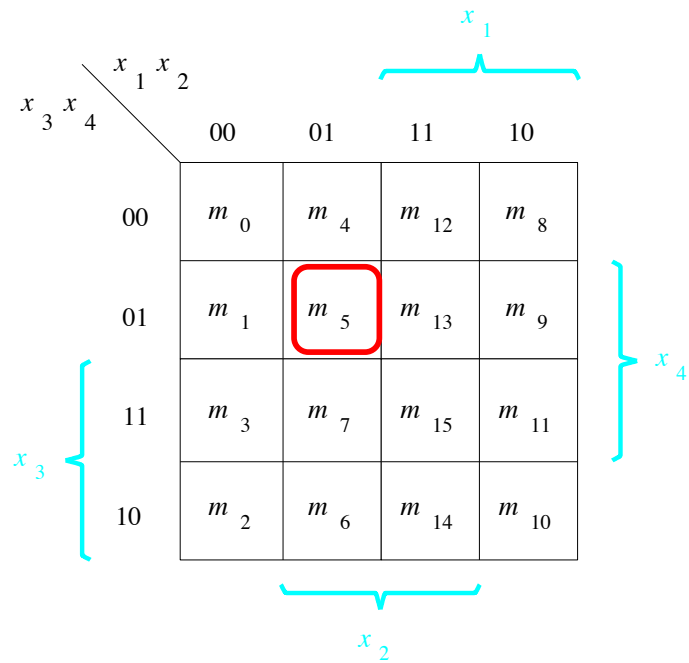
adjacent
rows

adjacent
columns

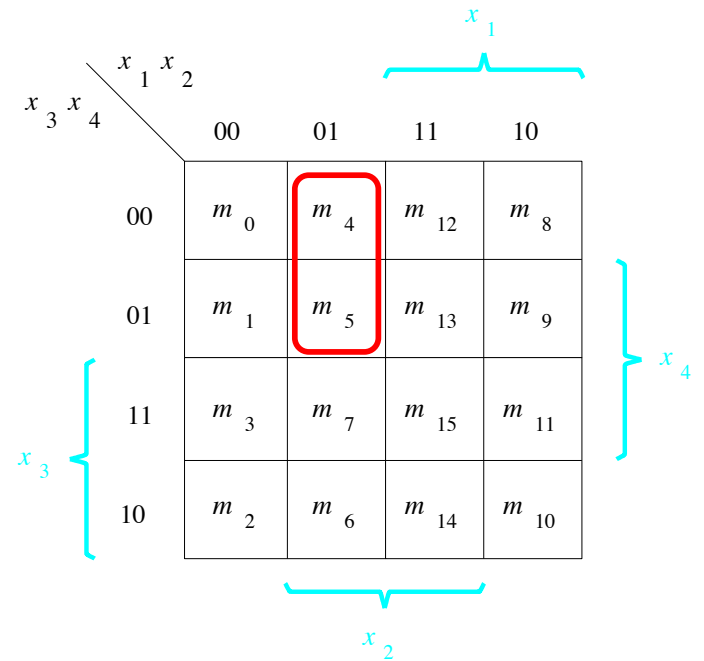
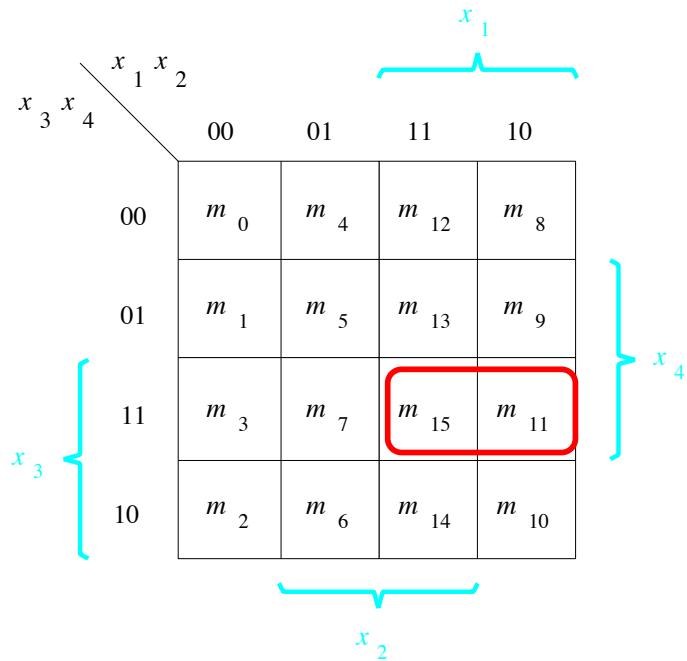
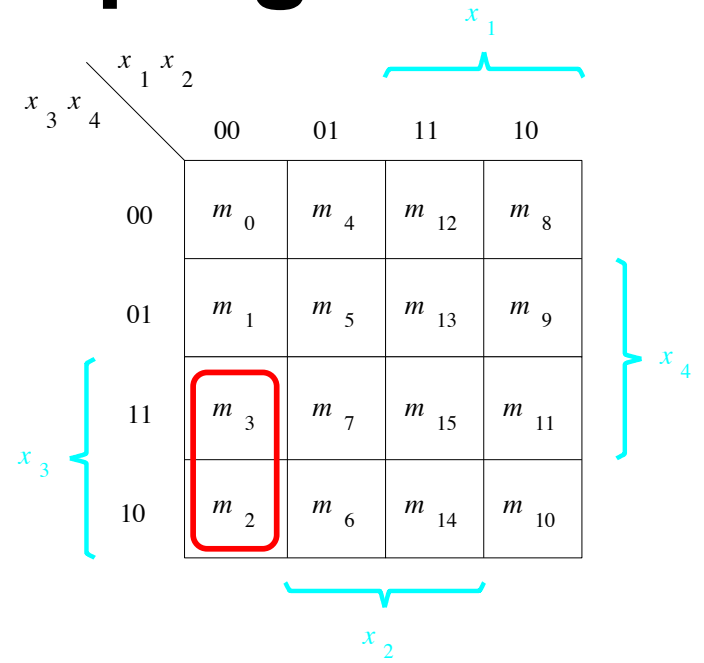
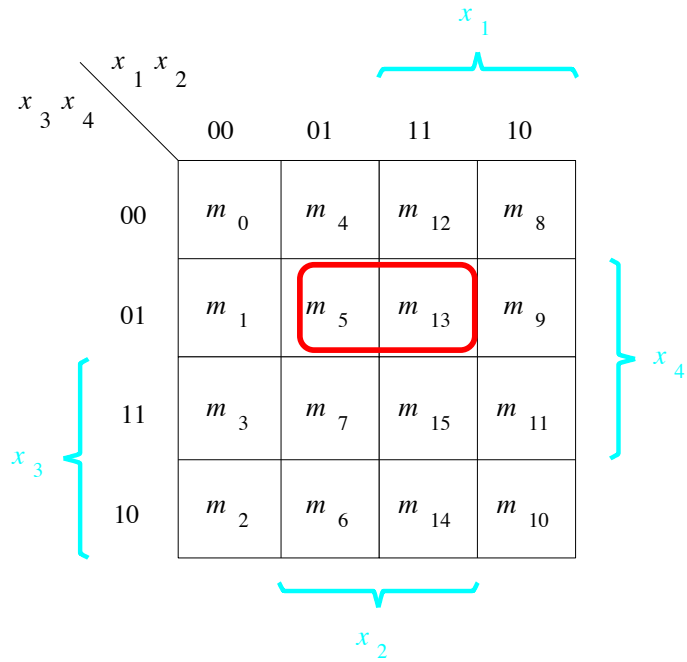


As if the K-map were
drawn on a torus

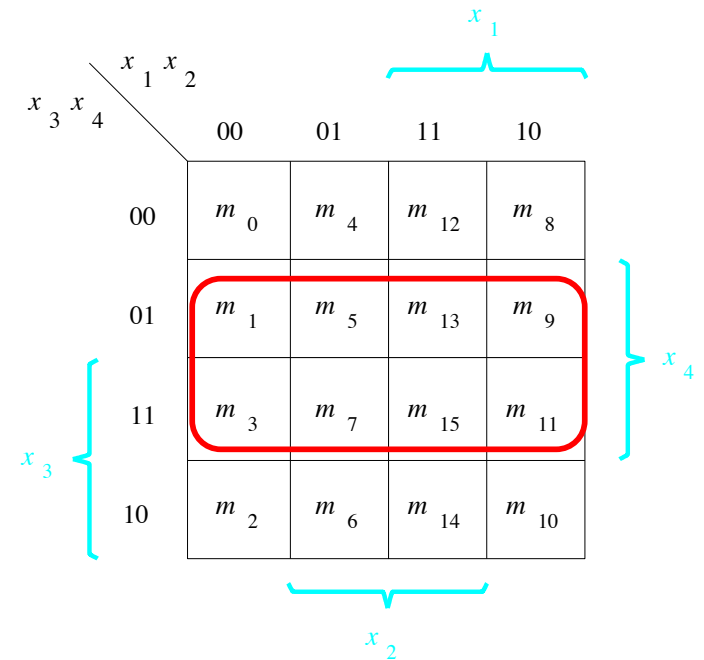
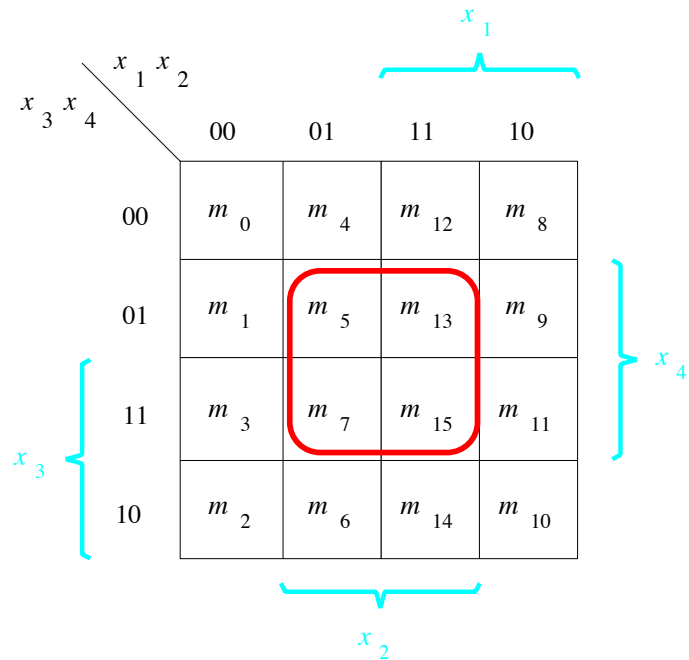
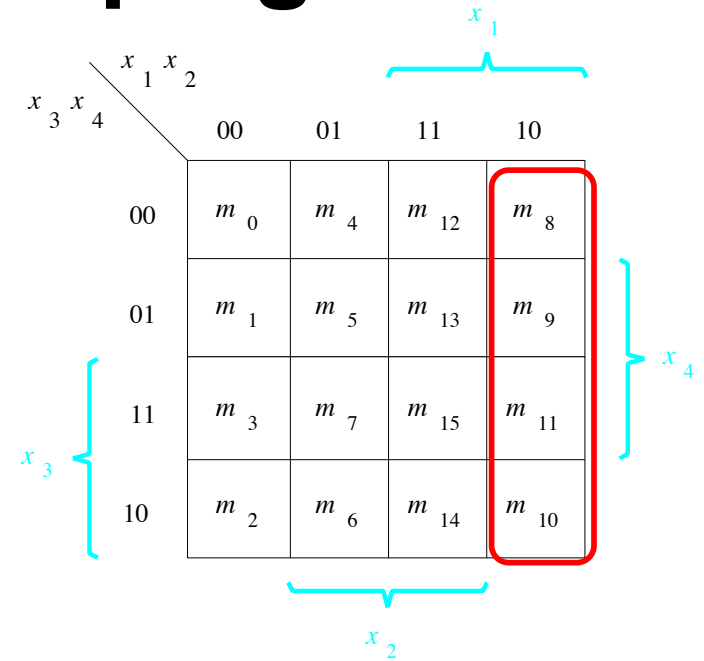
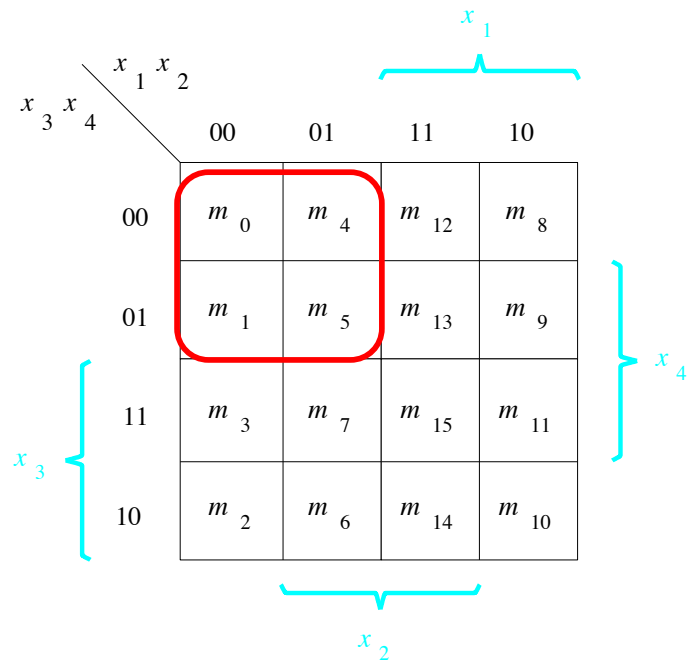
Some Valid Groupings



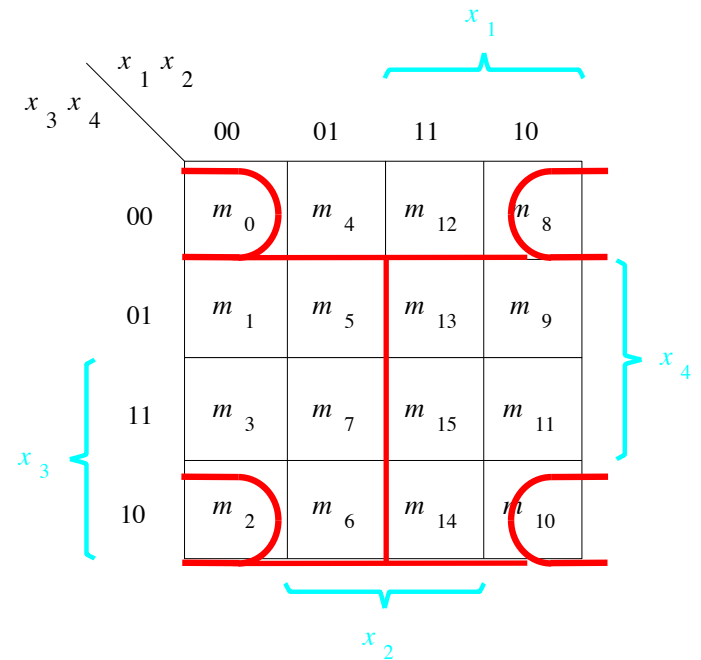
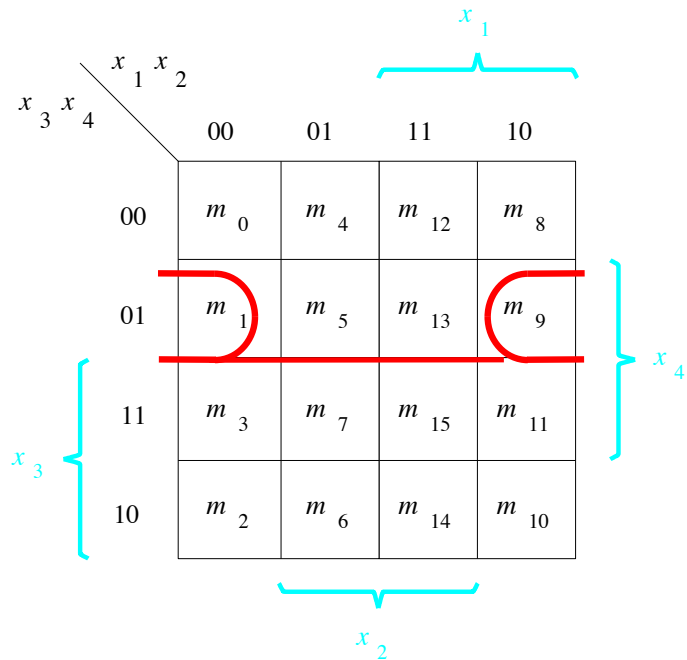
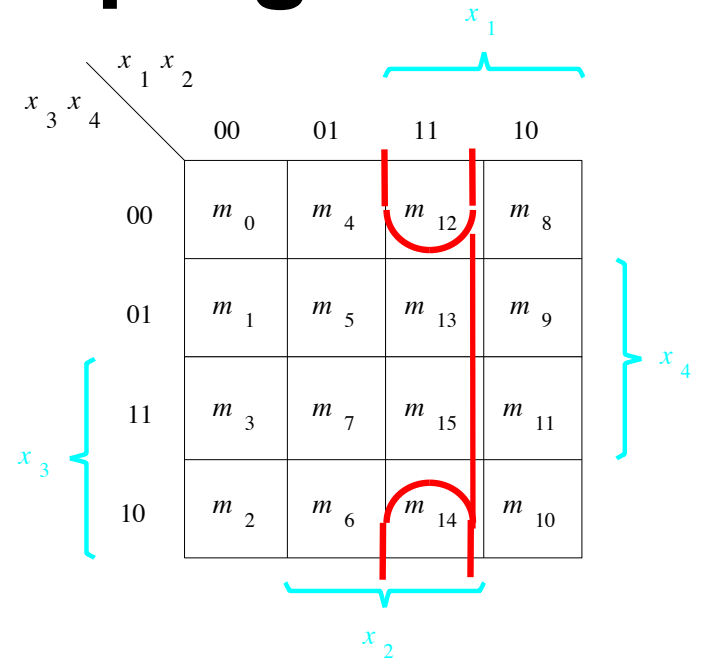
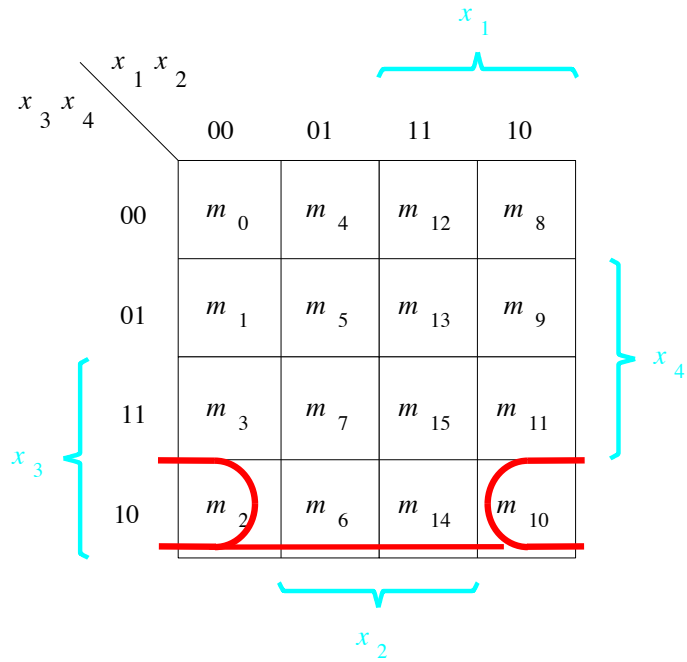
Some Valid Groupings



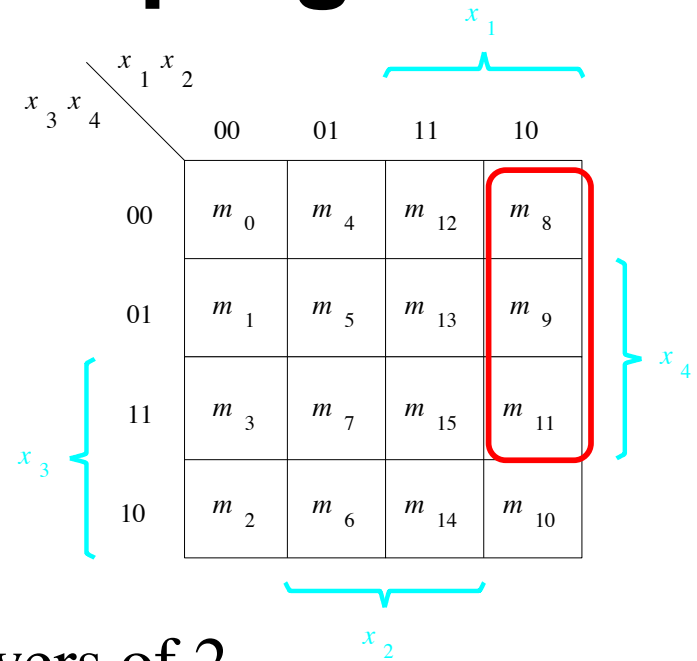
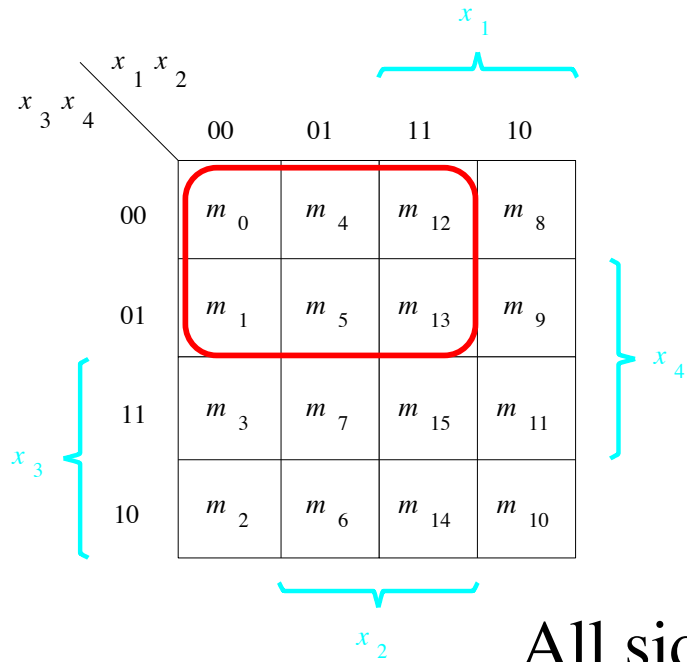
Some Valid Groupings



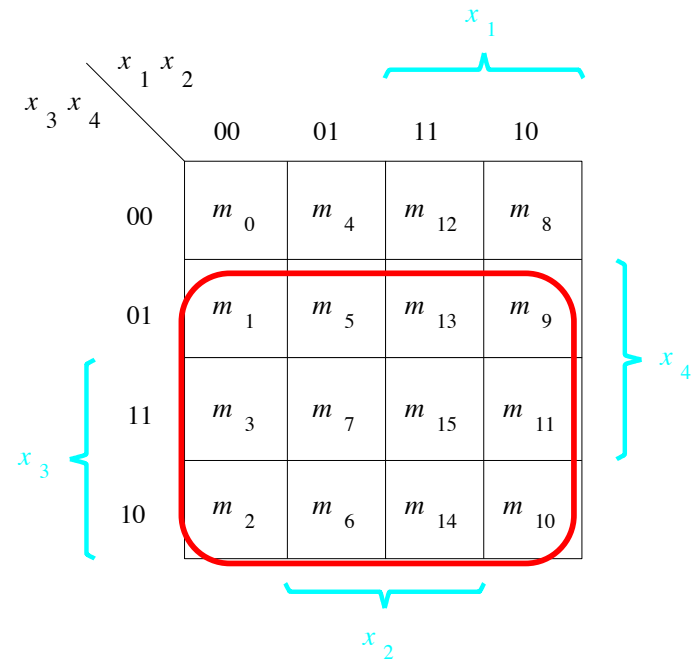
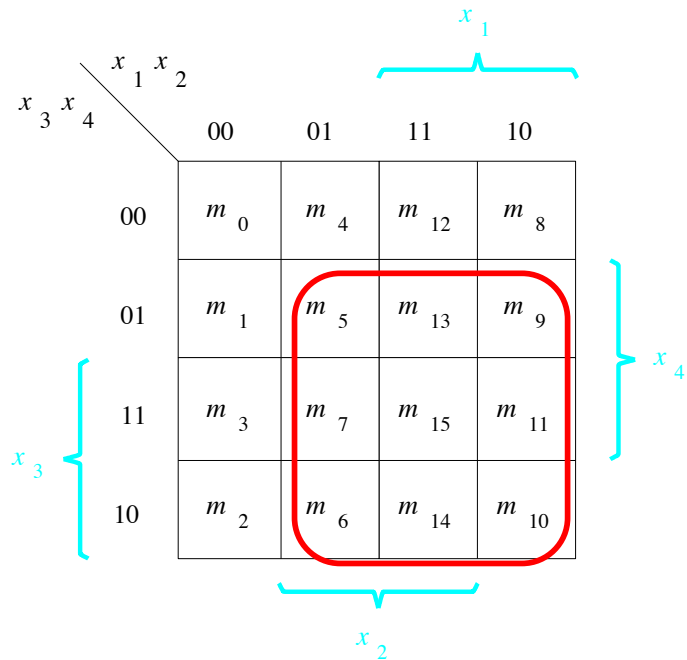
Some Valid Groupings



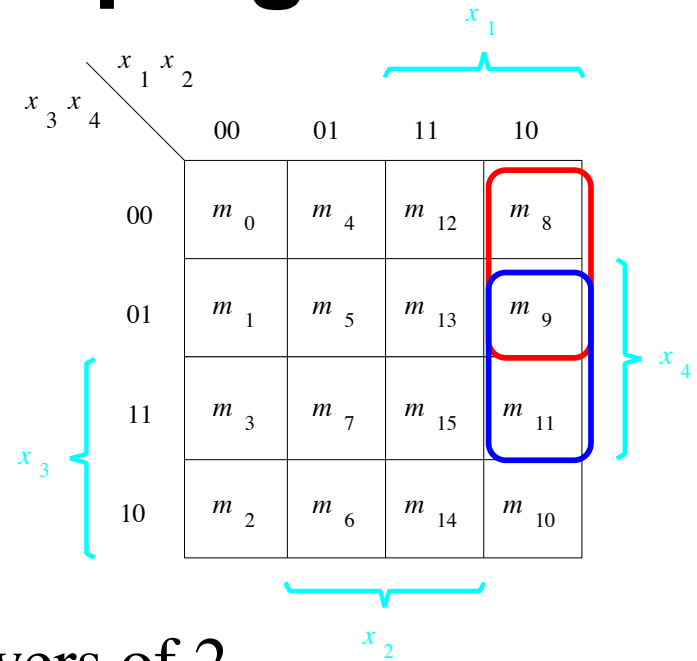
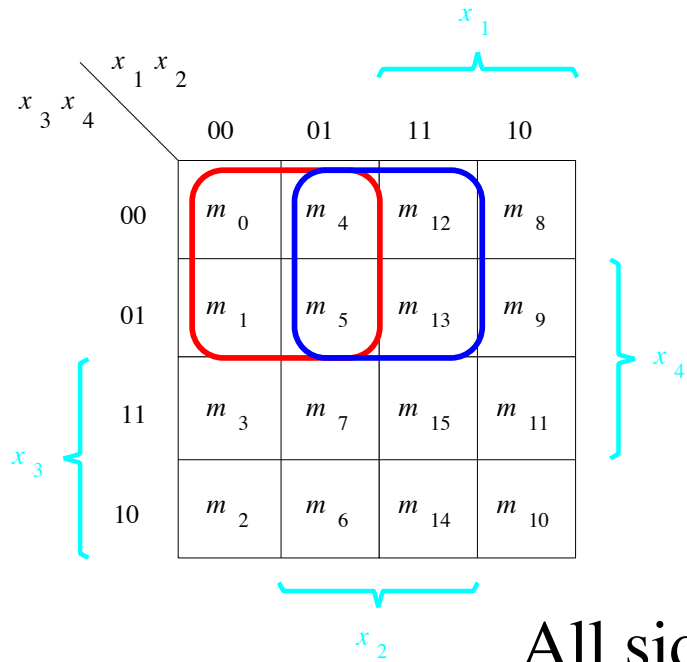
Some Invalid Groupings



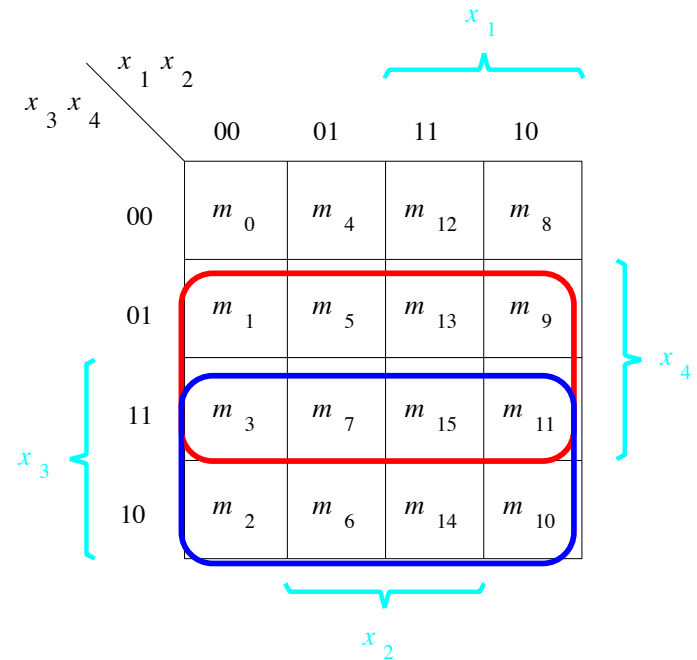
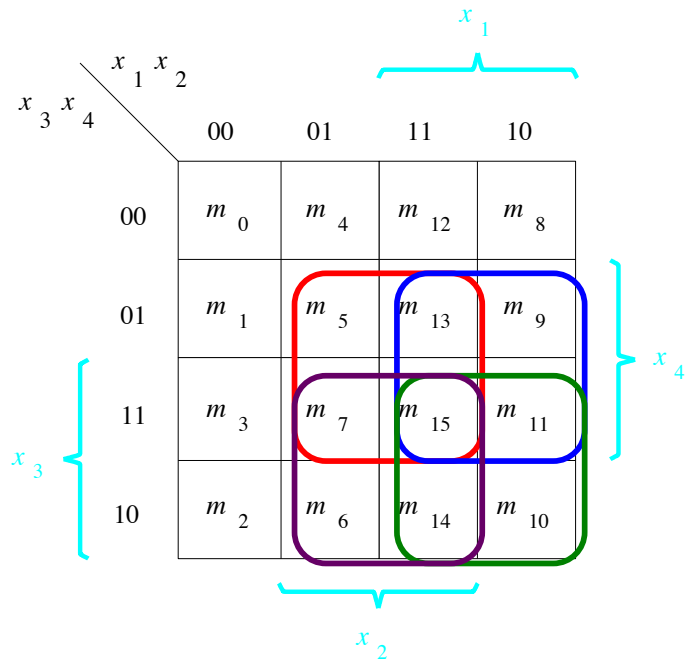
All sides must be powers of 2.



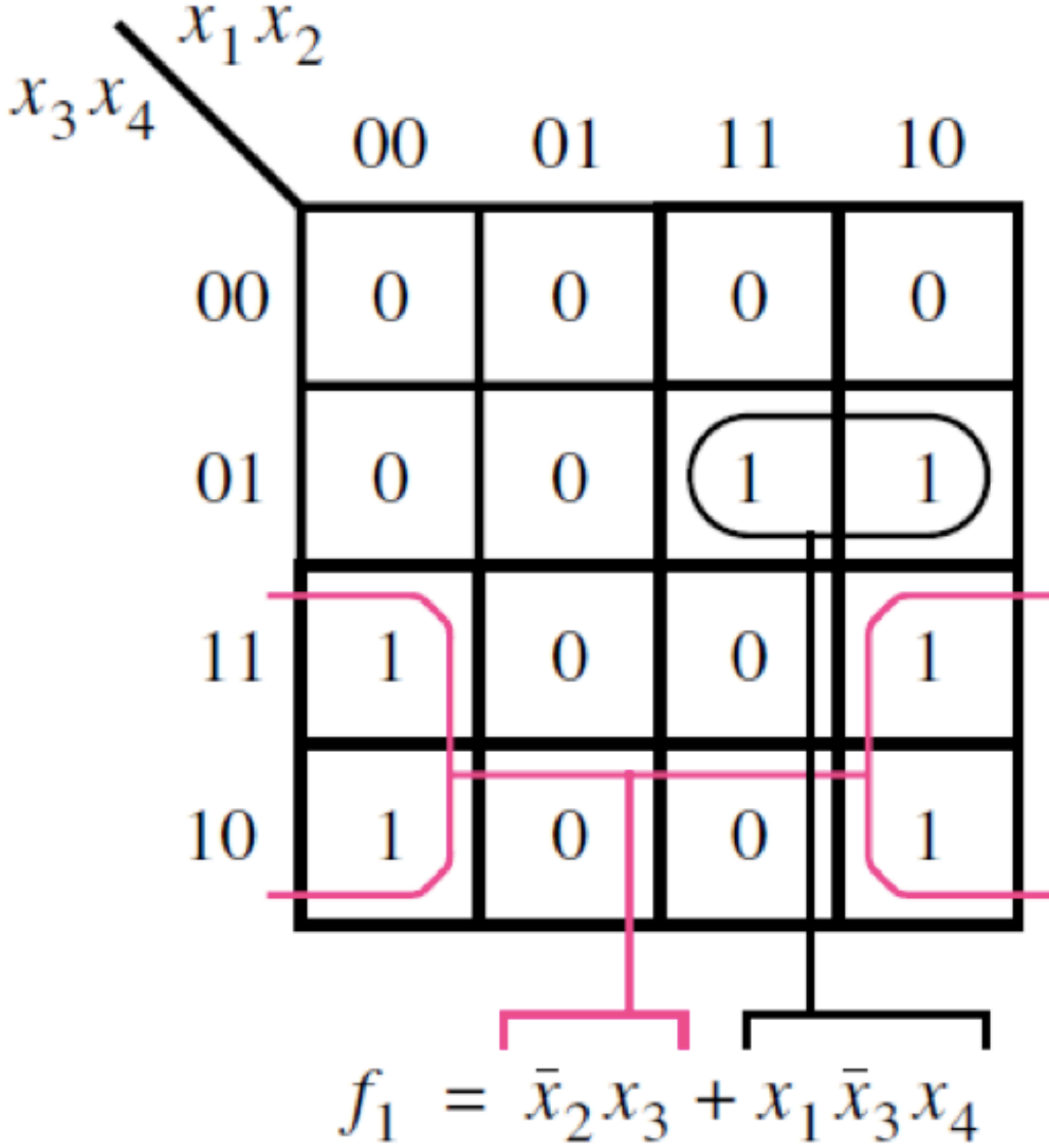
Some **valid** Groupings



All sides must be powers of 2.

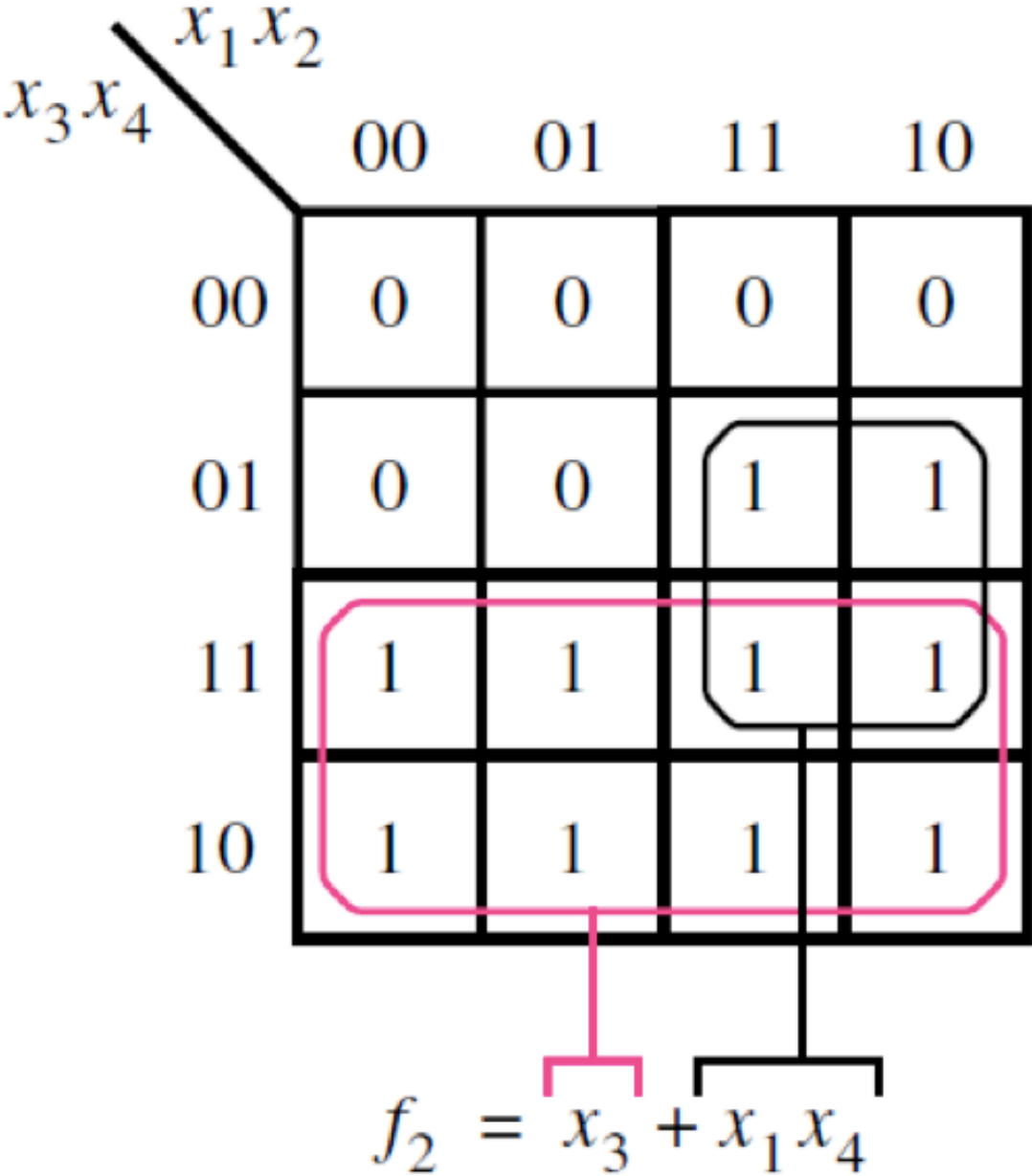


Example of a four-variable Karnaugh map



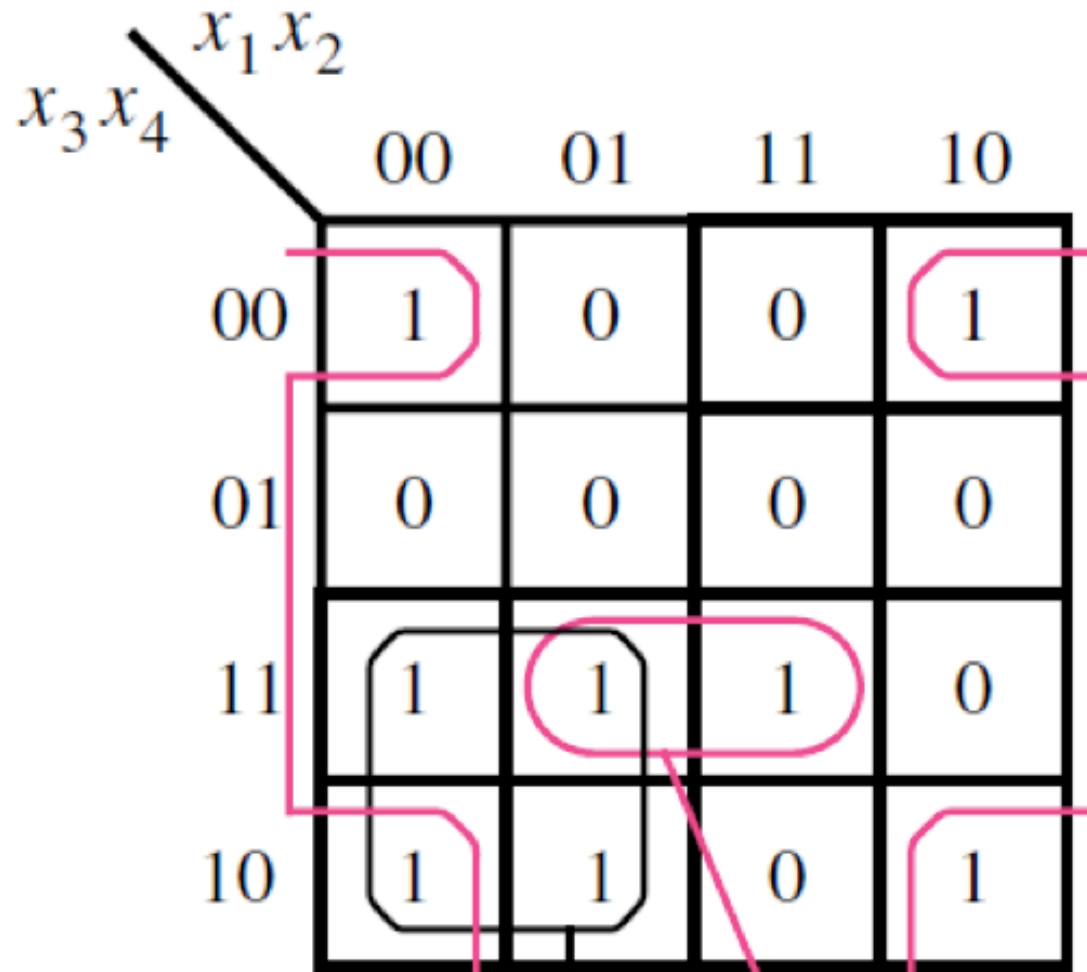
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



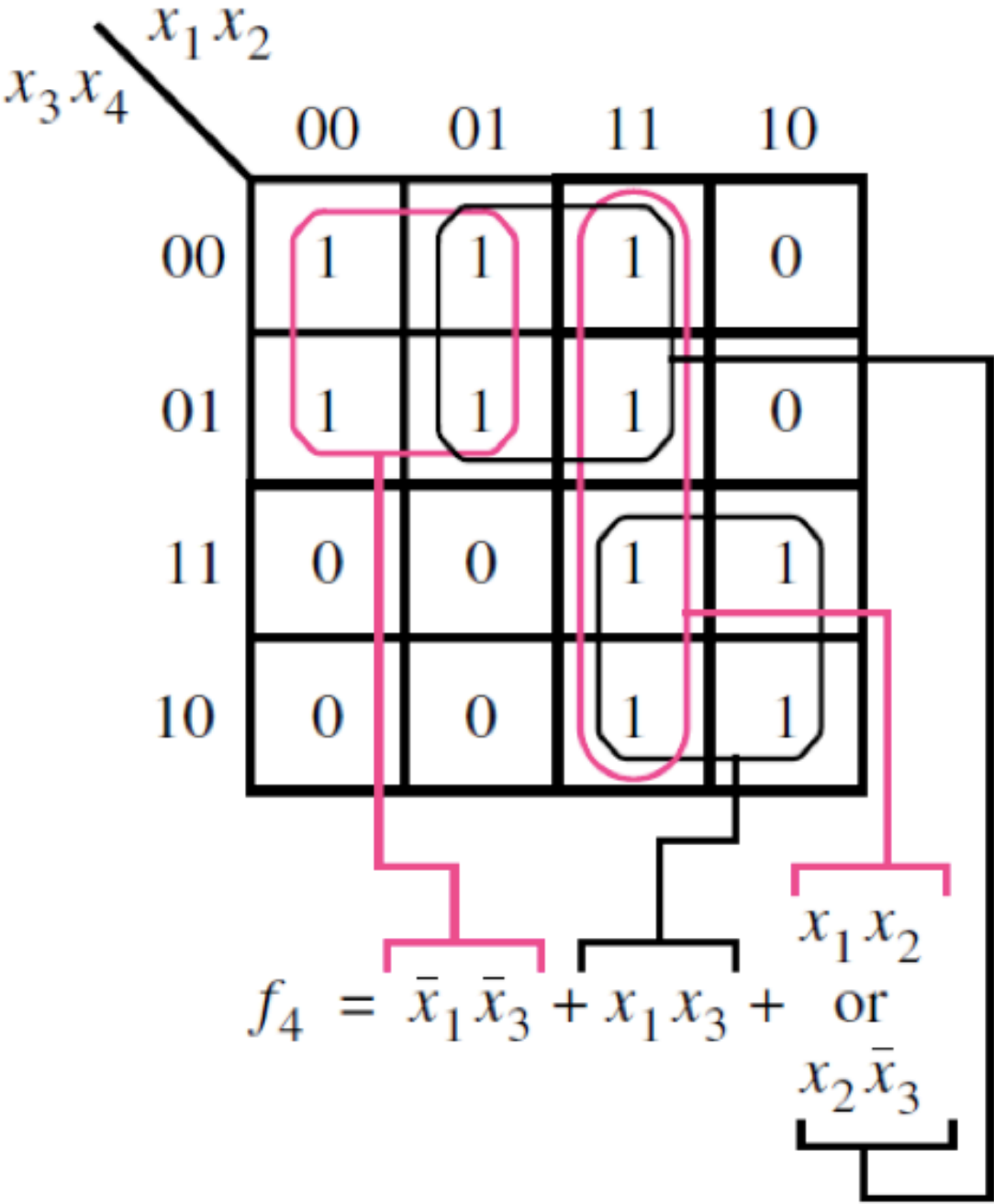
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



$$f_3 = \bar{x}_2\bar{x}_4 + \bar{x}_1x_3 + x_2x_3x_4$$

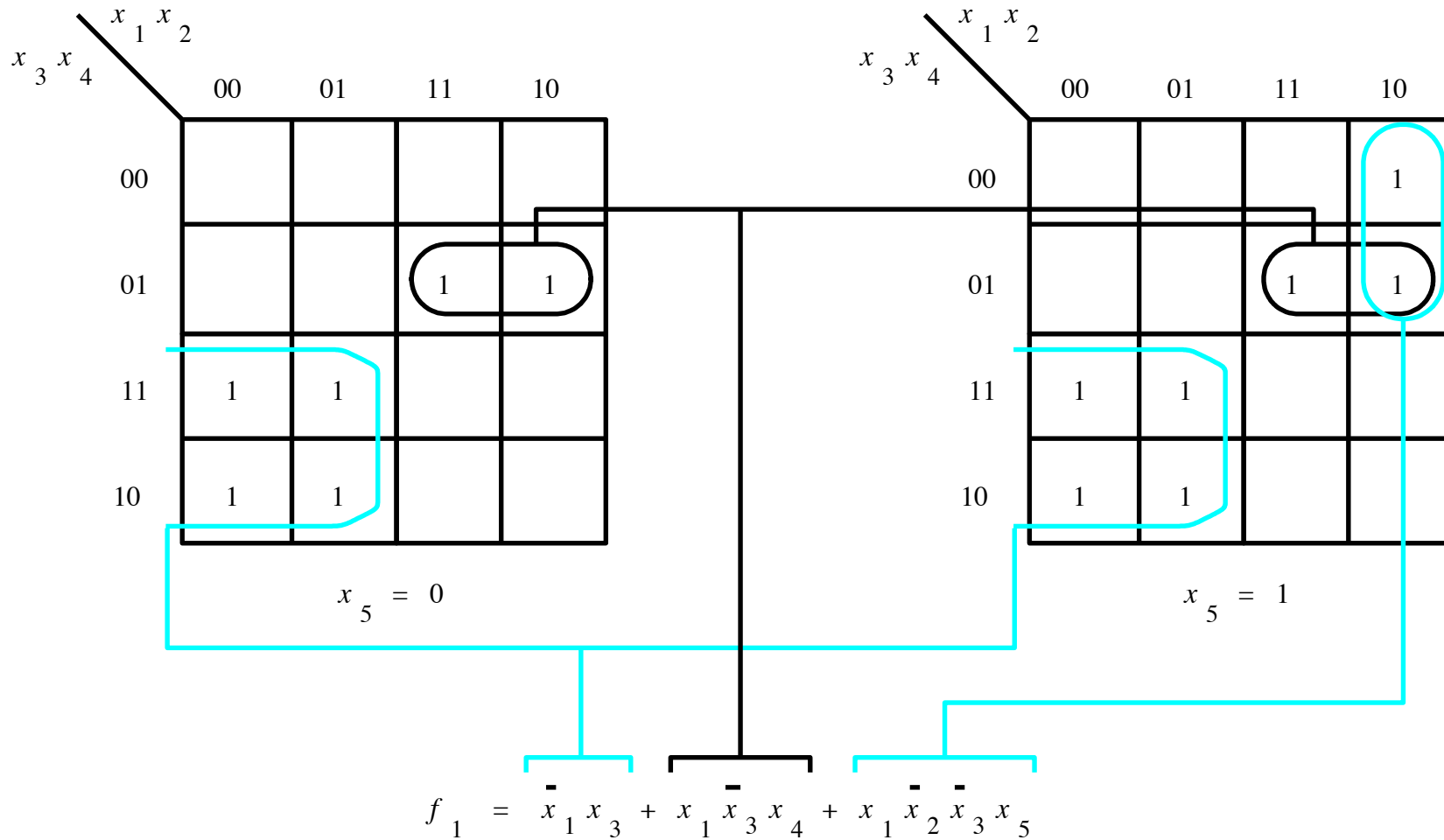
Example of a four-variable Karnaugh map



[Figure 2.54 from the textbook]

Five-Variable K-Map

A five-variable Karnaugh map



[Figure 2.55 from the textbook]

Questions?

THE END