HCI/CprE/ComS 575: Computational Perception
Homework 1 (part 1)
Out: Thursday Jan 17, 2019
Due: Thursday Jan 31, 2019

For extra credit submit part 1 (only part 1) by Saturday Jan 19 before 4pm.
(submit a scanned copy, or cellphone pictures of your pages, via e-mail to the instructor)

Note: Not all homeworks are created equal :(

These problems are designed to test your background knowledge in linear algebra and math in general. Please show the intermediary steps of your calculations (not just the final answers).

1. What is 2+2?
2. What is 2+2 in the ternary number system (i.e., one with base 3)?
3. Convert the decimal number 153 to binary.
4. In the hexadecimal system, what is the value of A + B?
5. Convert the hexadecimal number BEEF to binary.
6. What is the remainder when 123456789 is divided by 9?
7. The whole number N is divisible by 7. If N is divided by 2 or 3 or 4 or 5, then the remainder is 1 in each case. What is the smallest value that N can be?
8. Three brothers have the same birthday, but were born in different years. Four years ago, the oldest brother's age was 5 times the age of the youngest brother at that time, and the age of the middle brother was 3 times the age of the youngest brother at that time. Today, the middle brother's age is twice the age of the youngest brother. Today, how old is the oldest brother?
9. Two standard dice are rolled. One is red and one is green. What is the probability that the product of the two numbers on top is divisible by 3?
10. Compute the value of "n choose k", i.e., $\left(\begin{array}{c} n \\ k \end{array} \right)$, for $n = 5$ and $k = 3$.
11. What are the numbers in the 12-th row of Pascal's triangle?
12. Find a formula for the sum $\sum_{k=0}^{n} 2^k$.

13. Derive a formula for the following sum $1 + 2 + 3 + \ldots + n$.

14. Given that $1^2+2^2+3^2+\ldots+n^2 = \dfrac{n(n+1)(2n+1)}{6}$ derive a similar formula for $1^2+2^2+3^2+\ldots+(n-1)^2$.

15. What is the value of $\log_2 1024$?

16. Compute $\displaystyle\int_0^1 x \, \mathrm{d}x$.

17. Compute $\displaystyle\int_0^1 x^2 \, \mathrm{d}x$.

18. Compute $\displaystyle\int_0^1 (\cos^2 x + \sin^2 x) \, \mathrm{d}x$.

19. Prove the Pythagorean theorem in two different ways.

20. Let A be an $m \times n$ matrix and let x be a column vector of size n. How many additions and multiplications are required to compute the matrix-vector product Ax?

21. Let A be an $m \times n$ matrix and let B be an $n \times p$ matrix. How many additions and multiplications are required to compute the matrix product AB?

22. Let A be an $n \times n$ square matrix. How many additions and multiplications are required to compute A^k, i.e., the matrix multiplied by itself k times? Assume that the direct multiplication approach without any optimizations is taken.

23. What is the computational complexity, using big-O notation, for the following algorithms: Bubble sort, Insertion sort, Quicksort, Linear search, Binary search, Matrix-Vector product, Matrix-Matrix product, Matrix inversion, FFT, IFFT? Assume that all arrays and vectors are of size N and all matrices are $N \times N$.

24. Calculate the following determinants.

$$(a) \begin{vmatrix} 1 & 3 \\ -6 & 5 \end{vmatrix} \quad (b) \begin{vmatrix} \alpha - 3 & 7 \\ -4 & \alpha - 2 \end{vmatrix} \quad (c) \begin{vmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{vmatrix} \quad (d) \begin{vmatrix} -2 & 4 & 2 \\ 2 & 1 & -1 \\ 4 & 7 & 3 \end{vmatrix}$$

25. Show that the value of the determinant does not depend on θ.

$$\begin{vmatrix} sin\theta & cos\theta & 0 \\ -cos\theta & sin\theta & 0 \\ sin\theta - cos\theta & sin\theta + cos\theta & 1 \end{vmatrix}$$

26. Calculate the inverse of the given matrix.

$$(a)\quad \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad (b)\quad \begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix} \quad (c)\quad \begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix} \quad (d)\quad \begin{bmatrix} 0 & 0 & 0 & k_1 \\ 0 & 0 & k_2 & 0 \\ 0 & k_3 & 0 & 0 \\ k_4 & 0 & 0 & 0 \end{bmatrix}$$

27. Solve the following systems of equations.

$$(a)\quad \begin{aligned} 5x_1 - 3x_2 &= -1 \\ 2x_1 + 5x_2 &= 12 \end{aligned} \qquad (b)\quad \begin{aligned} 5x_1 + 3x_2 + 2x_3 &= 4 \\ 3x_1 + 3x_2 + 2x_3 &= 2 \\ x_2 + x_3 &= 5 \end{aligned}$$

28. Let $u = (1, 3, -2)$, $v = (2, 0, 1)$, and $w = (1, 3, 4)$. Compute:

$$(a)\ u \cdot v \qquad (b)\|u\| \qquad (b)\ v \times w \qquad (c)\ u \times (v \times w) \qquad (d)\ (u \times v) \times w$$

29. Find all values of λ for which the determinant of the matrix is equal to 0.

$$(a)\quad \begin{vmatrix} \lambda - 3 & 0 \\ 8 & \lambda + 1 \end{vmatrix} \quad (b)\quad \begin{vmatrix} \lambda & 3 \\ 4 & \lambda \end{vmatrix} \quad (c)\quad \begin{vmatrix} \lambda - 5 & 0 & 1 \\ 1 & \lambda - 1 & 0 \\ -7 & 1 & \lambda \end{vmatrix}$$

30. Find the eigenvalues of the following matrices.

$$(a)\quad \begin{bmatrix} -1 & 6 \\ 0 & 5 \end{bmatrix} \quad (a)\quad \begin{bmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{bmatrix} \quad (b)\quad \begin{bmatrix} 3 & 0 & 0 \\ -2 & 7 & 0 \\ 4 & 8 & 1 \end{bmatrix}$$

31. Given the vector $v = [x, y]^T$ find a rotation matrix R that rotates the vector by $60°$ counterclockwise. Give the values for the vector $v' = [x', y']^T$ in terms of x and y, where $v' = Rv$.

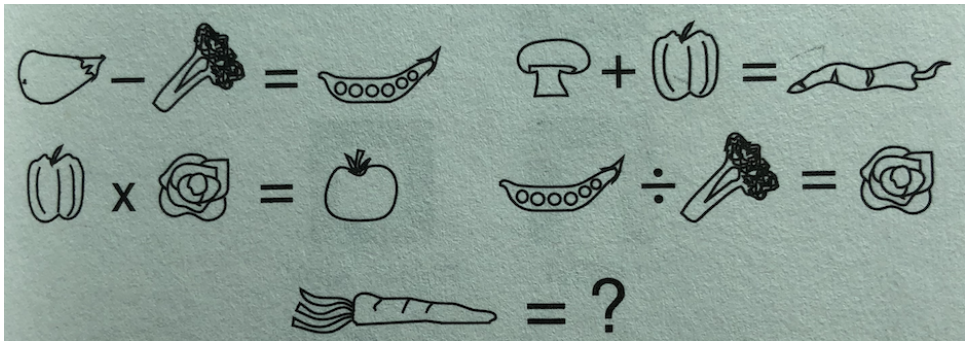
32. By example, show that vector cross-product is not commutative.

33. By example, show that matrix multiplication is not commutative.

34. Give the formulas for the 1D and 2D probability density function of the normal distribution.

35. Give the formulas for the discrete Fourier transform and its inverse. Explain their meaning with one paragraph for each.

36. The numbers one through nine are represented by nine vegetables in the equations below. Each veggie represents the same number throughout. If the broccoli equals three, what is the identity of the carrot?



37. Find the locations of the words in the following word search puzzle.

Algebra Word Search

C	R	A	M	B	A	L	I	N	E	A	R	E	I
R	O	T	L	Q	C	T	L	L	R	L	A	N	R
E	O	O	A	U	A	A	R	R	L	A	A	M	I
I	T	E	I	A	L	A	I	M	O	N	O	M	R
I	A	L	M	D	T	E	A	R	A	N	E	Z	B
L	R	B	O	R	A	R	B	E	G	L	A	L	O
R	T	A	N	A	E	R	Q	O	M	T	I	I	B
O	L	I	I	T	C	O	N	S	T	A	N	T	I
R	T	R	R	I	A	D	C	I	B	U	C	E	N
E	G	A	T	C	A	A	E	R	I	D	M	C	O
Z	L	V	R	M	E	A	B	G	C	U	N	O	M
O	L	O	R	R	Z	R	L	A	R	E	L	R	I
A	A	Z	A	I	R	O	I	A	N	E	A	E	A
E	Q	U	A	T	I	O	N	G	I	D	E	C	L

CUBIC
VARIABLE
QUADRATIC
ROOT
TRINOMIAL
BINOMIAL
ZERO
DEGREE
MONOMIAL
CONSTANT
LINEAR
ALGEBRA
EQUATION

Play this puzzle online at : http://thewordsearch.com/puzzle/352/

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The Matlab book, which is a required textbook for this class, shows several plot types on pages 170-174 (old edition of the book) or pages 184-188 (new edition).

Pick one plot type from each page and try to draw something similar, but it should not be the exact same plot as in the book. For each problem submit your matlab file and an output image. Name your files: myplot1.m, myplot2.m, ... , myplot5.m. Similarly, name your output images myoutput1.jpg, myoutput2.jpg, ... , myoutput5.jpg. (PNG or GIF file format is also acceptable).

Submit a ZIP file with all of your files to the TA.

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Solve 10 of the following 19 problems (see the following pages). You can pick any 10. Your grade will not depend on your problem selection. Submit a separate matlab file (and any supporting files if needed) for each problem. Name your files using the problem and page number. For example, your solution file for exercise 3 on page 84 should be named e3_page84.m

Submit a ZIP file with all of your files to the TA.