

**P1 (10 points):** A given circuit takes  $V$ , a 6-bit binary number, divides  $V$  by 9, and stores the quotient and remainder into  $Q$  and  $R$ , respectively. (e.g. if  $V=21$ , then  $Q=2$  and  $R=3$ ).

A: How many bits are needed to represent all possible values of  $Q$ ?

B: How many bits are needed to represent  $R$ ?

**P2 (10 points):** Draw the circuit for the following expressions:

$$F = \bar{a}\bar{b} + (\bar{a}b)$$

$$G = \overline{(w + \bar{x} + y + \bar{z})(\bar{w} + x + \bar{y} + z)}$$

**P3 (10 points):** Using a Venn diagram, show that  $X + YZ = (X + Y)(X + Z)$

**P4 (15 points):** Use Boolean Algebra to simplify the following expressions:

A:  $w + x + \bar{w} + x$

B:  $wx\bar{y}z + wx\bar{y}\bar{z} + wxy\bar{z} + wxyz + w\bar{x}y\bar{y}$

C:  $(\bar{p} + \bar{q} + r)(\bar{q} + r + \bar{s})(\bar{p} + q + r)(\bar{q} + \bar{r} + \bar{s})$

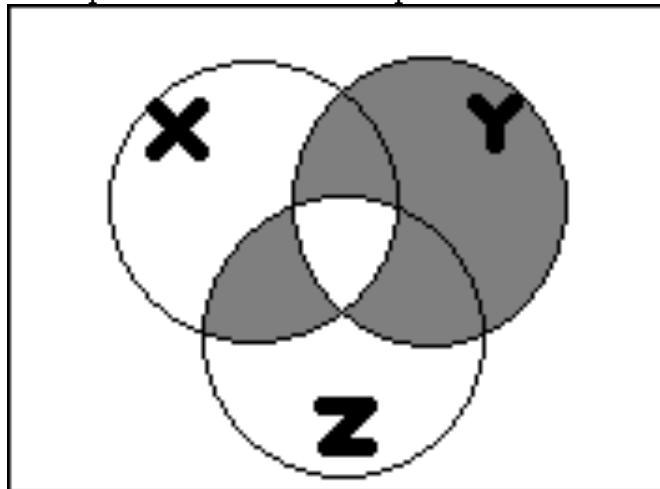
D:  $w + wx\bar{y} + wx\bar{z} + w\bar{x}y + w\bar{x}z$

**P5 (15 points):** Given the following Venn Diagram for  $F$ , show the following:

A: The truth table.

B: The canonical sum-of-products expression for  $F$

C: The canonical product-of-sums expression for  $F$ .



**P6 (20 points):** Use Boolean Algebra to prove the following expressions as equivalent, and show each rule of Boolean Algebra used to perform each step:

I:  $XY\bar{Z} + X\bar{Y}Z + XYZ + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} = X + Y\bar{Z}$

II:  $(\bar{A} + B + \bar{B}\bar{C})(\bar{A}\bar{B} + B + C) = \bar{A} + B + \bar{A}C$

III:  $\overline{(p + q) + r} + \bar{x}_3 + \bar{x}_2 + \bar{x}_1 = (p + \bar{q})\bar{r} + \overline{(x_3x_2x_1)}$

**P7 (20 points):** Given the following expression

$$G = \bar{x}\bar{y}(w + z) + x\bar{y}(\bar{w} + z) + xy(\bar{w} + \bar{z}):$$

- A. Let the circuit cost be defined as the number of gates plus the number of gate inputs. Draw the circuit for G, then show that the cost of this circuit is 33. You may have to reuse a gate to reduce the cost; the circuit should be drawn appropriately to reflect the cost.
- B. Use Boolean algebra to simplify the expression for G.
- C. Draw the circuit for G and state the new cost of the circuit.