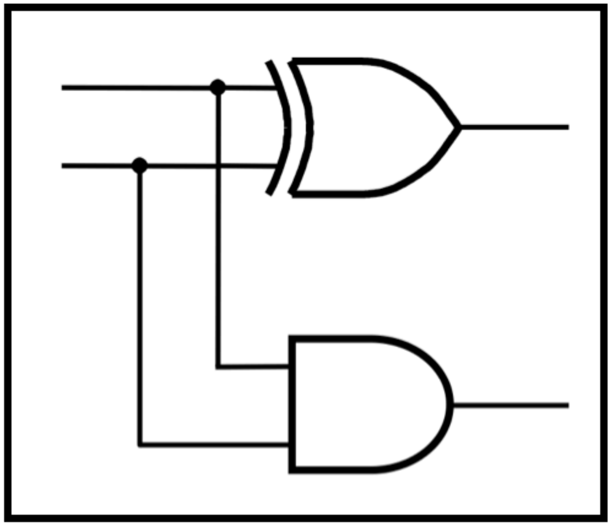


CprE 281: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

Karnaugh Maps

CprE 281: Digital Logic
Iowa State University, Ames, IA

Administrative Stuff

- **HW4 is out**
- **It is due on Monday Sep 23 @ 4 pm**
- **It is posted on the class web page**
- **I also sent you an e-mail with the link.**

Administrative Stuff

- **Sample homework solutions are posted on Canvas**
- **Look under ‘Files’**

Quick Review

Do You Still Remember This Boolean Algebra Theorem?

14a. $x \cdot y + x \cdot \overline{y} = x$ Combining

14b. $(x + y) \cdot (x + \overline{y}) = x$

Let's prove 14.a

x	y	$x \cdot y + x \cdot \overline{y} = x$
0	0	
0	1	
1	0	
1	1	

Let's prove 14.a

x	y	$\mathbf{x \cdot y + x \cdot \overline{y} = x}$
0	0	0
0	1	0
1	0	0
1	1	1

Let’s prove 14.a

x	y	$x \cdot y$	$+$	$x \cdot \bar{y}$	$= x$
0	0	0		0	
0	1	0		0	
1	0	0		1	
1	1	1		0	

Let’s prove 14.a

x	y	$\mathbf{x} \bullet$	$\mathbf{y} +$	$\mathbf{x} \bullet$	$\overline{\mathbf{y}} = \mathbf{x}$
0	0	0	0	0	
0	1	0	0	0	
1	0	0	1	1	
1	1	1	1	0	

Let's prove 14.a

x	y	$x \cdot y$	$+$	$x \cdot \overline{y}$	$= x$
0	0	0	0	0	0
0	1	0	0	0	0
1	0	0	1	1	1
1	1	1	1	0	1

Let's prove 14.a

x	y	$\mathbf{x \cdot y}$	$\mathbf{+}$	$\mathbf{x \cdot \overline{y}}$	$\mathbf{=}$	$\mathbf{x}$
0	0	0	0	0		0
0	1	0	0	0		0
1	0	0	1	1		1
1	1	1	1	0		1

They are equal.

Motivation

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

An approach for simplifying logic expressions.

How do we guarantee we have reached the minimum SOP/POS representation?

Two-Variable K-Map

Karnaugh Map (K-map)

- **View the function in a visual form**
- **Same information as truth table**
- **Easier to group minterms**

x_1	x_2	
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

(a) Truth table

x_2 \ x_1	0	1
0	m_0	m_2
1	m_1	m_3

(b) Karnaugh map

[Figure 2.49 from the textbook]

Minterms

x_1	x_2	
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

x_1	x_2	m_0	m_1	m_2	m_3
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

Minterm Example

x_1	x_2	
0	0	0
0	1	1
1	0	0
1	1	1

x_1	x_2	m_0	m_1	m_2	m_3	$m_1 + m_3$
0	0	1	0	0	0	0
0	1	0	1	0	0	1
1	0	0	0	1	0	0
1	1	0	0	0	1	1

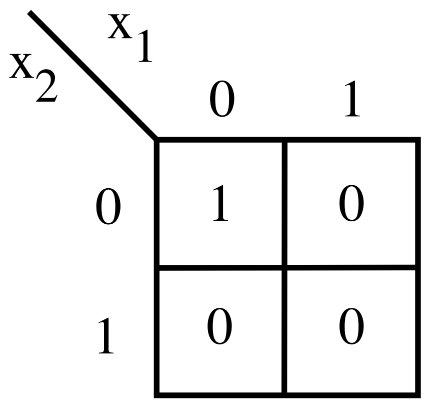
Minterm Example

x_1	x_2	
0	0	0
0	1	1
1	0	0
1	1	1

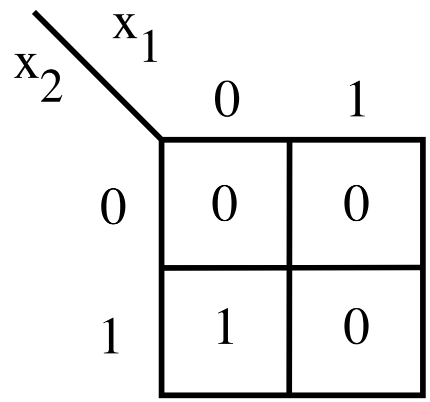
x_1	x_2	m_0	m_1	m_2	m_3	$m_1 + m_3$
0	0	1	0	0	0	0
0	1	0	1	0	0	1
1	0	0	0	1	0	0
1	1	0	0	0	1	1

$$\overline{x}_1 x_2 + x_1 x_2 = x_2$$

Grouping Example



m_0



m_1

Grouping Example

$x_2 \backslash x_1$	0	1
0	1	0
1	0	0

+

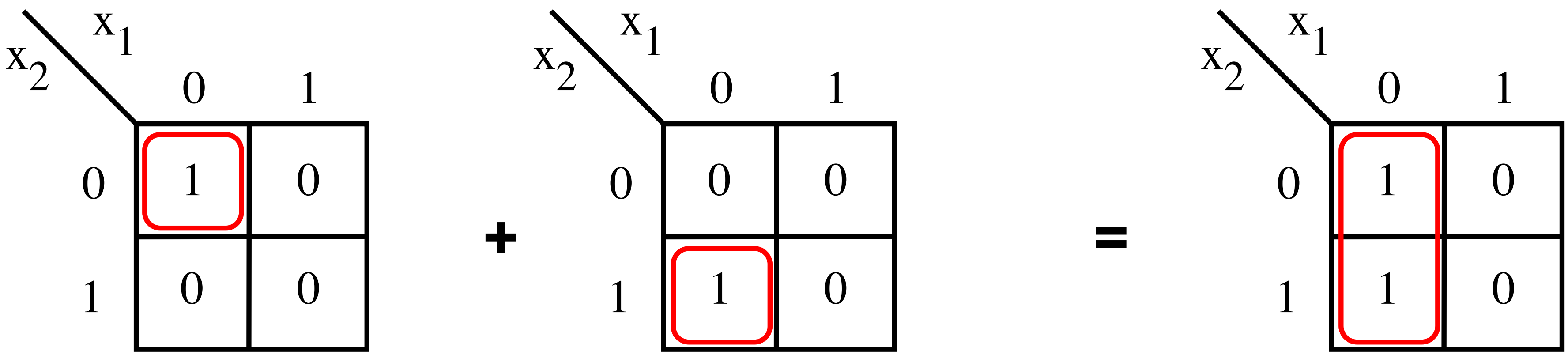
$x_2 \backslash x_1$	0	1
0	0	0
1	1	0

=

$x_2 \backslash x_1$	0	1
0	1	0
1	1	0

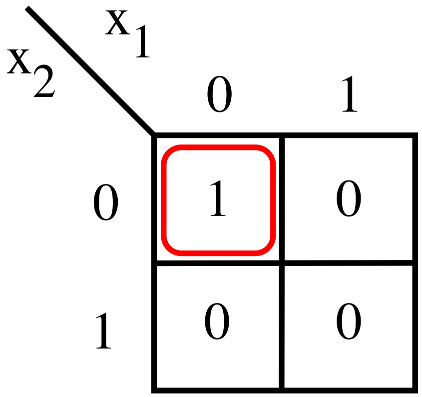
$$\mathbf{m_0 \quad + \quad m_1 \quad = \quad m_0 + m_1}$$

Grouping Example

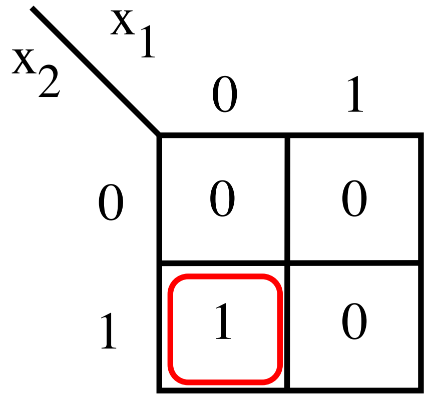


m_0 + m_1 = $m_0 + m_1$

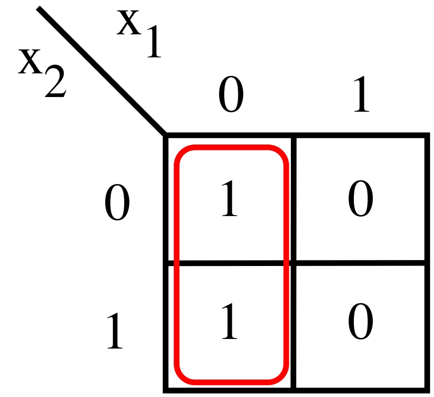
Grouping Example



+



=



$$m_0 \quad + \quad m_1 \quad = \quad m_0 + m_1$$

$$\bar{x}_1\bar{x}_2 \quad + \quad \bar{x}_1 x_2 \quad = \quad \bar{x}_1$$

Property 14a (Combining)

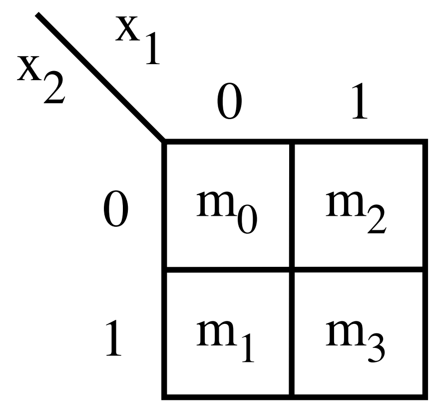
Grouping Rules

- Group “1”s with rectangles
- Both sides a power of 2:
  - 1x1, 1x2, 2x1, 2x2, 1x4, 4x1, 2x4, 4x2, 4x4
- Can use the same minterm more than once
- Can wrap around the edges of the map
- Some rules in selecting groups:
  - Try to use as few groups as possible to cover all “1”s.
  - For each group, try to make it as large as you can (i.e., if you can use a 2x2, don’t use a 2x1 even if that is enough).

Two-Variable K-map

x_1	x_2	
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

(a) Truth table



(b) Karnaugh map

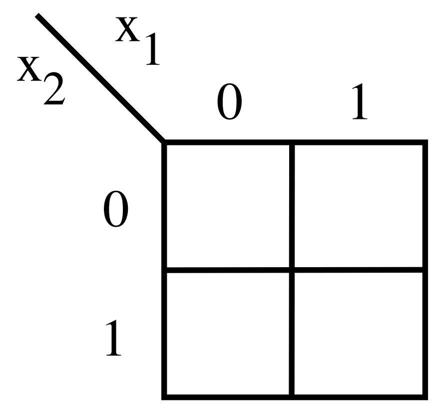
[Figure 2.49 from the textbook]

Step-By-Step Example

x_1	x_2	
0	0	1
0	1	1
1	0	0
1	1	1

1. Draw The Map

x_1	x_2	
0	0	1
0	1	1
1	0	0
1	1	1



2. Fill The Map

	x_1	x_2	
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

x_2 \ x_1	0	1
0	m_0	m_2
1	m_1	m_3

2. Fill The Map

	x_1	x_2	
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

x_2 \ x_1	0	1
0	1	0
1	1	1

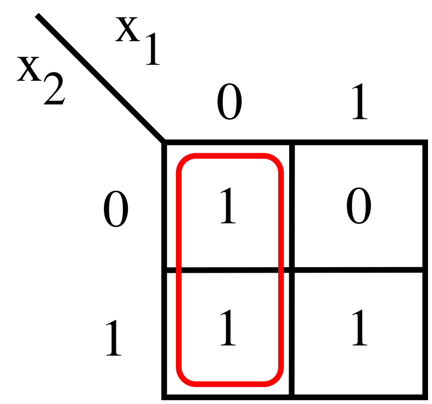
3. Group

	x_1	x_2	
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

$x_2 \backslash x_1$	0	1
0	1	0
1	1	1

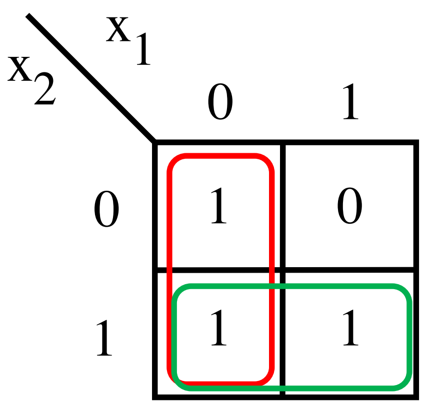
3. Group

	x_1	x_2	
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1



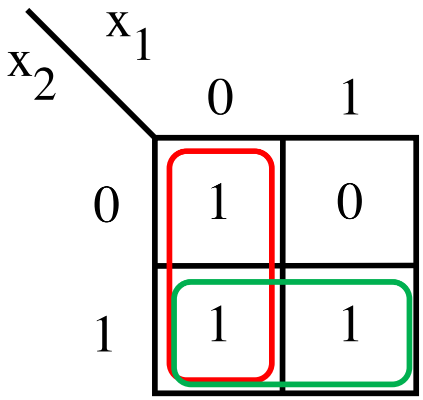
3. Group

	x_1	x_2	
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1



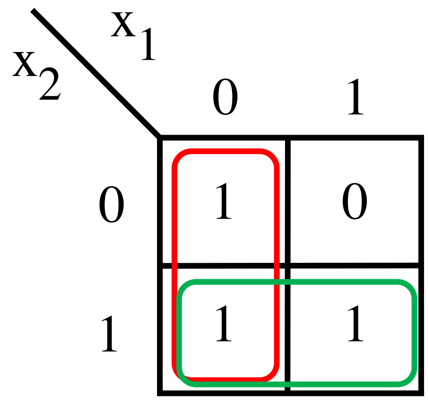
4. Write The Expression

x_1	x_2	
0	0	1
0	1	1
1	0	0
1	1	1



4. Write The Expression

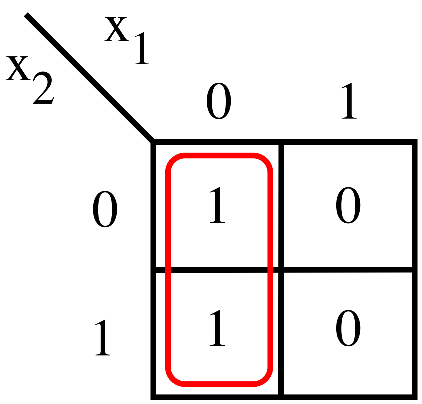
x_1	x_2	
0	0	1
0	1	1
1	0	0
1	1	1



$\overline{x}_1 + x_2$

Writing The Expression

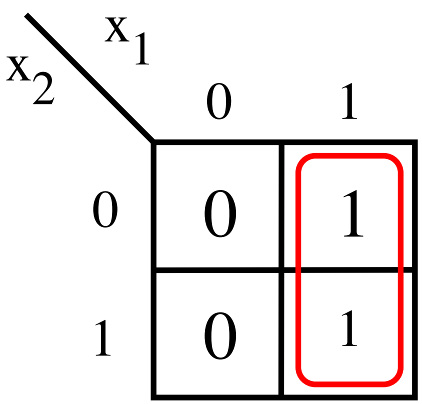
- **Find which variable is constant**



$\bar{x}_1$ is constant

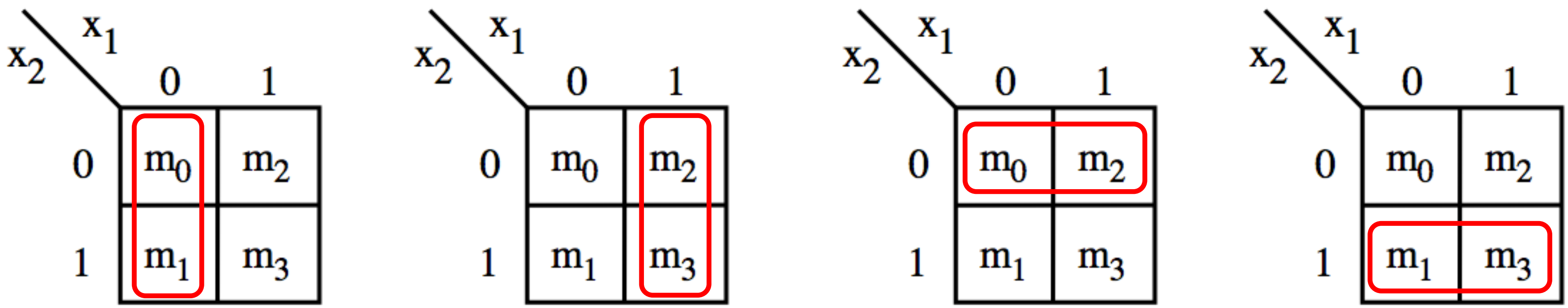
Writing The Expression

- **Find which variable is constant**

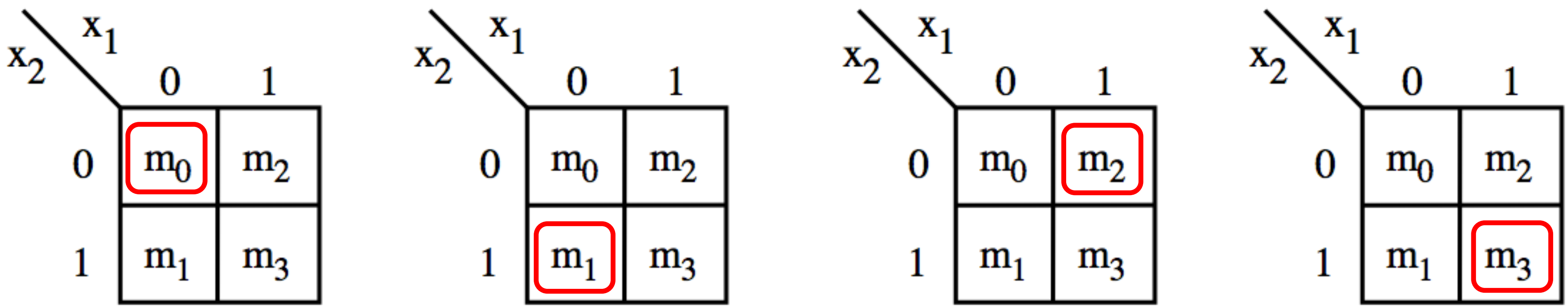


x_1 is constant

These are all valid groupings

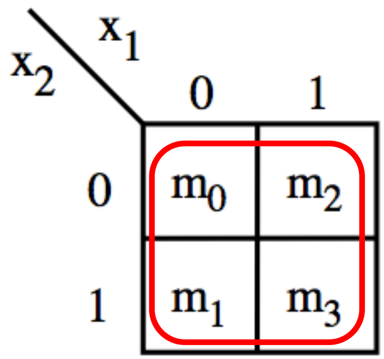


These are also valid



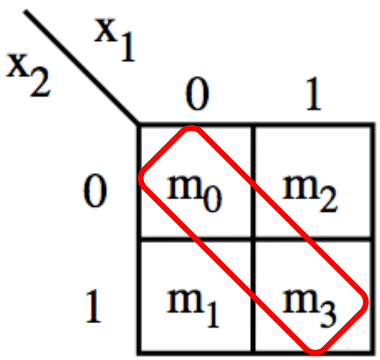
But try to use larger rectangles if possible.

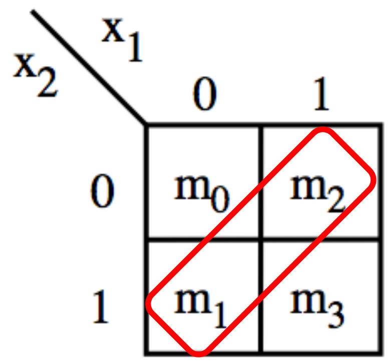
This one is valid too



In this case the result is the constant function 1.

Why are these two not valid?





Let's Find Out

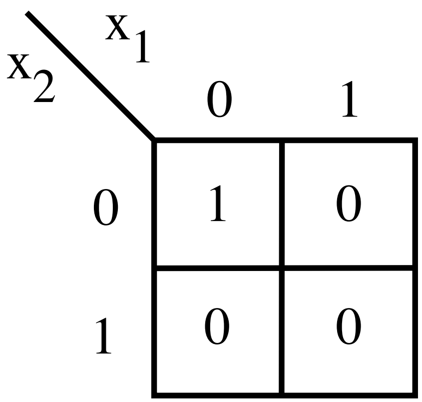
$x_2 \backslash x_1$	0	1
0	1	0
1	0	0

m_0

$x_2 \backslash x_1$	0	1
0	0	0
1	0	1

m_3

Let’s Find Out



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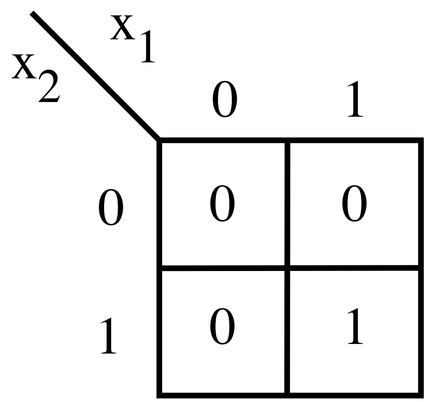

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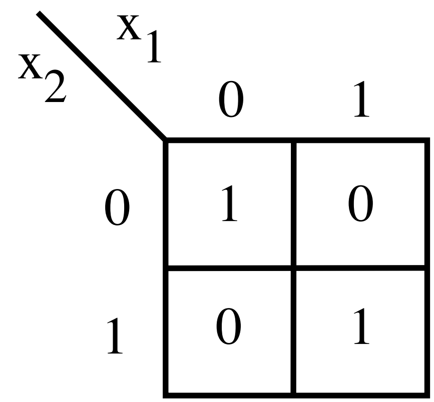
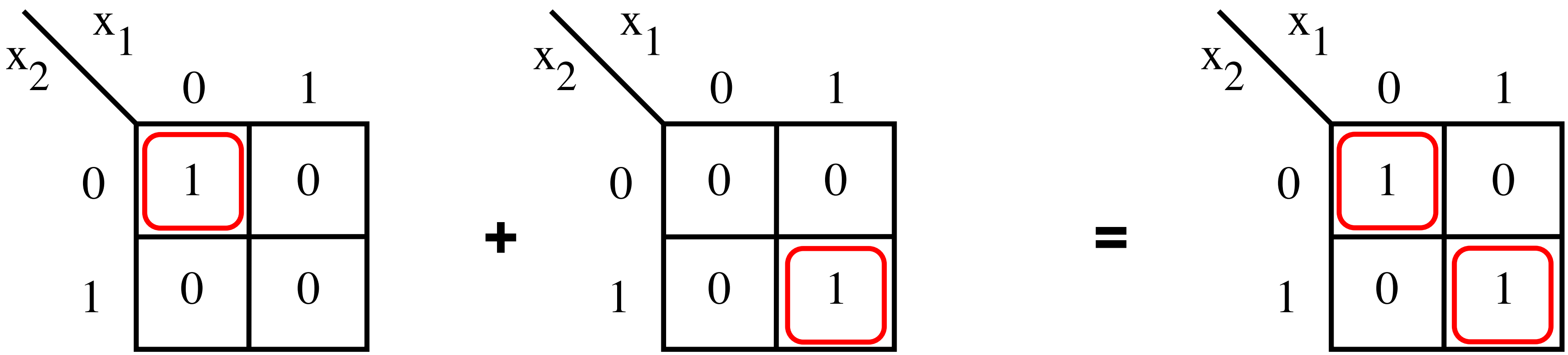


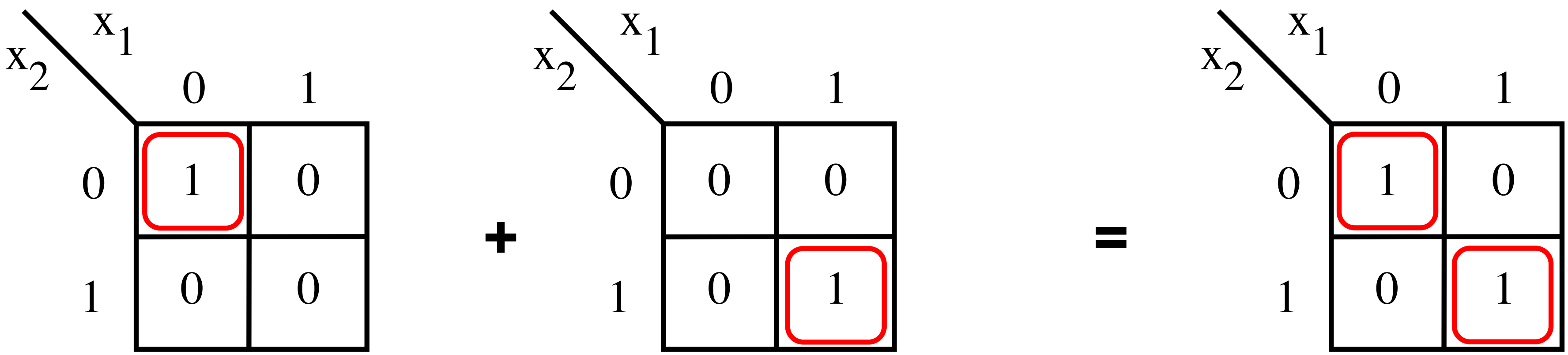
m_0 + m_3 = $m_0 + m_3$

Let's Find Out



m_0 + m_3 = $m_0 + m_3$

Let's Find Out



$$m_0 \quad + \quad m_3 \quad = \quad m_0 + m_3$$

$$\bar{x}_1\bar{x}_2 \quad + \quad x_1x_2 \quad = \quad \bar{x}_1\bar{x}_2 \ + \ x_1x_2$$

We can't use Property 14a here. This can't be simplified.

Three-Variable K-Map

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3 \ $x_1 x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

[Figure 2.51 from the textbook]

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3 \ $x_1 x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

Notice the placement of

- **Variables**
- **Binary pair values**
- **Minterms**

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3 \ $x_1 x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

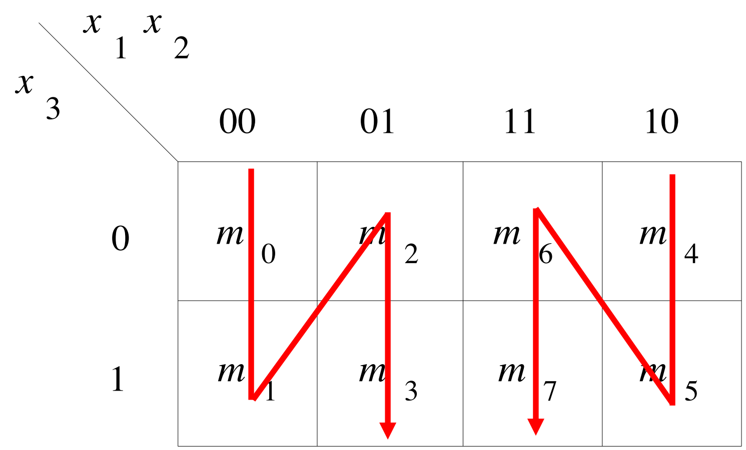
Notice the placement of

- **Variables**
- **Binary pair values**
- **Minterms**

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table



(b) Karnaugh map

Notice the placement of

- **Variables**
- **Binary pair values**
- **Minterms**

Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

	000
	001
00	**011**
01	**010**
11	**110**
10	**111**
	101
	100

Gray Code & K-map

	$s\ x_1\ x_2$
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ $s\ x_1$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

Gray Code & K-map

	$s\ x_1\ x_2$
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ $s\ x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

Gray Code & K-map

	s x_1 x_2
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ $s\ x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

Gray Code & K-map

	$s\ x_1\ x_2$
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ $s\ x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

Gray Code & K-map

	$s\ x_1\ x_2$
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ $s\ x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

Gray Code & K-map

	$s\ x_1\ x_2$
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ $s\ x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

Gray Code & K-map

	$s\ x_1\ x_2$
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ $s\ x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These two neighbors
differ only in the LAST bit

Gray Code & K-map

	$s\ x_1\ x_2$
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ $s\ x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These two neighbors differ only in the LAST bit

Gray Code & K-map

	$s\ x_1\ x_2$
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ $s\ x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These two neighbors
differ only in the FIRST bit

Gray Code & K-map

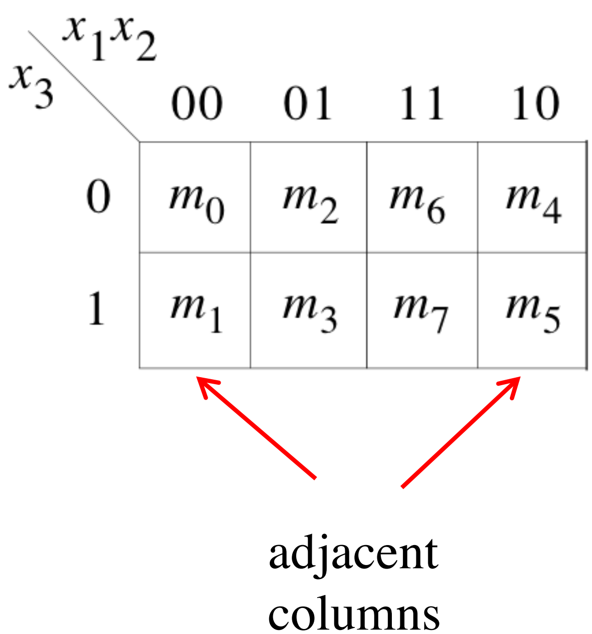
	$s\ x_1\ x_2$
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ $s\ x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These four neighbors
differ in the FIRST and LAST bit

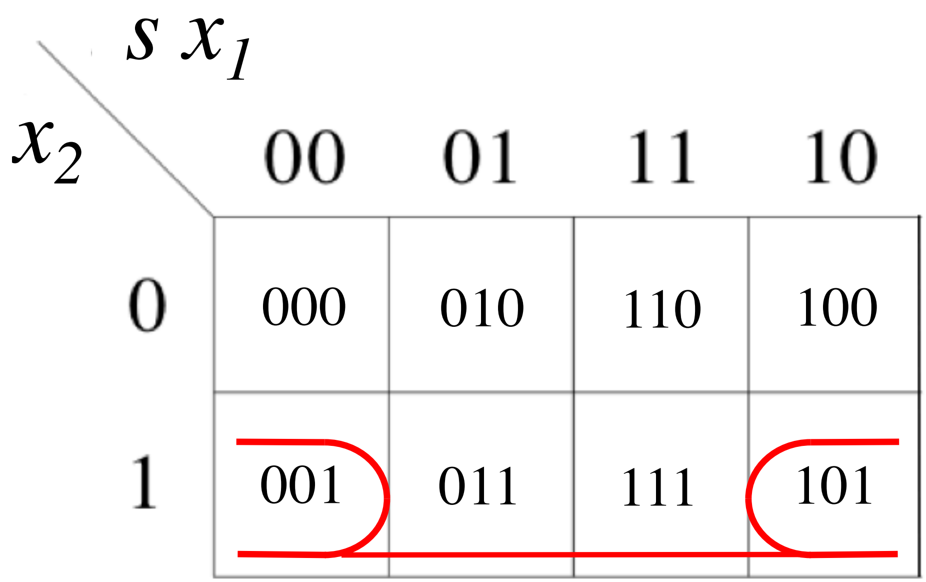
They are similar in their MIDDLE bit

Adjacency Rules



Gray Code & K-map

	$s\ x_1\ x_2$
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1



These two neighbors
differ only in the FIRST bit

Gray Code & K-map

	$s\ x_1\ x_2$
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ $s\ x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

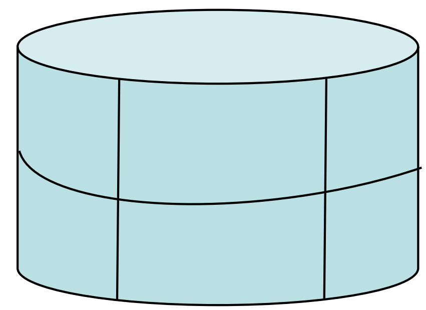
These four neighbors
differ in the FIRST and LAST bit

They are similar in their MIDDLE bit

Adjacency Rules

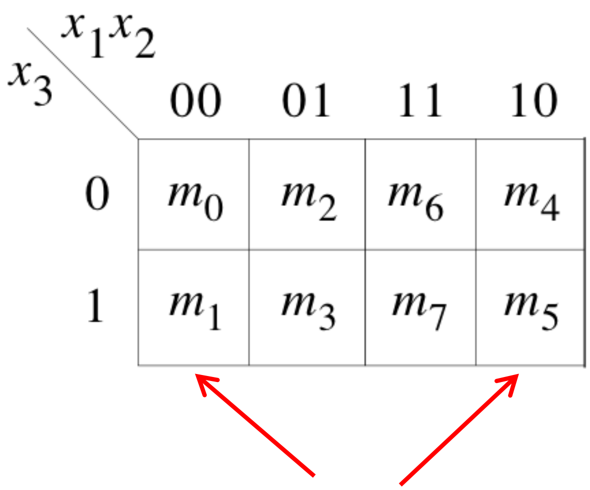
$x_3 \backslash x_1x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

adjacent
columns

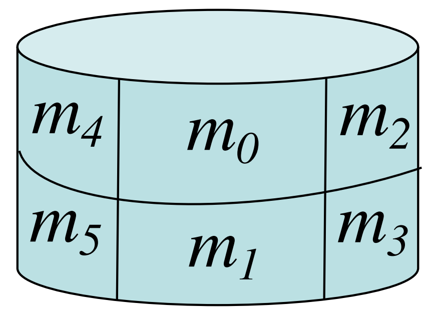


As if the K-map were
drawn on a cylinder

Adjacency Rules

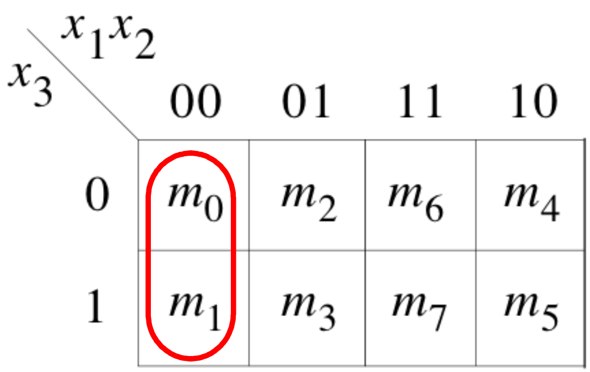


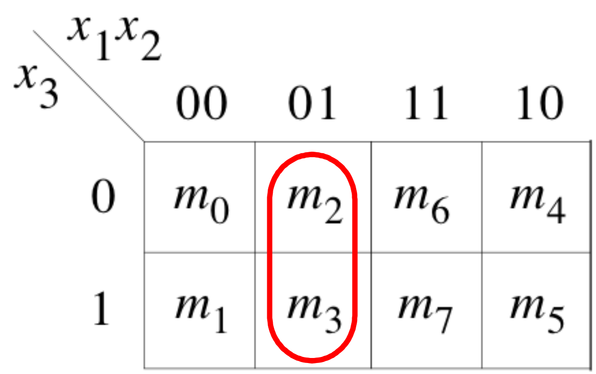
adjacent
columns

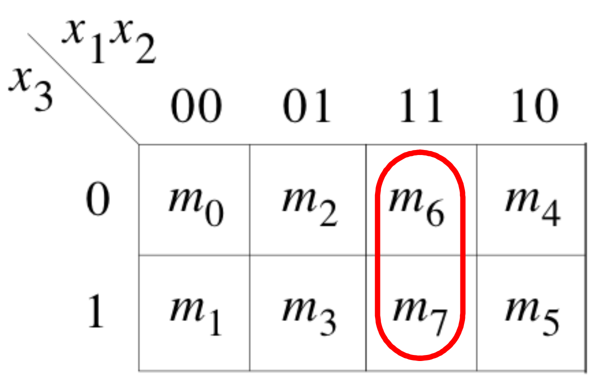


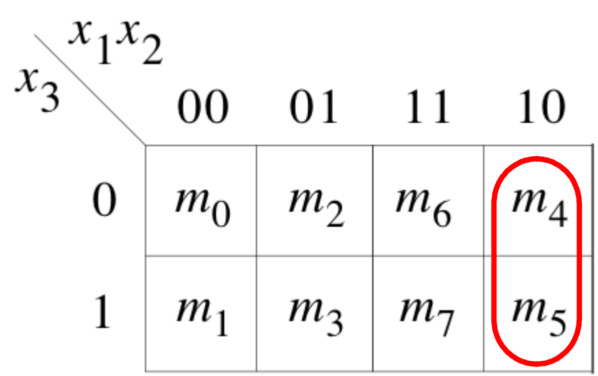
As if the K-map were
drawn on a cylinder

These are valid groupings

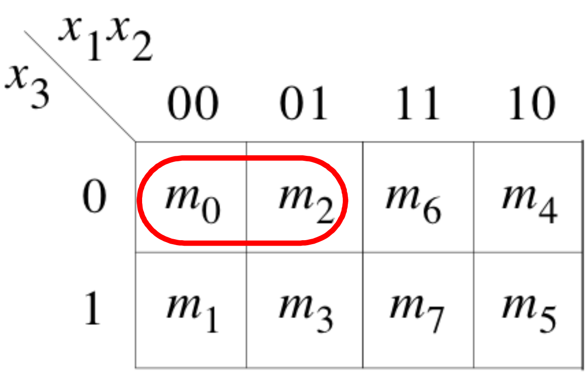


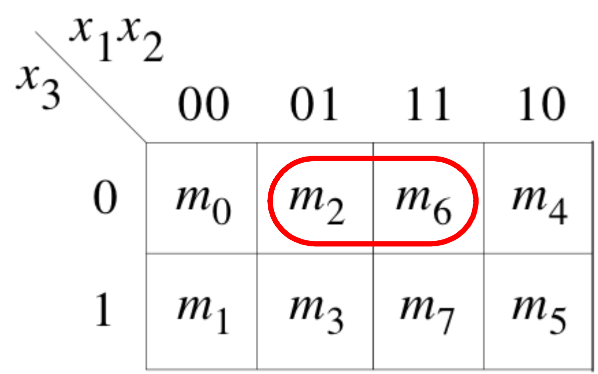


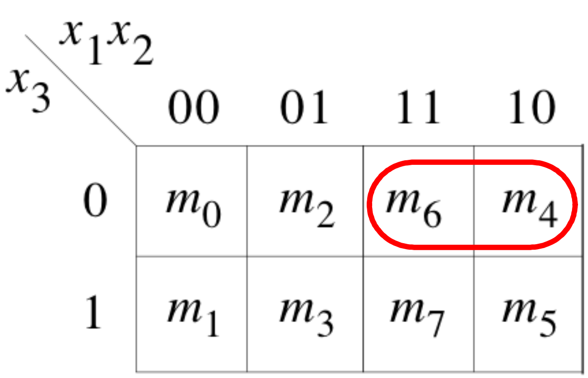


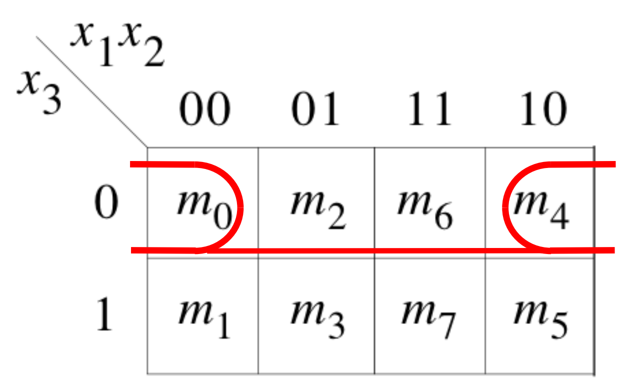


These are valid groupings

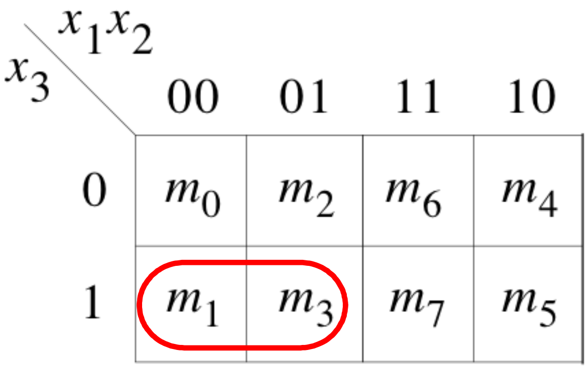


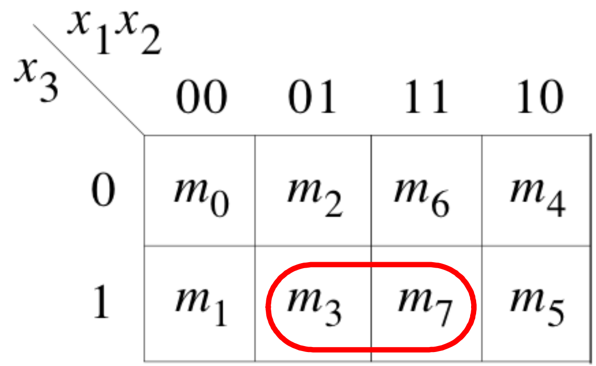


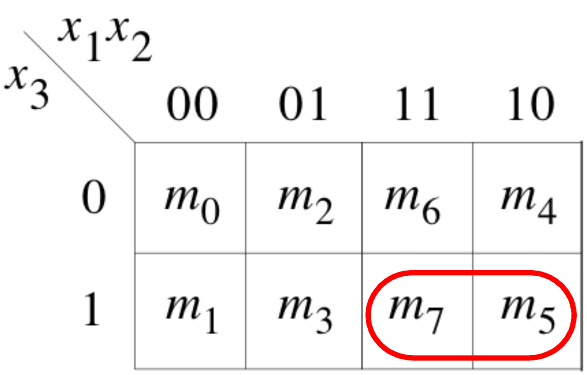


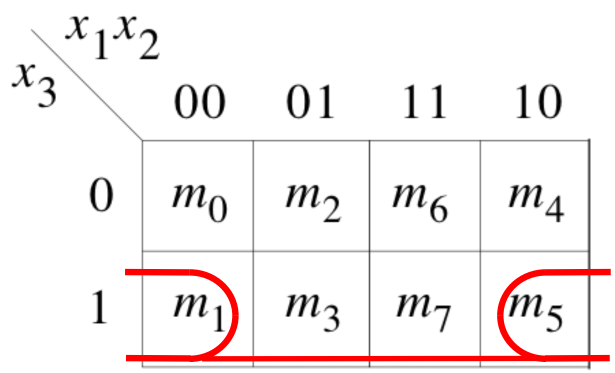


These are valid groupings

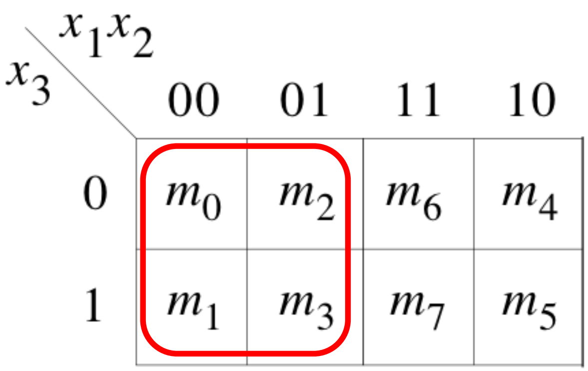


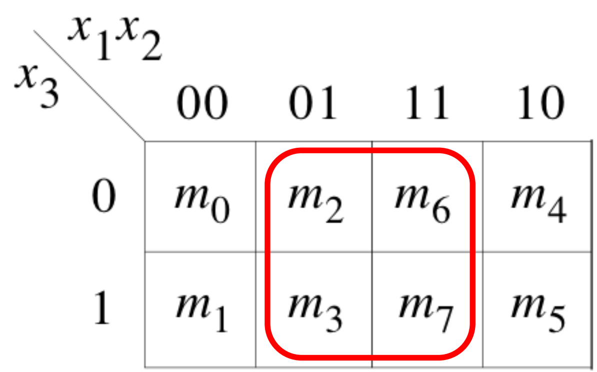


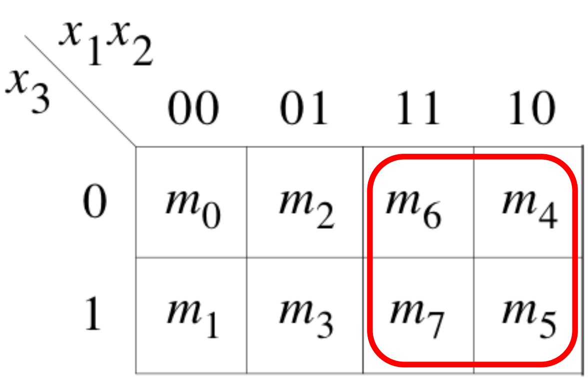


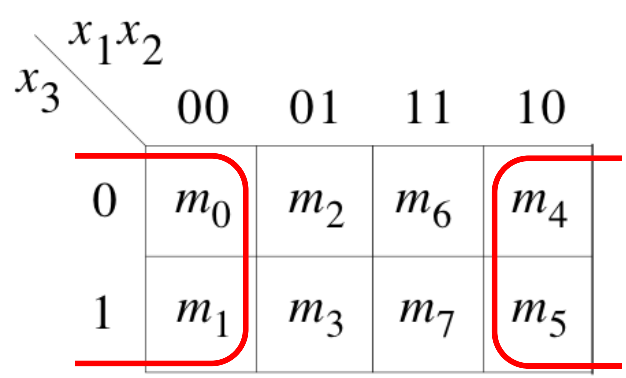


These are valid groupings

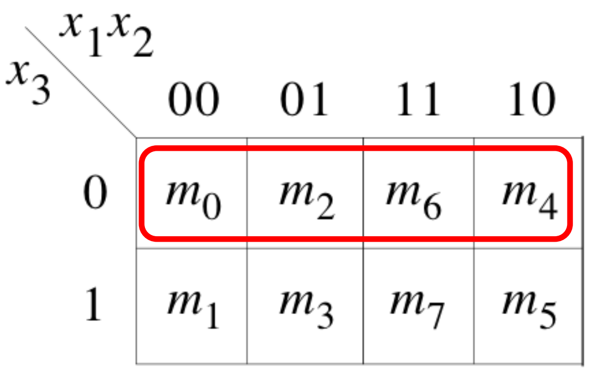


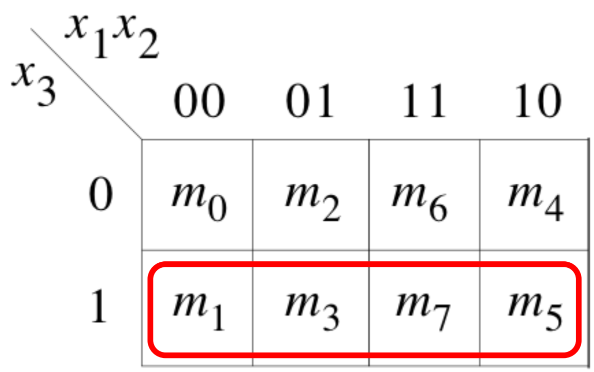




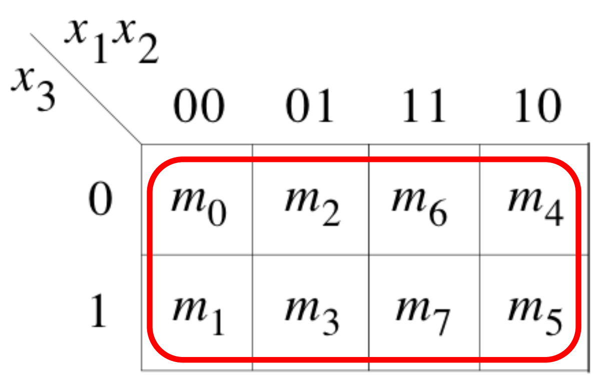


These are valid groupings

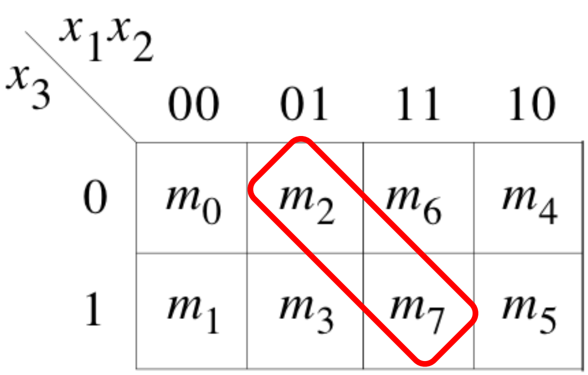


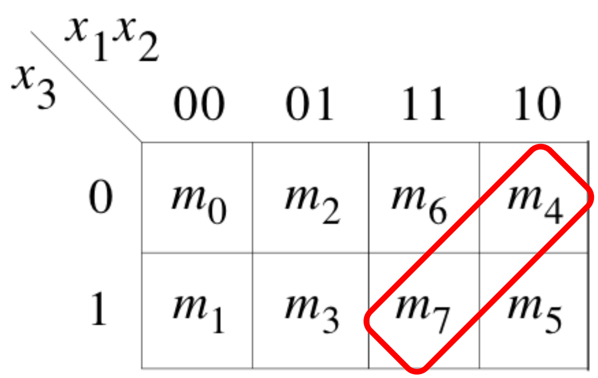


This is a valid grouping

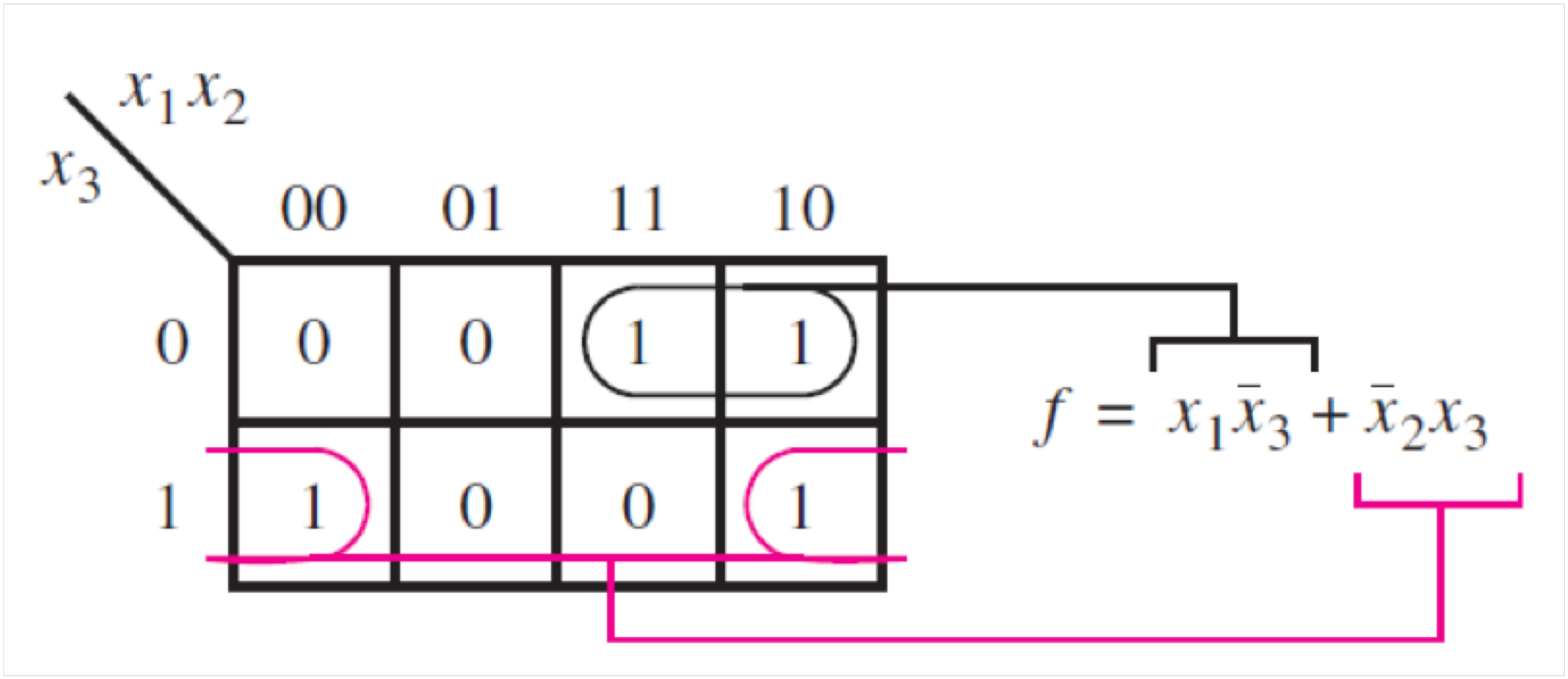


Some invalid groupings



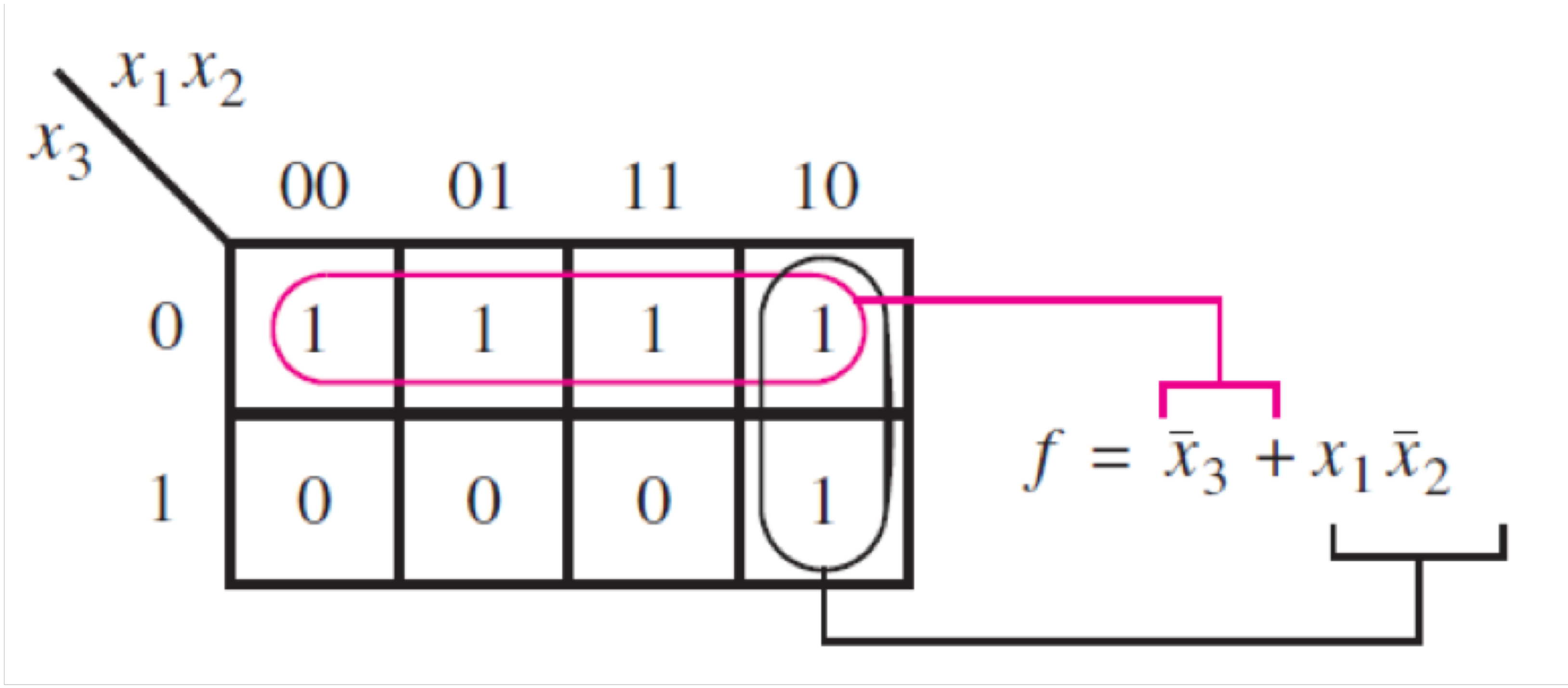


Examples of three-variable Karnaugh maps



[Figure 2.52a from the textbook]

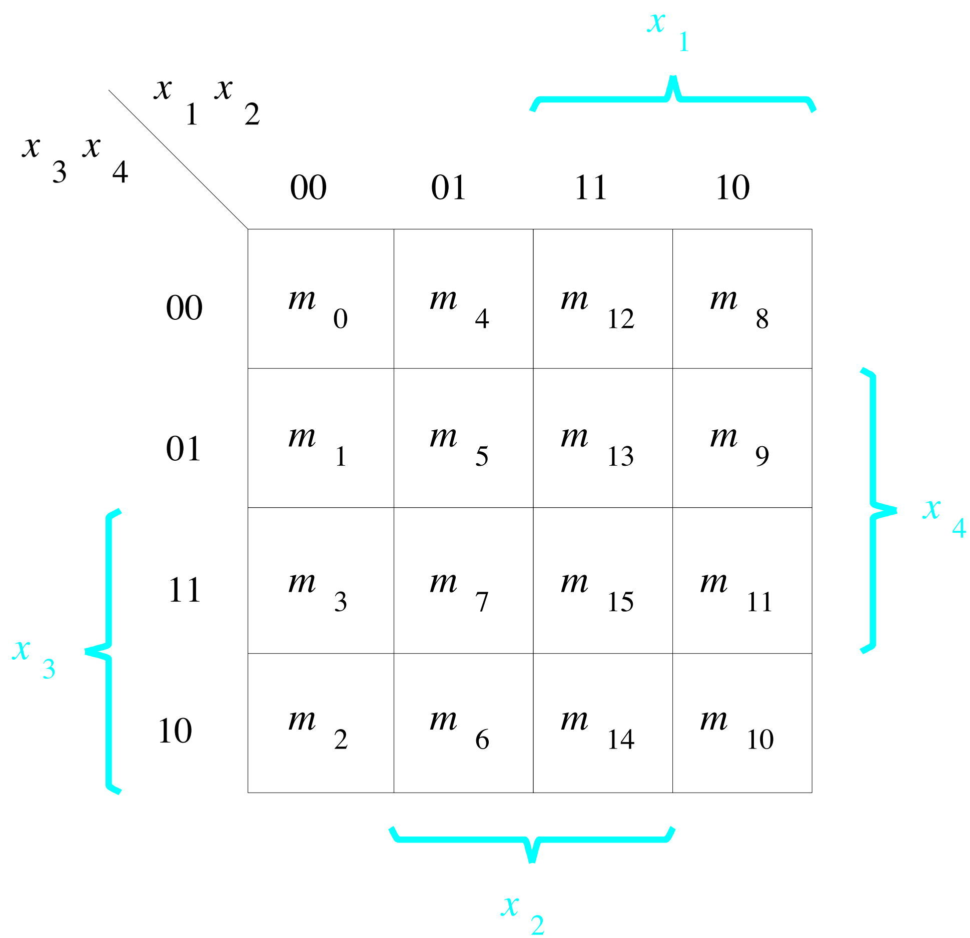
Examples of three-variable Karnaugh maps



[Figure 2.52b from the textbook]

Four-Variable K-Map

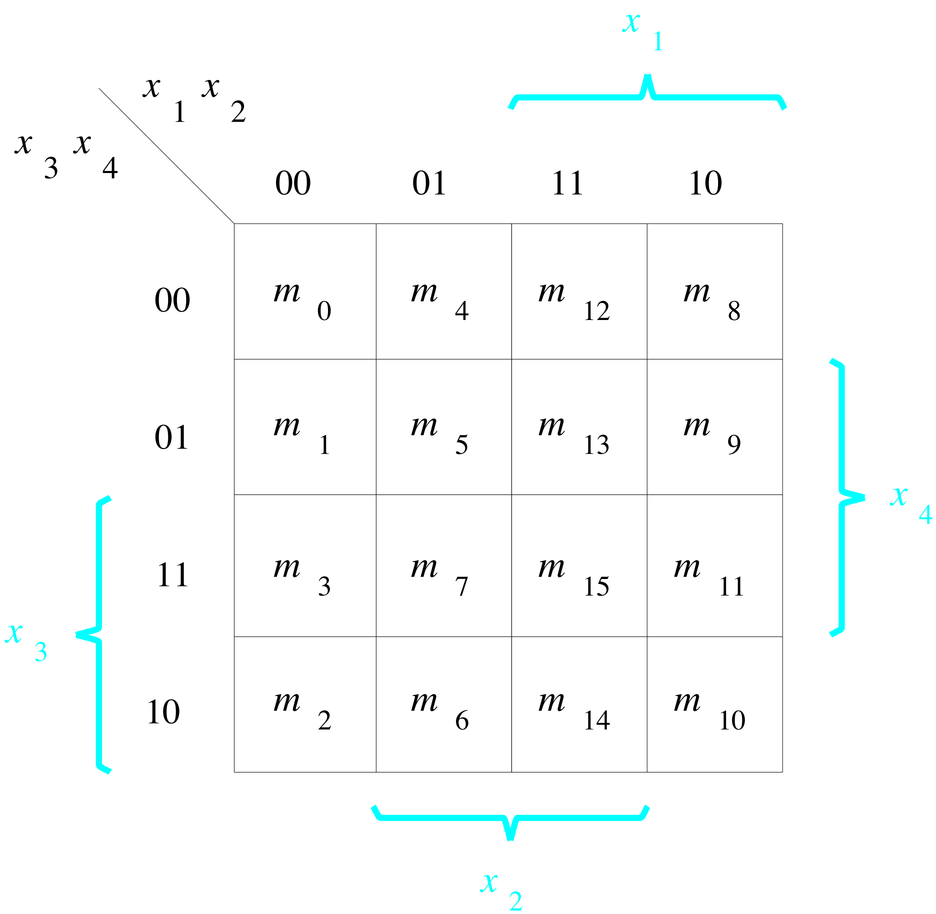
A four-variable Karnaugh map



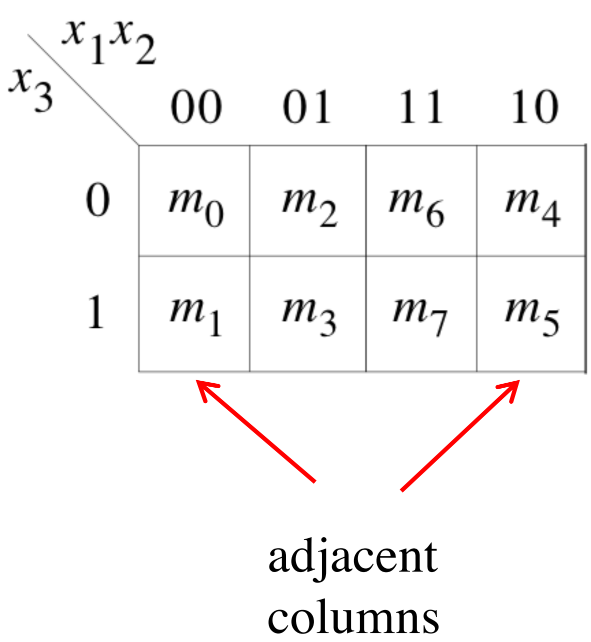
[Figure 2.53 from the textbook]

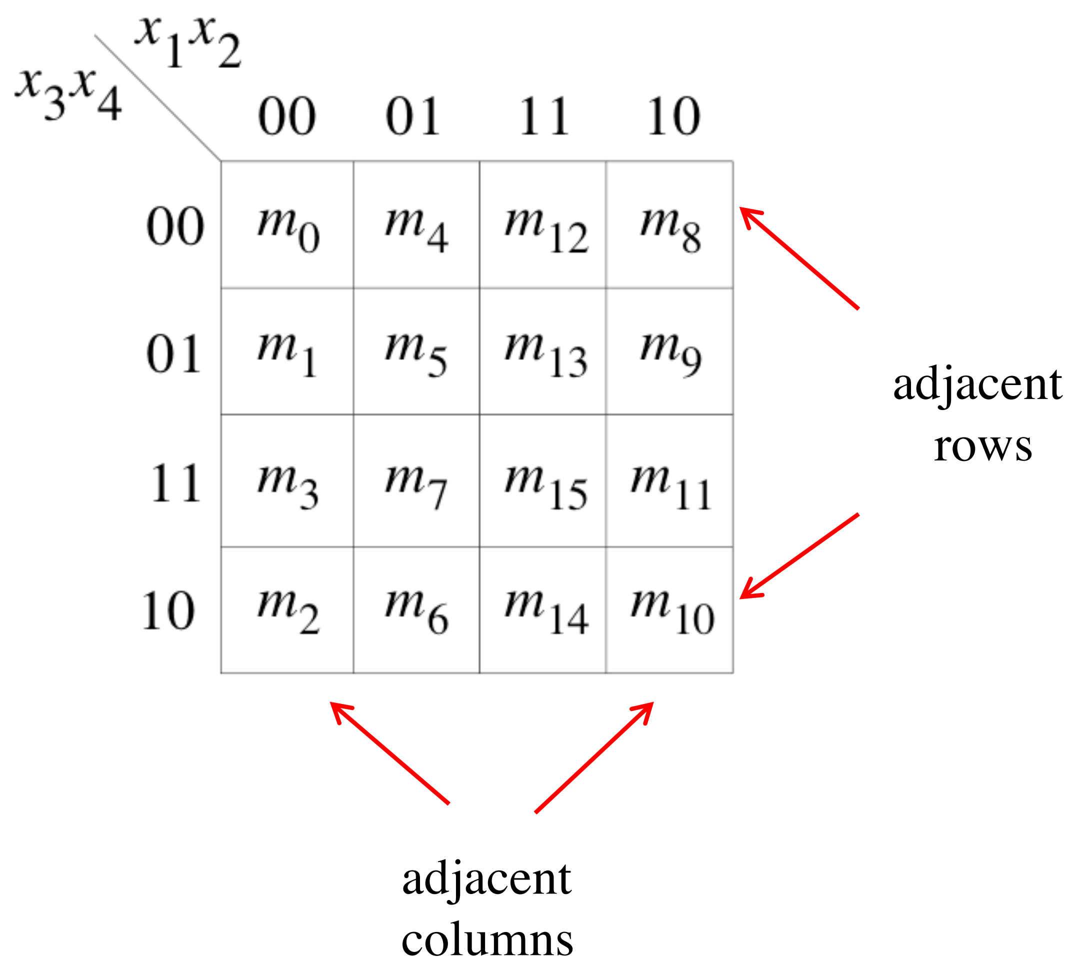
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



Adjacency Rules



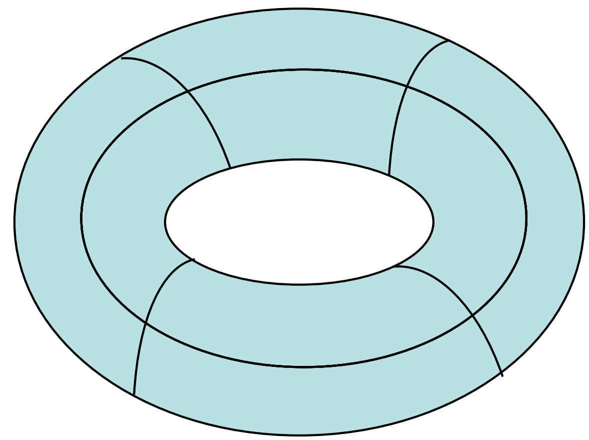


Adjacency Rules

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

adjacent rows

adjacent columns



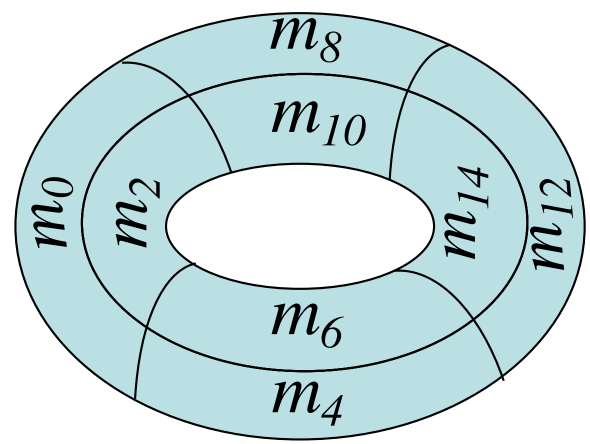
As if the K-map were drawn on a torus

Adjacency Rules

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

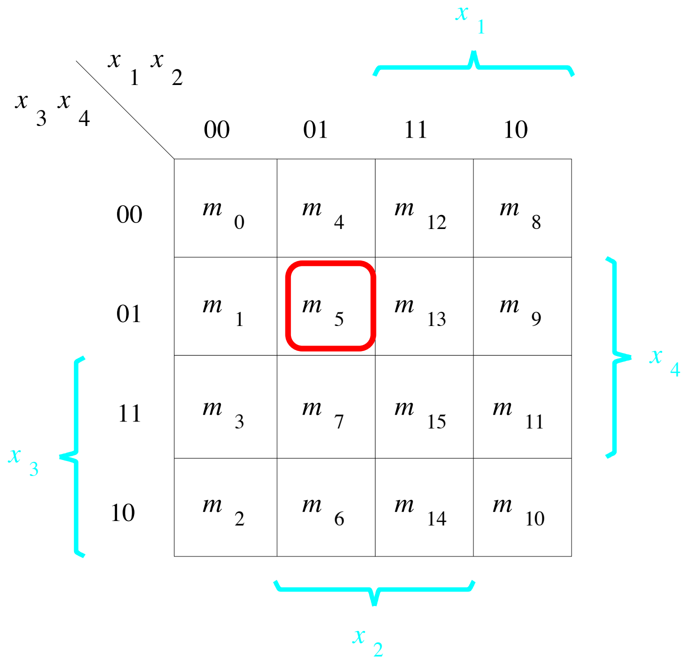
adjacent rows

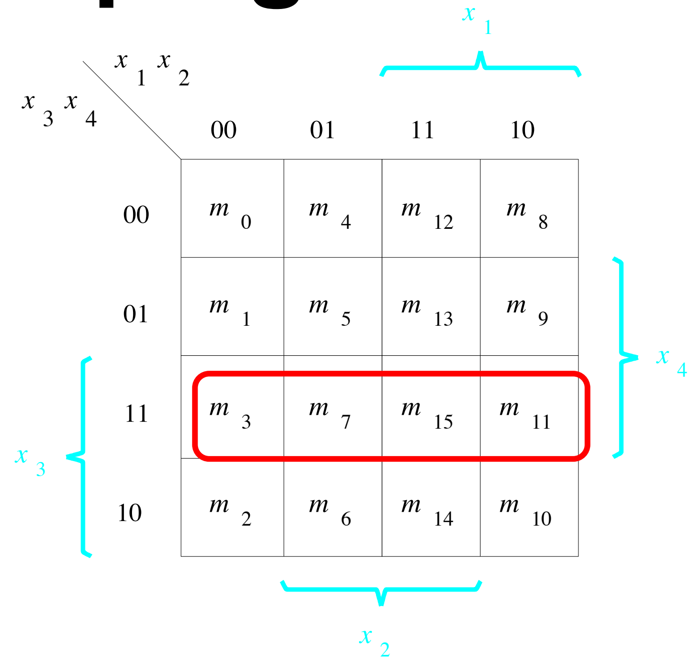
adjacent columns

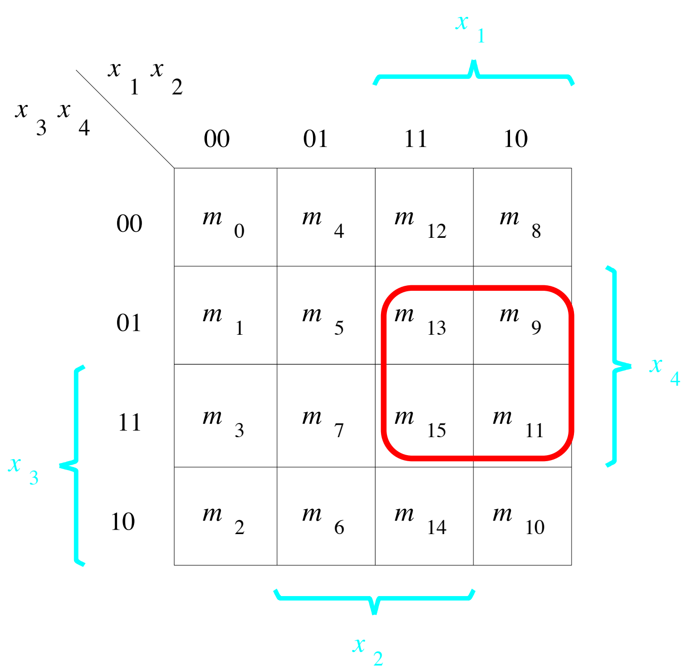


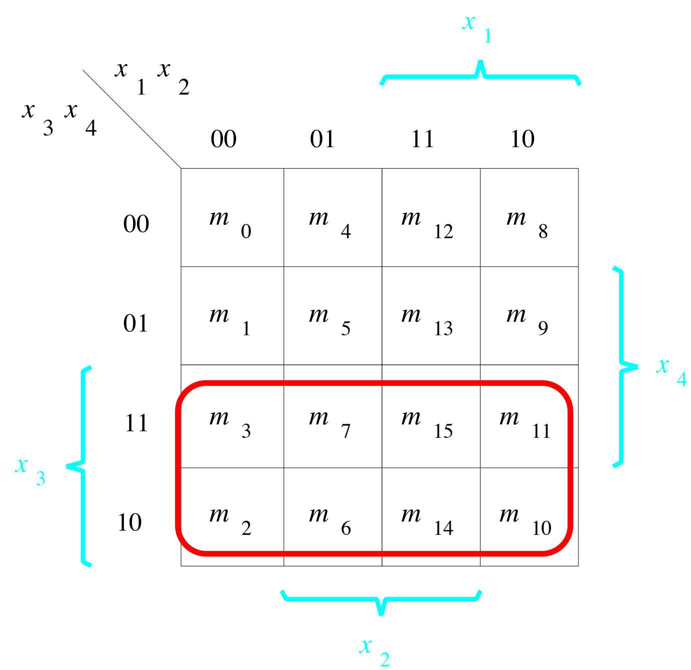
As if the K-map were drawn on a torus

Some Valid Groupings

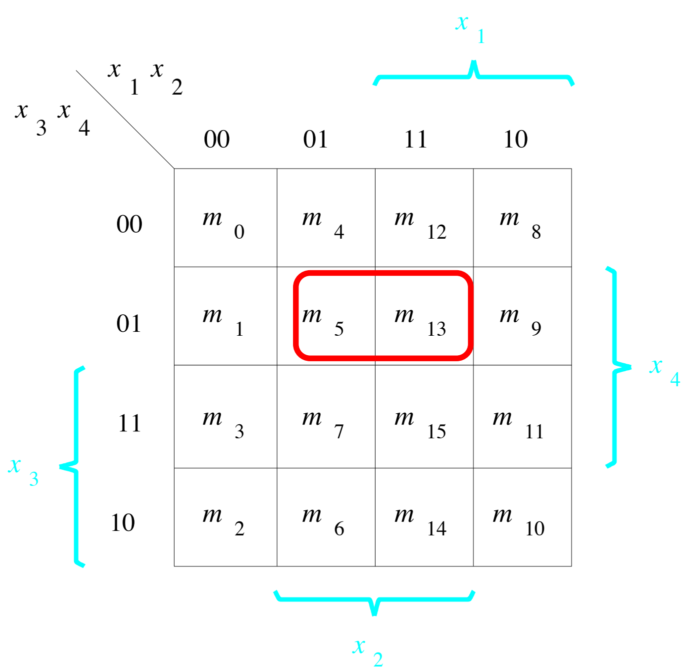


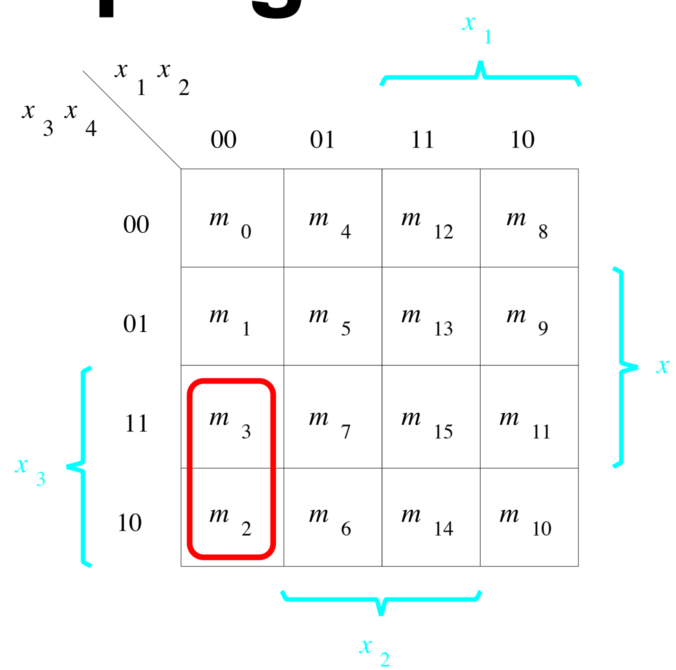


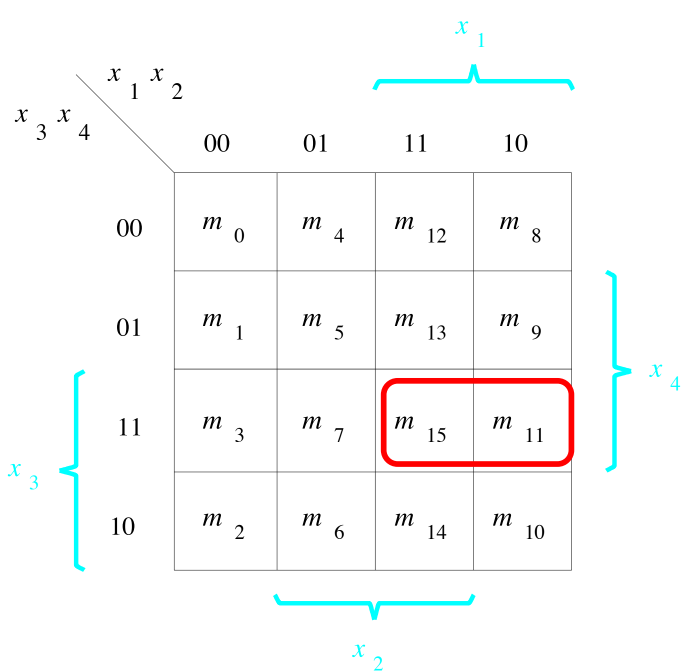


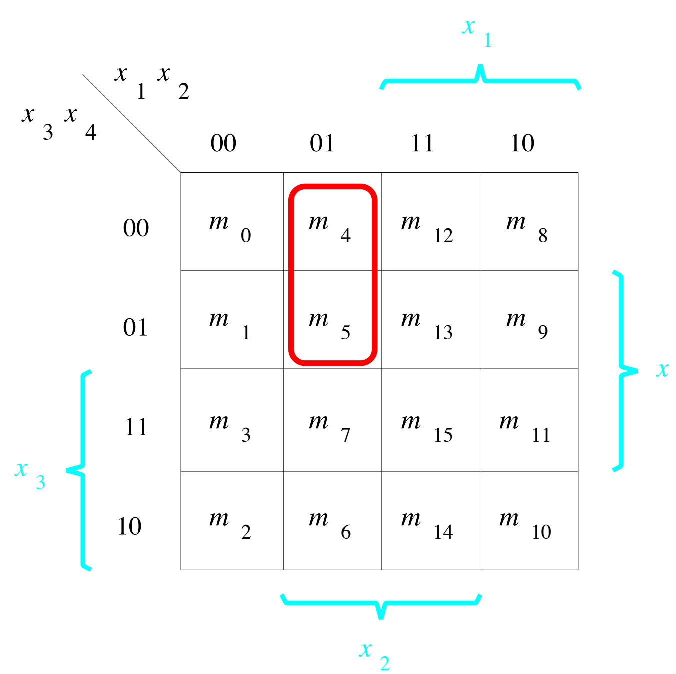


Some Valid Groupings

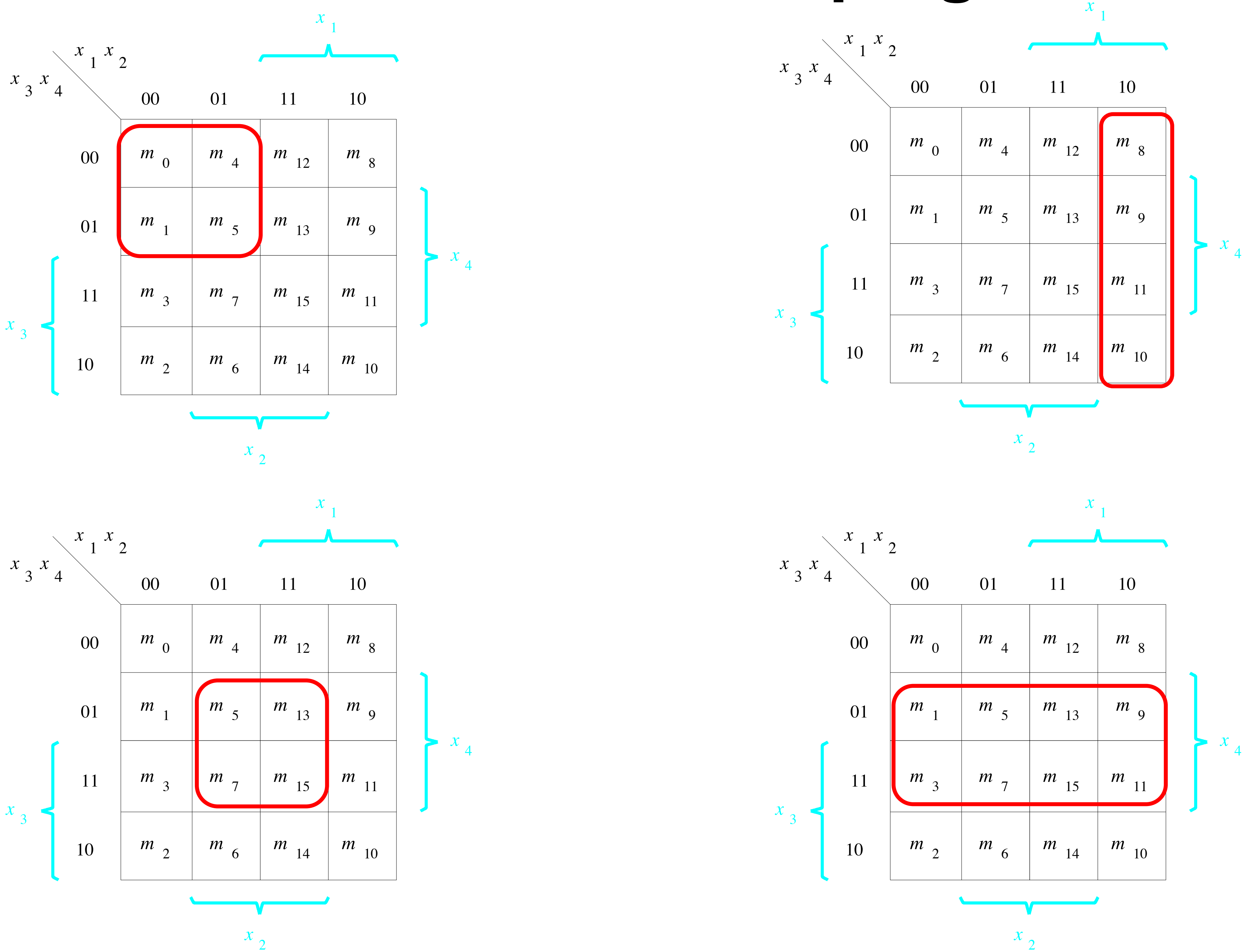




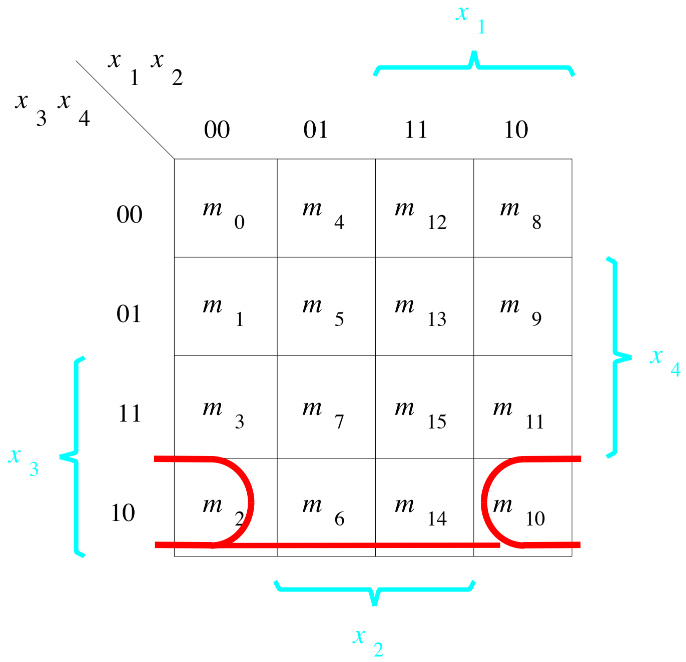


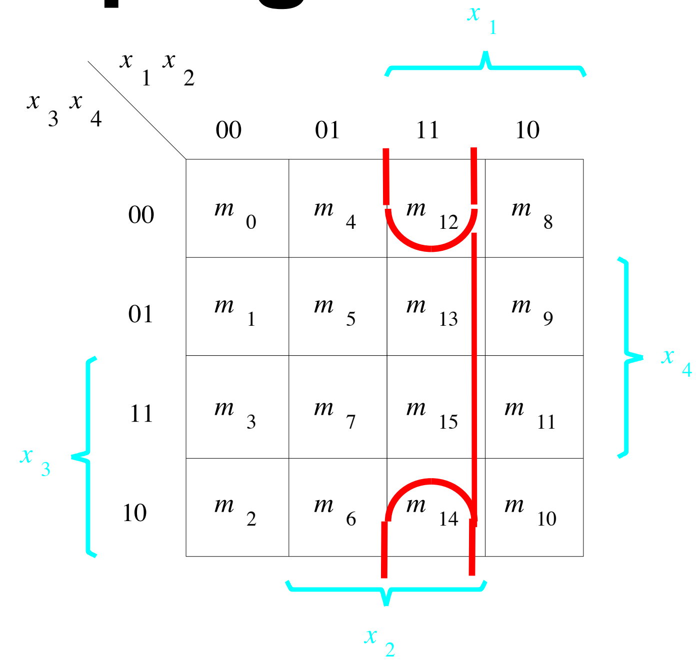


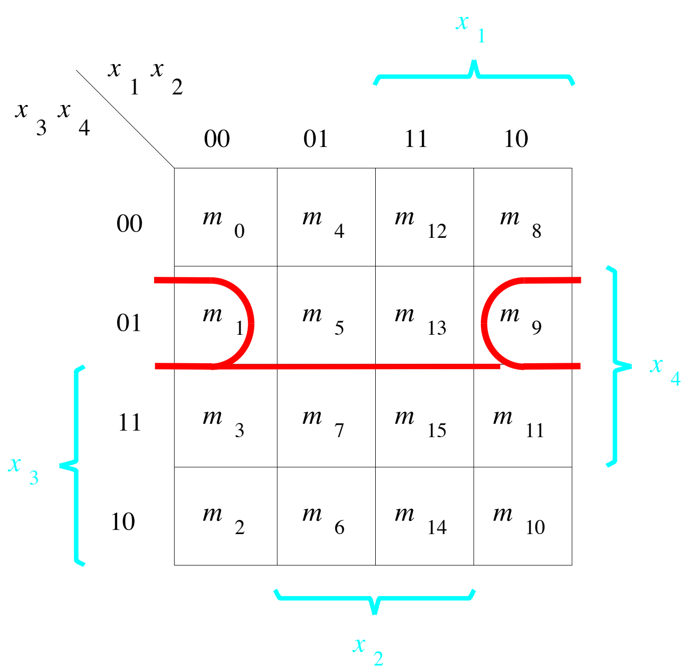
Some Valid Groupings

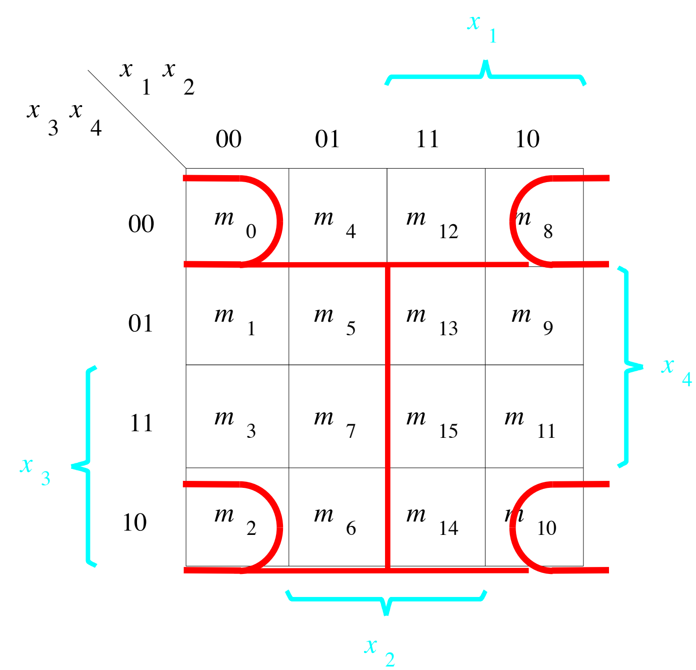


Some Valid Groupings

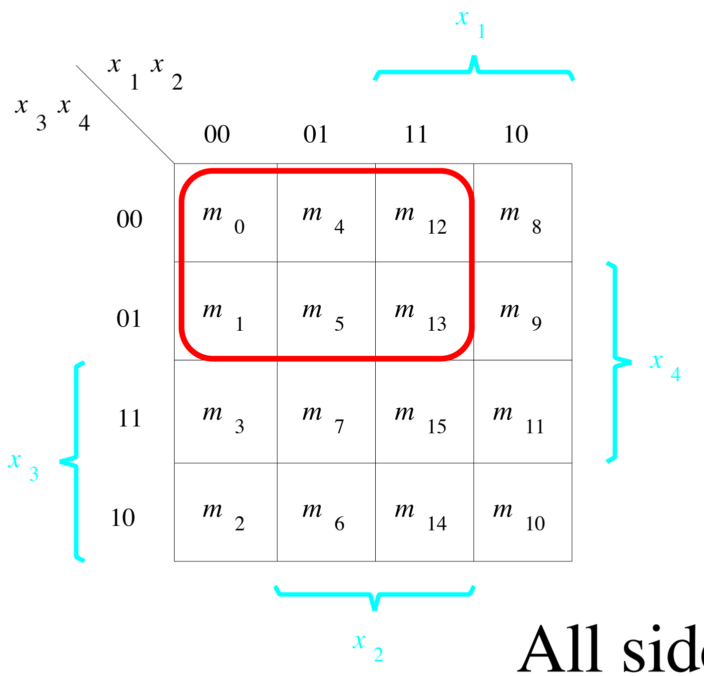


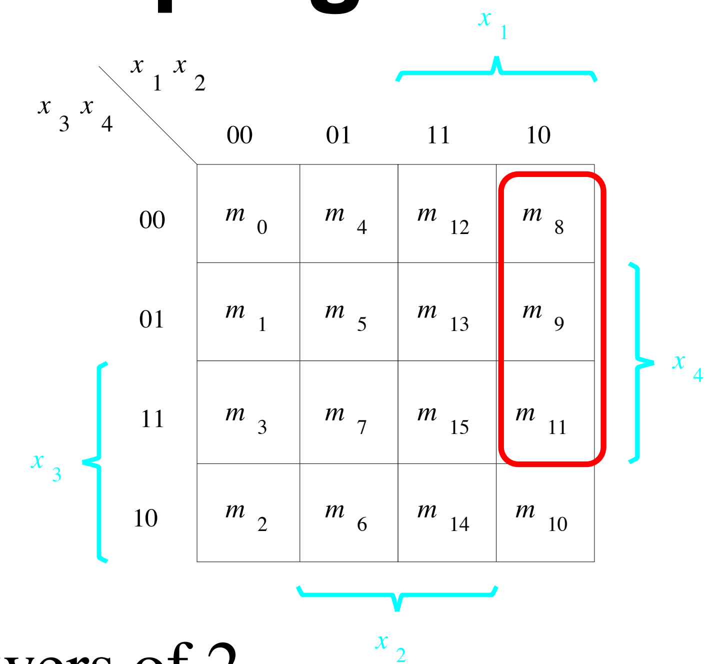




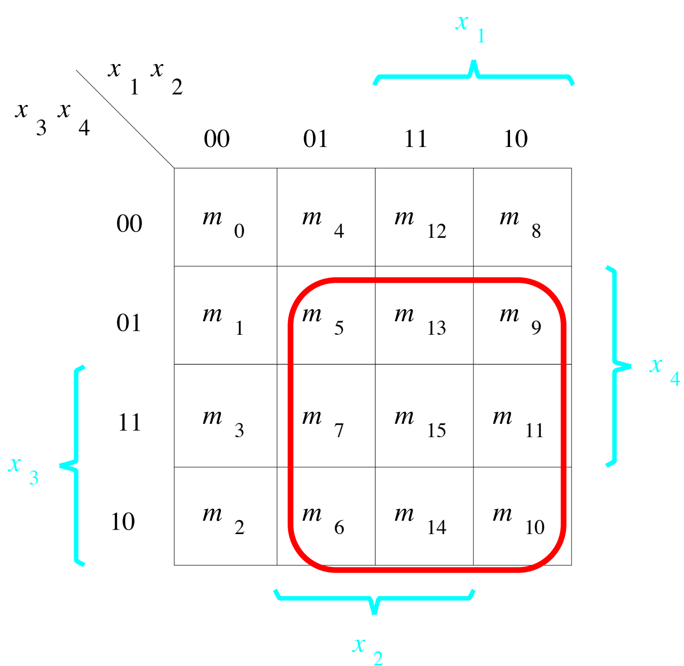


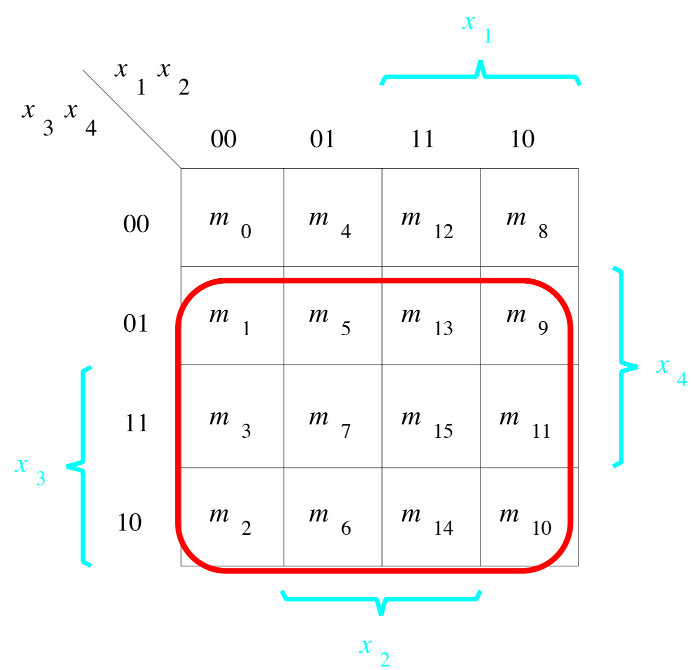
Some Invalid Groupings



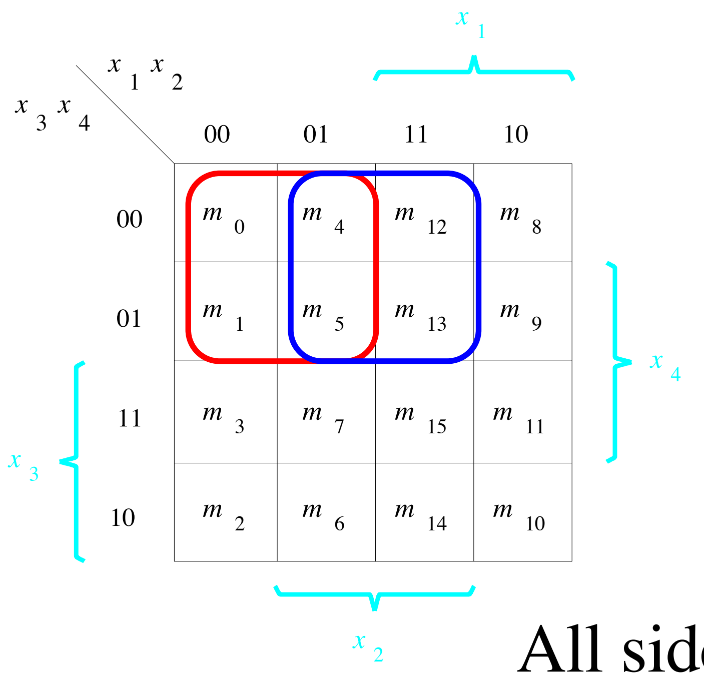


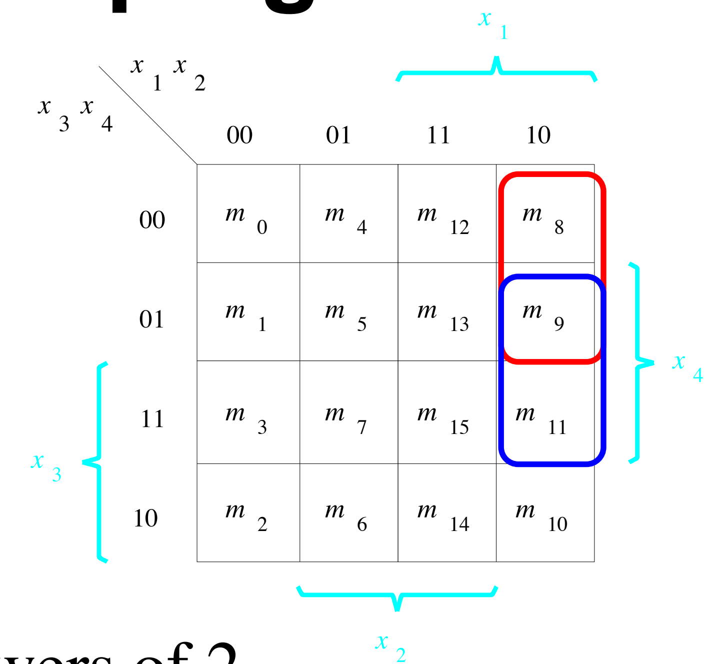
All sides must be powers of 2.



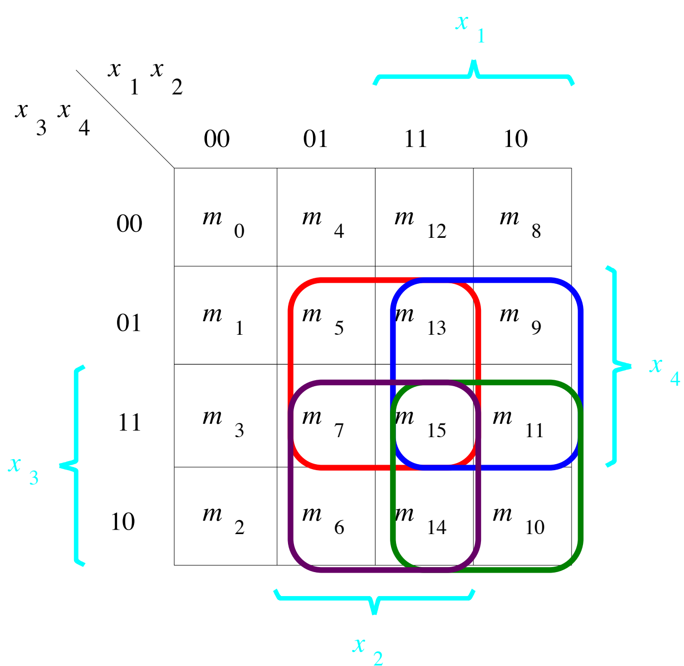


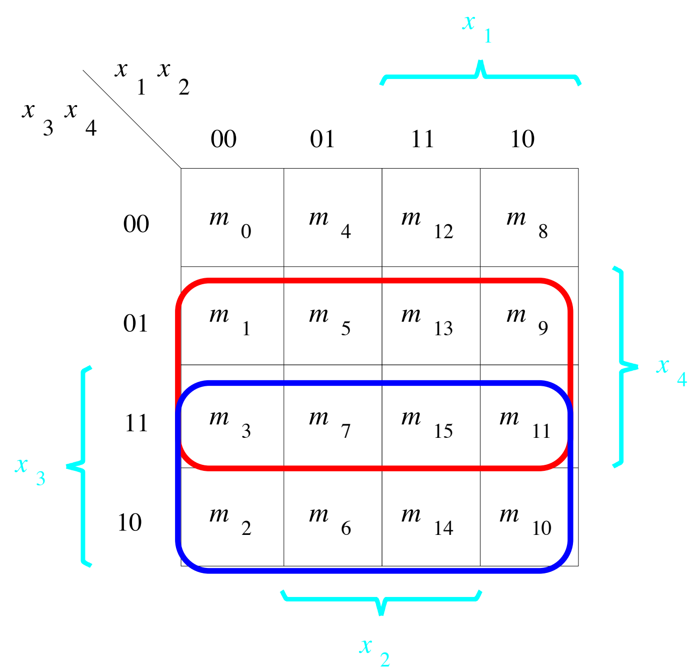
Some valid Groupings



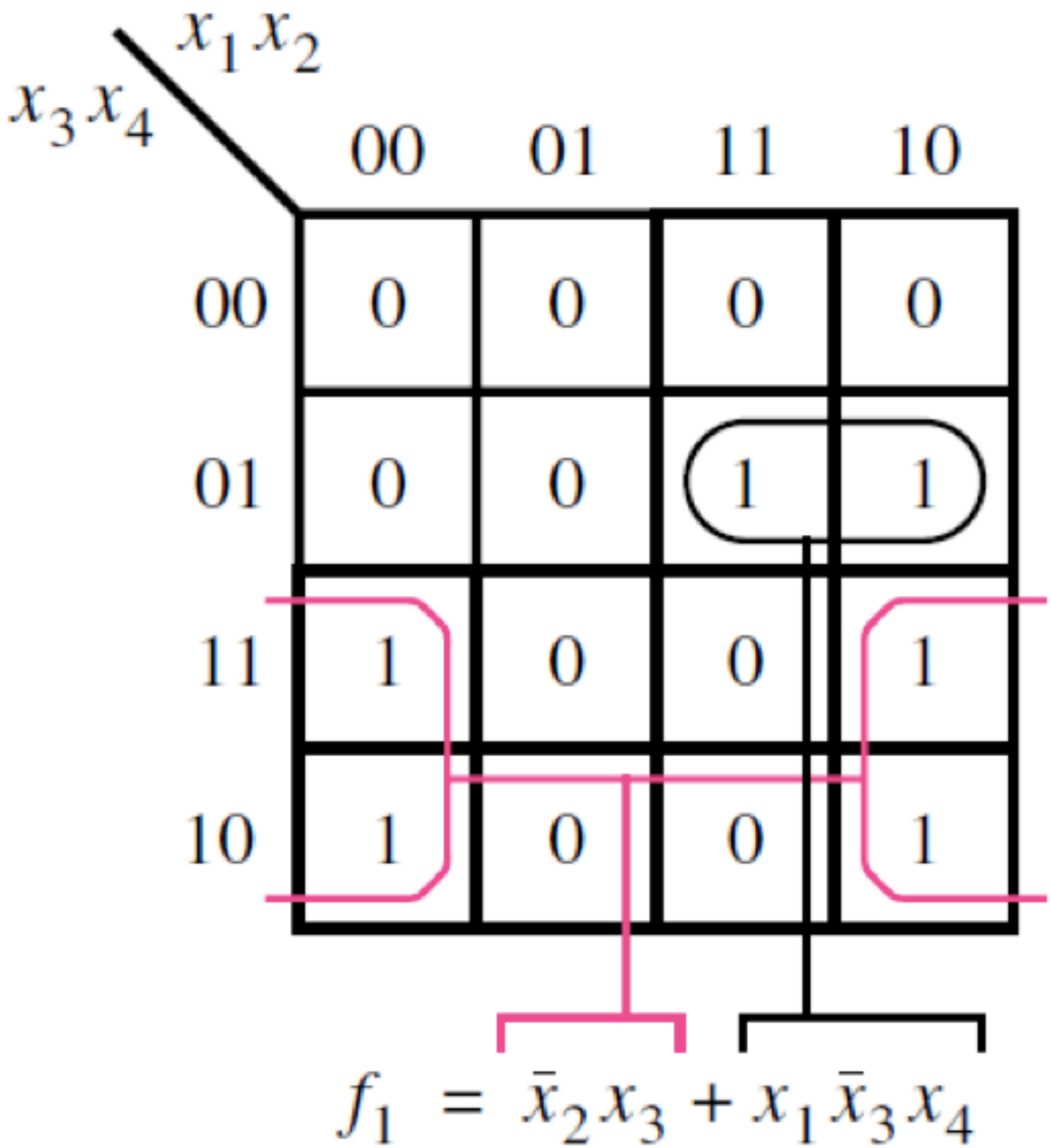


All sides must be powers of 2.



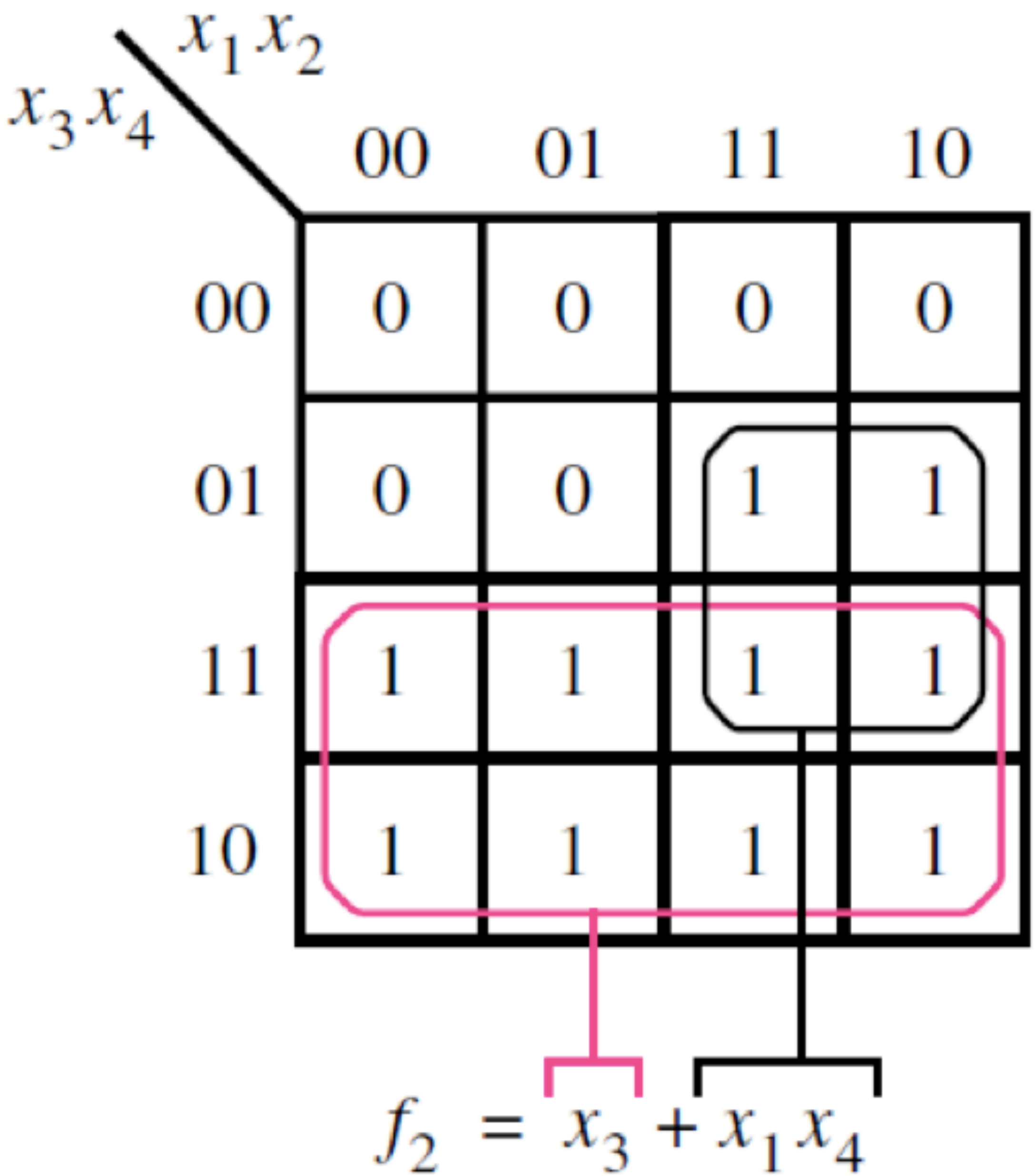


Example of a four-variable Karnaugh map



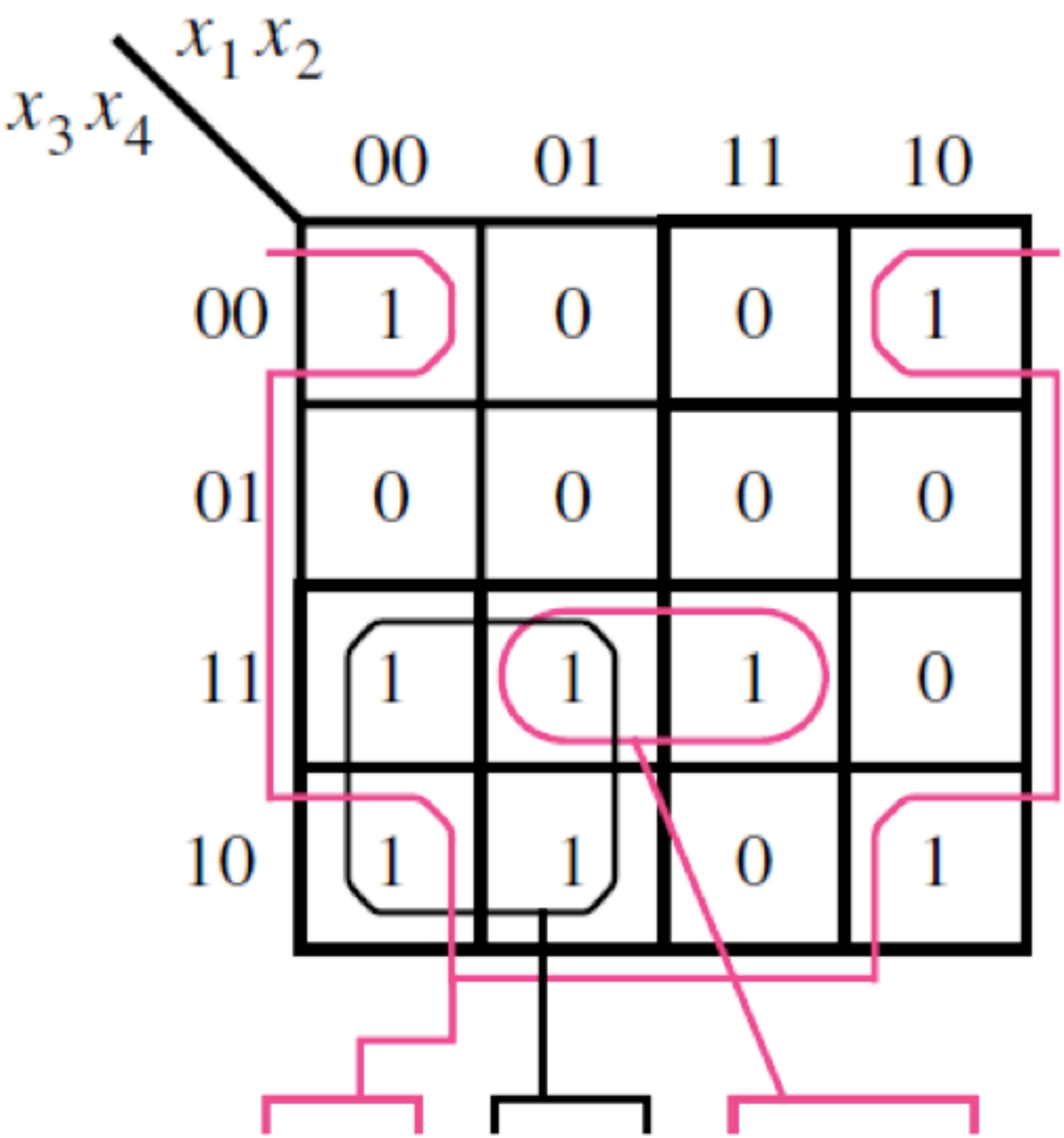
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



[Figure 2.54 from the textbook]

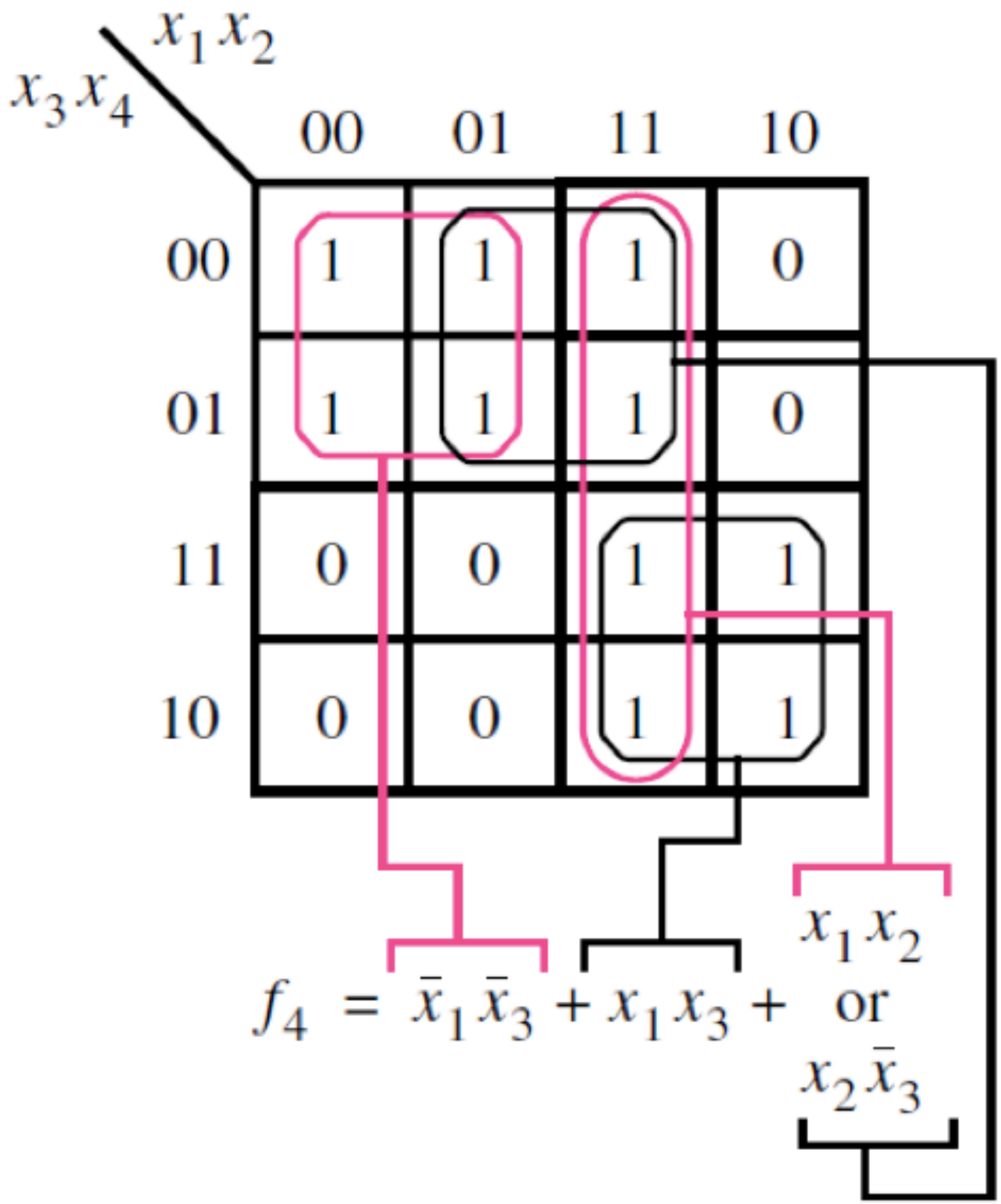
Example of a four-variable Karnaugh map



$$f_3 = \bar{x}_2 \bar{x}_4 + \bar{x}_1 x_3 + x_2 x_3 x_4$$

[Figure 2.54 from the textbook]

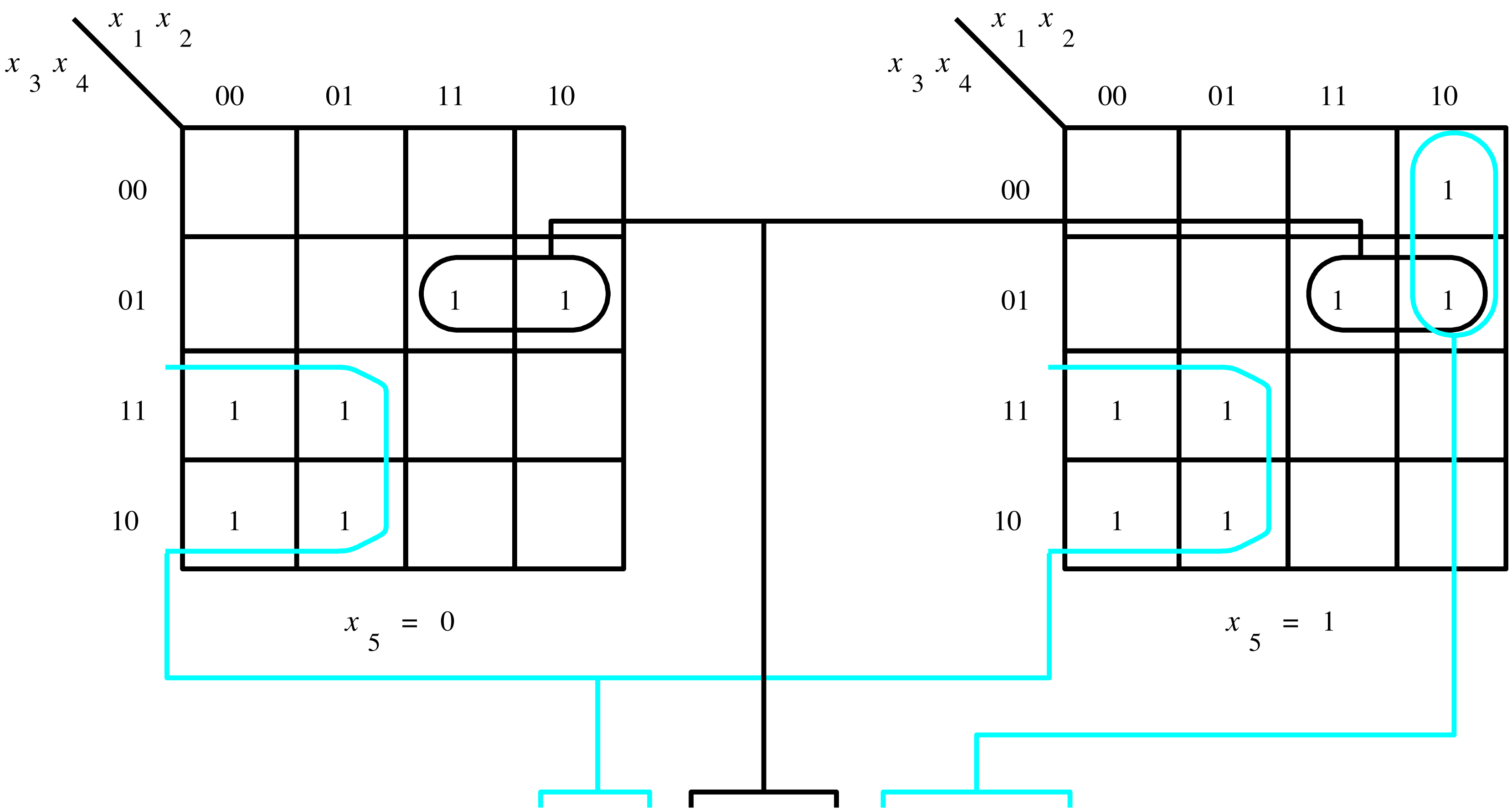
Example of a four-variable Karnaugh map



[Figure 2.54 from the textbook]

Five-Variable K-Map

A five-variable Karnaugh map



$$f_1 = \bar{x}_1 x_3 + x_1 \bar{x}_3 x_4 + x_1 \bar{x}_2 \bar{x}_3 x_5$$

[Figure 2.55 from the textbook]

Questions?

THE END