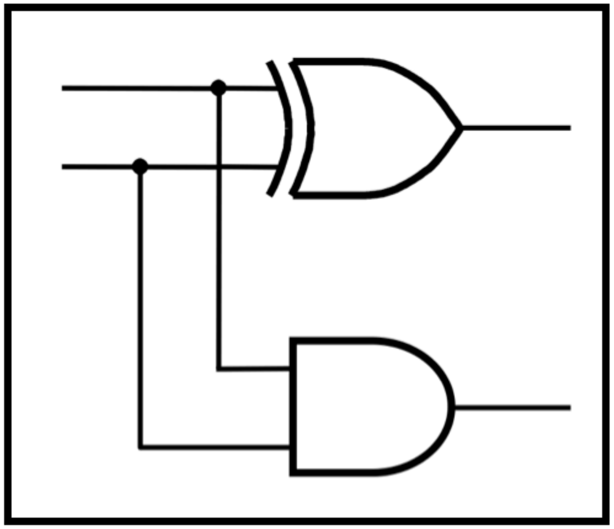


CprE 281: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

Incompletely Specified Functions & Multiple-Output Circuits

CprE 281: Digital Logic
Iowa State University, Ames, IA

Administrative Stuff

- **HW4 is out**

- **It is due on Monday Sep 18 @ 4pm.**

- **Please write clearly on the first page (in block capital letters) the following three things:**

  - **Your First and Last Name**
  - **Your Student ID Number**
  - **Your Lab Section Letter**

- **Also, staple all of your pages together**

Administrative Stuff

- **Midterm Exam #1**
- **When: Friday Sep 22.**
- **Where: This classroom**
- **What: Chapter 1 and Chapter 2 plus number systems**
- **The exam will be open book and open notes (you can bring up to 3 pages of handwritten notes).**
- **Sample exams are posted on the class web page**

Topics for the Midterm Exam

- **Binary Numbers**
- **Octal Numbers**
- **Hexadecimal Numbers**
- **Conversion between the different number systems**
- **Truth Tables**
- **Boolean Algebra**
- **Logic Gates**
- **Circuit Synthesis with AND, OR, NOT**
- **Circuit Synthesis with NAND, NOR**
- **Converting an AND/OR/NOT circuit to NAND circuit**
- **Converting an AND/OR/NOT circuit to NOR circuit**
- **SOP and POS expressions**

Topics for the Midterm Exam

- **Mapping a Circuit to Verilog code**
- **Mapping Verilog code to a circuit**

- **Multiplexers**
- **Venn Diagrams**
- **K-maps for 2, 3, and 4 variables**

- **Minimization of Boolean expressions using theorems**
- **Minimization of Boolean expressions with K-maps**

- **Incompletely specified functions (with don’t cares)**
- **Functions with multiple outputs**

Quick Review

The Combining Theorems of Boolean Algebra

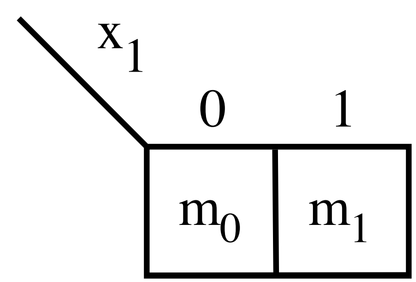
14a. $x \cdot y + x \cdot \overline{y} = x$

14b. $(x + y) \cdot (x + \overline{y}) = x$

One-Variable K-map

x_1	
0	m_0
1	m_1

(a) Truth table

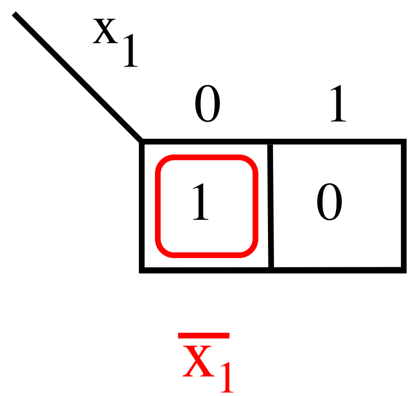


(b) Karnaugh map

One-Variable K-map

x_1	
0	1
1	0

(a) Truth table



(b) Karnaugh map

Two-Variable K-map

x_1	x_2	
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

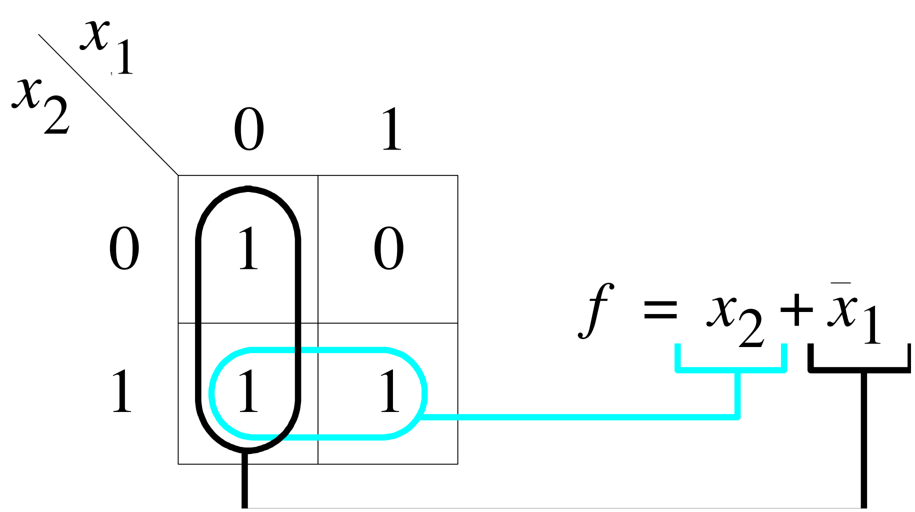
(a) Truth table

x_2 \ x_1	0	1
0	m_0	m_2
1	m_1	m_3

(b) Karnaugh map

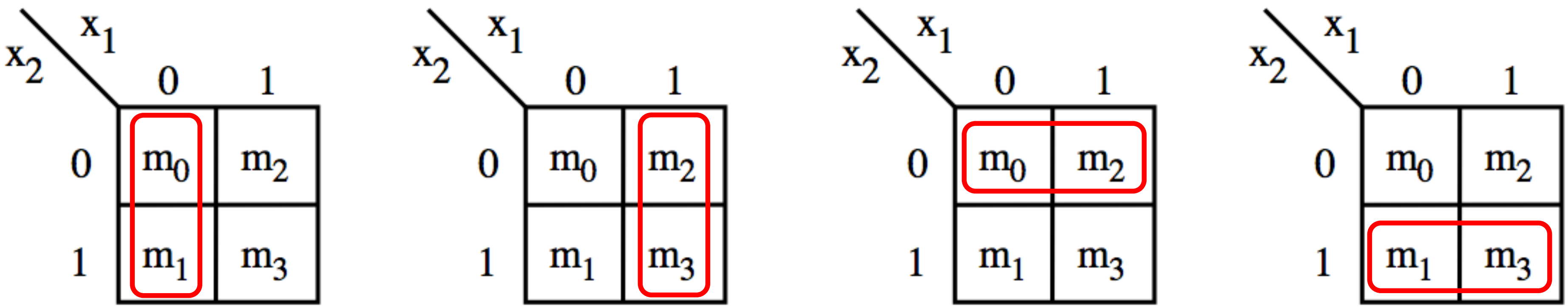
[Figure 2.49 from the textbook]

Two-Variable K-map

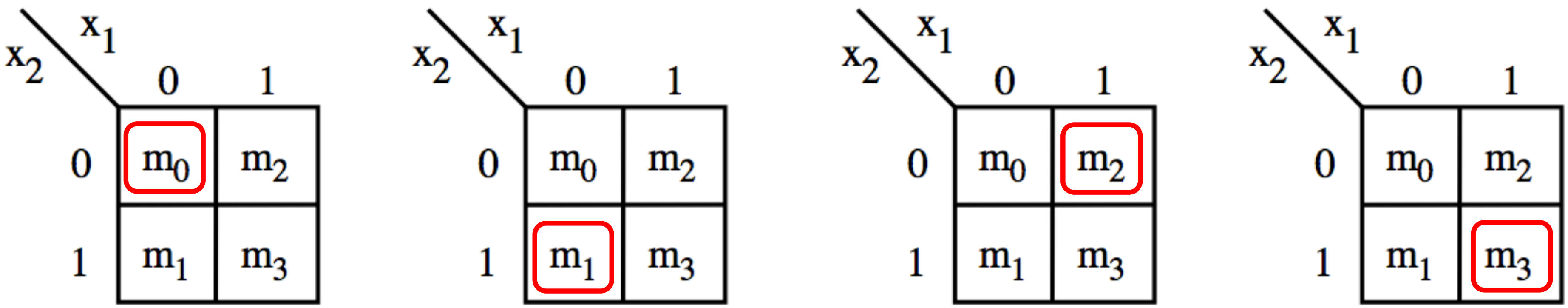


[Figure 2.50 from the textbook]

These are all valid groupings

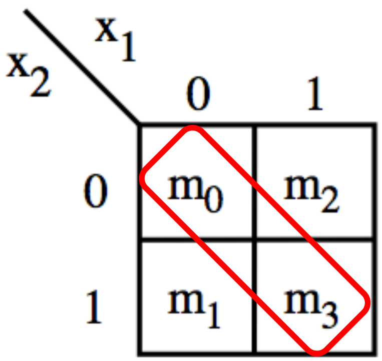


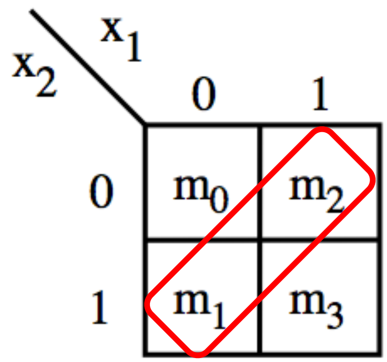
These are also valid



But try to use larger rectangles if possible.

These two are not valid





Three-Variable K-map

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

[Figure 2.51 from the textbook]

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

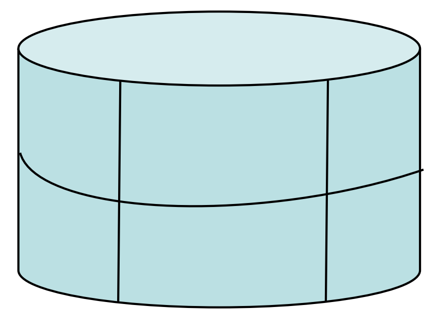
Notice the placement of

- **Variables**
- **Binary pair values**
- **Minterms**

Adjacency Rules

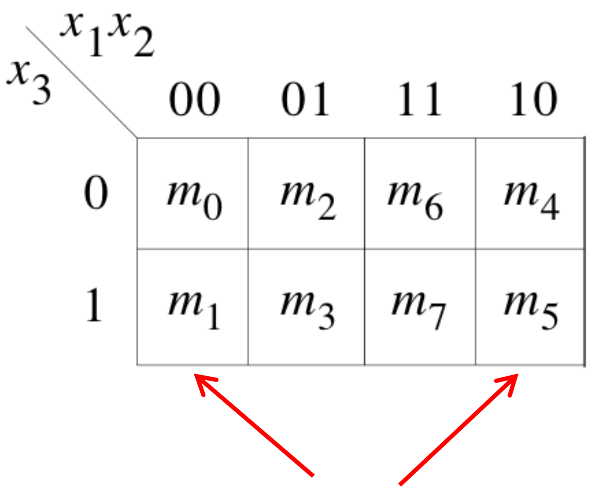
x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

adjacent
columns

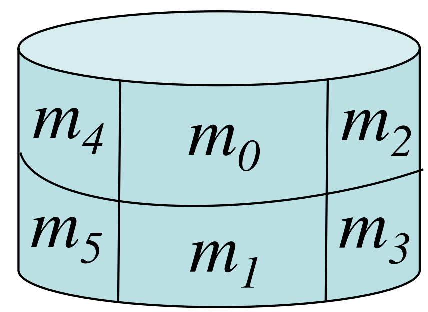


As if the K-map were
drawn on a cylinder

Adjacency Rules

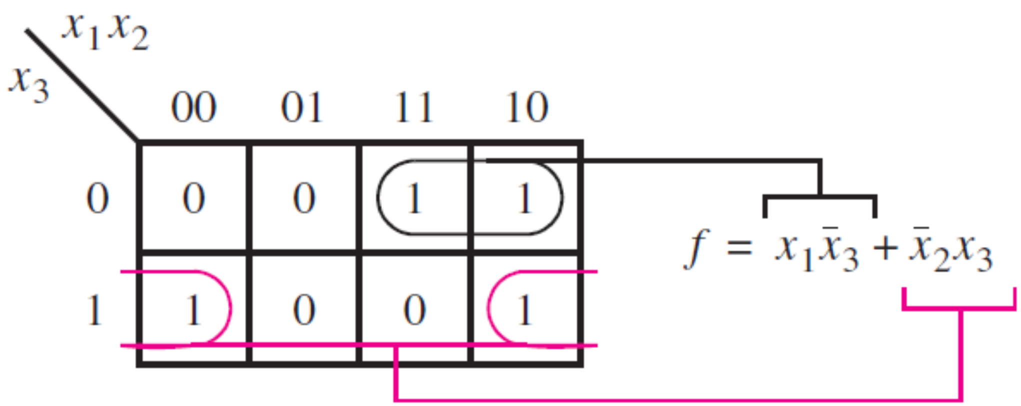


adjacent
columns

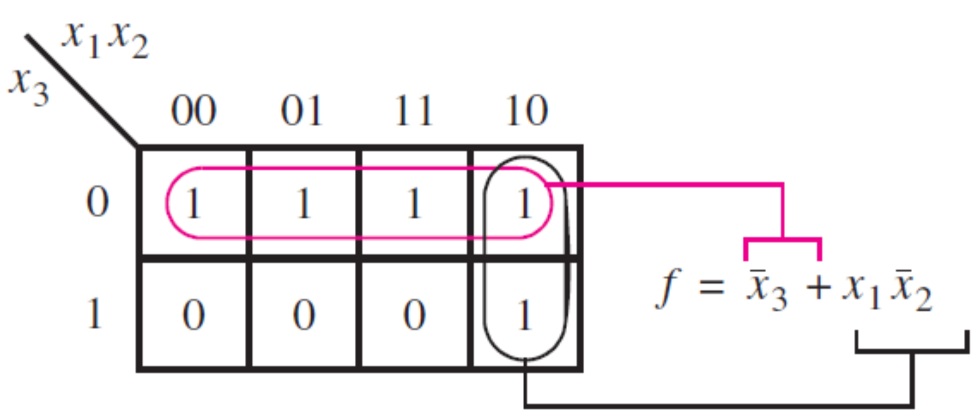


As if the K-map were
drawn on a cylinder

Three-Variable K-map



(a) The function of Figure 2.23



(b) The function of Figure 2.48

[Figure 2.52 from the textbook]

Two Different Ways to Draw the K-map

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3 \ x_1x_2	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

x_1 \ x_2x_3	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Another Way to Draw 3-variable K-map

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3 \ $x_1 x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

$x_2 x_3$ \ x_1	0	1
00	m_0	m_4
01	m_1	m_5
11	m_3	m_7
10	m_2	m_6

Gray Code

- **Sequence of binary codes**
- **Consecutive lines vary by only 1 bit**

	000
	001
00	011
01	010
11	110
10	111
	101
	100

Gray Code & K-map

	s x_1 x_2
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ s x_1	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

Gray Code & K-map

	s x_1 x_2
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ $s\ x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

Gray Code & K-map

	$s\ x_1\ x_2$
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ $s\ x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These two neighbors
differ only in the LAST bit

Gray Code & K-map

	$s\ x_1\ x_2$
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ $s\ x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These two neighbors
differ only in the LAST bit

Gray Code & K-map

	$s\ x_1\ x_2$
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ $s\ x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These two neighbors differ only in the FIRST bit

Gray Code & K-map

	$s\ x_1\ x_2$
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ $s\ x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These two neighbors differ only in the FIRST bit

Gray Code & K-map

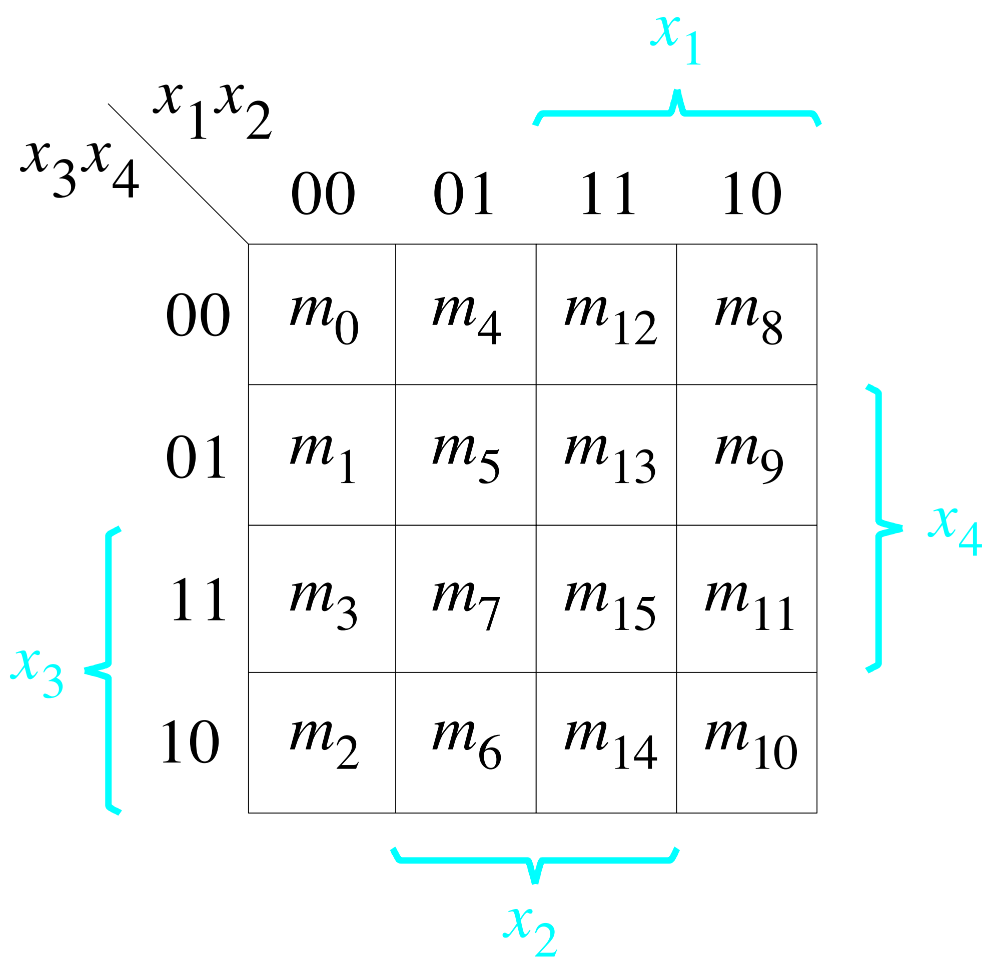
	$s\ x_1\ x_2$
m_0	0 0 0
m_1	0 0 1
m_2	0 1 0
m_3	0 1 1
m_4	1 0 0
m_5	1 0 1
m_6	1 1 0
m_7	1 1 1

x_2 \ $s\ x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These four neighbors
differ in the FIRST and LAST bit

They are similar in their MIDDLE bit

A four-variable Karnaugh map



[Figure 2.53 from the textbook]

A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

x_1: columns 11, 10; x_2: columns 01, 11; x_3: rows 11, 10; x_4: rows 01, 11

Gray Code & K-map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15

x_1

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

x_4

x_3

x_2

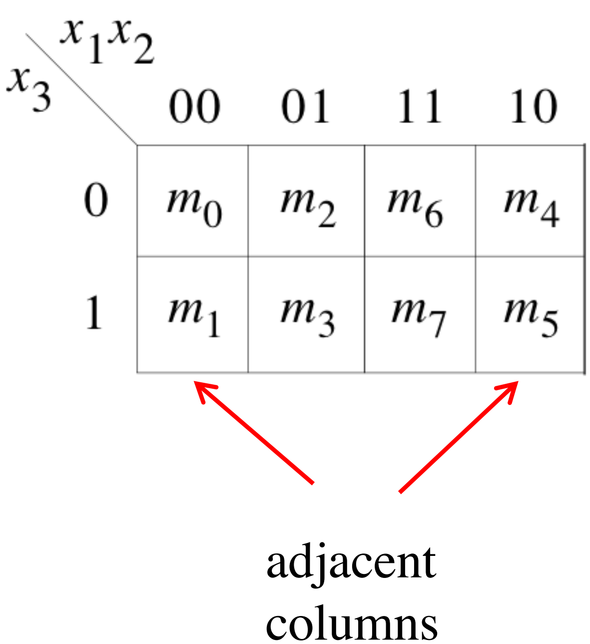
Gray Code & K-map

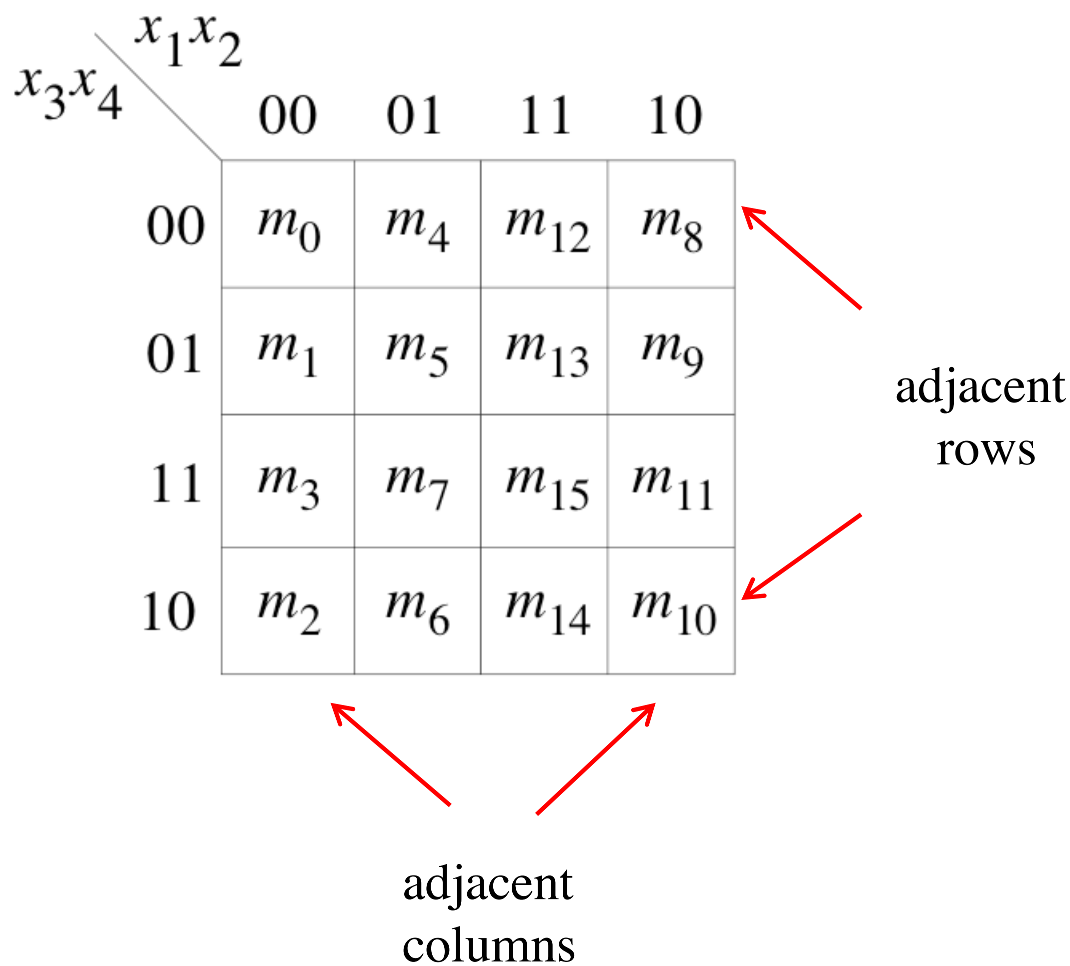
x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15

x_3x_4 \ x_1x_2	00	01	11	10
00	0000	0100	1100	1000
01	0001	0101	1101	1001
11	0011	0111	1111	1011
10	0010	0110	1110	1010

x_1 (columns 11, 10); x_2 (columns 01, 11); x_3 (rows 11, 10); x_4 (rows 01, 11)

Adjacency Rules



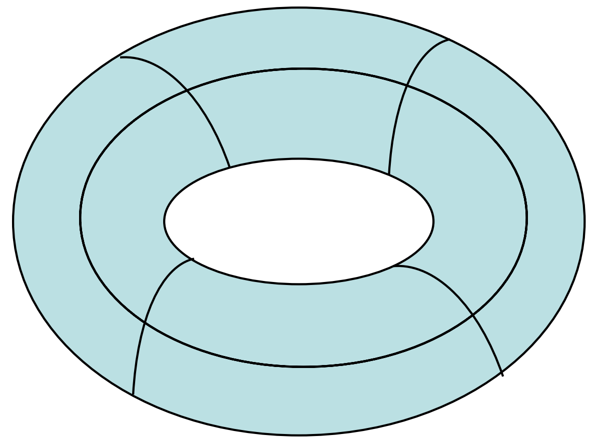


Adjacency Rules

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

adjacent rows

adjacent columns



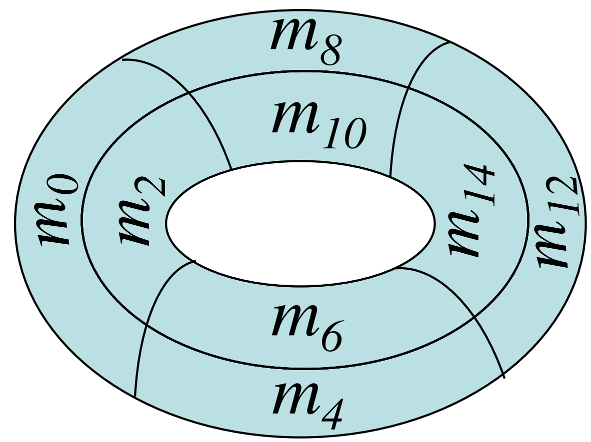
As if the K-map were drawn on a torus

Adjacency Rules

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

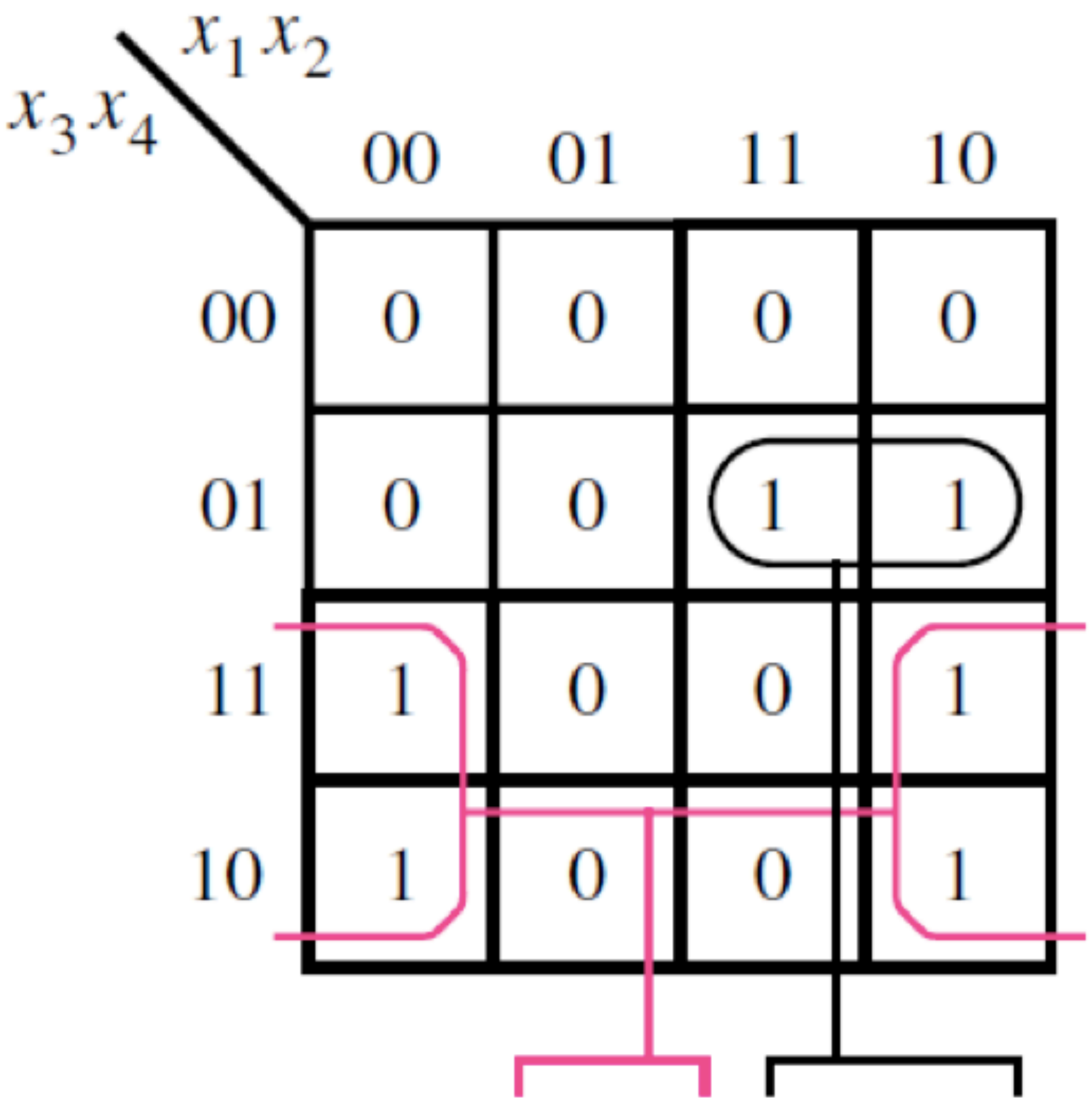
adjacent rows

adjacent columns



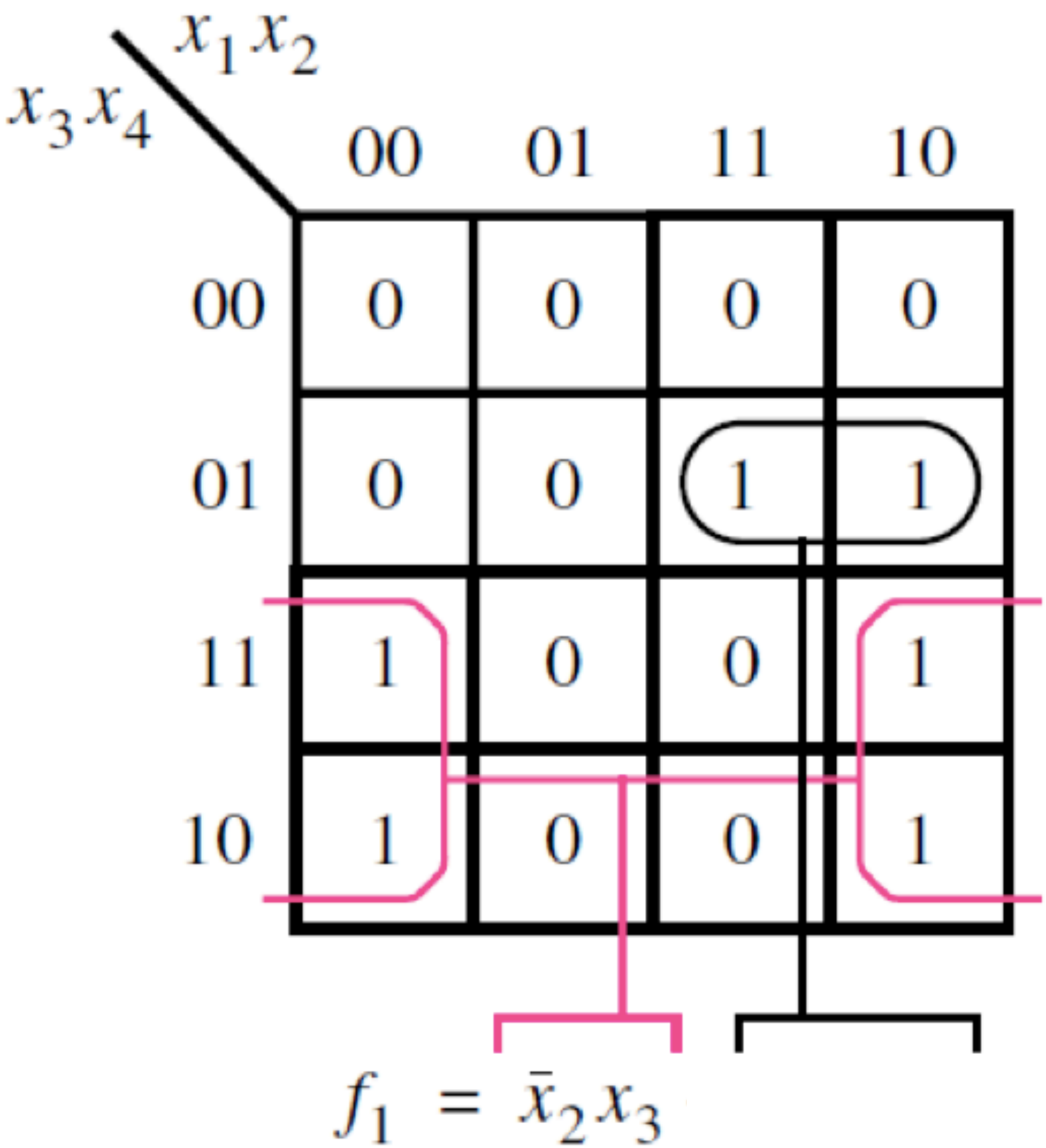
As if the K-map were drawn on a torus

Example of a four-variable Karnaugh map



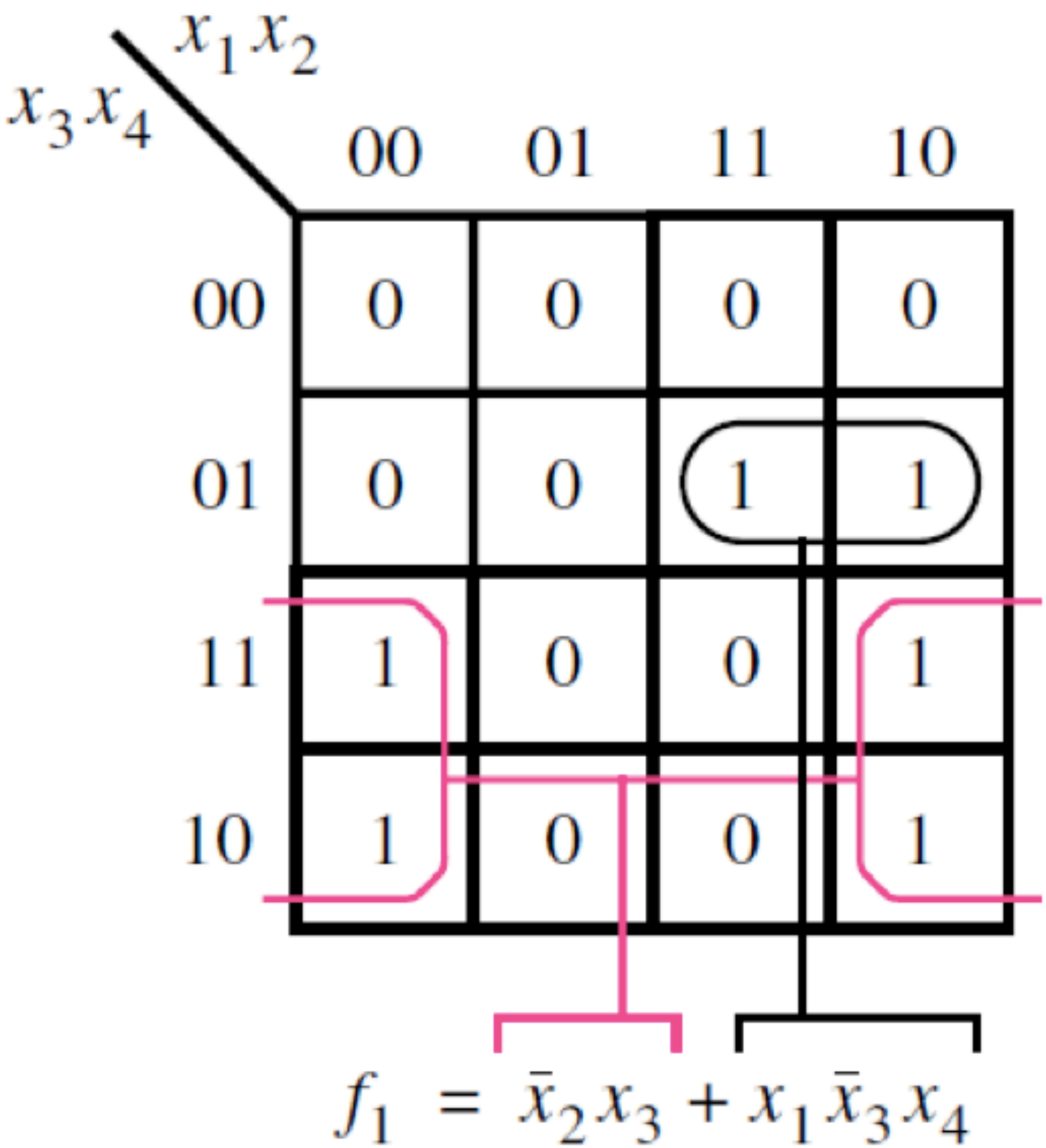
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



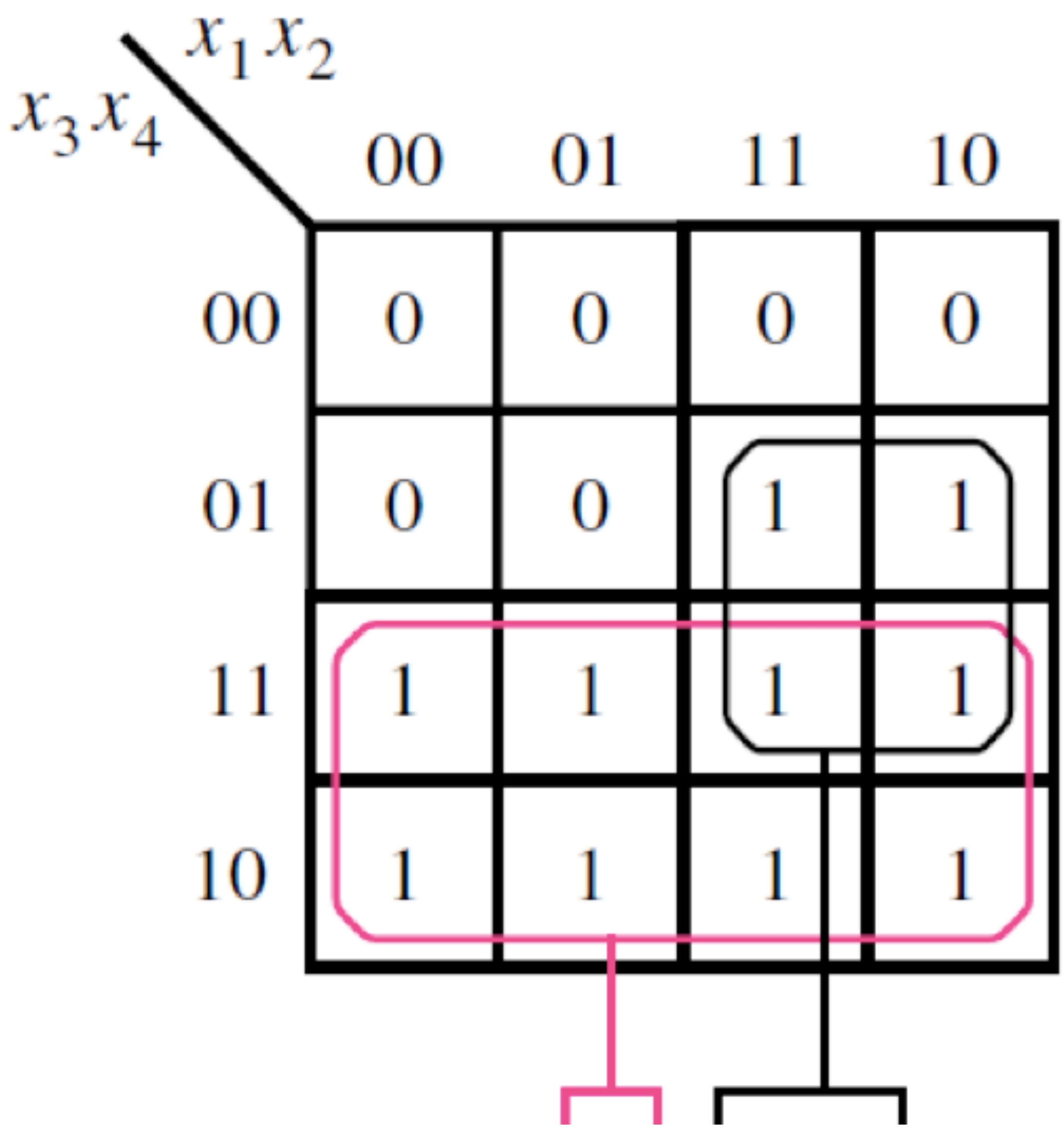
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



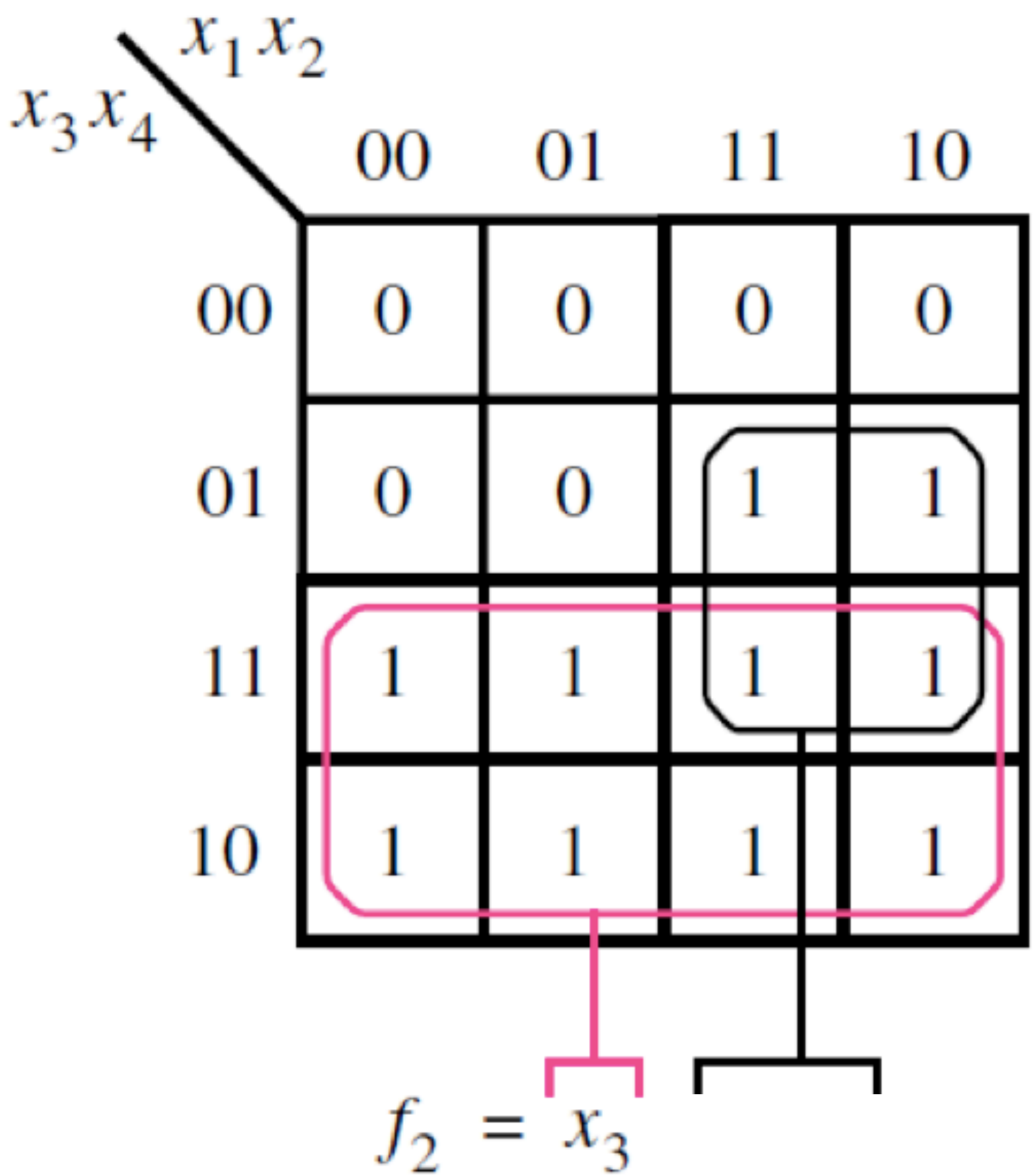
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



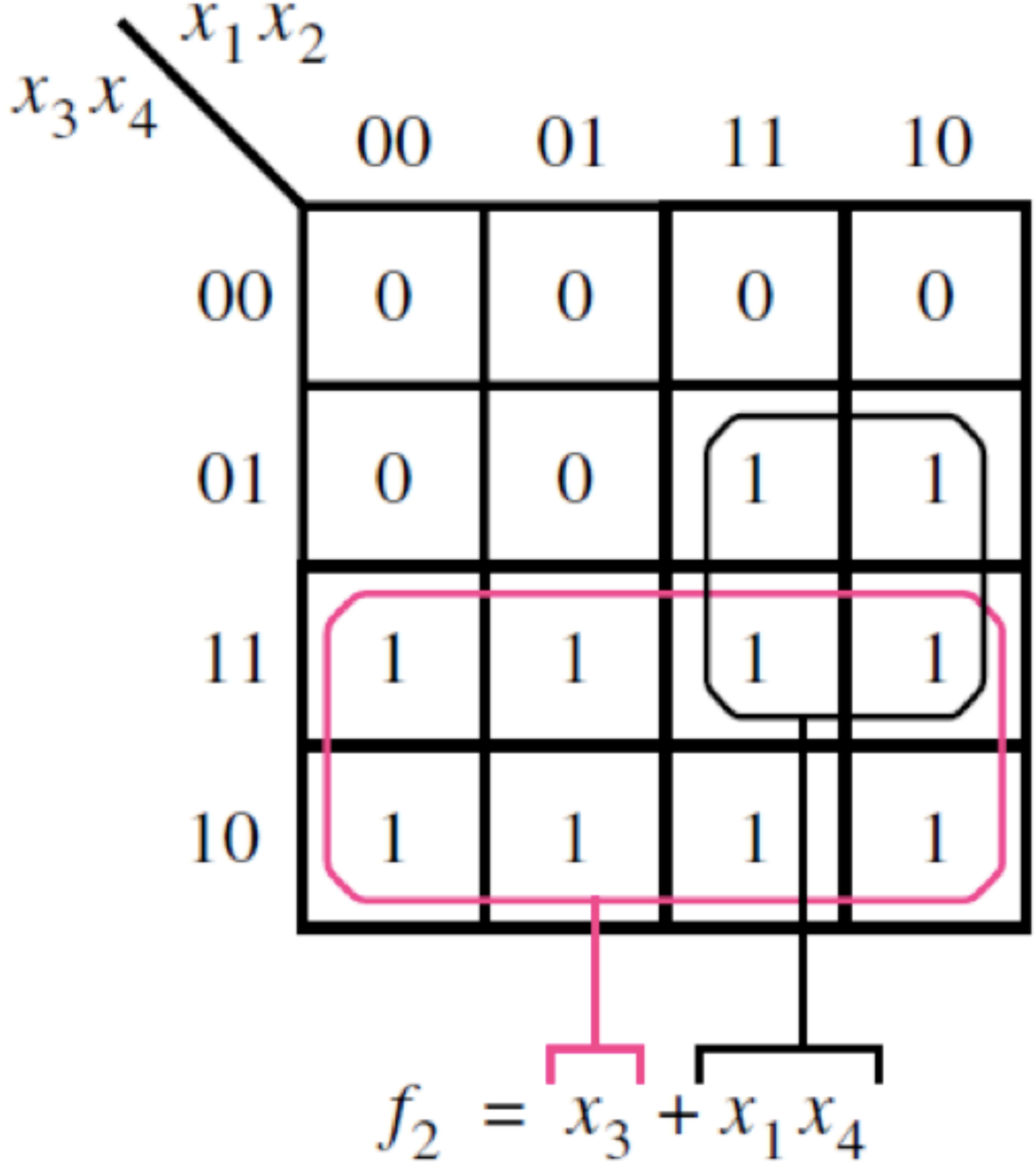
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



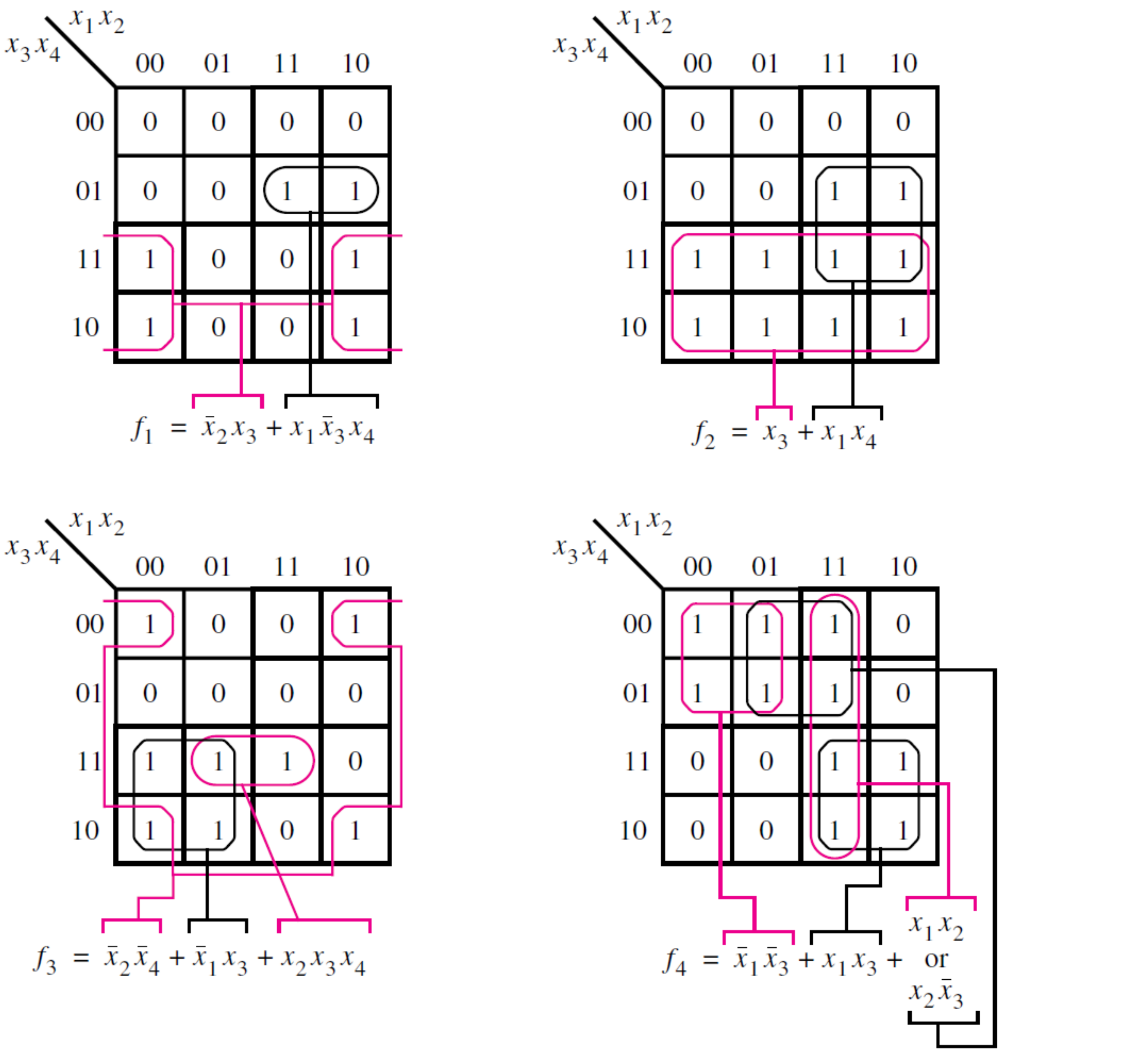
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



[Figure 2.54 from the textbook]

Other Four-Variable K-map Examples



[Figure 2.54 from the textbook]

Strategy For Minimization

Grouping Rules

- Group "1"s with rectangles
- Both sides a power of 2:
  - 1x1, 1x2, 2x1, 2x2, 1x4, 4x1, 2x4, 4x2, 4x4
- Can use the same minterm more than once
- Can wrap around the edges of the map
- Some rules in selecting groups:
  - Try to use as few groups as possible to cover all "1"s.
  - For each group, try to make it as large as you can (i.e., if you can use a 2x2, don't use a 2x1 even if that is enough).

Terminology

Literal: a variable, complemented or uncomplemented

Some Examples:

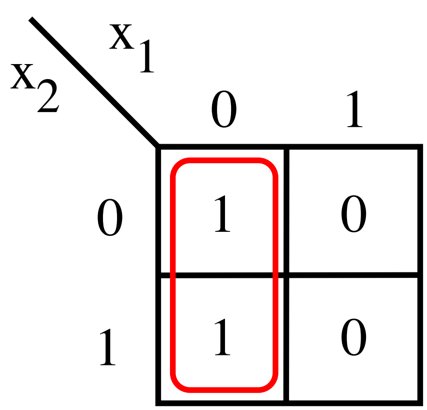
- X_1
- X_2

Terminology

- **Implicant: product term that indicates the input combinations for which the function output is 1**

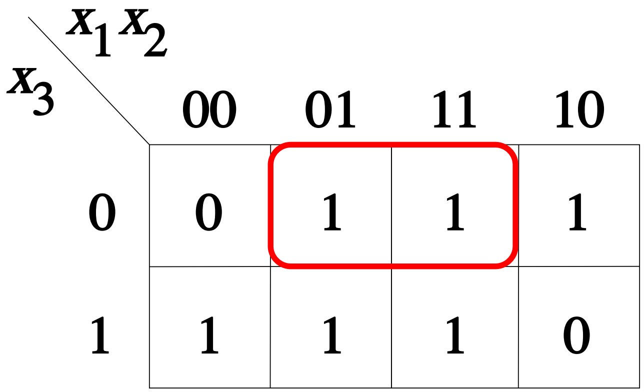
- **Example**

  - $\bar{x}_1$ **- indicates that $\bar{x}_1\bar{x}_2$ and $\bar{x}_1 x_2$ yield output of 1**

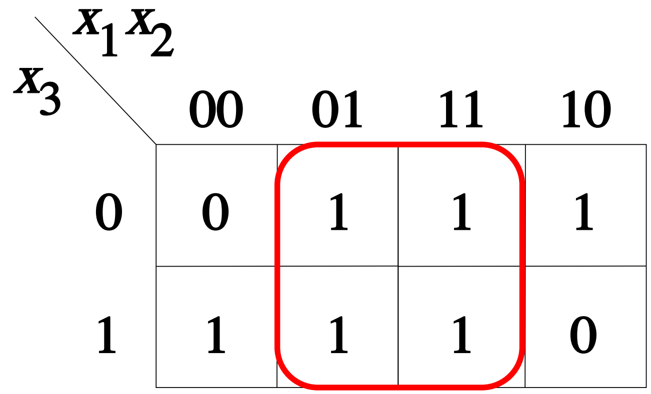


Terminology

- **Prime Implicant**
  - **Implicant that cannot be combined into another implicant with fewer literals**
  - **Some Examples**



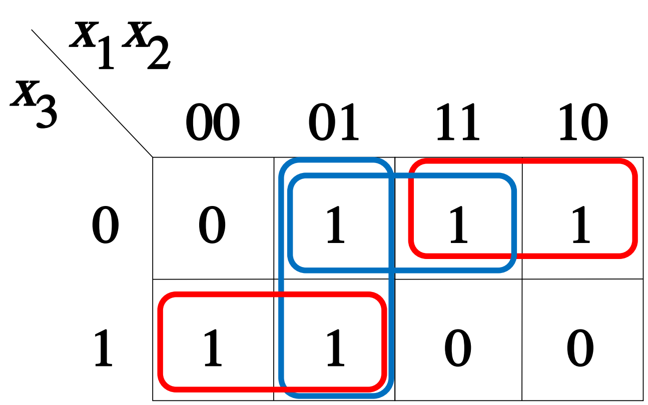
Not prime



Prime

Terminology

- **Essential Prime Implicant**
  - **Prime implicant that includes a minterm not covered by any other prime implicant**
  - **Some Examples**



Terminology

- **Cover**
  - **Collection of implicants that account for all possible input valuations where output is 1**

  - **Ex.** $x_1' x_2 x_3 + x_1 x_2 x_3' + x_1 x_2' x_3'$
  - **Ex.** $x_1' x_2 x_3 + x_1 x_3'$

x_3 \ $x_1 x_2$	00	01	11	10
0	0	0	1	1
1	0	1	0	0

Example

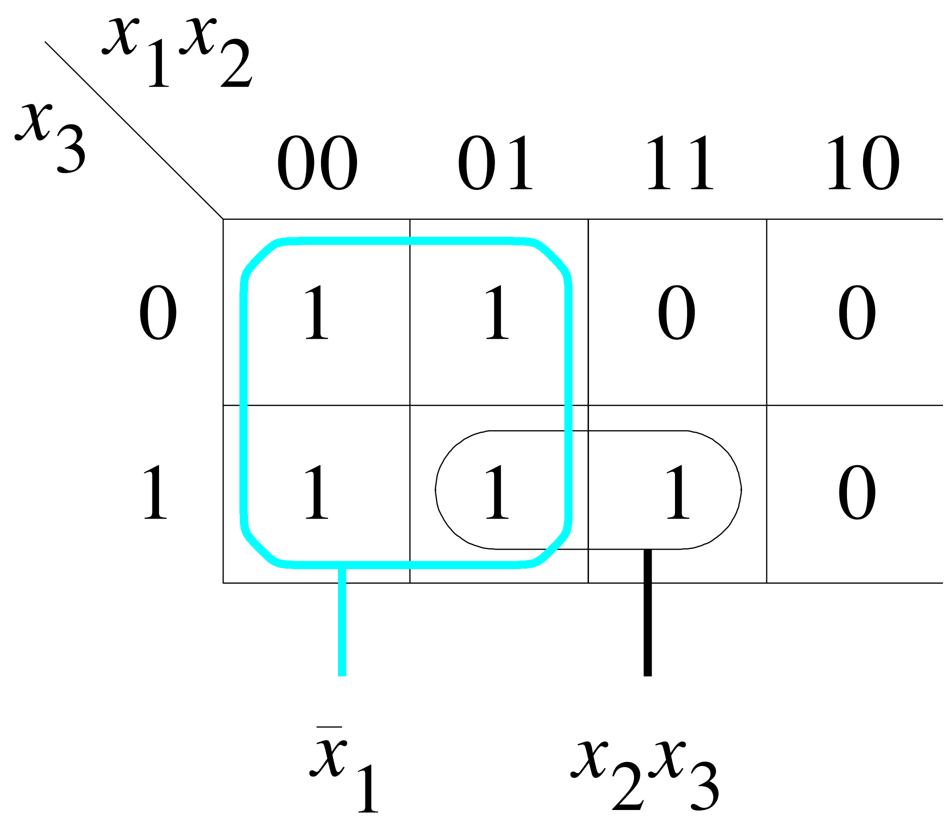
- **Give the Number of**
  - **Implicants?**
  - **Prime Implicants?**
  - **Essential Prime Implicants?**

x_3 \ $x_1 x_2$	00	01	11	10
0	1	1	0	0
1	1	1	1	0

Why concerned with minimization?

- Simplified function
- Reduce the cost of the circuit
  - Cost: Gates + Inputs
  - Transistors

Three-variable function $f(x_1, x_2, x_3) = \sum m(0, 1, 2, 3, 7)$

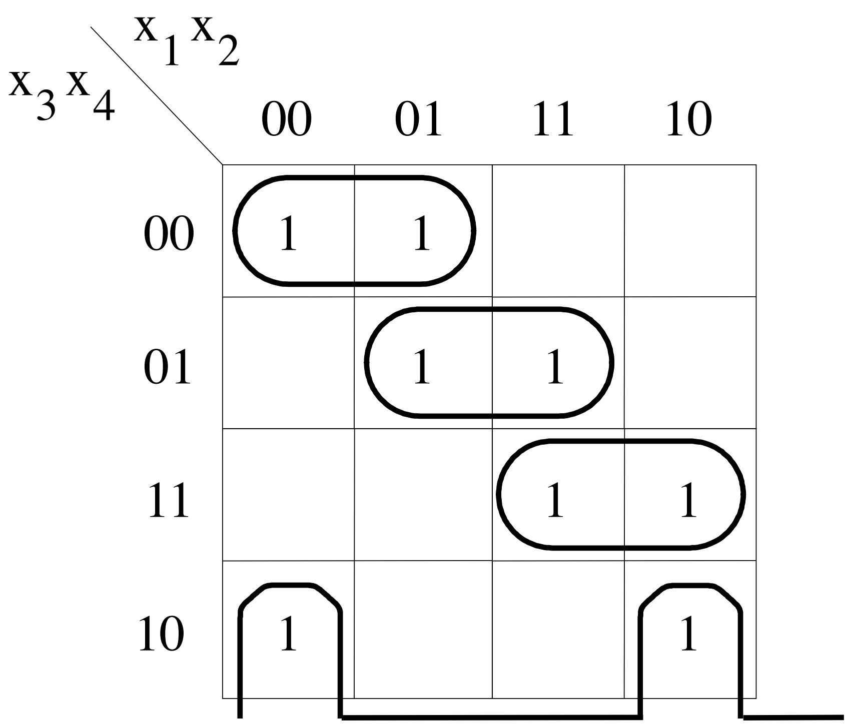


[Figure 2.56 from the textbook]

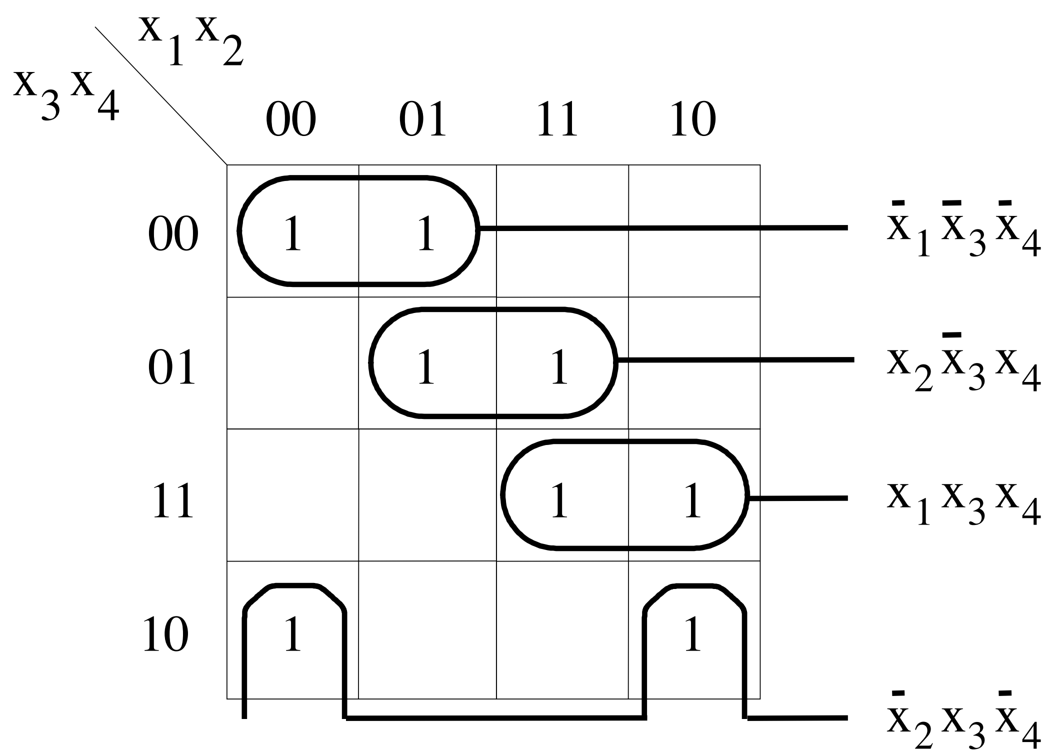
Example

x_3x_4 \ x_1x_2	00	01	11	10
00	1	1		
01		1	1	
11			1	1
10	1			1

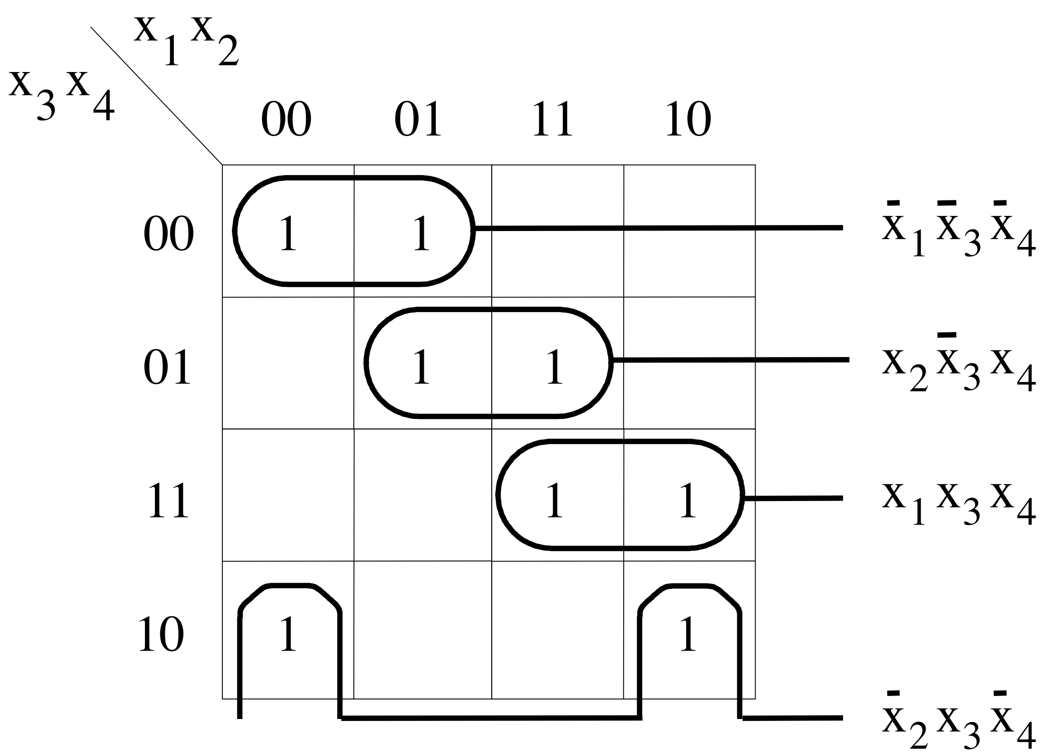
Example



Example


$x_1 x_2$
$x_3 x_4$
00
01
11
10
00
01
11
10
1
1
1
1
1
1
1
1
$\bar{x}_1 \bar{x}_3 \bar{x}_4$
$x_2 \bar{x}_3 x_4$
$x_1 x_3 x_4$
$\bar{x}_2 x_3 \bar{x}_4$

Example

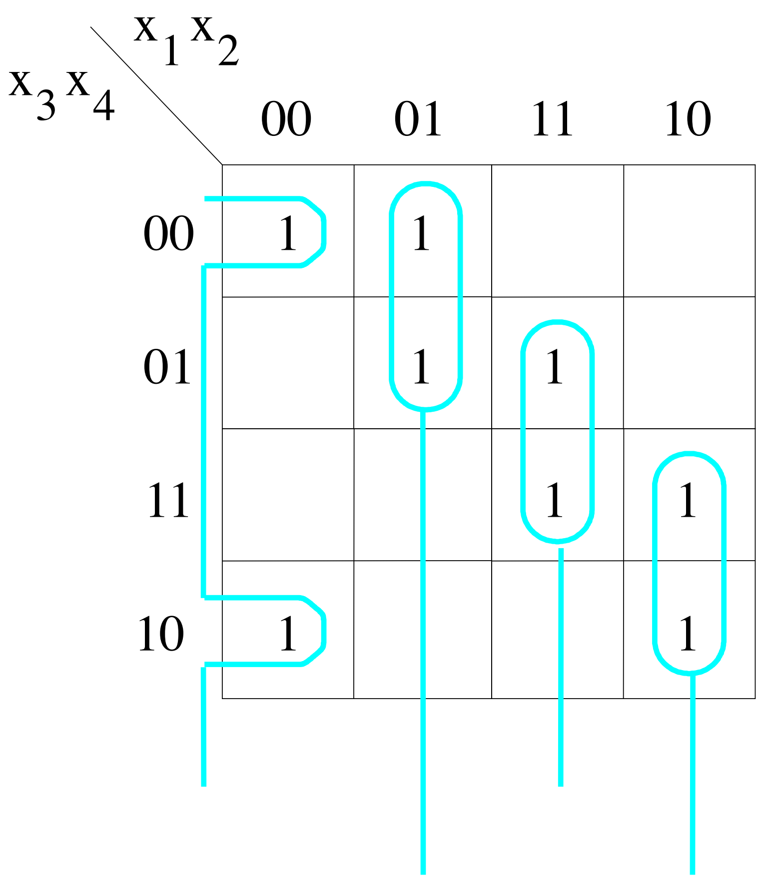


$$f = \bar{x}_1 \bar{x}_3 \bar{x}_4 + x_2 \bar{x}_3 x_4 + x_1 x_3 x_4 + \bar{x}_2 x_3 \bar{x}_4$$

Example: Another Solution

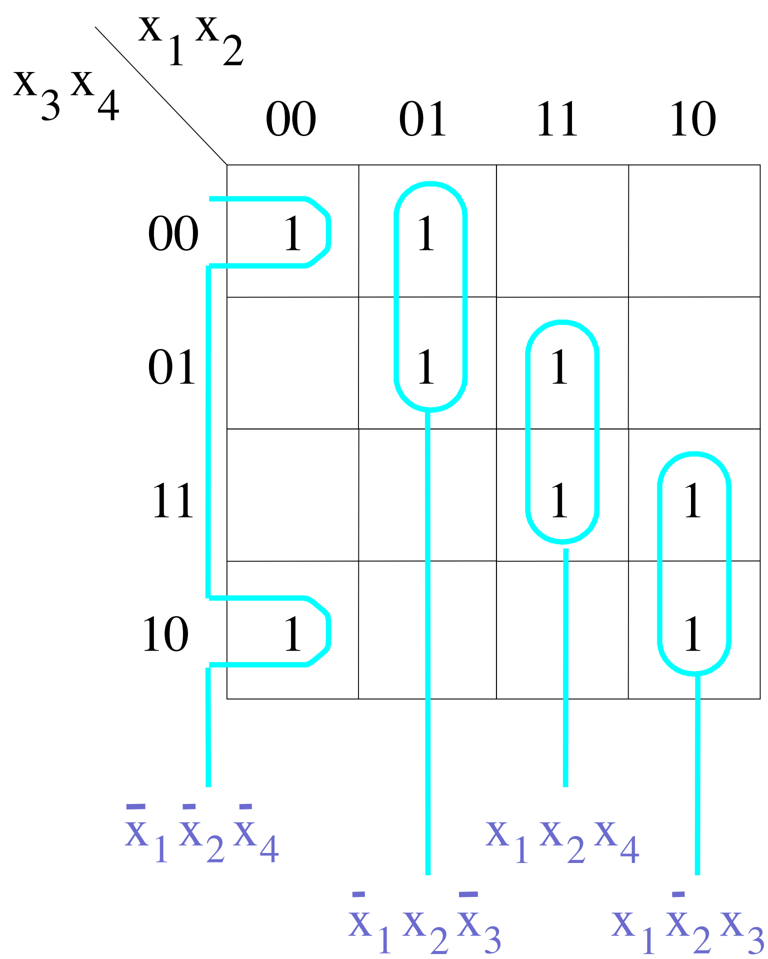
x_3x_4 \ x_1x_2	00	01	11	10
00	1	1		
01		1	1	
11			1	1
10	1			1

Example: Another Solution

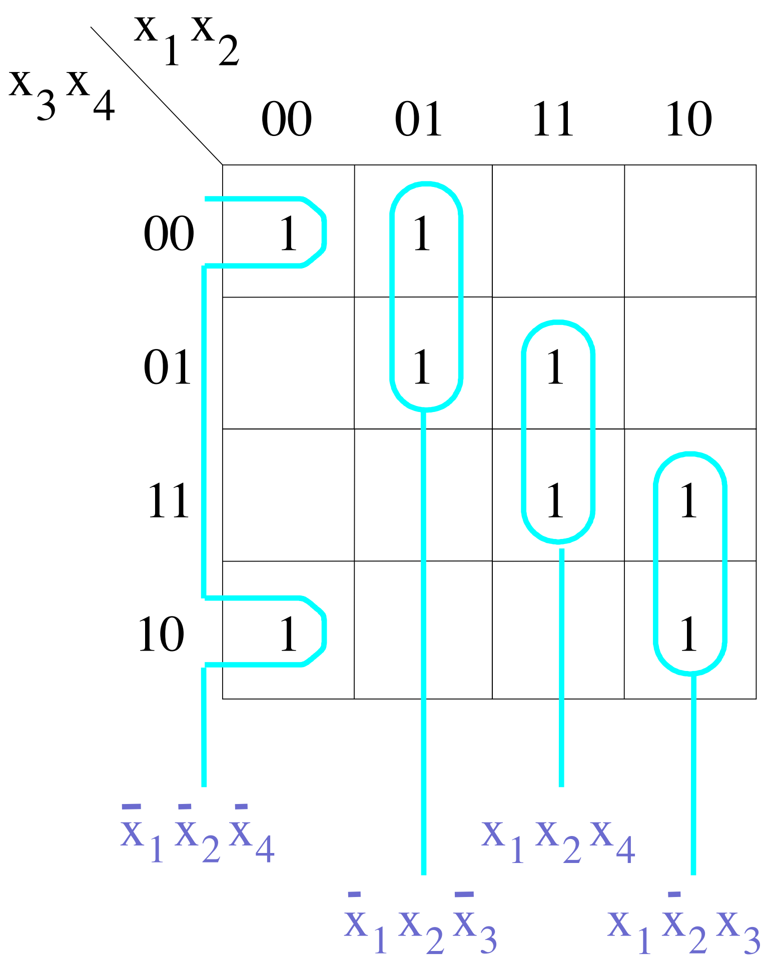


[Figure 2.59 from the textbook]

Example: Another Solution

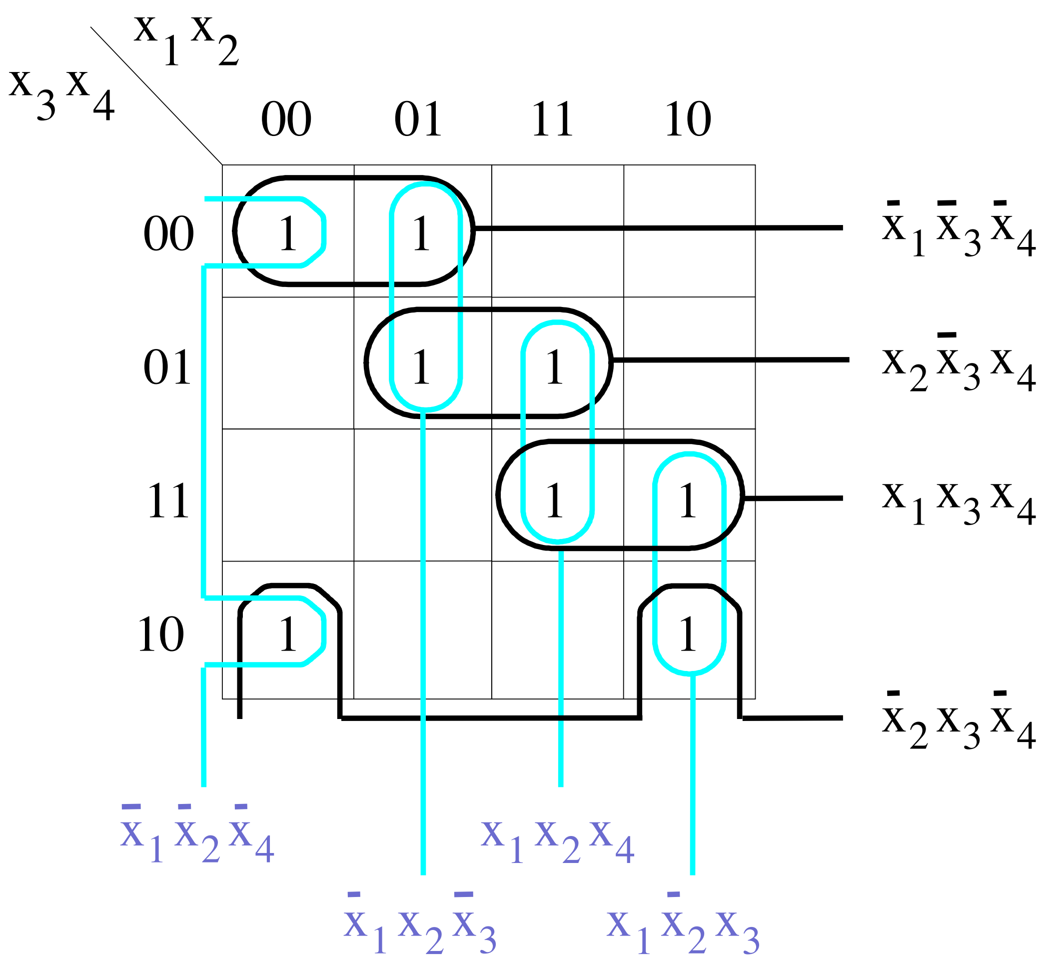


Example: Another Solution



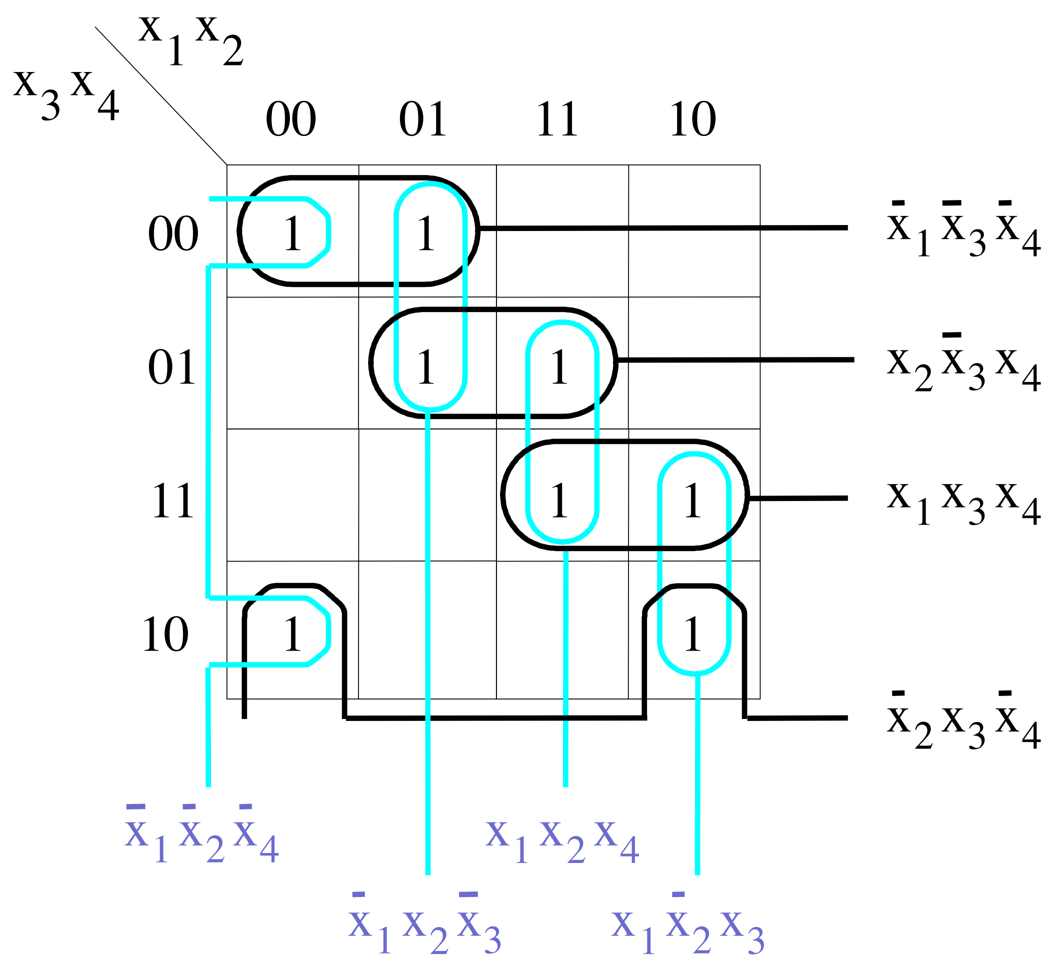
$$f = \bar{x}_1\bar{x}_2\bar{x}_4 + \bar{x}_1x_2\bar{x}_3 + x_1x_2x_4 + x_1\bar{x}_2x_3$$

Example: Both Are Valid Solutions



[Figure 2.59 from the textbook]

Example: Both Are Valid Solutions



$$f = \bar{x}_1 \bar{x}_3 \bar{x}_4 + x_2 \bar{x}_3 x_4 + x_1 x_3 x_4 + \bar{x}_2 x_3 \bar{x}_4$$

$$f = \bar{x}_1 \bar{x}_2 \bar{x}_4 + \bar{x}_1 x_2 \bar{x}_3 + x_1 x_2 x_4 + x_1 \bar{x}_2 x_3$$

Example:
Incompletely Specified Function

Three Ways to Specify the Function

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

x_1	x_2	x_3	x_4	f
0	0	0	0	m_0
0	0	0	1	m_1
0	0	1	0	m_2
0	0	1	1	m_3
0	1	0	0	m_4
0	1	0	1	m_5
0	1	1	0	m_6
0	1	1	1	m_7
1	0	0	0	m_8
1	0	0	1	m_9
1	0	1	0	m_{10}
1	0	1	1	m_{11}
1	1	0	0	m_{12}
1	1	0	1	m_{13}
1	1	1	0	m_{14}
1	1	1	1	m_{15}

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

x_1 (columns 11, 10); x_2 (columns 01, 11); x_3 (rows 11, 10); x_4 (rows 01, 11)

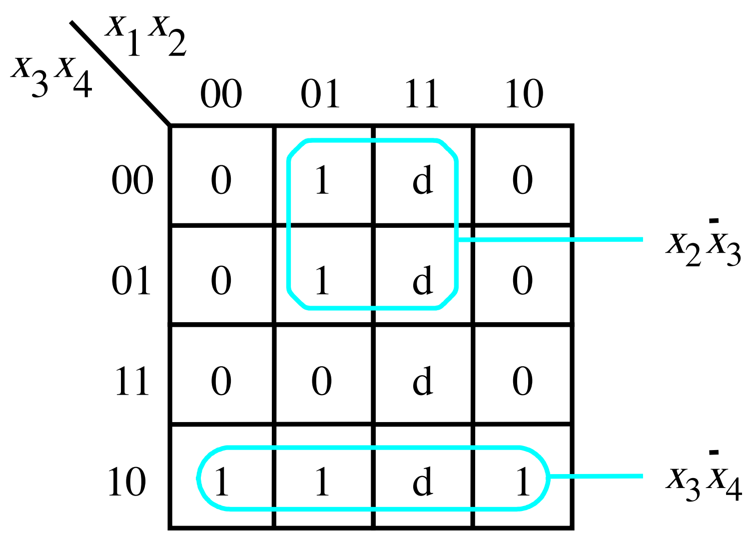
Three Ways to Specify the Function

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

x_1 x_2 x_3 x_4	f
0 0 0 0	m_0
0 0 0 1	m_1
0 0 1 0	m_2
0 0 1 1	m_3
0 1 0 0	m_4
0 1 0 1	m_5
0 1 1 0	m_6
0 1 1 1	m_7
1 0 0 0	m_8
1 0 0 1	m_9
1 0 1 0	m_{10}
1 0 1 1	m_{11}
1 1 0 0	m_{12}
1 1 0 1	m_{13}
1 1 1 0	m_{14}
1 1 1 1	m_{15}

x_3x_4 \ x_1x_2	00	01	11	10
00	0	1	d	0
01	0	1	d	0
11	0	0	d	0
10	1	1	d	1

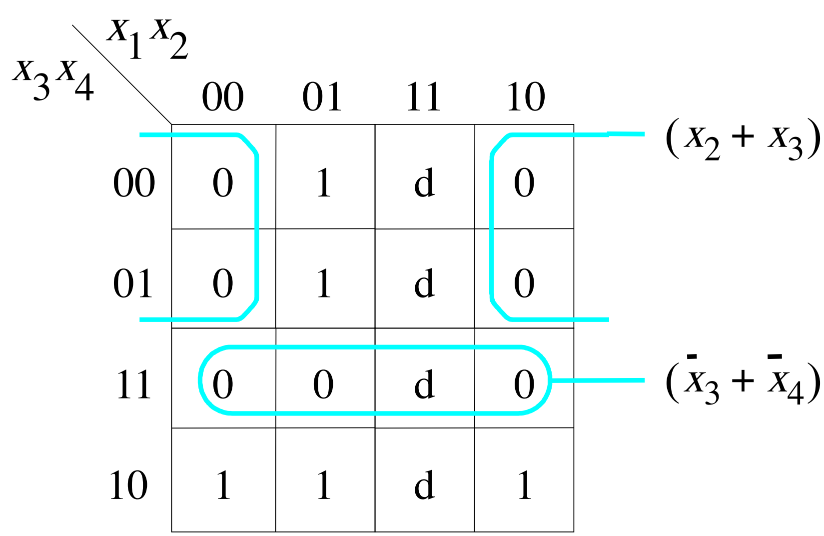
SOP implementation



(a) SOP implementation

[Figure 2.62 from the textbook]

POS implementation

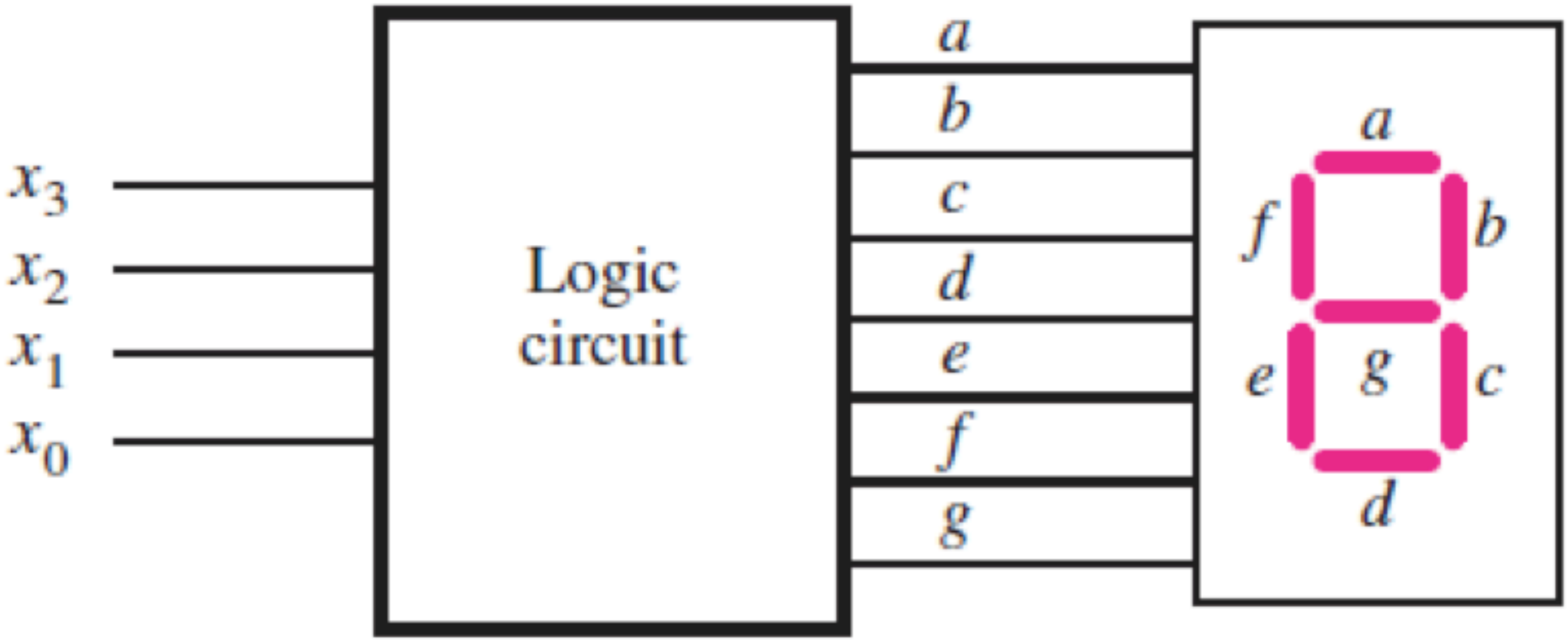


(b) POS implementation

[Figure 2.62 from the textbook]

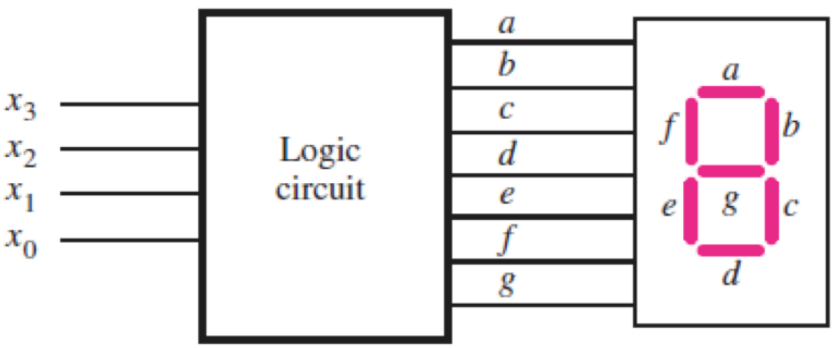
Example:
A circuit with multiple outputs

Seven-Segment Indicator



(a) Logic circuit and 7-segment display

Seven-Segment Indicator



(a) Logic circuit and 7-segment display

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1

(b) Truth table

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0							
	1	0	1	1							
	1	1	0	0							
	1	1	0	1							
	1	1	1	0							
	1	1	1	1							

Seven-Segment Indicator

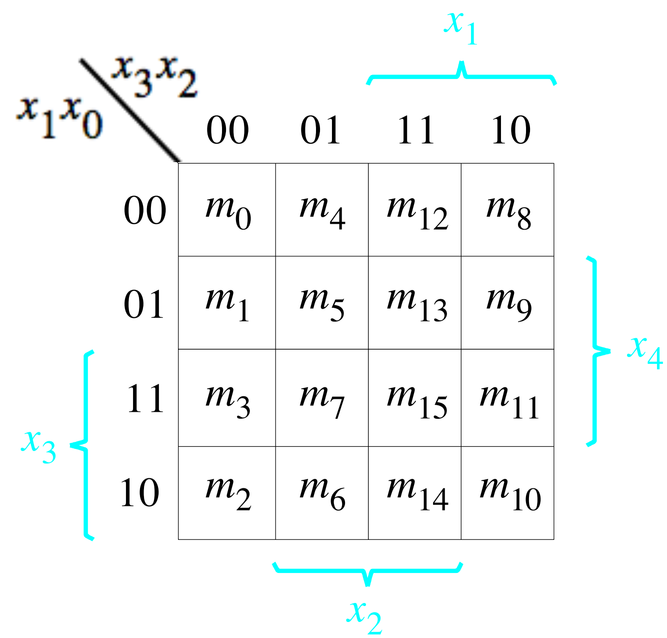
	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	d	d	d	d	d	d	d
	1	0	1	1	d	d	d	d	d	d	d
	1	1	0	0	d	d	d	d	d	d	d
	1	1	0	1	d	d	d	d	d	d	d
	1	1	1	0	d	d	d	d	d	d	d
	1	1	1	1	d	d	d	d	d	d	d

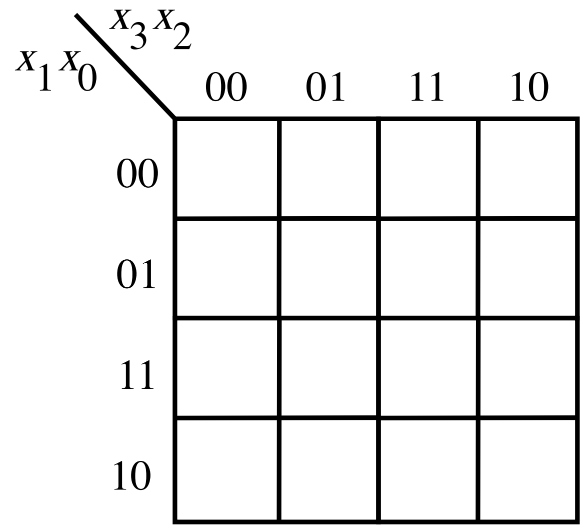
Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	d	d	d	d	d	d	d
	1	0	1	1	d	d	d	d	d	d	d
	1	1	0	0	d	d	d	d	d	d	d
	1	1	0	1	d	d	d	d	d	d	d
	1	1	1	0	d	d	d	d	d	d	d
	1	1	1	1	d	d	d	d	d	d	d

Seven-Segment Indicator

x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	d	d	d	d	d	d	d
1	0	1	1	d	d	d	d	d	d	d
1	1	0	0	d	d	d	d	d	d	d
1	1	0	1	d	d	d	d	d	d	d
1	1	1	0	d	d	d	d	d	d	d
1	1	1	1	d	d	d	d	d	d	d





Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	d	d	d	d	d	d	d
	1	0	1	1	d	d	d	d	d	d	d
	1	1	0	0	d	d	d	d	d	d	d
	1	1	0	1	d	d	d	d	d	d	d
	1	1	1	0	d	d	d	d	d	d	d
	1	1	1	1	d	d	d	d	d	d	d

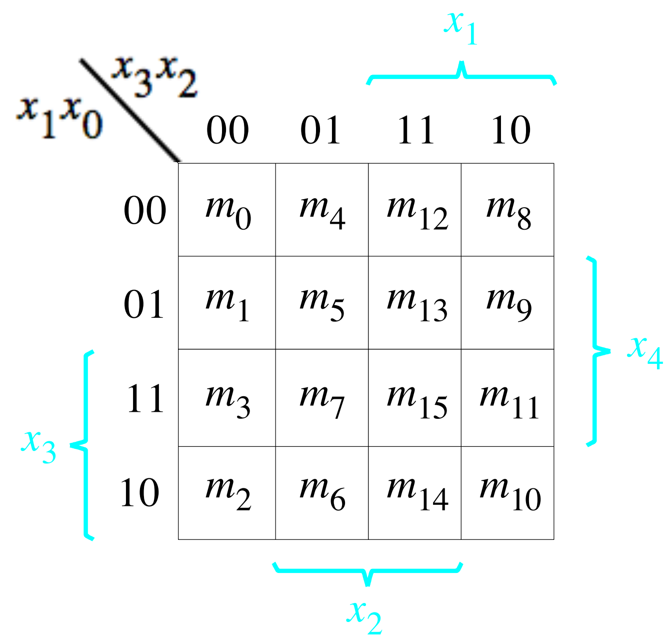
x_1x_0 \ x_3x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

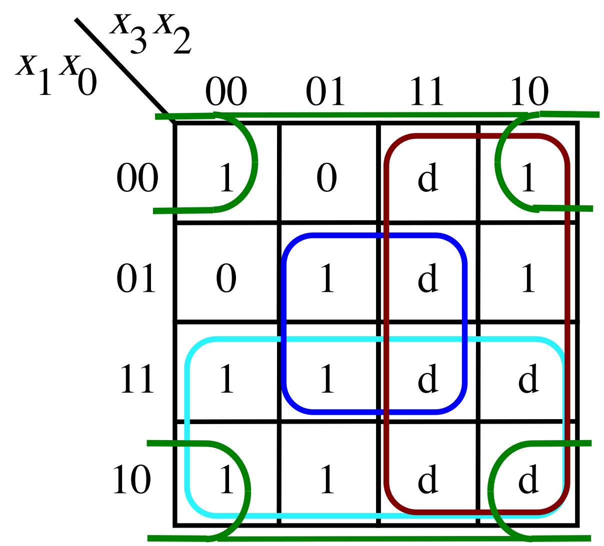
x_1 (columns 11, 10); x_2 (columns 01, 11); x_4 (rows 01, 11); x_3 (rows 11, 10)

x_1x_0 \ x_3x_2	00	01	11	10
00	1	0	d	1
01	0	1	d	1
11	1	1	d	d
10	1	1	d	d

Seven-Segment Indicator

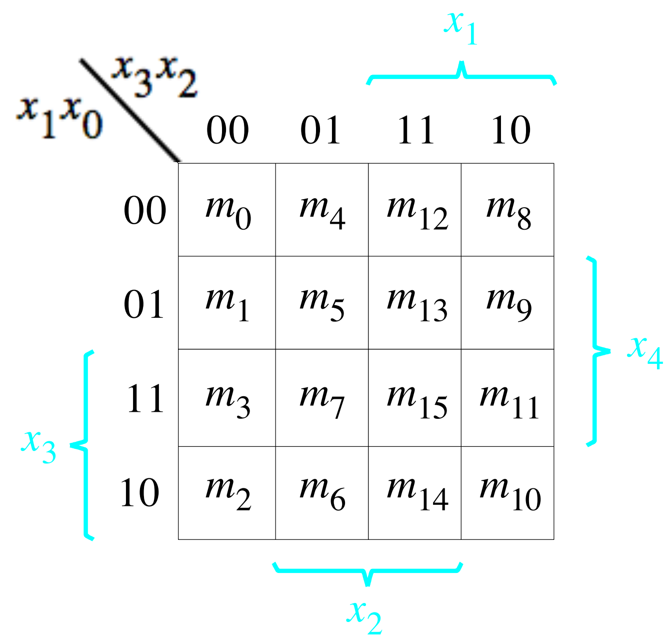
	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	d	d	d	d	d	d	d
	1	0	1	1	d	d	d	d	d	d	d
	1	1	0	0	d	d	d	d	d	d	d
	1	1	0	1	d	d	d	d	d	d	d
	1	1	1	0	d	d	d	d	d	d	d
	1	1	1	1	d	d	d	d	d	d	d

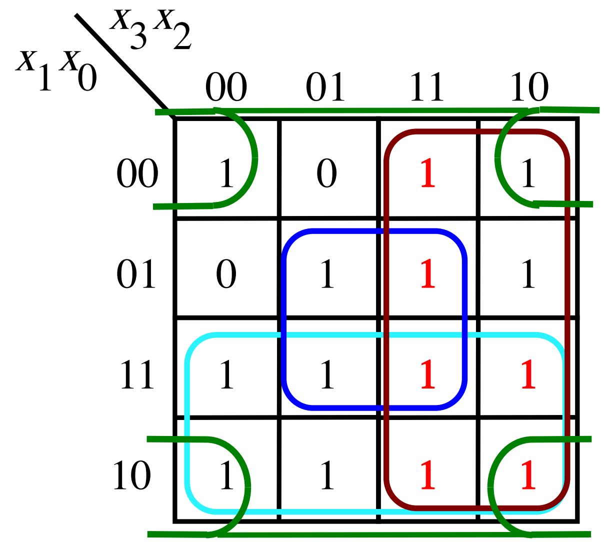




Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	1	d	d	d	d	d	d
	1	0	1	1	1	d	d	d	d	d	d
	1	1	0	0	1	d	d	d	d	d	d
	1	1	0	1	1	d	d	d	d	d	d
	1	1	1	0	1	d	d	d	d	d	d
	1	1	1	1	1	d	d	d	d	d	d





In this case all d's were treated as 1's.

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	1	d	d	d	d	d	d
	1	0	1	1	1	d	d	d	d	d	d
	1	1	0	0	1	d	d	d	d	d	d
	1	1	0	1	1	d	d	d	d	d	d
	1	1	1	0	1	d	d	d	d	d	d
	1	1	1	1	1	d	d	d	d	d	d

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	1	d	d	d	d	d	d
	1	0	1	1	1	d	d	d	d	d	d
	1	1	0	0	1	d	d	d	d	d	d
	1	1	0	1	1	d	d	d	d	d	d
	1	1	1	0	1	d	d	d	d	d	d
	1	1	1	1	1	d	d	d	d	d	d

x_1x_0 \ x_3x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

x_1 (columns 11, 10); x_2 (columns 01, 11); x_3 (rows 11, 10); x_4 (rows 01, 11)

x_1x_0 \ x_3x_2	00	01	11	10
00				
01				
11				
10				

Seven-Segment Indicator

x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	d	d	d	d	d	d
1	0	1	1	1	d	d	d	d	d	d
1	1	0	0	1	d	d	d	d	d	d
1	1	0	1	1	d	d	d	d	d	d
1	1	1	0	1	d	d	d	d	d	d
1	1	1	1	1	d	d	d	d	d	d

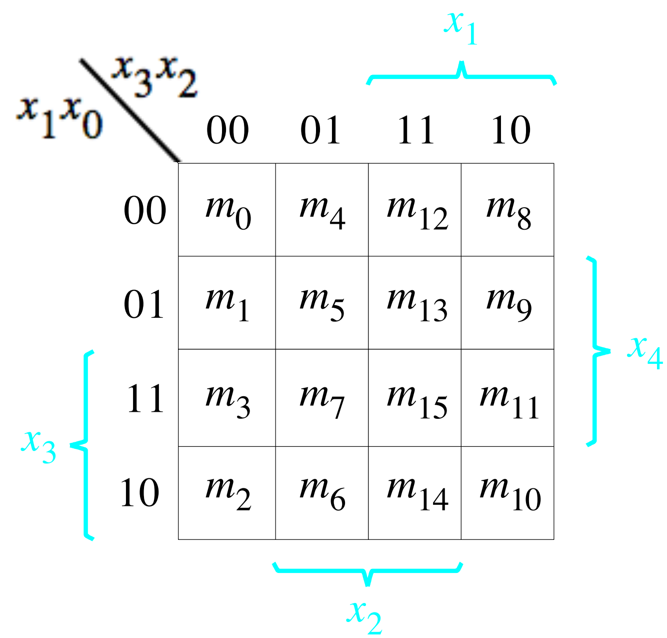
x_1x_0 \ x_3x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

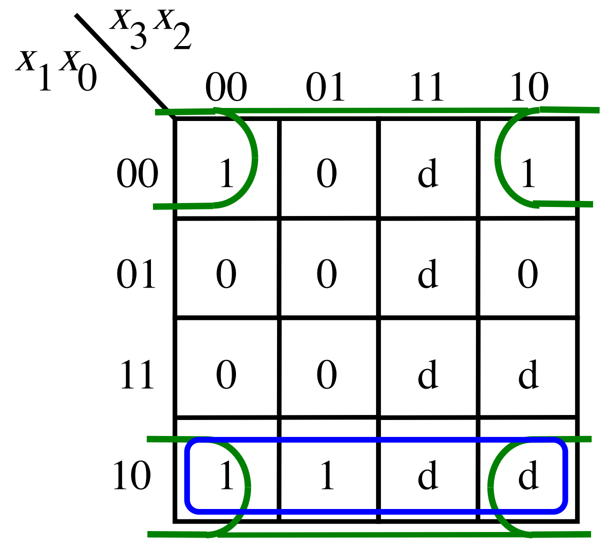
x_1 (columns 11, 10); x_2 (columns 01, 11); x_3 (rows 11, 10); x_4 (rows 01, 11)

x_1x_0 \ x_3x_2	00	01	11	10
00	1	0	d	1
01	0	0	d	0
11	0	0	d	d
10	1	1	d	d

Seven-Segment Indicator

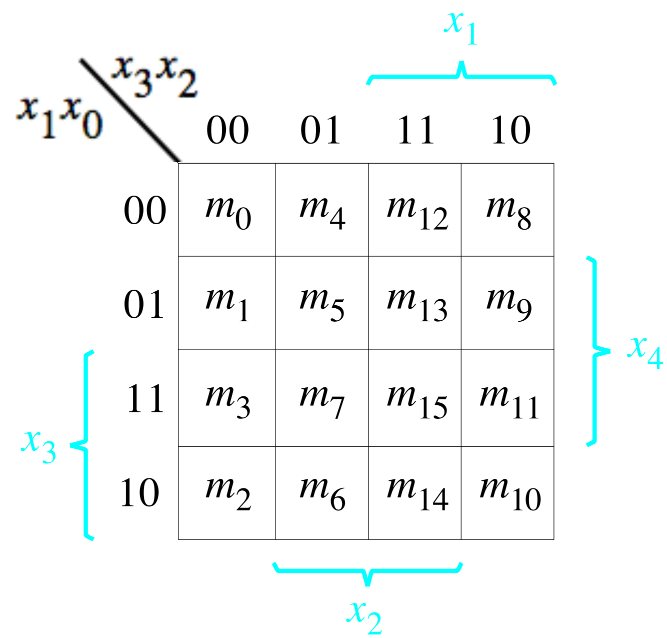
x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	d	d	d	d	d	d
1	0	1	1	1	d	d	d	d	d	d
1	1	0	0	1	d	d	d	d	d	d
1	1	0	1	1	d	d	d	d	d	d
1	1	1	0	1	d	d	d	d	d	d
1	1	1	1	1	d	d	d	d	d	d

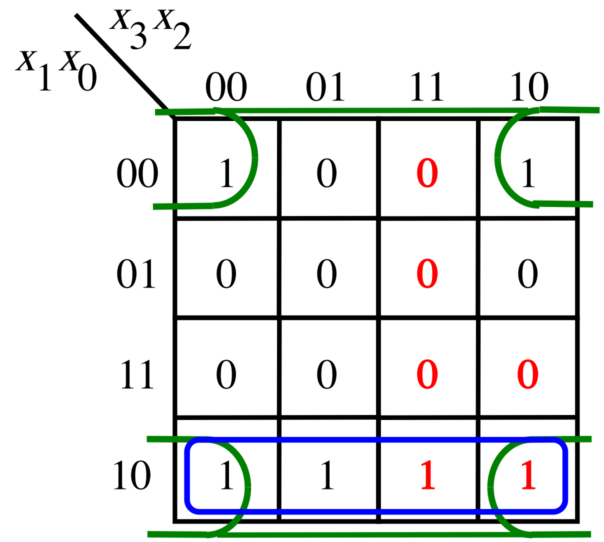




Seven-Segment Indicator

x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	d	d	d	1	d	d
1	0	1	1	1	d	d	d	0	d	d
1	1	0	0	1	d	d	d	0	d	d
1	1	0	1	1	d	d	d	0	d	d
1	1	1	0	1	d	d	d	1	d	d
1	1	1	1	1	d	d	d	0	d	d



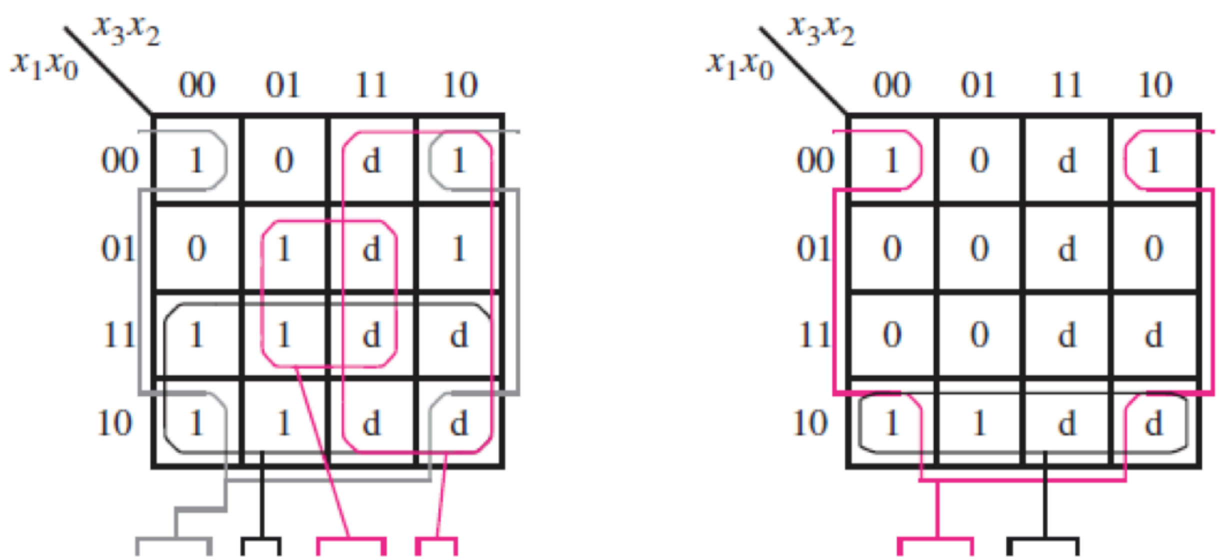


In this case some d's were treated as 1's, others as 0's.

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1

(b) Truth table



$a = \bar{x}_2\bar{x}_0 + x_1 + x_2x_0 + x_3$

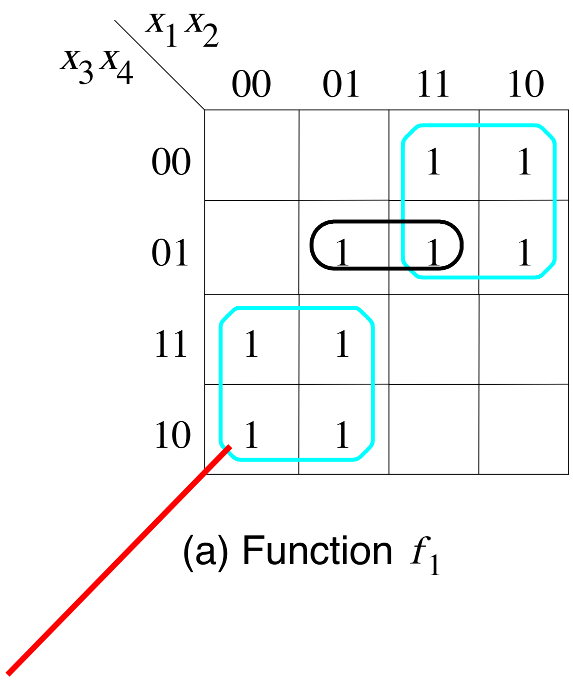
$e = \bar{x}_2\bar{x}_0 + x_1\bar{x}_0$

(c) The Karnaugh maps for outputs a and e.

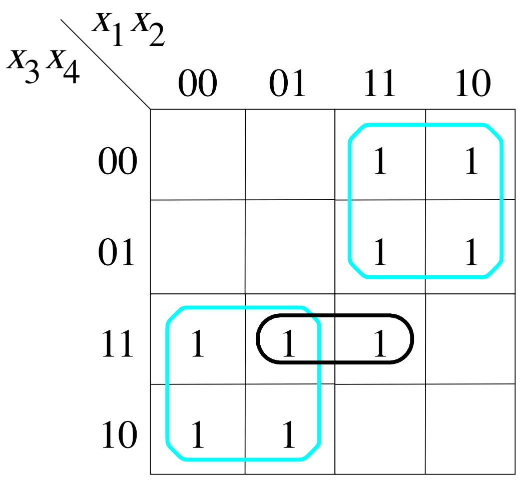
Another Example

(a) Function f_1 (b) Function f_2

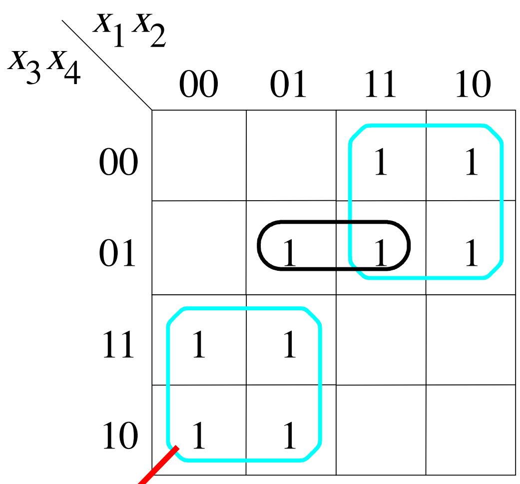
[Figure 2.64 from the textbook]



(a) Function f_1

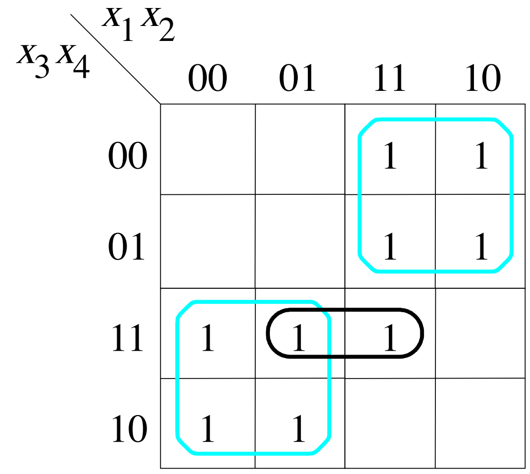


(b) Function f_2

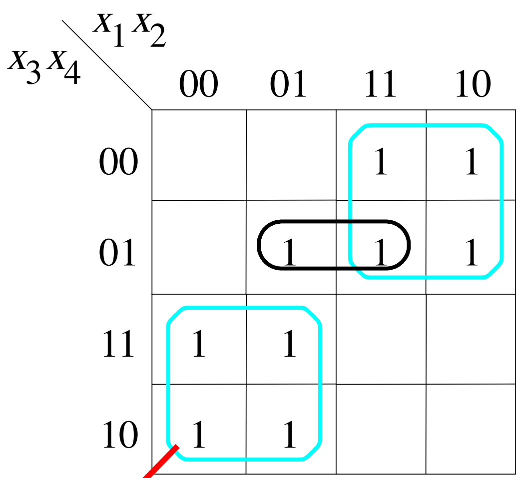


(a) Function f_1

$\bar{x}_1 x_3$

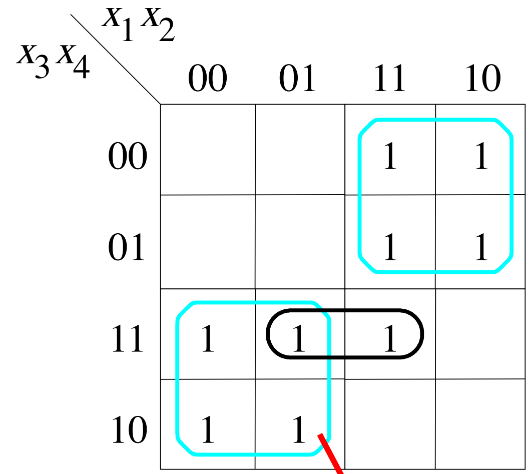


(b) Function f_2

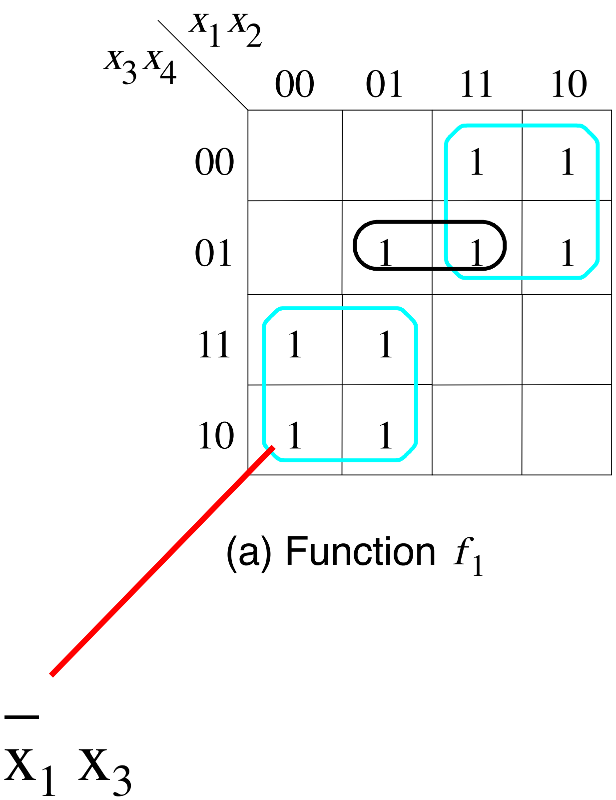


(a) Function f_1

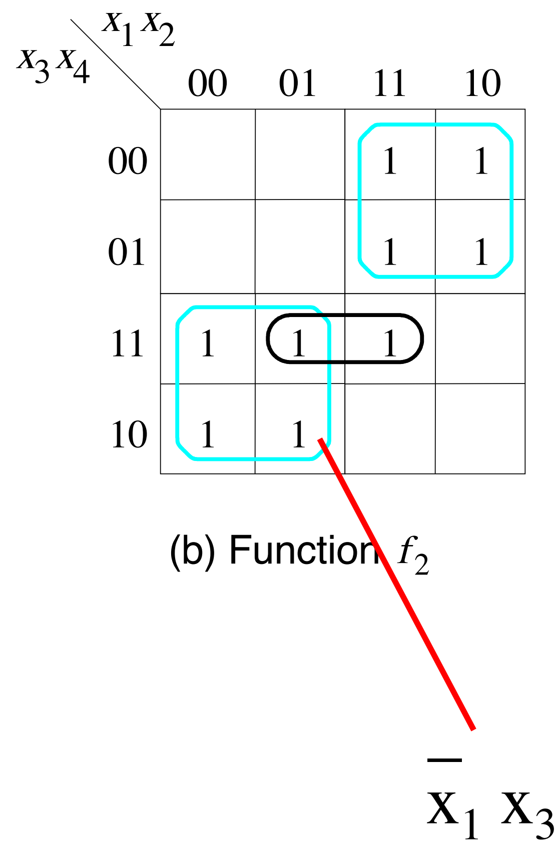
$\bar{x}_1 \; x_3$



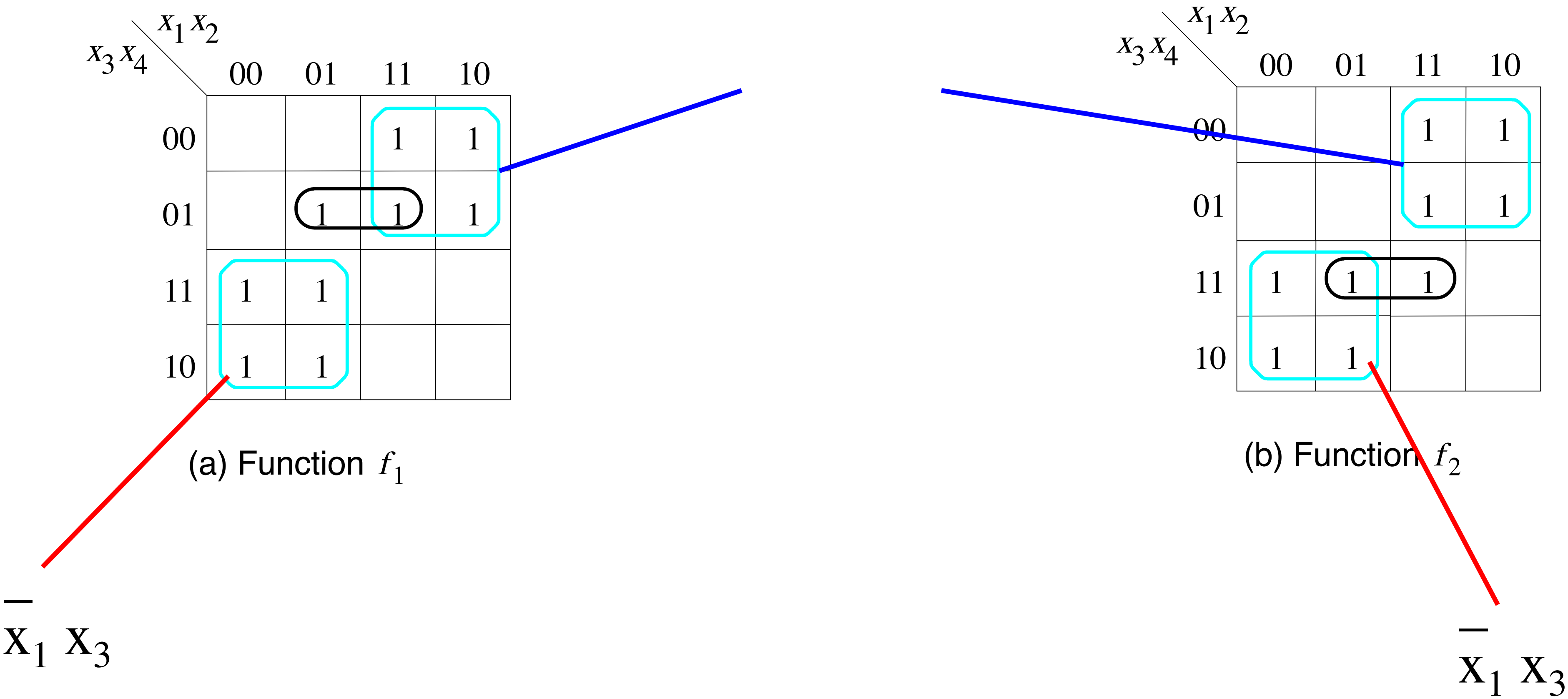
(b) Function f_2



(a) Function f_1

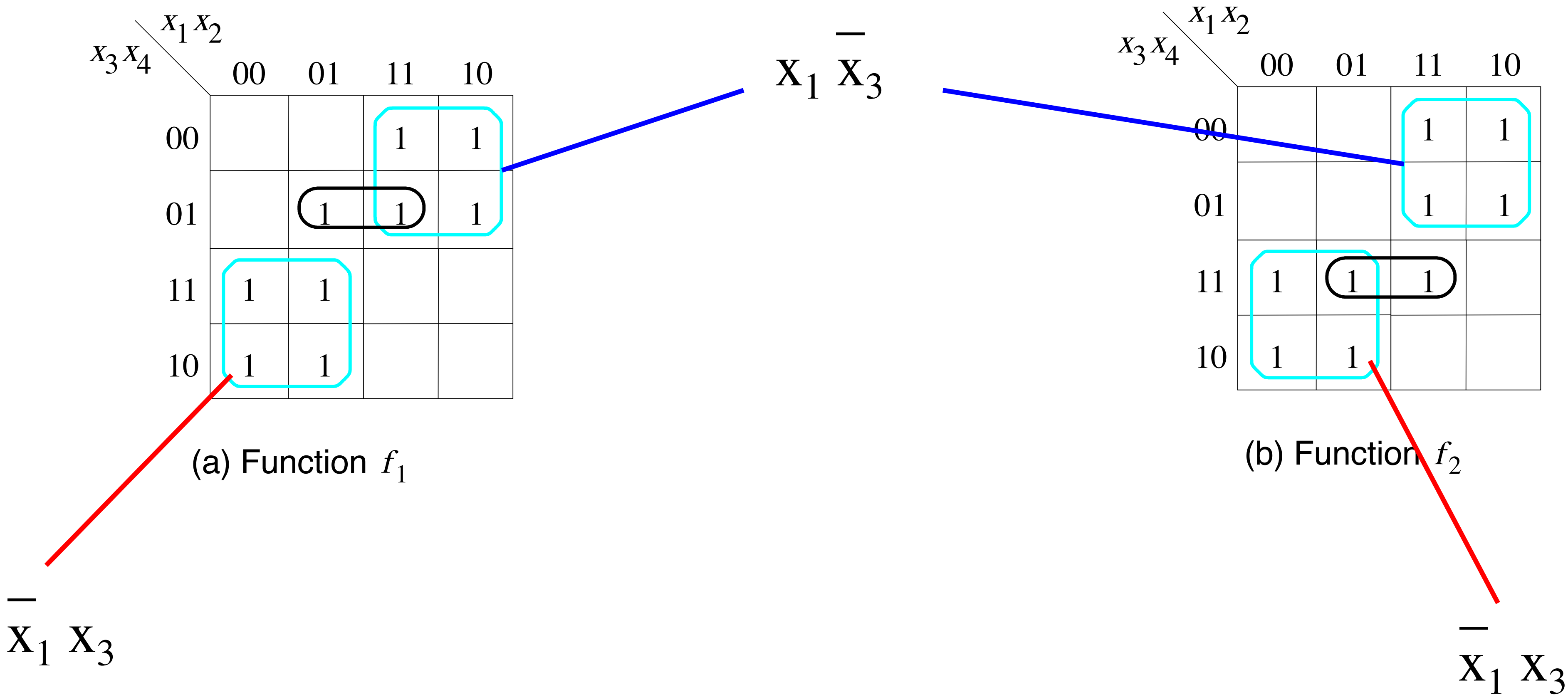


(b) Function f_2



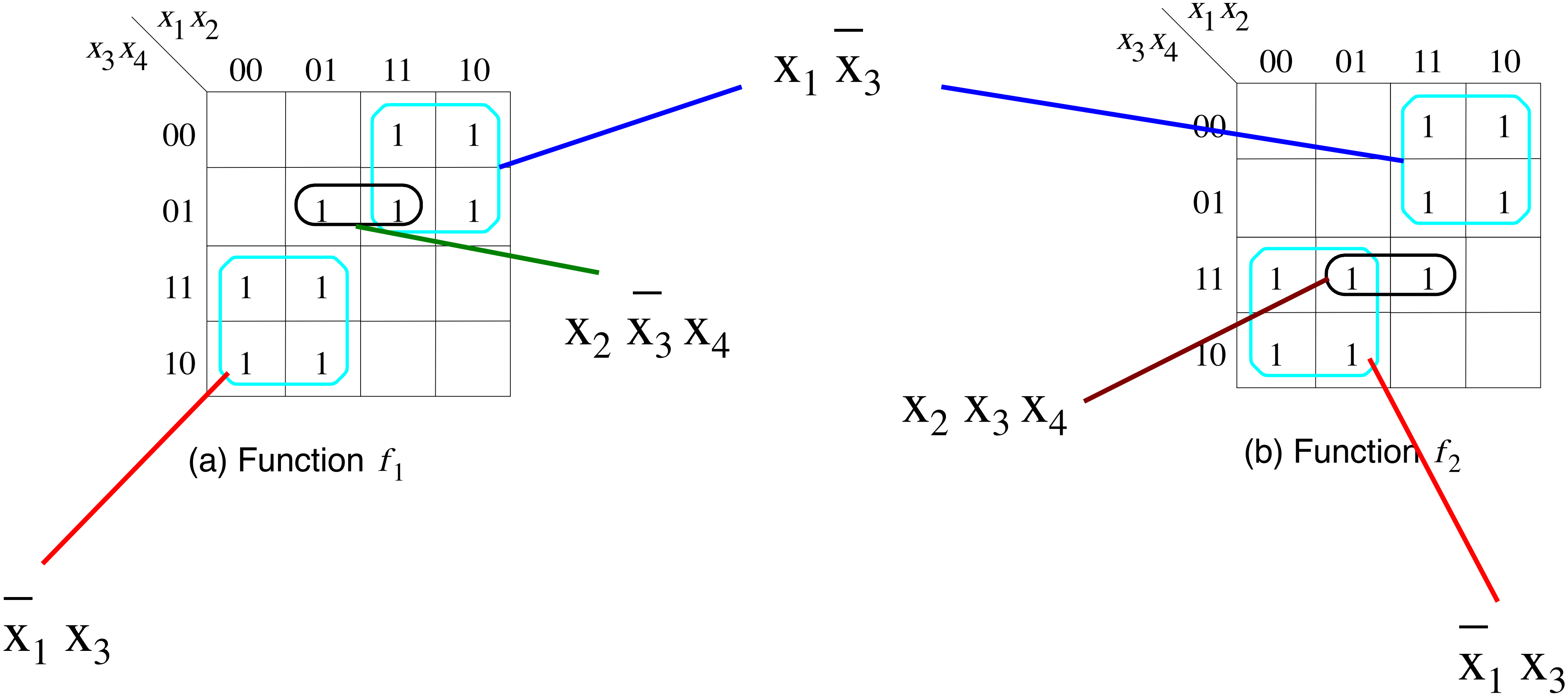
(a) Function f_1

(b) Function f_2



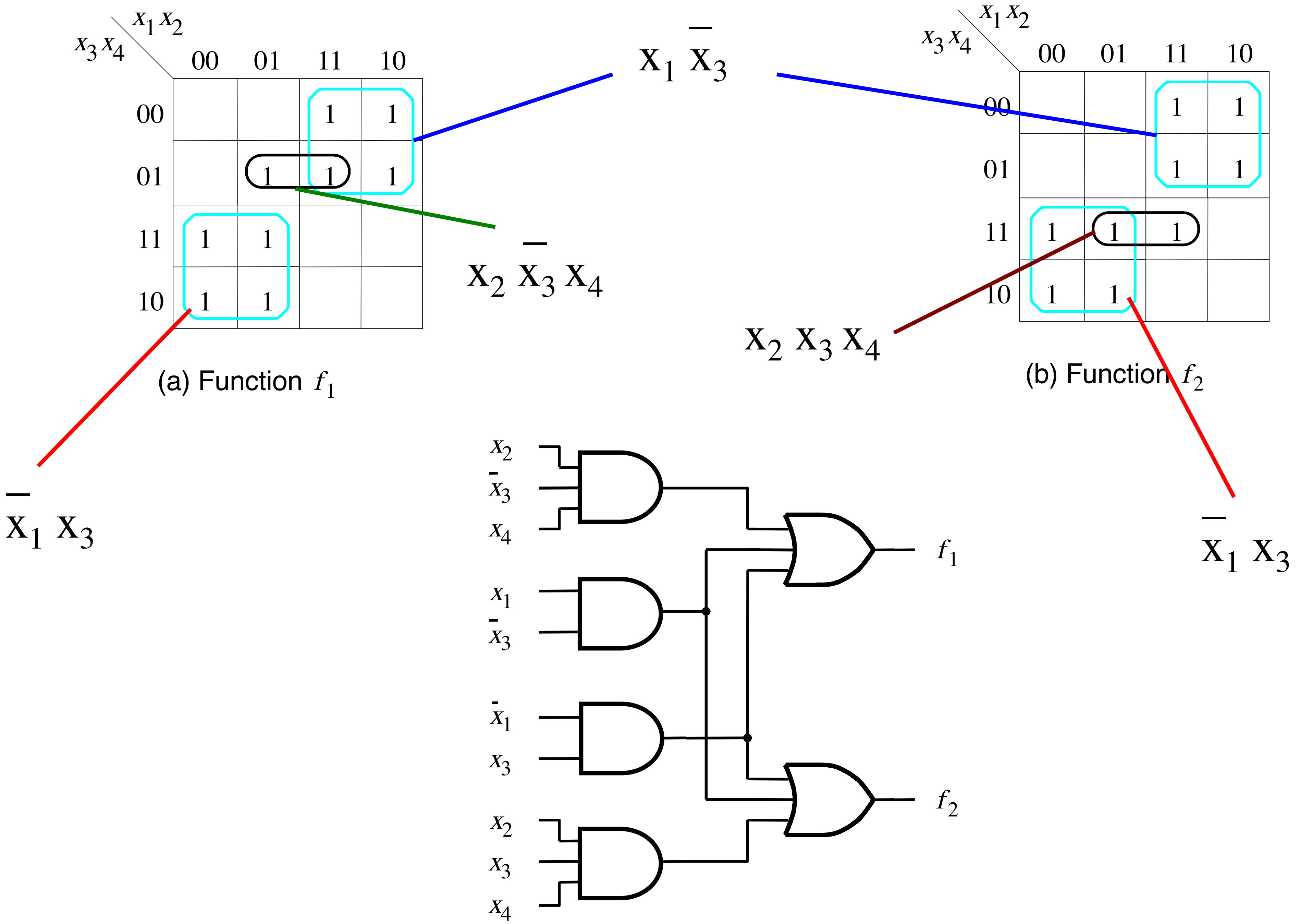
(a) Function f_1

(b) Function f_2



(a) Function f_1

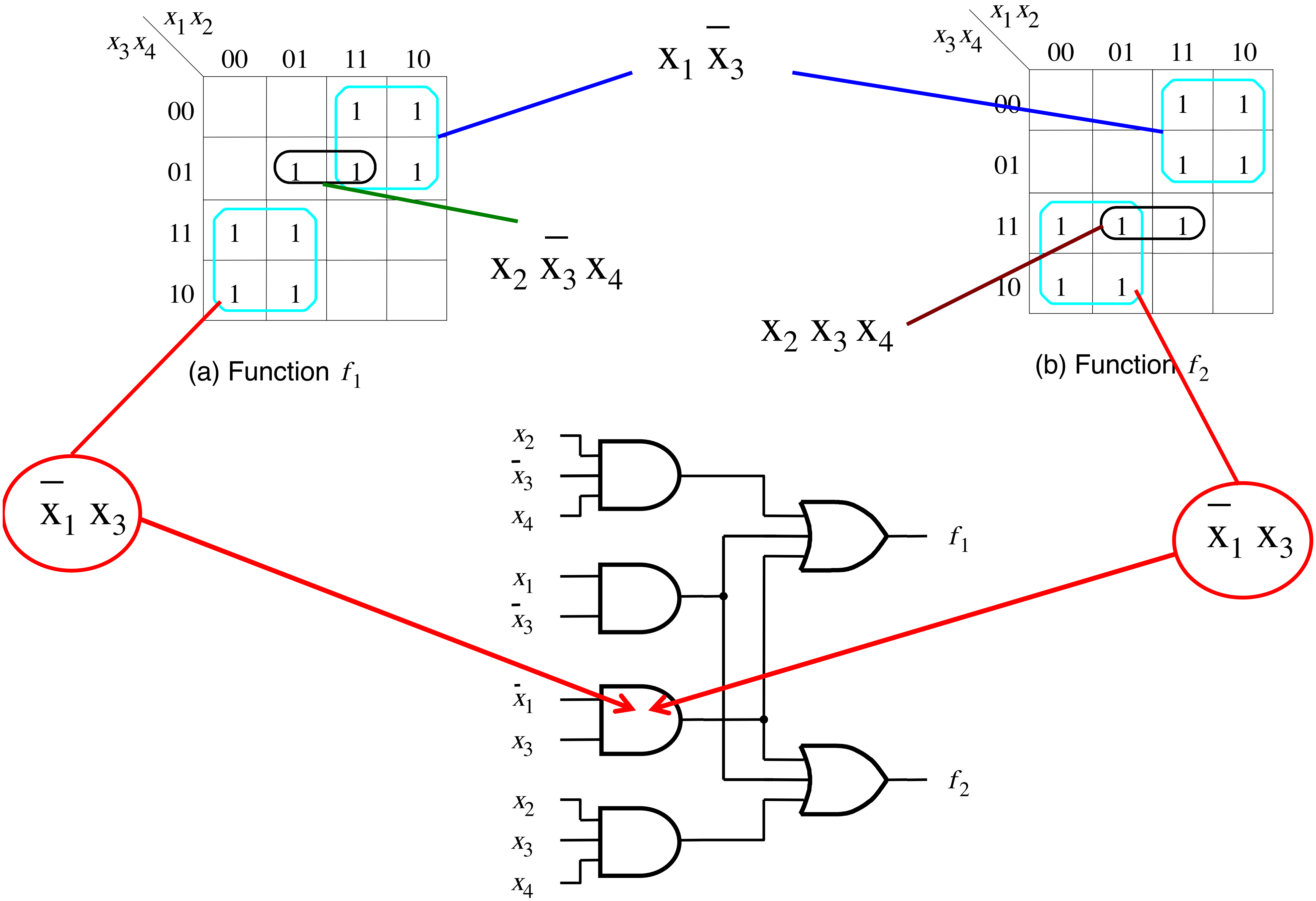
(b) Function f_2



(a) Function f_1

(b) Function f_2

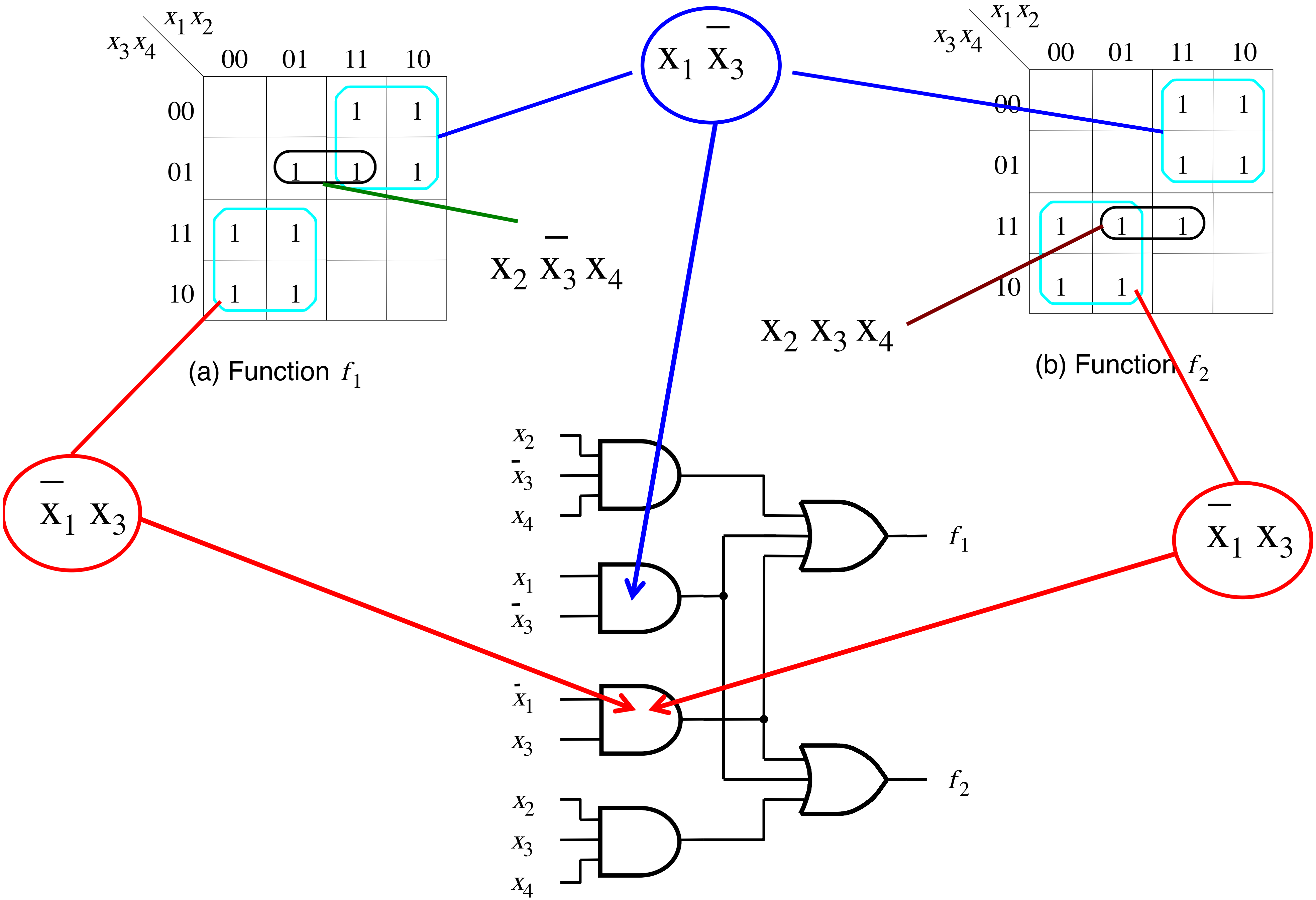
(c) Combined circuit for f_1 and f_2



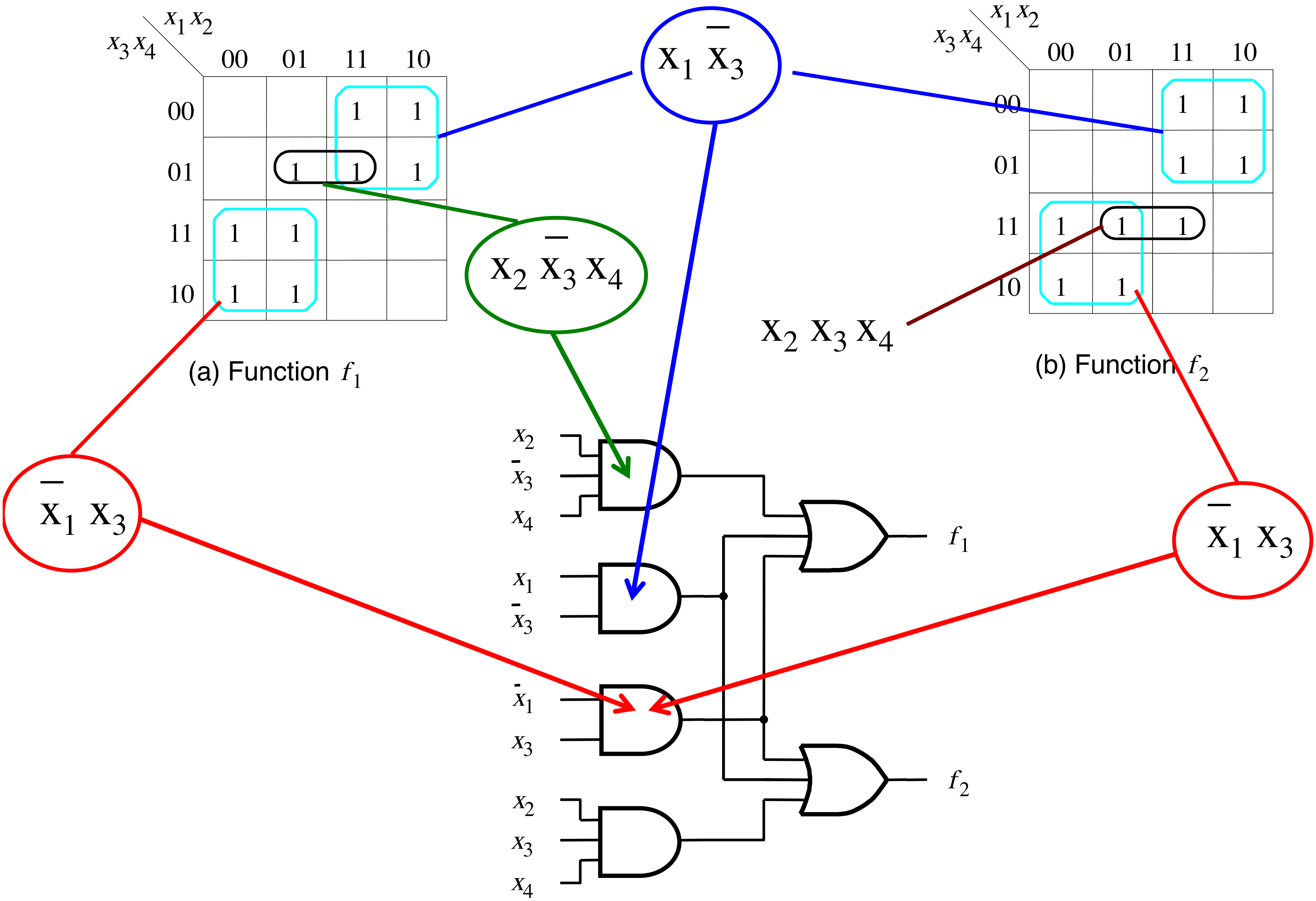
(a) Function f_1

(b) Function f_2

(c) Combined circuit for f_1 and f_2



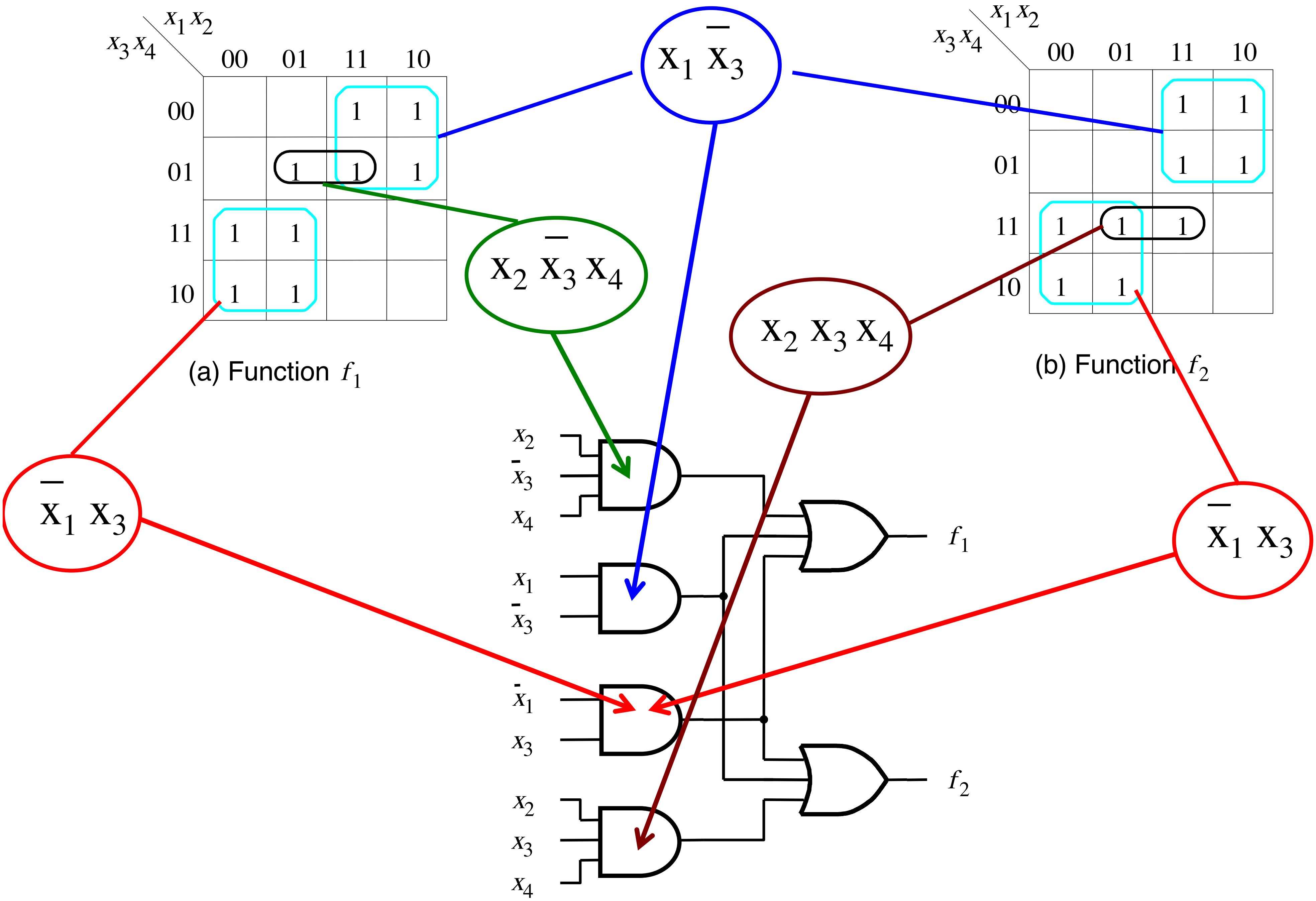
(c) Combined circuit for f_1 and f_2



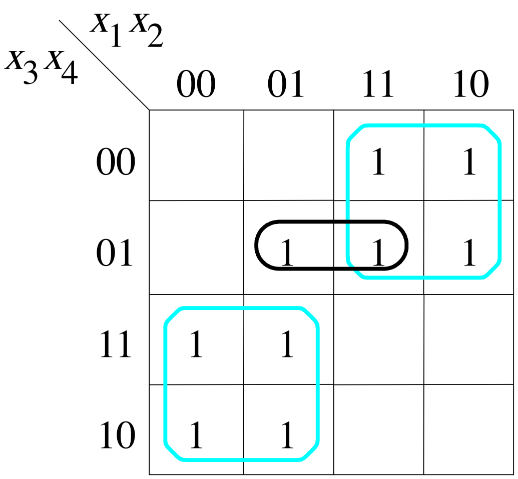
(a) Function f_1

(b) Function f_2

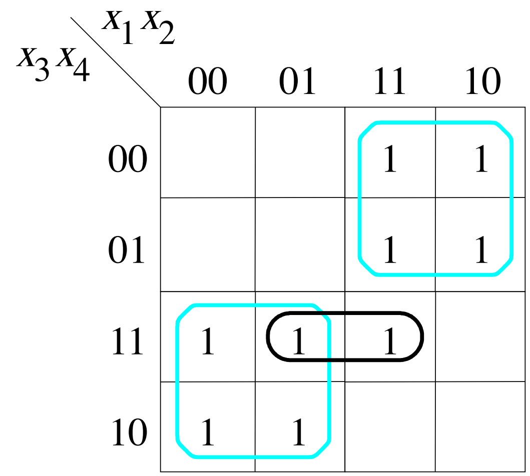
(c) Combined circuit for f_1 and f_2



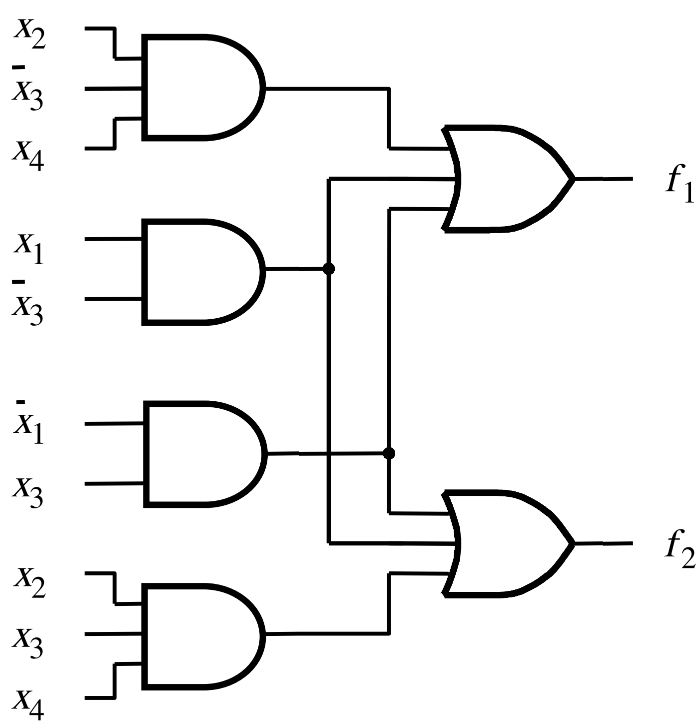
(c) Combined circuit for f_1 and f_2



(a) Function f_1



(b) Function f_2

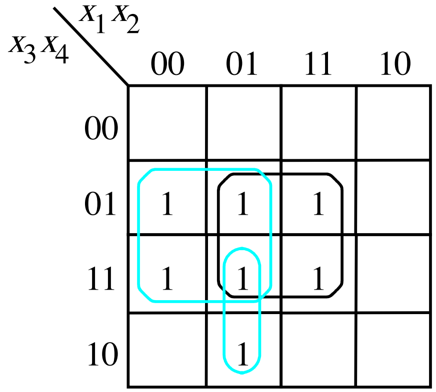


(c) Combined circuit for f_1 and f_2

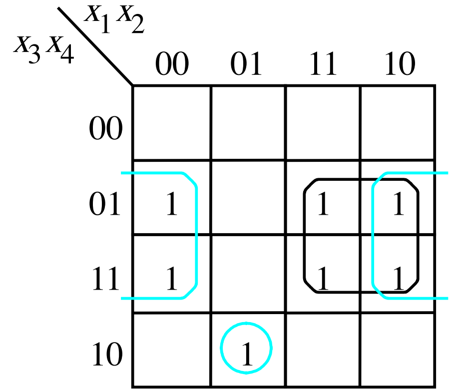
[Figure 2.64 from the textbook]

Yet Another Example

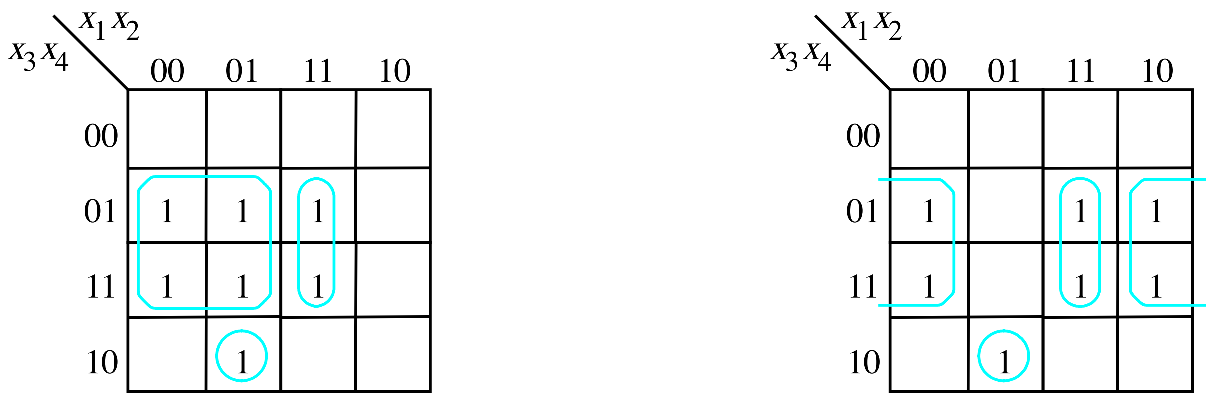
Individual vs Joint Optimization



(a) Optimal realization of f_3



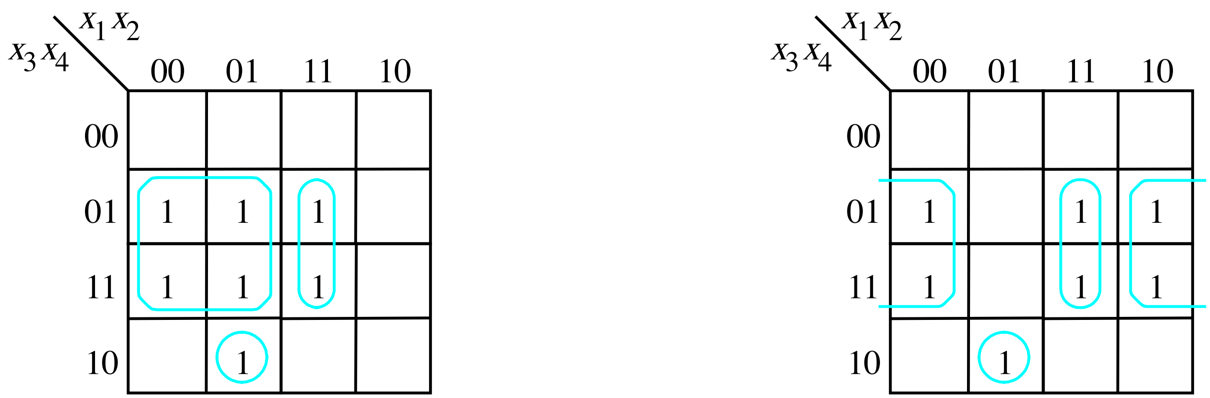
(b) Optimal realization of f_4



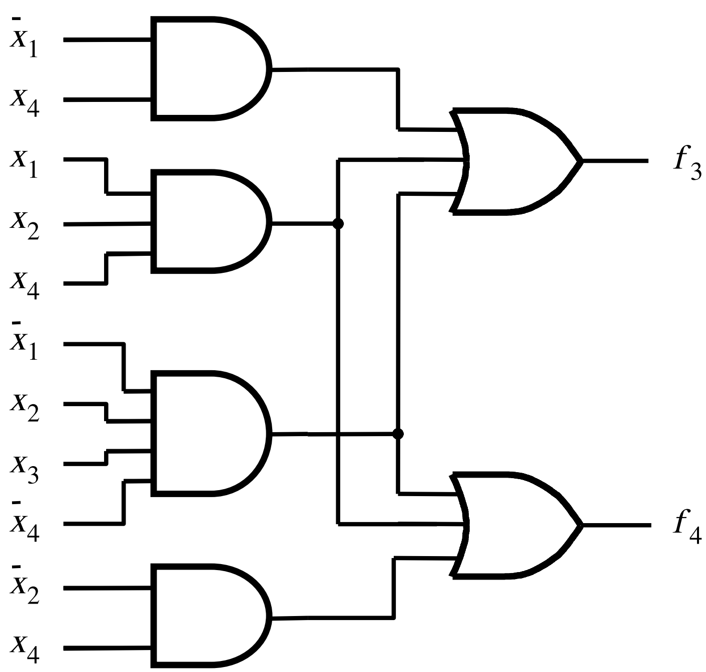
(c) Optimal realization of f_3 and f_4 together

[Figure 2.65 from the textbook]

Individual vs Joint Optimization



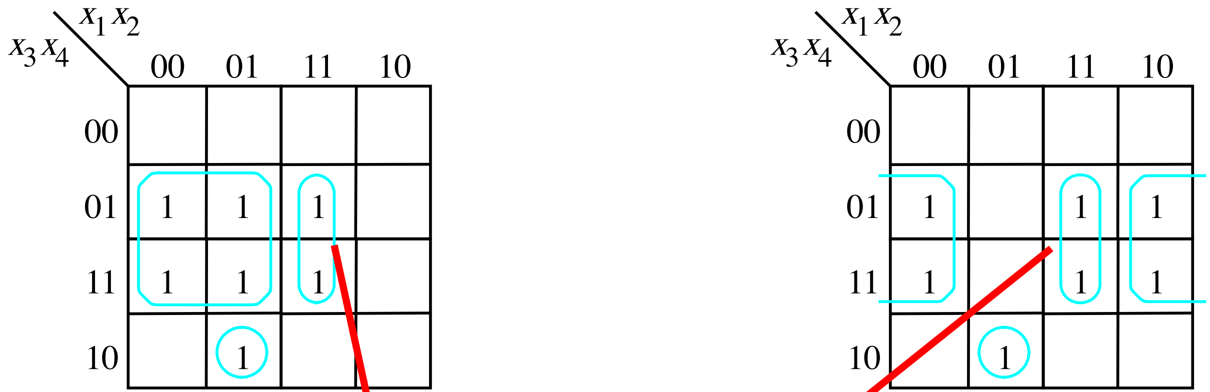
(c) Optimal realization of f_3 and f_4 together



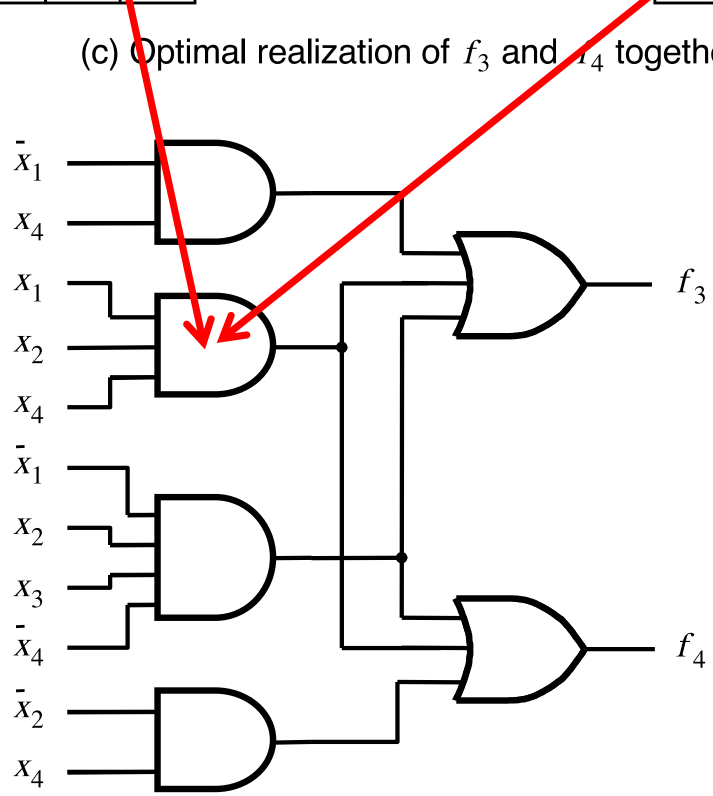
(d) Combined circuit for f_3 and f_4

[Figure 2.65 from the textbook]

Individual vs Joint Optimization



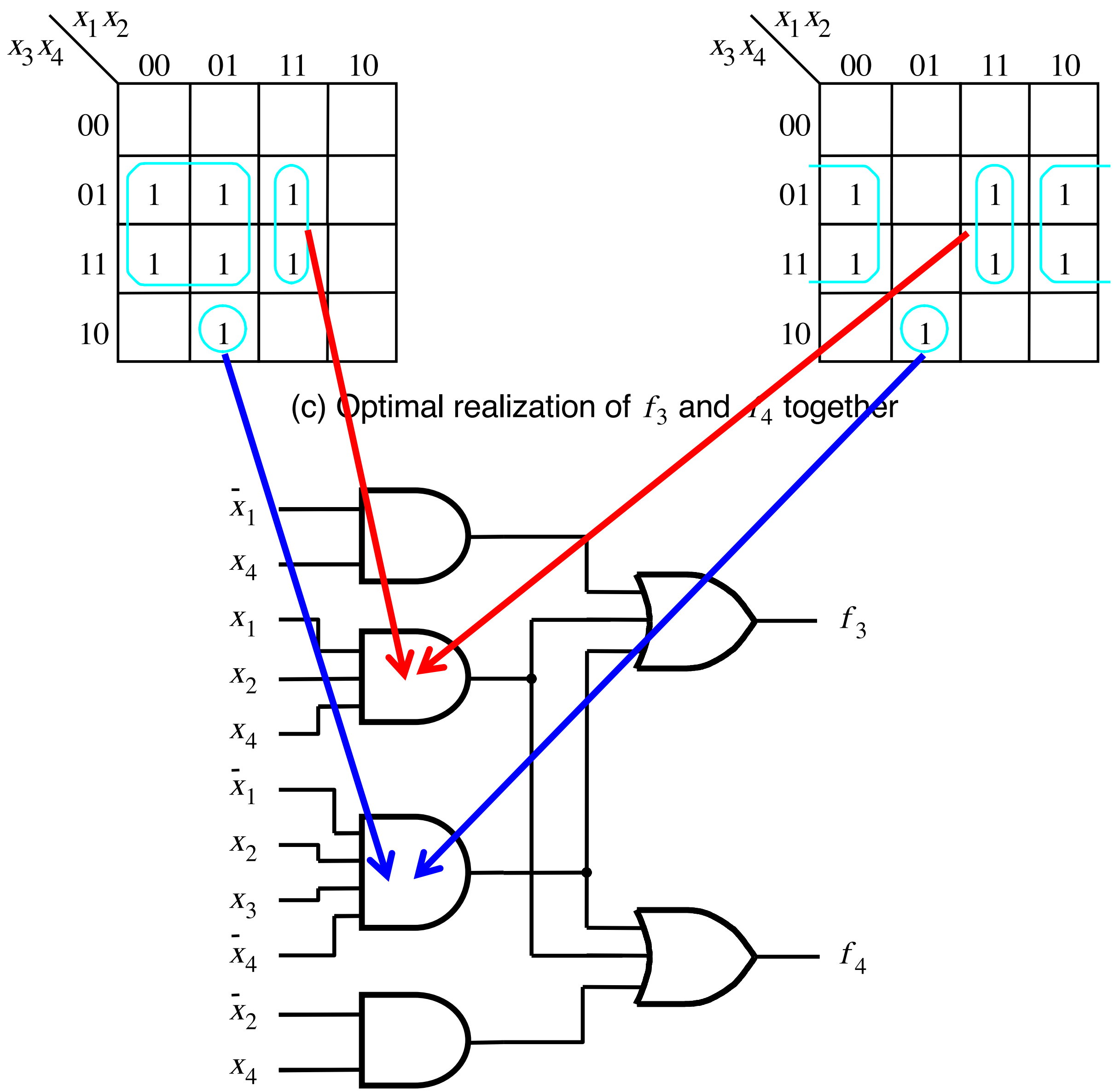
(c) Optimal realization of f_3 and f_4 together



(d) Combined circuit for f_3 and f_4

[Figure 2.65 from the textbook]

Individual vs Joint Optimization



(c) Optimal realization of f_3 and f_4 together

(d) Combined circuit for f_3 and f_4

[Figure 2.65 from the textbook]

Questions?

THE END