

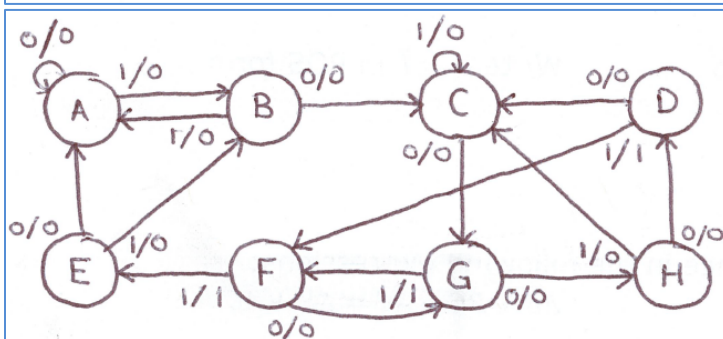
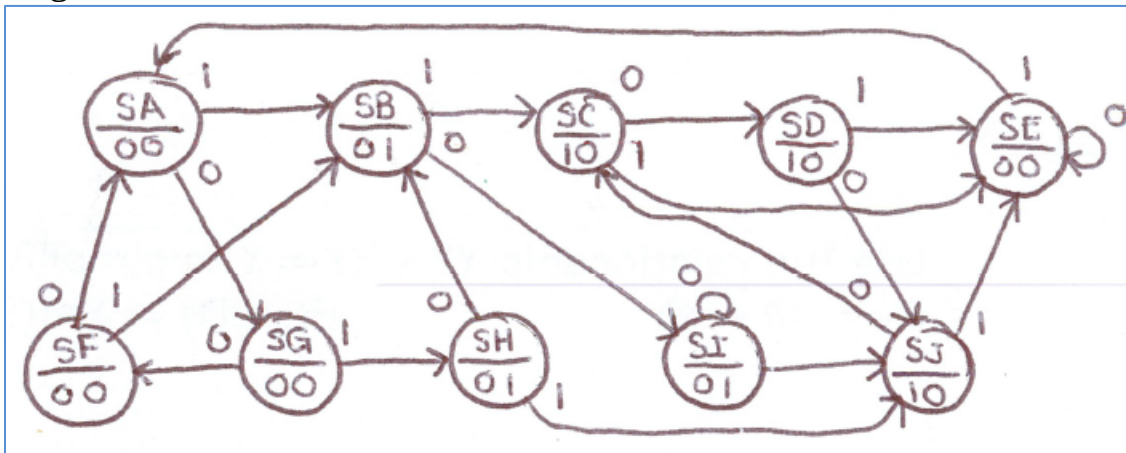
P1 (15 points): The following all concern state machines that detect if a number is a multiple of three (3, 6, 9, 12...). Draw the specified state diagrams that will output $Z=1$ based on the input W .

A: Let F be the number of clock cycles for which the input W has been one. Draw a state diagram for a Moore FSM that outputs $Z=1$ if F is a multiple of 3.

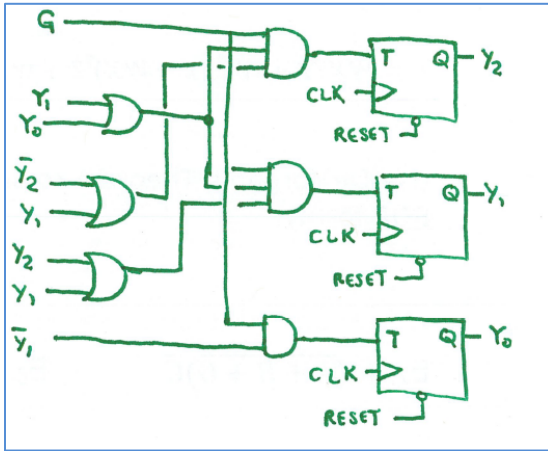
B: Let G be the three-bit binary representation of the last three values of W , with the most significant bit as the earliest observed value of W . Assume that G is initially zero. Draw a state diagram for a Mealy FSM that outputs $Z=1$ if G is a multiple of 3.

C: For every clock cycle, let the new value of H be twice its previous value plus the value of W ($H^* = 2H_0 + W$). This means that H is the binary representation of all values observed on W (a bit string of unlimited length). Assume that H is initially zero. Draw a state diagram for a Moore FSM that outputs $Z=1$ if H is a multiple of 3.

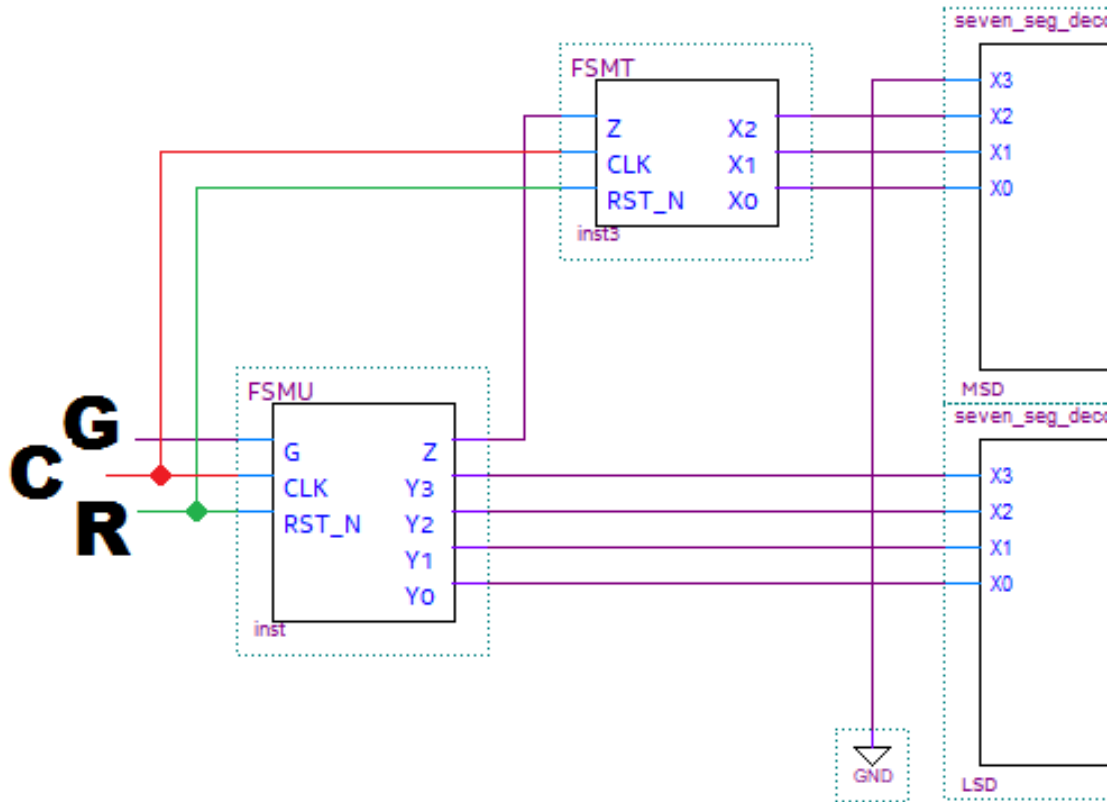
P2 (15 points): Perform state minimization on the following state diagrams.



P3 (10 points): The circuit below looks like a counter. Draw a state diagram which illustrates its counting sequence.



P4 (20 points): For this problem, you are given two FSMs: FSM T has six states, one bit input Z, and three bit output $X_2 X_1 X_0$. FSM U has ten states, one bit input G, and five bit output Z $Y_3 Y_2 Y_1 Y_0$. These FSMs are connected as shown below:



Each FSM shown here has states determined by a sufficient number of positive-edge-triggered DFFs.

A: Draw the state table for FSM U, a modulo-10 counter that counts *down* when $G=1$ and holds its value constant when $G=0$. Upon reset, FSM U should return to a state which outputs $Y=9$. The output $Z=1$ if $Y=0$ and $G=1$. Make state assignments such that the output Y is equal to the current state (note: the state variable and the output should not have the same name).

B: FSM U has one input and four state variables, so each expression for the next state is a function of five inputs. To avoid using a 5-variable K-map, we will instead implement FSM U using four 2-to-1 MUXes with G as the select line. Let us assume that $G=1$. Derive the next state expressions assuming that $G=1$. Show that the K-maps for the next state variables produce these expressions.

$$\text{For } G = 1: S_3^{new} = \bar{S}_3\bar{S}_2\bar{S}_1\bar{S}_0 + S_3S_0, S_2^{new} = S_3\bar{S}_0 + S_2S_1 + S_2S_0 \\ S_1^{new} = S_2\bar{S}_1\bar{S}_0 + S_3\bar{S}_0 + S_1S_0, S_0^{new} = \bar{S}_0$$

C: FSM T is a modulo-6 counter that also counts down when $Z=1$ and resets to a state which outputs $X=5$. Draw the state table for FSM T.

D: If both FSMs are treated as a single FSM as shown in the initial diagram, what type of circuit does it create? Please specify the direction, encoding, and modulus.

P5 (20 points): The following Moore FSM state table is incomplete. The clock for this FSM (FSM 1) has a period of 100 microseconds such that the button for the input X , controlled by the user, cannot be pressed for only one clock cycle. In addition, button X , when pressed, will output $X=0$.

Current State	Next State		Output
	X=0	X=1	W
A (reset)	B	A	0
B	B	C	0
C	D	C	0
D	E	--	1
E	F	--	1
F	G	--	1
G	G	A	0

I: Draw a state diagram for this state table.

II: How many full button press cycles (push and release) will cause this FSM to pass through all seven states and return to state A?

III: Use the following state assignments: $A=000$, $B=001$, $C=010$, $D=100$, $E=101$, $F=110$, $G=111$. Use a K-map to show that the next state expressions for FSM 1 are:

$$Y_2^{new} = (\bar{X})(Y_2 + Y_1), Y_1^{new} = (Y_1 + Y_0)(X + Y_2)(\bar{X} + \bar{Y}_2), Y_0^{new} = (\bar{X})(\bar{Y}_2 + Y_1 + \bar{Y}_0)(Y_2 + \bar{Y}_1)$$

Assume that DFFs are used to hold the state values.

P6 (20 points): Design a FSM with no inputs (other than CLK and RESETN) and four-bit output Z such that the FSM outputs the sequence 2, 3, 4, 5, 9, 13. The state assignments should be equal to the output and your circuit should use four positive-edge-triggered JKFFs and a minimal number of other gates.

A: Draw a state diagram. Don't forget the reset signal.

B: Draw the state-assigned table. This table should also include the excitation for the JKFFs (the values for J and K along with the next state values).

C: Draw K-maps to show that the inputs to the JK FF are as follows:

$$J_3 = s_2s_0, K_3 = s_2, J_2 = s_0, K_2 = s_0, J_1 = s_3s_2, K_1 = s_0, J_0 = 1, K_0 = s_3s_2 + s_1$$

D: How might JKFF 2 be simplified given that both of its inputs are the same?