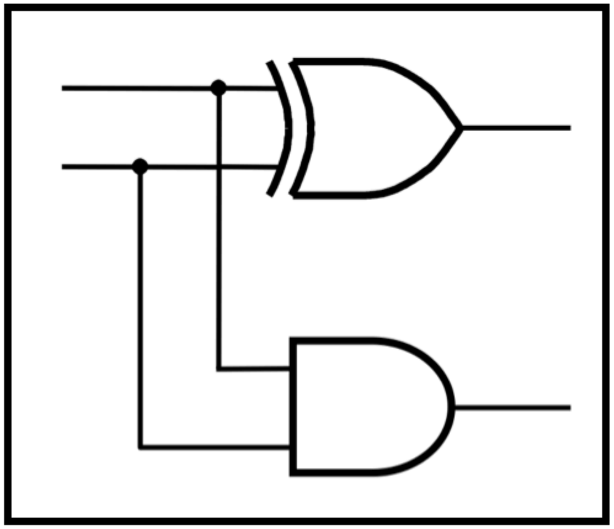


CprE 281: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

State Minimization

CprE 281: Digital Logic
Iowa State University, Ames, IA

Administrative Stuff

- **Final Project**

- **Posted on the class web page (Labs section)**

- **Pick one of the problems and solve it.**

- **Your grade will not depend on which project you pick**

- **By next Wednesday you need to select your project and send an e-mail to your lab TAs**

Sample E-mail

Hello TAs,

I decided to pick problem number x for my final project in CprE 281.

Thanks,

[your name, your lab section]

Equivalence of states

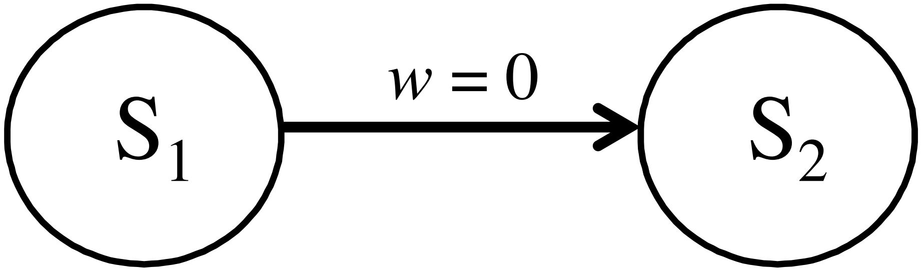
“Two states S_i and S_j are said to be equivalent if and only if for every possible input sequence, the same output sequence will be produced regardless of whether S_i or S_j is the initial state.”

[Page 374 in the textbook]

Partition Minimization Procedure

0-successor

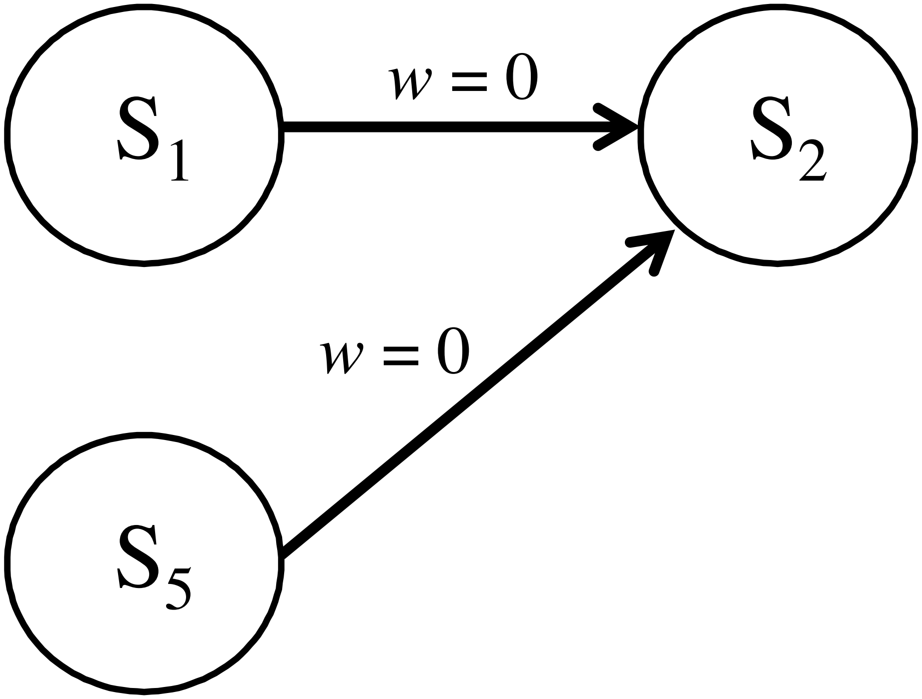
Assuming that we have only one input signal w



S_2 is a 0-successor of S_1

0-successor

Assuming that we have only one input signal w

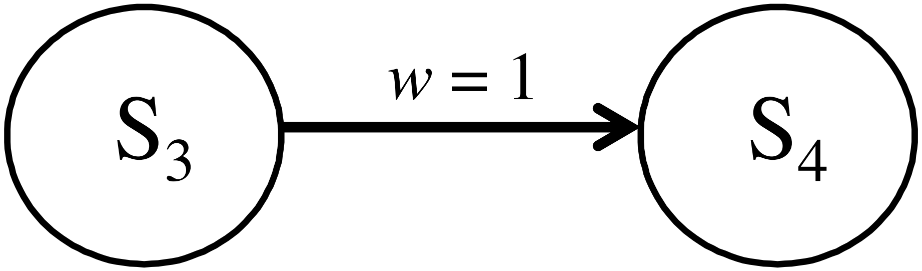


S_2 is a 0-successor of S_1

S_2 is a 0-successor of S_5

1-successor

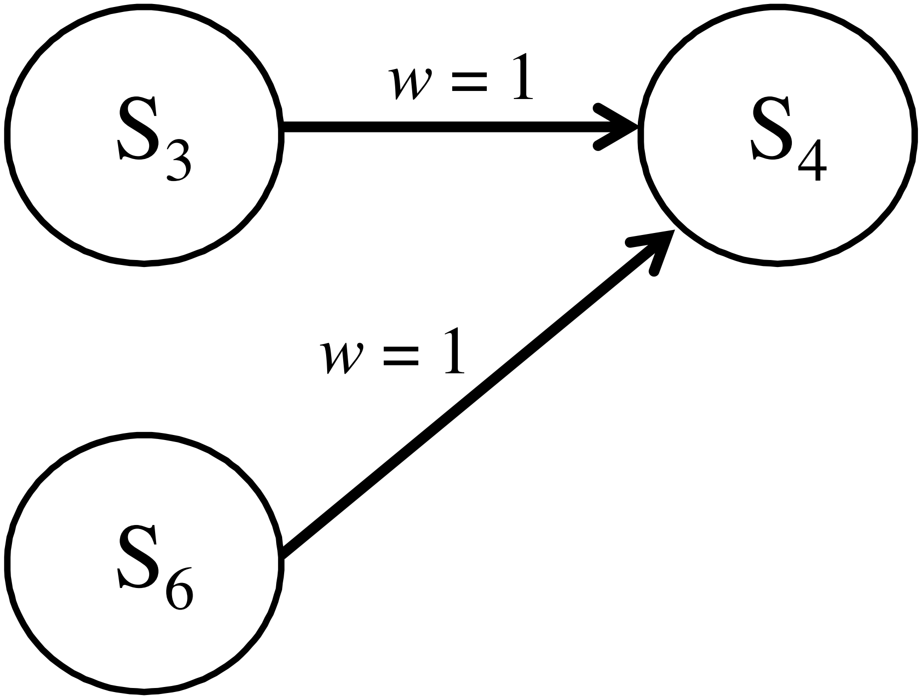
Assuming that we have only one input signal w



S_4 is a 1-successor of S_3

1-successor

Assuming that we have only one input signal w

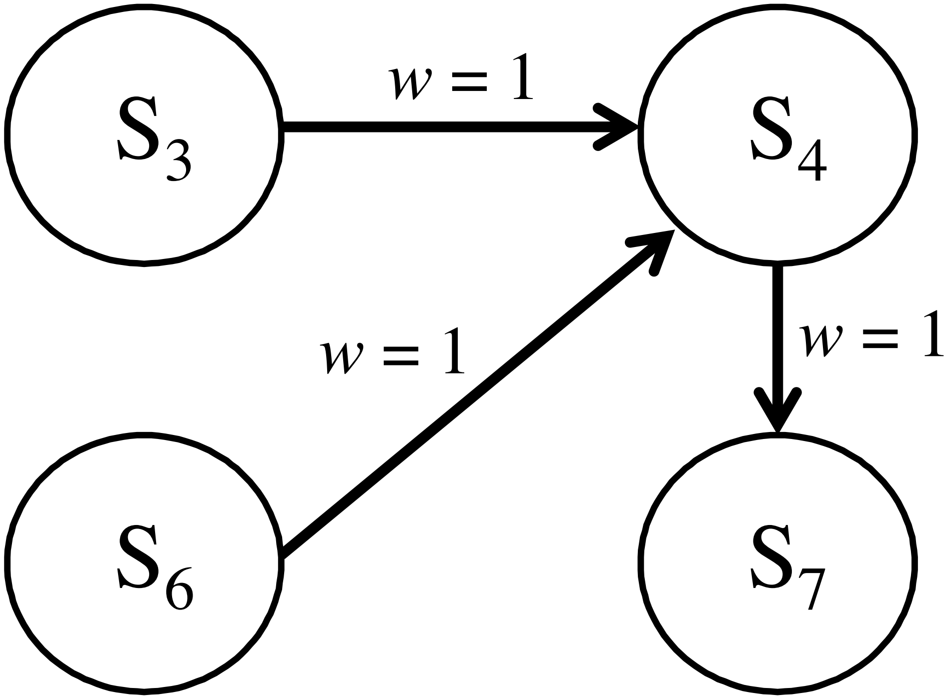


S_4 is a 1-successor of S_3

S_4 is a 1-successor of S_6

1-successor

Assuming that we have only one input signal w



S_4 is a 1-successor of S_3

S_4 is a 1-successor of S_6

S_7 is a 1-successor of S_4

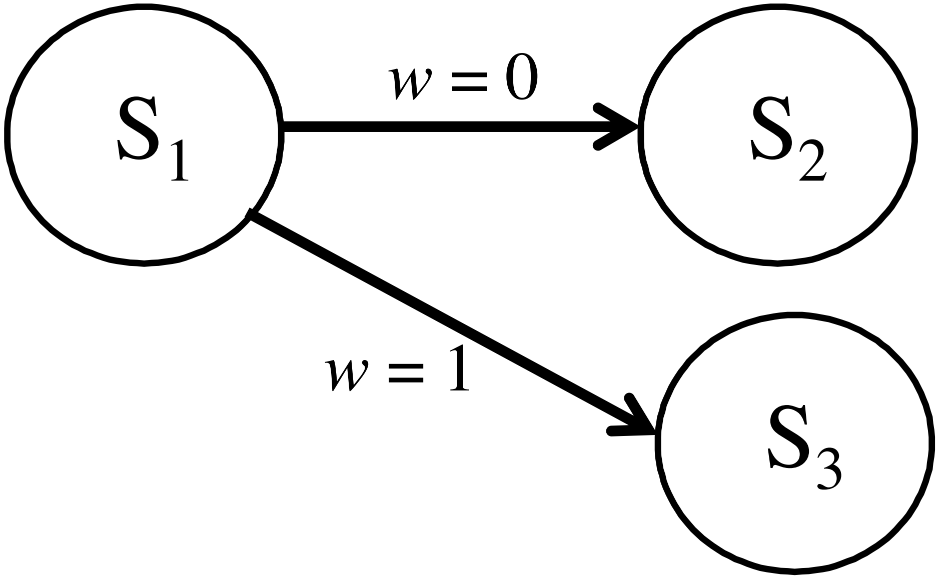
k-successors of a State

Assuming that we have only one input signal w, then k can only be equal to 0 or 1.

k-successors of a State

Assuming that we have only one input signal w, then k can only be equal to 0 or 1.

In other words, this is the familiar 0-successor or 1-successor case.



S_2 is a 0-successor of S_1

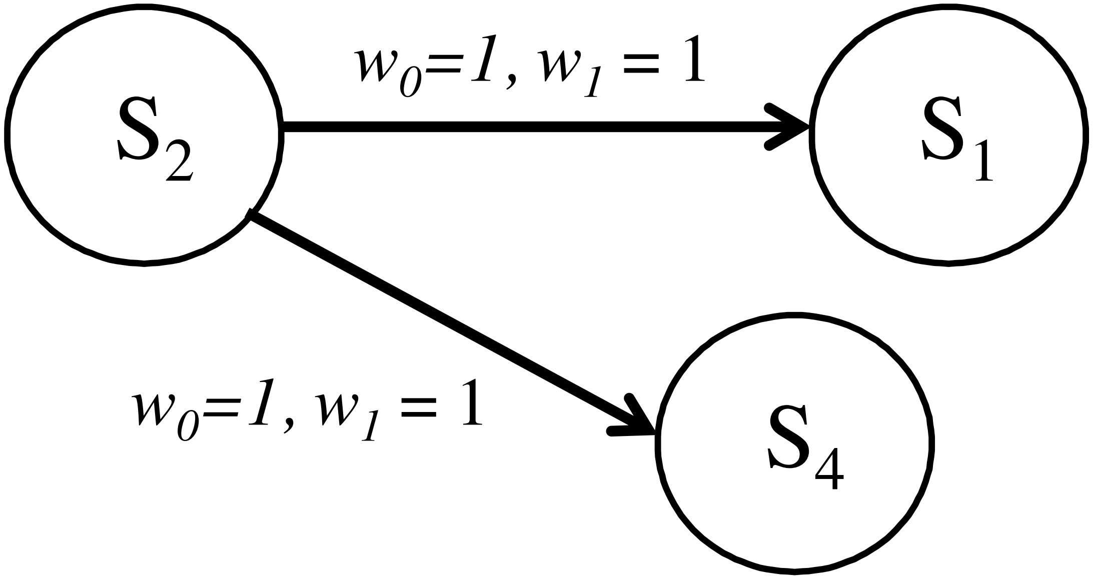
S_3 is a 1-successor of S_1

k-successors of a State

If we have two input signals, e.g., w_0 and w_1, then k can only be equal to 0,1, 2, or 3.

k-successors of a State

If we have two input signals, e.g., w_0 and w_1, then k can only be equal to 0,1, 2, or 3.



S_1 is a 3-successor of S_2

S_4 is a 3-successor of S_2

Equivalence of states

“If states S_i and S_j are equivalent, then their corresponding k-successors (for all k) are also equivalent.”

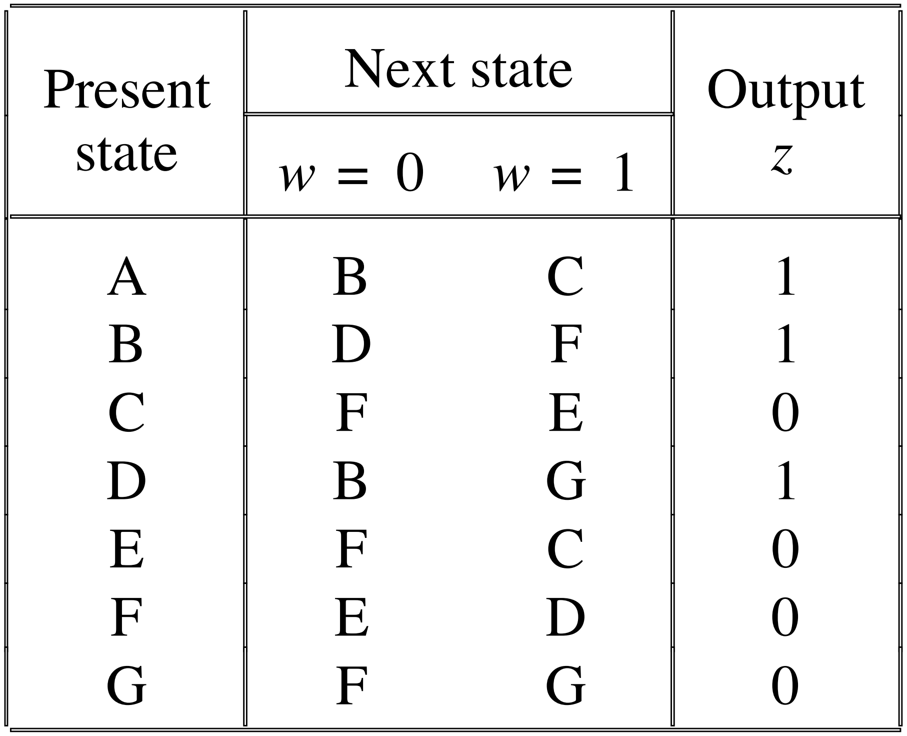
[Page 374 in the textbook]

Partition

“A partition consists of one or more blocks, where each block comprises a subset of states that may be equivalent, but the states in a given block are definitely not equivalent to the states in other blocks.”

[Page 374 in the textbook]

State Table for This Example

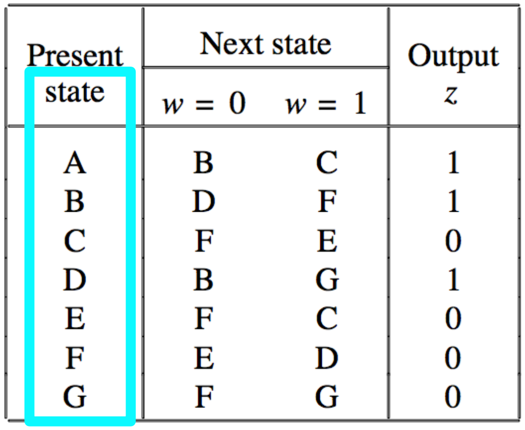


Present state	Next state		Output
	$w = 0$	$w = 1$	z
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0

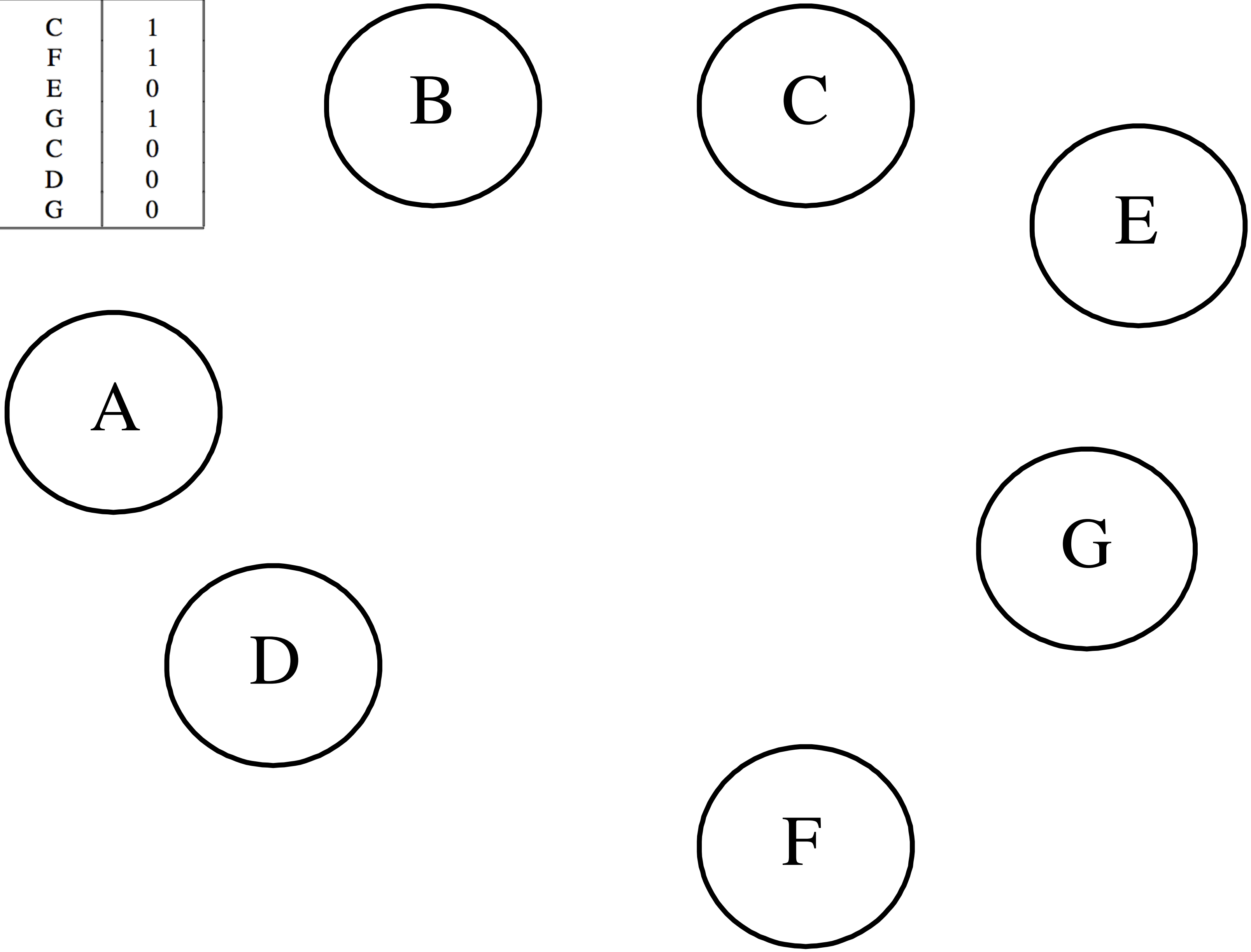
[Figure 6.51 from the textbook]

State Diagram

(just the states)



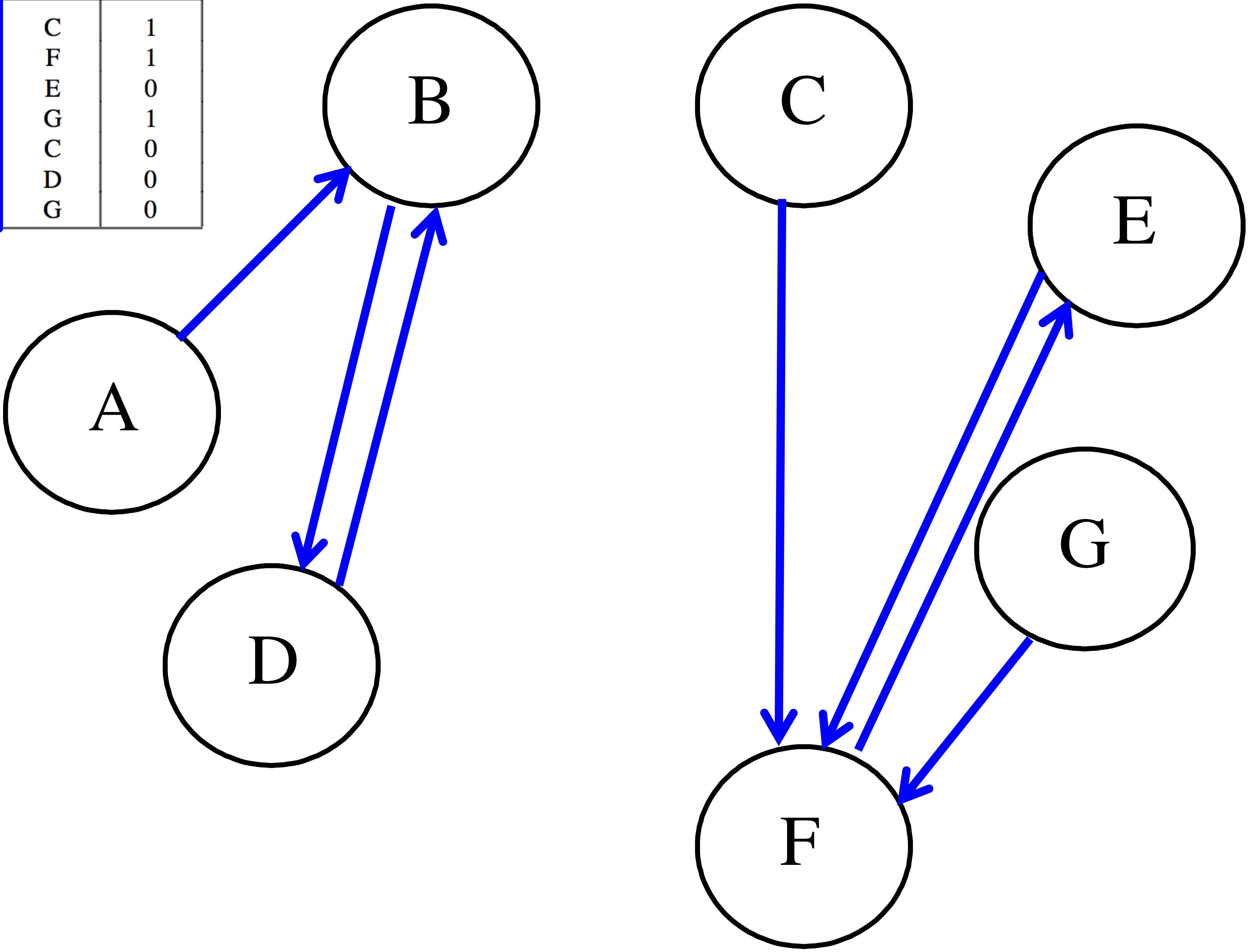
Present state	Next state		Output
	$w = 0$	$w = 1$	z
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0



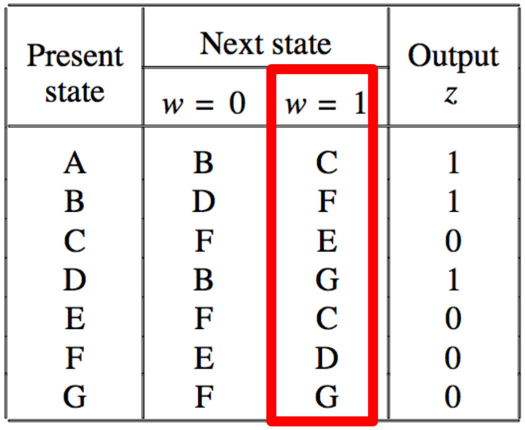
State Diagram

(transitions when w=0)

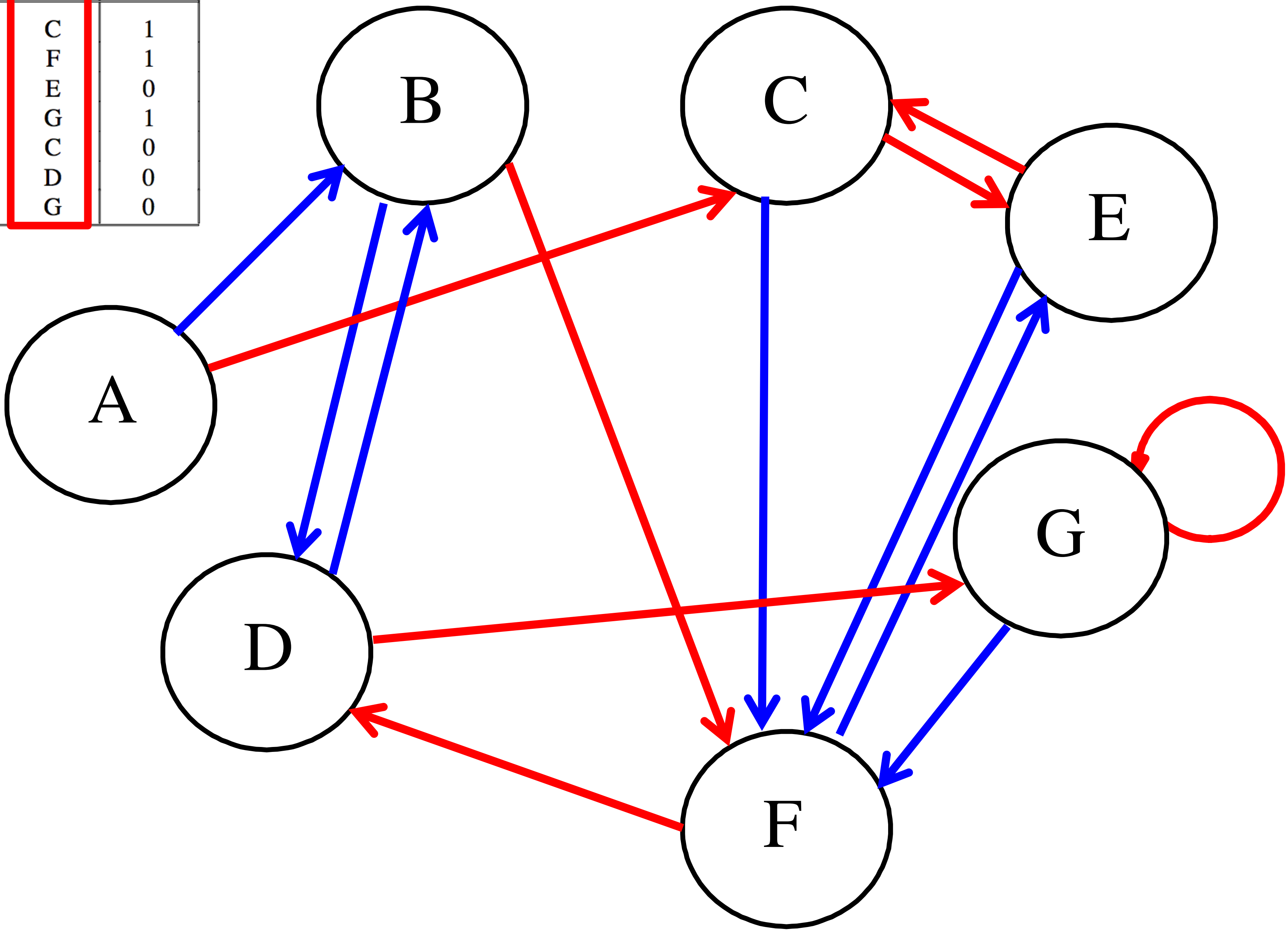
Present state	Next state		Output
	$w = 0$	$w = 1$	z
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0



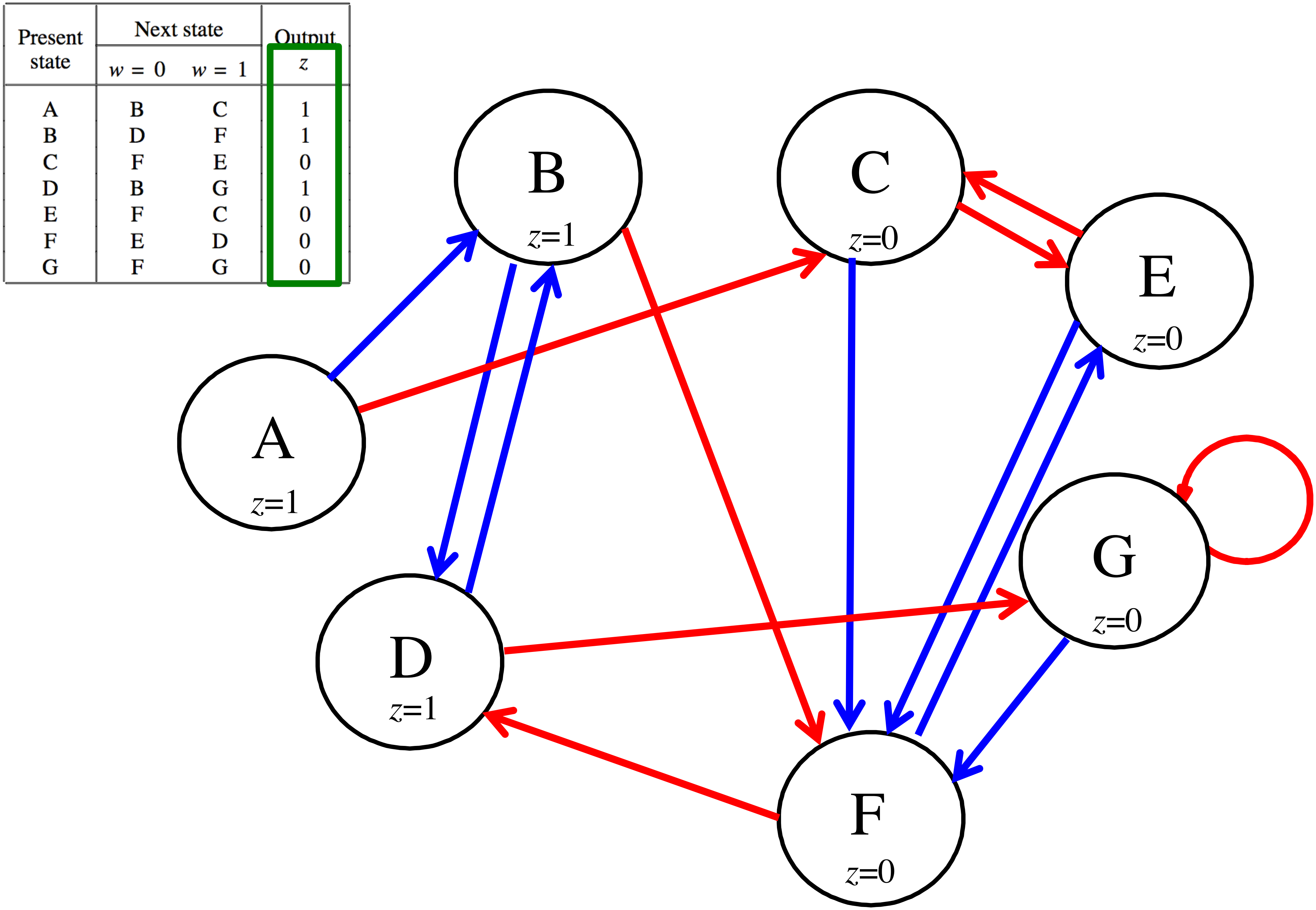
State Diagram
(transitions when w=1)



Present state	Next state		Output
	$w = 0$	$w = 1$	z
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0



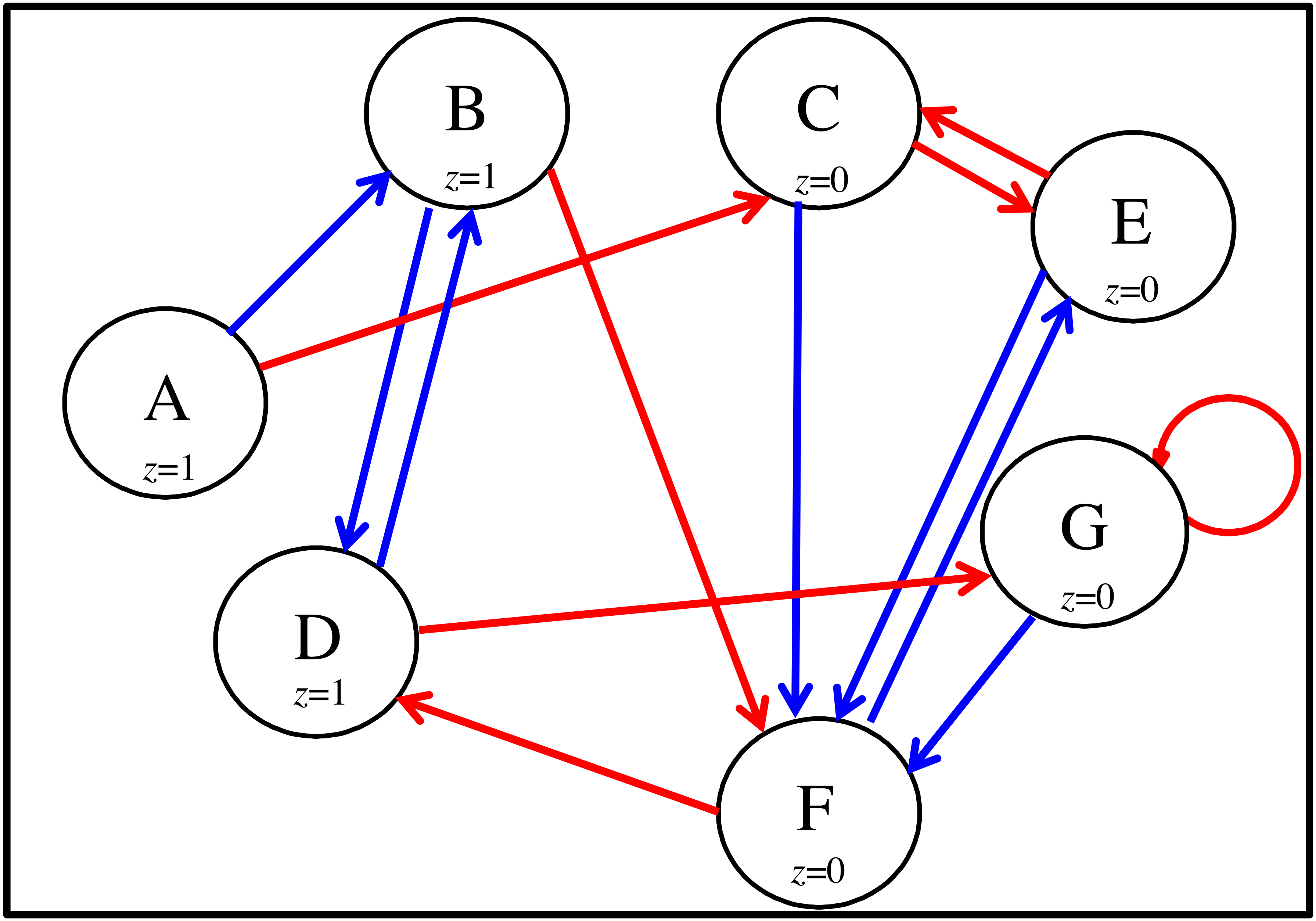
Outputs



Present state	Next state		Output
	$w = 0$	$w = 1$	z
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0

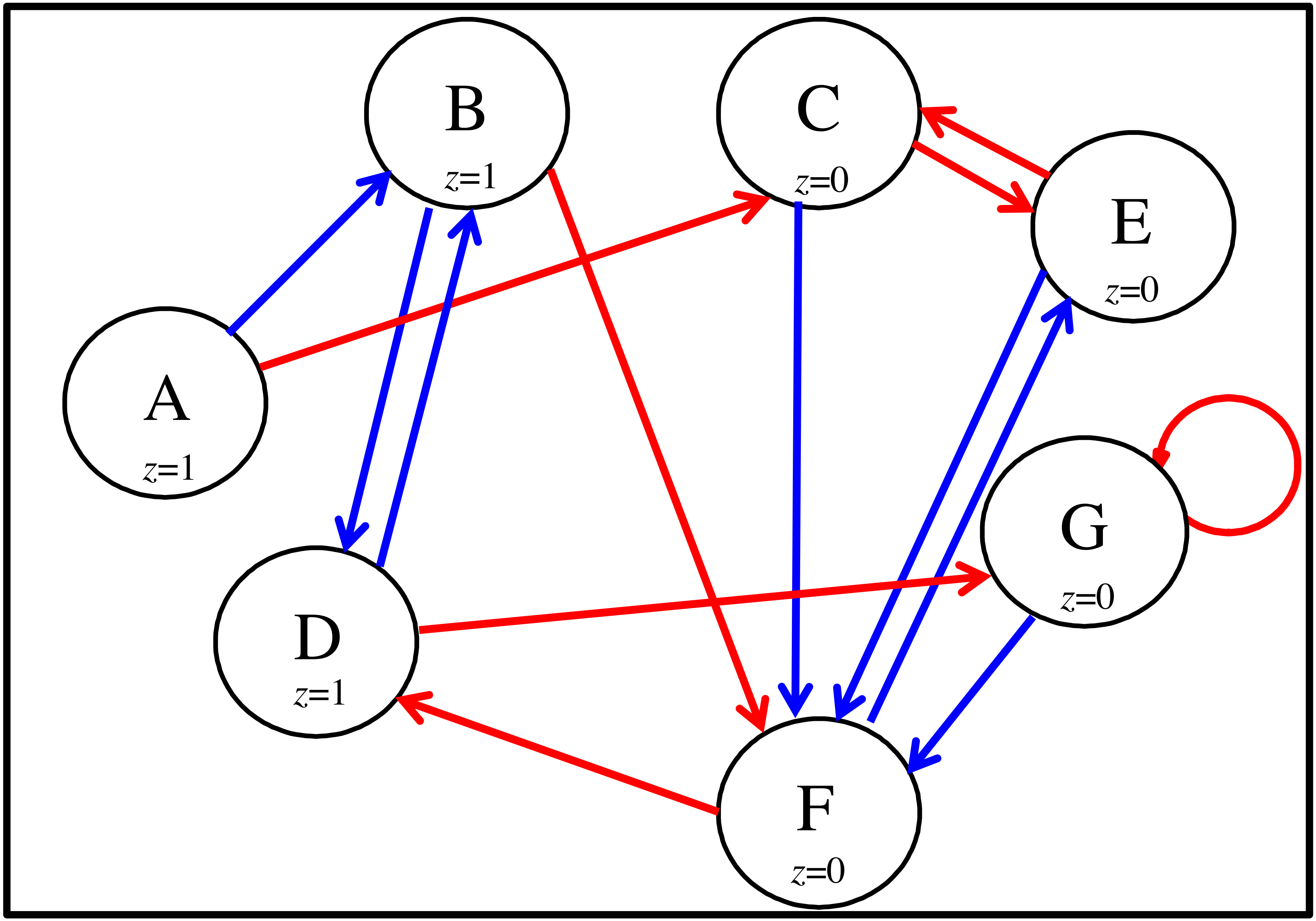
Partition #1

(All states in the same partition)



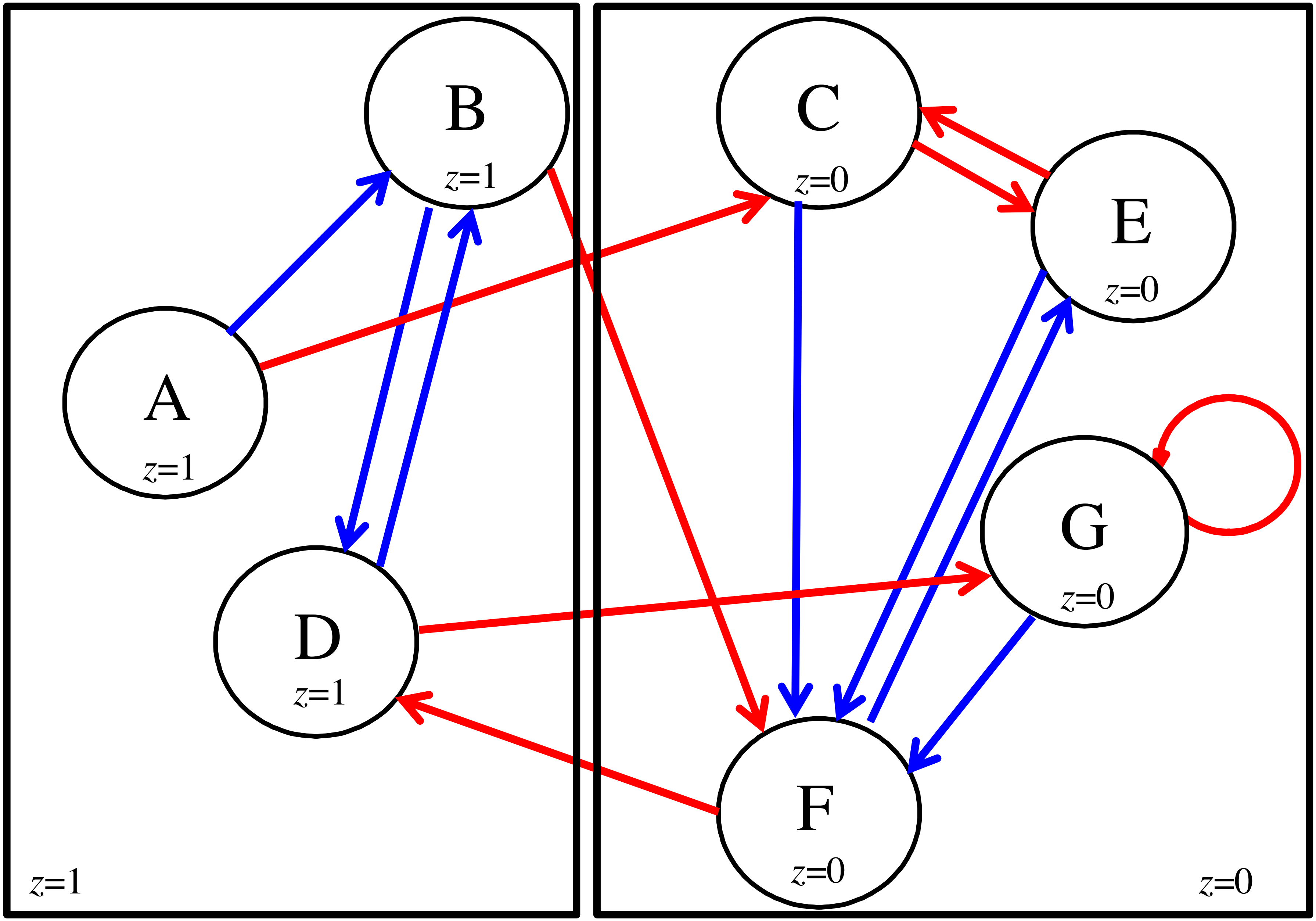
Partition #1

(ABCDEFG)



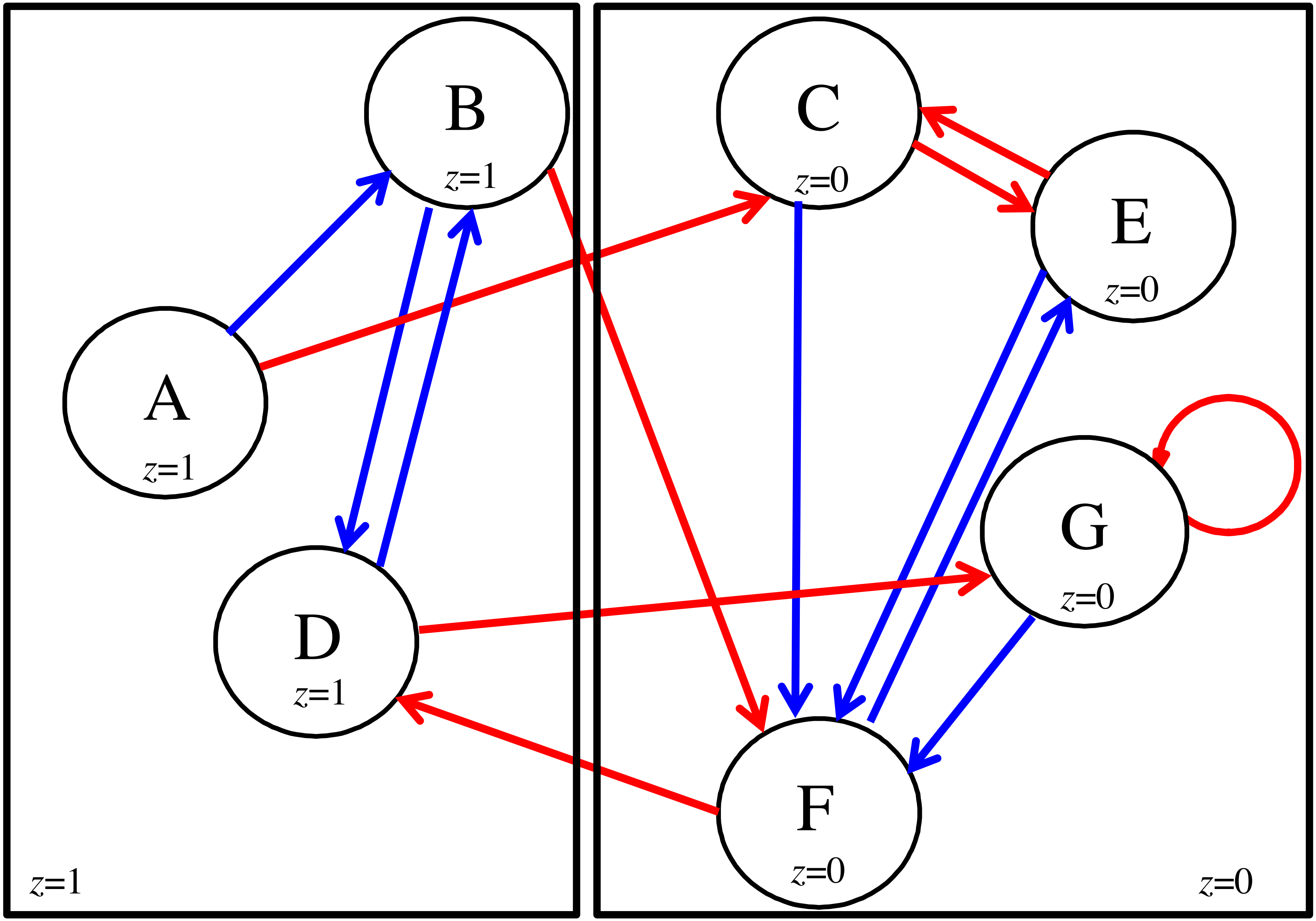
Partition #2

(based on outputs)



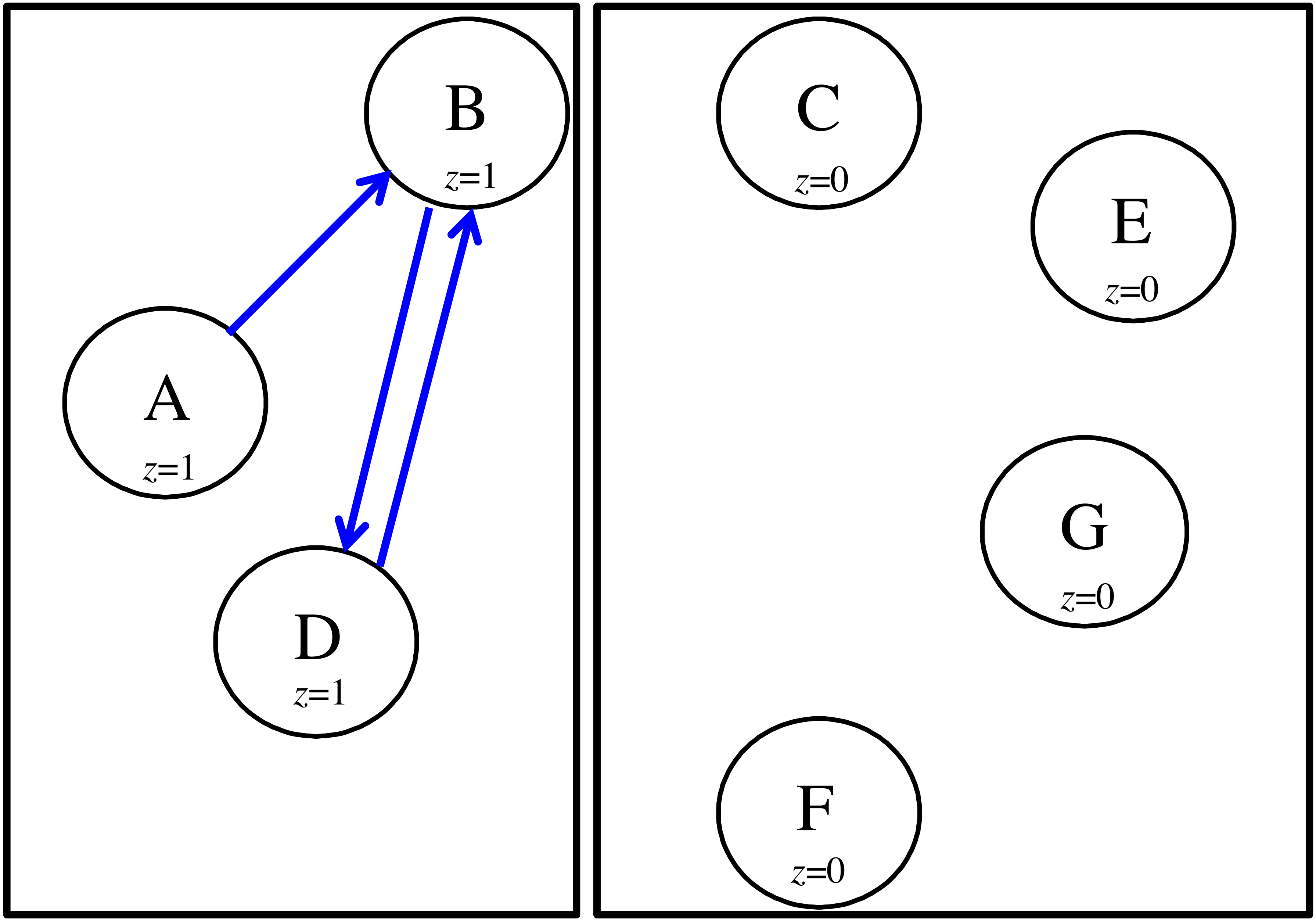
Partition #2

(ABD)(CEFG)



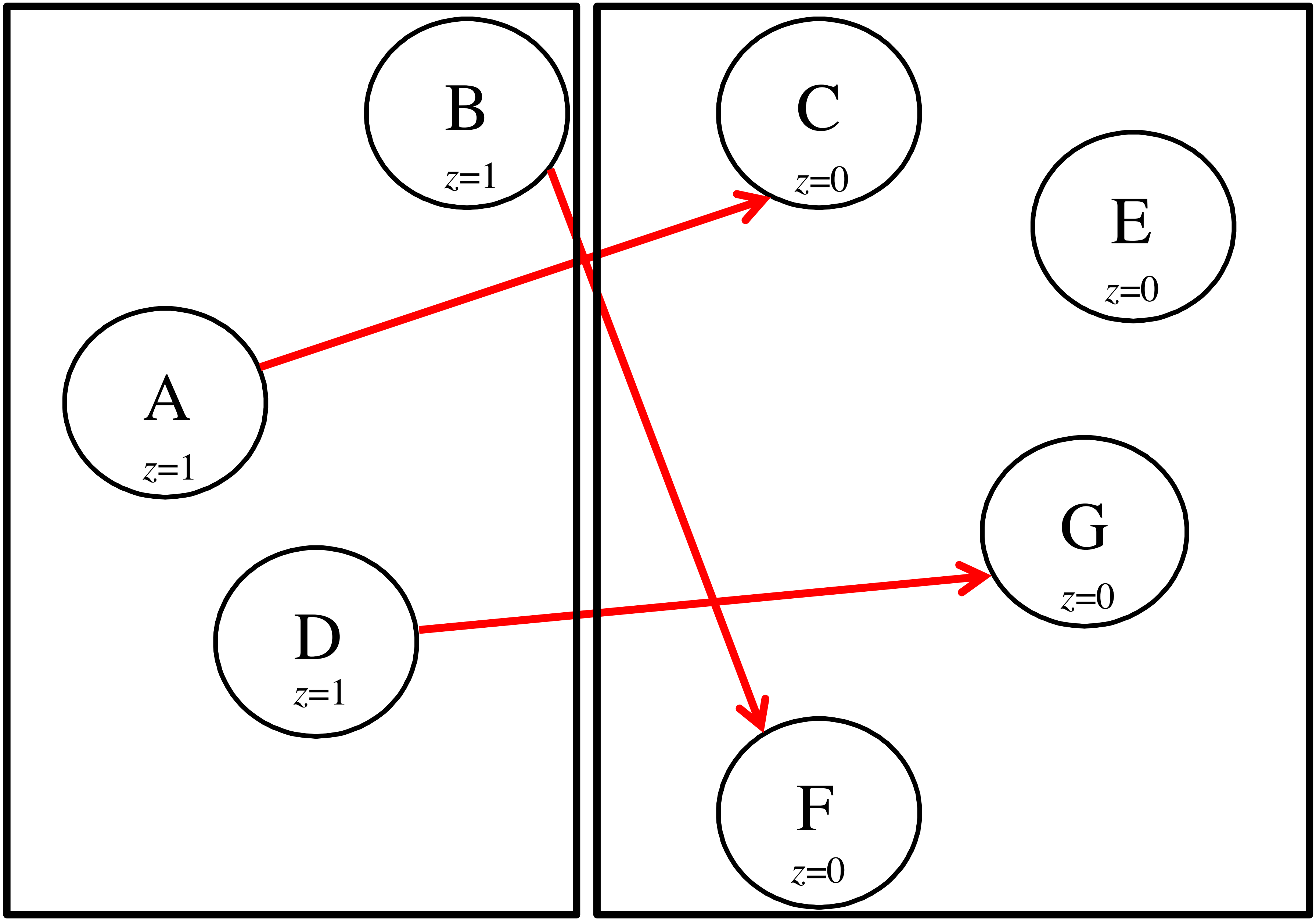
Partition #3.1

(Examine the 0-successors of ABD)



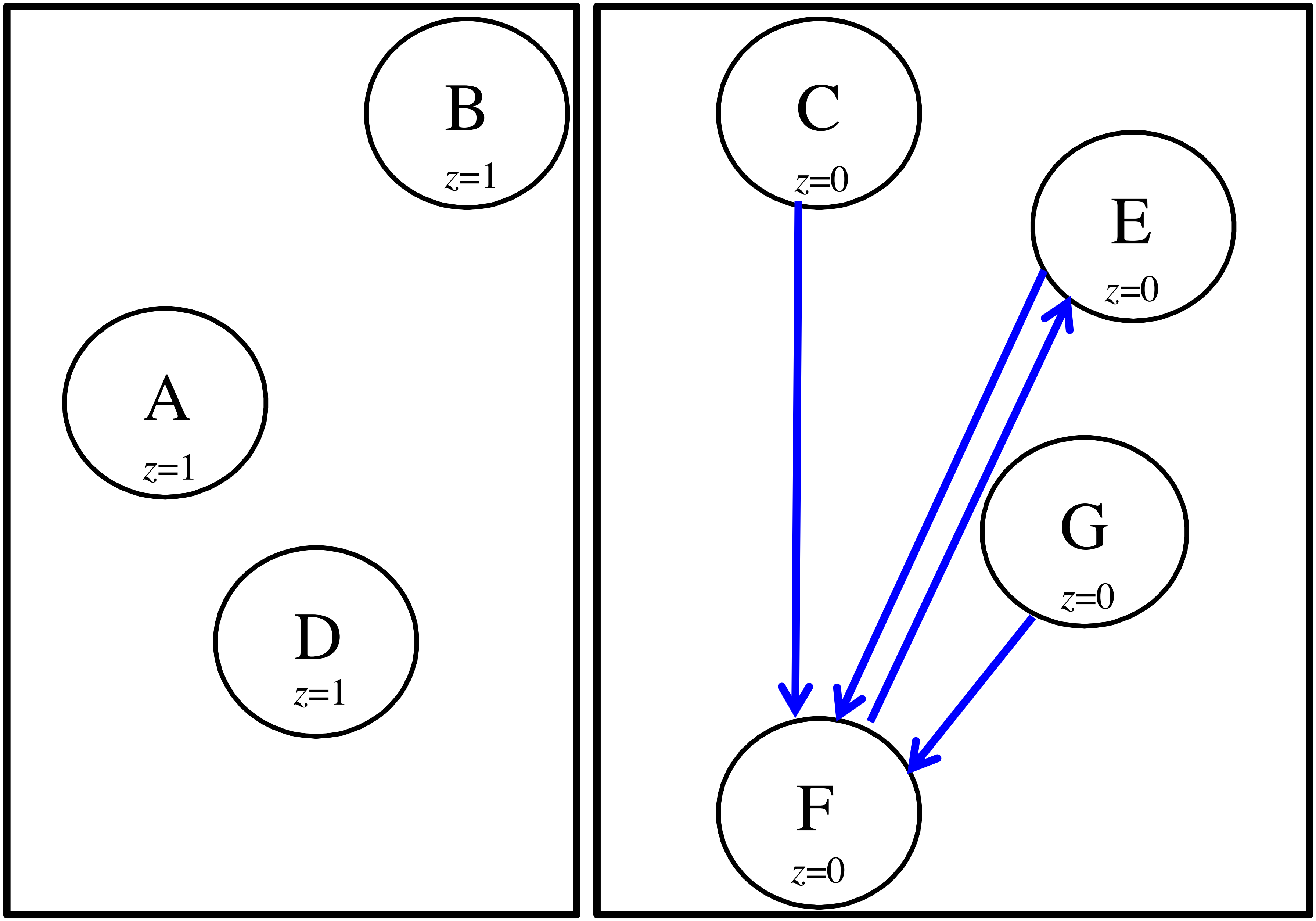
Partition #3.1

(Examine the 1-successors of ABD)



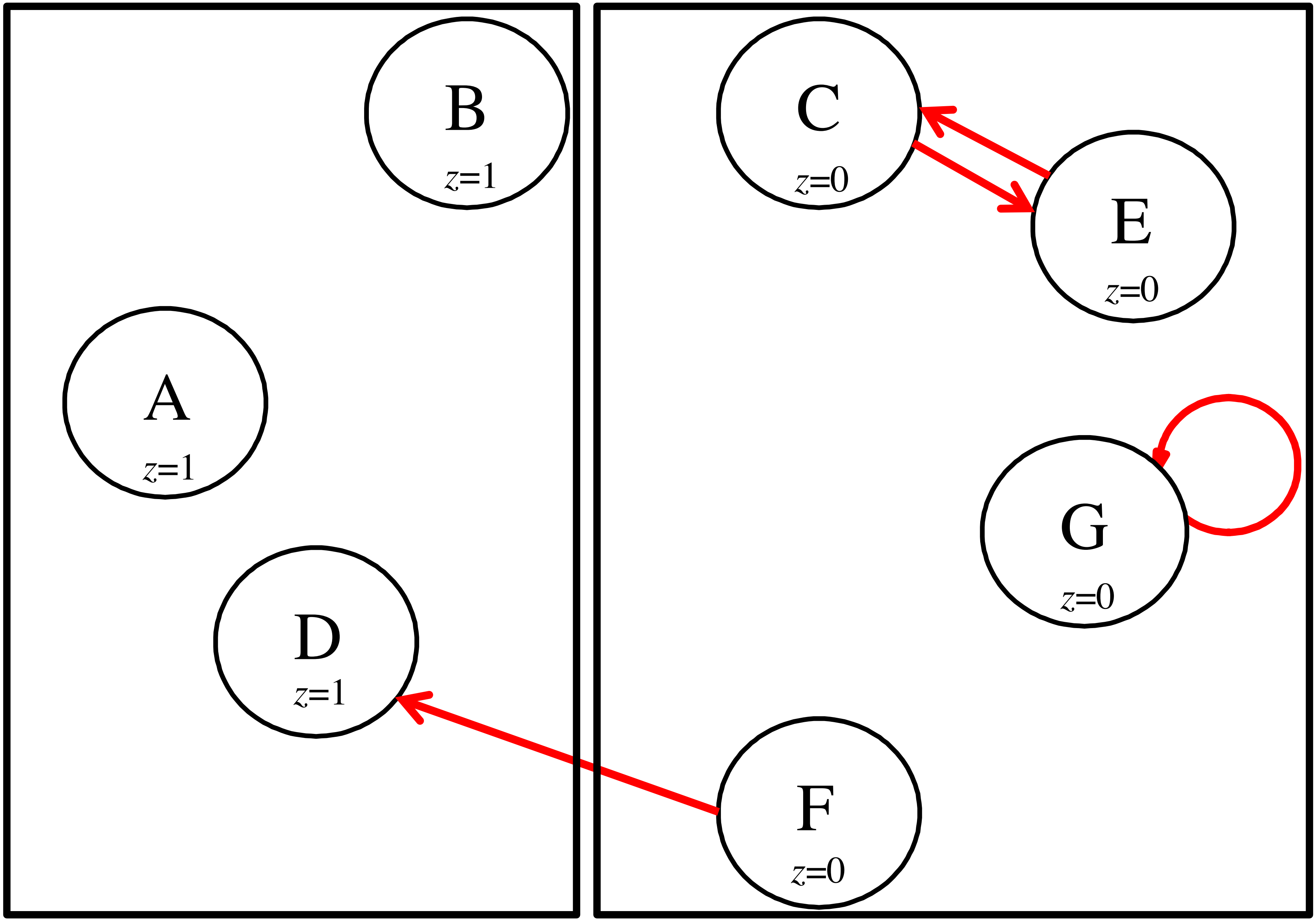
Partition #3.2

(Examine the 0-successors of CEFG)



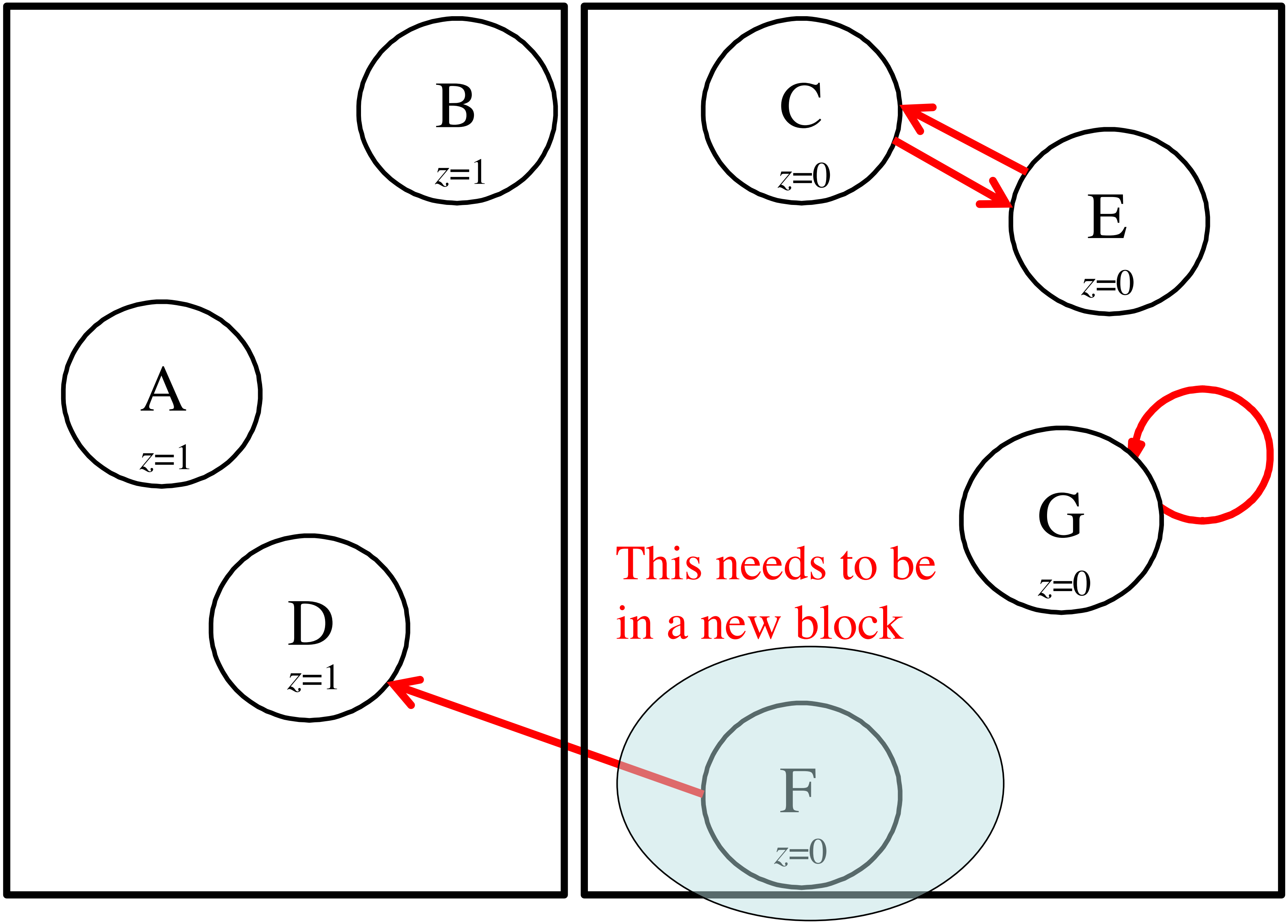
Partition #3.2

(Examine the 1-successors of CEFG)



Partition #3.2

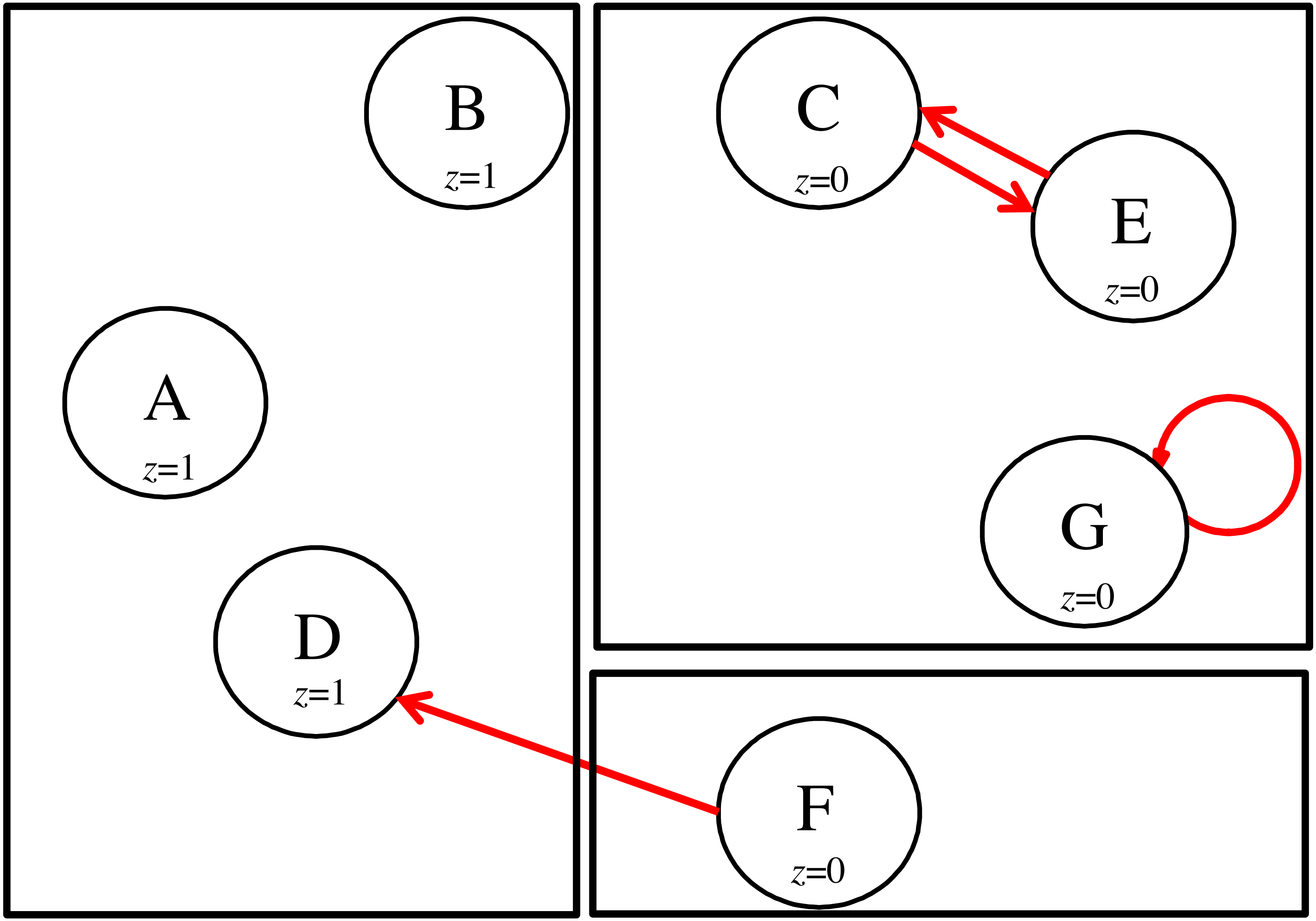
(Examine the 1-successors of CEFG)



This needs to be in a new block

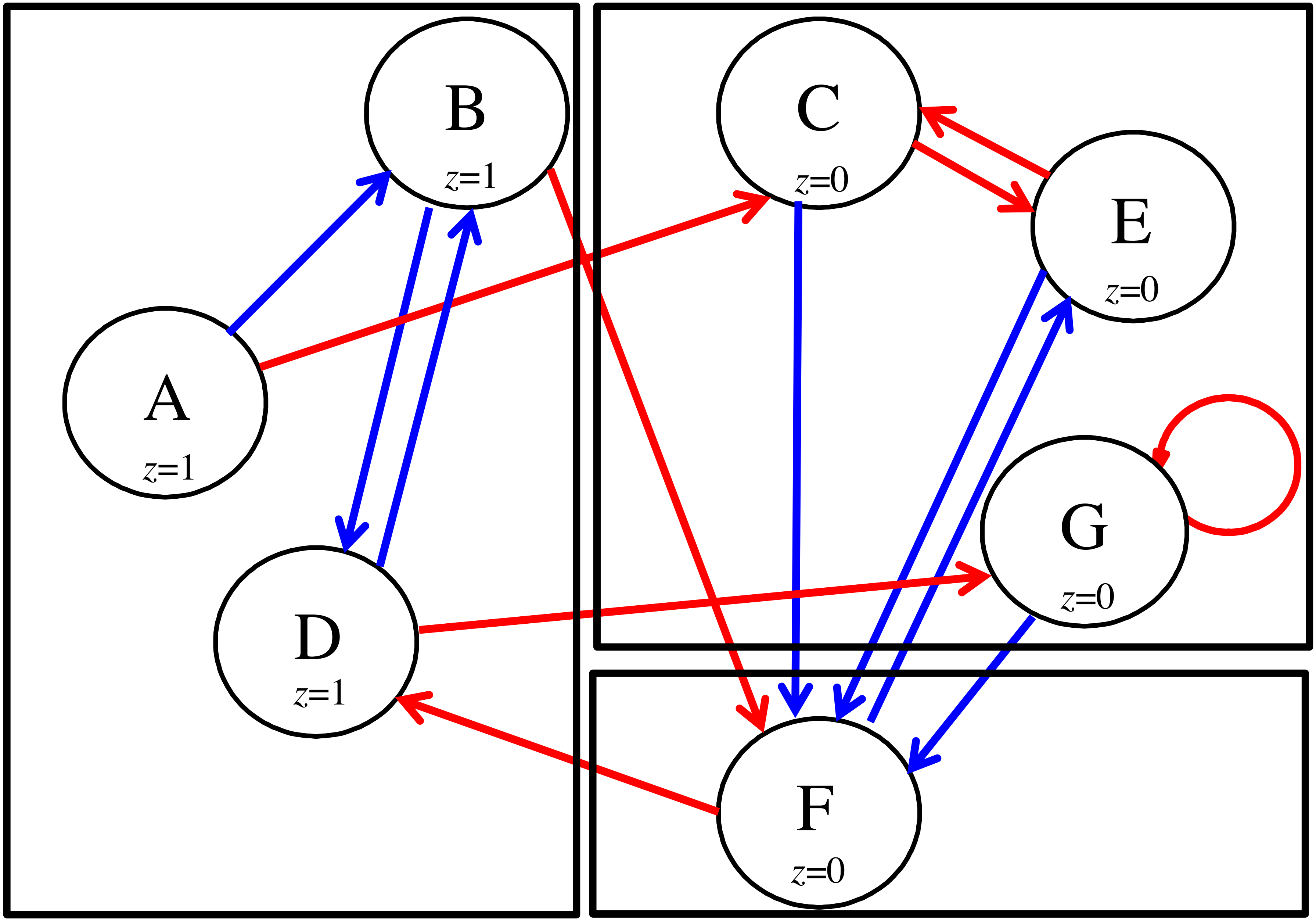
Partition #3

(ABD)(CEG)(F)



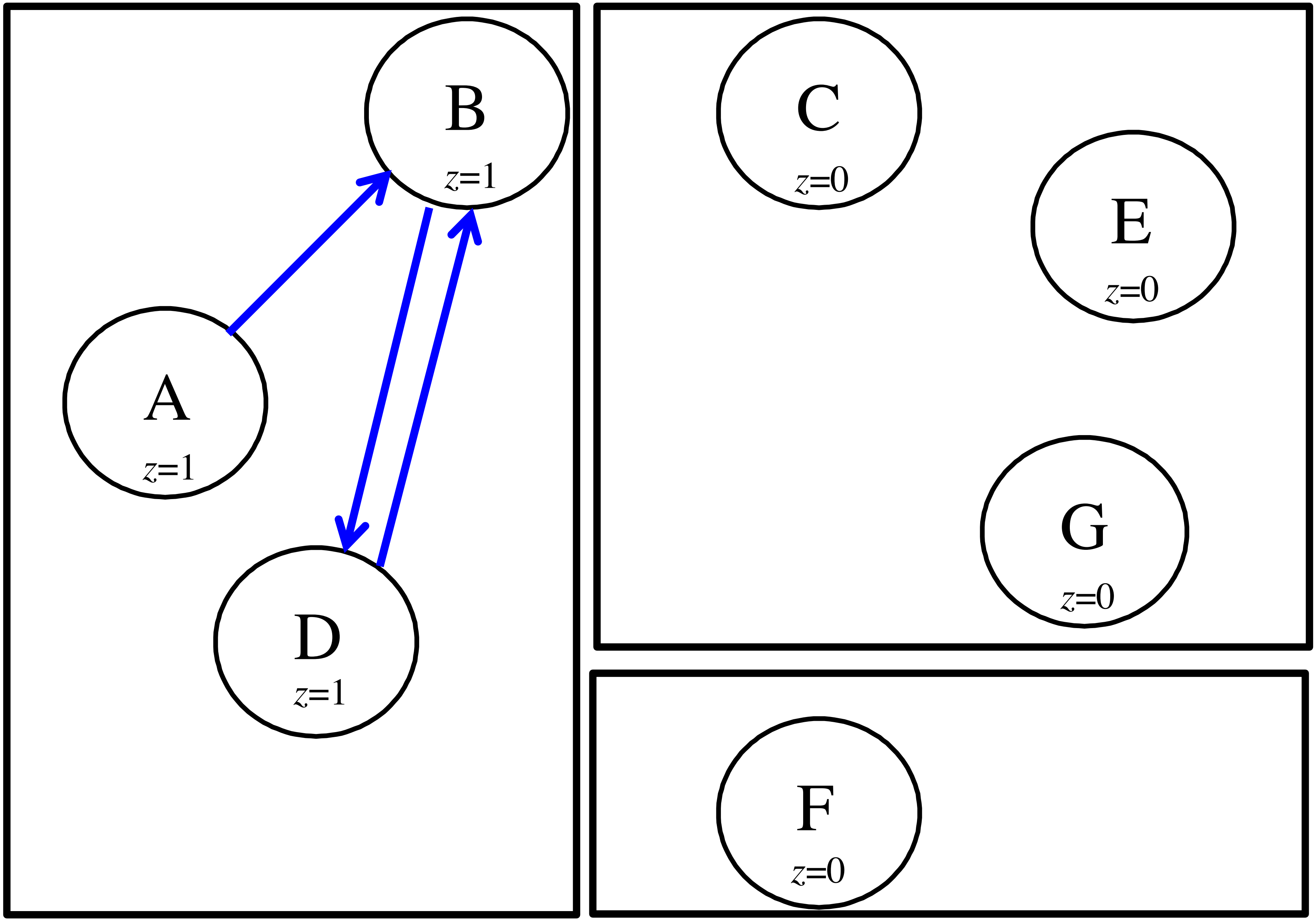
Partition #3

(ABD)(CEG)(F)



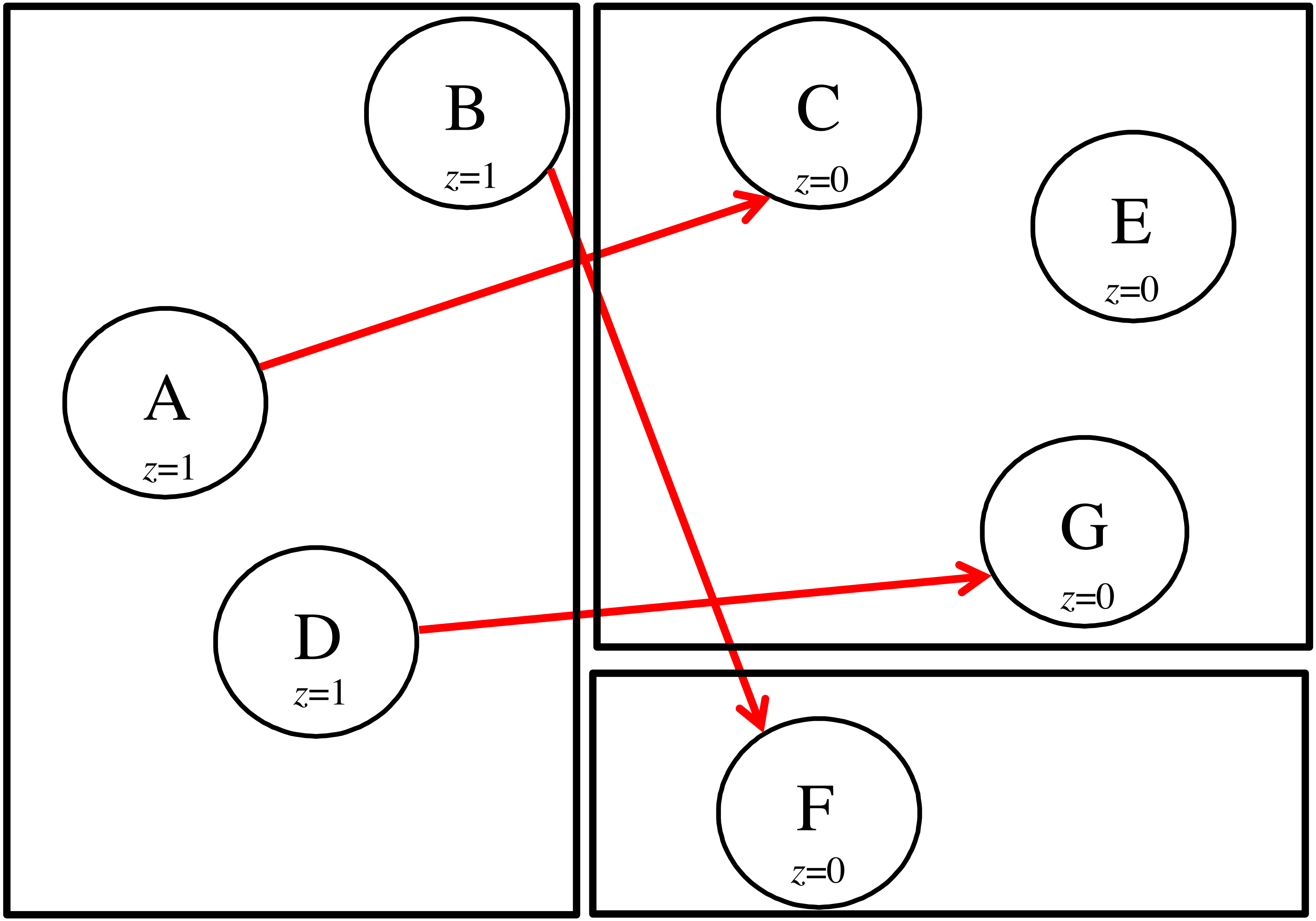
Partition #4.1

(Examine the 0-successors of ABD)



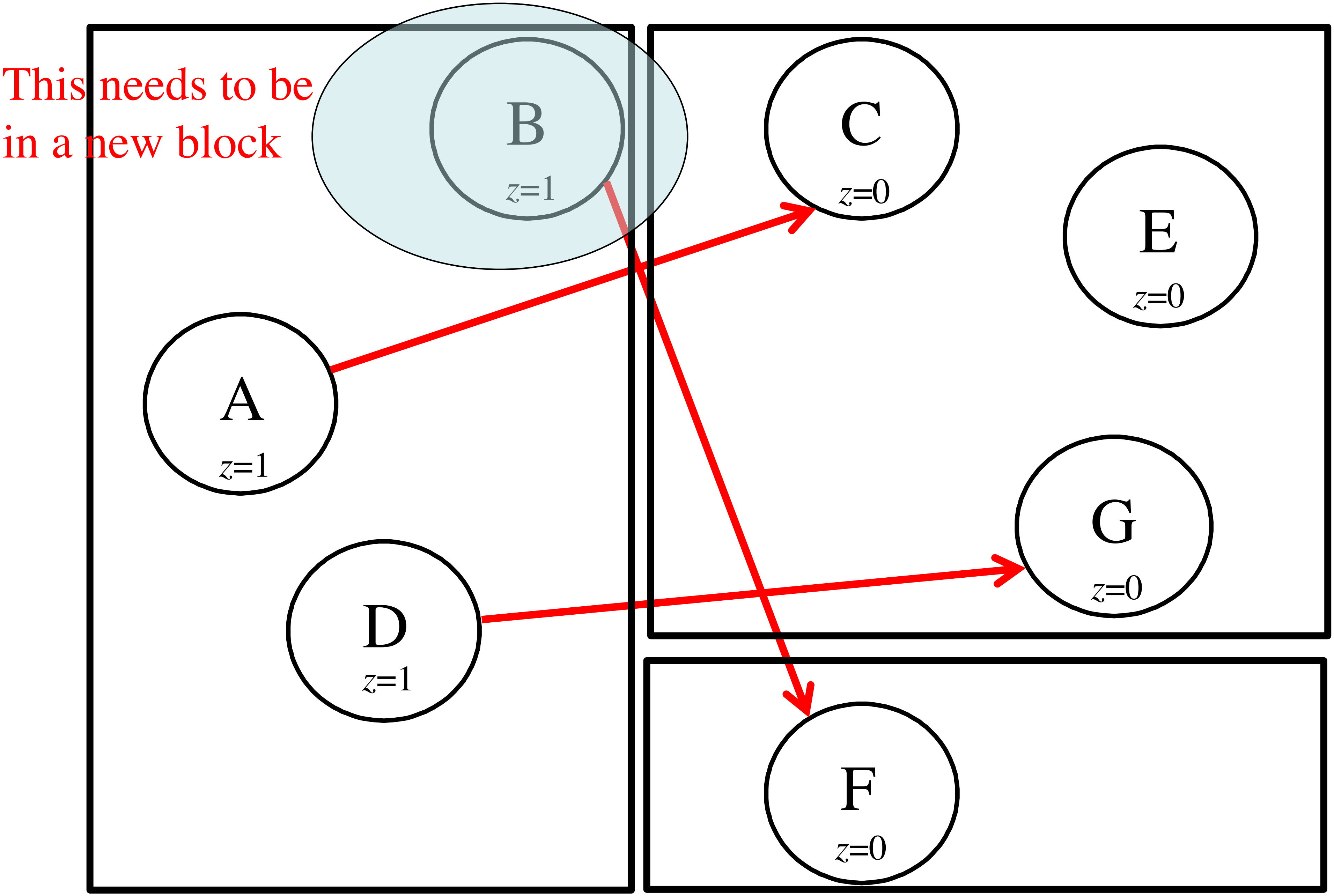
Partition #4.1

(Examine the 1-successors of ABD)



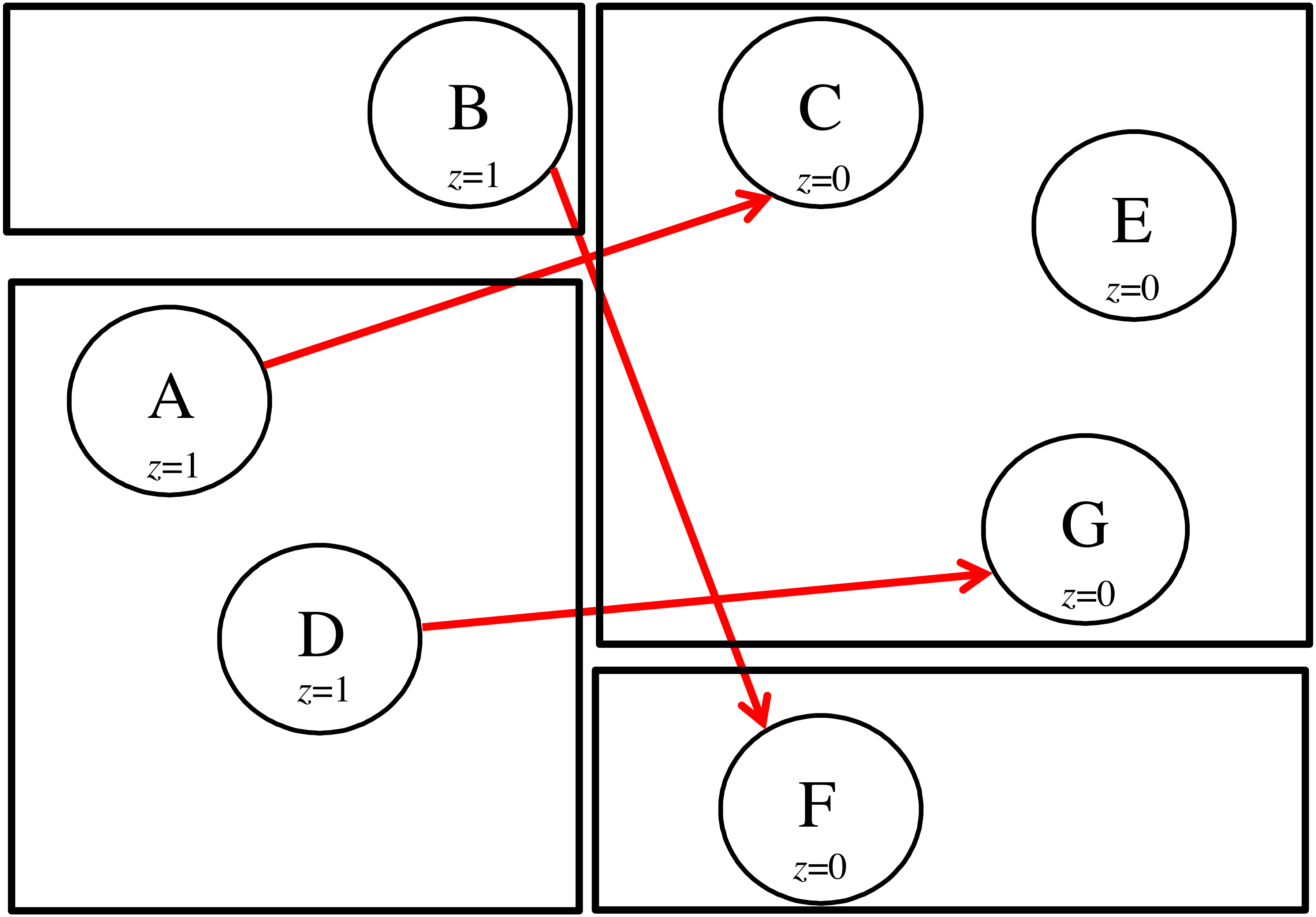
Partition #4.1

(Examine the 1-successors of ABD)



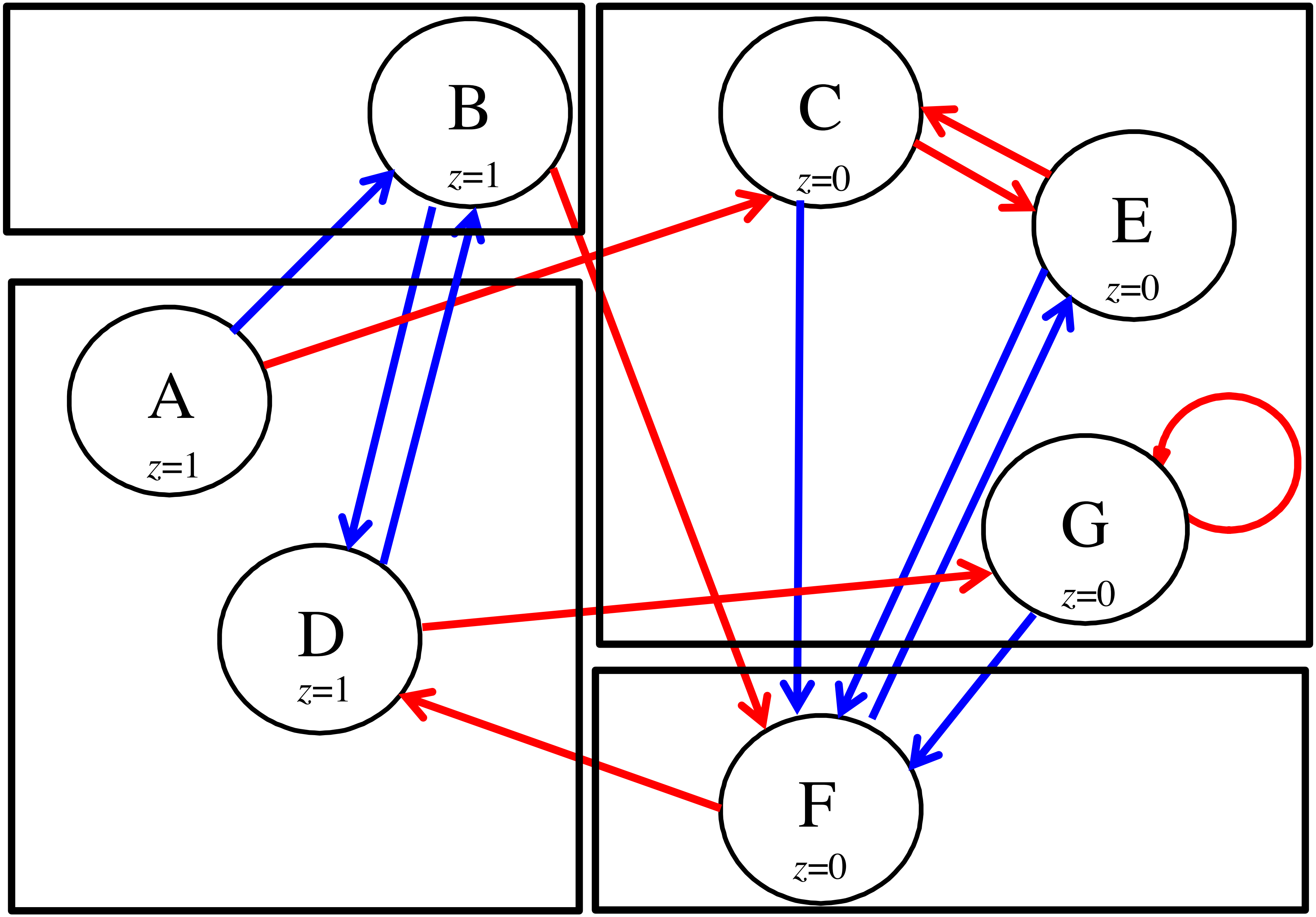
Partition #4

(AD)(B)(CEG)(F)



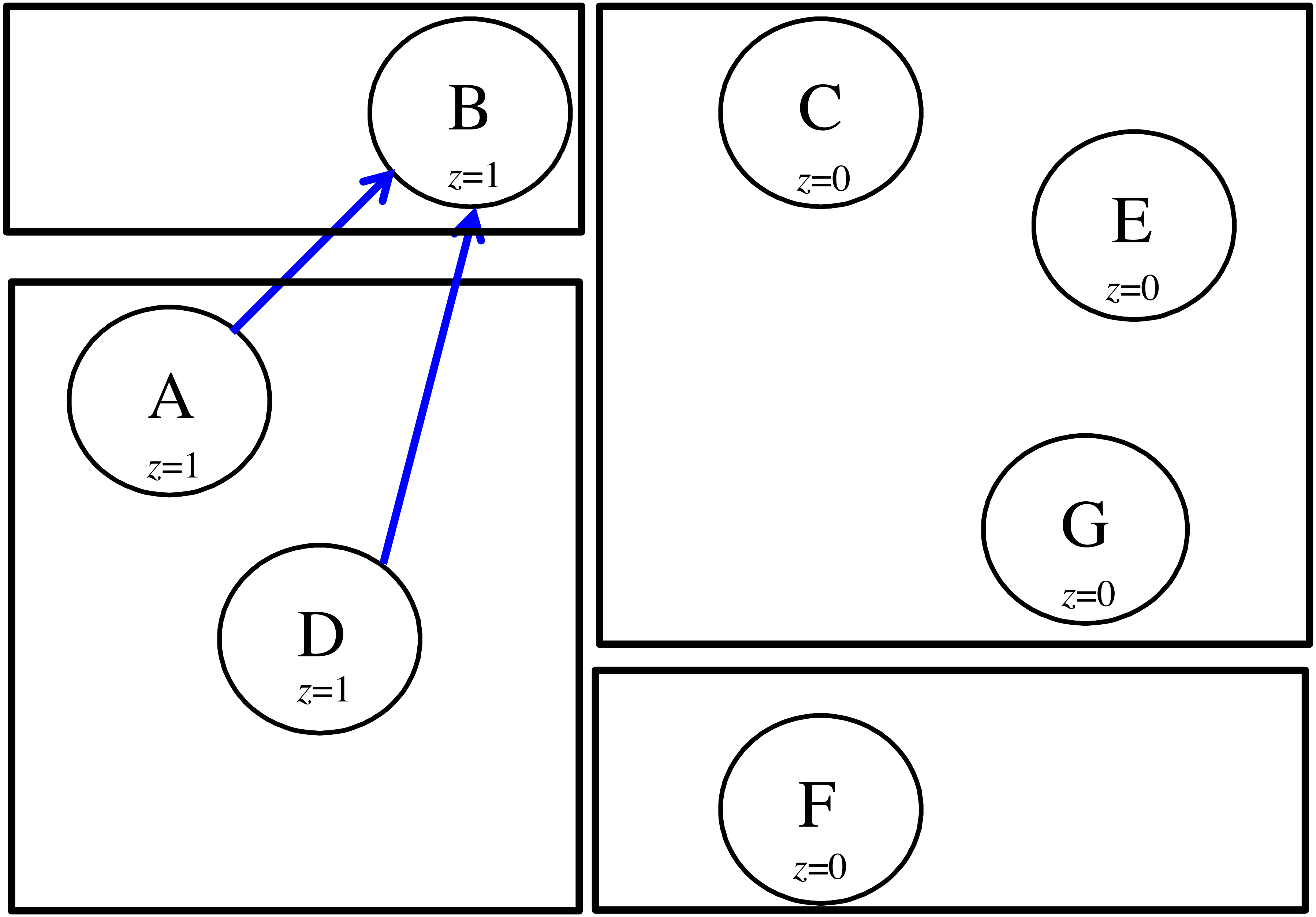
Partition #4

(AD)(B)(CEG)(F)



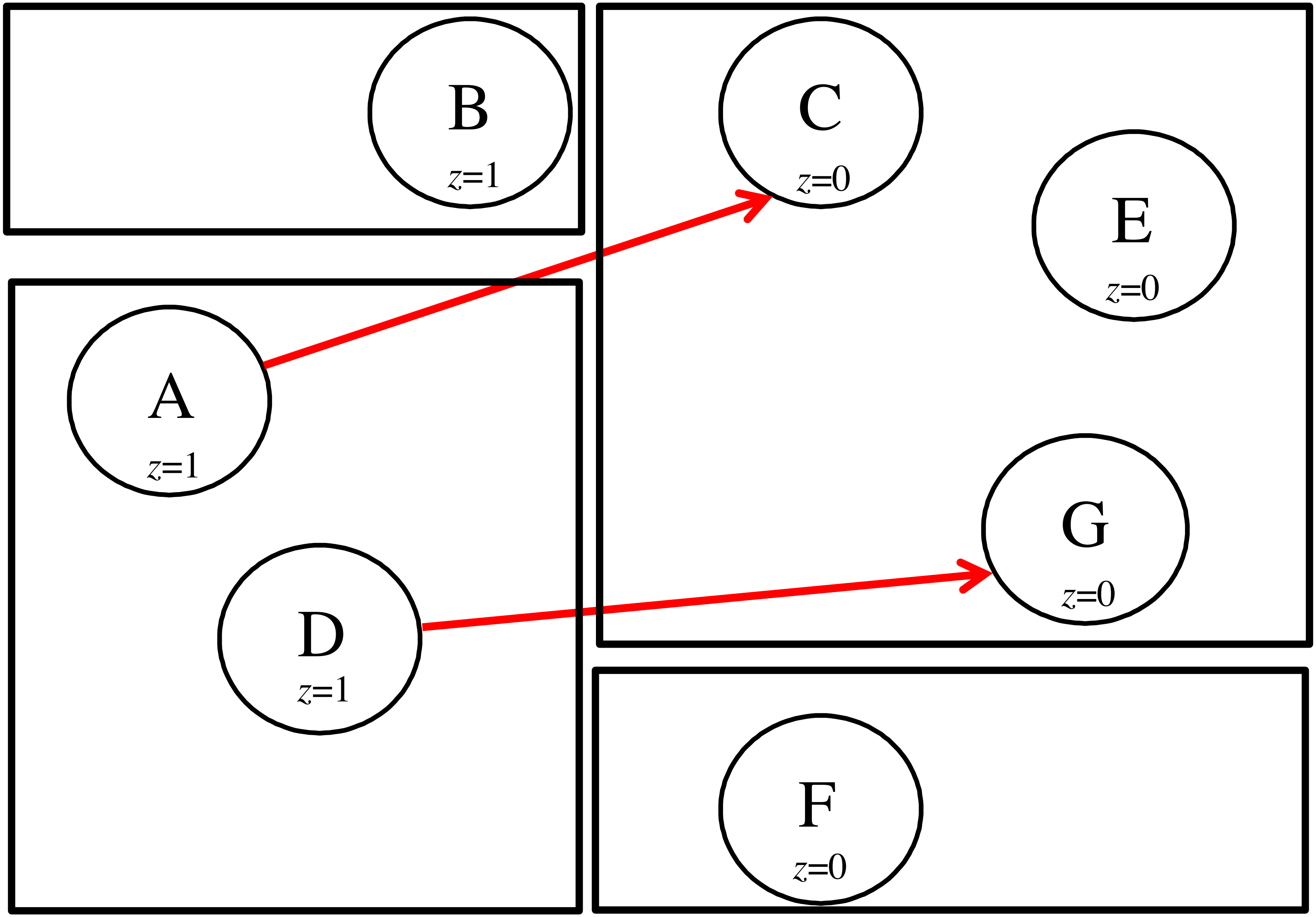
Partition #5.1

(Examine the 0-successors of AD)



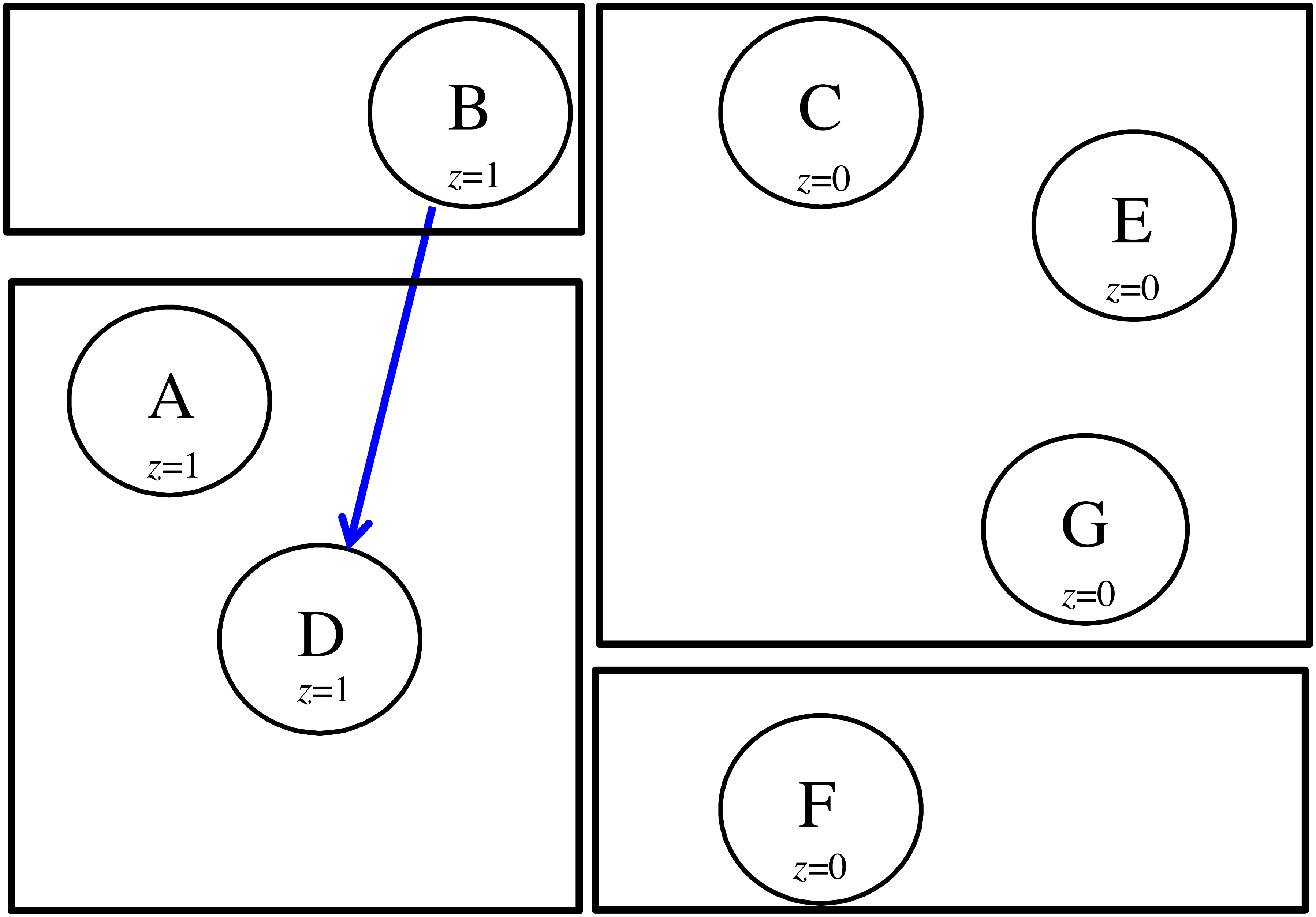
Partition #5.1

(Examine the 1-successors of AD)



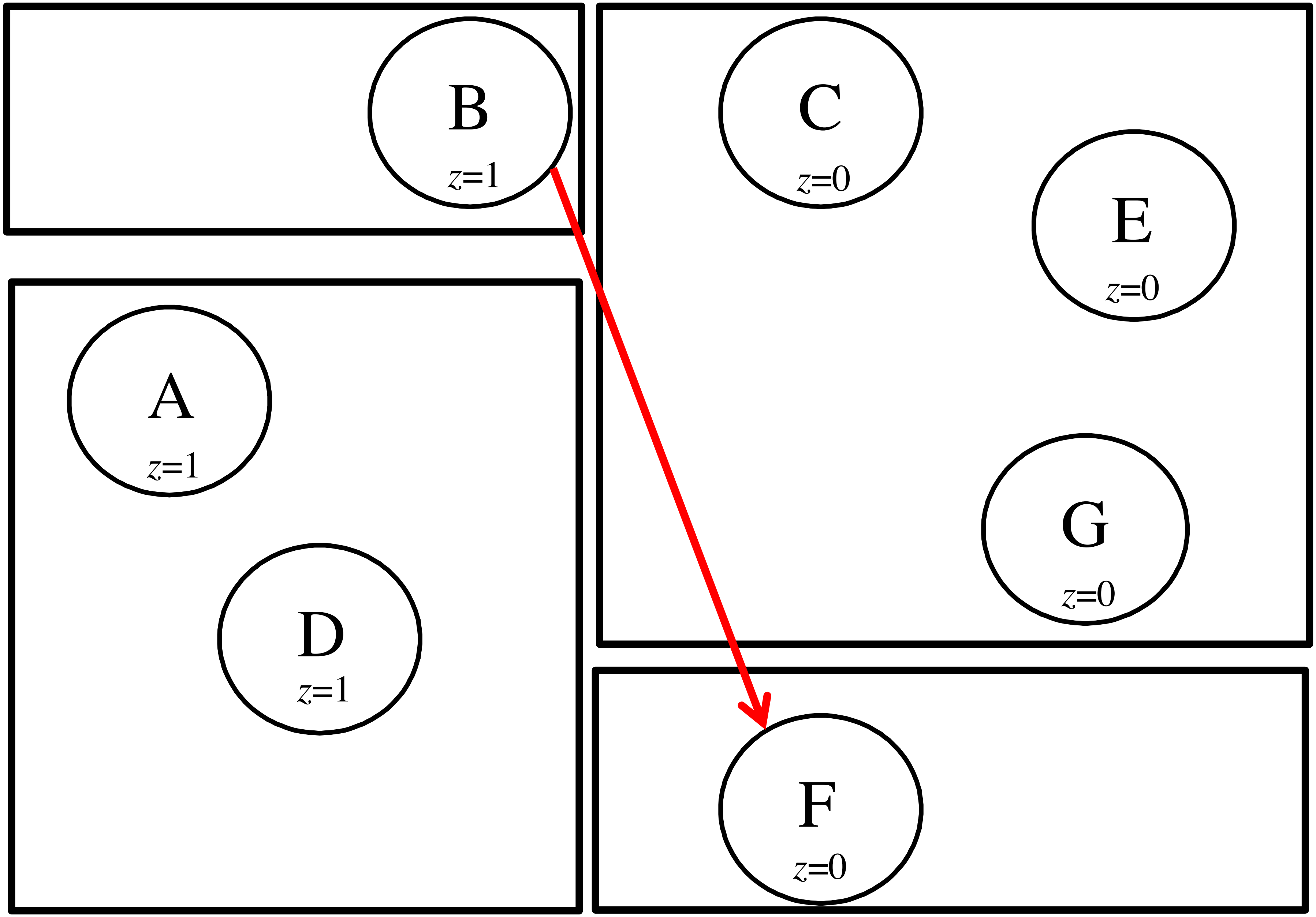
Partition #5.2

(Examine the 0-successors of B)



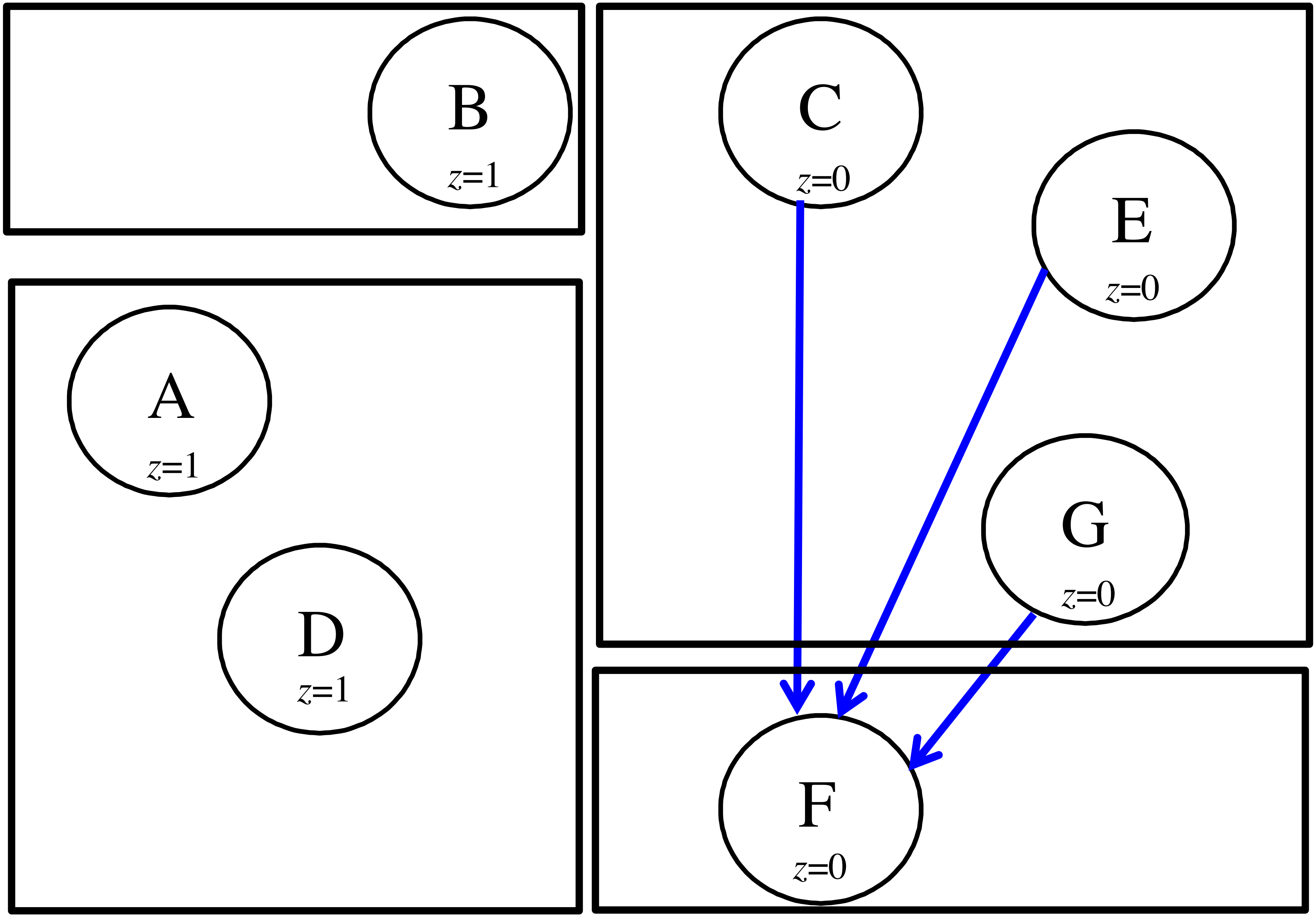
Partition #5.2

(Examine the 1-successors of B)



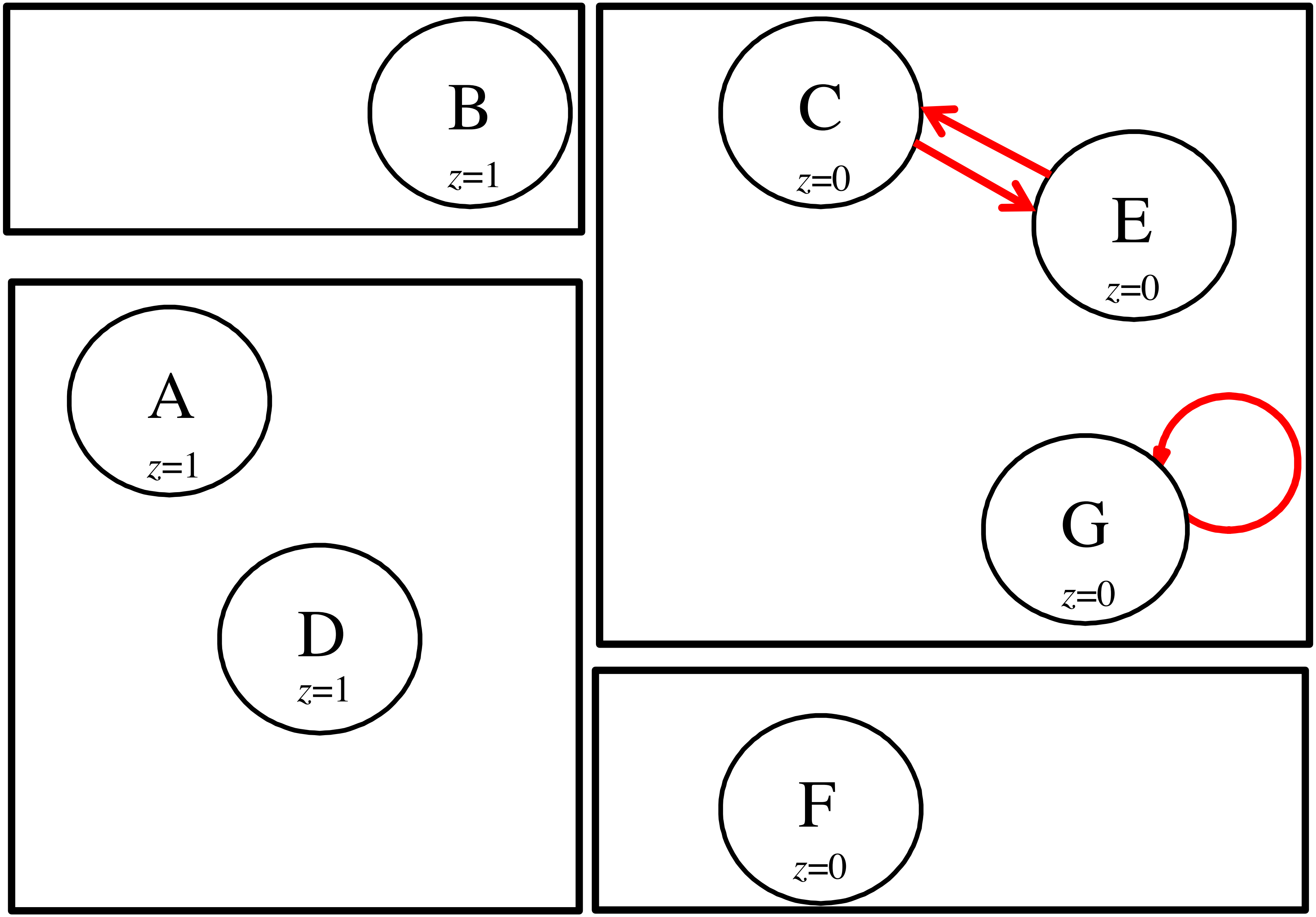
Partition #5.3

(Examine the 0-successors of CEG)



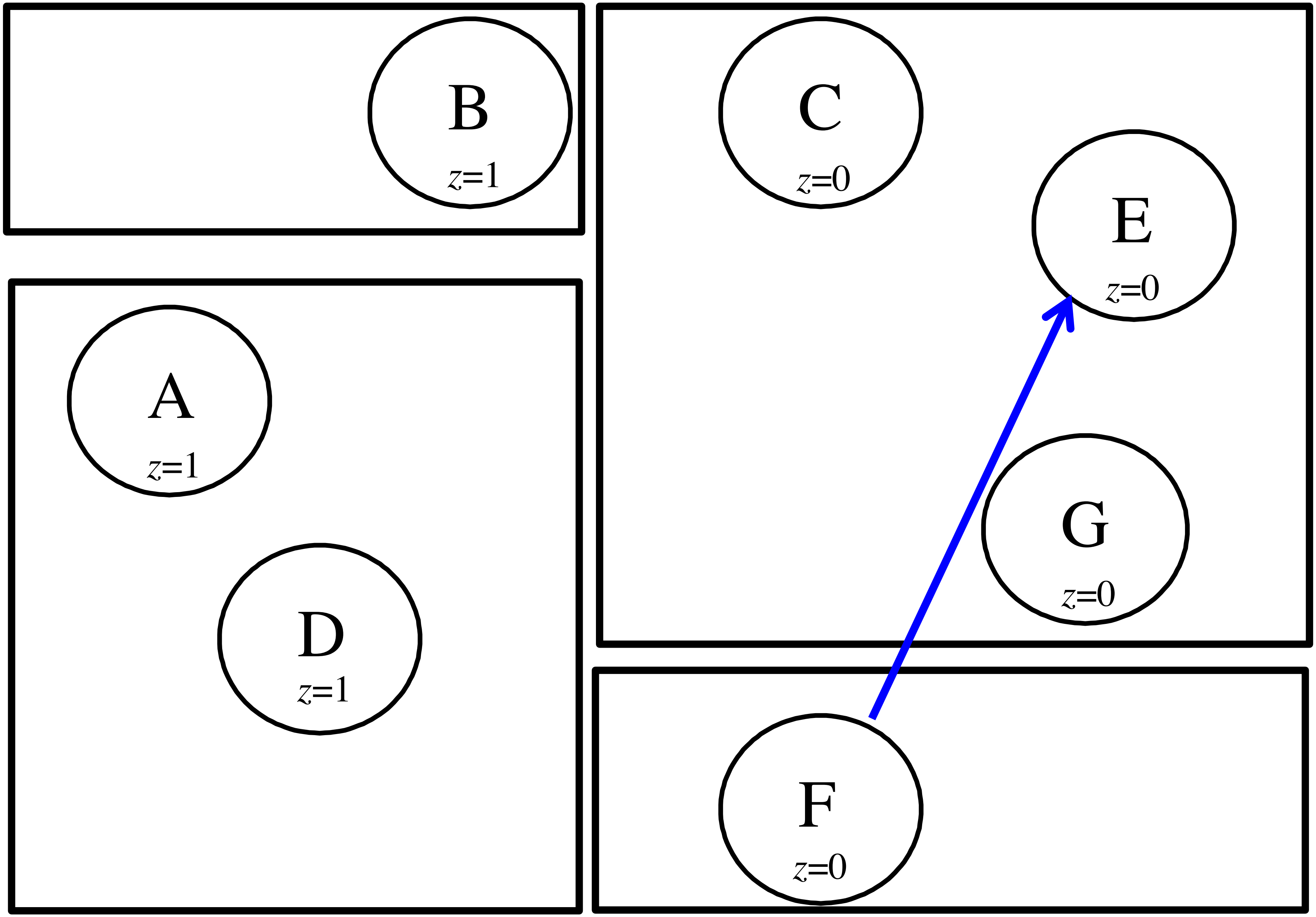
Partition #5.3

(Examine the 1-successors of CEG)



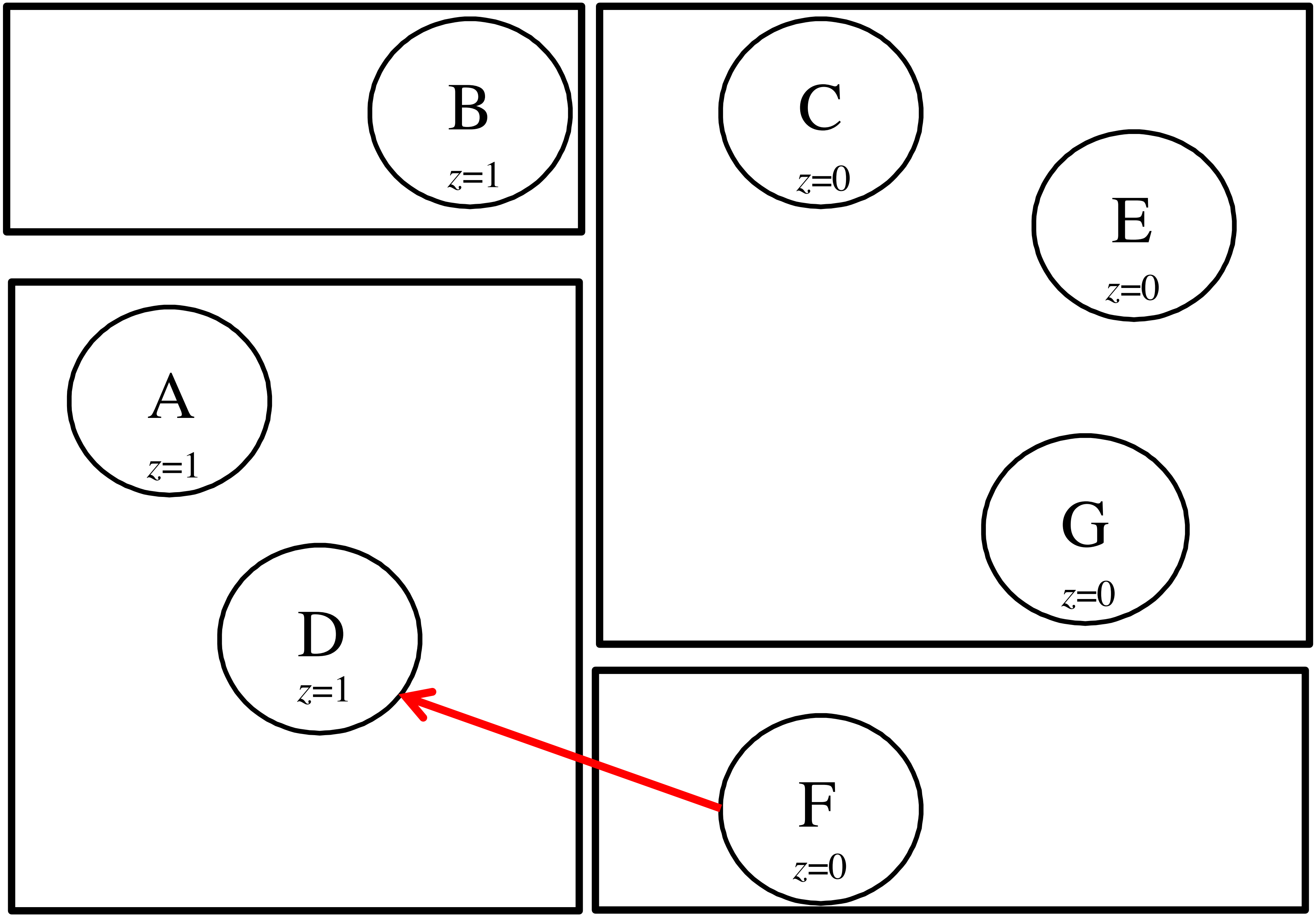
Partition #5.4

(Examine the 0-successors of F)



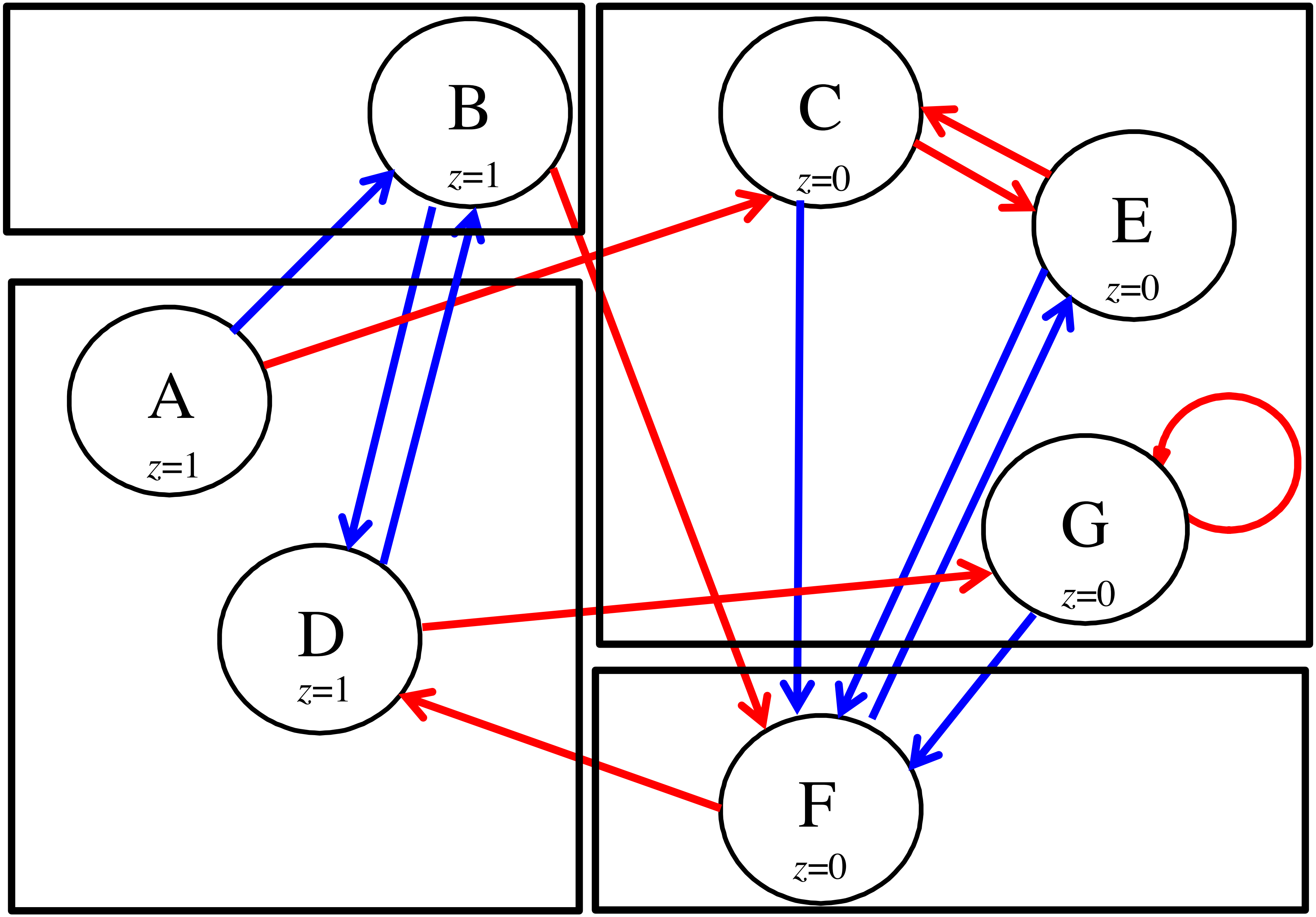
Partition #5.4

(Examine the 1-successors of F)



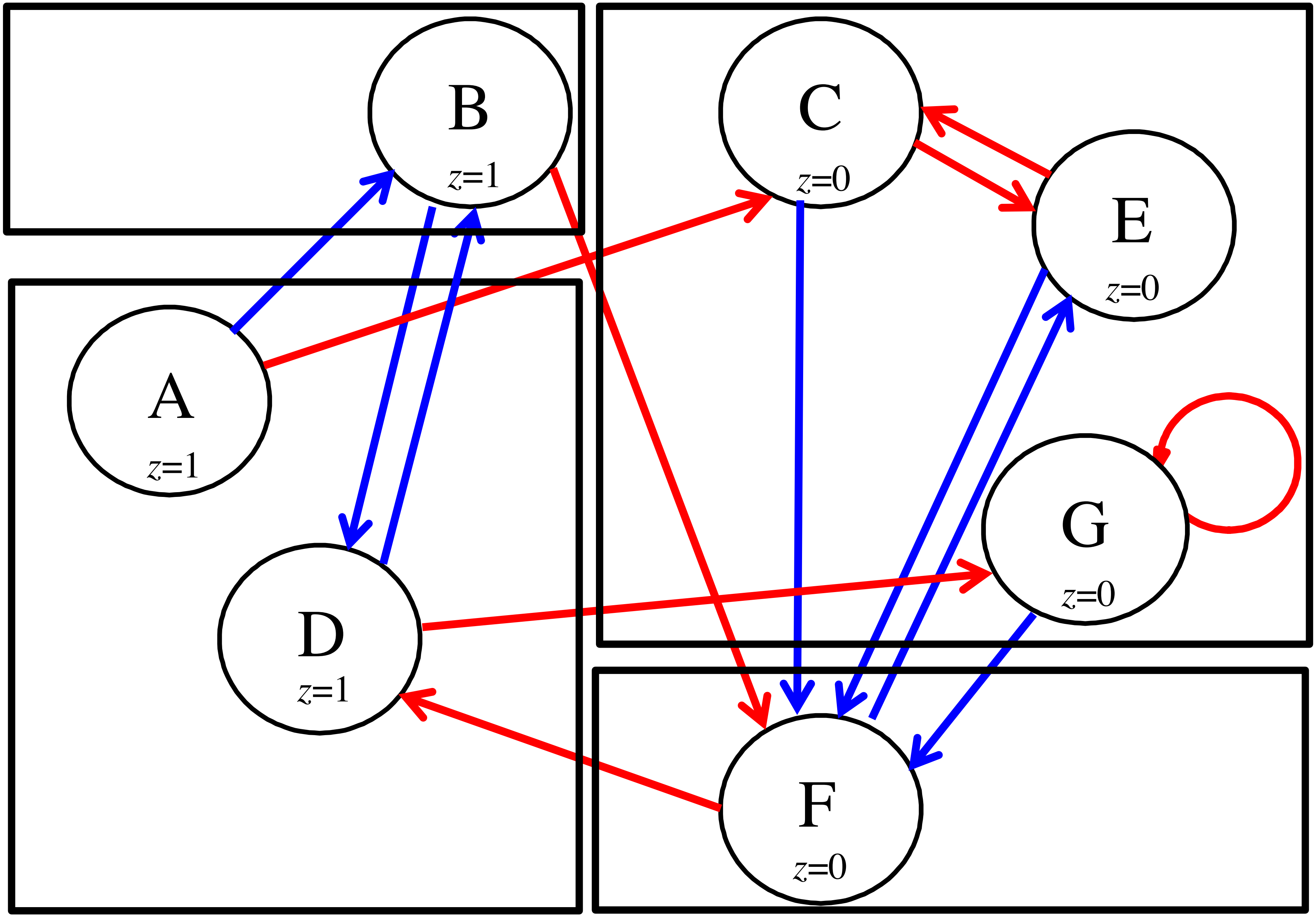
Partition #5

(AD)(B)(CEG)(F)



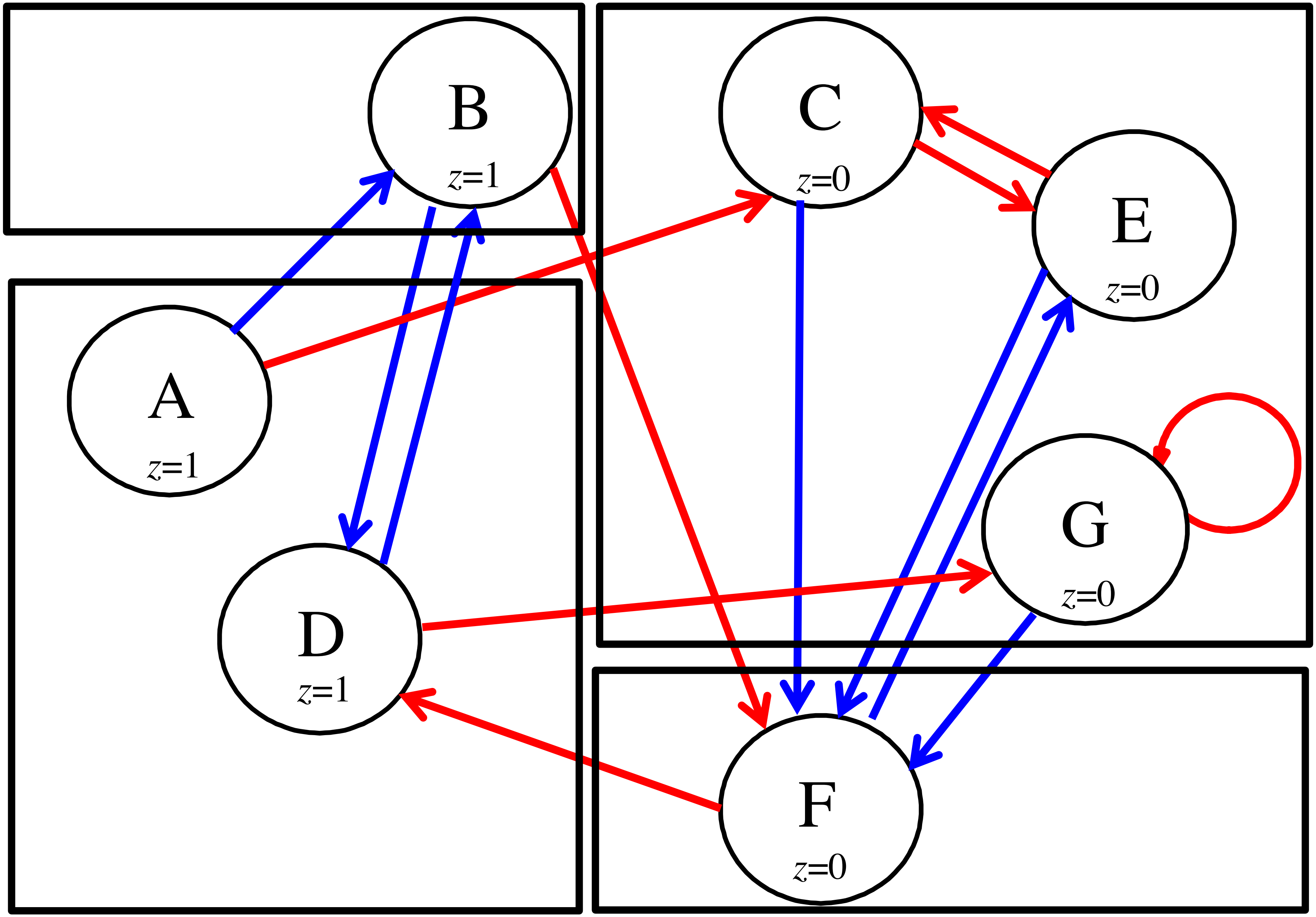
Partition #4

(AD)(B)(CEG)(F)



Partition #5

(This is the same as #4 so we can stop here)



Minimized state table

Present state	Nextstate		Output z
	w = 0	w = 1	
A	B	C	1
B	A	F	1
C	F	C	0
F	C	A	0

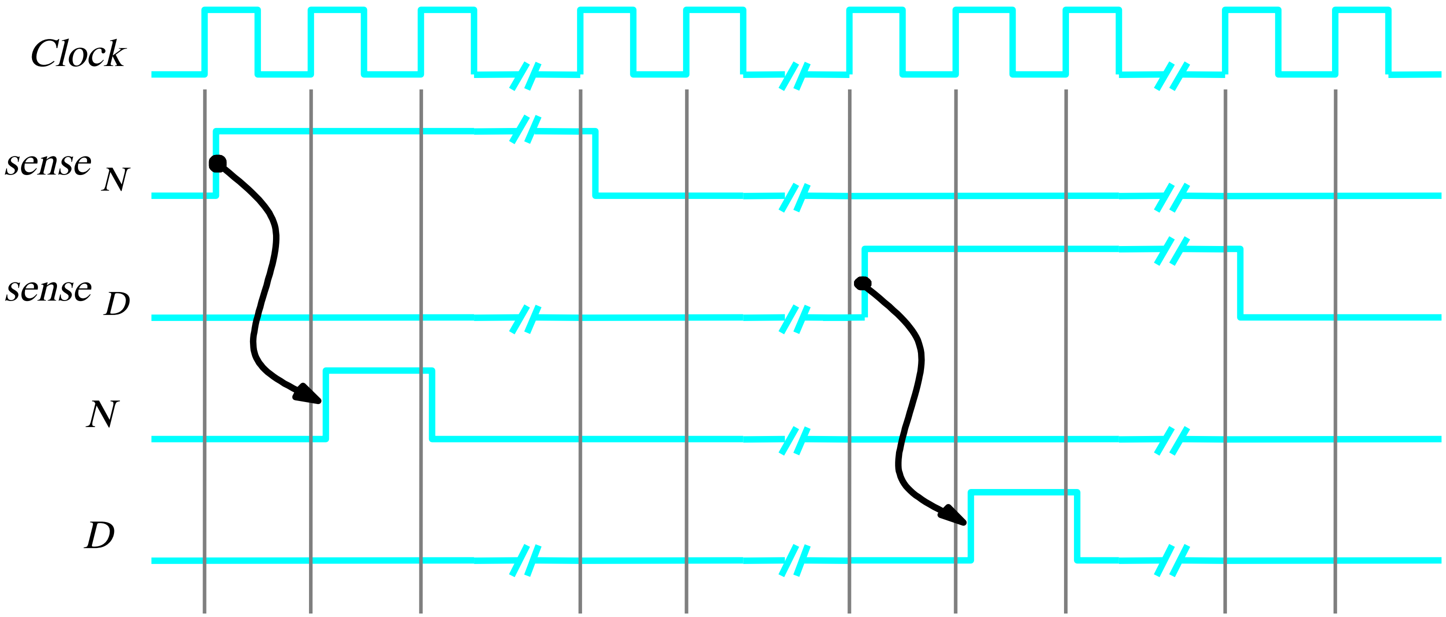
[Figure 6.52 from the textbook]

Vending Machine Example

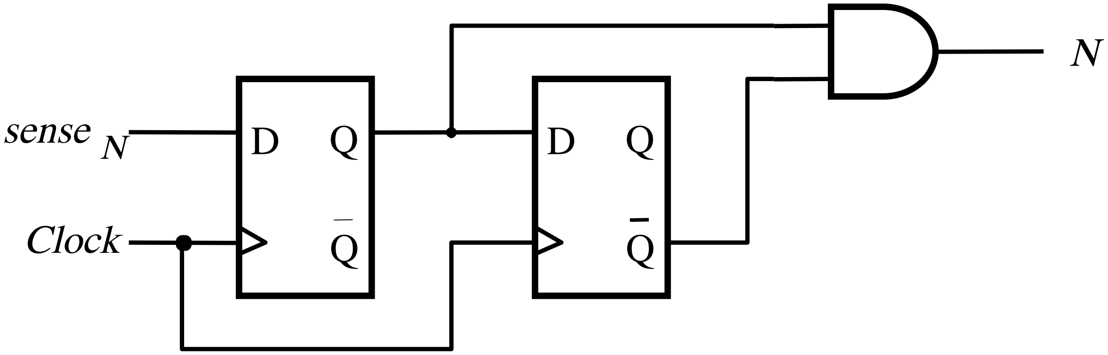
Vending Machine Example

- The machine accepts nickels and dimes
- It takes 15 cents for a piece of candy to be released from the machine
- If 20 cents is deposited, the machine will not return the change, but it will credit the buyer with 5 cents and wait for the buyer to make a second purchase.

Signals for the vending machine



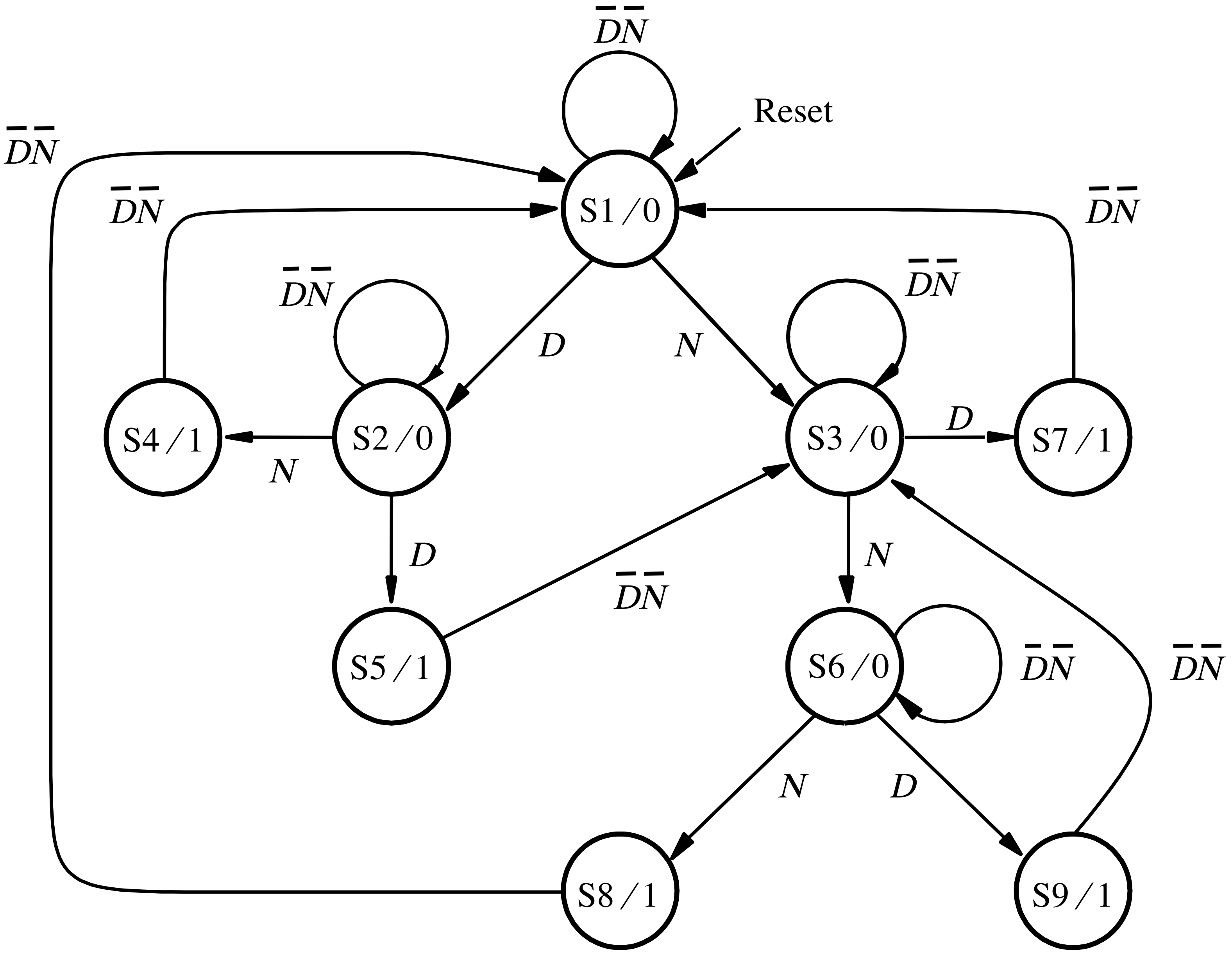
(a) Timing diagram



(b) Circuit that generates *N*

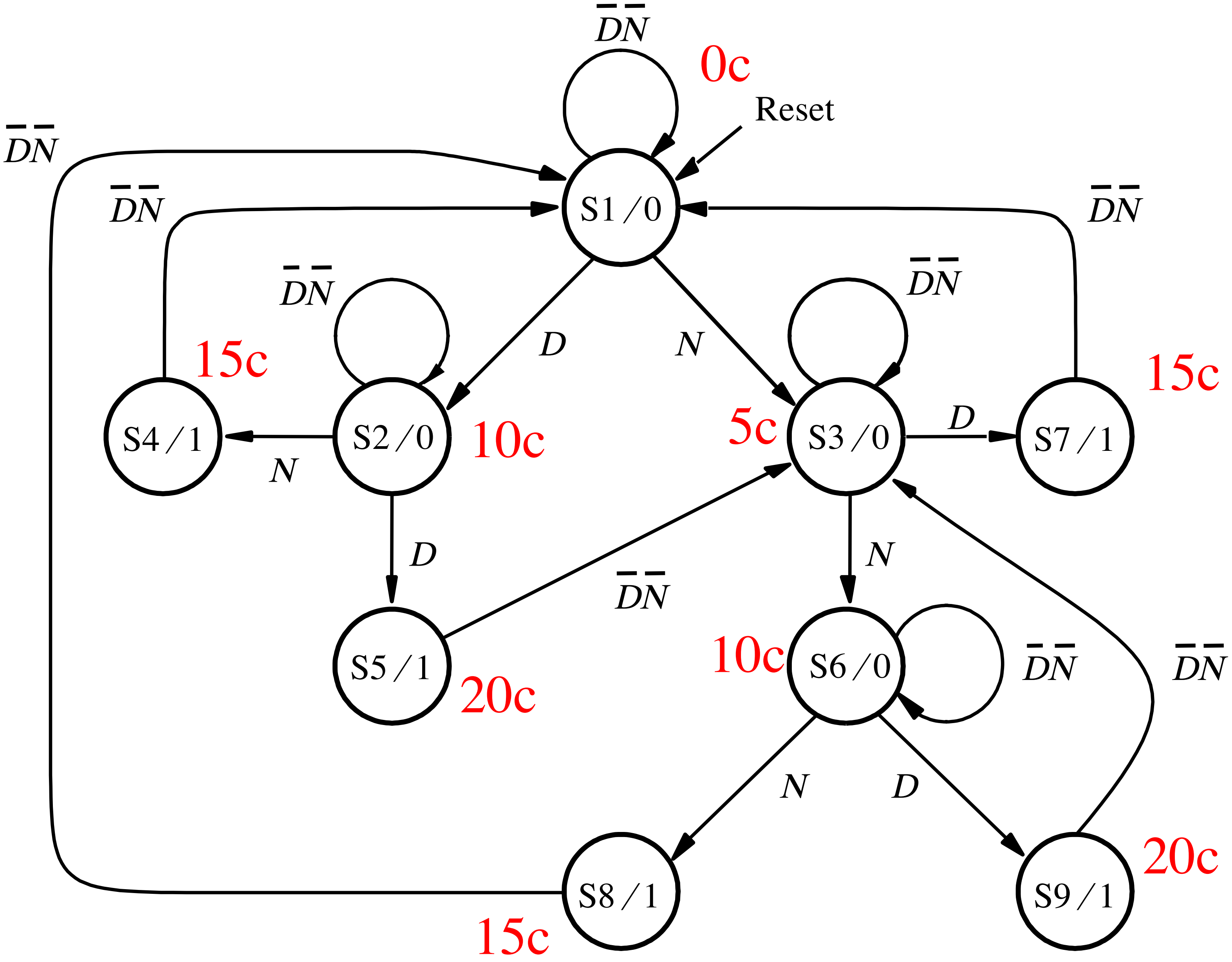
[Figure 6.53 from the textbook]

State Diagram for the vending machine


$\overline{D}\,\overline{N}$
Reset
$\overline{D}\,\overline{N}$
$\overline{D}\,\overline{N}$
S1 / 0
$\overline{D}\,\overline{N}$
$\overline{D}\,\overline{N}$
$\overline{D}\,\overline{N}$
D
N
S4 / 1
S2 / 0
S3 / 0
D
S7 / 1
N
D
N
$\overline{D}\,\overline{N}$
S5 / 1
S6 / 0
$\overline{D}\,\overline{N}$
$\overline{D}\,\overline{N}$
N
D
S8 / 1
S9 / 1

[Figure 6.54 from the textbook]

State Diagram for the vending machine



[Figure 6.54 from the textbook]

State Table for the vending machine

Present state	Next state				Output
	DN =00	01	10	11	*z*
S1	S1	S3	S2	–	0
S2	S2	S4	S5	–	0
S3	S3	S6	S7	–	0
S4	S1	–	–	–	1
S5	S3	–	–	–	1
S6	S6	S8	S9	–	0
S7	S1	–	–	–	1
S8	S1	–	–	–	1
S9	S3	–	–	–	1

Incompletely specified state table

[Figure 6.55 from the textbook]

Partition for Vending Machine FSM

Present state	Next state				Output
	00	01	10	11	z
S1	S1	S3	S2	-	0
S3	S3	S6	S7	-	0
S2	S2	S4	S5	-	0
S6	S6	S8	S9	-	0
S4	S1	-	-	-	1
S7	S1	-	-	-	1
S8	S1	-	-	-	1
S5	S3	-	-	-	1
S9	S3	-	-	-	1

P1=(S1,S2,S3,S4,S5,S6,S7,S8,S9)

Partition for Vending Machine FSM

Present	Next state				Output
state	00	01	10	11	z
S1	S1	S3	S2	-	0
S3	S3	S6	S7	-	0
S2	S2	S4	S5	-	0
S6	S6	S8	S9	-	0
S4	S1	-	-	-	1
S7	S1	-	-	-	1
S8	S1	-	-	-	1
S5	S3	-	-	-	1
S9	S3	-	-	-	1

P1=(S1,S2,S3,S4,S5,S6,S7,S8,S9)
P2=(S1,S2,S3,S6) (S4,S5,S7,S8,S9)

Partition for Vending Machine FSM

Present state	Next state				Output
	00	01	10	11	z
S1	S1	S3	S2	-	0
S3	S3	S6	S7	-	0
S2	S2	S4	S5	-	0
S6	S6	S8	S9	-	0
S4	S1	-	-	-	1
S7	S1	-	-	-	1
S8	S1	-	-	-	1
S5	S3	-	-	-	1
S9	S3	-	-	-	1

P1=(S1,S2,S3,S4,S5,S6,S7,S8,S9)
P2=(S1,S2,S3,S6) (S4,S5,S7,S8,S9)
P3=(S1) (S3) (S2,S6) (S4,S5,S7,S8,S9)

Partition for Vending Machine FSM

Present state	Next state				Output
	00	01	10	11	z
S1	S1	S3	S2	-	0
S3	S3	S6	S7	-	0
S2	S2	S4	S5	-	0
S6	S6	S8	S9	-	0
S4	S1	-	-	-	1
S7	S1	-	-	-	1
S8	S1	-	-	-	1
S5	S3	-	-	-	1
S9	S3	-	-	-	1

P1=(S1,S2,S3,S4,S5,S6,S7,S8,S9)
P2=(S1,S2,S3,S6) (S4,S5,S7,S8,S9)
P3=(S1) (S3) (S2,S6) (S4,S5,S7,S8,S9)
P4=(S1) (S3) (S2,S6) (S4,S7,S8) (S5,S9)

Partition for Vending Machine FSM

Present state	Next state				Output
	00	01	10	11	z
S1	S1	S3	S2	-	0
S3	S3	S6	S7	-	0
S2	S2	S4	S5	-	0
S6	S6	S8	S9	-	0
S4	S1	-	-	-	1
S7	S1	-	-	-	1
S8	S1	-	-	-	1
S5	S3	-	-	-	1
S9	S3	-	-	-	1

P1=(S1,S2,S3,S4,S5,S6,S7,S8,S9)
P2=(S1,S2,S3,S6) (S4,S5,S7,S8,S9)
P3=(S1) (S3) (S2,S6) (S4,S5,S7,S8,S9)
P4=(S1) (S3) (S2,S6) (S4,S7,S8) (S5,S9)
P5=(S1) (S3) (S2,S6) (S4,S7,S8) (S5,S9)

Partition for Vending Machine FSM

Present state	Next state				Output
	00	01	10	11	z
S1	S1	S3	S2	-	0
S3	S3	S6	S7	-	0
S2	S2	S4	S5	-	0
S6	S6	S8	S9	-	0
S4	S1	-	-	-	1
S7	S1	-	-	-	1
S8	S1	-	-	-	1
S5	S3	-	-	-	1
S9	S3	-	-	-	1

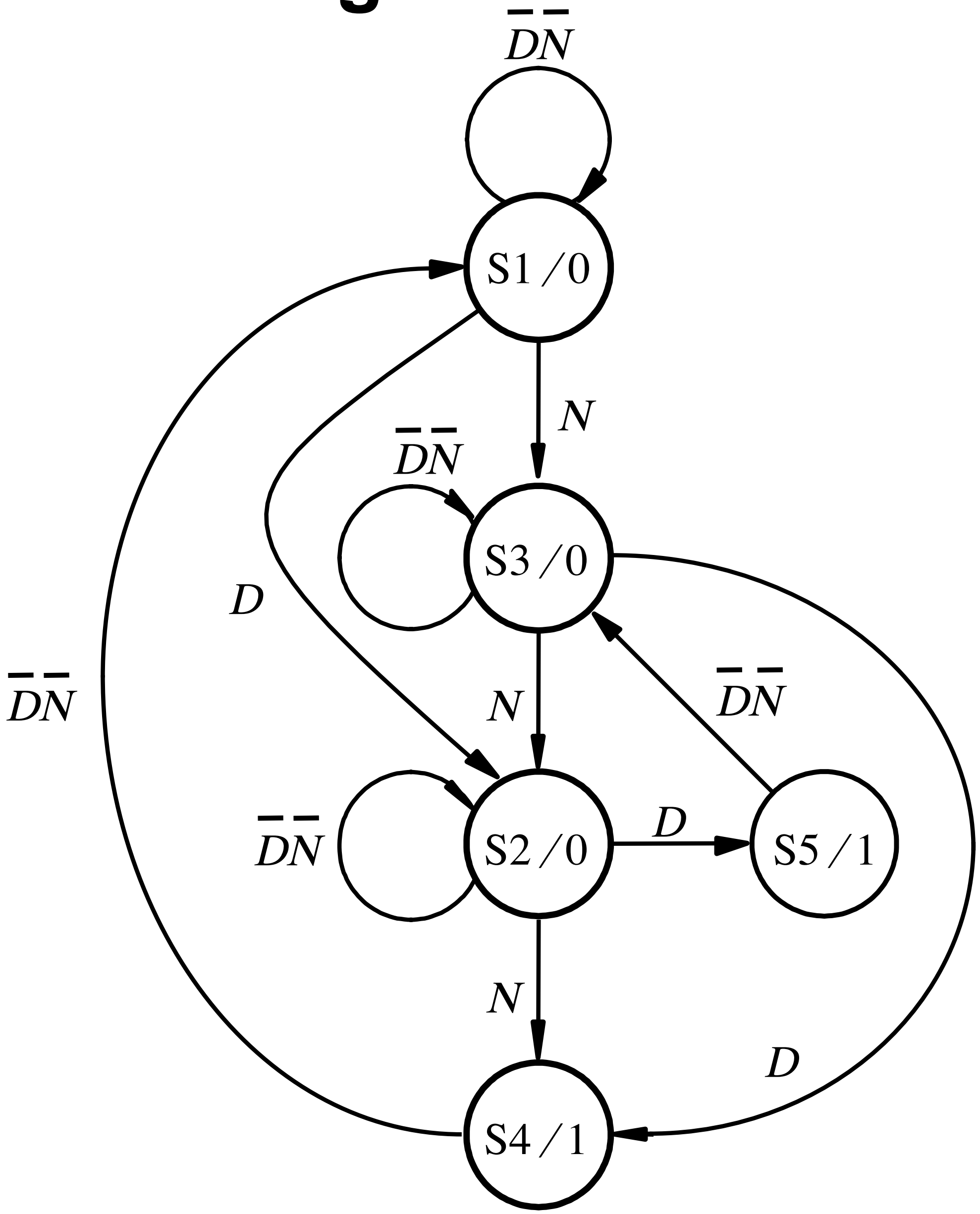
P1=(S1,S2,S3,S4,S5,S6,S7,S8,S9)
P2=(S1,S2,S3,S6) (S4,S5,S7,S8,S9)
P3=(S1) (S3) (S2,S6) (S4,S5,S7,S8,S9)
P4=(S1) (S3) (S2,S6) (S4,S7,S8) (S5,S9)
P5=(S1) (S3) (S2,S6) (S4,S7,S8) (S5,S9)

Minimized State Table for the vending machine

Present state	Next state				Output
	DN =00	01	10	11	*z*
S1	S1	S3	S2	–	0
S2	S2	S4	S5	–	0
S3	S3	S2	S4	–	0
S4	S1	–	–	–	1
S5	S3	–	–	–	1

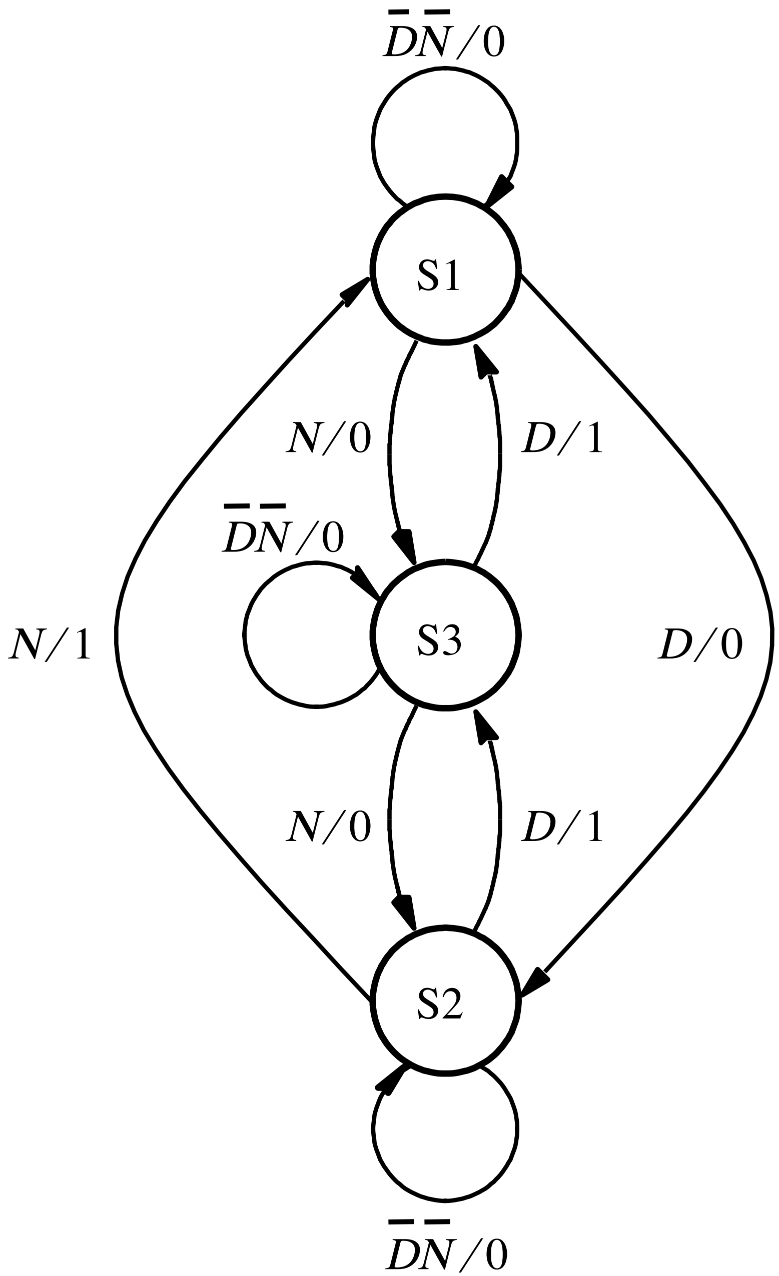
[Figure 6.56 from the textbook]

Minimized State Diagram for the vending machine


$\overline{D}\overline{N}$
S1/0
N
$\overline{D}\overline{N}$
S3/0
D
$\overline{D}\overline{N}$
N
$\overline{D}\overline{N}$
$\overline{D}\overline{N}$
S2/0
D
S5/1
N
D
S4/1

[Figure 6.57 from the textbook]

Mealy-type FSM for the vending machine


$\overline{D}\overline{N}/0$
S1
$N/0$
$D/1$
$\overline{D}\overline{N}/0$
S3
$N/1$
$D/0$
$N/0$
$D/1$
S2
$\overline{D}\overline{N}/0$

[Figure 6.58 from the textbook]

Another Example of Incompletely specified state table

Present state	Next state		Output z	
	$w = 0$	$w = 1$	$w = 0$	$w = 1$
A	B	C	0	0
B	D	–	0	–
C	F	E	0	1
D	B	G	0	0
E	F	C	0	1
F	E	D	0	1
G	F	–	0	–

[Figure 6.59 from the textbook]

Questions?

THE END