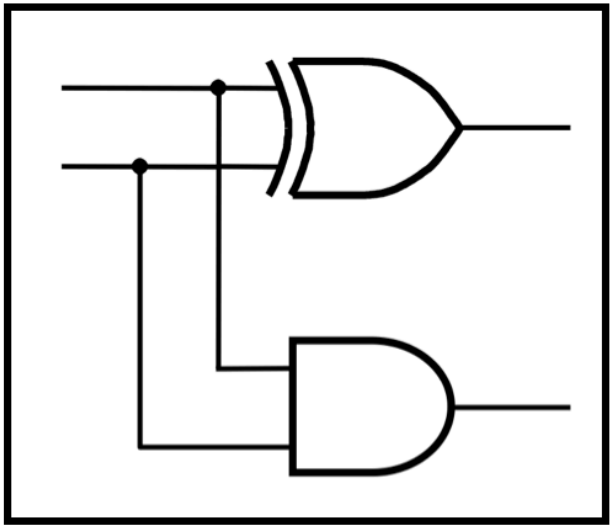


# CprE 281: Digital Logic

## Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

# Examples of Solved Problems

*CprE 281: Digital Logic*
*Iowa State University, Ames, IA*

# Administrative Stuff

- HW5 is out
- It is due on Monday Oct 1 @ 4pm.
- Please write clearly on the first page (in block capital letters) the following three things:
  - Your First and Last Name
  - Your Student ID Number
  - Your Lab Section Letter
- Also, staple all of your pages together

# Administrative Stuff

- No homework is due next week.

# Administrative Stuff

- **Midterm Exam #1**
- **When: Friday Sep 21.**
- **Where: This classroom**
- **What: Chapter 1 and Chapter 2 plus number systems**
- **The exam will be open book and open notes (you can bring up to 3 pages of handwritten notes).**

# Topics for the Midterm Exam

- Binary Numbers
- Octal Numbers
- Hexadecimal Numbers
- Conversion between the different number systems
- Truth Tables
- Boolean Algebra
- Logic Gates
- Circuit Synthesis with AND, OR, NOT
- Circuit Synthesis with NAND, NOR
- Converting an AND/OR/NOT circuit to NAND circuit
- Converting an AND/OR/NOT circuit to NOR circuit
- SOP and POS expressions

# Topics for the Midterm Exam

- **Mapping a Circuit to Verilog code**
- **Mapping Verilog code to a circuit**

- **Multiplexers**
- **Venn Diagrams**
- **K-maps for 2, 3, and 4 variables**

- **Minimization of Boolean expressions using theorems**
- **Minimization of Boolean expressions with K-maps**

- **Incompletely specified functions (with don't cares)**
- **Functions with multiple outputs**

# Example 1

**Determine if the following equation is valid**

$$\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2 = \bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3$$

$$\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2 \stackrel{?}{=} \bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3$$

$$\underbrace{\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2}_{\text{LHS}} \stackrel{?}{=} \underbrace{\bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3}_{\text{RHS}}$$

# Left-Hand Side (LHS)

| Row number | $x_1$ | $x_2$ | $x_3$ | $\overline{x_1}\,\overline{x_3}$ | $x_2 x_3$ | $x_1 \overline{x_2}$ | $f$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | | | | |
| 1 | 0 | 0 | 1 | | | | |
| 2 | 0 | 1 | 0 | | | | |
| 3 | 0 | 1 | 1 | | | | |
| 4 | 1 | 0 | 0 | | | | |
| 5 | 1 | 0 | 1 | | | | |
| 6 | 1 | 1 | 0 | | | | |
| 7 | 1 | 1 | 1 | | | | |

# Left-Hand Side (LHS)

| Row number | $x_1$ | $x_2$ | $x_3$ | $\overline{x_1}\,\overline{x_3}$ | $x_2 x_3$ | $x_1 \overline{x_2}$ | $f$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | |
| 2 | 0 | 1 | 0 | 1 | 0 | 0 | |
| 3 | 0 | 1 | 1 | 0 | 1 | 0 | |
| 4 | 1 | 0 | 0 | 0 | 0 | 1 | |
| 5 | 1 | 0 | 1 | 0 | 0 | 1 | |
| 6 | 1 | 1 | 0 | 0 | 0 | 0 | |
| 7 | 1 | 1 | 1 | 0 | 1 | 0 | |

# Left-Hand Side (LHS)

| Row number | $x_1$ | $x_2$ | $x_3$ | $\overline{x_1}\,\overline{x_3}$ | $x_2 x_3$ | $x_1 \overline{x_2}$ | $f$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 3 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 4 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 5 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 6 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 7 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |

# Right-Hand Side (RHS)

| Row number | $x_1$ | $x_2$ | $x_3$ | $\overline{x_1} x_2$ | $x_1 x_3$ | $\overline{x_2}\ \overline{x_3}$ | $f$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | | | | |
| 1 | 0 | 0 | 1 | | | | |
| 2 | 0 | 1 | 0 | | | | |
| 3 | 0 | 1 | 1 | | | | |
| 4 | 1 | 0 | 0 | | | | |
| 5 | 1 | 0 | 1 | | | | |
| 6 | 1 | 1 | 0 | | | | |
| 7 | 1 | 1 | 1 | | | | |

# Right-Hand Side (RHS)

| Row number | $x_1$ | $x_2$ | $x_3$ | $\overline{x_1} x_2$ | $x_1 x_3$ | $\overline{x_2}\ \overline{x_3}$ | $f$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | |
| 2 | 0 | 1 | 0 | 1 | 0 | 0 | |
| 3 | 0 | 1 | 1 | 1 | 0 | 0 | |
| 4 | 1 | 0 | 0 | 0 | 0 | 1 | |
| 5 | 1 | 0 | 1 | 0 | 1 | 0 | |
| 6 | 1 | 1 | 0 | 0 | 0 | 0 | |
| 7 | 1 | 1 | 1 | 0 | 1 | 0 | |

# Right-Hand Side (RHS)

| Row number | $x_1$ | $x_2$ | $x_3$ | $\overline{x_1}x_2$ | $x_1x_3$ | $\overline{x_2}\,\overline{x_3}$ | $f$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 3 | 0 | 1 | 1 | 1 | 0 | 0 | 1 |
| 4 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 5 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 6 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 7 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |

$$\underbrace{\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2}_{\text{LHS}} \overset{?}{=} \underbrace{\bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3}_{\text{RHS}}$$

| $f$ |
|---|
| 1 |
| 0 |
| 1 |
| 1 |
| 1 |
| 1 |
| 0 |
| 1 |

They are equal.

| $f$ |
|---|
| 1 |
| 0 |
| 1 |
| 1 |
| 1 |
| 1 |
| 0 |
| 1 |

# Example 2

**Design the minimum-cost product-of-sums expression for the function**

$$f(x_1, x_2, x_3) = \Sigma\, m(0, 2, 4, 5, 6, 7)$$

# Minterms and Maxterms (with three variables)

| Row number | $x_1$ | $x_2$ | $x_3$ | Minterm | Maxterm |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$ | $M_0 = x_1 + x_2 + x_3$ |
| 1 | 0 | 0 | 1 | $m_1 = \overline{x}_1\overline{x}_2 x_3$ | $M_1 = x_1 + x_2 + \overline{x}_3$ |
| 2 | 0 | 1 | 0 | $m_2 = \overline{x}_1 x_2\overline{x}_3$ | $M_2 = x_1 + \overline{x}_2 + x_3$ |
| 3 | 0 | 1 | 1 | $m_3 = \overline{x}_1 x_2 x_3$ | $M_3 = x_1 + \overline{x}_2 + \overline{x}_3$ |
| 4 | 1 | 0 | 0 | $m_4 = x_1\overline{x}_2\overline{x}_3$ | $M_4 = \overline{x}_1 + x_2 + x_3$ |
| 5 | 1 | 0 | 1 | $m_5 = x_1\overline{x}_2 x_3$ | $M_5 = \overline{x}_1 + x_2 + \overline{x}_3$ |
| 6 | 1 | 1 | 0 | $m_6 = x_1 x_2\overline{x}_3$ | $M_6 = \overline{x}_1 + \overline{x}_2 + x_3$ |
| 7 | 1 | 1 | 1 | $m_7 = x_1 x_2 x_3$ | $M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$ |

[ Figure 2.22 from the textbook ]

# Minterms and Maxterms (with three variables)

| Row number | $x_1$ | $x_2$ | $x_3$ | Minterm | Maxterm |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$ | $M_0 = x_1 + x_2 + x_3$ |
| 1 | 0 | 0 | 1 | $m_1 = \overline{x}_1\overline{x}_2 x_3$ | $M_1 = x_1 + x_2 + \overline{x}_3$ |
| 2 | 0 | 1 | 0 | $m_2 = \overline{x}_1 x_2\overline{x}_3$ | $M_2 = x_1 + \overline{x}_2 + x_3$ |
| 3 | 0 | 1 | 1 | $m_3 = \overline{x}_1 x_2 x_3$ | $M_3 = x_1 + \overline{x}_2 + \overline{x}_3$ |
| 4 | 1 | 0 | 0 | $m_4 = x_1\overline{x}_2\overline{x}_3$ | $M_4 = \overline{x}_1 + x_2 + x_3$ |
| 5 | 1 | 0 | 1 | $m_5 = x_1\overline{x}_2 x_3$ | $M_5 = \overline{x}_1 + x_2 + \overline{x}_3$ |
| 6 | 1 | 1 | 0 | $m_6 = x_1 x_2\overline{x}_3$ | $M_6 = \overline{x}_1 + \overline{x}_2 + x_3$ |
| 7 | 1 | 1 | 1 | $m_7 = x_1 x_2 x_3$ | $M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$ |

**The function is 1 for these rows**

# Minterms and Maxterms (with three variables)

| Row number | $x_1$ | $x_2$ | $x_3$ | Minterm | Maxterm |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$ | $M_0 = x_1 + x_2 + x_3$ |
| 1 | 0 | 0 | 1 | $m_1 = \overline{x}_1\overline{x}_2 x_3$ | $M_1 = x_1 + x_2 + \overline{x}_3$ |
| 2 | 0 | 1 | 0 | $m_2 = \overline{x}_1 x_2\overline{x}_3$ | $M_2 = x_1 + \overline{x}_2 + x_3$ |
| 3 | 0 | 1 | 1 | $m_3 = \overline{x}_1 x_2 x_3$ | $M_3 = x_1 + \overline{x}_2 + \overline{x}_3$ |
| 4 | 1 | 0 | 0 | $m_4 = x_1\overline{x}_2\overline{x}_3$ | $M_4 = \overline{x}_1 + x_2 + x_3$ |
| 5 | 1 | 0 | 1 | $m_5 = x_1\overline{x}_2 x_3$ | $M_5 = \overline{x}_1 + x_2 + \overline{x}_3$ |
| 6 | 1 | 1 | 0 | $m_6 = x_1 x_2\overline{x}_3$ | $M_6 = \overline{x}_1 + \overline{x}_2 + x_3$ |
| 7 | 1 | 1 | 1 | $m_7 = x_1 x_2 x_3$ | $M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$ |

**The function is 1 for these rows**

**The function is 0 for these rows**

# Two different ways to specify the same function f of three variables

$$f(x_1, x_2, x_3) = \Sigma\, m(0, 2, 4, 5, 6, 7)$$

$$f(x_1, x_2, x_3) = \Pi\, M(1, 3)$$

# The POS Expression

$$M_1 = x_1 + x_2 + \overline{x}_3 \qquad\qquad M_3 = x_1 + \overline{x}_2 + \overline{x}_3$$

$$f(x_1, x_2, x_3) = \Pi\, M(1, 3)$$

$$= M_1 \bullet M_3$$

$$= ( x_1 + x_2 + \overline{x_3}) \bullet ( x_1 + \overline{x_2} + \overline{x_3})$$

# The Minimum POS Expression

$$
\begin{aligned}
f(x_1, x_2, x_3) &= (x_1 + x_2 + \overline{x_3}) \bullet (x_1 + \overline{x_2} + \overline{x_3}) \\
&= (x_1 + \overline{x_3} + x_2) \bullet (x_1 + \overline{x_3} + \overline{x_2}) \\
&= (x_1 + \overline{x_3})
\end{aligned}
$$

**Hint: Use the following Boolean Algebra theorem**

**14b.** $(x + y) \bullet (x + \overline{y}) = x$

# Alternative Solution Using K-Maps

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| 0 | 0 | 0 | $m_0$ |
| 0 | 0 | 1 | $m_1$ |
| 0 | 1 | 0 | $m_2$ |
| 0 | 1 | 1 | $m_3$ |
| 1 | 0 | 0 | $m_4$ |
| 1 | 0 | 1 | $m_5$ |
| 1 | 1 | 0 | $m_6$ |
| 1 | 1 | 1 | $m_7$ |

(a) Truth table

| $x_3$ \ $x_1 x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $m_0$ | $m_2$ | $m_6$ | $m_4$ |
| 1 | $m_1$ | $m_3$ | $m_7$ | $m_5$ |

(b) Karnaugh map

# Alternative Solution Using K-Maps

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| 0 | 0 | 0 | $m_0$ |
| 0 | 0 | 1 | $m_1$ |
| 0 | 1 | 0 | $m_2$ |
| 0 | 1 | 1 | $m_3$ |
| 1 | 0 | 0 | $m_4$ |
| 1 | 0 | 1 | $m_5$ |
| 1 | 1 | 0 | $m_6$ |
| 1 | 1 | 1 | $m_7$ |

(a) Truth table

| $x_3$ \ $x_1 x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $m_0$ | $m_2$ | $m_6$ | $m_4$ |
| 1 | $m_1$ | $m_3$ | $m_7$ | $m_5$ |

(b) Karnaugh map

# Alternative Solution Using K-Maps

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| 0 | 0 | 0 | $m_0$ |
| 0 | 0 | 1 | $m_1$ |
| 0 | 1 | 0 | $m_2$ |
| 0 | 1 | 1 | $m_3$ |
| 1 | 0 | 0 | $m_4$ |
| 1 | 0 | 1 | $m_5$ |
| 1 | 1 | 0 | $m_6$ |
| 1 | 1 | 1 | $m_7$ |

(a) Truth table

| $x_3$ \ $x_1 x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $m_0$ | $m_2$ | $m_6$ | $m_4$ |
| 1 | $m_1$ | $m_3$ | $m_7$ | $m_5$ |

(b) Karnaugh map

# Alternative Solution Using K-Maps

| $x_3 \backslash x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $m_0$ | $m_2$ | $m_6$ | $m_4$ |
| 1 | $m_1$ | $m_3$ | $m_7$ | $m_5$ |

# Alternative Solution Using K-Maps

| $x_3 \backslash x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |

# Alternative Solution Using K-Maps

| $x_3$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |

$(x_1 + \overline{x_3})$

# Example 3

**Problem:** A circuit that controls a given digital system has three inputs: $x_1$, $x_2$, and $x_3$. It has to recognize three different conditions:

- Condition $A$ is true if $x_3$ is true and either $x_1$ is true or $x_2$ is false
- Condition $B$ is true if $x_1$ is true and either $x_2$ or $x_3$ is false
- Condition $C$ is true if $x_2$ is true and either $x_1$ is true or $x_3$ is false

The control circuit must produce an output of 1 if at least two of the conditions $A$, $B$, and $C$ are true. Design the simplest circuit that can be used for this purpose.

# Condition A

Condition $A$ is true if $x_3$ is true and either $x_1$ is true or $x_2$ is false

# Condition A

Condition $A$ is true if $x_3$ is true and either $x_1$ is true or $x_2$ is false

$$A = x_3(x_1 + \overline{x}_2) = x_3x_1 + x_3\overline{x}_2$$

# Condition B

Condition $B$ is true if $x_1$ is true and either $x_2$ or $x_3$ is false

# Condition B

Condition $B$ is true if $x_1$ is true and either $x_2$ or $x_3$ is false

$$B = x_1(\overline{x}_2 + \overline{x}_3) = x_1\overline{x}_2 + x_1\overline{x}_3$$

# Condition C

Condition $C$ is true if $x_2$ is true and either $x_1$ is true or $x_3$ is false

# Condition C

Condition $C$ is true if $x_2$ is true and either $x_1$ is true or $x_3$ is false

$$C = x_2(x_1 + \bar{x}_3) \; = \; x_2 x_1 + x_2 \bar{x}_3$$

**The output of the circuit can be expressed as**

$$f = \boxed{AB} + AC + BC$$

$$\begin{aligned}
AB &= (x_3x_1 + x_3\bar{x}_2)(x_1\bar{x}_2 + x_1\bar{x}_3) \\
&= x_3x_1x_1\bar{x}_2 + x_3x_1x_1\bar{x}_3 + x_3\bar{x}_2x_1\bar{x}_2 + x_3\bar{x}_2x_1\bar{x}_3 \\
&= x_3x_1\bar{x}_2 + 0 + x_3\bar{x}_2x_1 + 0 \\
&= x_1\bar{x}_2x_3
\end{aligned}$$

## The output of the circuit can be expressed as

## $f = AB + \boxed{AC} + BC$

$$
\begin{aligned}
AC &= (x_3x_1 + x_3\bar{x}_2)(x_2x_1 + x_2\bar{x}_3) \\
&= x_3x_1x_2x_1 + x_3x_1x_2\bar{x}_3 + x_3\bar{x}_2x_2x_1 + x_3\bar{x}_2x_2\bar{x}_3 \\
&= x_3x_1x_2 + 0 + 0 + 0 \\
&= x_1x_2x_3
\end{aligned}
$$

# The output of the circuit can be expressed as
# f = AB + AC + BC

$$
\begin{aligned}
BC &= (x_1\bar{x}_2 + x_1\bar{x}_3)(x_2x_1 + x_2\bar{x}_3) \\
&= x_1\bar{x}_2x_2x_1 + x_1\bar{x}_2x_2\bar{x}_3 + x_1\bar{x}_3x_2x_1 + x_1\bar{x}_3x_2\bar{x}_3 \\
&= 0 + 0 + x_1\bar{x}_3x_2 + x_1\bar{x}_3x_2 \\
&= x_1x_2\bar{x}_3
\end{aligned}
$$

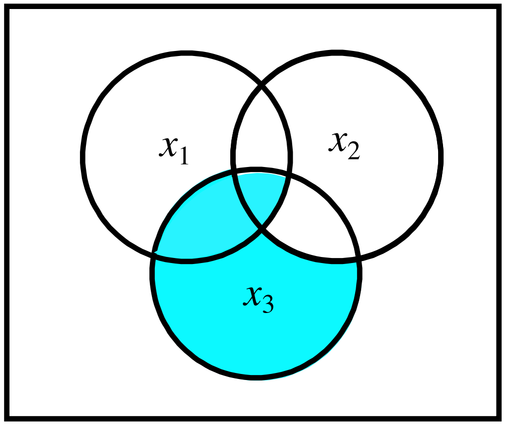
# Finally, we get

$$
\begin{aligned}
f &= x_1\bar{x}_2x_3 + x_1x_2x_3 + x_1x_2\bar{x}_3 \\
&= x_1(\bar{x}_2 + x_2)x_3 + x_1x_2(x_3 + \bar{x}_3) \\
&= x_1x_3 + x_1x_2 \\
&= x_1(x_3 + x_2)
\end{aligned}
$$

# Example 4

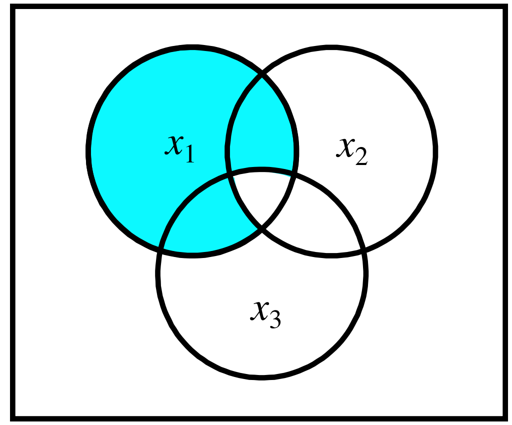
**Solve the previous problem using Venn diagrams.**

# Venn Diagrams
# (find the areas that are shaded at least two times)



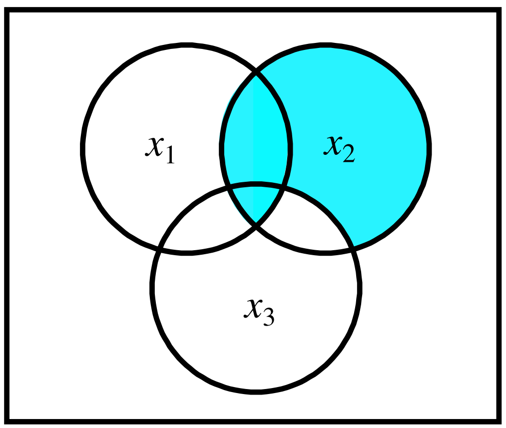

(a) Function *A:*

$$x_3x_1 + x_3\bar{x}_2$$



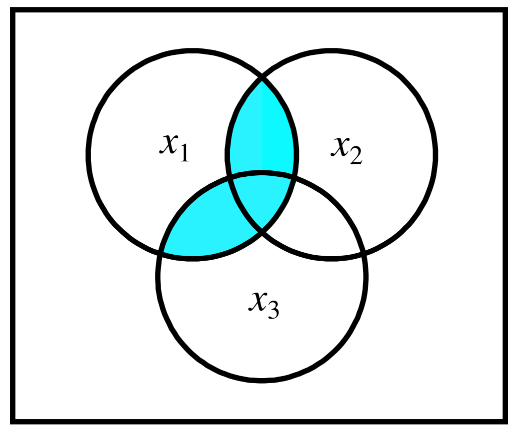

(b) Function *B*

$$x_1\bar{x}_2 + x_1\bar{x}_3$$




(c) Function *C*

$$x_2x_1 + x_2\bar{x}_3$$




(d) Function *f*

$$x_1(x_3 + x_2)$$

[ Figure 2.66 from the textbook ]

# Example 5

**Design the minimum-cost SOP and POS expression for the function**

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

# Let’s Use a K-Map

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

| | $x_3x_4 \backslash x_1x_2$ | 00 | 01 | 11 | 10 | |
|---|---|---|---|---|---|---|
| | 00 | $m_0$ | $m_4$ | $m_{12}$ | $m_8$ | |
| | 01 | $m_1$ | $m_5$ | $m_{13}$ | $m_9$ | $x_4$ |
| $x_3$ | 11 | $m_3$ | $m_7$ | $m_{15}$ | $m_{11}$ | $x_4$ |
| $x_3$ | 10 | $m_2$ | $m_6$ | $m_{14}$ | $m_{10}$ | |
| | | | $x_2$ | $x_2$ | | |

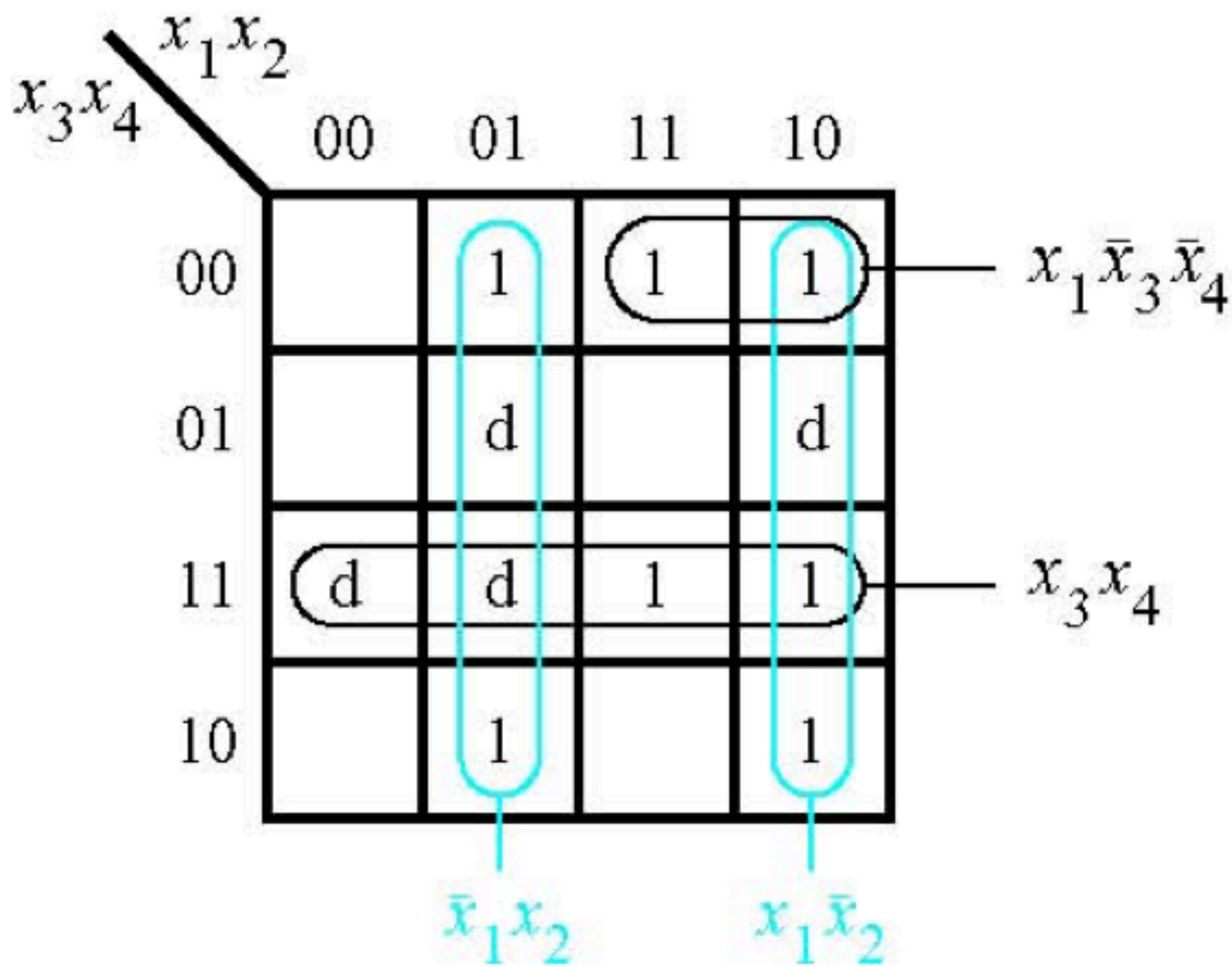
$x_1$: columns 11, 10

# Let's Use a K-Map

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

| $x_3x_4 \backslash x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 1 | 1 |
| 01 | 0 | d | 0 | d |
| 11 | d | d | 1 | 1 |
| 10 | 0 | 1 | 0 | 1 |

# The SOP Expression



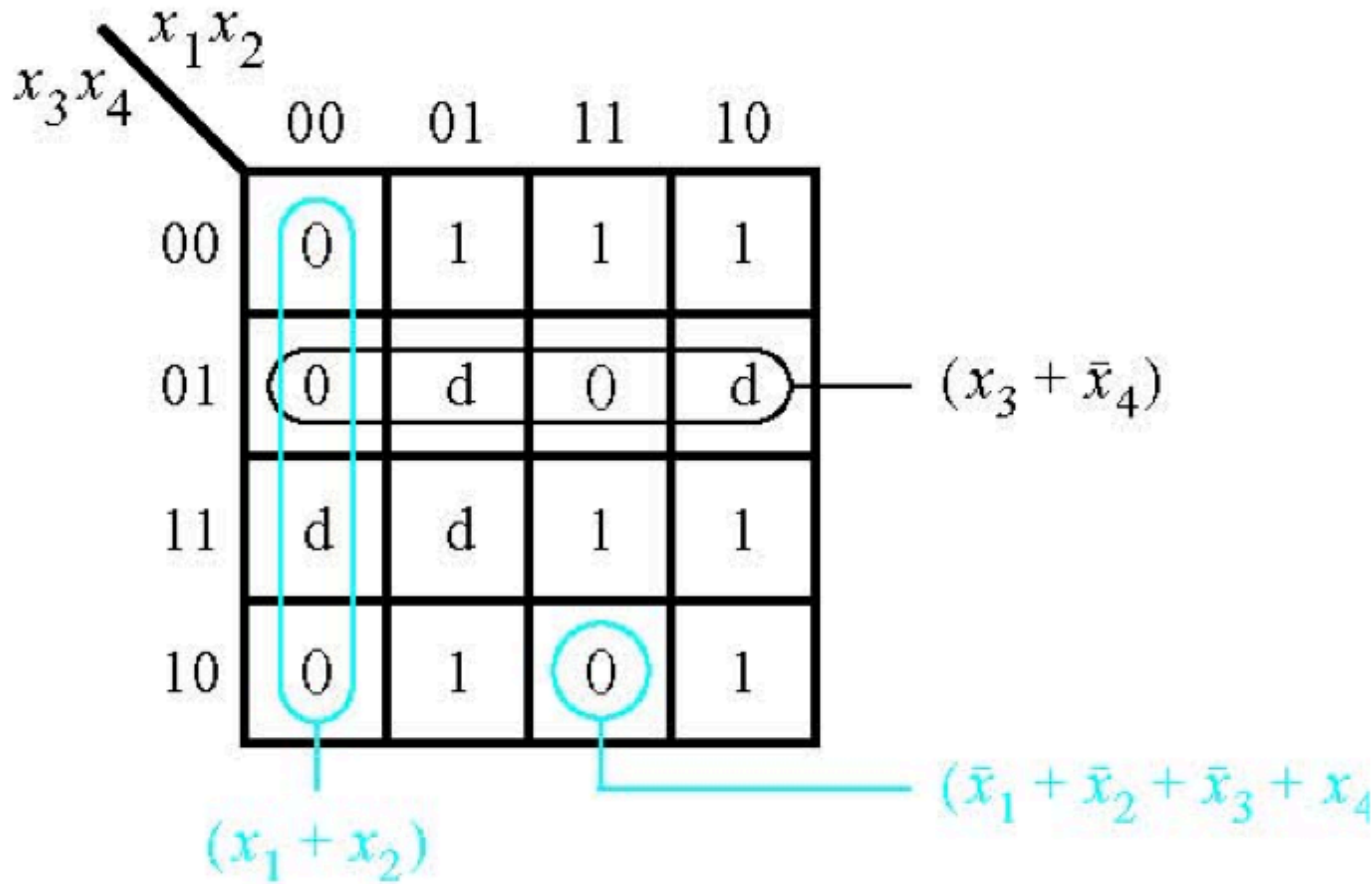

[ Figure 2.67a from the textbook ]

# What about the POS Expression?

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

| $x_3x_4$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 1 | 1 |
| 01 | 0 | d | 0 | d |
| 11 | d | d | 1 | 1 |
| 10 | 0 | 1 | 0 | 1 |

# The POS Expression




[ Figure 2.67b from the textbook ]

# Example 6

**Use K-maps to find the minimum-cost SOP and POS expression for the function**

$$f(x_1, \ldots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

assuming that there are also don't-cares defined as $D = \sum(9, 12, 14)$.

# Let’s map the expression to the K-Map

$$f(x_1, \ldots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

$$D = \sum(9, 12, 14).$$

| $x_3x_4$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ | $m_4$ | $m_{12}$ | $m_8$ |
| 01 | $m_1$ | $m_5$ | $m_{13}$ | $m_9$ |
| 11 | $m_3$ | $m_7$ | $m_{15}$ | $m_{11}$ |
| 10 | $m_2$ | $m_6$ | $m_{14}$ | $m_{10}$ |

# Let’s map the expression to the K-Map

$$f(x_1, \ldots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

$$D = \sum(9, 12, 14).$$

| $x_3x_4$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ | $m_4$ | d | $m_8$ |
| 01 | $m_1$ | $m_5$ | $m_{13}$ | d |
| 11 | $m_3$ | $m_7$ | $m_{15}$ | $m_{11}$ |
| 10 | $m_2$ | $m_6$ | d | $m_{10}$ |

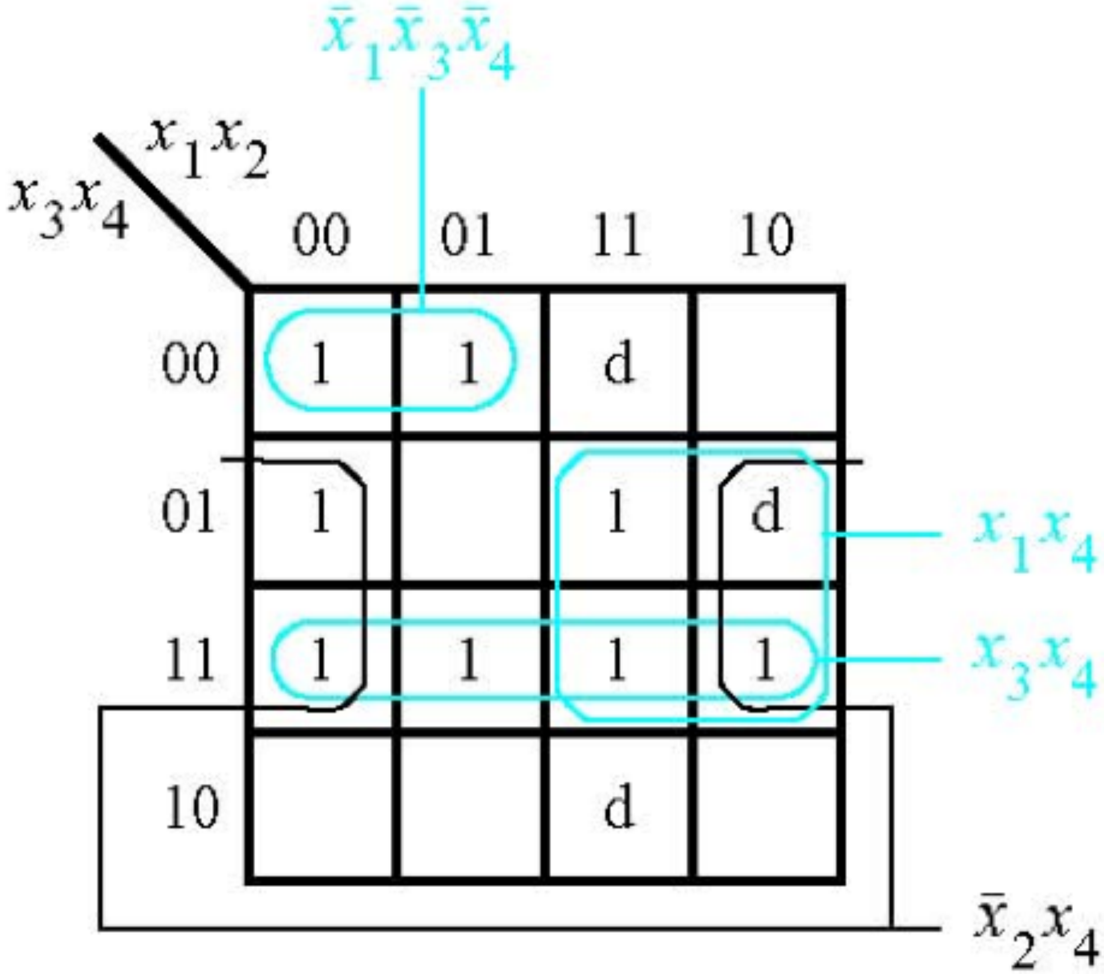
# Let’s map the expression to the K-Map

$$f(x_1, \ldots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

| $x_3x_4$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | $m_0$ | $m_4$ | d | $m_8$ |
| 01 | $m_1$ | $m_5$ | $m_{13}$ | d |
| 11 | $m_3$ | $m_7$ | $m_{15}$ | $m_{11}$ |
| 10 | $m_2$ | $m_6$ | d | $m_{10}$ |

# The SOP Expression

$$f(x_1, \ldots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$



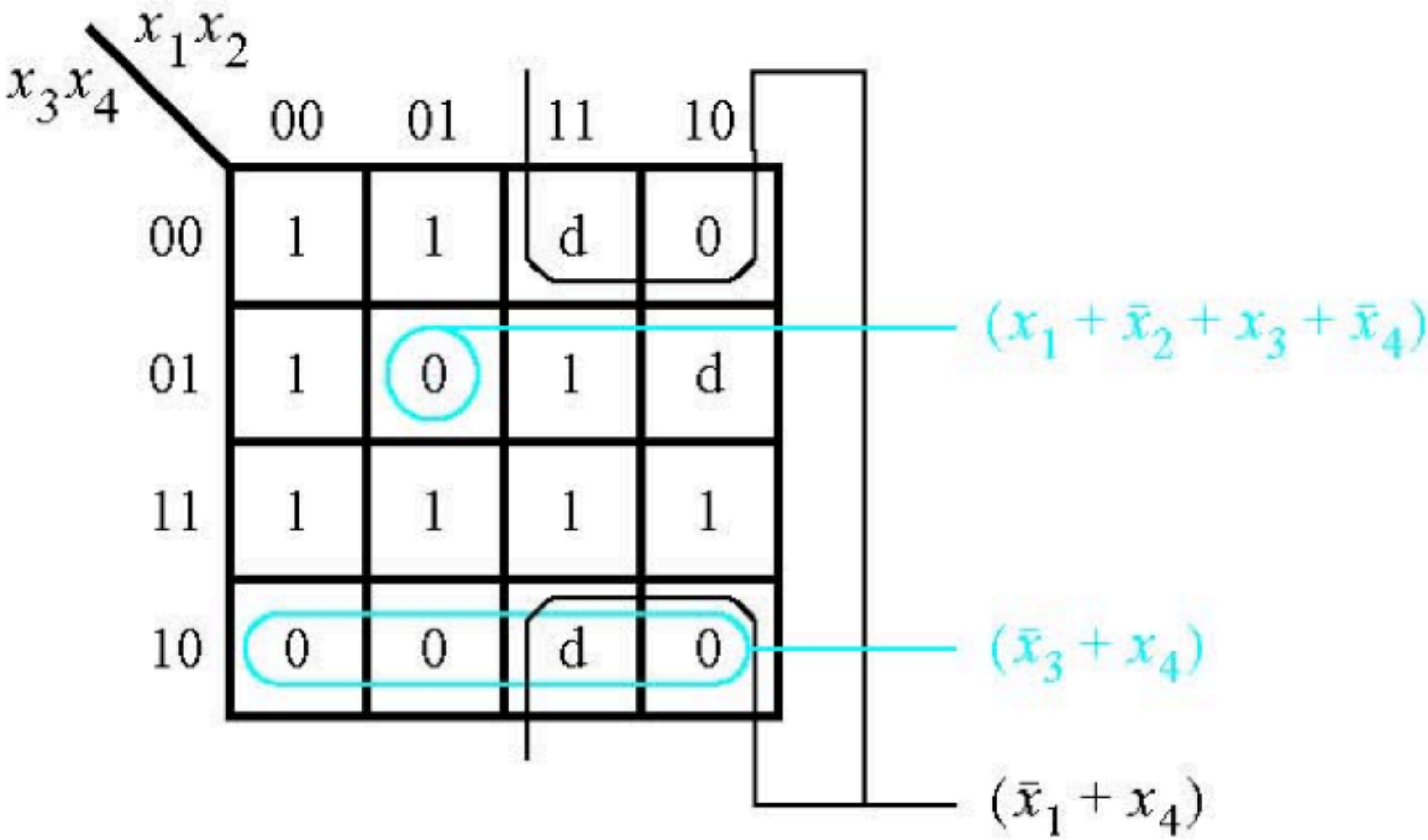

$$f = x_3x_4 + \bar{x}_1\bar{x}_3\bar{x}_4 + \bar{x}_2x_4 + x_1x_4$$

[ Figure 2.68a from the textbook ]

# What about the POS Expression?

| $x_3x_4$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | d | 0 |
| 01 | 1 | 0 | 1 | d |
| 11 | 1 | 1 | 1 | 1 |
| 10 | 0 | 0 | d | 0 |

# The POS Expression




$$f = (\bar{x}_3 + x_4)(\bar{x}_1 + x_4)(x_1 + \bar{x}_2 + x_3 + \bar{x}_4)$$

[ Figure 2.68b from the textbook ]

# Example 7

**Derive the minimum-cost SOP expression for**

$$f = s_3(\bar{s}_1 + \bar{s}_2) + s_1 s_2$$

# First, expand the expression using property 12a

$$f = s_3(\bar{s}_1 + \bar{s}_2) + s_1 s_2$$

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

# Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

| $s_1$ | $s_2$ | $s_3$ | |
|---|---|---|---|
| 0 | 0 | 0 | $m_0$ |
| 0 | 0 | 1 | $m_1$ |
| 0 | 1 | 0 | $m_2$ |
| 0 | 1 | 1 | $m_3$ |
| 1 | 0 | 0 | $m_4$ |
| 1 | 0 | 1 | $m_5$ |
| 1 | 1 | 0 | $m_6$ |
| 1 | 1 | 1 | $m_7$ |

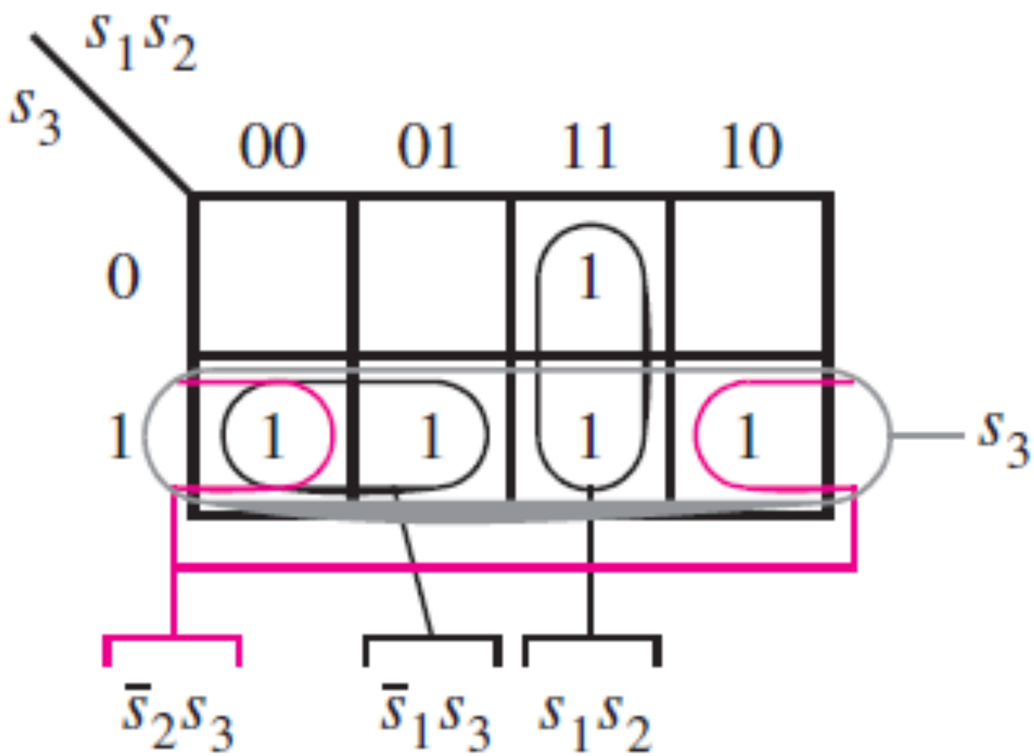
(a) Truth table

| $s_3$ \ $s_1 s_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | $m_0$ | $m_2$ | $m_6$ | $m_4$ |
| 1 | $m_1$ | $m_3$ | $m_7$ | $m_5$ |

(b) Karnaugh map

# Construct the K-Map for this expression

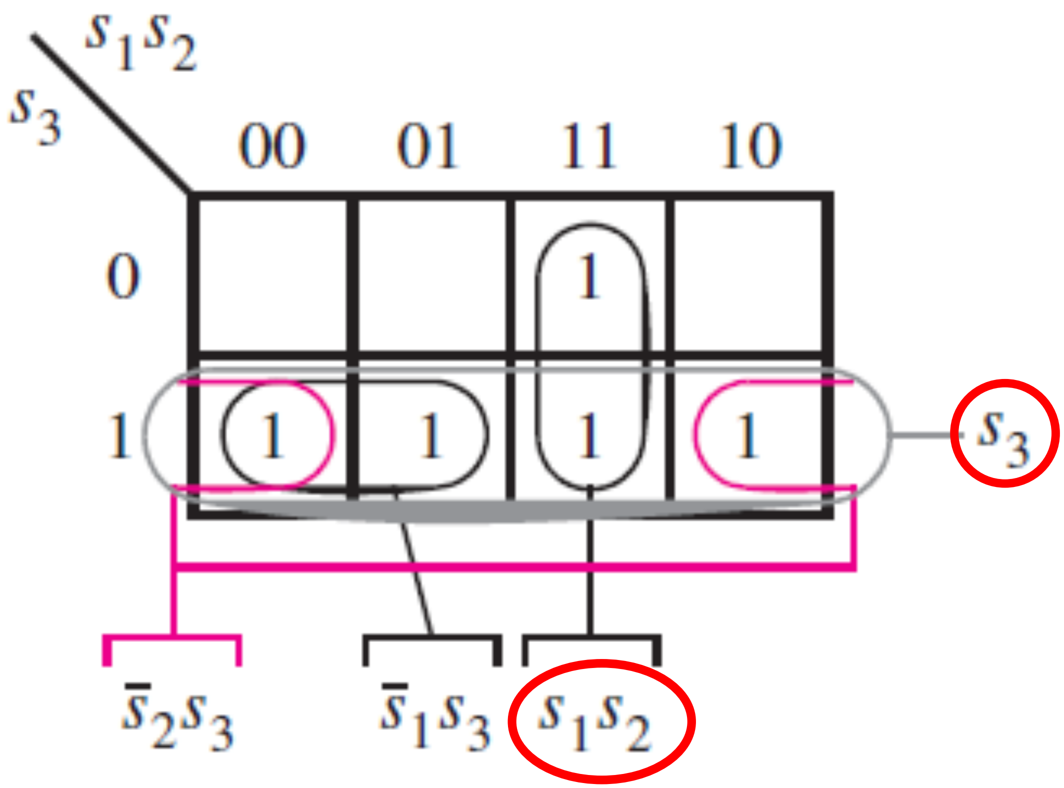
$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$




[ Figure 2.69 from the textbook ]

# Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$



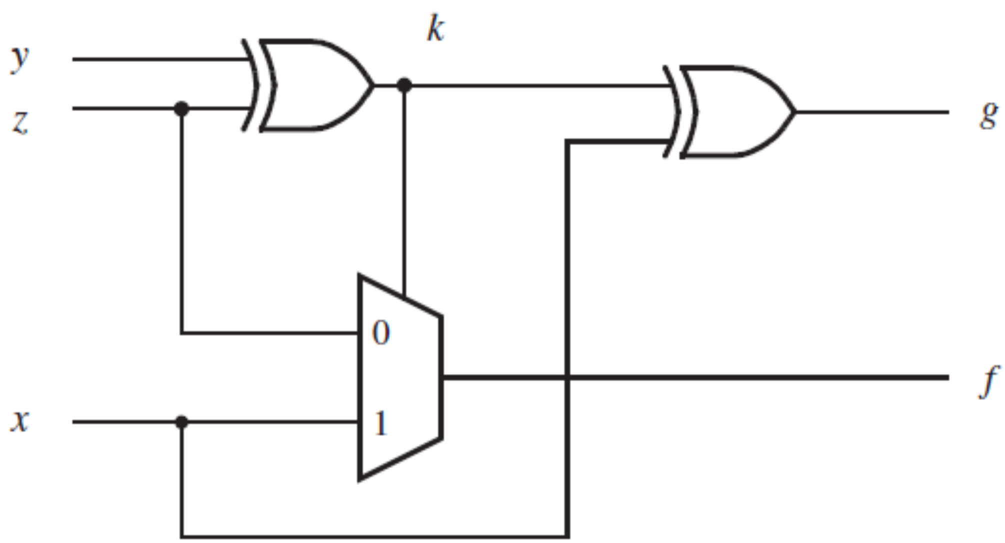

**Simplified Expression: $f = s_3 + s_1 s_2$**

[ Figure 2.69 from the textbook ]

# Example 8

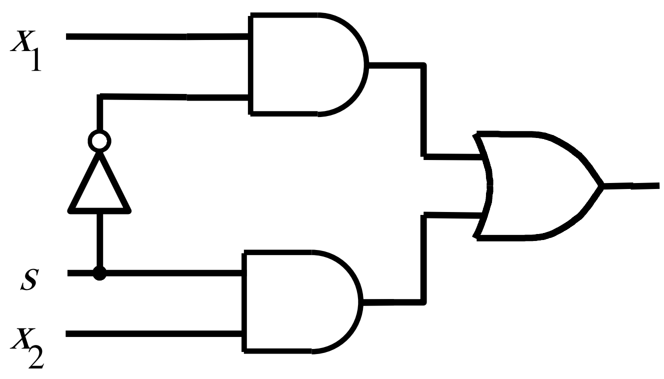
**Write the Verilog code for the following circuit …**

# Logic Circuit

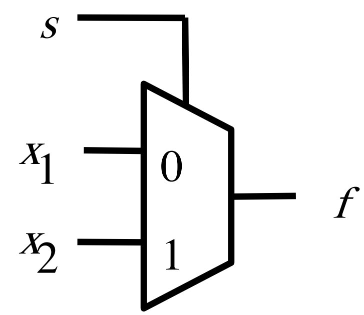



[ Figure 2.70 from the textbook ]

# Circuit for 2-1 Multiplexer




(b) Circuit

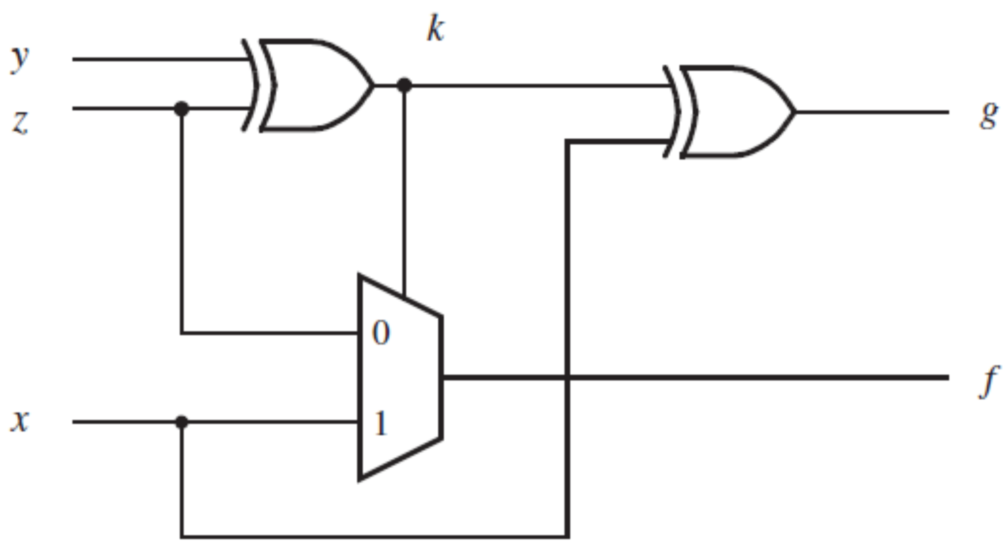



(c) Graphical symbol

$$f(s, x_1, x_2) = \overline{s}\, x_1 + s\, x_2$$

[ Figure 2.33b-c from the textbook ]

# Logic Circuit vs Verilog Code




```
module f_g (x, y, z, f, g);
    input x, y, z;
    output f, g;
    wire k;

    assign k = y ^ z;
    assign g = k ^ x;
    assign f = (~k & z) | (k & x);

endmodule
```

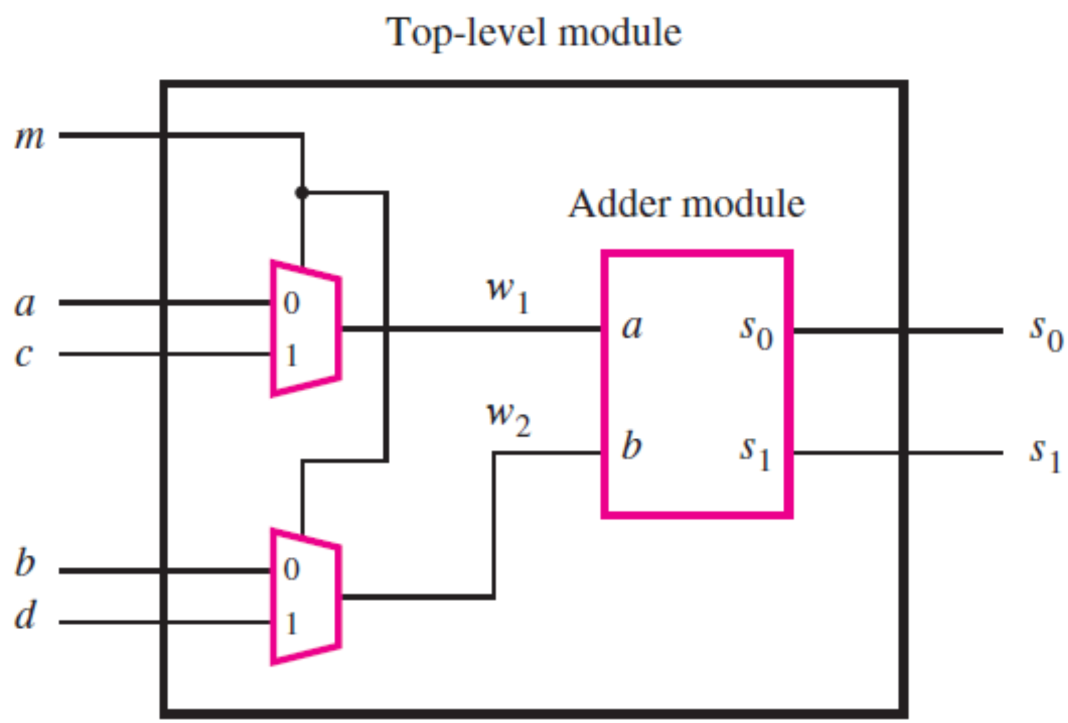
[ Figure 2.70 from the textbook ]

[ Figure 2.71 from the textbook ]

# Example 9

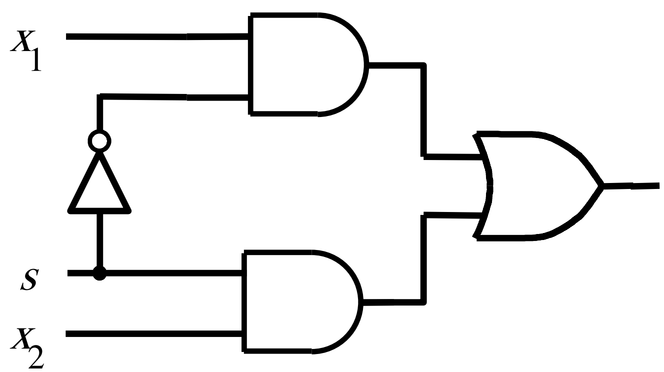
**Write the Verilog code for the following circuit …**

# The Logic Circuit for this Example

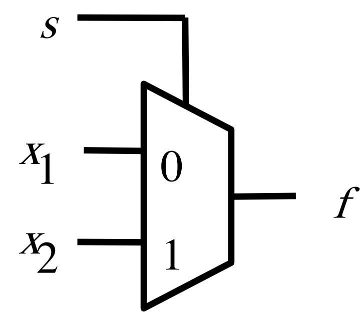



[ Figure 2.72 from the textbook ]

# Circuit for 2-1 Multiplexer




(b) Circuit

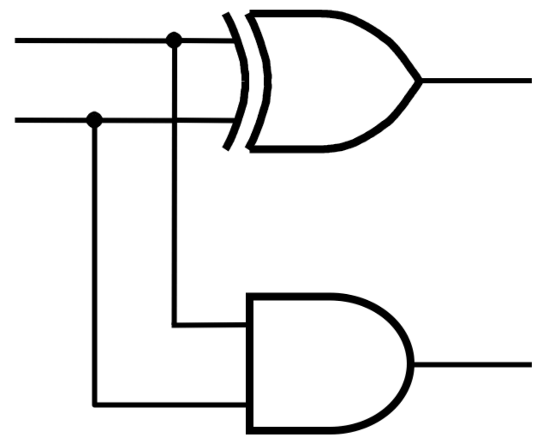



(c) Graphical symbol

$$f(s, x_1, x_2) = \overline{s}\, x_1 + s\, x_2$$

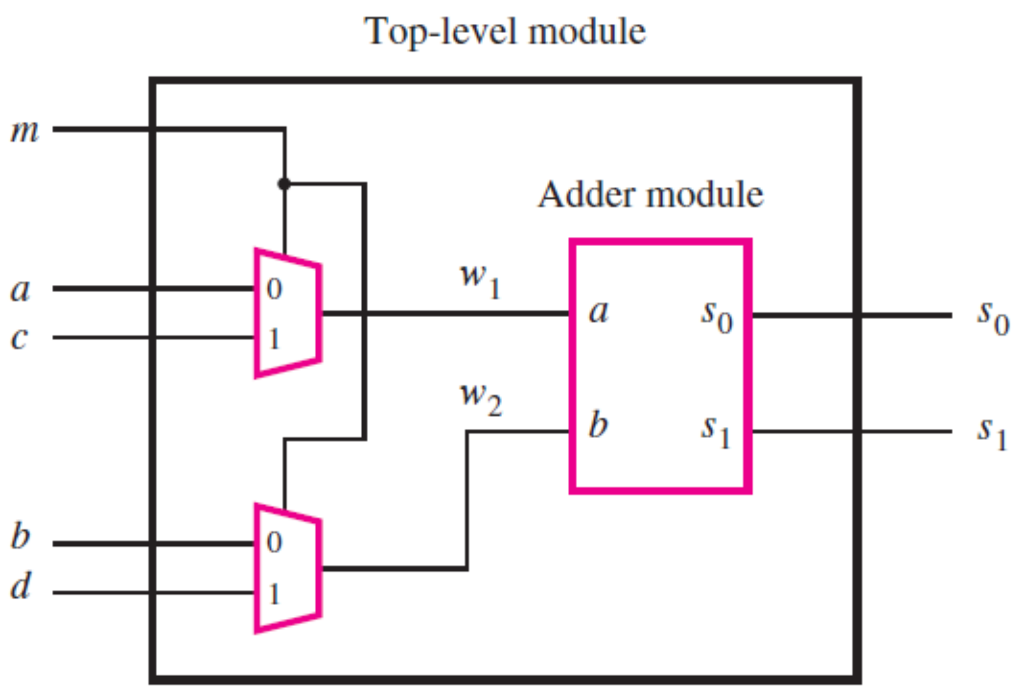
[ Figure 2.33b-c from the textbook ]

# Addition of Binary Numbers



| $a$ | $b$ | $s_1$ | $s_0$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# Logic Circuit vs Verilog Code

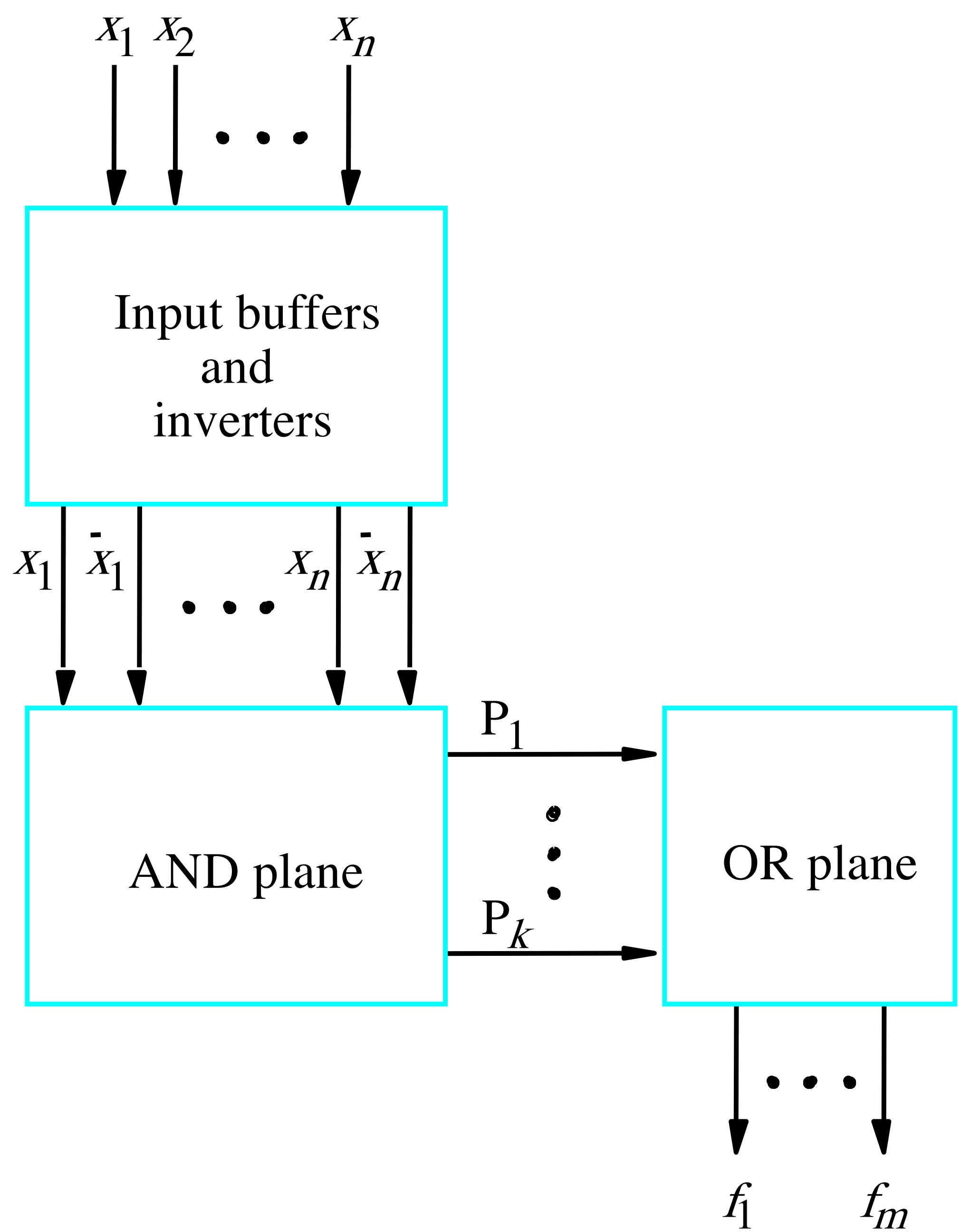



```
module shared (a, b, c, d, m, s1, s0);
    input a, b, c, d, m;
    output s1, s0;
    wire w1, w2;
    mux2to1 U1 (a, c, m, w1);
    mux2to1 U2 (b, d, m, w2);
    adder U3 (w1, w2, s1, s0);
endmodule

module mux2to1 (x1, x2, s, f);
    input x1, x2, s;
    output f;
    assign f = (~s & x1) | (s & x2);
endmodule

module adder (a, b, s1, s0);
    input a, b;
    output s1, s0;
    assign s1 = a & b;
    assign s0 = a ^ b;
endmodule
```

[ Figure 2.73 from the textbook ]

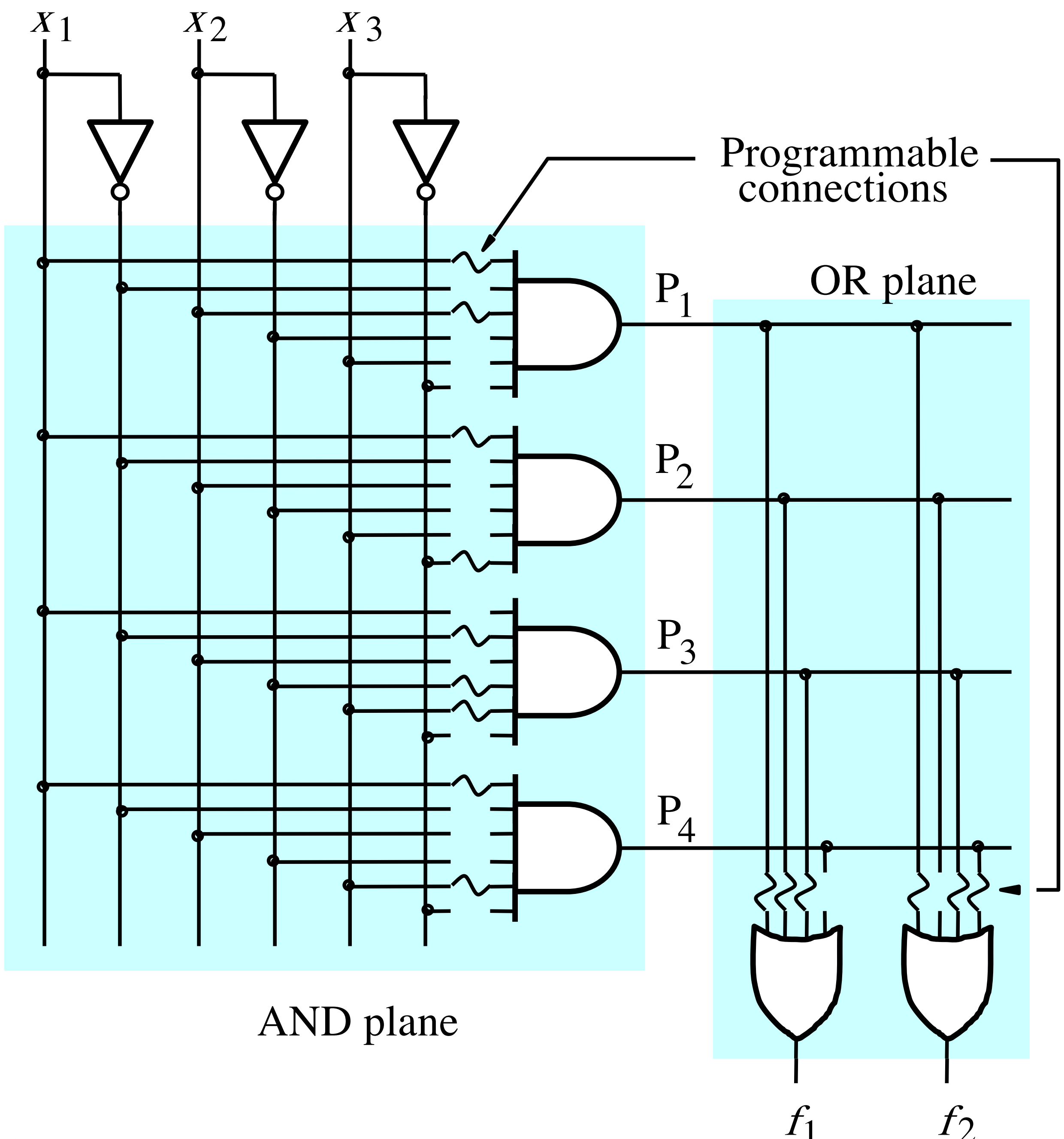
# Some material form Appendix B

# Programmable Logic Array (PLA)



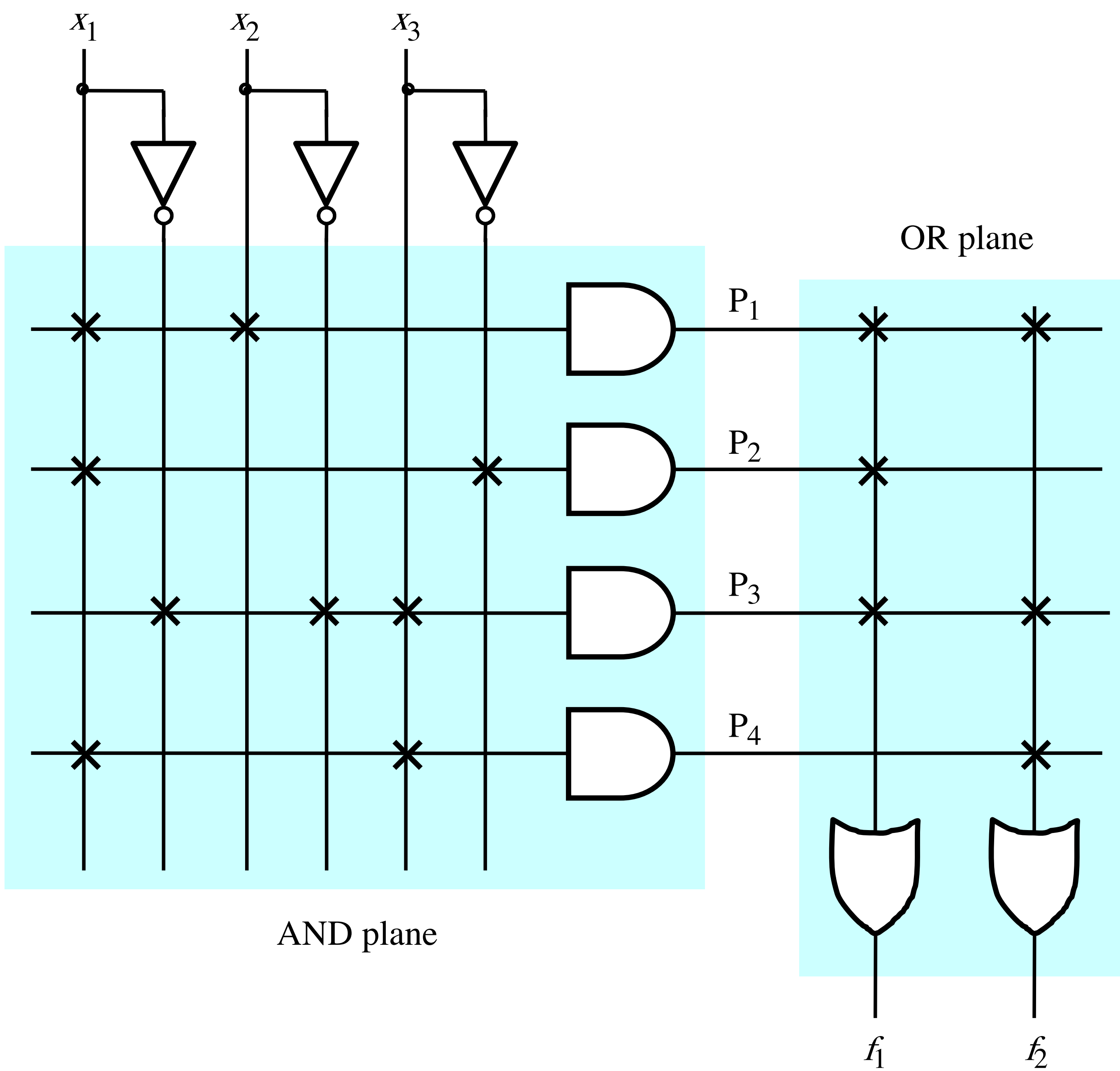

[ Figure B.25 from textbook ]

# Gate-Level Diagram of a PLA



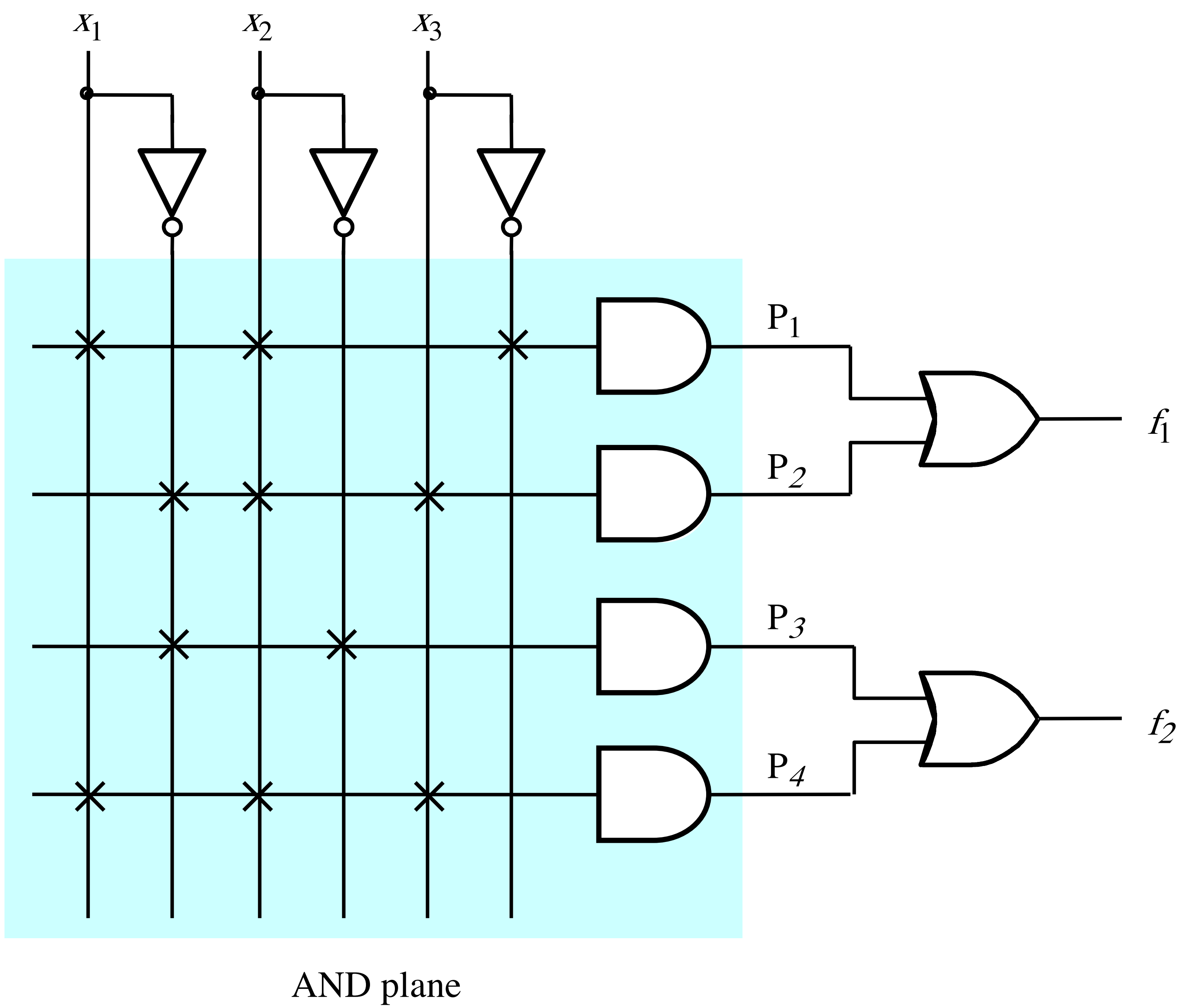

[ Figure B.26 from textbook ]

# Customary Schematic for PLA



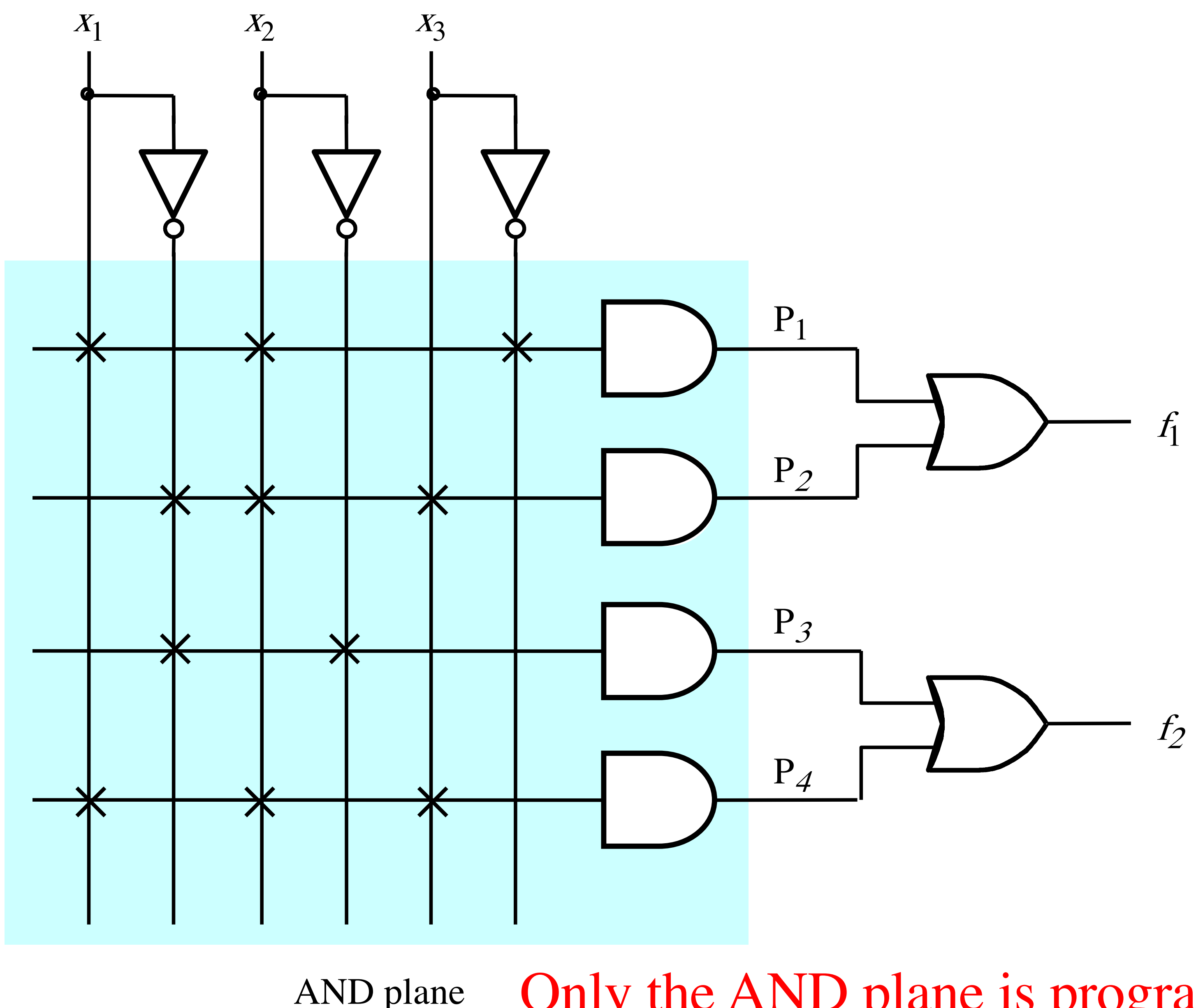

[ Figure B.27 from textbook ]

# Programmable Array Logic (PAL)




[ Figure B.28 from textbook ]

# Programmable Array Logic (PAL)




Only the AND plane is programmable.
The OR plane is fixed.

[ Figure B.28 from textbook ]

# Questions?

THE END