Cpr E 281 HW04

ELECTRICAL AND COMPUTER ENGINEERING

IOWA STATE UNIVERSITY

# Minimization and Karnaugh Maps

## Assigned: Week 4

## Due Date: Sep. 14, 2020

**P1 (15 points):** Produce the simplified sum-of-products (SOP) expressions for the following K-maps:

$F_1$

| C \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 |

$F_2$

| YZ \ WX | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 1 |
| 01 | 0 | 0 | 0 | 0 |
| 11 | 1 | 1 | 1 | 1 |
| 10 | 0 | 0 | 0 | 0 |

$F_3$

| YZ \ WX | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 1 |
| 01 | 1 | 1 | 0 | 0 |
| 11 | 0 | 0 | 0 | 1 |
| 10 | 0 | 0 | 0 | 0 |

**P2 (15 points):** For each shorthand expression below, derive the simplest SOP expression:

a. $G_1(w,x,y,z) = \sum m(1,3,4,5,6,7,12,13,15)$
b. $G_2(x,y,z) = \sum m(0,1,2,6)$
c. $G_3(w,x,y,z) = \sum m(0,1,3,8,15)$

**P3. (20 points)** A four-variable function that is equal to 1 if any three or all four of its variables are equal to 1 is called a *majority* function.

a. Write the truth table for the majority function.

b. Use a K-map to derive the simplest SOP expression for this majority function.

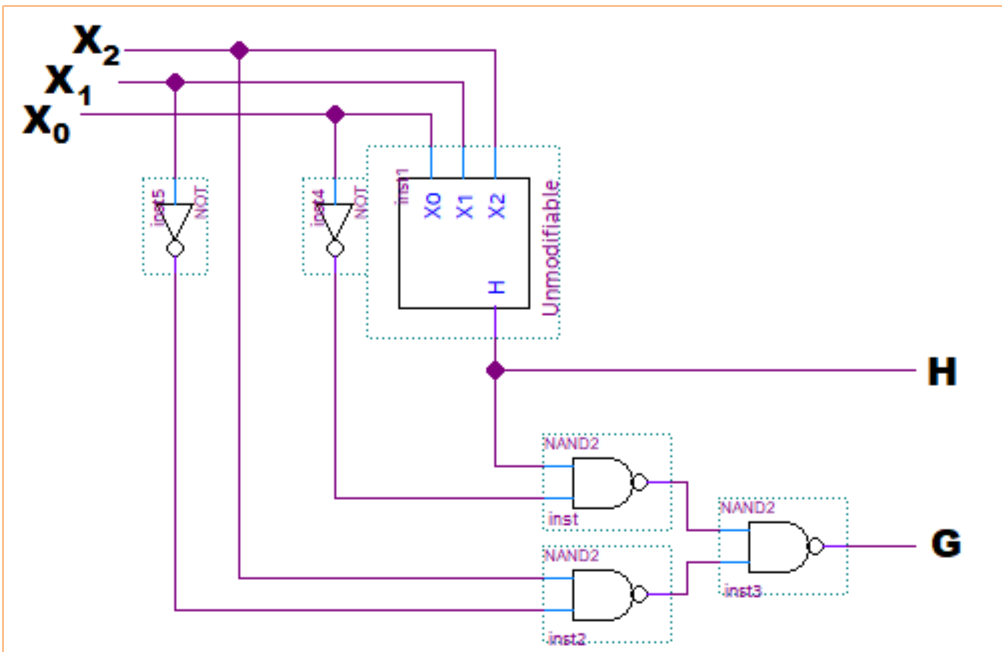
c. Use a K-map to derive the simplest POS expression for this majority function.

d. Compare the costs of the circuits implementing the expressions in part(b) and part(c) in terms of the total number of gates plus the total number of inputs.

**P4 (15 points):** For each expression below, derive the simplest POS expression:

a. $H_1(a,b,c,d) = \prod M(0,2,5,7,8,10,13,15)$
b. $H_2(a,b,c) = \prod M(2,3,6,7)$
c. $H_3(a,b,c,d) = \prod M(0,4,5,7,8,11,12)$

**P5 (20 points):** In the circuit below, there is a block of "unmodifiable circuitry" with three inputs and one output H. The expression for H is as follows: $H = \bar{x}_1\bar{x}_0 + x_2x_1 + \bar{x}_2x_1x_0$. Note that the circuit output H and the output of the unmodifiable block have the same name. This is not strictly required in the circuits that will be analyzed in the future, but, for ease in the reader's understanding of this problem, have been given the same name.




a. Write the expression for H in shorthand.
b. Using Boolean algebra, show that the expression for G in shorthand is $G(x_2, x_1, x_0) = \sum m(0,4,5,6)$
c. The three-bit inputs to this circuit are the binary representation of a three-bit non-negative (unsigned) integer X, where (by convention) $x_0$ represents the least-significant bit (LSB) of the binary representation of the integer X. If X=3, what are the values of H and G, respectively?
d. What values of X will produce a result such that $\overline{H + G} = 1$?

**P6 (15 points):** Use Karnaugh Maps to convert the following expressions to <u>simplified SOP expressions</u>:

I: $Q_1(A,B,C,D) = \bar{A}\bar{C}D + \bar{A}B\bar{C} + BD + A\bar{C}D + A\bar{B}C$

II: $Q_2(A,B,C,D) = \prod M(5,13)$

III: $Q_3(A,B,C,D) = (B + \bar{C} + D)(A + C + D)(\bar{B} + \bar{C} + D)$