



CprE 281: Digital Logic

Instructor: Alexander Stoytchev

<http://www.ece.iastate.edu/~alexs/classes/>

Review for the Final Exam

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Iowa State University, Ames, IA
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Administrative Stuff

- **The FINAL exam is scheduled for**
- **Monday Dec 12 @ 2:15 – 4:15 PM**
- **It will be in this room.**

Final Exam Format

- **The exam will cover: Chapter 1 to Chapter 6, and Sections 7.1-7.2**
- **Emphasis will be on Chapter 5, 6, and 7**
- **The exam will be open book and open notes (you can bring up to 5 pages of handwritten/typed notes) plus your textbook.**

Final Exam Format

- The exam will be out of 130 points
- You need 95 points to get an A
- It will be great if you can score more than 100 points.
 - but you can't roll over your extra points 😞

Topics for the Final Exam

- **K-maps for 2, 3, and 4 variables**
- **Multiplexers (circuits and function)**
- **Synthesis of logic functions using multiplexers**
- **Shannon's Expansion Theorem**
- **1's complement and 2's complement representation**
- **Addition and subtraction of binary numbers**
- **Circuits for adding and subtracting**
- **Serial adder**
- **Latches (circuits, behavior, timing diagrams)**
- **Flip-Flops (circuits, behavior, timing diagrams)**
- **Counters (up, down, synchronous, asynchronous)**
- **Registers and Register Files**

Topics for the Final Exam

- **Synchronous Sequential Circuits**
- **FSMs**
- **Moore Machines**
- **Mealy Machines**
- **State diagrams, state tables, state-assigned tables**
- **State minimization**
- **Designing a counter**
- **Arbiter Circuits**
- **Reverse engineering a circuit**
- **ASM Charts**
- **Register Machines**
- **Bus structure and Simple Processors**
- **Something from Star Wars**

How to Study for the Final Exam

- **Form a study group**
- **Go over the slides for this class**
- **Go over the homeworks again**
- **Go over the problems at the end of Ch 5 & 6**
- **Exercise**
- **Get some sleep**

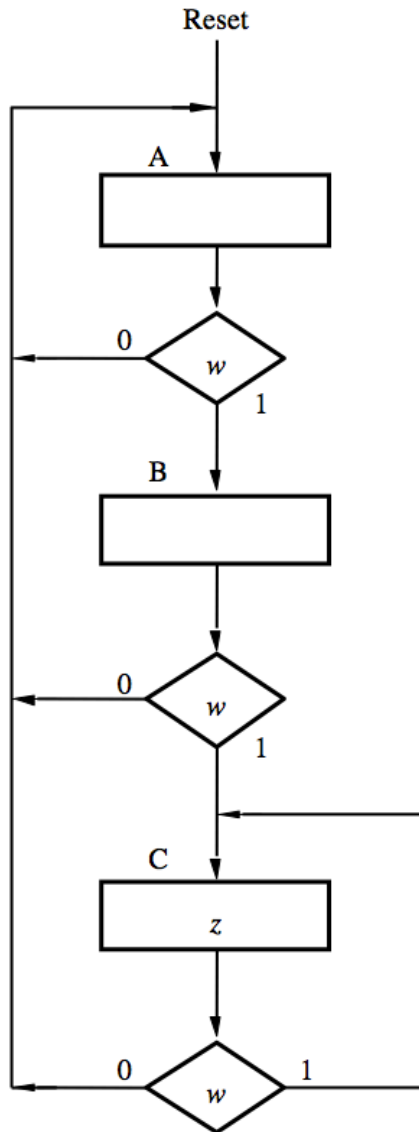
Administrative Stuff

- **Please check your grades on BlackBoard**
- **Let me know if something is wrong or missing**

Sample Problems

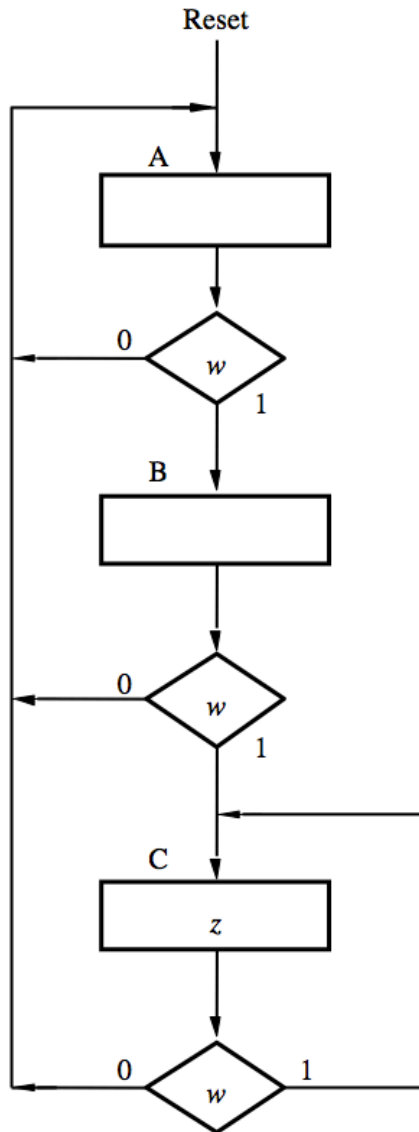
ASM Charts

Given an ASM chart draw the corresponding FSM

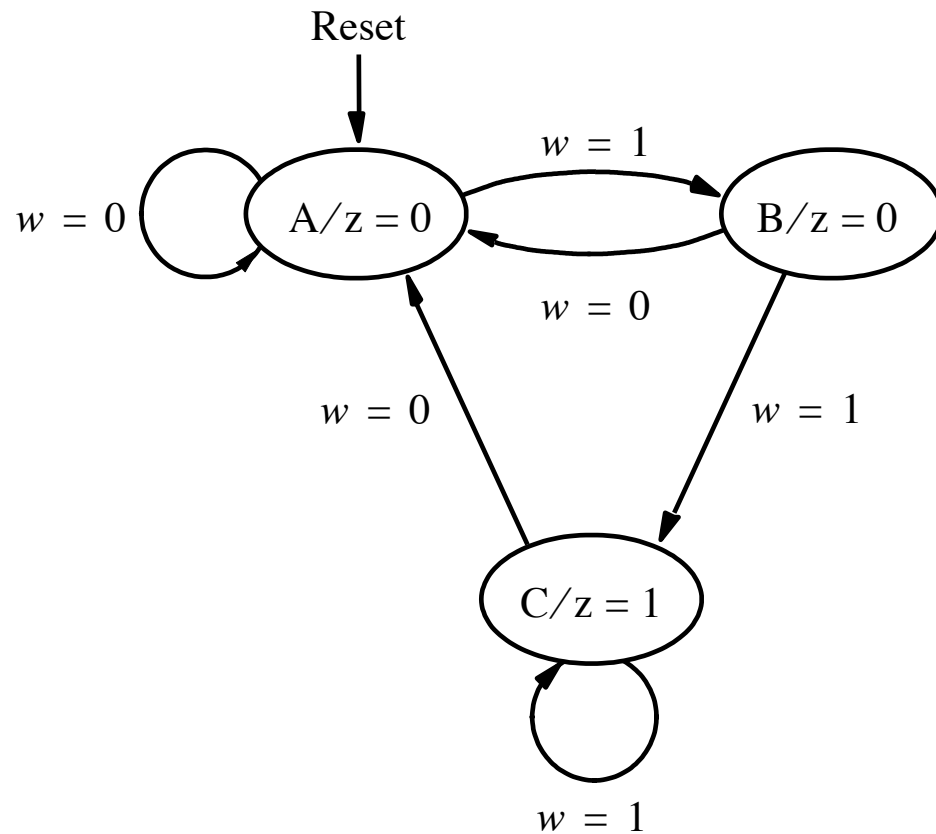


ASM Charts

Given an ASM chart draw the corresponding FSM



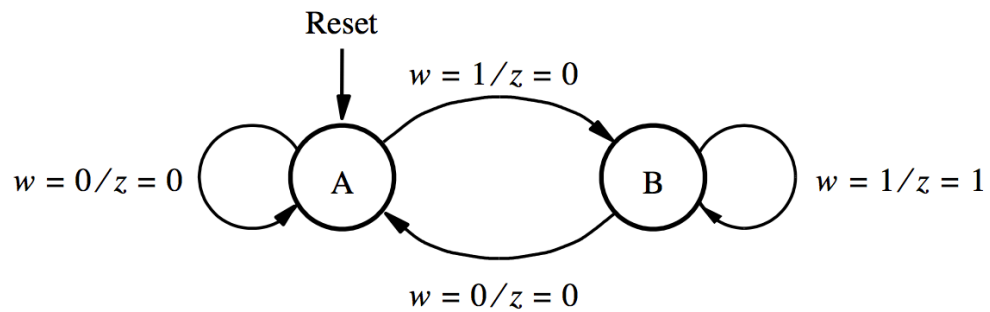
[Figure 6.82 from the textbook]



[Figure 6.3 from the textbook]

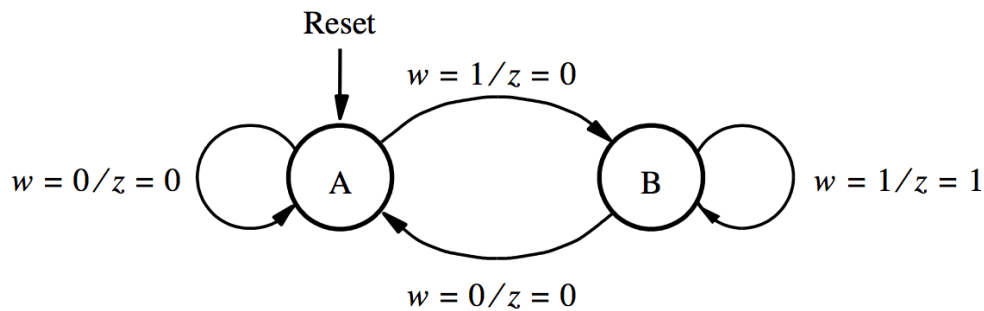
ASM Charts

Given an FSM draw the corresponding ASM Chart

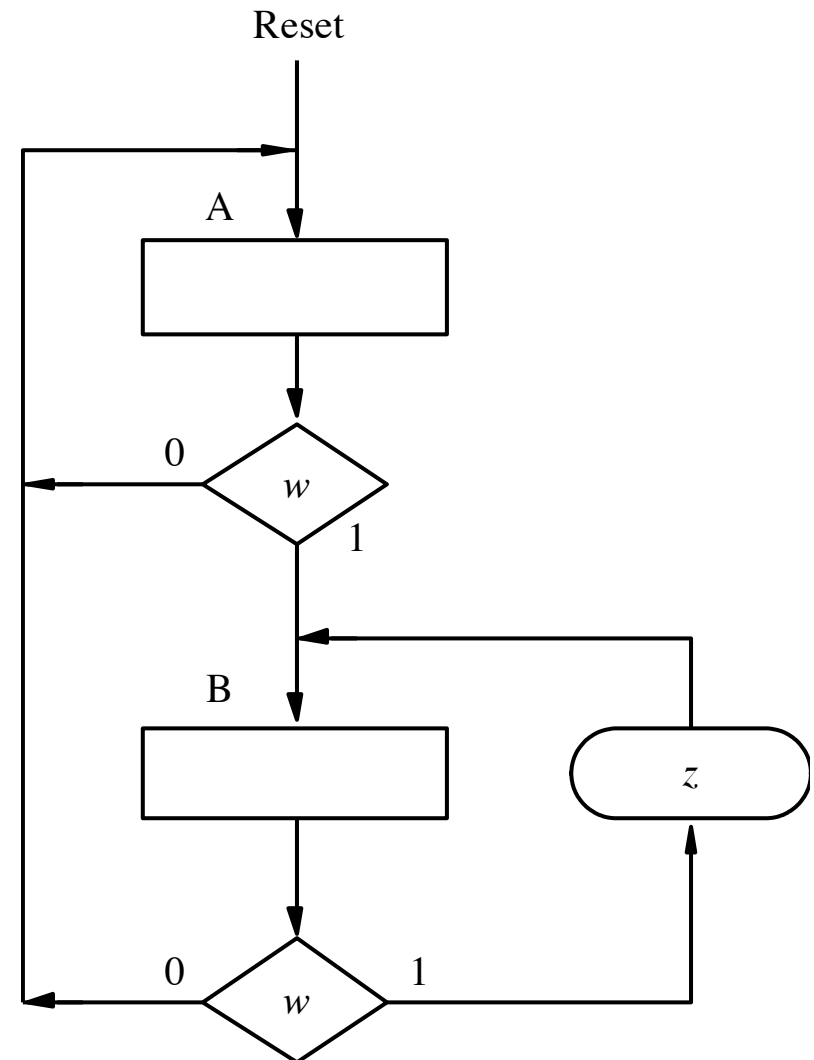


ASM Charts

Given an FSM draw the corresponding ASM Chart



[Figure 6.23 from the textbook]



[Figure 6.83 from the textbook]

Circuit Implementation of FSMs

Implement this state-assigned Table using JK flip-flops

	Present state $y_3y_2y_1$	Next state		Output z
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	100	110	0
B	100	101	110	0
C	101	101	110	1
D	110	100	111	0
E	111	100	111	1

Circuit Implementation of FSMs

Implement this state-assigned Table using JK flip-flops

$$J_1 = wy_2 + \bar{w}y_3\bar{y}_2$$

$$K_1 = \bar{w}y_2 + wy_1\bar{y}_2$$

$$J_2 = w$$

$$K_2 = \bar{w}$$

$$J_3 = 1$$

$$K_3 = 0$$

$$z = y_1$$

	Present state $y_3y_2y_1$	Next state		Output z
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	100	110	0
B	100	101	110	0
C	101	101	110	1
D	110	100	111	0
E	111	100	111	1

Circuit Implementation of FSMs

Implement this state-assigned Table using JK flip-flops

	Present state $y_3y_2y_1$	Flip-flop inputs							Output z	
		$w = 0$			$w = 1$					
		$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	$Y_3Y_2Y_1$	J_3K_3	J_2K_2		J_1K_1
A	000	100	$1d$	$0d$	$0d$	110	$1d$	$1d$	$0d$	0
B	100	101	$d0$	$0d$	$1d$	110	$d0$	$1d$	$0d$	0
C	101	101	$d0$	$0d$	$d0$	110	$d0$	$1d$	$d1$	1
D	110	100	$d0$	$d1$	$0d$	111	$d0$	$d0$	$1d$	0
E	111	100	$d0$	$d1$	$d1$	111	$d0$	$d0$	$d0$	1

Excitation table with JK flip-flops

Register Machines:

What does this program do?

How many balls are left in each register at the end of the program?



STEP	INSTRUCTION	REGISTER	GO TO STEP	[BRANCH TO STEP]
1.	Deb	3	1	2
2.	Deb	2	3	4
3.	Inc	3	2	
4.	End			

Register Machines:

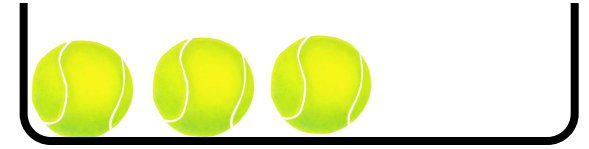
Move the contents of register 2 to register 3



Register 1



Register 2



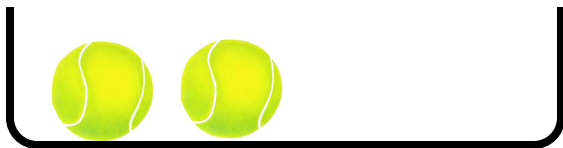
Register 3

STEP	INSTRUCTION	REGISTER	GO TO STEP	[BRANCH TO STEP]
1.	Deb	3	1	2
2.	Deb	2	3	4
3.	Inc	3	2	
4.	End			

Register Machines:

What does this program do?

How many balls are left in each register at the end of the program?



Register 1



Register 2

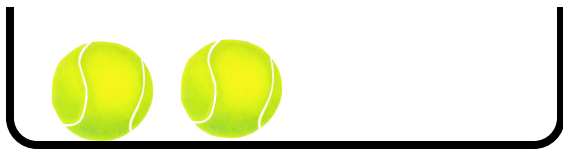


Register 3

STEP	INSTRUCTION	REGISTER	GO TO STEP	[BRANCH TO STEP]
1.	Deb	3	1	2
2.	Deb	2	2	3
3.	Deb	1	4	6
4.	Inc	3	5	
5.	Inc	2	3	
6.	Deb	2	7	8
7.	Inc	1	6	
8.	End			

Register Machines:

Copy the contents of register 1 to register 3
using register 2 as a temporary storage



Register 1



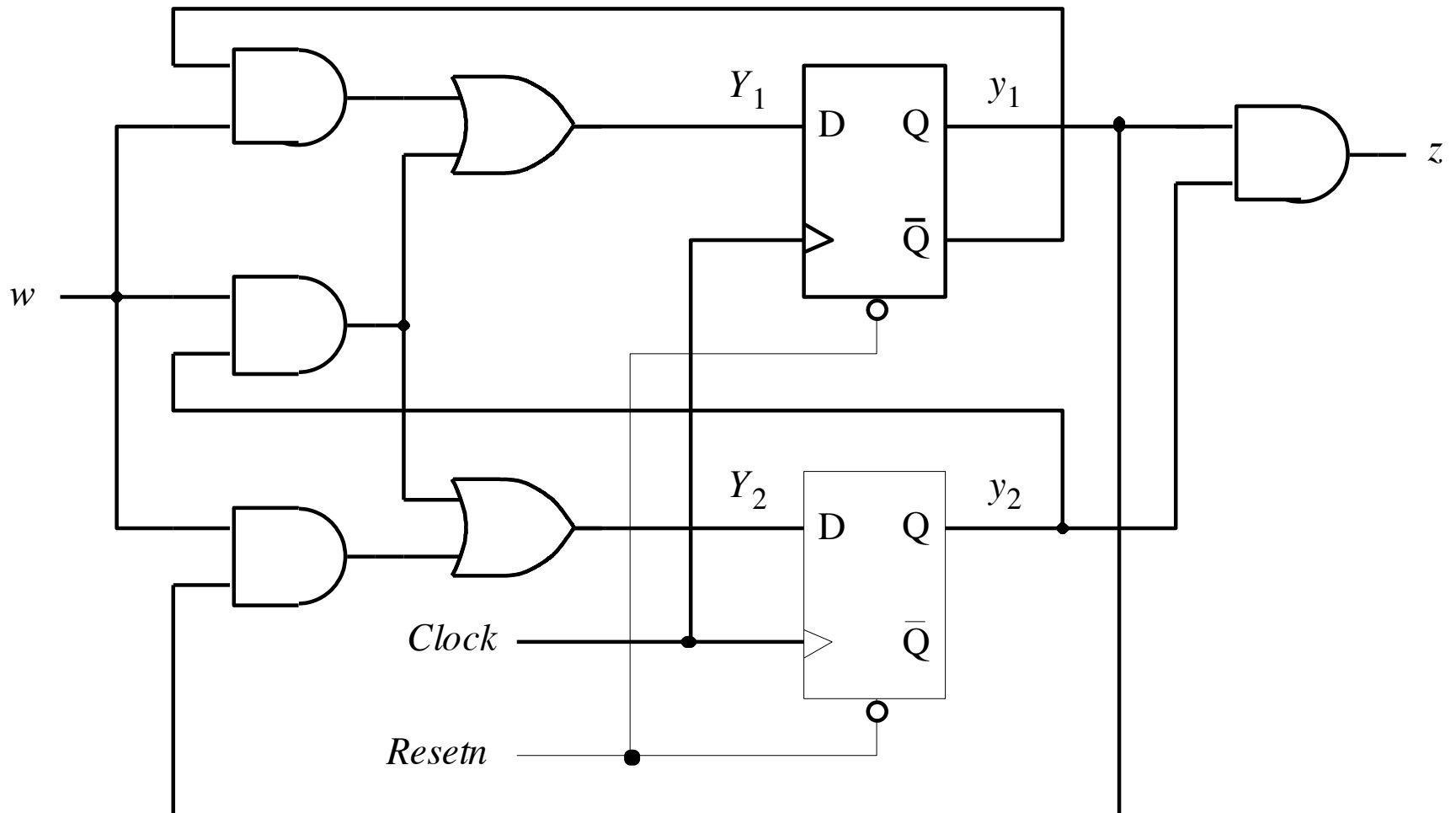
Register 2



Register 3

STEP	INSTRUCTION	REGISTER	GO TO STEP	[BRANCH TO STEP]
1.	Deb	3	1	2
2.	Deb	2	2	3
3.	Deb	1	4	6
4.	Inc	3	5	
5.	Inc	2	3	
6.	Deb	2	7	8
7.	Inc	1	6	
8.	End			

What does this circuit do?



[Figure 6.75 from the textbook]

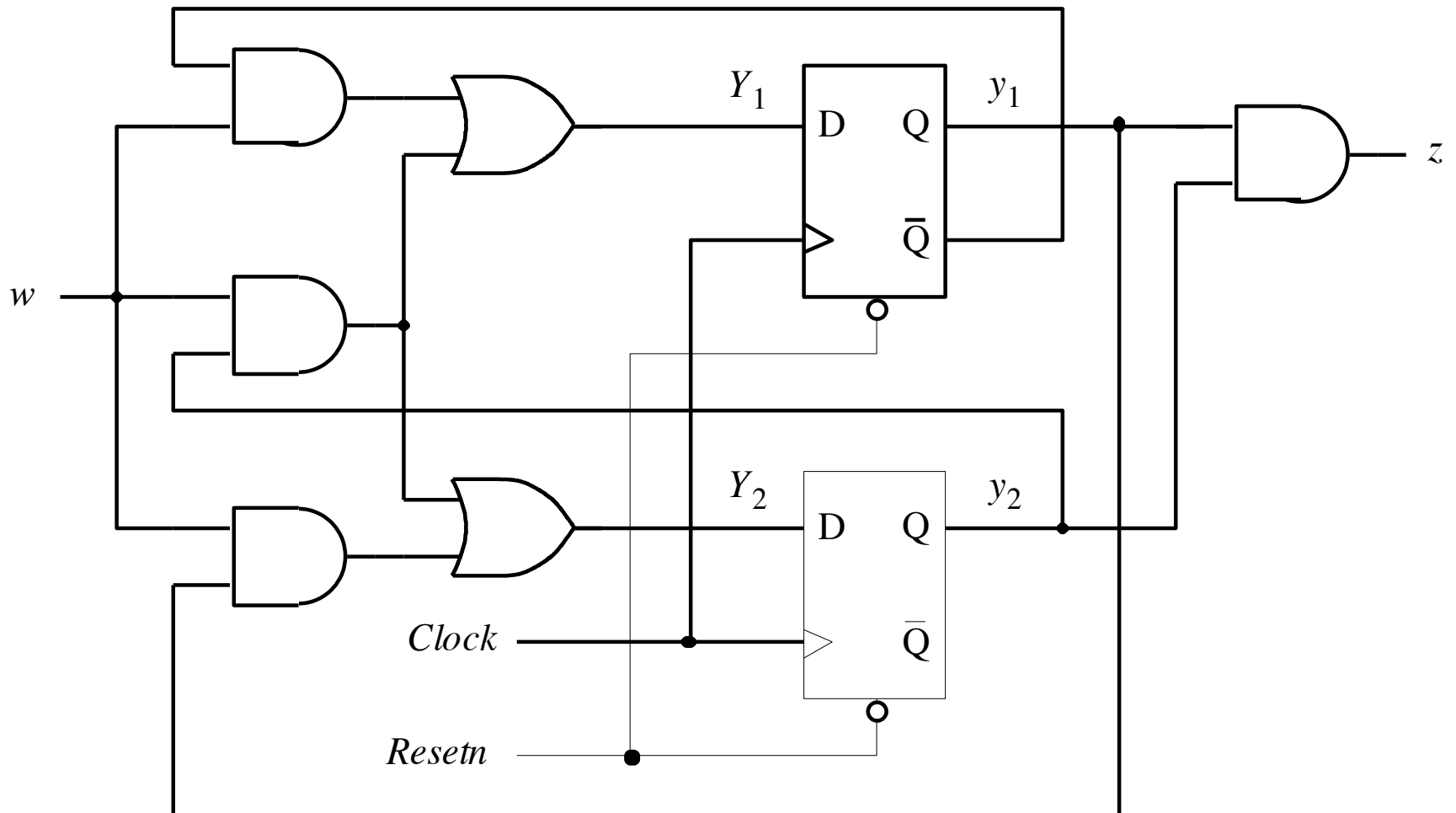
Approach

- **Find the flip-flops**
- **Outputs of the flip-flops = present state variables**
- **Inputs of the flip-flops determine the next state variables**
- **Determine the logical expressions for the outputs**
- **Given this info it is easy to do the state-assigned table**
- **Next do the state table**
- **Finally, draw the state diagram.**

Goal

- **Given a circuit diagram for a synchronous sequential circuit, the goal is to figure out the FSM**
- **Figure out the present state variables, the next state variables, the state-assigned table, the state table, and finally the state diagram.**
- **In other words, the goal is to reverse engineer the circuit.**

What does this circuit do?

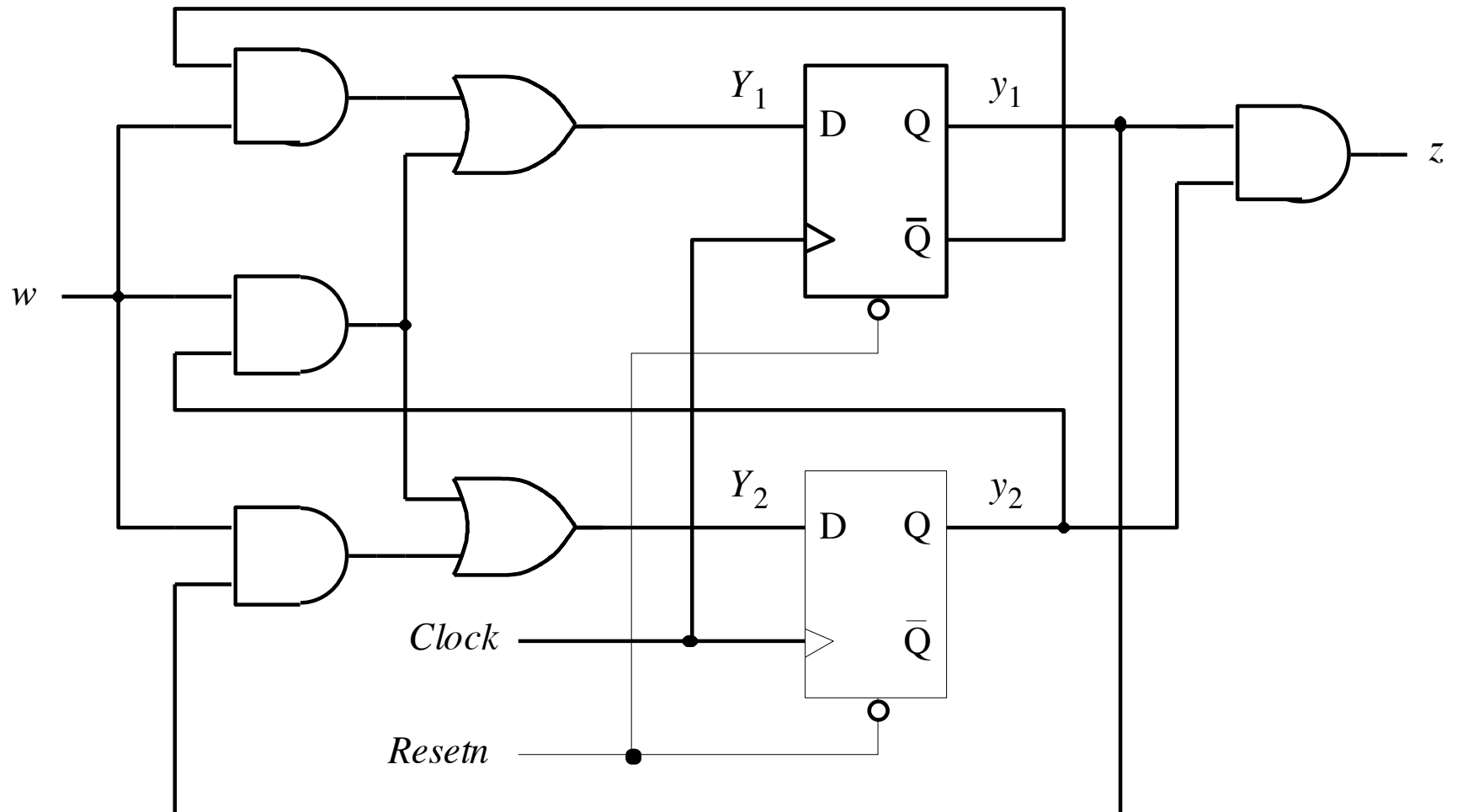


[Figure 6.75 from the textbook]

Approach

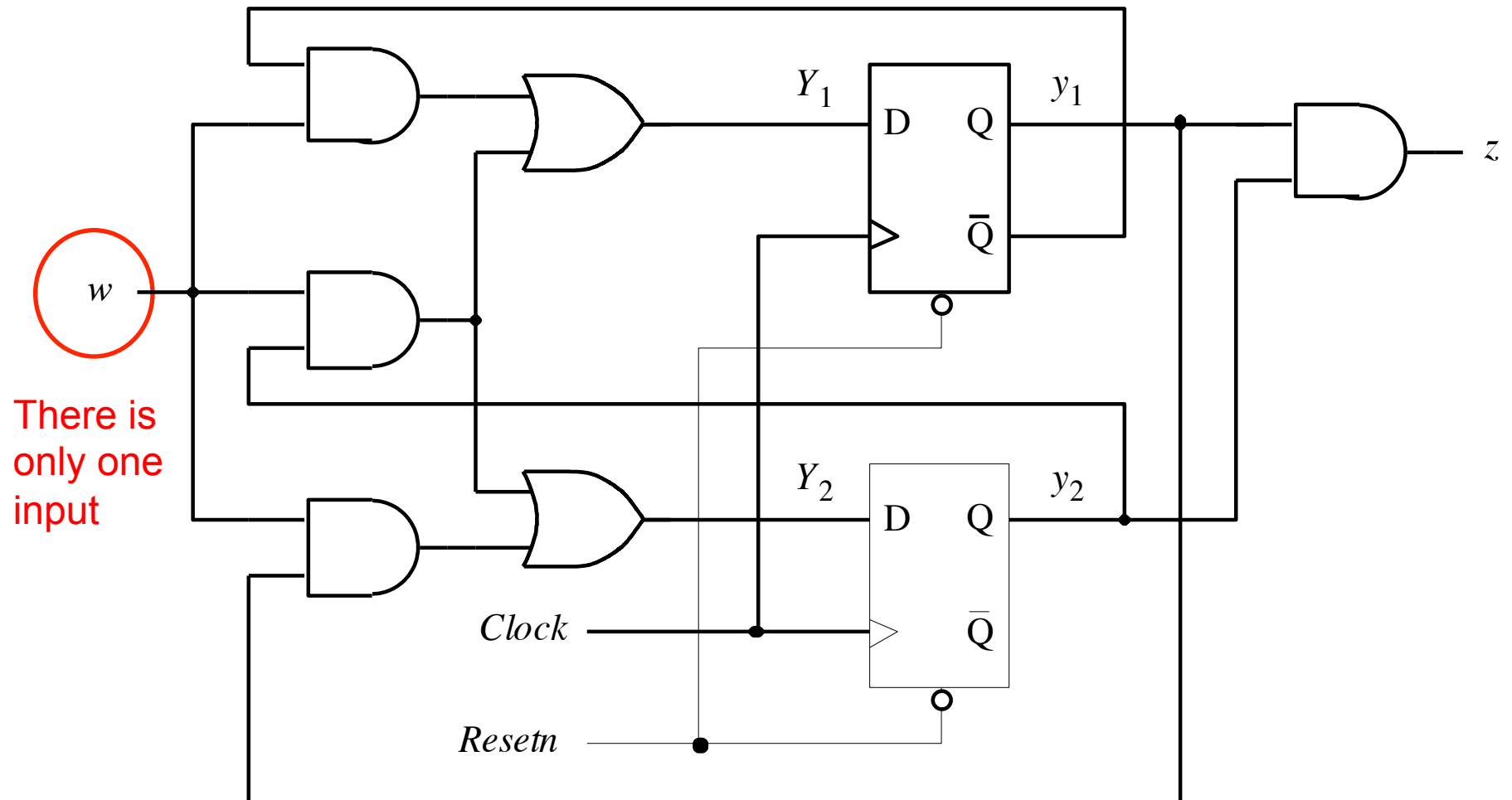
- **Find the flip-flops**
- **Outputs of the flip-flops = present state variables**
- **Inputs of the flip-flops determine the next state variables**
- **Determine the logical expressions for the outputs**
- **Given this info it is easy to do the state-assigned table**
- **Next do the state table**
- **Finally, draw the state diagram.**

Where are the inputs?



[Figure 6.75 from the textbook]

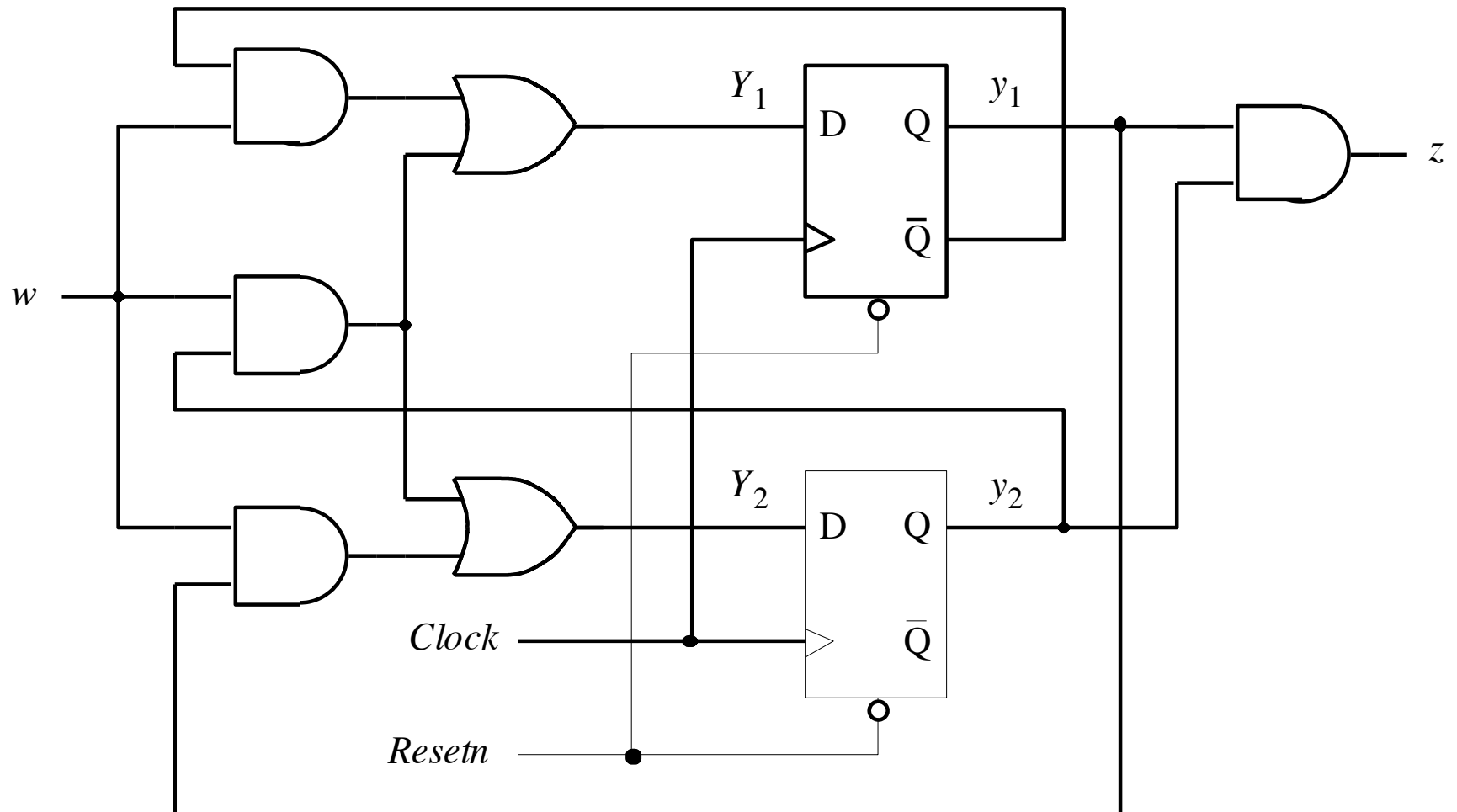
Where are the inputs?



There is only one input

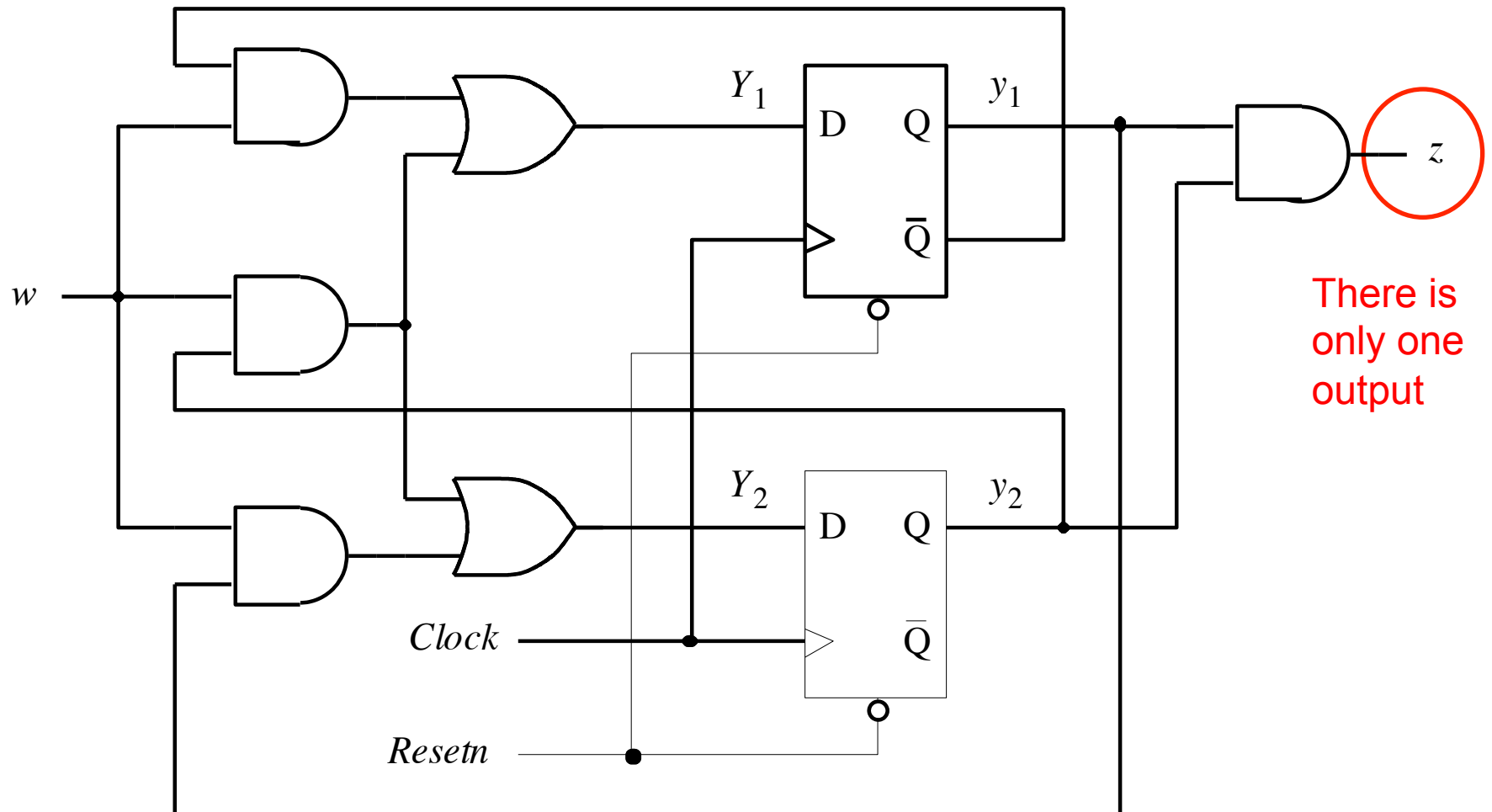
[Figure 6.75 from the textbook]

Where are the outputs?



[Figure 6.75 from the textbook]

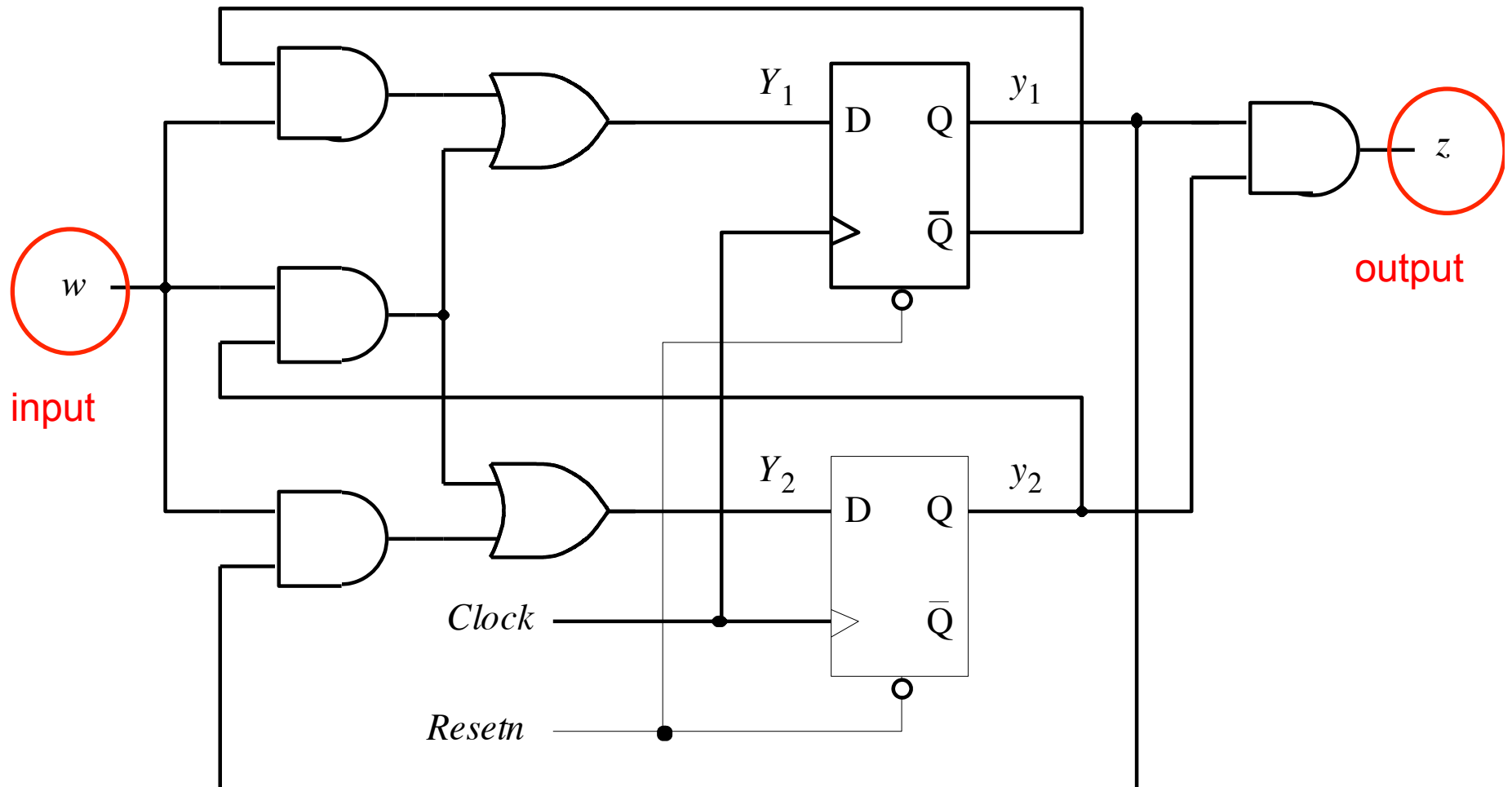
Where are the outputs?



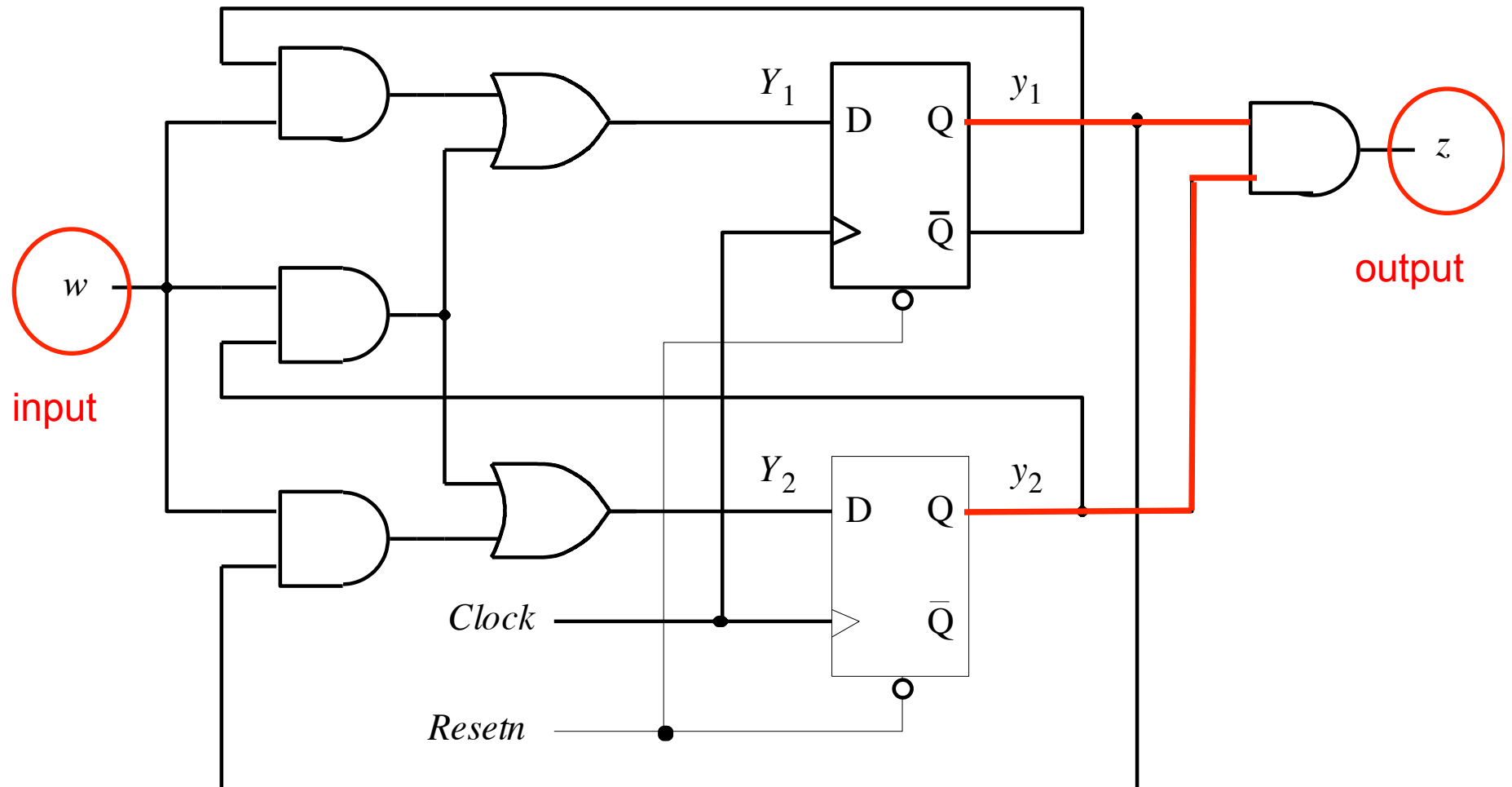
There is only one output

[Figure 6.75 from the textbook]

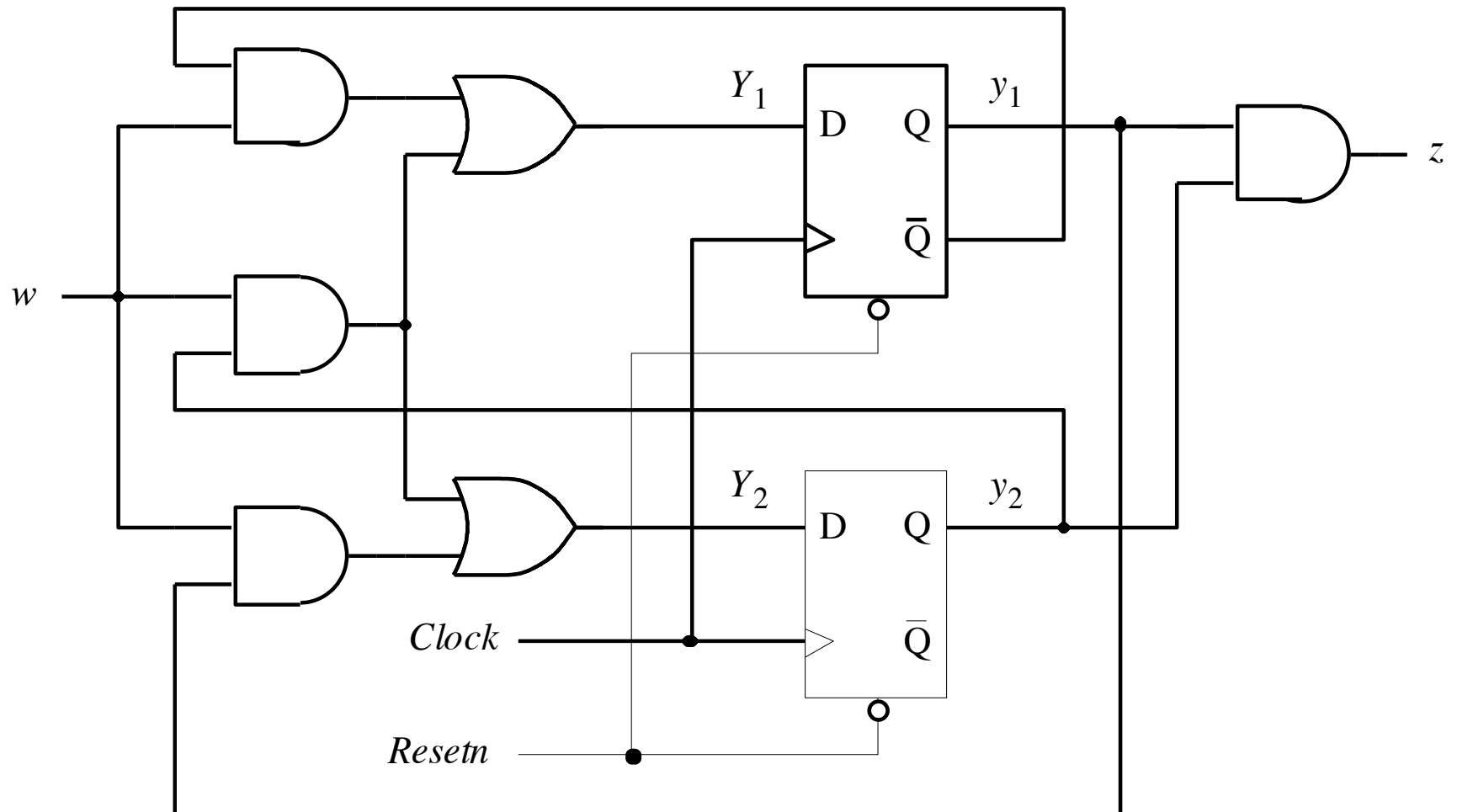
Where kind of machine is this? Moore or Mealy?



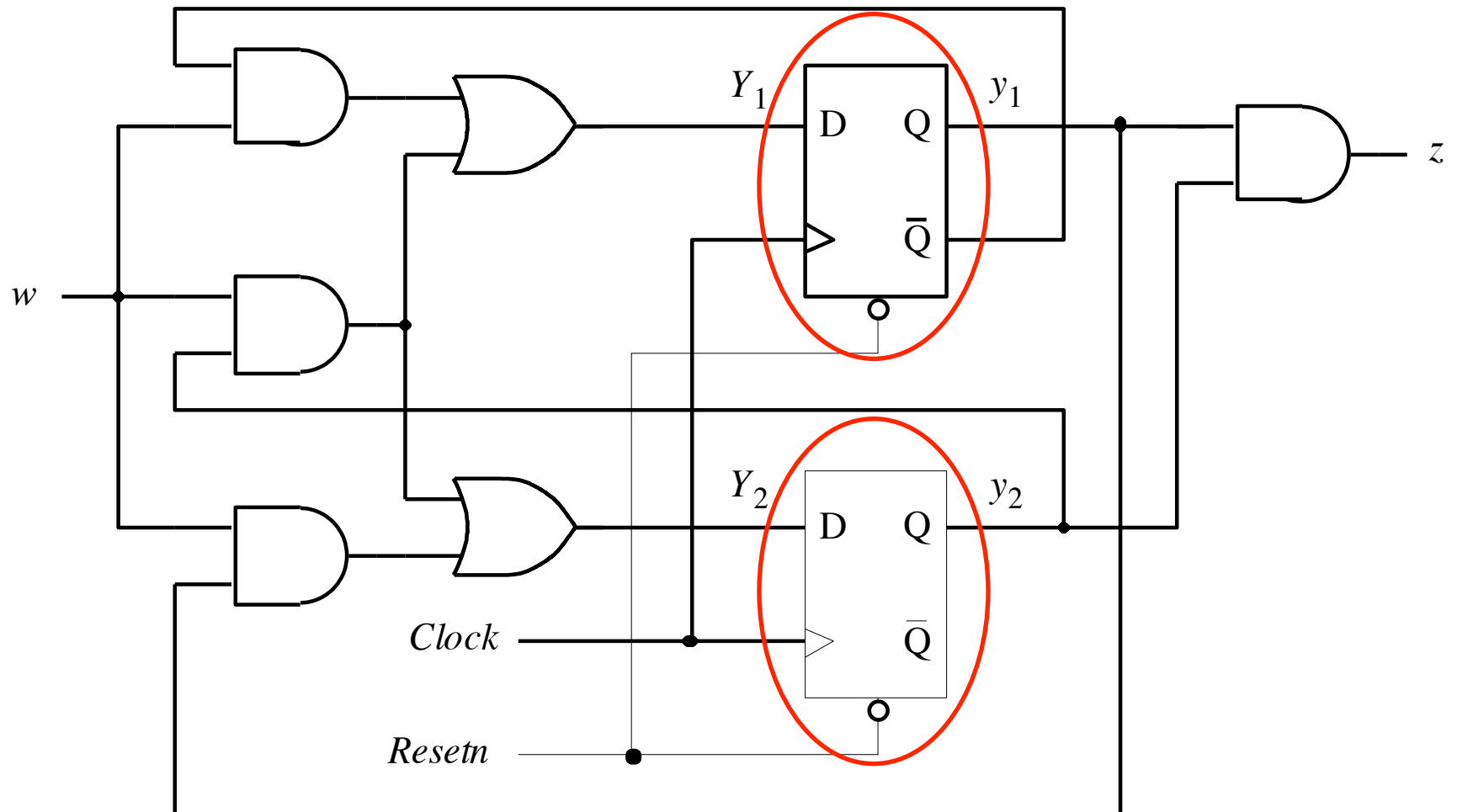
Moore: because the output does not depend directly on the primary input



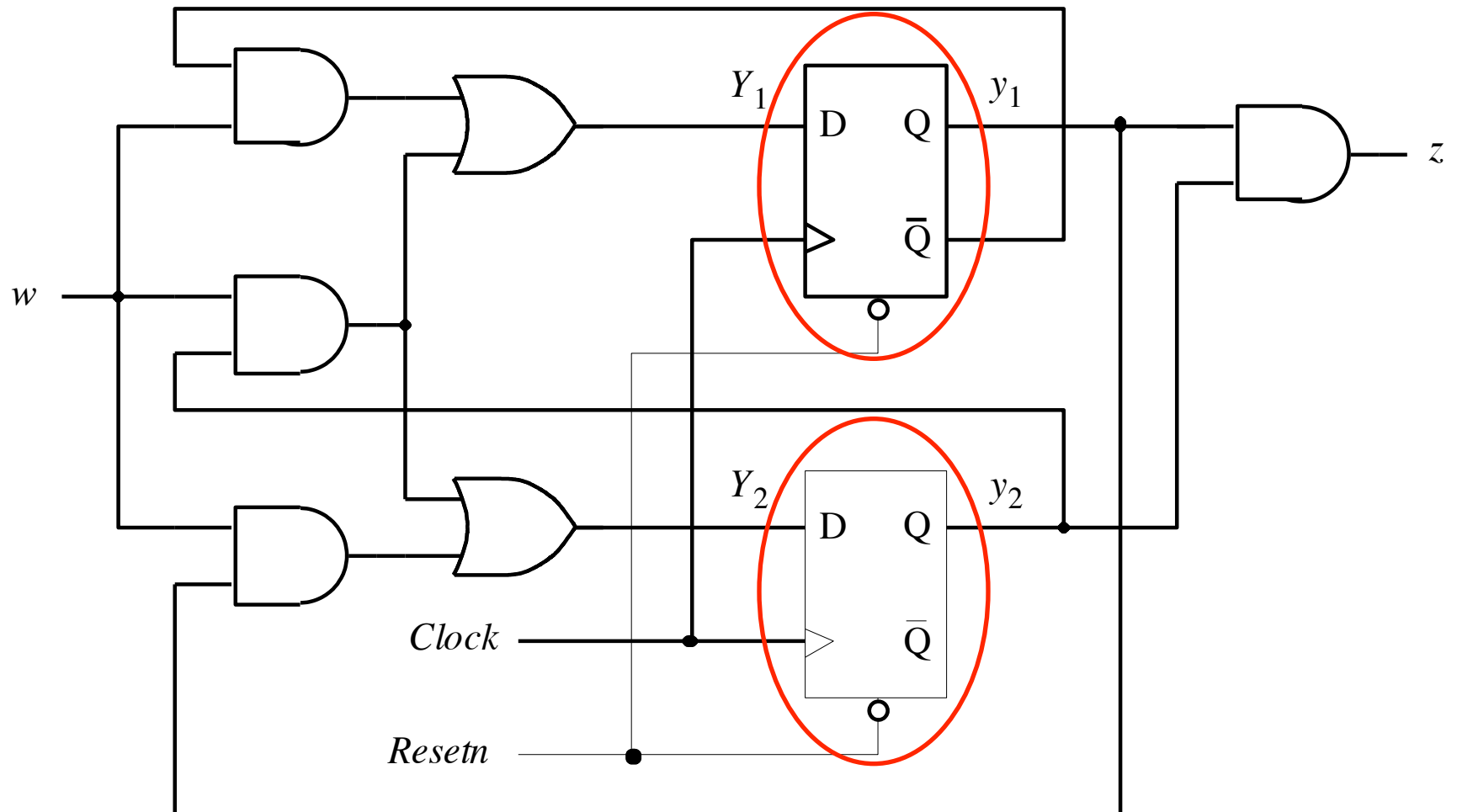
Where are the memory elements?



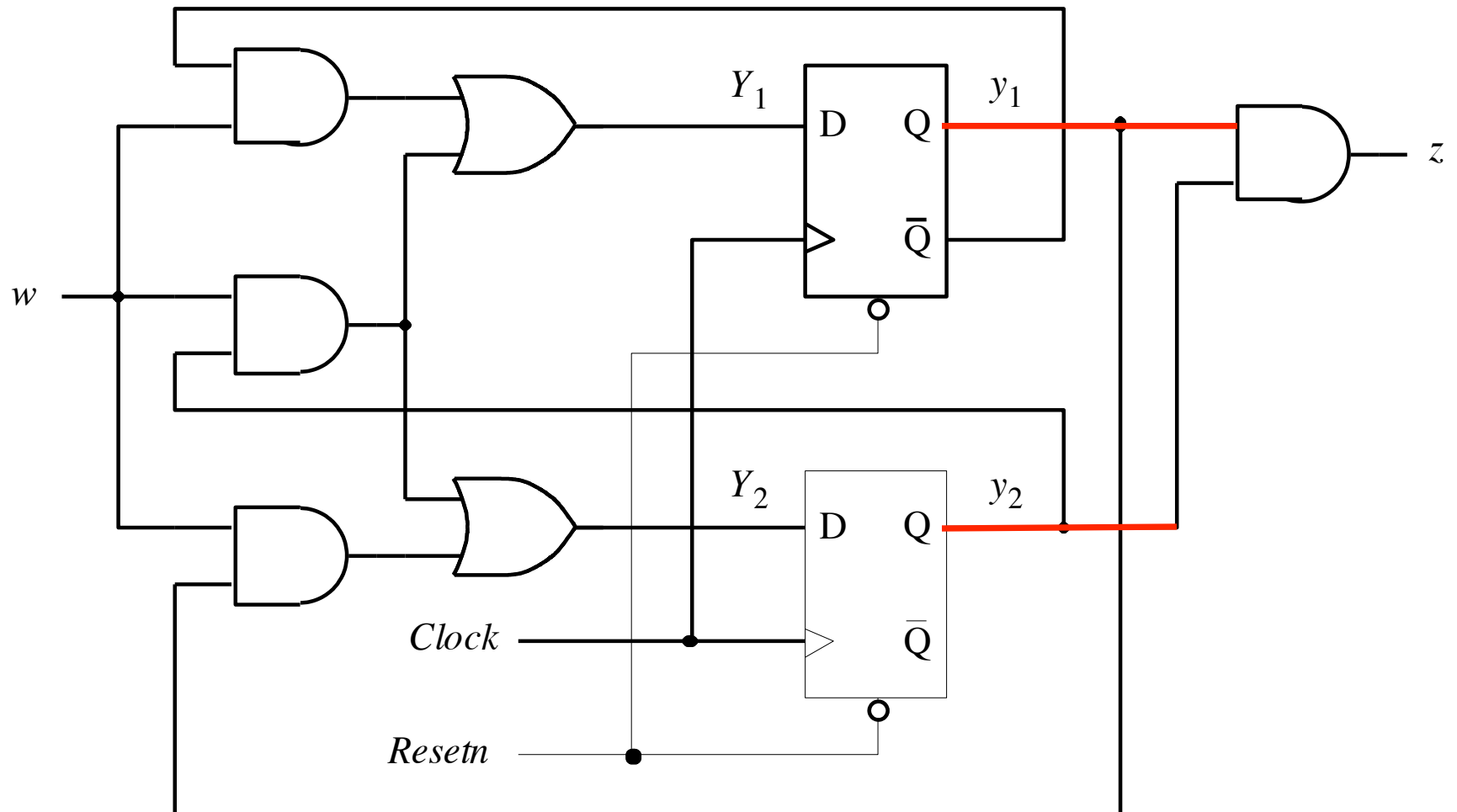
Where are the memory elements?



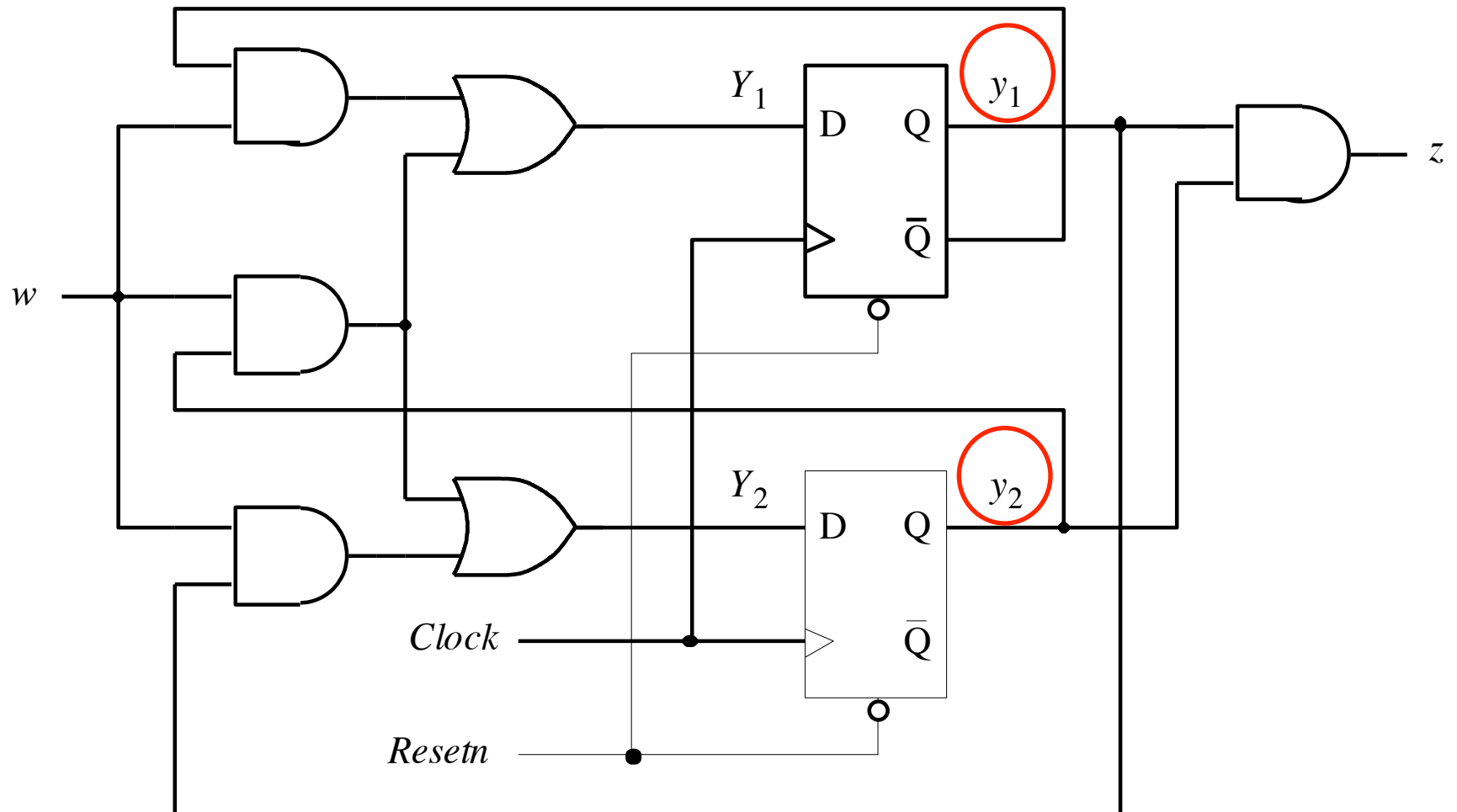
Where are the outputs of the flip-flops?



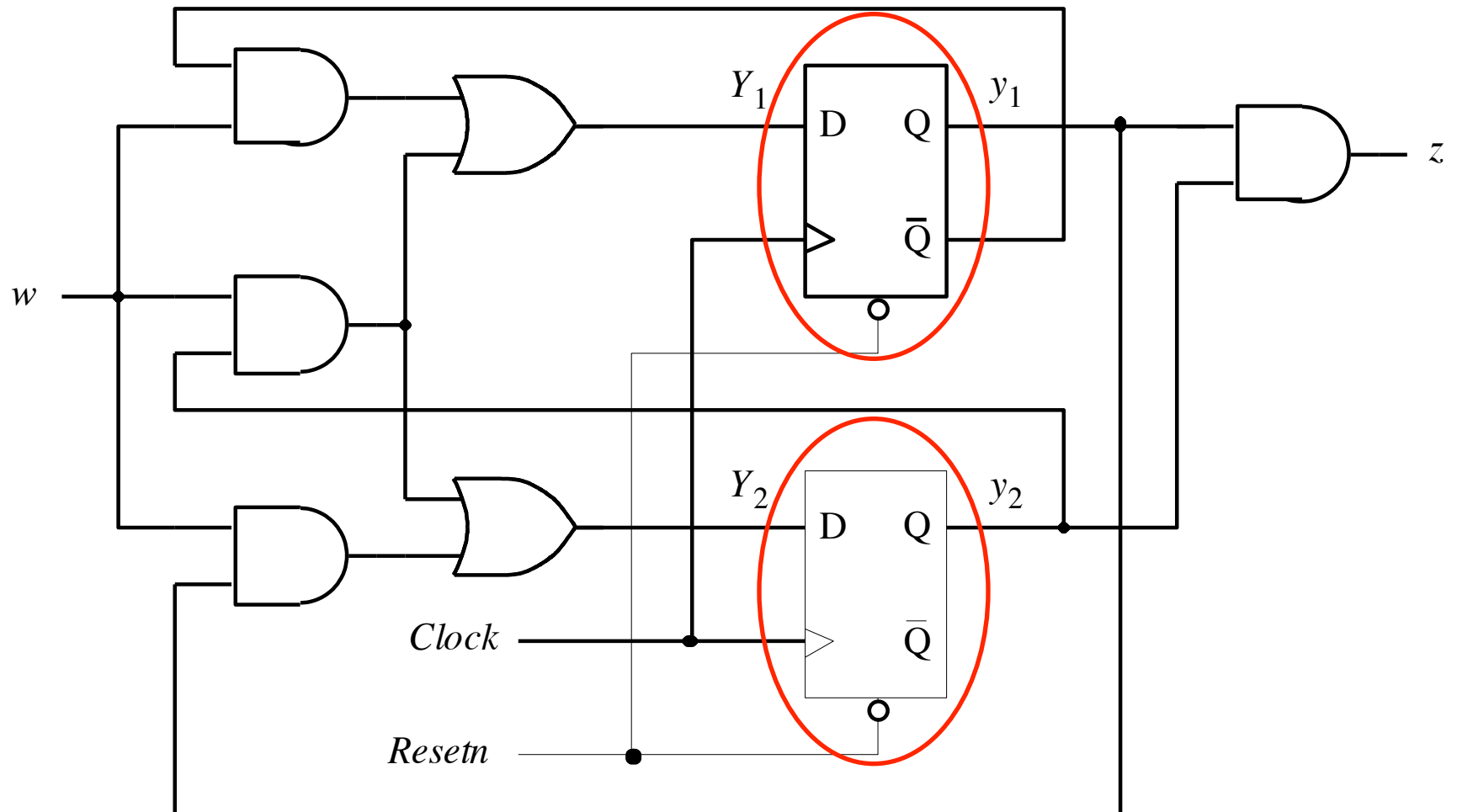
Where are the outputs of the flip-flops?



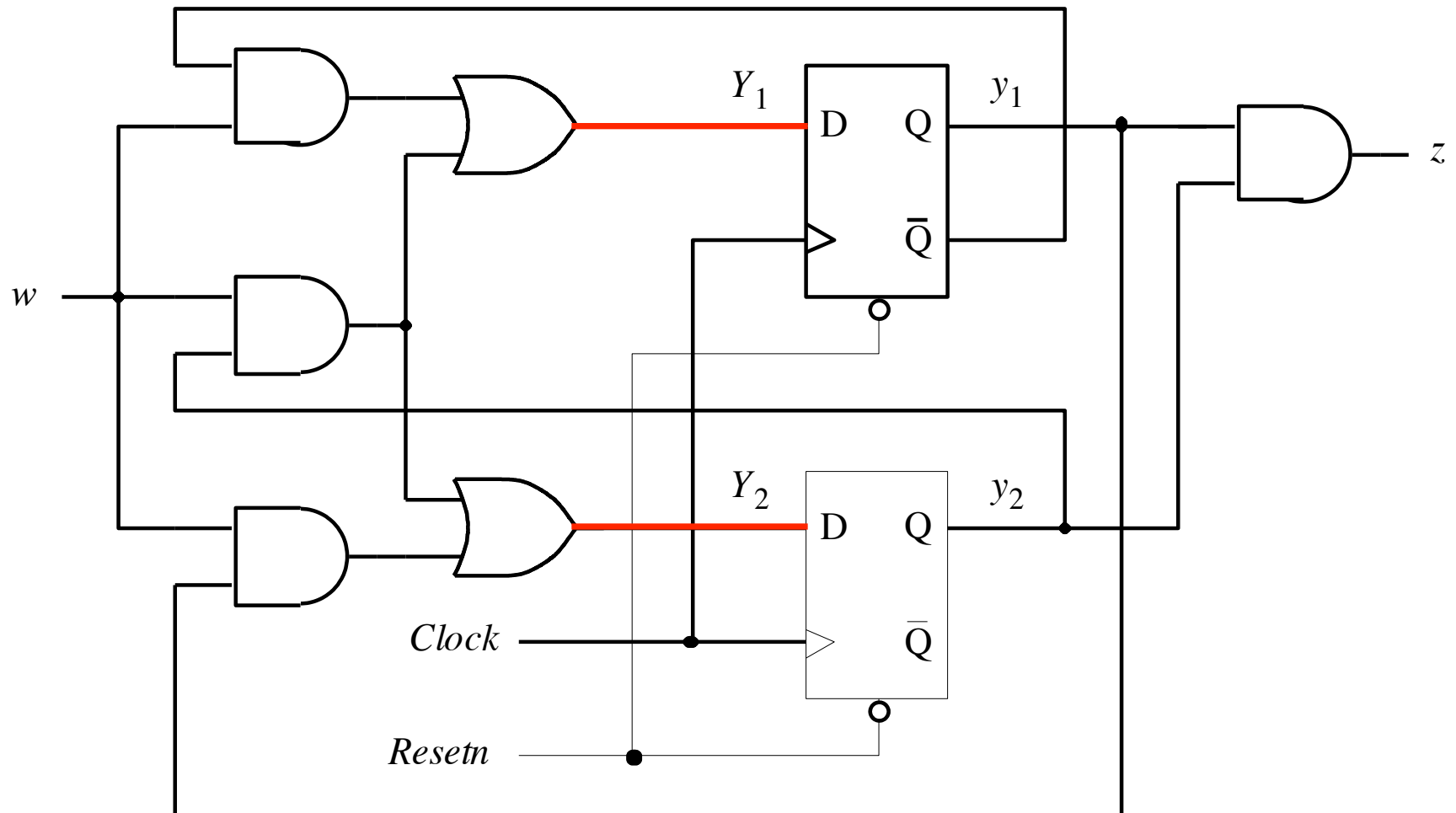
These are the present-state variables



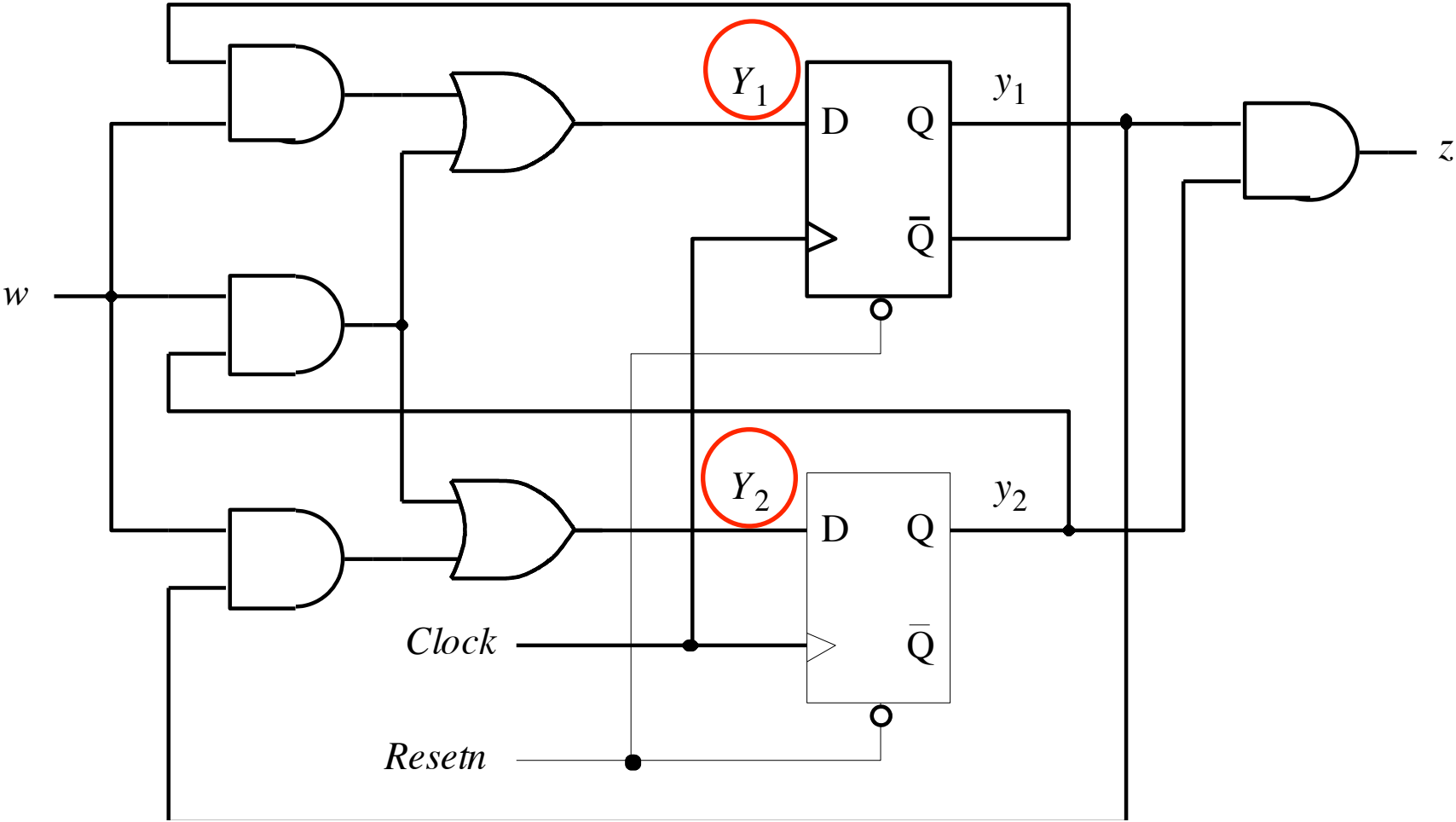
Where are the inputs of the flip-flops?



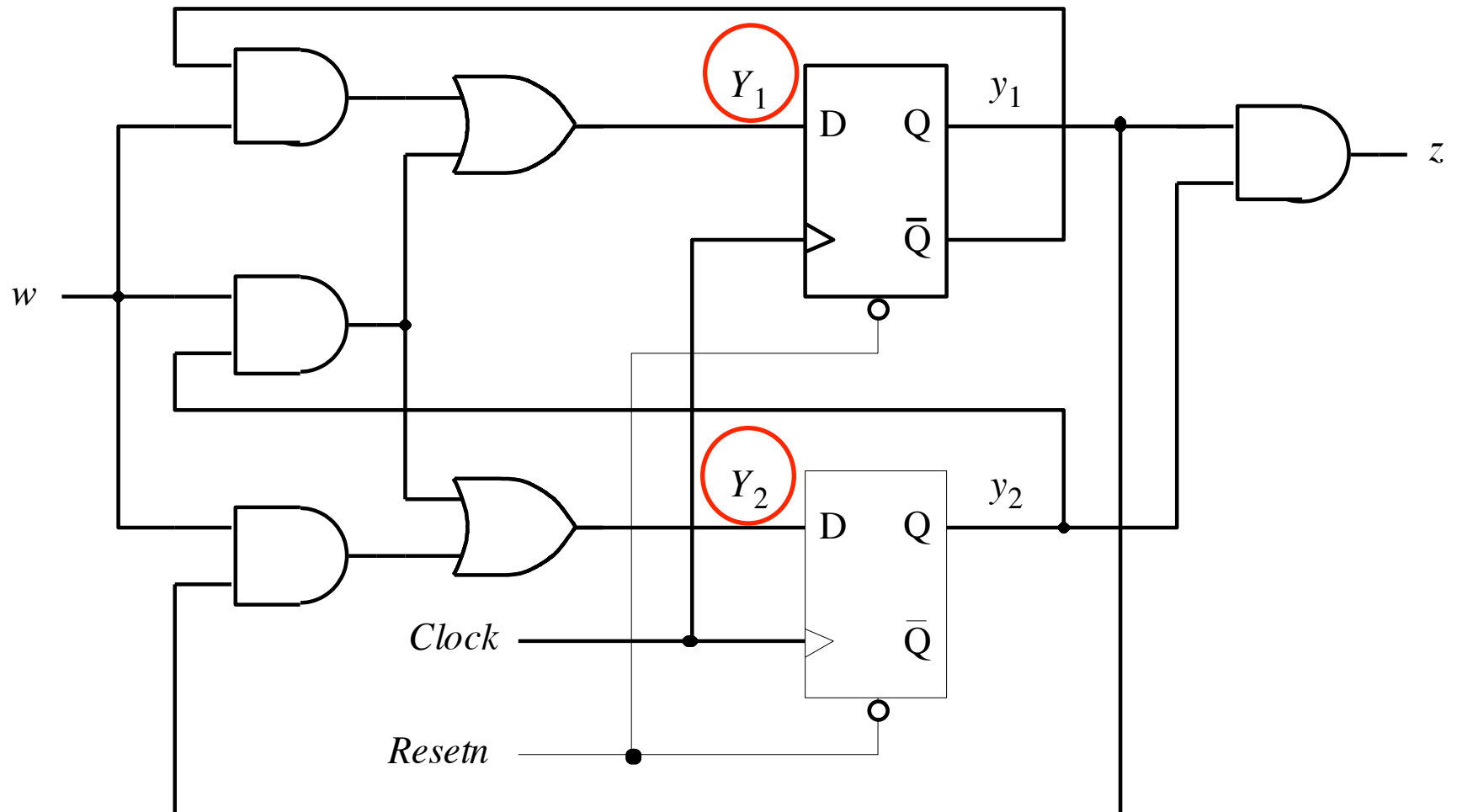
Where are the inputs of the flip-flops?



These are the next-state variables

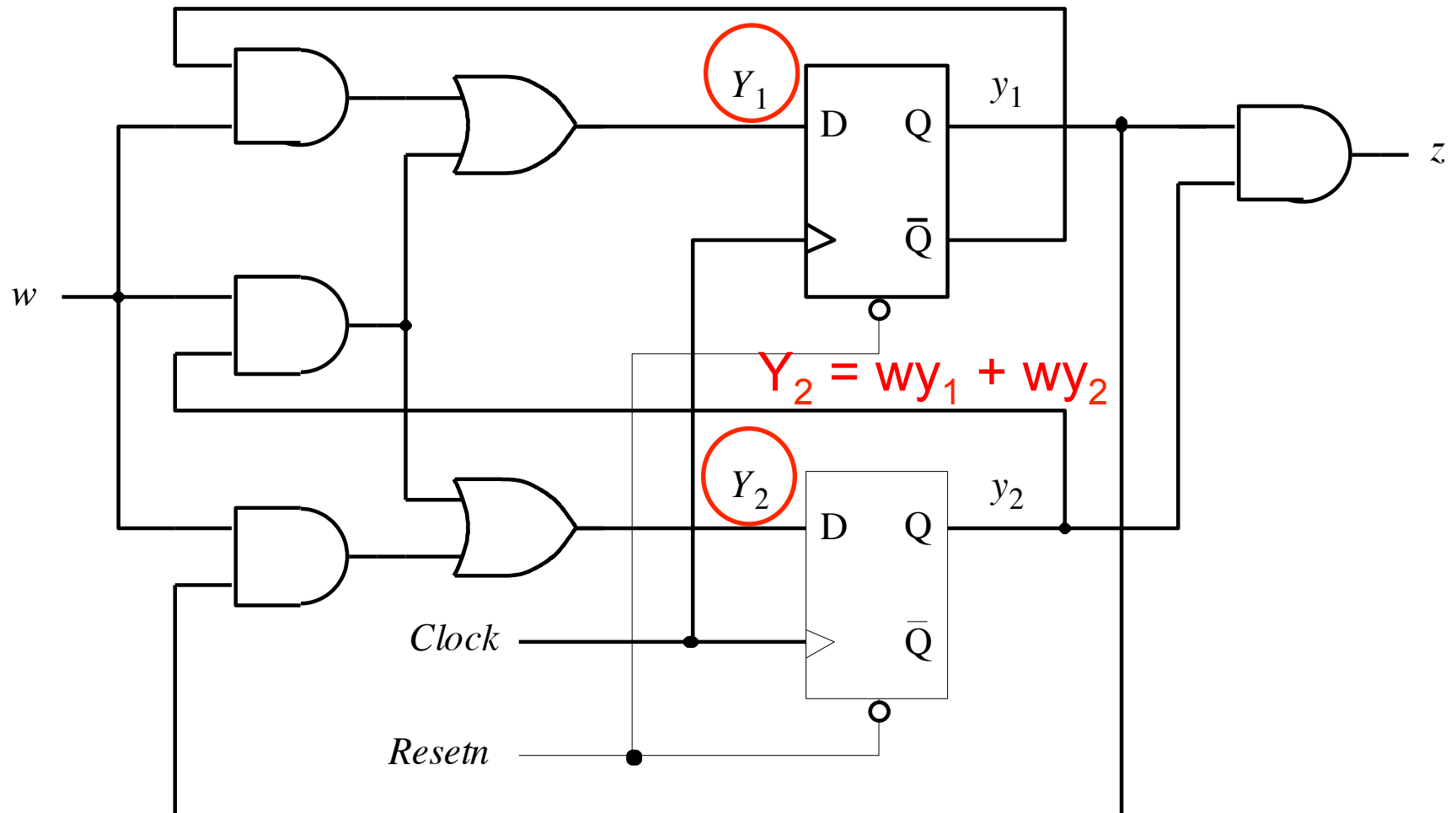


What are their logic expressions?



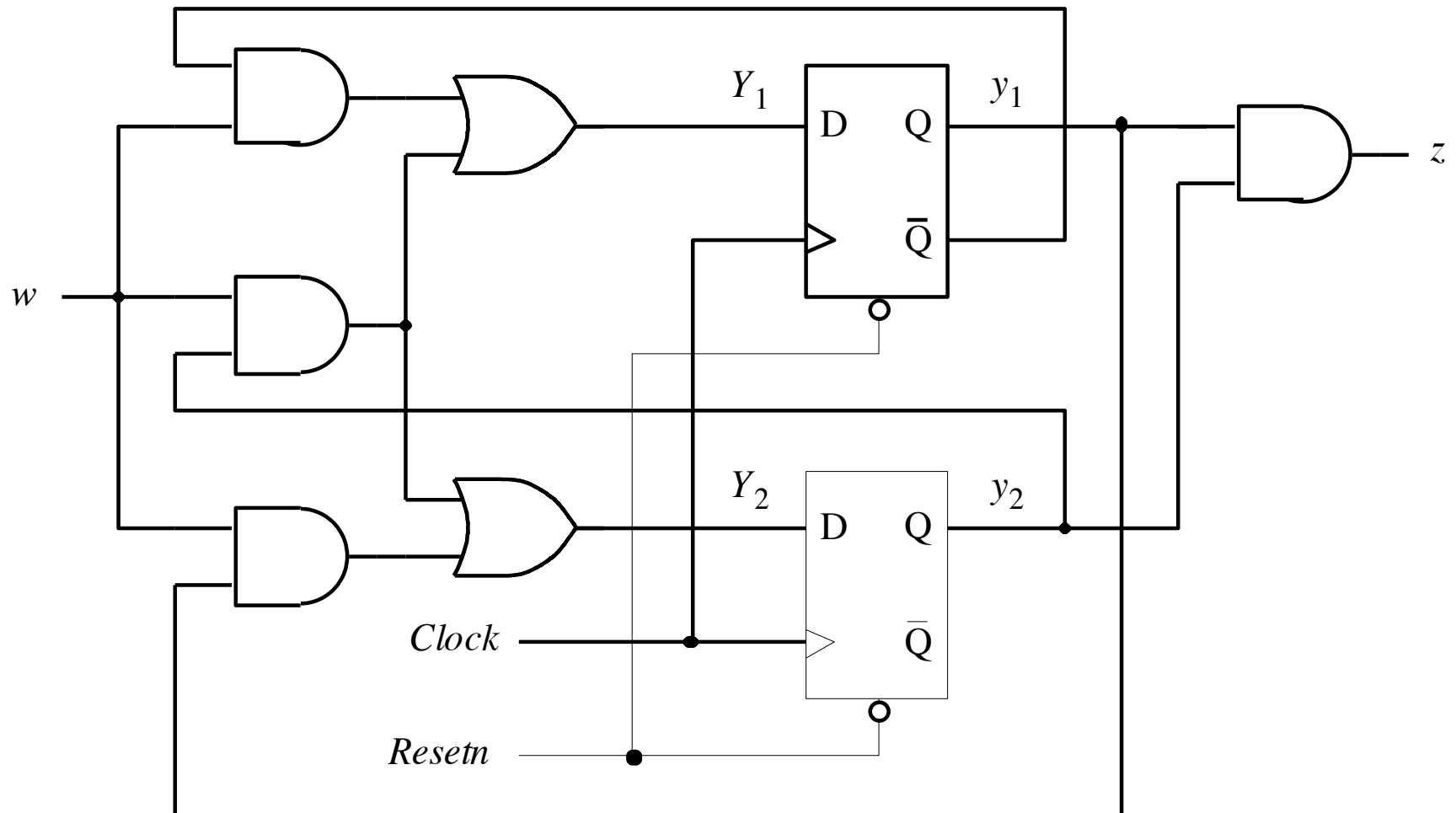
What are their logic expressions?

$$Y_1 = w\bar{y}_1 + wy_2$$

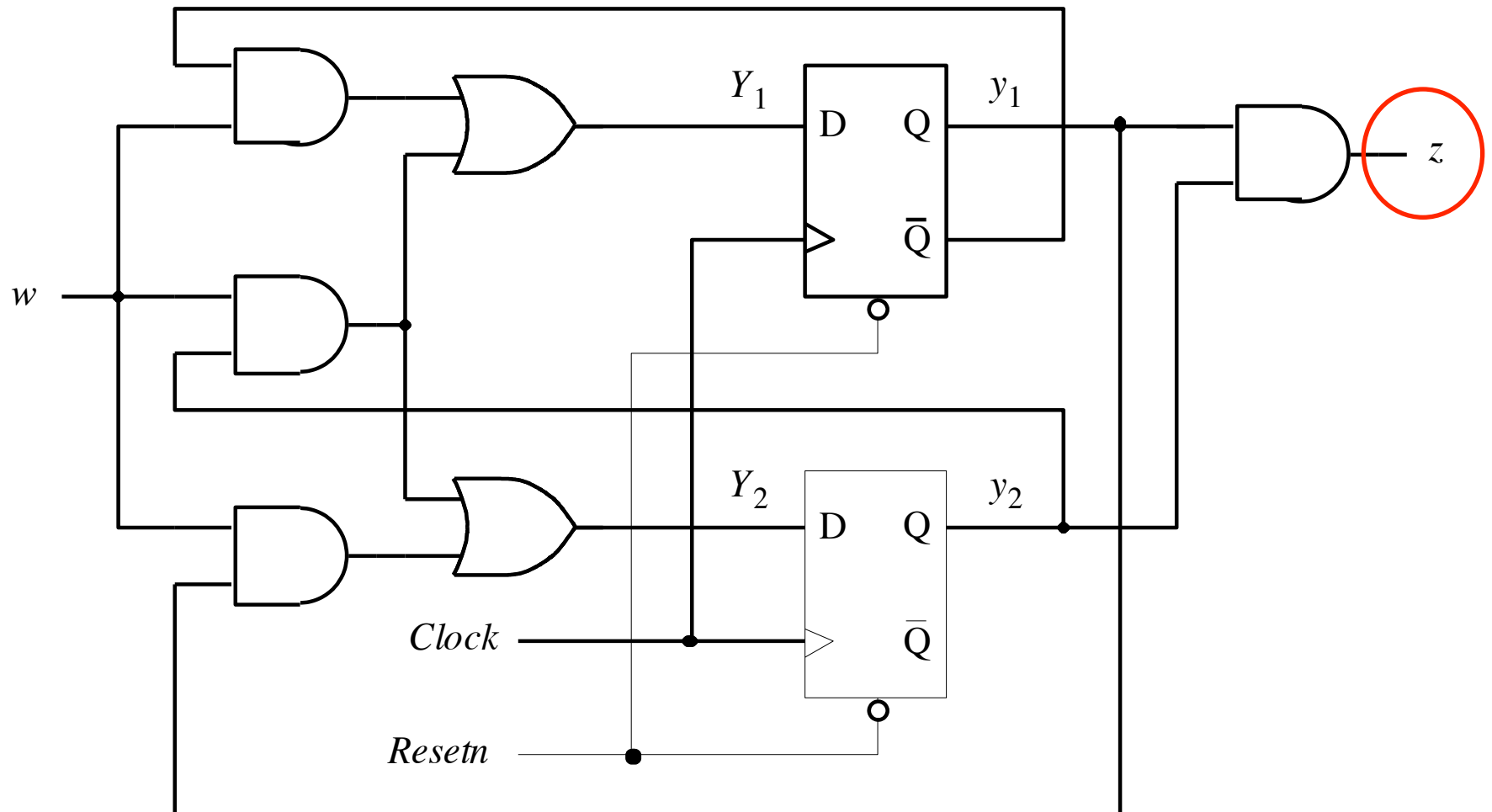


$$Y_2 = wy_1 + wy_2$$

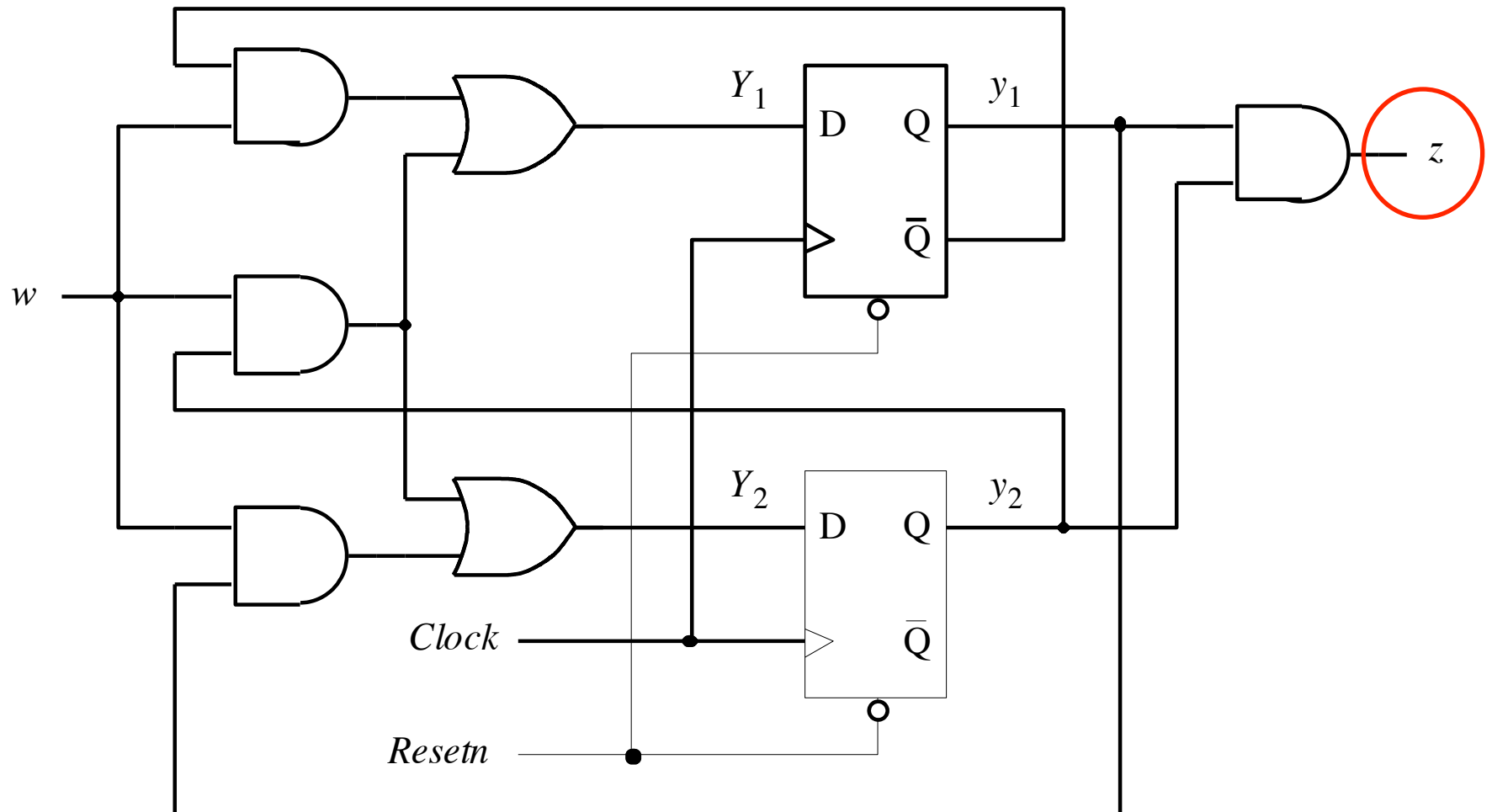
Where is the output, again?



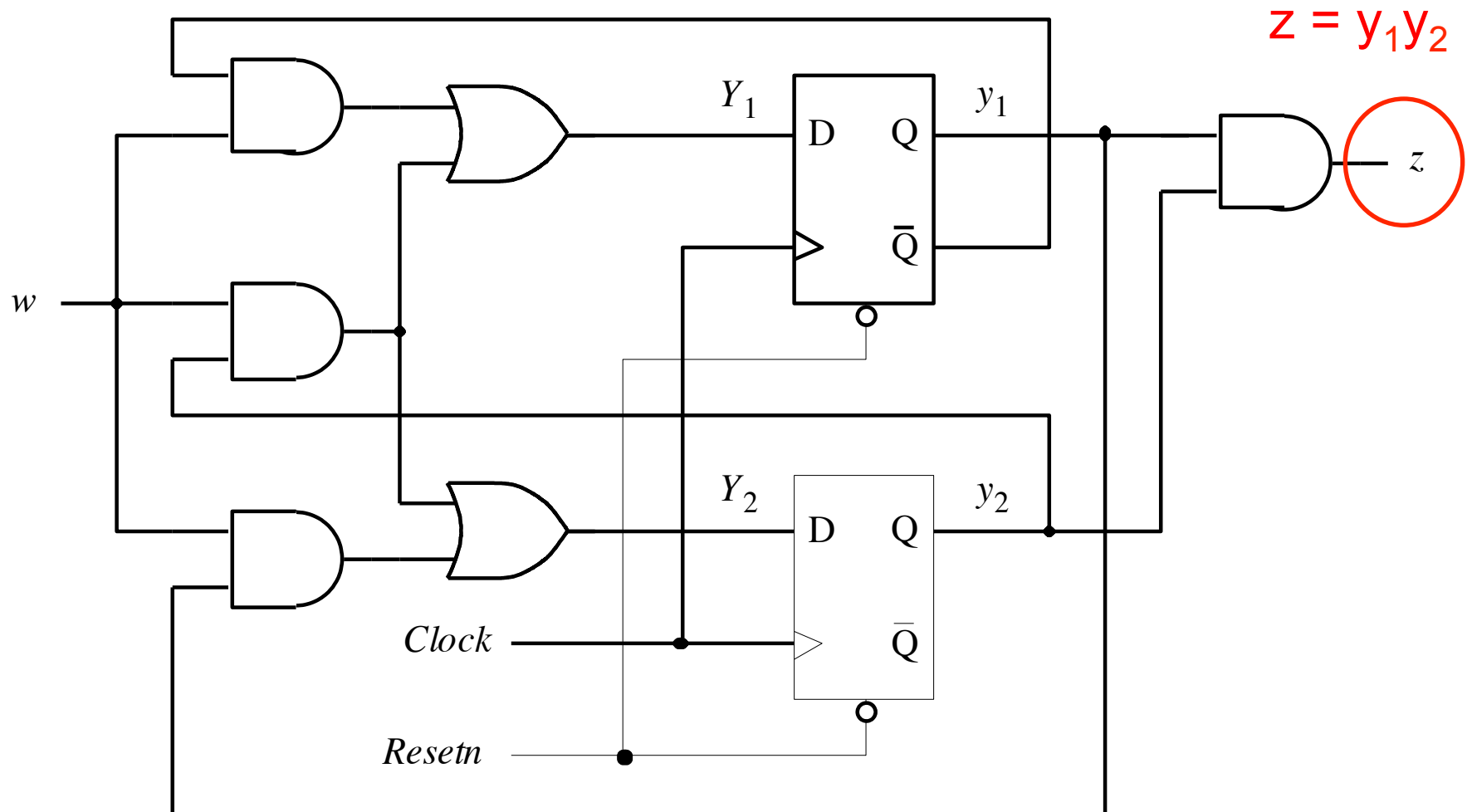
Where is the output, again?



What is its logic expression?



What is its logic expression?



**This is what we have to work with now
(we don't need the circuit anymore)**

$$Y_1 = w\bar{y}_1 + wy_2$$

$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

Let's derive the state-assigned table

$$Y_1 = w\bar{y}_1 + wy_2$$

$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

Present state y_2y_1	Next State		Output z
	$w = 0$	$w = 1$	
	Y_2Y_1	Y_2Y_1	
00			
01			
10			
11			

Let's derive the state-assigned table

$$Y_1 = w\bar{y}_1 + wy_2$$

$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

Present state y_2y_1	Next State		Output z
	$w = 0$	$w = 1$	
00			
01			
10			
11			

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$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

Present state y_2y_1	Next State		Output z
	$w = 0$	$w = 1$	
	Y_2Y_1	Y_2Y_1	
00			0
01			0
10			0
11			1

Let's derive the state-assigned table

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00			0
01			0
10			0
11			1

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	Y_2Y_1	Y_2Y_1	
00	0	1	0
01	0	0	0
10	0	1	0
11	0	1	1

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$$z = y_1y_2$$

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	Y_2Y_1	Y_2Y_1	
00	0	1	0
01	0	0	0
10	0	1	0
11	0	1	1

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	$w = 0$	$w = 1$	
	Y_2Y_1	Y_2Y_1	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

We don't need the logic expressions anymore

$$Y_1 = w\bar{y}_1 + wy_2$$

$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

Present state y_2y_1	Next State		Output z
	$w = 0$	$w = 1$	
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00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

We don't need the logic expressions anymore

Present state y_2y_1	Next State		Output z
	$w = 0$	$w = 1$	
	Y_2Y_1	Y_2Y_1	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	

State table

Present state y_2y_1	Next State		Output z
	w = 0 Y_2Y_1	w = 1 Y_2Y_1	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	

State table

Present state y_2y_1	Next State		Output z
	w = 0	w = 1	
	Y_2Y_1	Y_2Y_1	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table

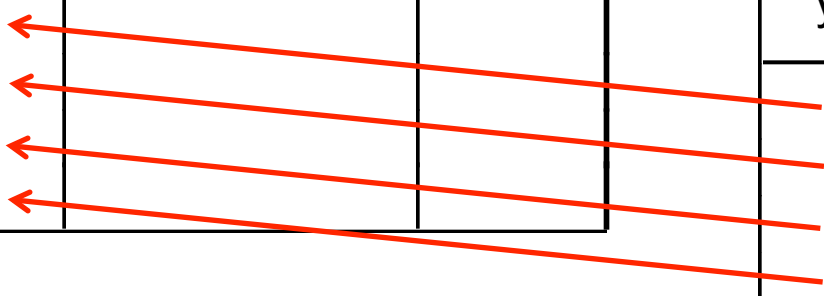
Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			
B			
C			
D			

State table

Present state	Next State		Output z
	w = 0	w = 1	
y_2y_1	Y_2Y_1	Y_2Y_1	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table



Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			
B			
C			
D			

State table

Present state y_2y_1	Next State		Output z
	w = 0 Y_2Y_1	w = 1 Y_2Y_1	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table

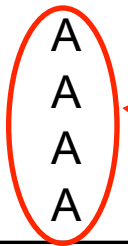
Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		
B	A		
C	A		
D	A		

State table

Present state y_2y_1	Next State		Output z
	w = 0 Y_2Y_1	w = 1 Y_2Y_1	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table



Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		
B	A		
C	A		
D	A		

State table

Present state y_2y_1	Next State		Output z
	w = 0 Y_2Y_1	w = 1 Y_2Y_1	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table

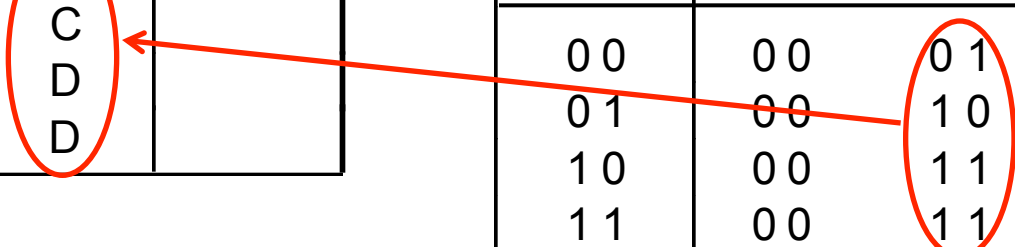
Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	
B	A	C	
C	A	D	
D	A	D	

State table

Present state	Next State		Output z
	w = 0	w = 1	
y_2y_1	Y_2Y_1	Y_2Y_1	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table



Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	
B	A	C	
C	A	D	
D	A	D	

State table

Present state y_2y_1	Next State		Output z
	w = 0	w = 1	
Y_2Y_1	Y_2Y_1	Y_2Y_1	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	
B	A	C	
C	A	D	
D	A	D	

State table

Present state y_2y_1	Next State		Output z
	w = 0	w = 1	
Y_2Y_1	Y_2Y_1	Y_2Y_1	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table

The output is the same in both tables

The two tables for the initial circuit

Present state	Next state		Output Z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table

Present state y_2y_1	Next State		Output Z
	w = 0	w = 1	
	Y_2Y_1	Y_2Y_1	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table

We don't need the state-assigned table anymore

Present state	Next state		Output Z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table

Present state y_2y_1	Next State		Output Z
	w = 0	w = 1	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table

We don't need the state-assigned table anymore

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table

Let's Draw the State Diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

Let's Draw the State Diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

Because this is a Moore machine the output is tied to the state

A / 0

B / 0

C / 0

D / 1

Let's Draw the State Diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

A / 0

B / 0

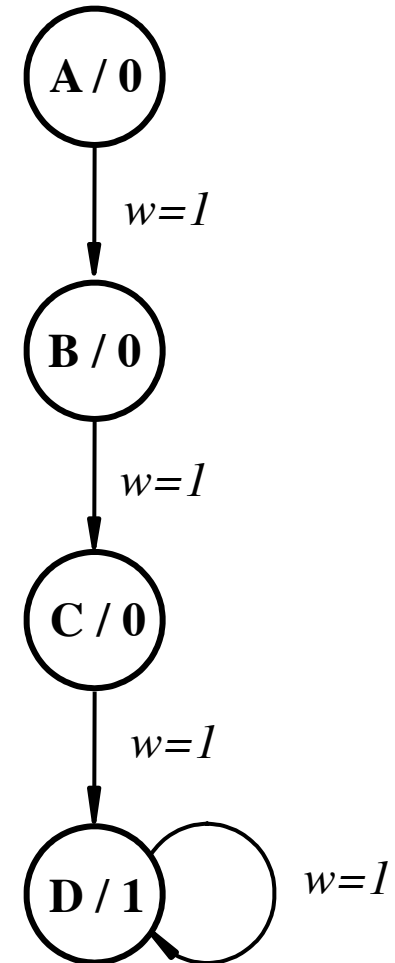
C / 0

D / 1

All transitions when the input w is equal to 1

Let's Draw the State Diagram

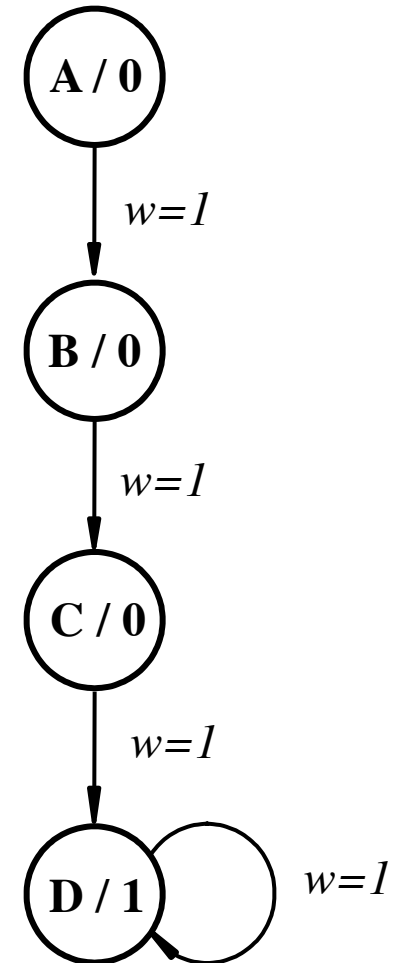
Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1



All transitions when the input w is equal to 1

Let's Draw the State Diagram

Present state	Next state		Output Z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

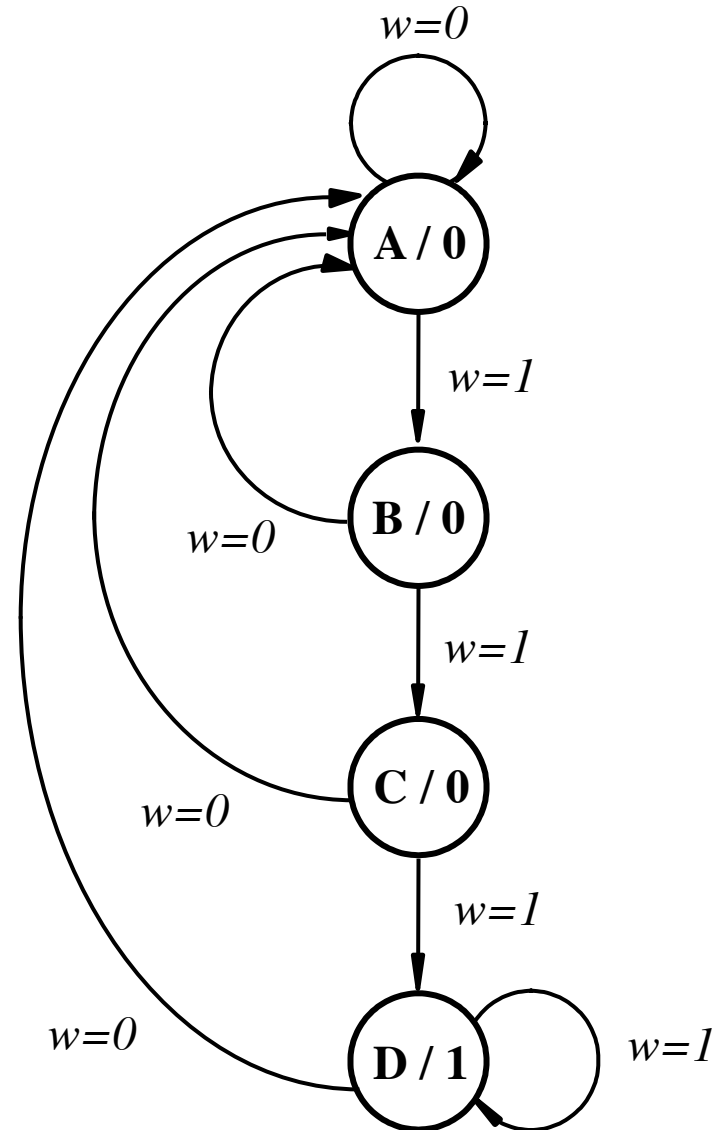


All transitions when the input w is equal to 0

Let's Draw the State Diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

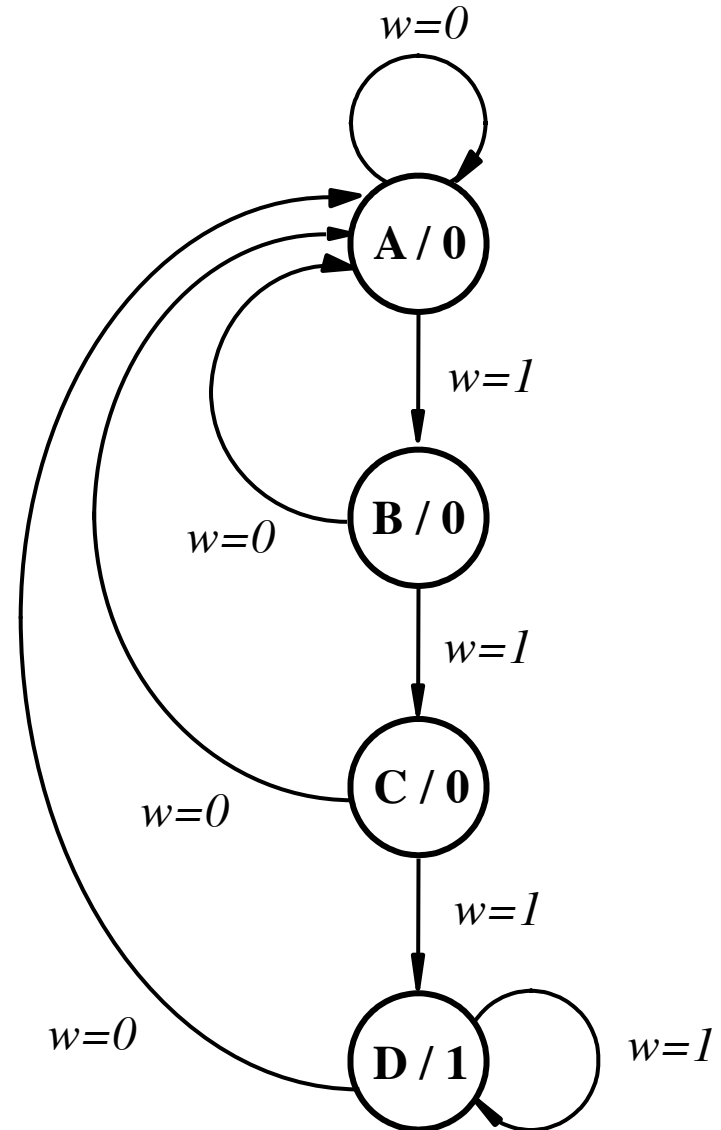
All transitions when the input w is equal to 0



We are done!

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table

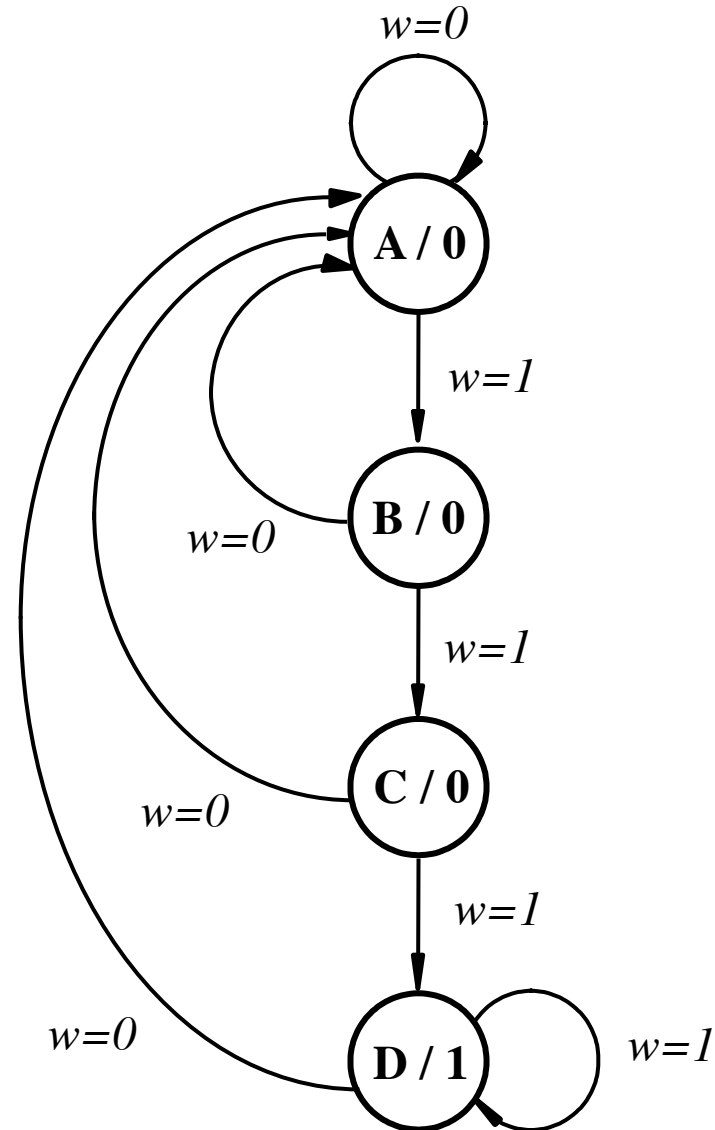


State diagram

Almost done. What does this FSM do?

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table



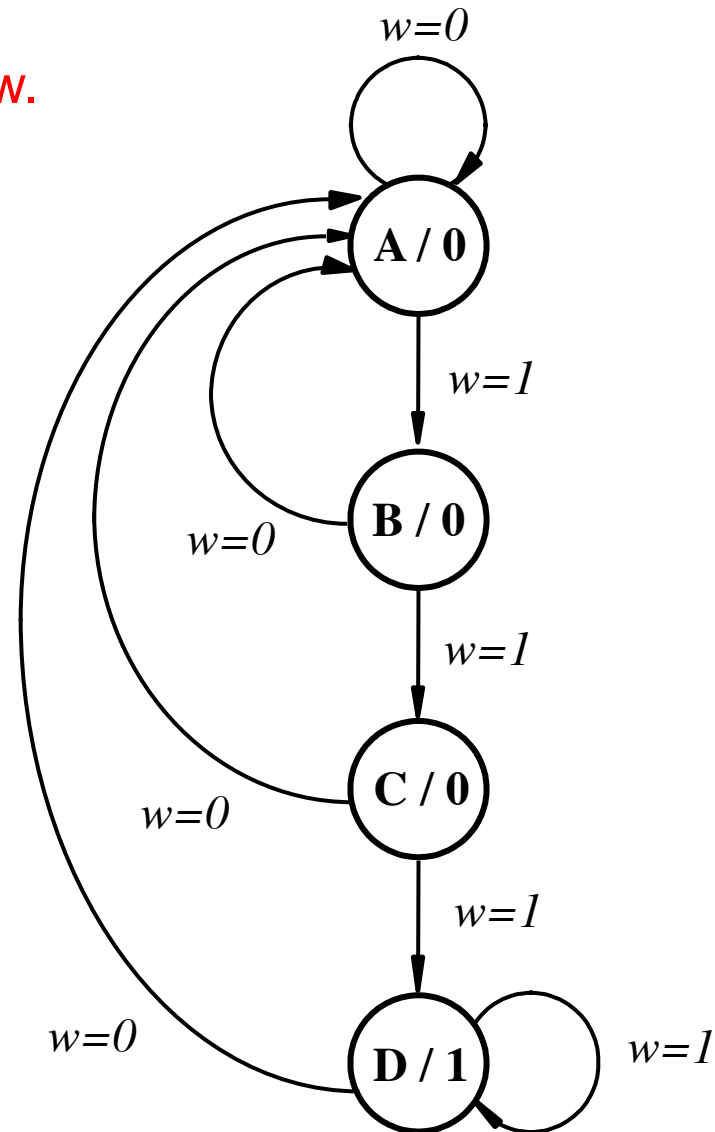
State diagram

Almost done. What does this FSM do?

It sets the output z to 1 when three consecutive 1's occur on the input w . In other words, it is a sequence detector for the input pattern 111.

Present state	Next state		Output z
	$w = 0$	$w = 1$	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

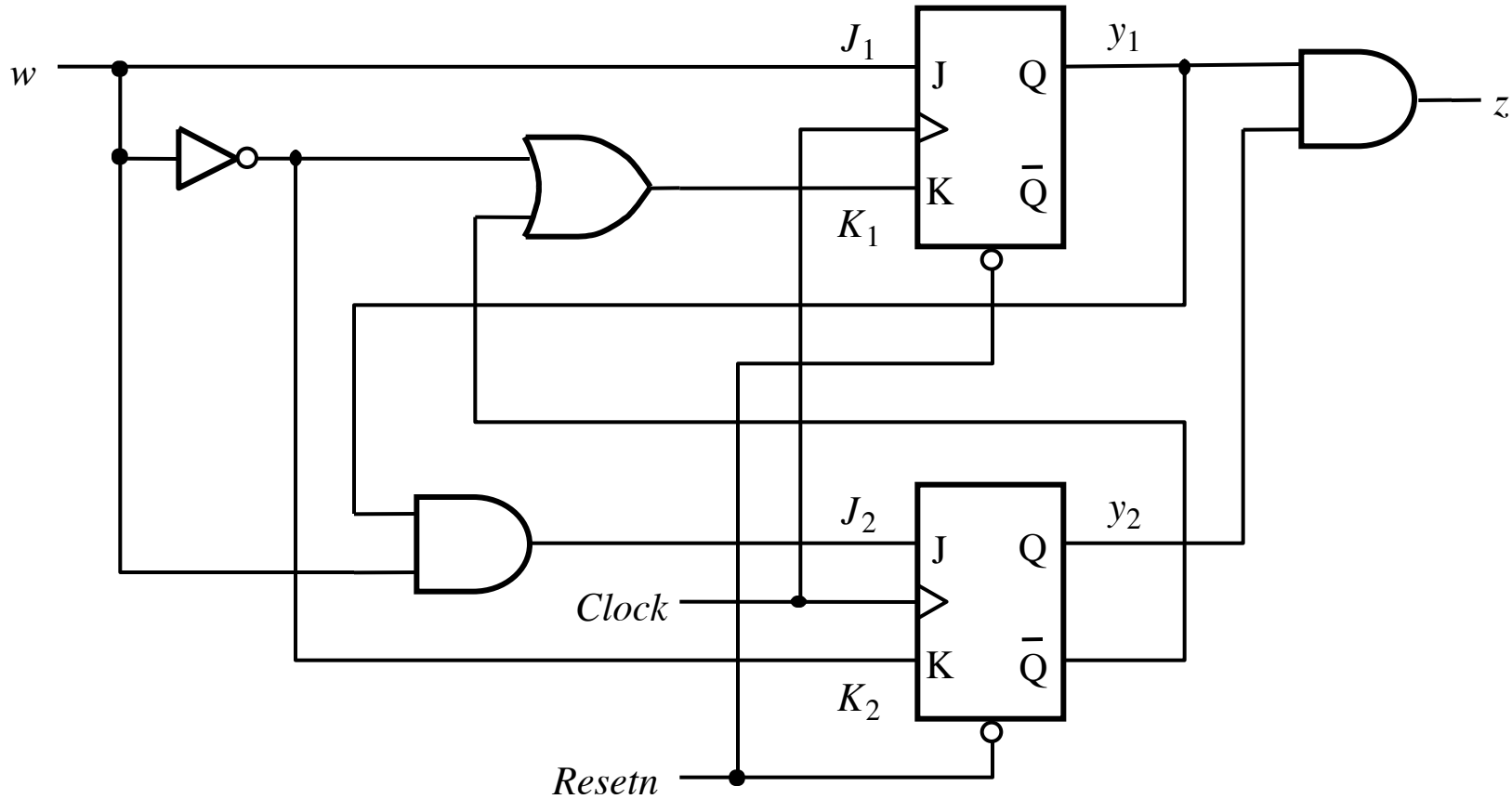
State table



State diagram

Another Example (with JK flip-flops)

What does this circuit do?

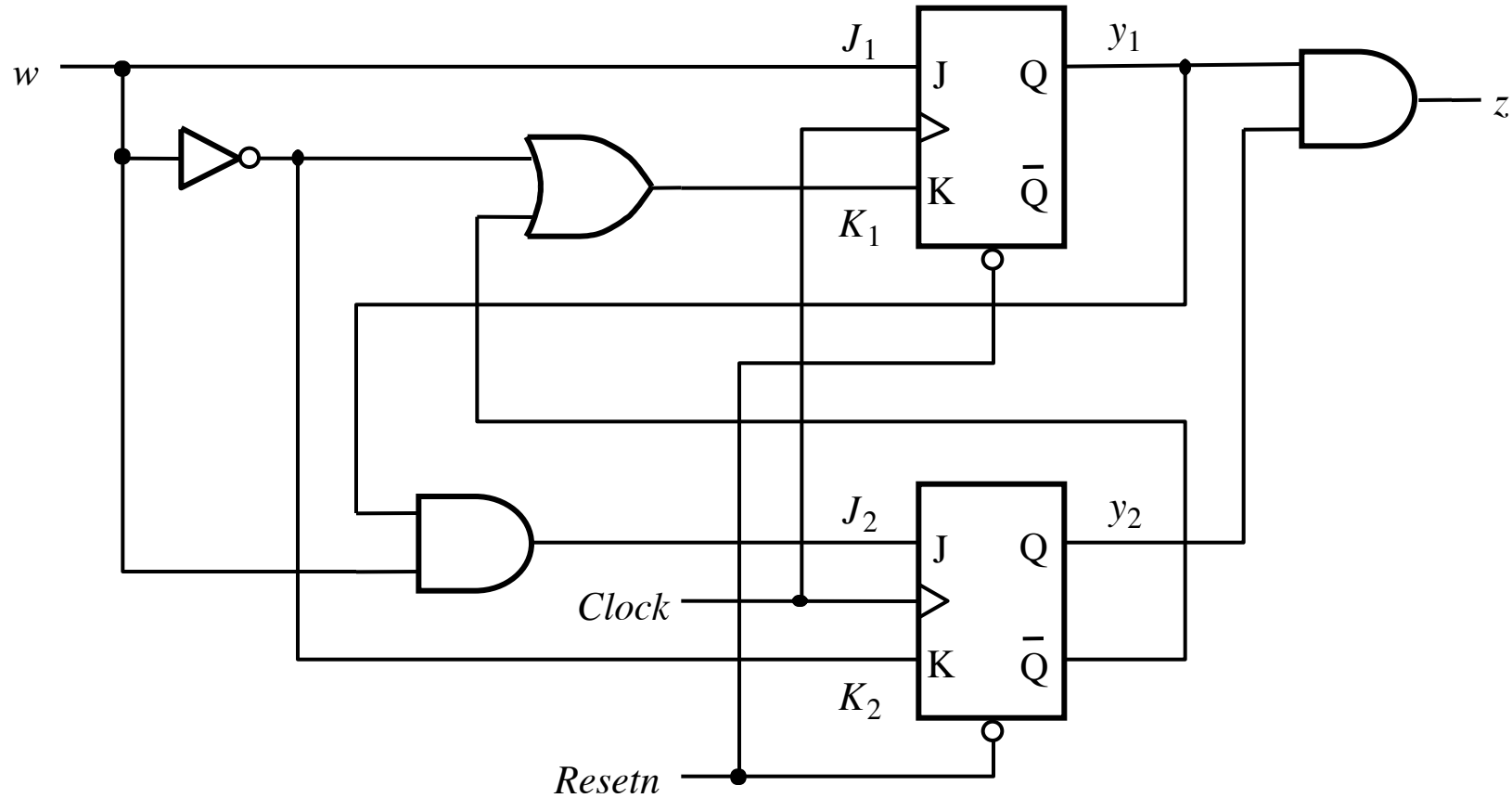


[Figure 6.77 from the textbook]

Approach

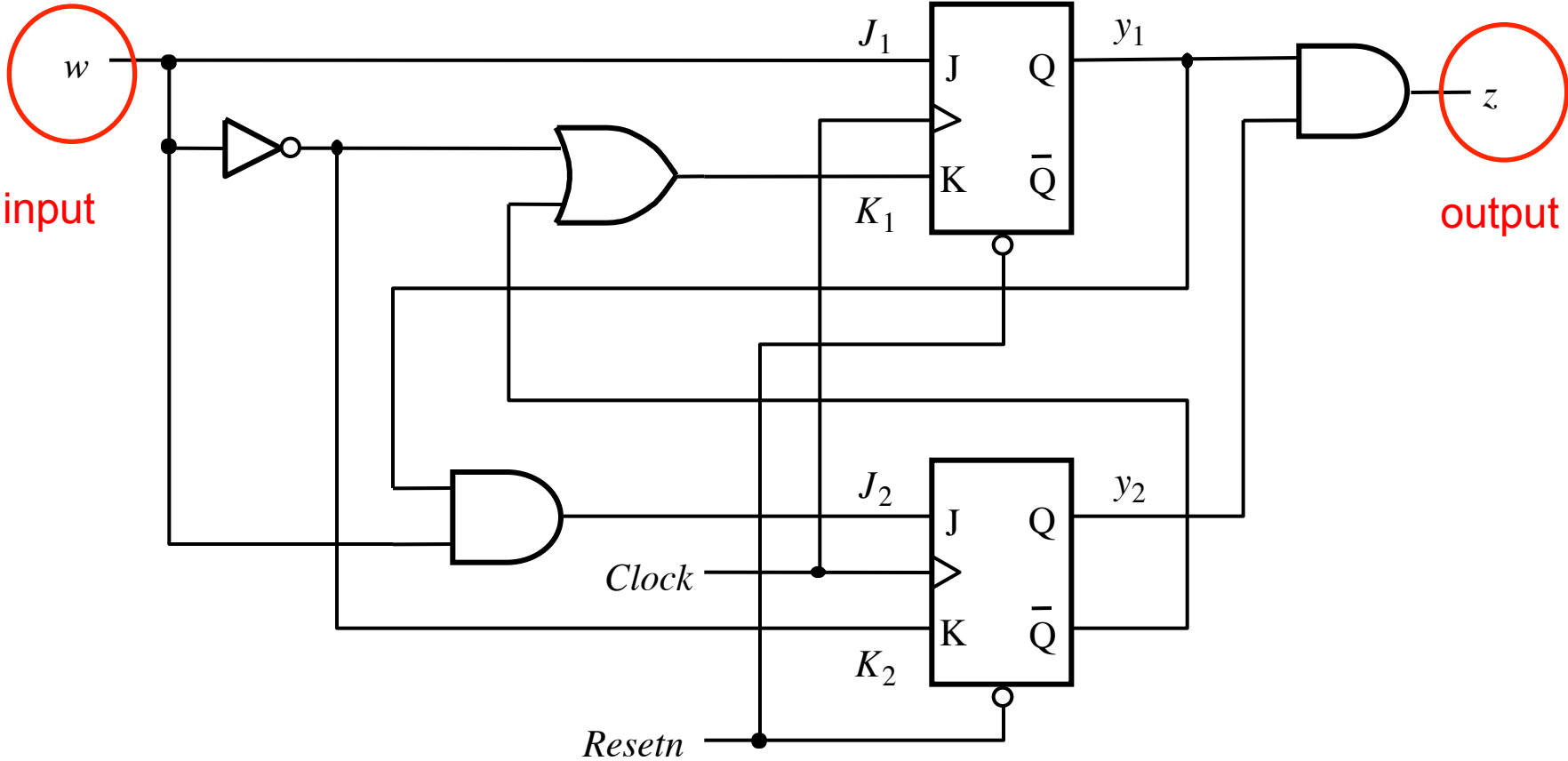
- **Find the flip-flops**
- **Outputs of the flip-flops = present state variables**
- **Inputs of the flip-flops determine the next state variables**
- **Determine the logical expressions for the outputs**
- **Given this info it is easy to do the state-assigned table**
- **Next do the state table**
- **Finally, draw the state diagram.**

Where are the inputs and outputs?

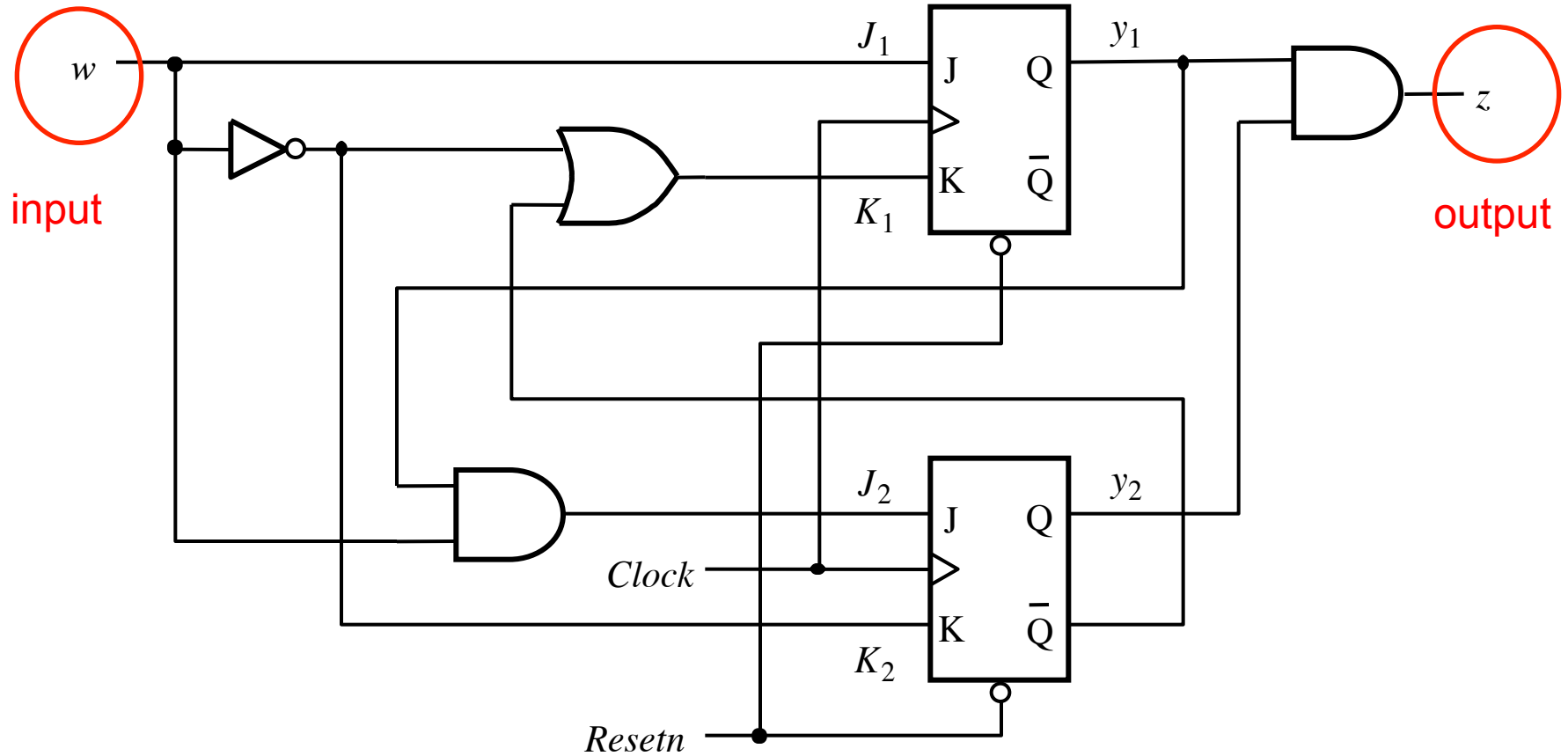


[Figure 6.77 from the textbook]

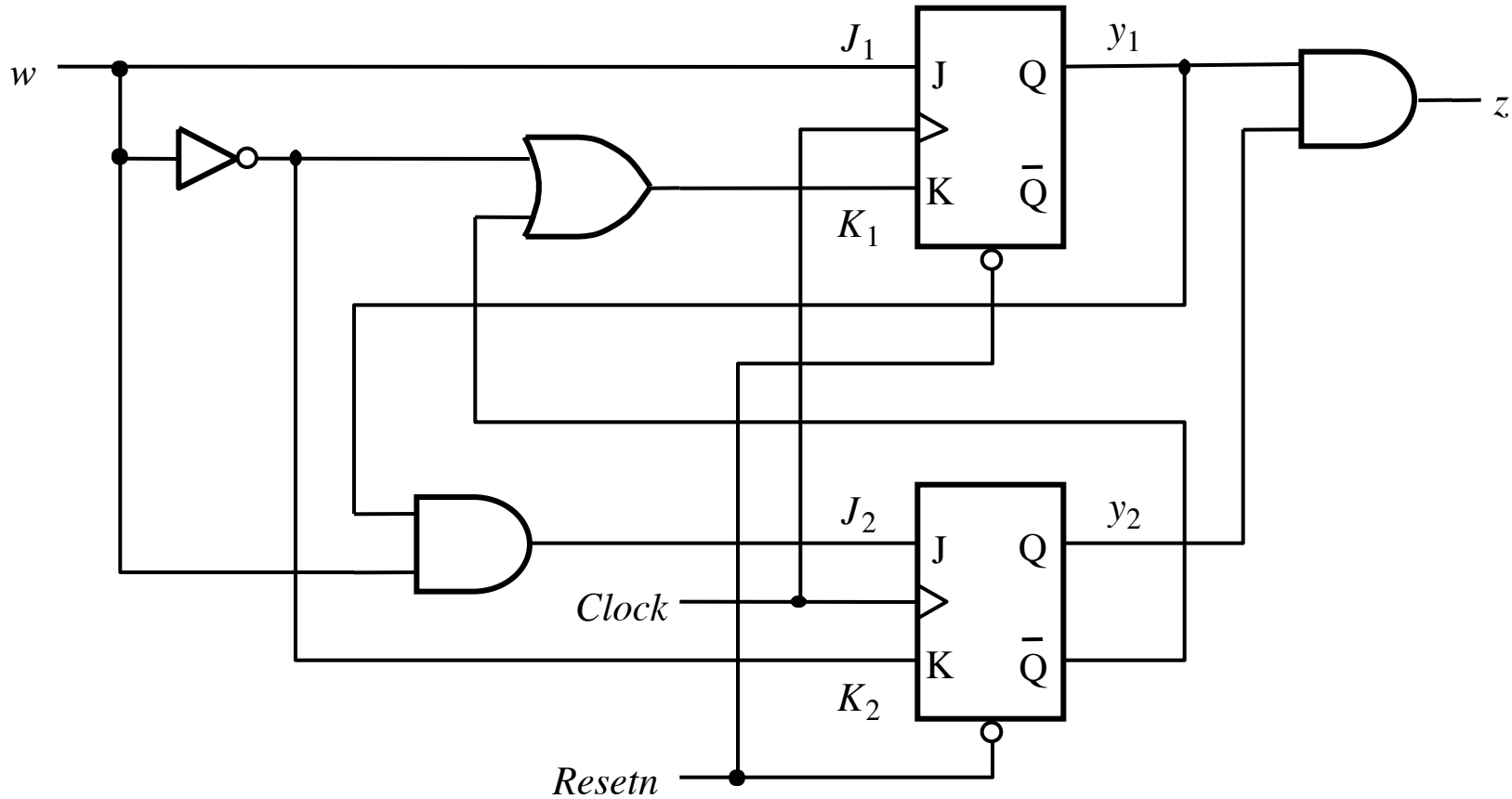
Where are the inputs and outputs?



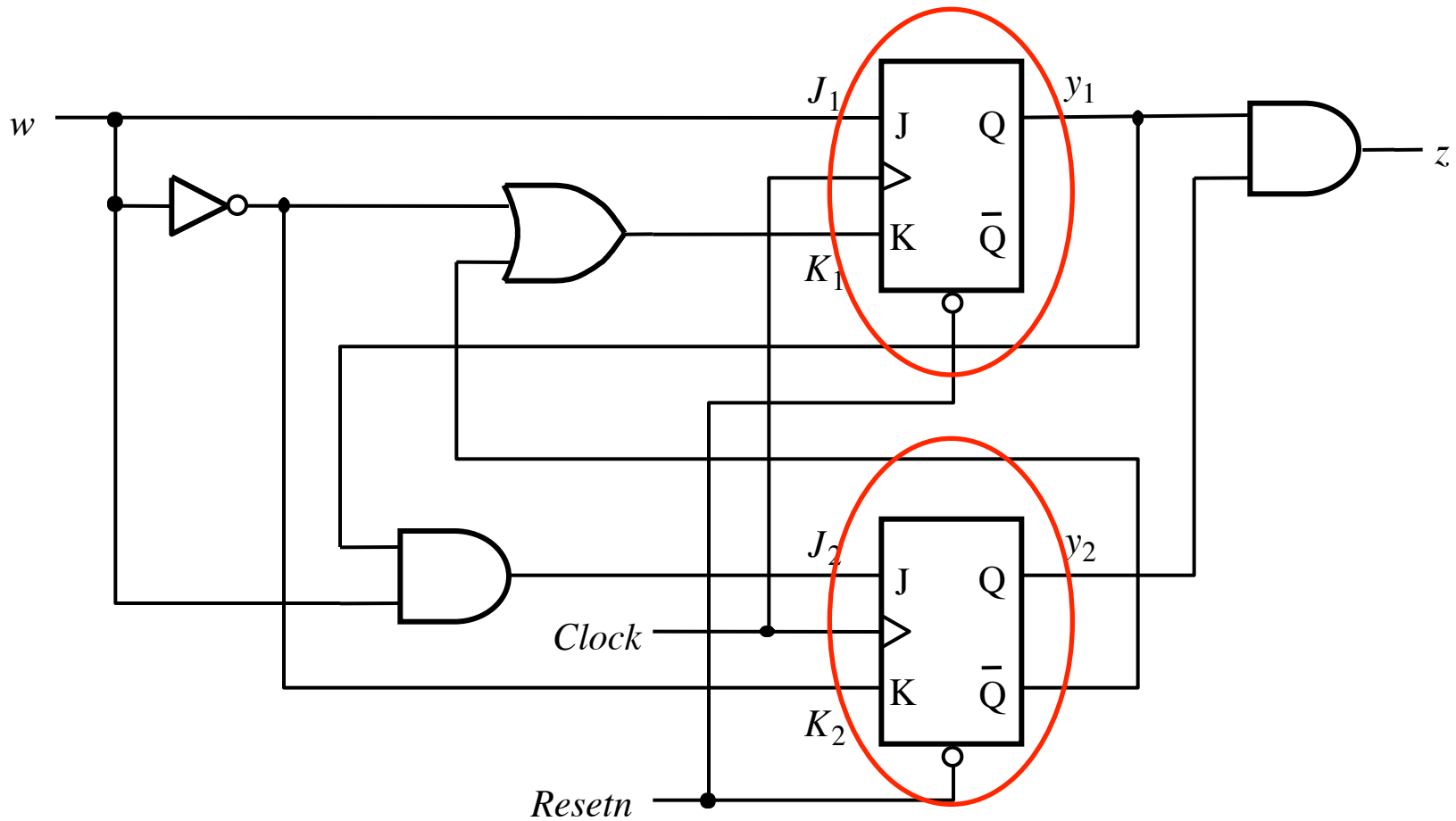
What kind of machine is this?



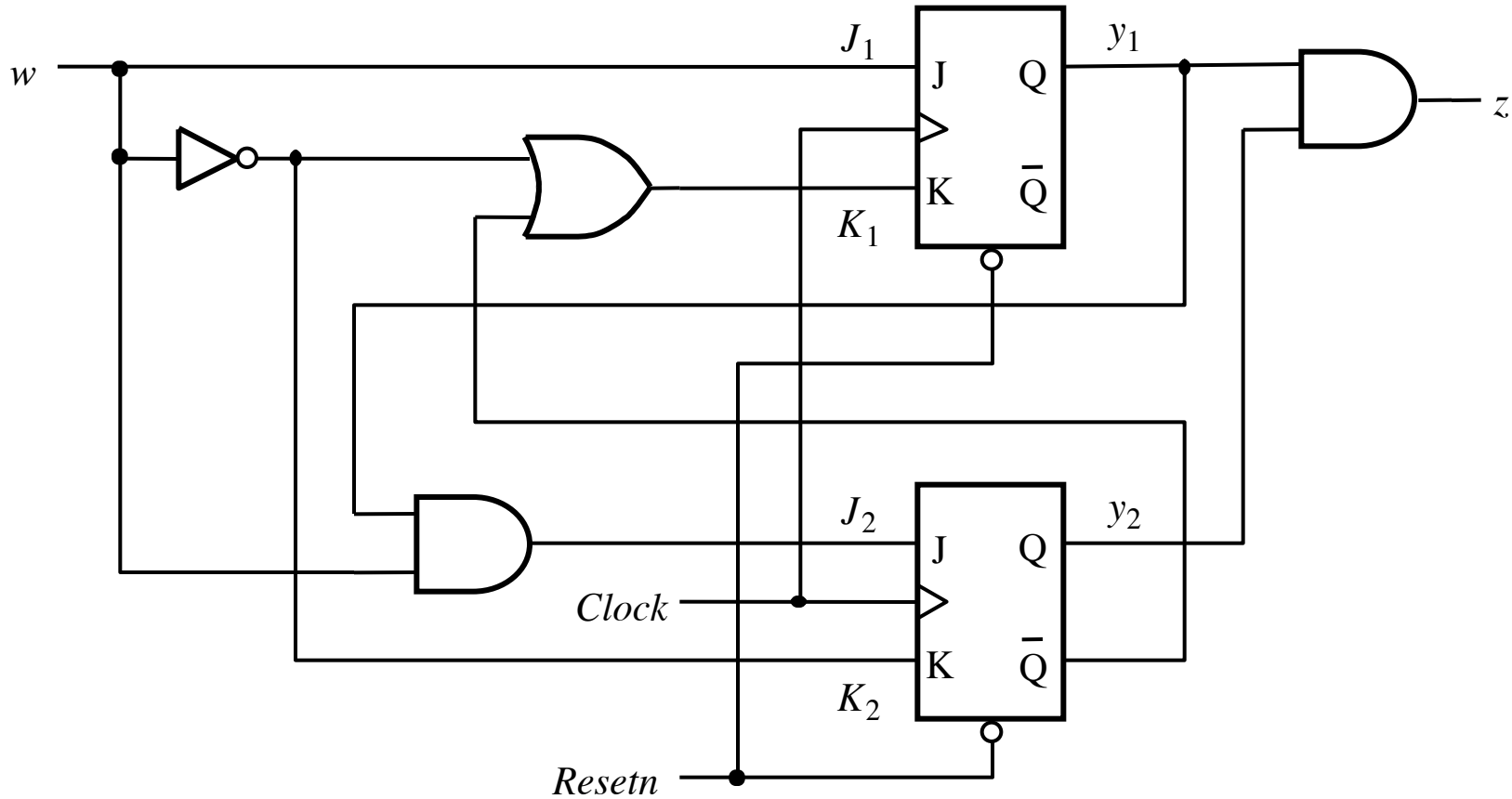
Where are the flip-flops?



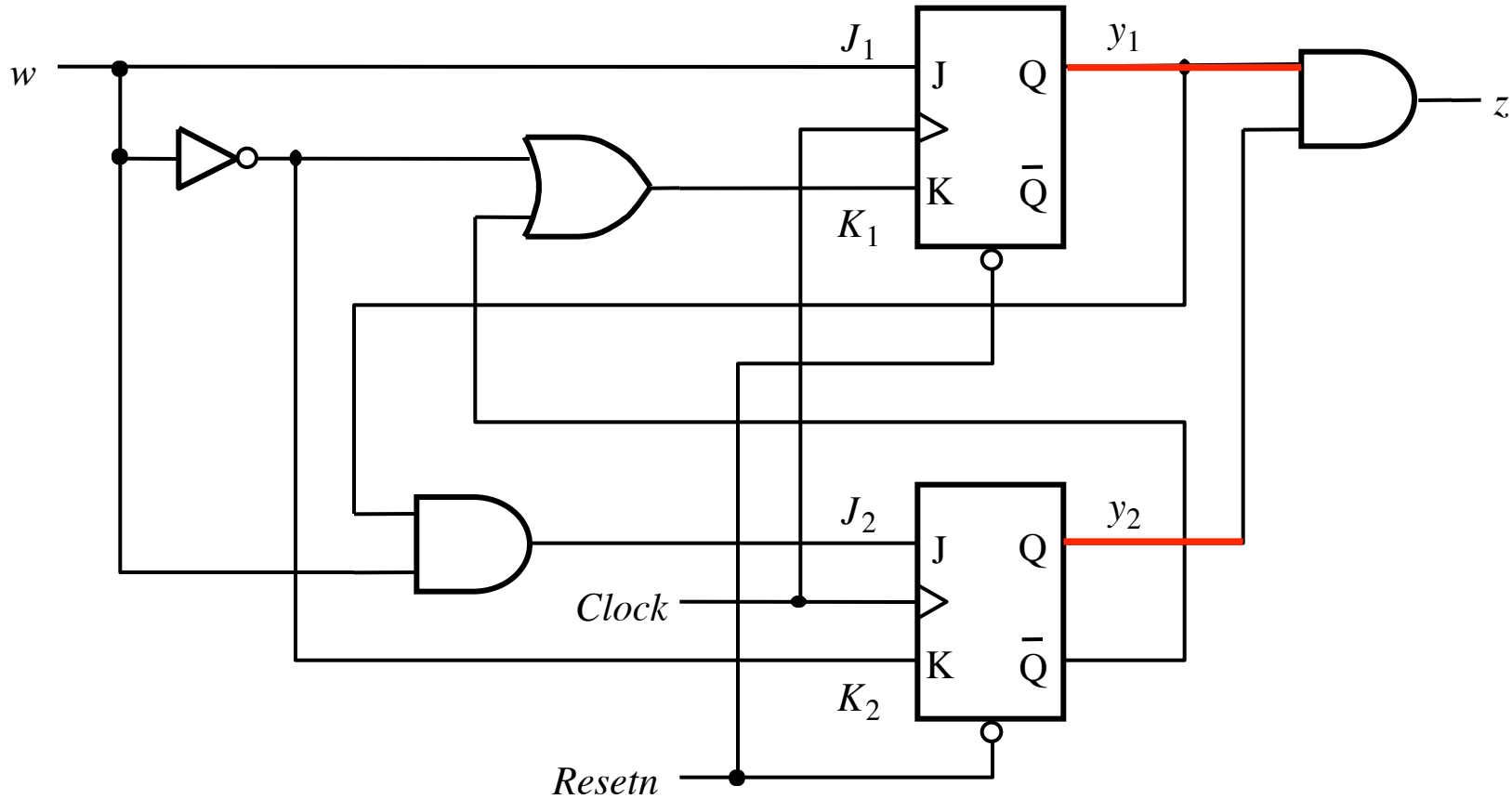
Where are the flip-flops?



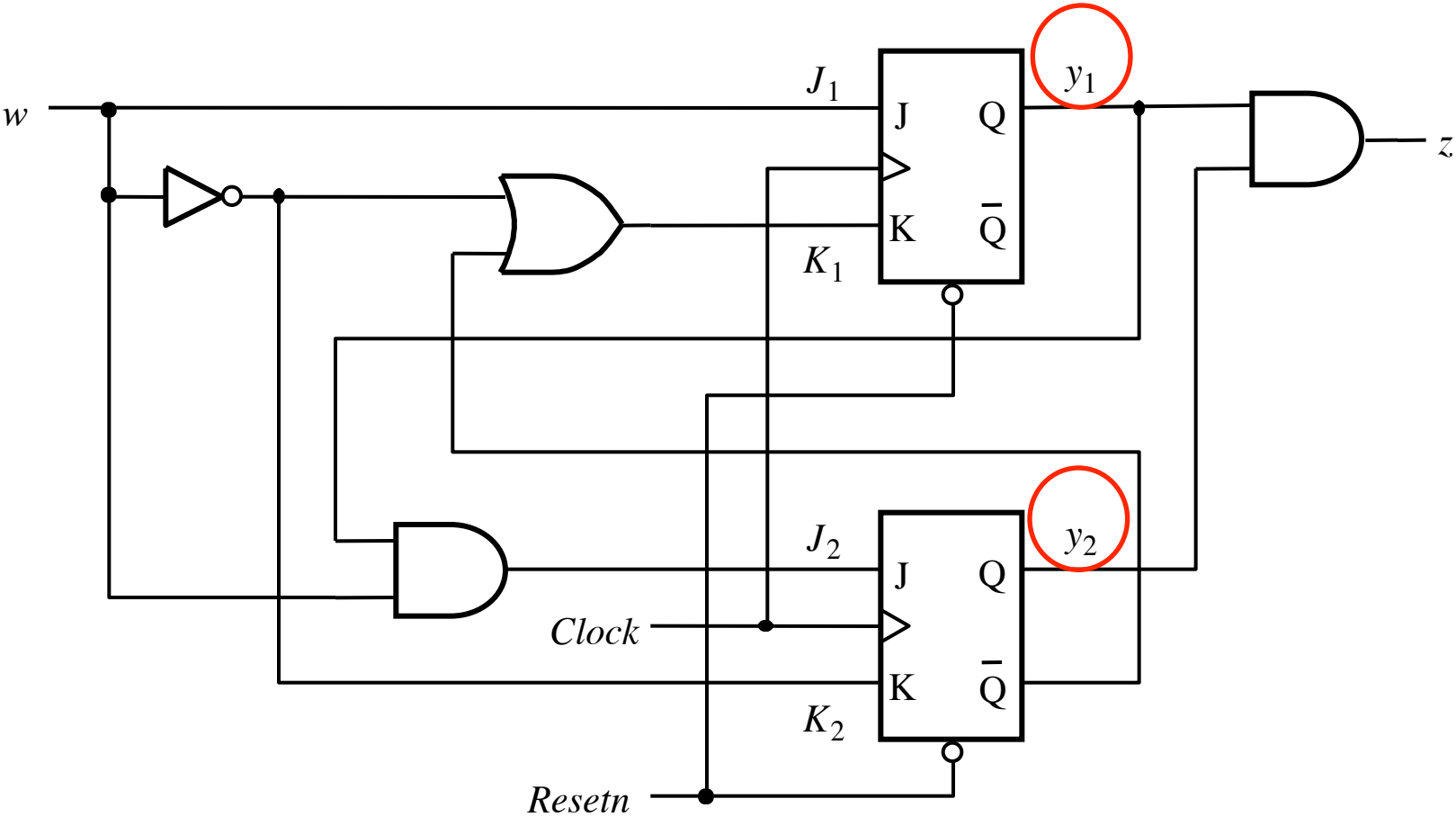
Where are the outputs of the flip-flops?



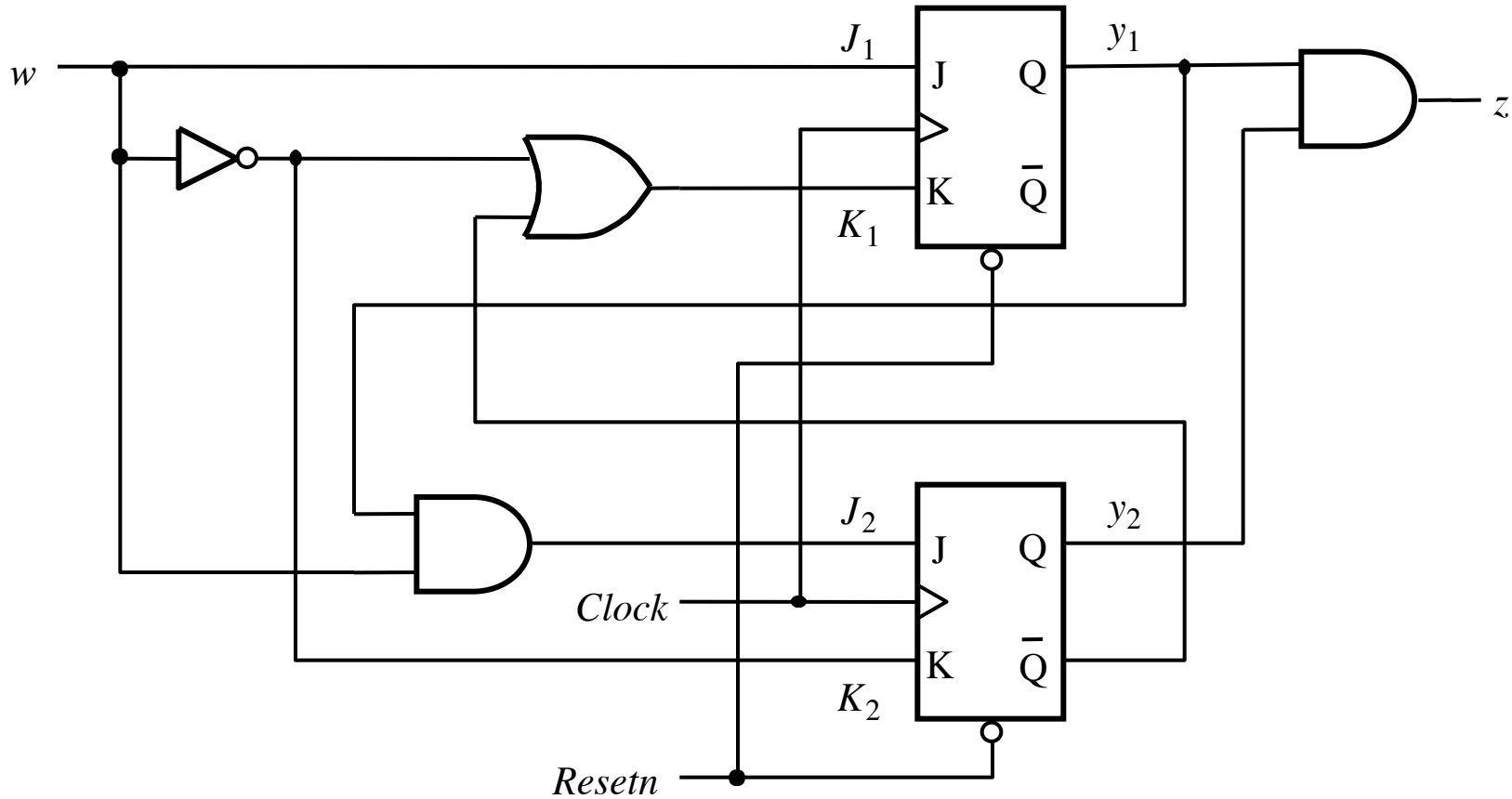
Where are the outputs of the flip-flops?



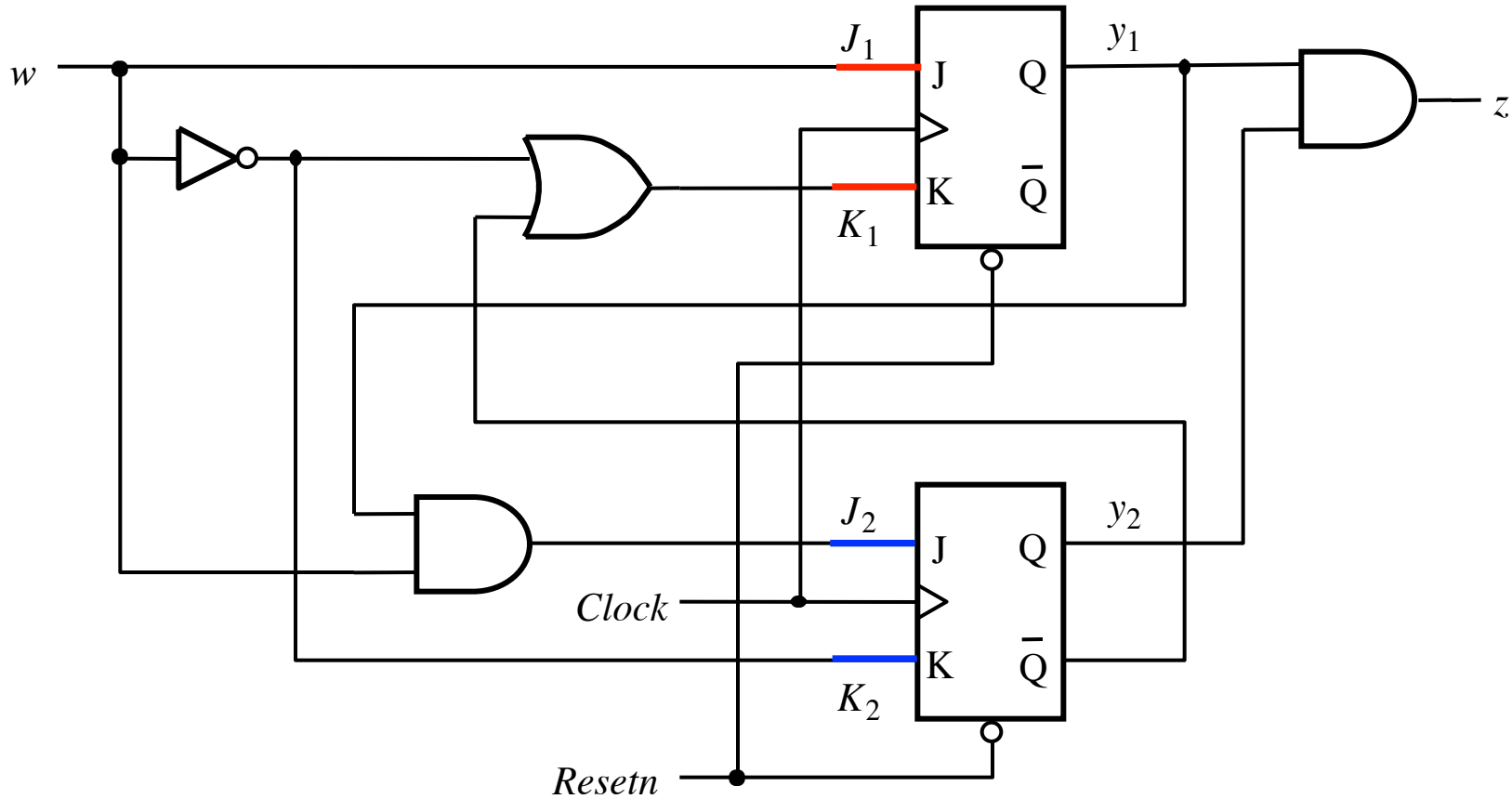
These are the next-state variables



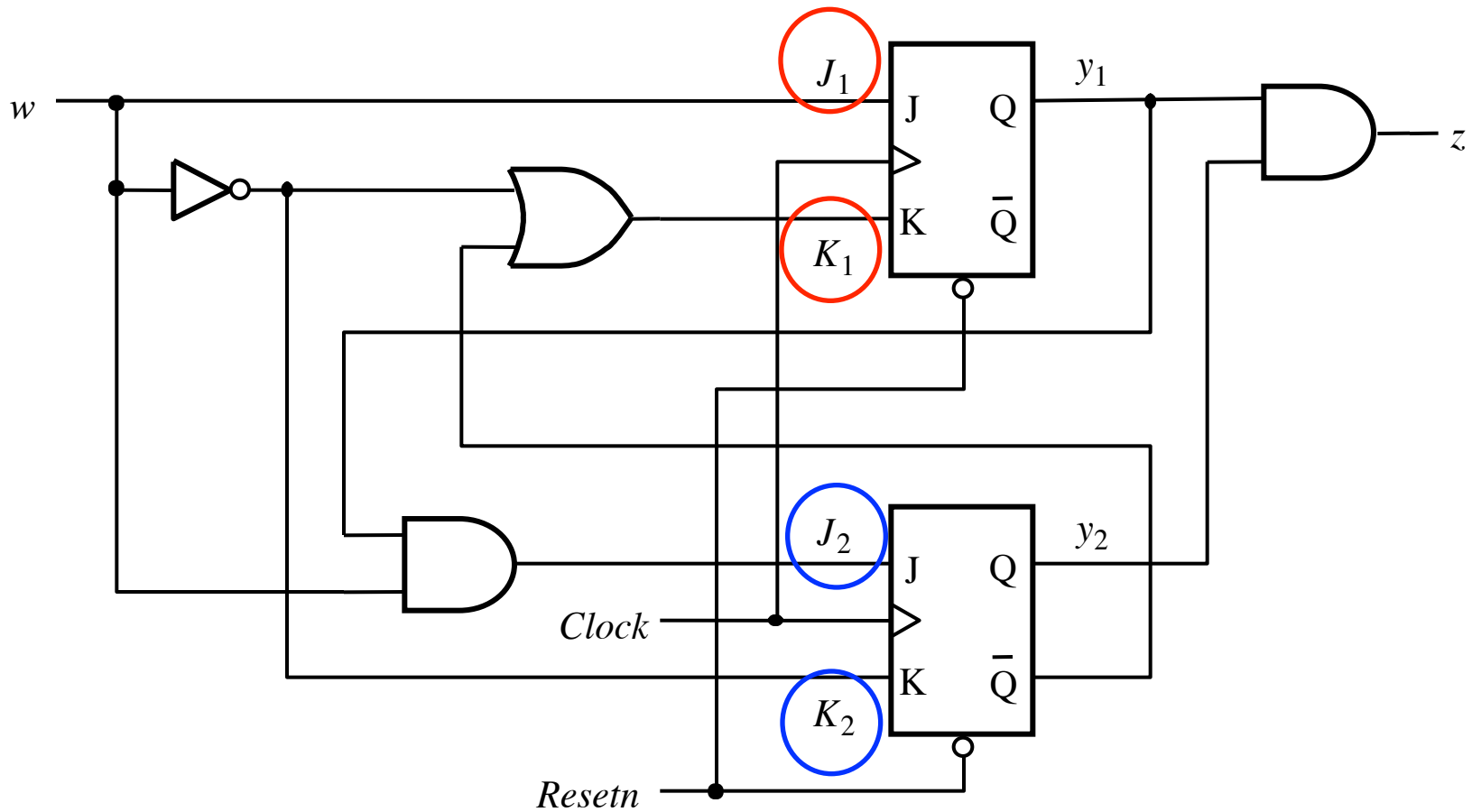
Where are the inputs of the flip-flops?



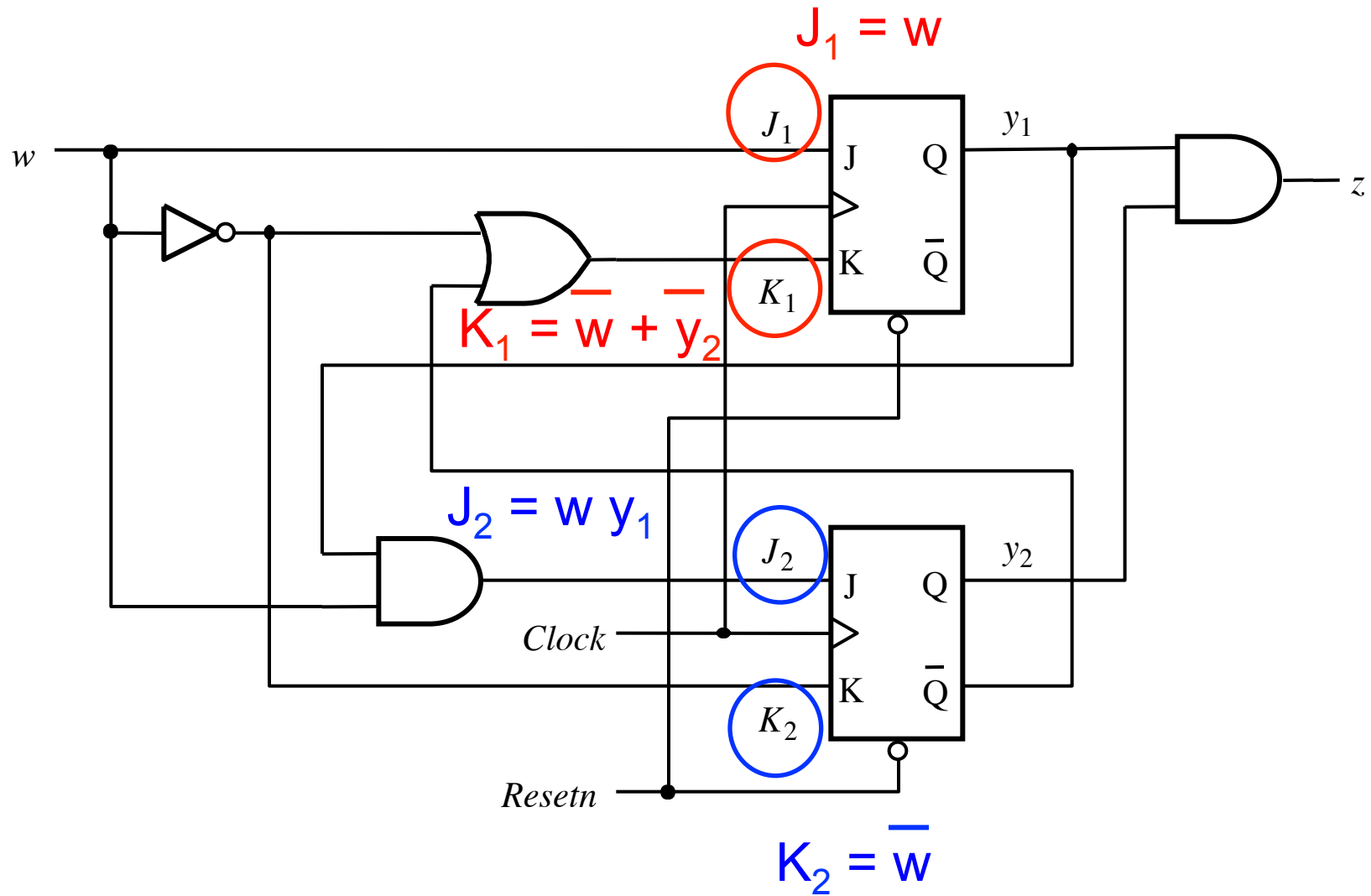
Where are the inputs of the flip-flops?



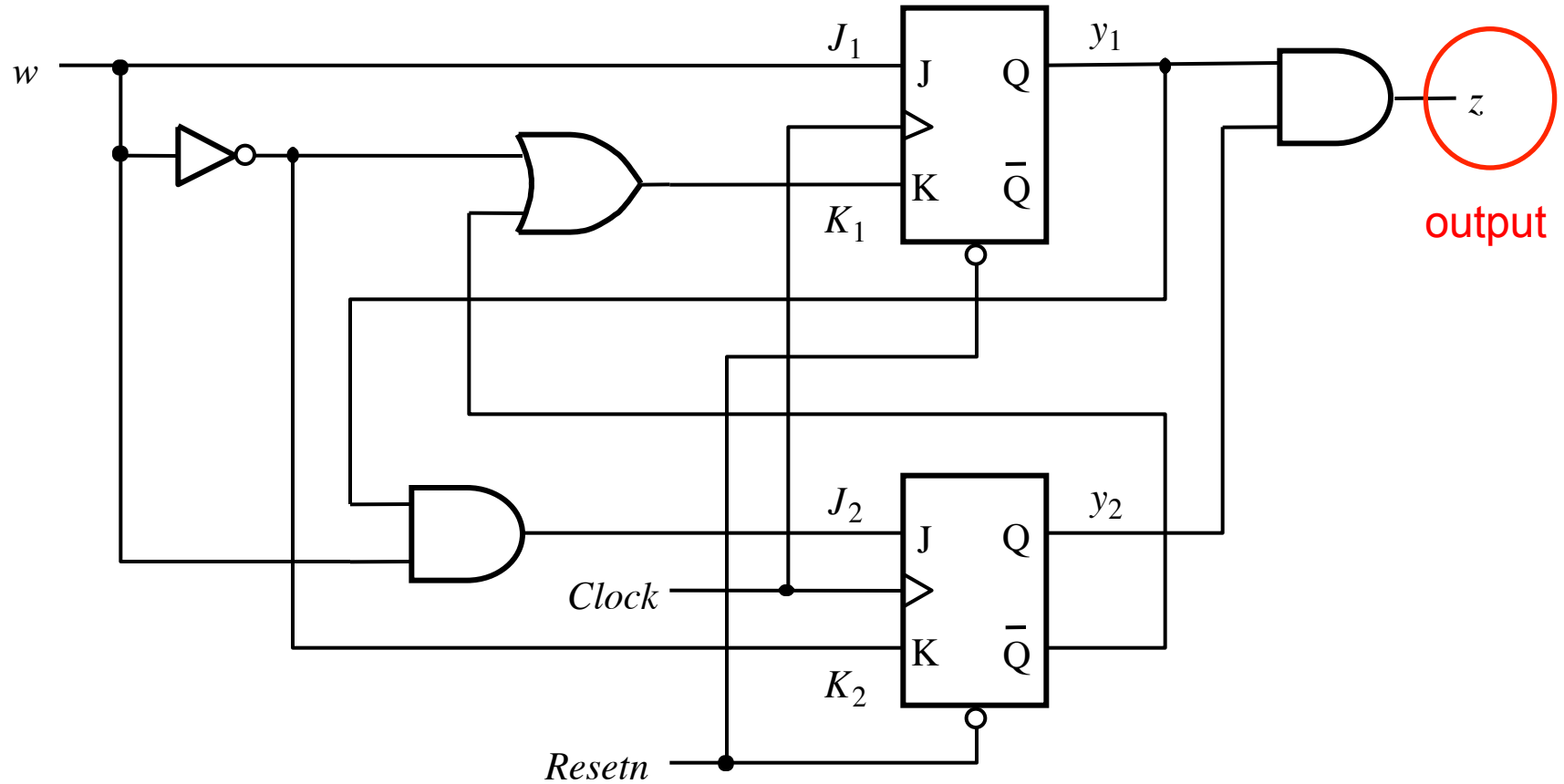
What are their logic expressions?



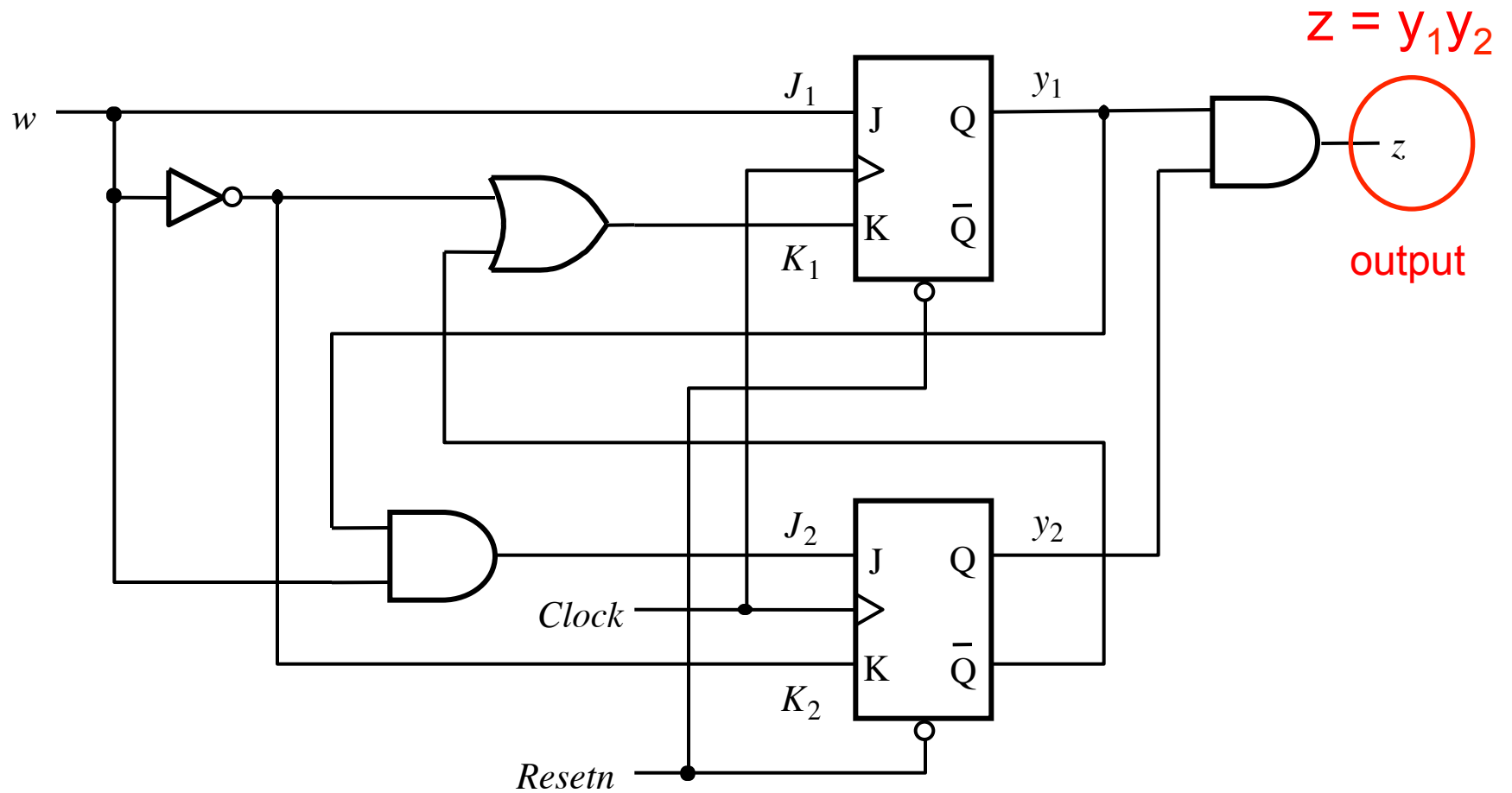
What are their logic expressions?



What is the logic expression of the output?



What is the logic expression of the output?



**This is what we have to work with now
(we don't need the circuit anymore)**

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

$$z = y_1 y_2$$

Let's derive the excitation table

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

$$z = y_1 y_2$$

Present state $y_2 y_1$	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	$J_2 K_2$	$J_1 K_1$	$J_2 K_2$	$J_1 K_1$	
00					
01					
10					
11					

Let's derive the excitation table

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

Present state y_2y_1	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00					
01					
10					
11					

$$z = y_1y_2$$

Let's derive the excitation table

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

Present state y_2y_1	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00					0
01					0
10					0
11					1

$$z = y_1y_2$$

Let's derive the excitation table

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

$$z = y_1 y_2$$

Present state $y_2 y_1$	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	$J_2 K_2$	$J_1 K_1$	$J_2 K_2$	$J_1 K_1$	
00					0
01					0
10					0
11					1

Let's derive the excitation table

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

$$z = y_1 y_2$$

Present state $y_2 y_1$	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	$J_2 K_2$	$J_1 K_1$	$J_2 K_2$	$J_1 K_1$	
00		01		11	0
01		01		11	0
10		01		10	0
11		01		10	1

Let's derive the excitation table

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

$$z = y_1 y_2$$

Present state $y_2 y_1$	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	$J_2 K_2$	$J_1 K_1$	$J_2 K_2$	$J_1 K_1$	
00		01		11	0
01		01		11	0
10		01		10	0
11		01		10	1

The excitation table

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

$$z = y_1 y_2$$

Present state $y_2 y_1$	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	$J_2 K_2$	$J_1 K_1$	$J_2 K_2$	$J_1 K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

We don't need the logic expressions anymore

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

$$z = y_1 y_2$$

Present state $y_2 y_1$	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	$J_2 K_2$	$J_1 K_1$	$J_2 K_2$	$J_1 K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

We don't need the logic expressions anymore

Present state y_2y_1	Flip-flop inputs				Output z
	$w = 0$		$w = 1$		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	

State table

Present state y_2y_1	Flip-flop inputs				Output z
	w = 0		w = 1		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Excitation table

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			
B			
C			
D			

Present state y_2y_1	Flip-flop inputs				Output z
	w = 0		w = 1		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

State table

Excitation table

This step is easy
(map 2-bit numbers to 4 letters)

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs				Output z
	w = 0		w = 1		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

State table

Excitation table

This step is easy too
(the outputs are the same in both tables)

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	?		0
B			0
C			0
D			1

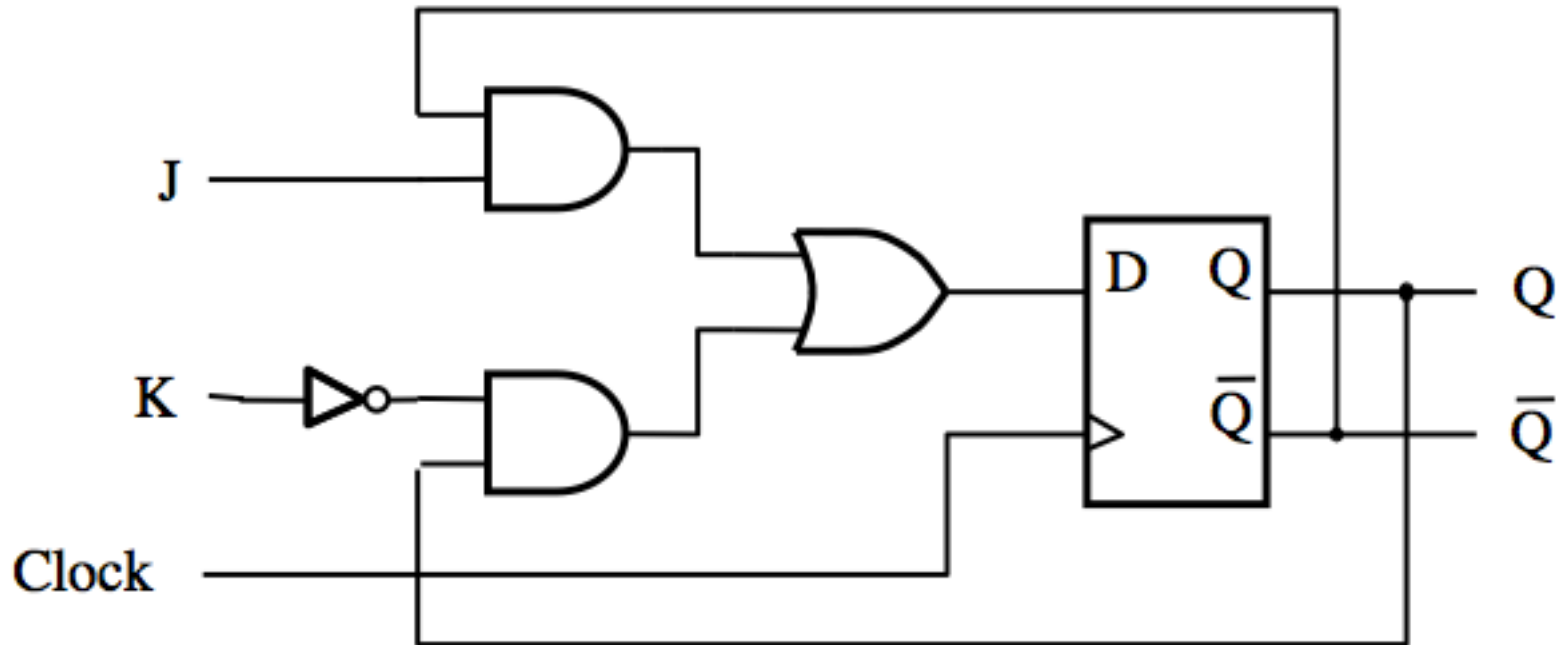
State table

Present state y_2y_1	Flip-flop inputs				Output z
	w = 0		w = 1		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Excitation table

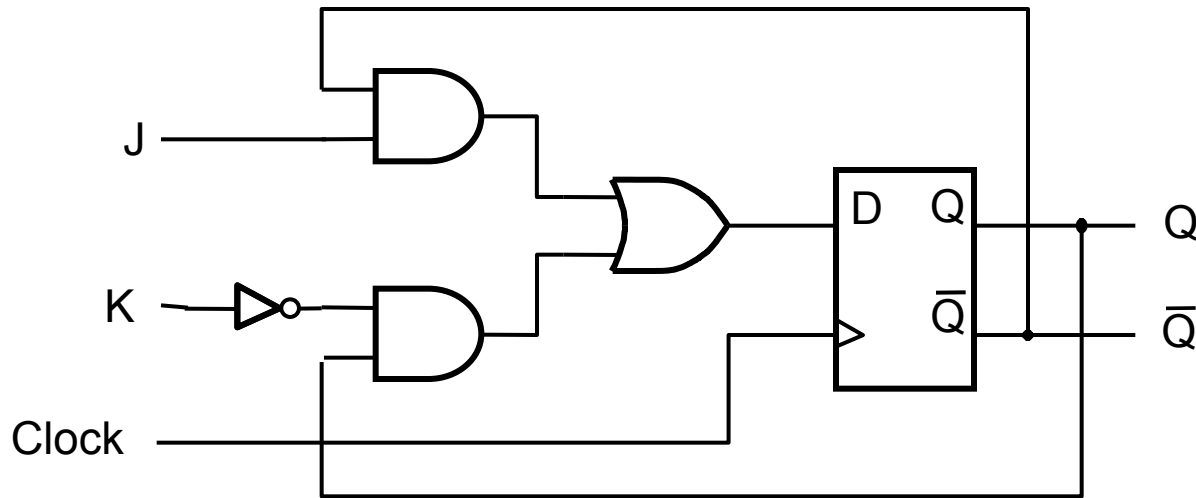
How should we do this?

JK Flip-Flop Refresher



$$D = \bar{J}\bar{Q} + \bar{K}Q$$

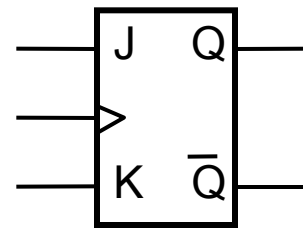
JK Flip-Flop Refresher



(a) Circuit

J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$\bar{Q}(t)$

(b) Truth table



(c) Graphical symbol

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	?		0
B			0
C			0
D			1

State table

Present state y_2y_1	Flip-flop inputs				Output z
	w = 0		w = 1		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Excitation table

How should we do this?

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs				Output z
	w = 0		w = 1		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J K	Q(t+1)
0 0	Q(t)
0 1	0
1 0	1
1 1	$\bar{Q}(t)$

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs				Output z
	w = 0		w = 1		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$\bar{Q}(t)$

J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$\bar{Q}(t)$

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs				Output z
	w = 0		w = 1		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Note that A = 00

J	K	$Q(t+1)$	J	K	$Q(t+1)$
0	0	$Q(t)$	0	0	$Q(t)$
0	1	0	0	1	0
1	0	1	1	0	1
1	1	$\bar{Q}(t)$	1	1	$\bar{Q}(t)$

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A	?	0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs				Output z
	w = 0		w = 1		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$\bar{Q}(t)$

J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$\bar{Q}(t)$

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs				Output z
	w = 0		w = 1		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J	K	$Q(t+1)$	J	K	$Q(t+1)$
0	0	$Q(t)$	0	0	$Q(t)$
0	1	0	0	1	0
1	0	1	1	0	1
1	1	$\bar{Q}(t)$	1	1	$\bar{Q}(t)$

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs				Output z
	w = 0		w = 1		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J	K	$Q(t+1)$	J	K	$Q(t+1)$
0	0	$Q(t)$	0	0	$Q(t)$
0	1	0	0	1	0
1	0	1	1	0	1
1	1	$\bar{Q}(t)$	1	1	$\bar{Q}(t)$

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs				Output z
	w = 0		w = 1		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J	K	$Q(t+1)$	J	K	$Q(t+1)$
0	0	$Q(t)$	0	0	$Q(t)$
0	1	0	0	1	0
1	0	1	1	0	1
1	1	$\bar{Q}(t)$	1	1	$\bar{Q}(t)$

01

10

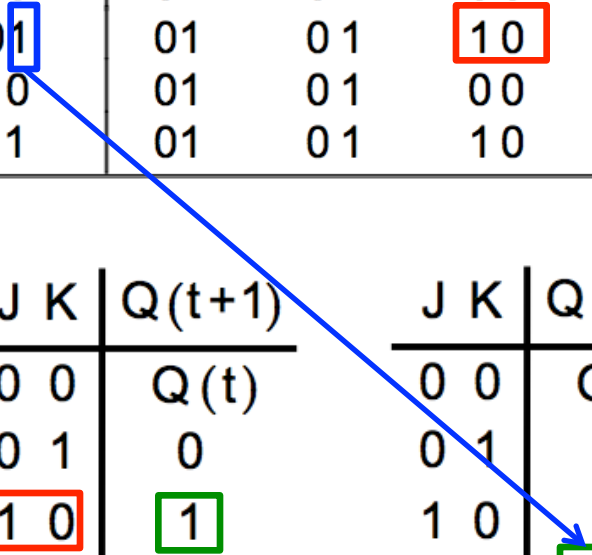
11

10

1

11

$\bar{Q}(t)$



Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs				Output z
	w = 0		w = 1		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J	K	$Q(t+1)$	J	K	$Q(t+1)$
0	0	$Q(t)$	0	0	$Q(t)$
0	1	0	0	1	0
1	0	1	1	0	1
1	1	$\bar{Q}(t)$	1	1	$\bar{Q}(t)$

$\bar{1} = 0$

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B		C	0
C			0
D			1

Present state y_2y_1	Flip-flop inputs				Output z
	w = 0		w = 1		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Note that C = 10

J	K	$Q(t+1)$	J	K	$Q(t+1)$
0	0	$Q(t)$	0	0	$Q(t)$
0	1	0	0	1	0
1	0	1	1	0	1
1	1	$\bar{Q}(t)$	1	1	$\bar{Q}(t)$ = 0

The two tables for the initial circuit

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table

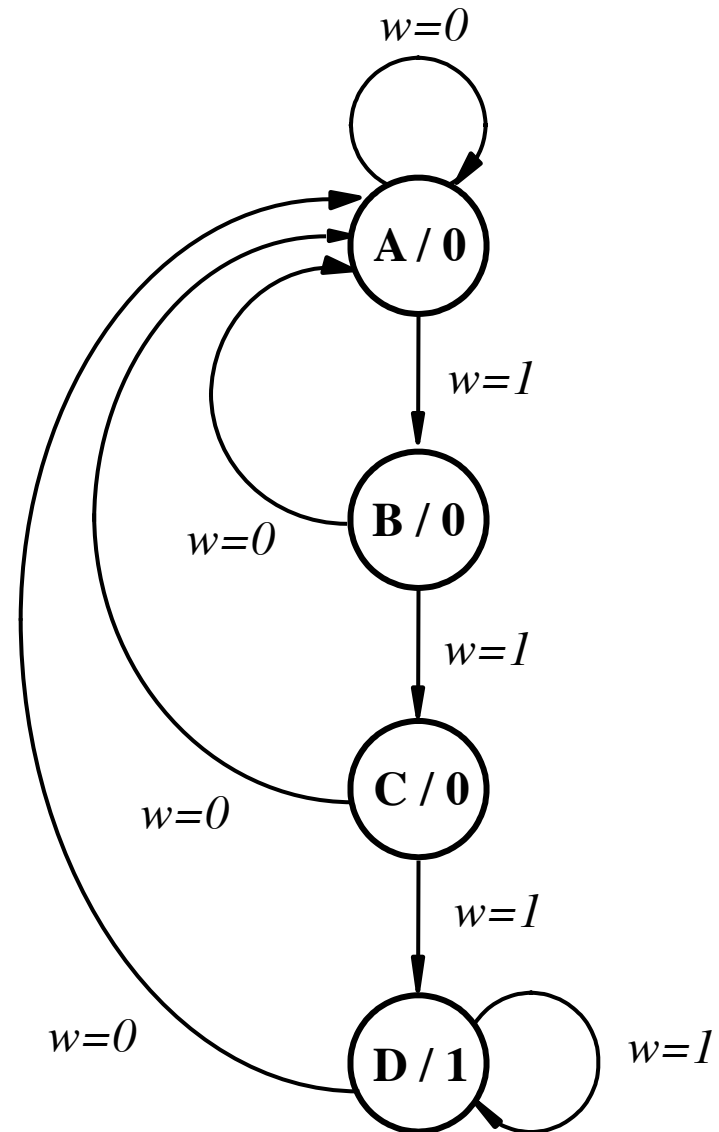
Present state y_2y_1	Flip-flop inputs				Output z
	w = 0		w = 1		
	J_2K_2	J_1K_1	J_2K_2	J_1K_1	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Excitation table

The state diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table



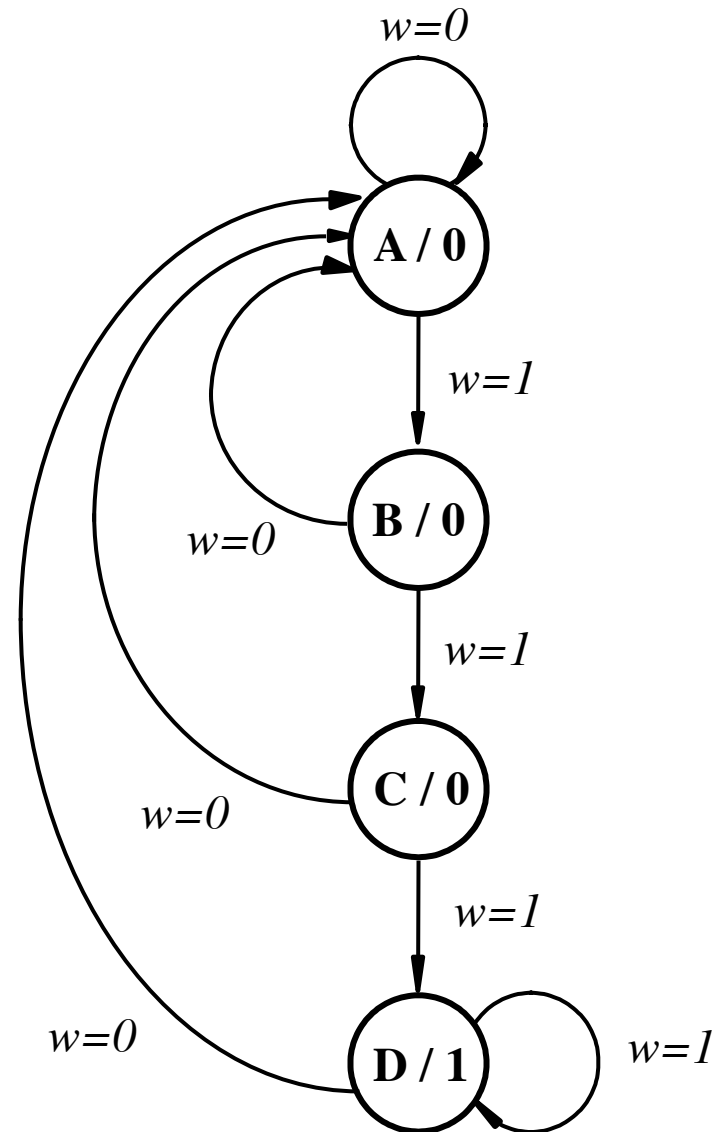
State diagram

The state diagram

Thus, this FSM is identical to the one in the previous example, even though the circuit uses JK flip-flops.

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

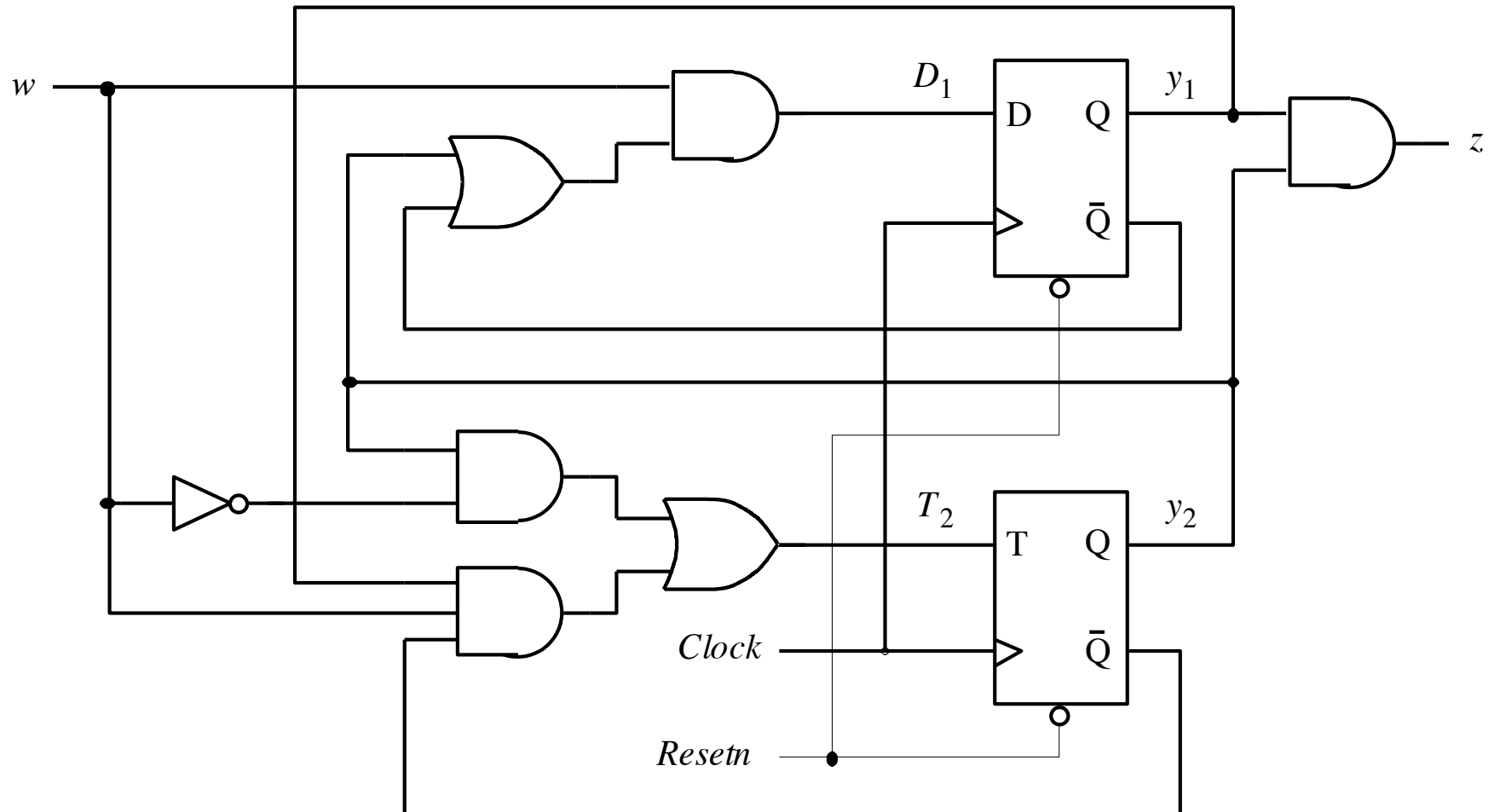
State table



State diagram

Yet Another Example (with mixed flip-flops)

What does this circuit do?

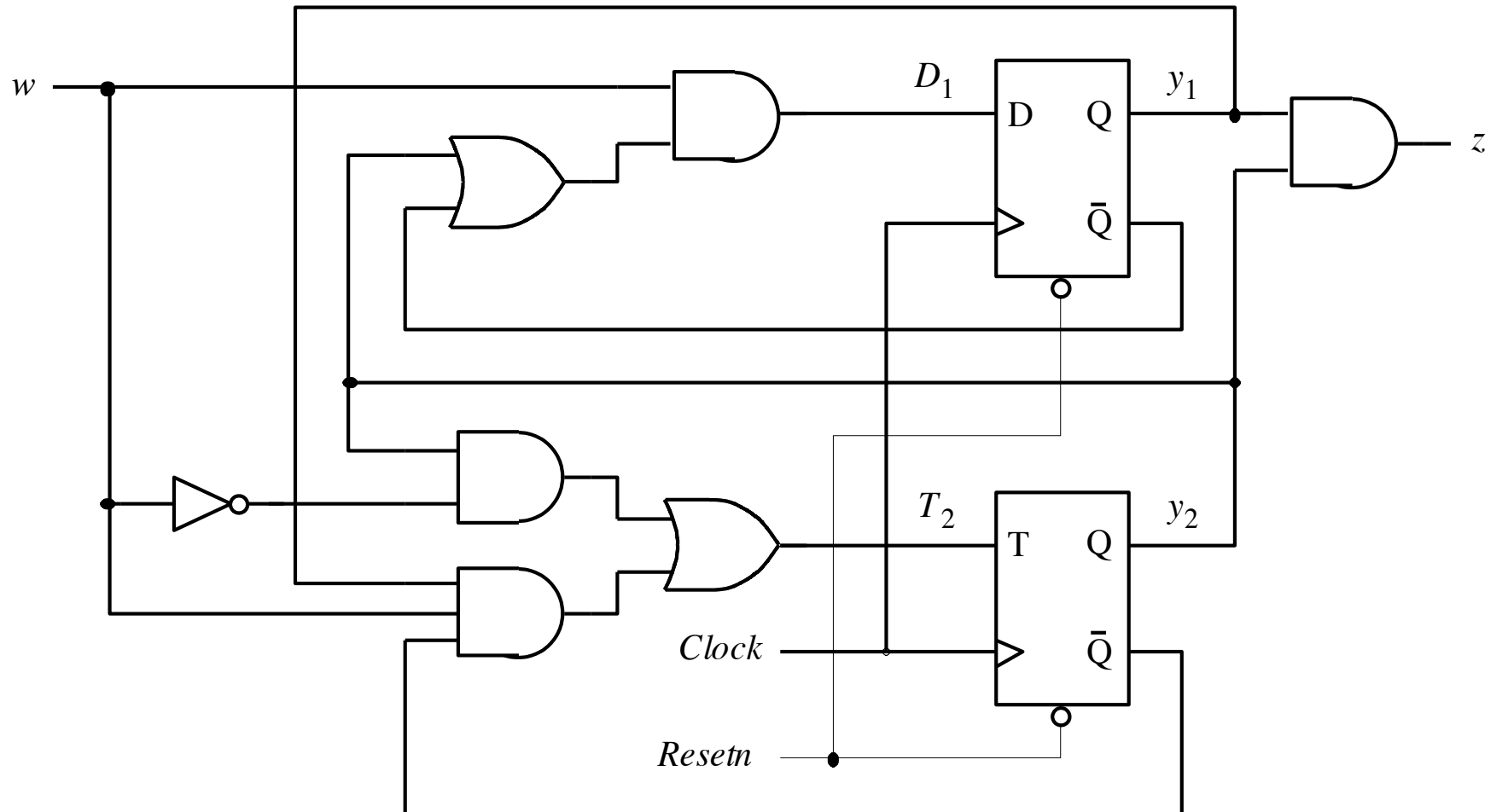


[Figure 6.79 from the textbook]

Approach

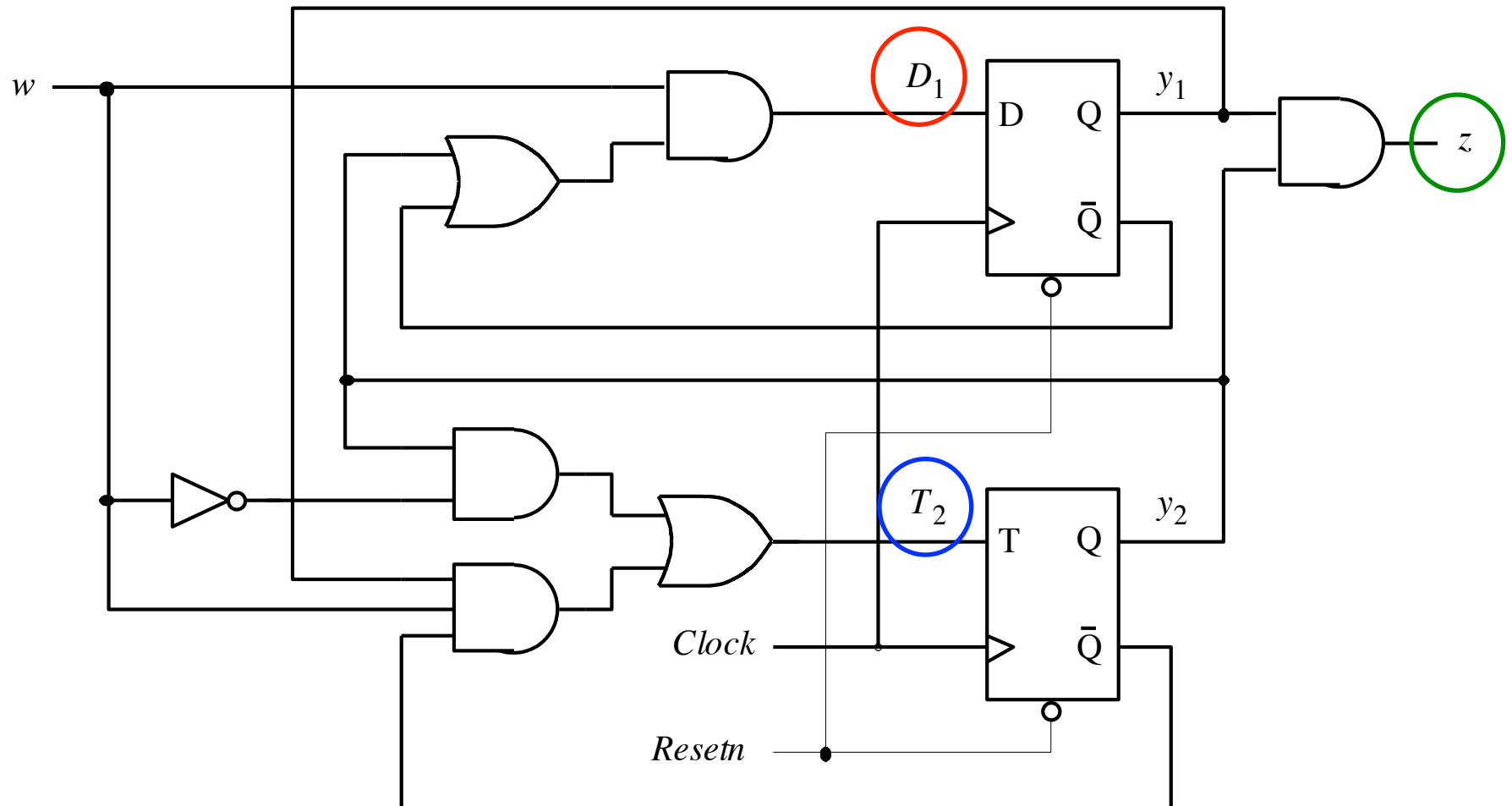
- **Find the flip-flops**
- **Outputs of the flip-flops = present state variables**
- **Inputs of the flip-flops determine the next state variables**
- **Determine the logical expressions for the outputs**
- **Given this info it is easy to do the state-assigned table**
- **Next do the state table**
- **Finally, draw the state diagram.**

What are the logic expressions?



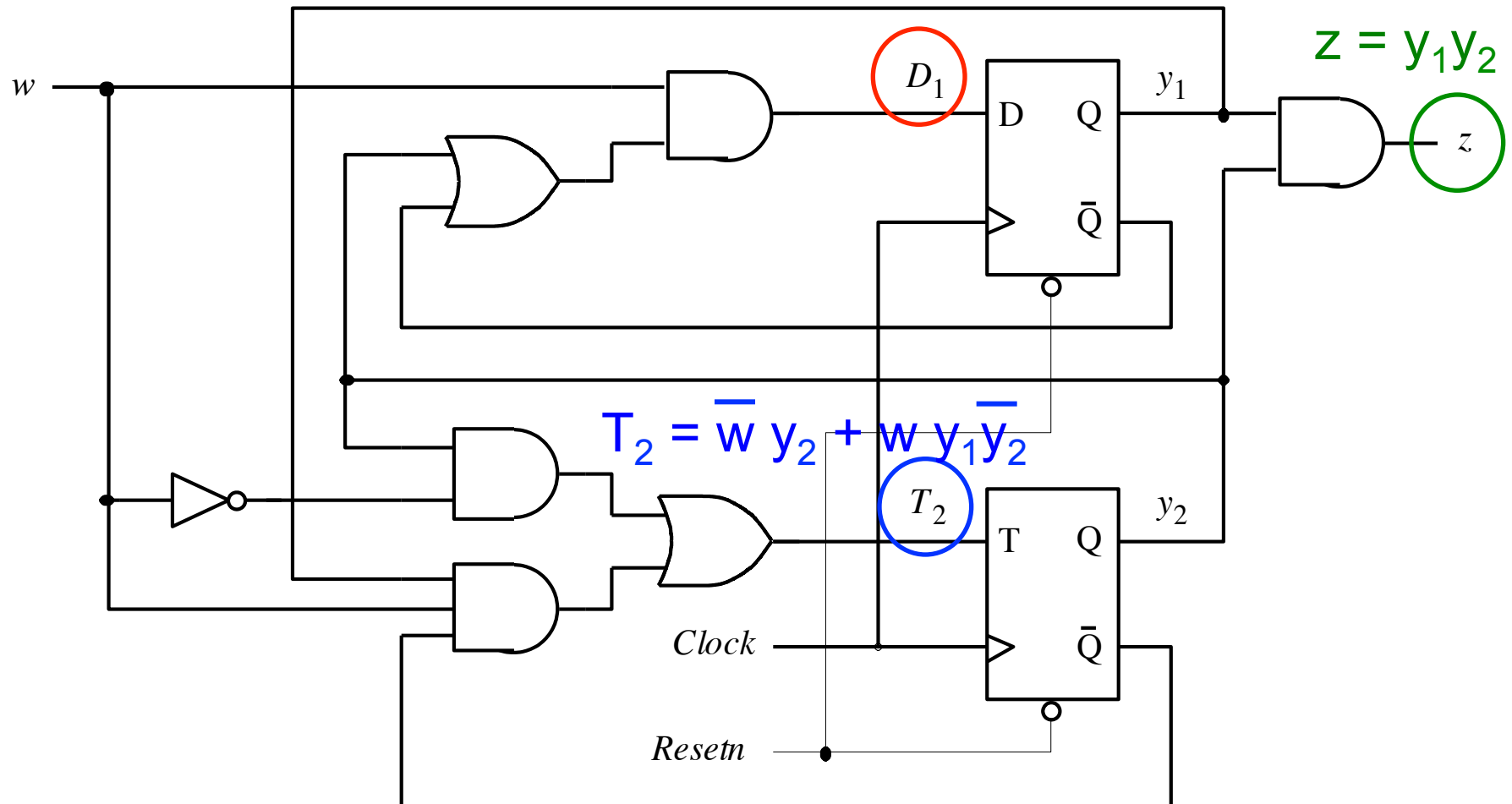
[Figure 6.79 from the textbook]

What are the logic expressions?



What are the logic expressions?

$$D_1 = w (\bar{y}_1 + y_2)$$



The Excitation Table

$$D_1 = w (\bar{y}_1 + y_2)$$

$$T_2 = \bar{w} y_2 + w y_1 \bar{y}_2$$

$$z = y_1 y_2$$

Present state $y_2 y_1$	Flip-flop inputs		Output z
	$w = 0$	$w = 1$	
	$T_2 D_1$	$T_2 D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

Excitation table

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			
B			
C			
D			

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

This step is easy
(map 2-bit numbers to 4 letters)

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

This step is easy too
(the outputs are the same in both tables)

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	?		0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

What should we do here?

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	?		0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

What should we do here?

T	$Q(t+1)$	D	$Q(t+1)$
0	$Q(t)$	0	0
1	$\overline{Q(t)}$	1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	$Q(t)$	0	0
1	$\overline{Q(t)}$	1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	$\overline{Q(t)}$	0	0
1	$Q(t)$	1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	$Q(t)$	0	0
1	$\overline{Q(t)}$	1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	0	0	0
1	$\overline{Q(t)}$	1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	0	0	0
1	$\overline{Q(t)}$	1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	0	0	0
1	$\overline{Q(t)}$	1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

Note that A = 00

T	$Q(t+1)$	D	$Q(t+1)$
0	0	0	0
1	$\overline{Q(t)}$	1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B	A		0
C	?		0
D	A		1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

What should we do here?

T	$Q(t+1)$	D	$Q(t+1)$
0	$Q(t)$	0	0
1	$\overline{Q(t)}$	1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C			0
D			1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	$Q(t)$	0	0
1	$\overline{Q(t)}$	1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B	A		0
C	A		0
D	A		1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	$Q(t)$	0	0
1	$\overline{Q(t)}$	1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B	A		0
C	A		0
D	A		1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	$Q(t)$	0	0
1	$\overline{Q(t)}$	1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B	A		0
C	A		0
D	A		1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	1	0	0
1	$\overline{Q(t)}$	1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B	A		0
C	A		0
D	A		1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	1	0	0
1	$\overline{Q(t)}$	1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B	A		0
C	A		0
D	A		1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	1	0	0
1	$\overline{Q(t)}$	1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B	A		0
C	D		0
D	D		1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

Note that D = 11

T	$Q(t+1)$	D	$Q(t+1)$
0	1	0	0
1	$\overline{Q}(t)$	1	1

Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$		D	$Q(t+1)$
0	$Q(t)$		0	0
1	$\overline{Q(t)}$		1	1

The two tables for the initial circuit

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table

[Figure 6.75b from the textbook]

Present state y_2y_1	Flip-flop inputs		Output z
	w = 0	w = 1	
	T_2D_1	T_2D_1	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

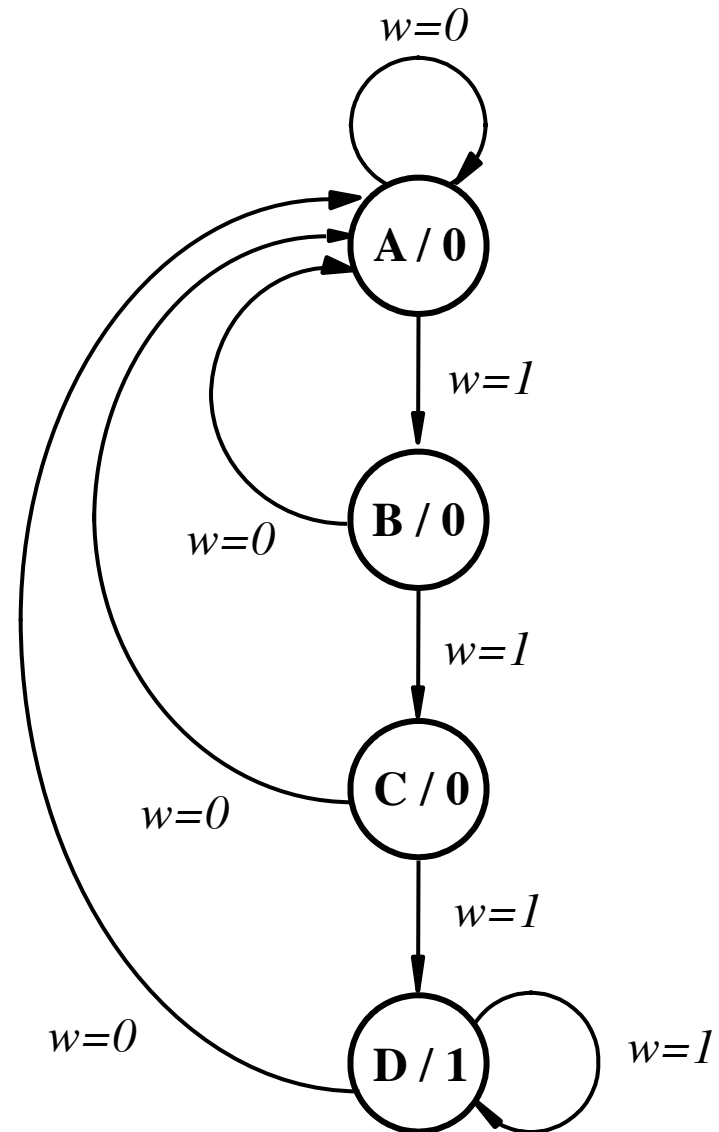
Excitation table

[Figure 6.80 from the textbook]

The state diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table



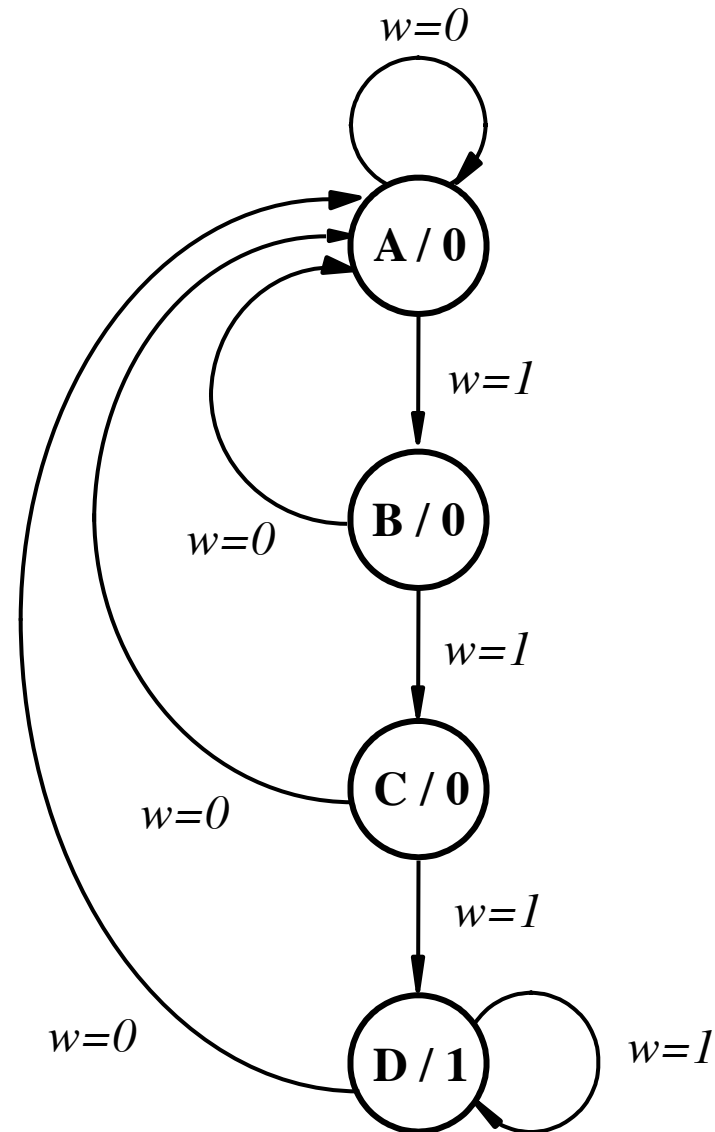
State diagram

The state diagram

Thus, this FSM is identical to the ones in the previous examples, even though the circuit uses JK flip-flops.

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table



State diagram

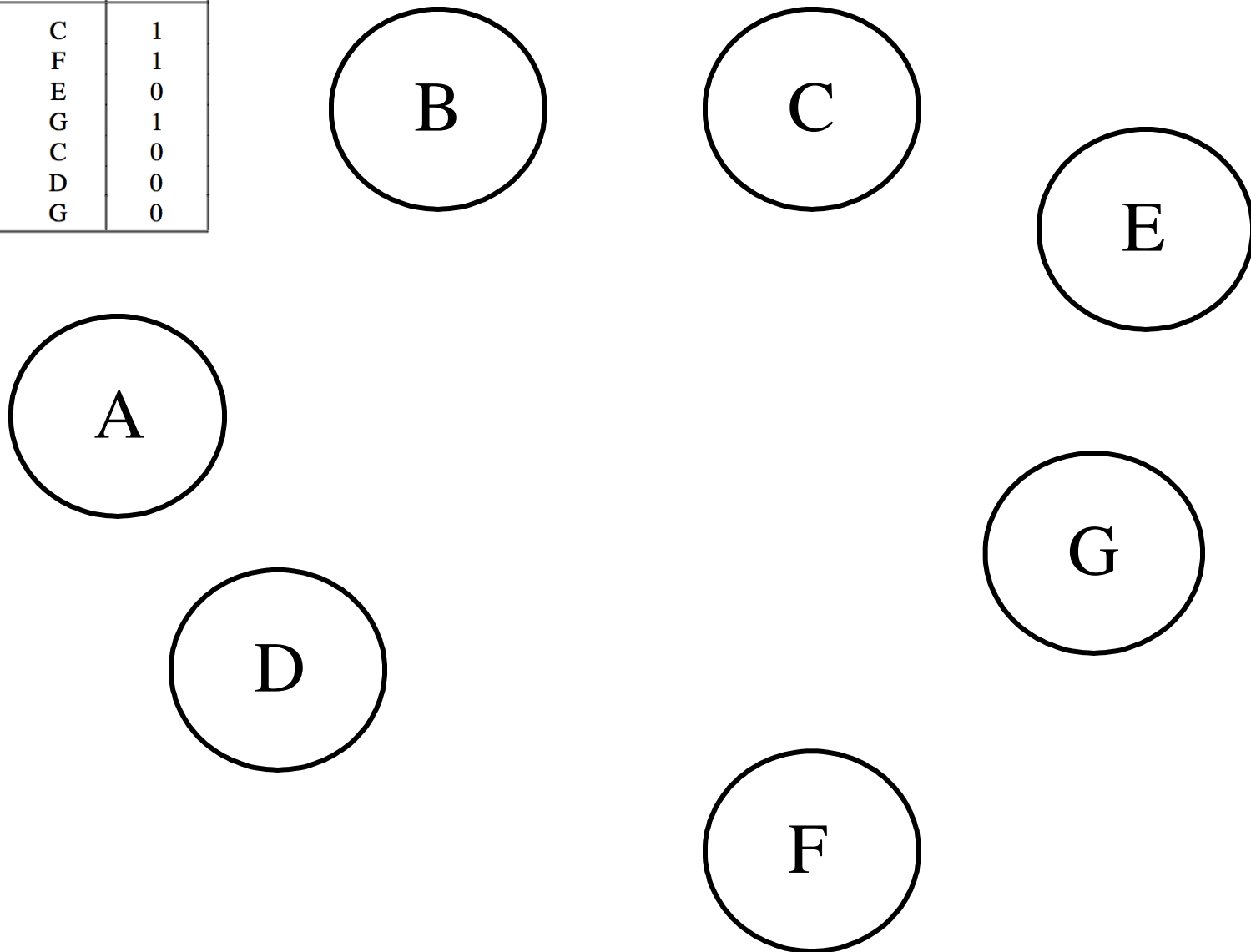
State Minimization

State Table for This Example

Present state	Next state		Output z
	$w = 0$	$w = 1$	
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0

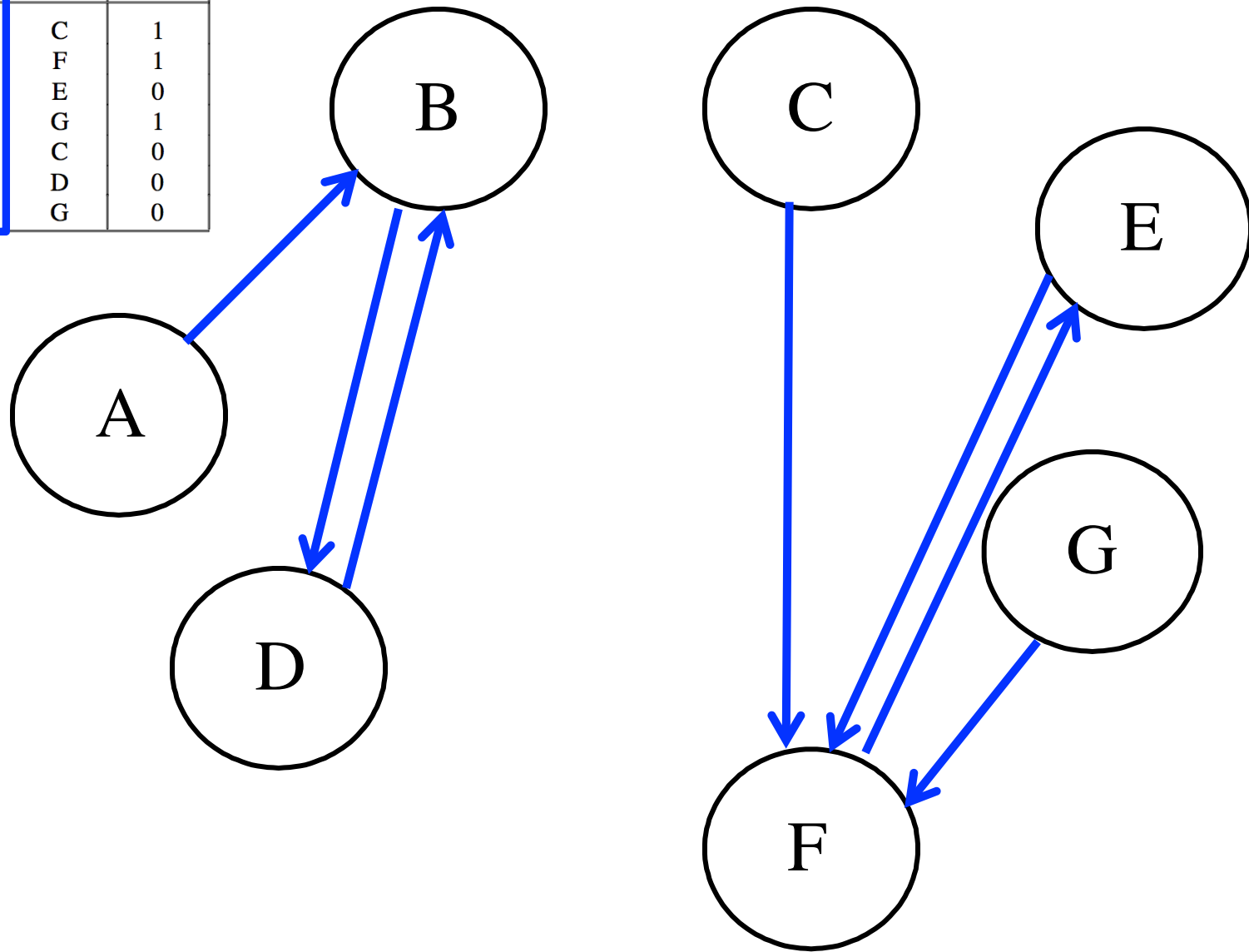
State Diagram (just the states)

Present state	Next state		Output z
	$w = 0$	$w = 1$	
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0



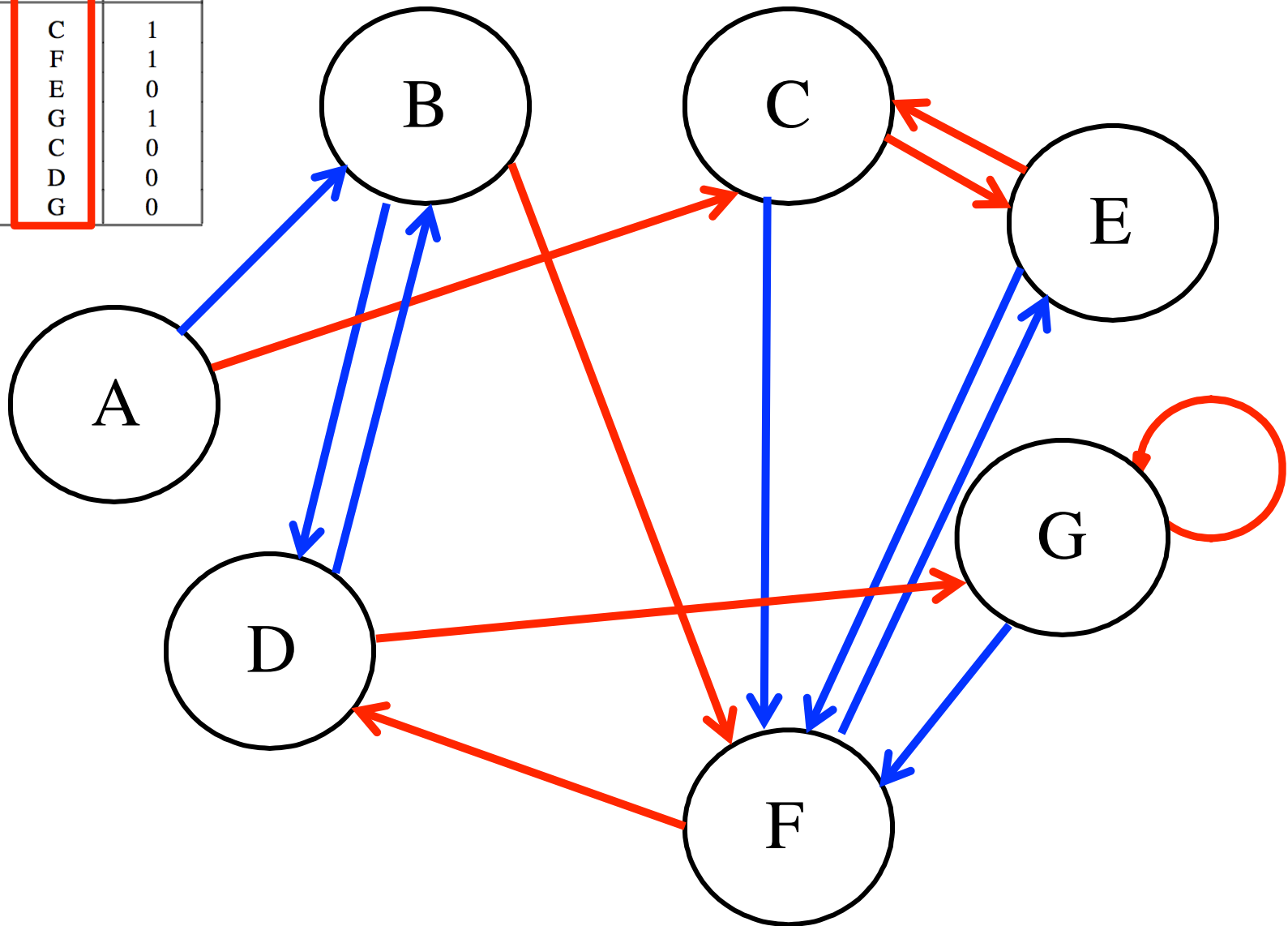
State Diagram (transitions when $w=0$)

Present state	Next state		Output z
	$w = 0$	$w = 1$	
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0



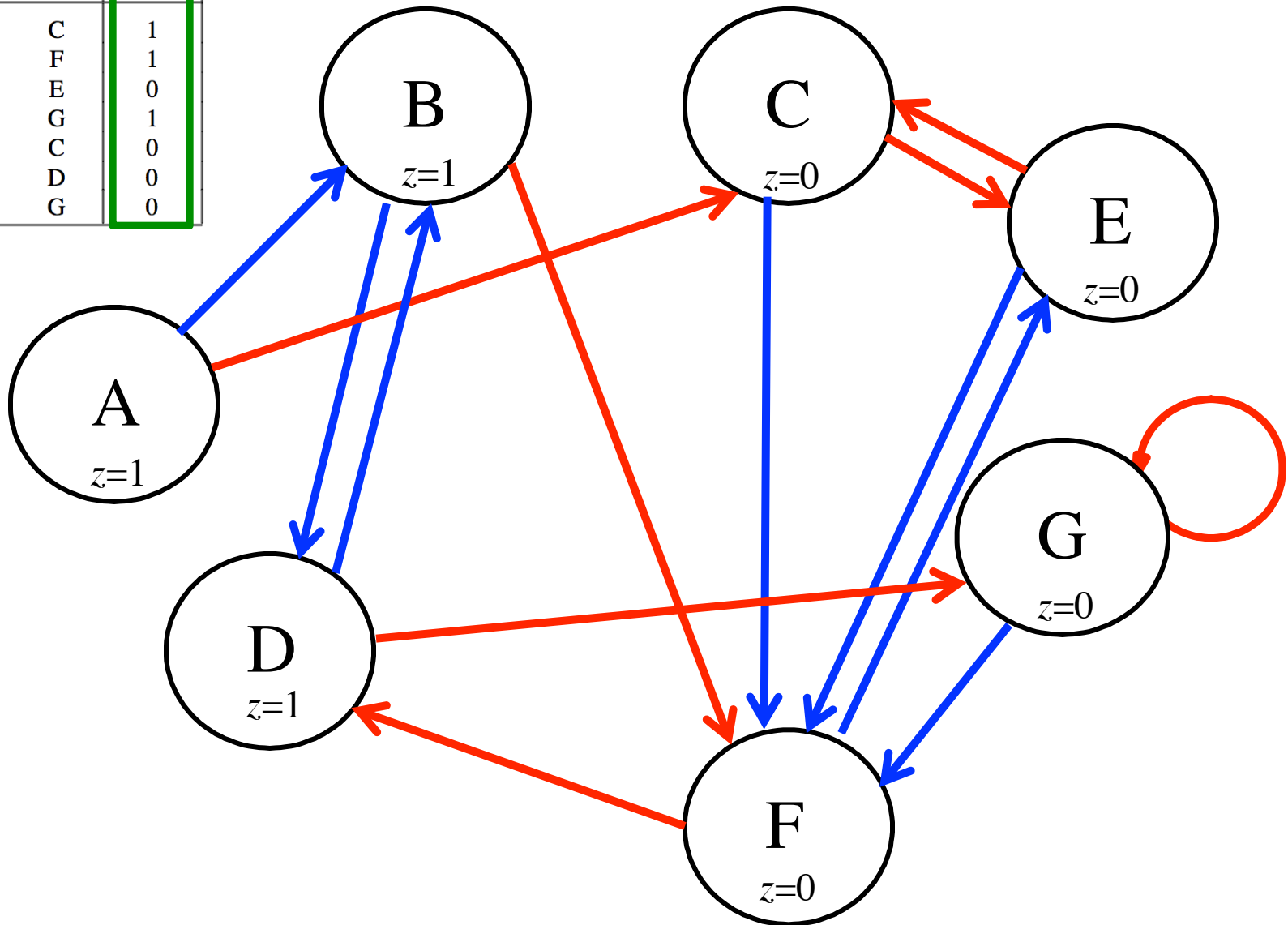
State Diagram (transitions when $w=1$)

Present state	Next state		Output z
	$w = 0$	$w = 1$	
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0



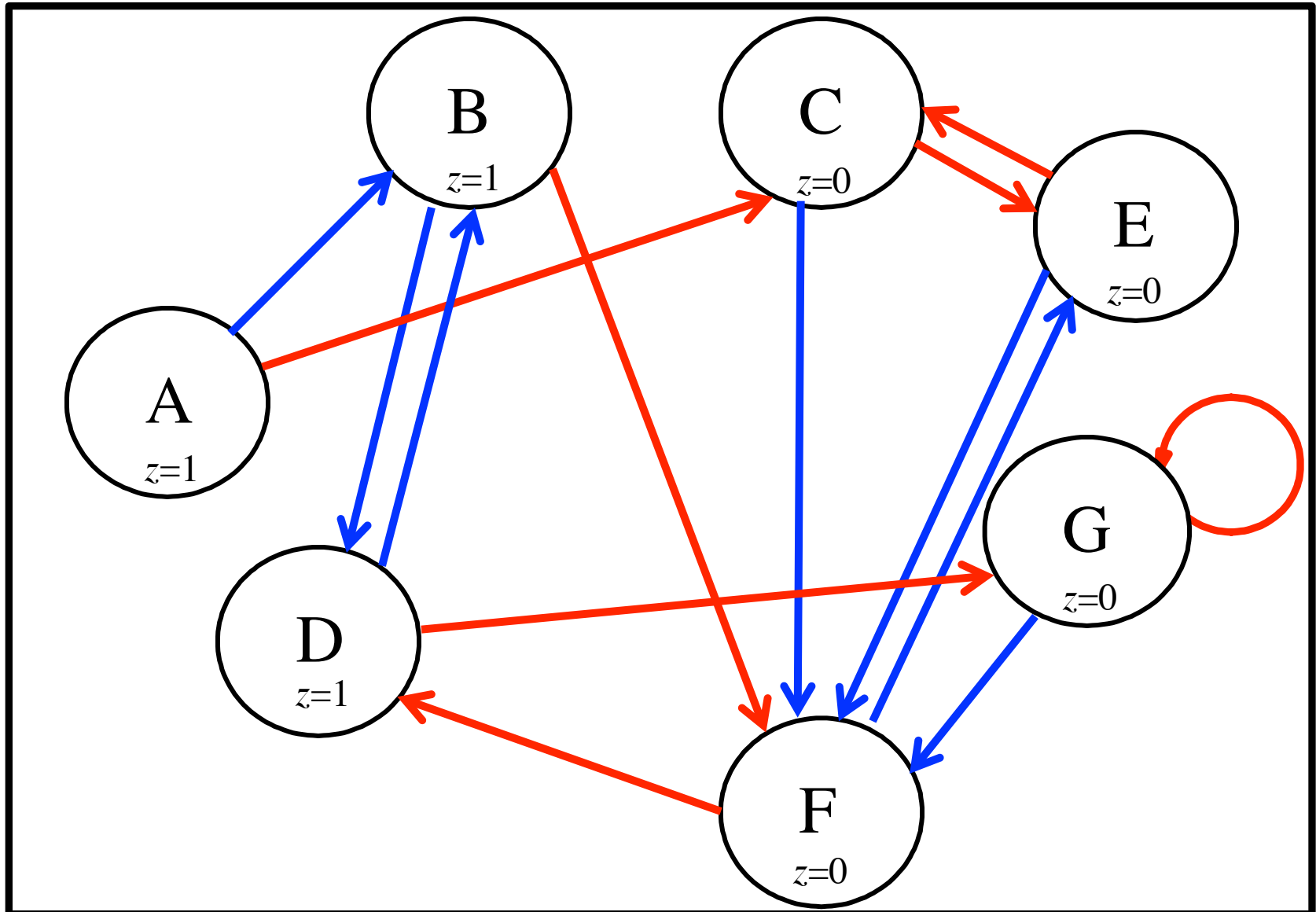
Outputs

Present state	Next state		Output
	$w = 0$	$w = 1$	z
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0



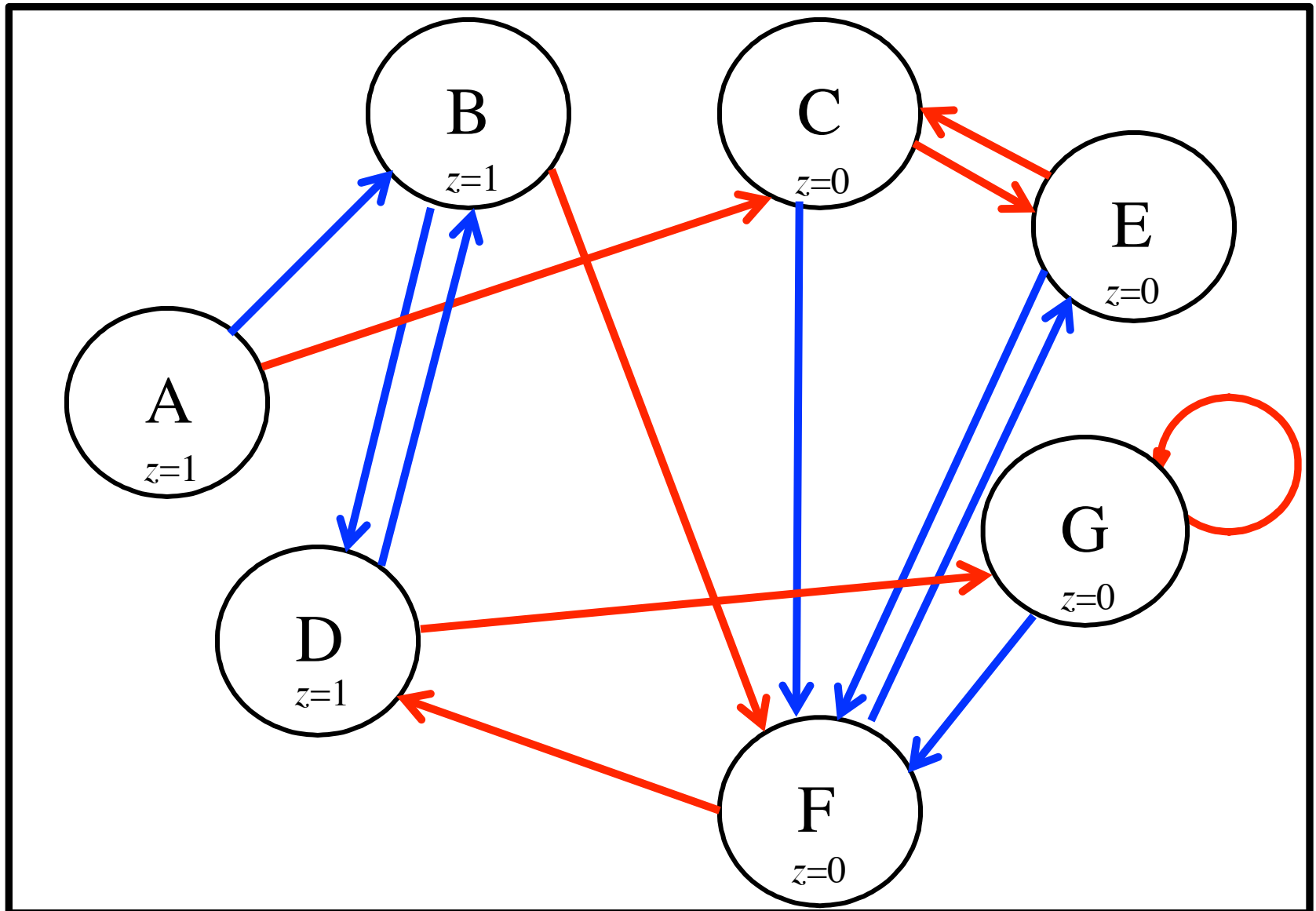
Partition #1

(All states in the same partition)



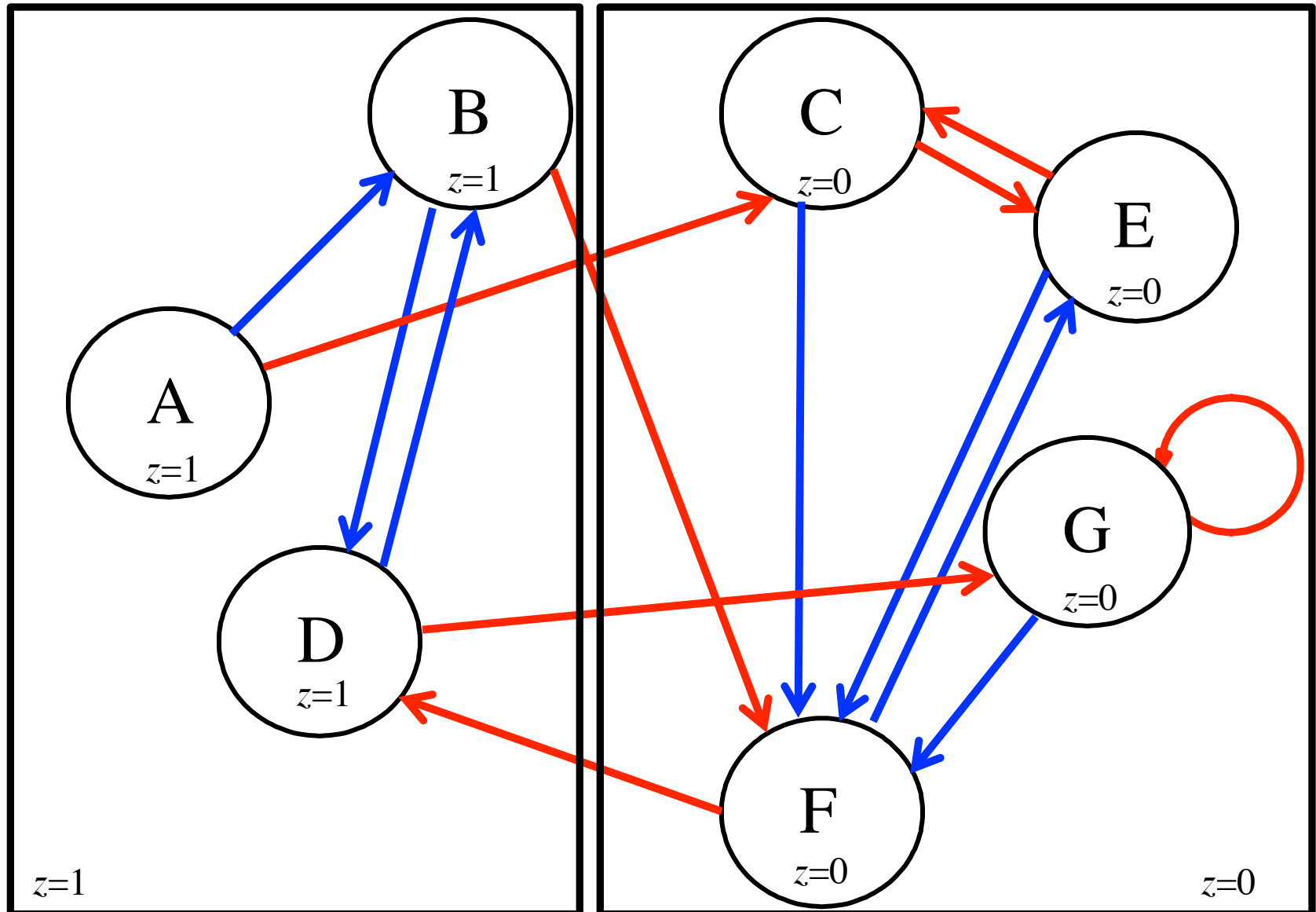
Partition #1

(ABCDEFGG)



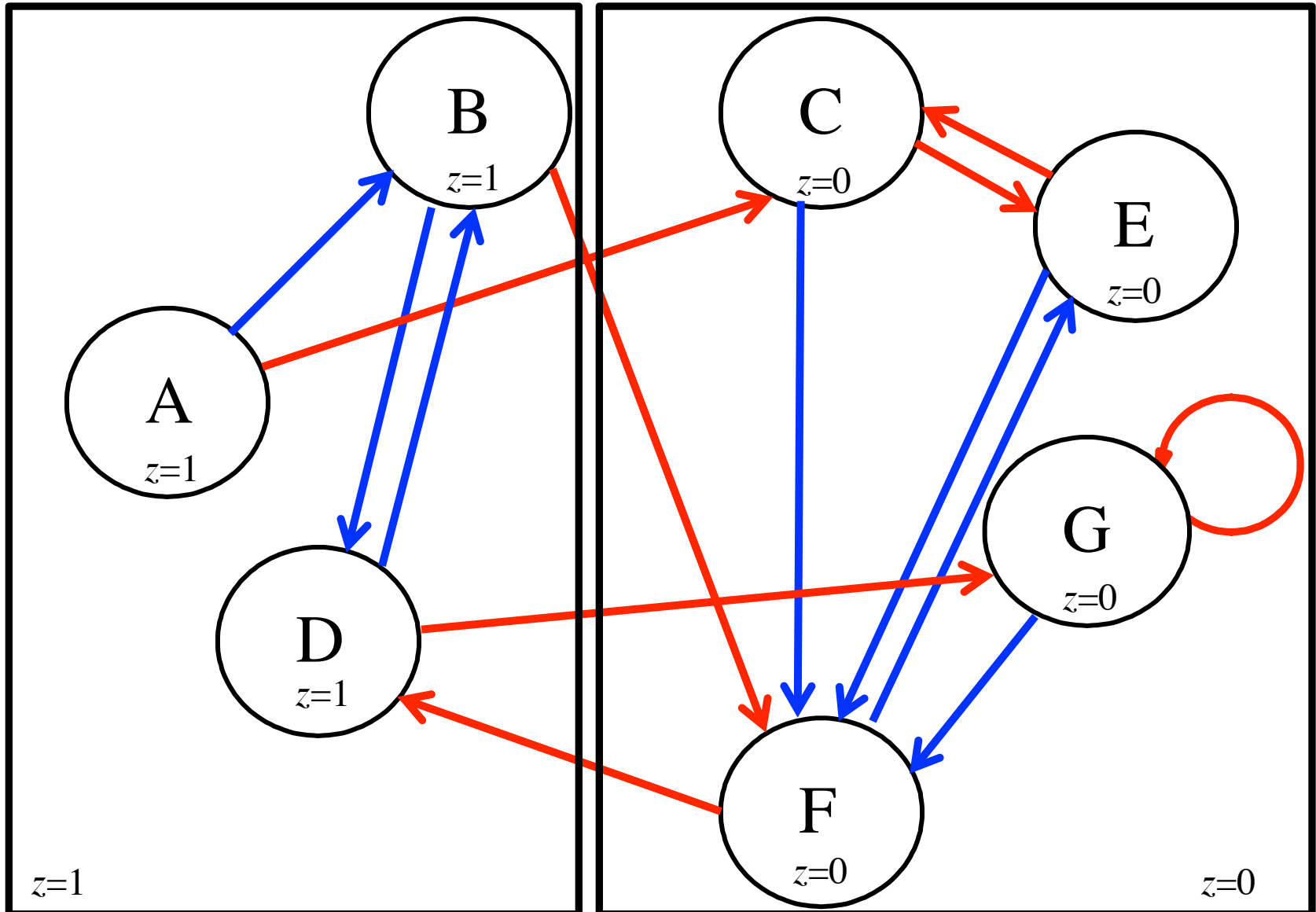
Partition #2

(based on outputs)



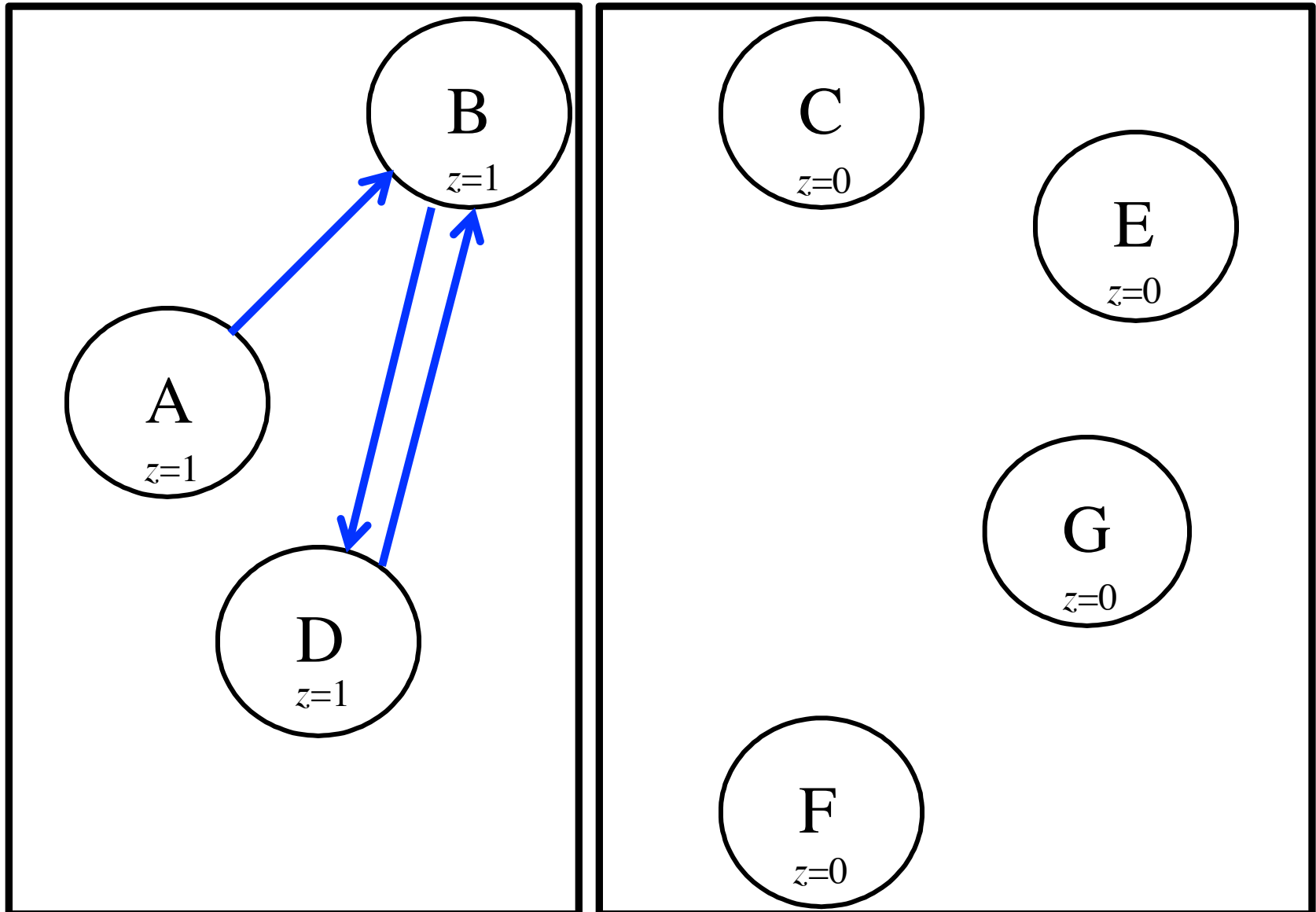
Partition #2

(ABD)(CEFG)



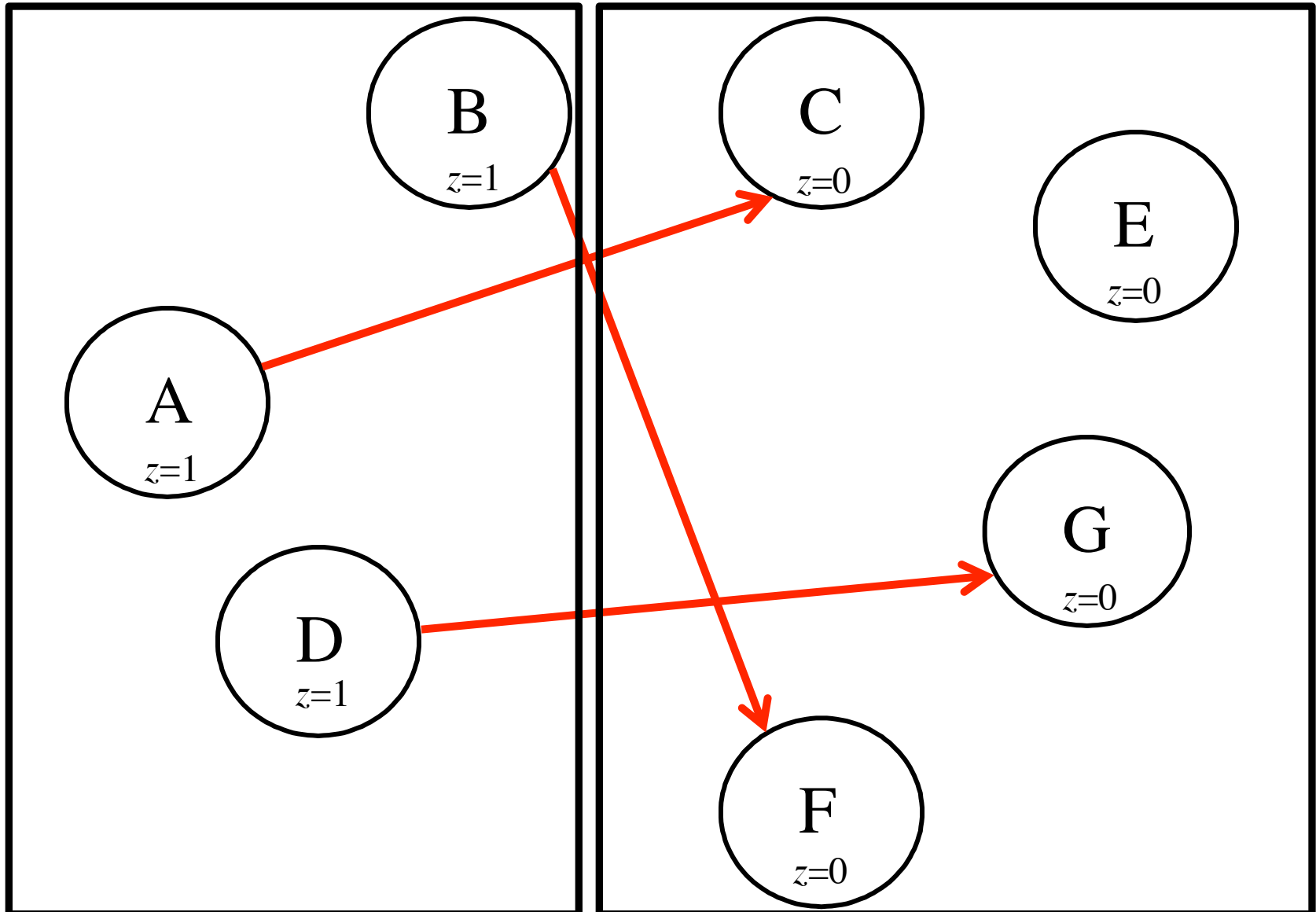
Partition #3.1

(Examine the 0-successors of ABD)



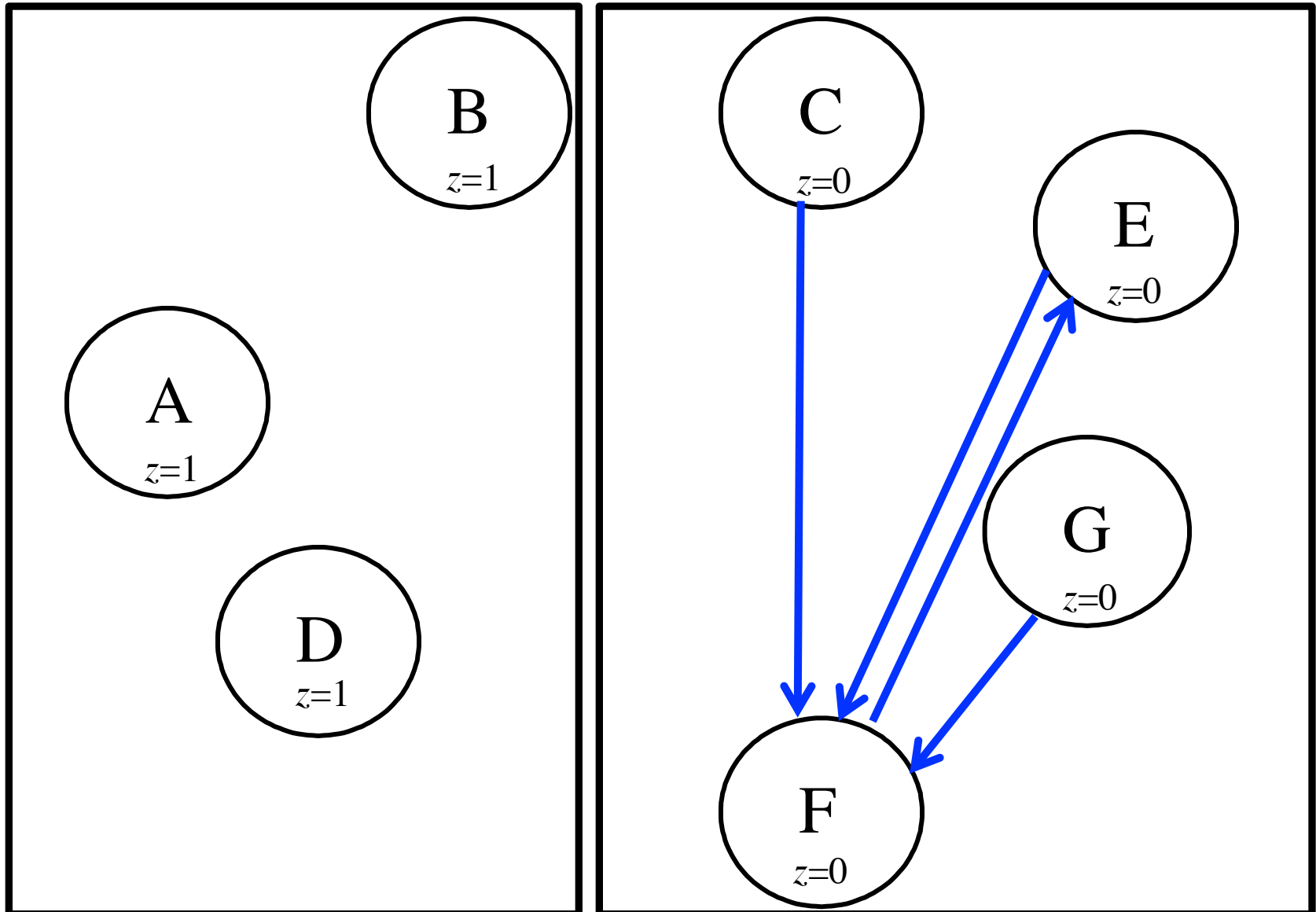
Partition #3.1

(Examine the 1-successors of ABD)



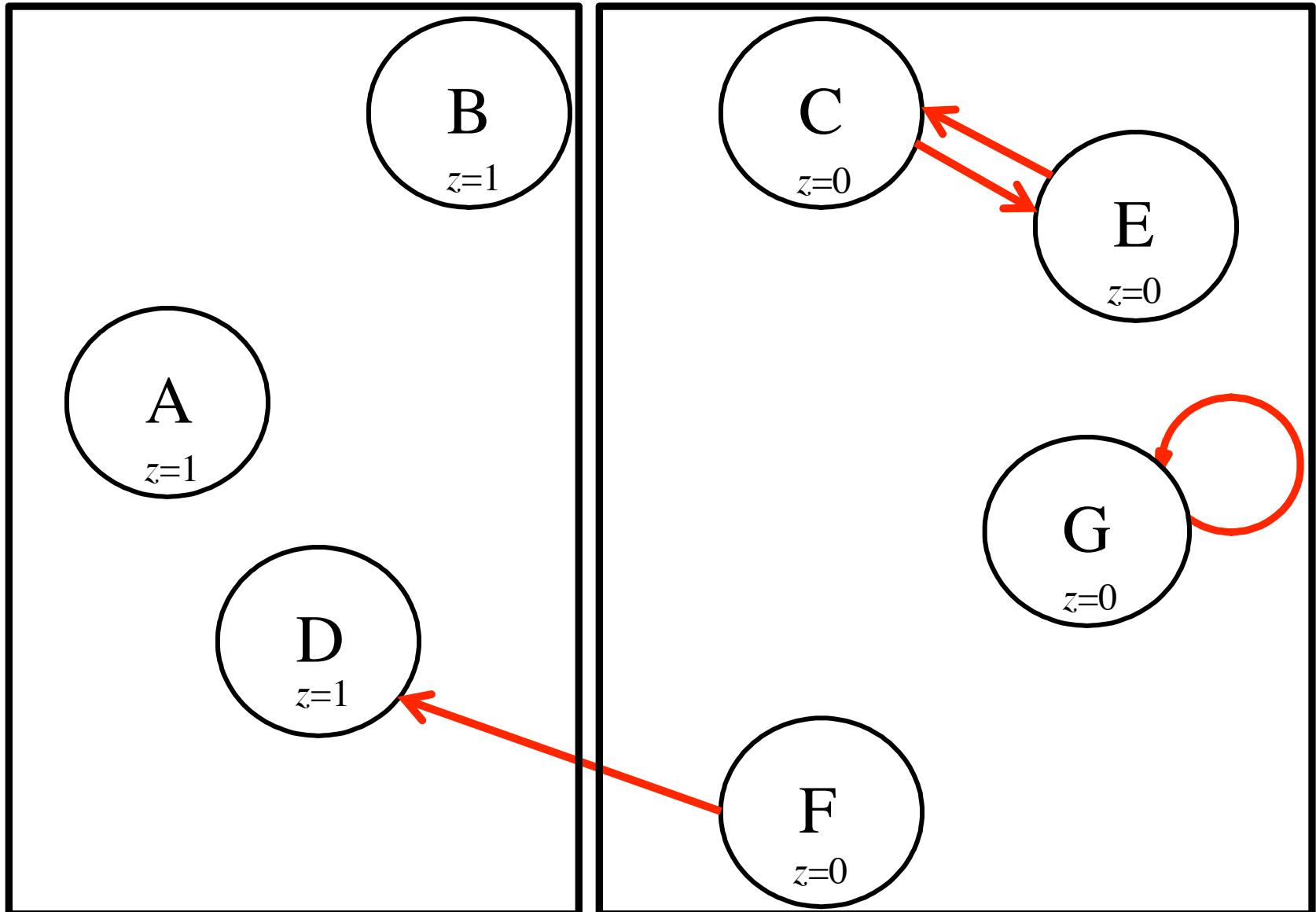
Partition #3.2

(Examine the 0-successors of CEFG)



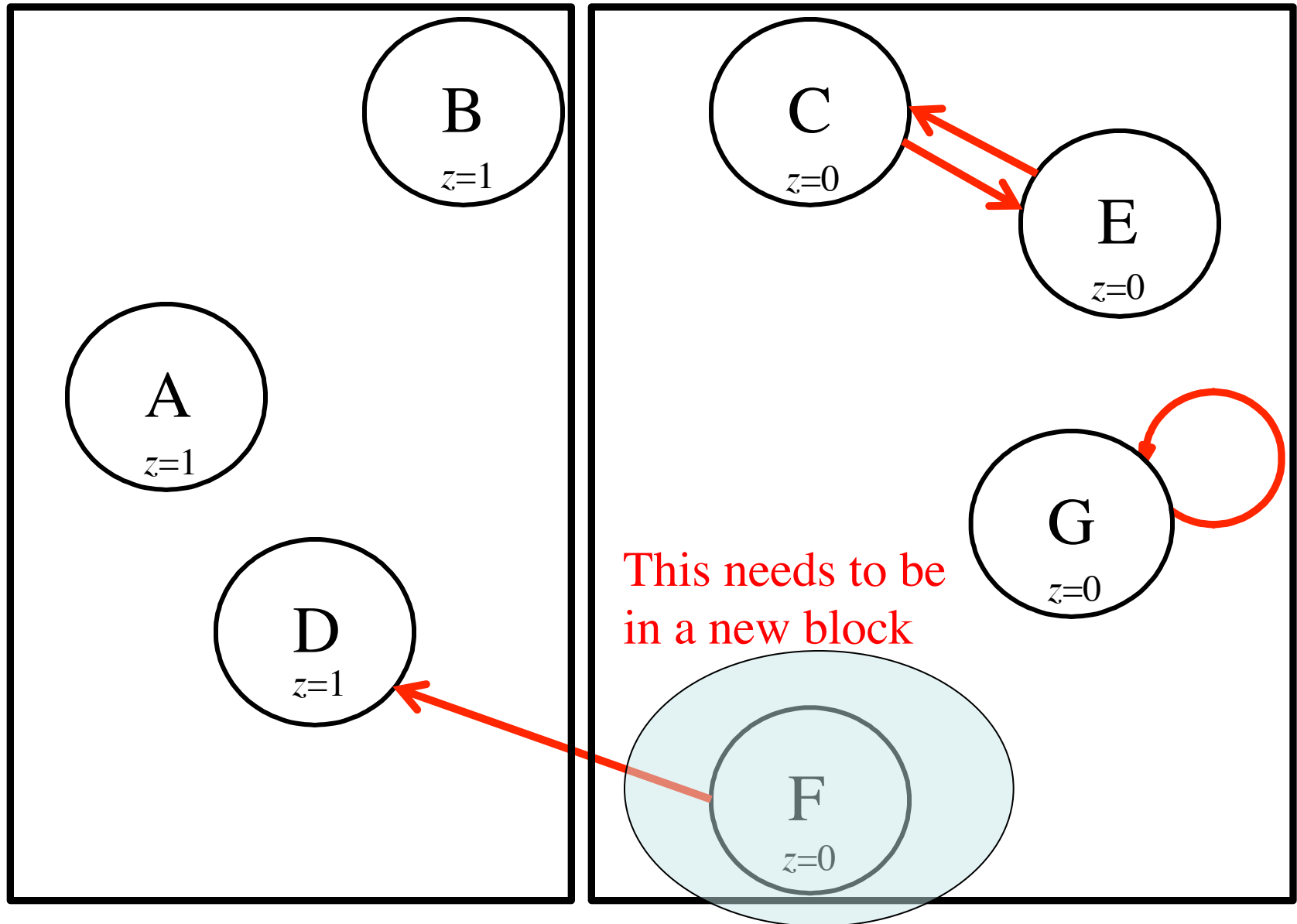
Partition #3.2

(Examine the 1-successors of CEFG)



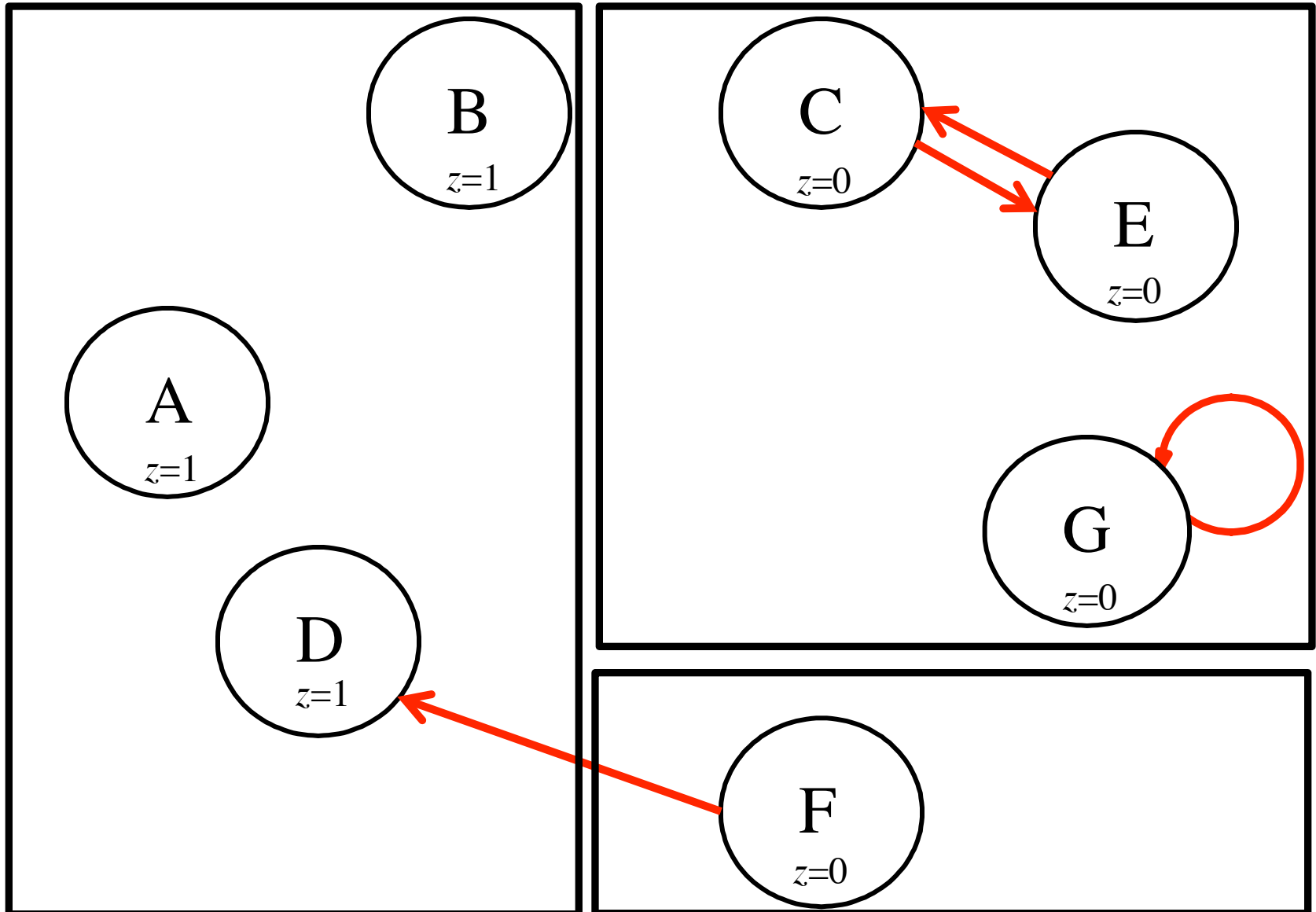
Partition #3.2

(Examine the 1-successors of CEF G)



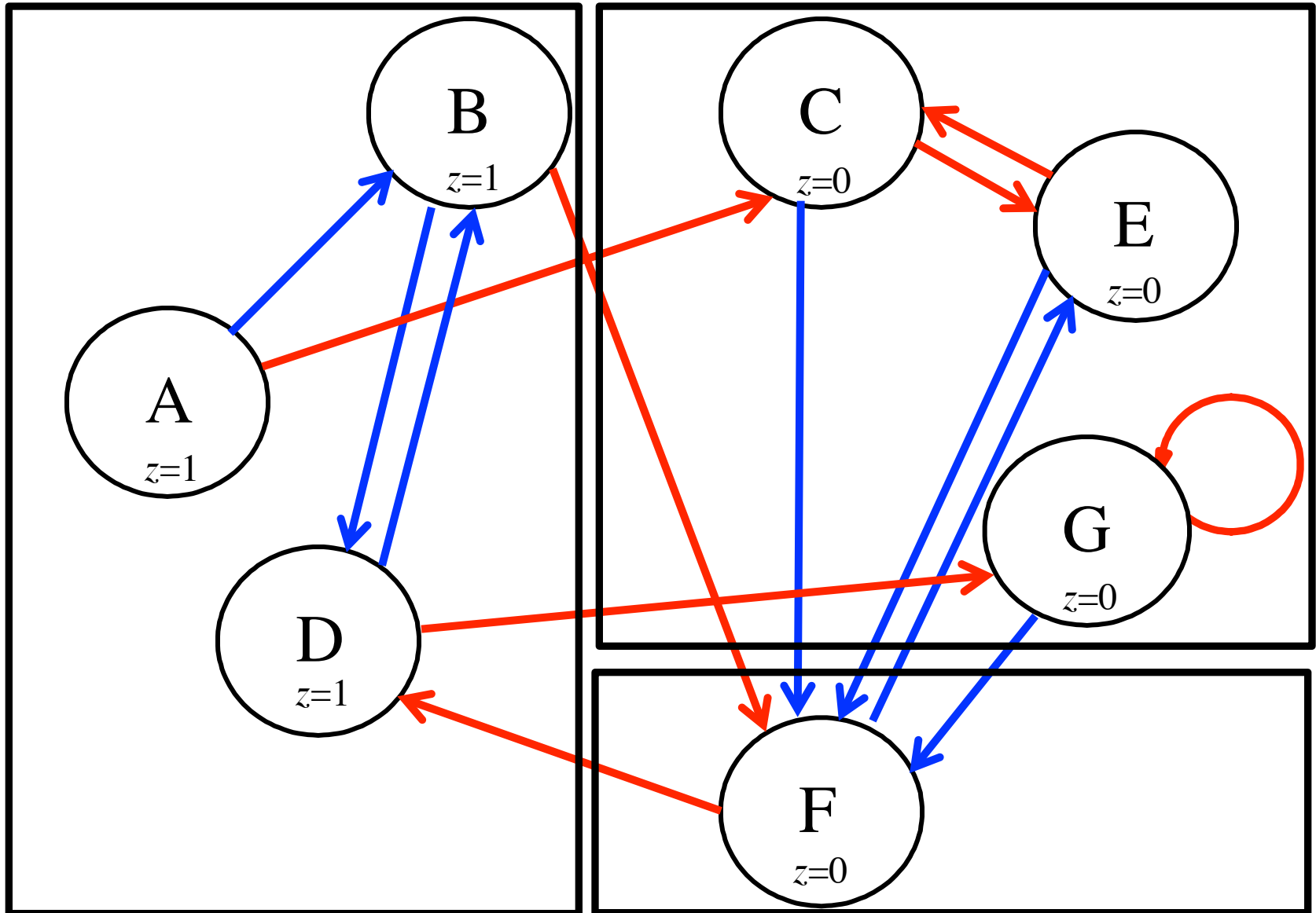
Partition #3

(ABD)(CEG)(F)



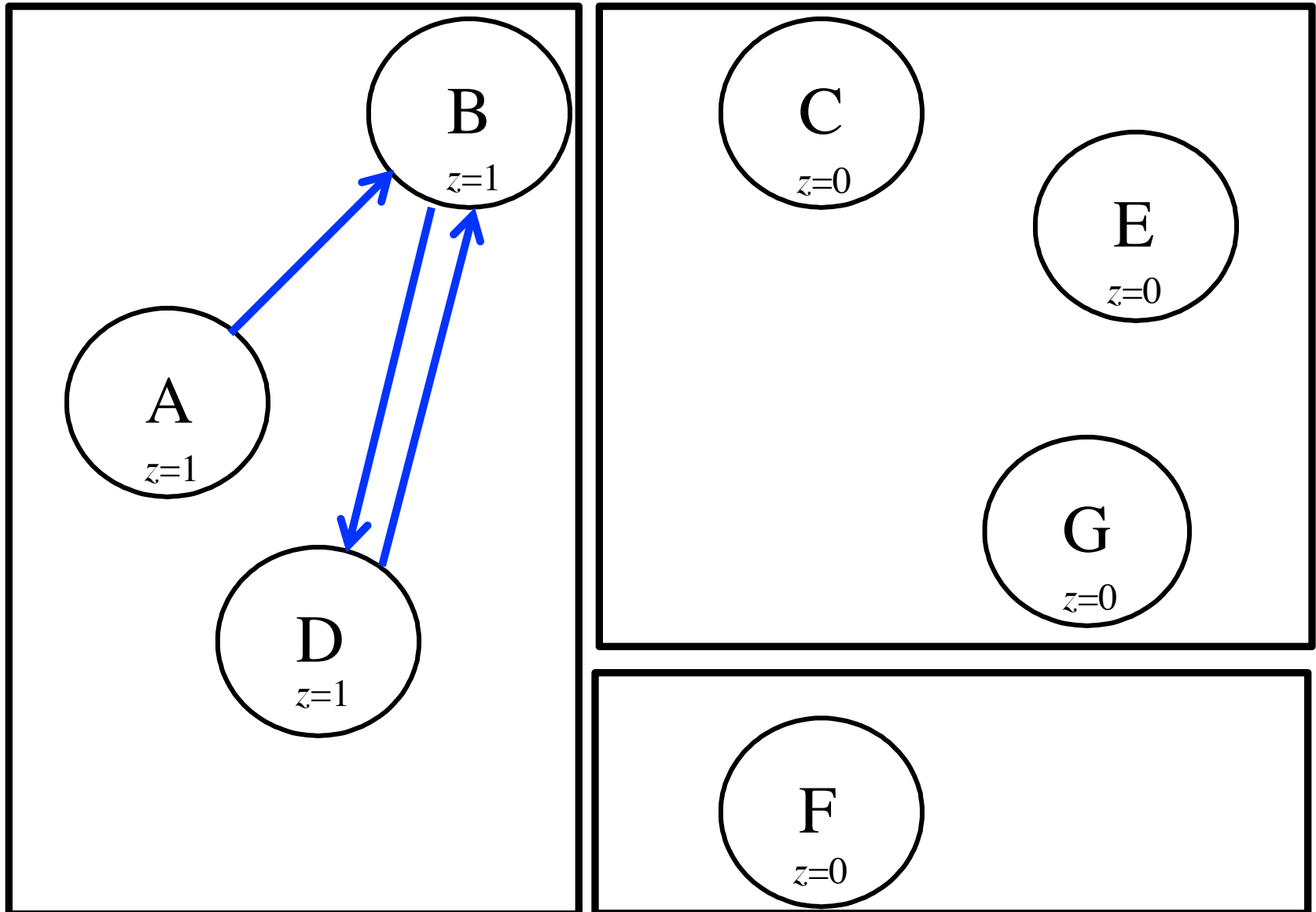
Partition #3

(ABD)(CEG)(F)



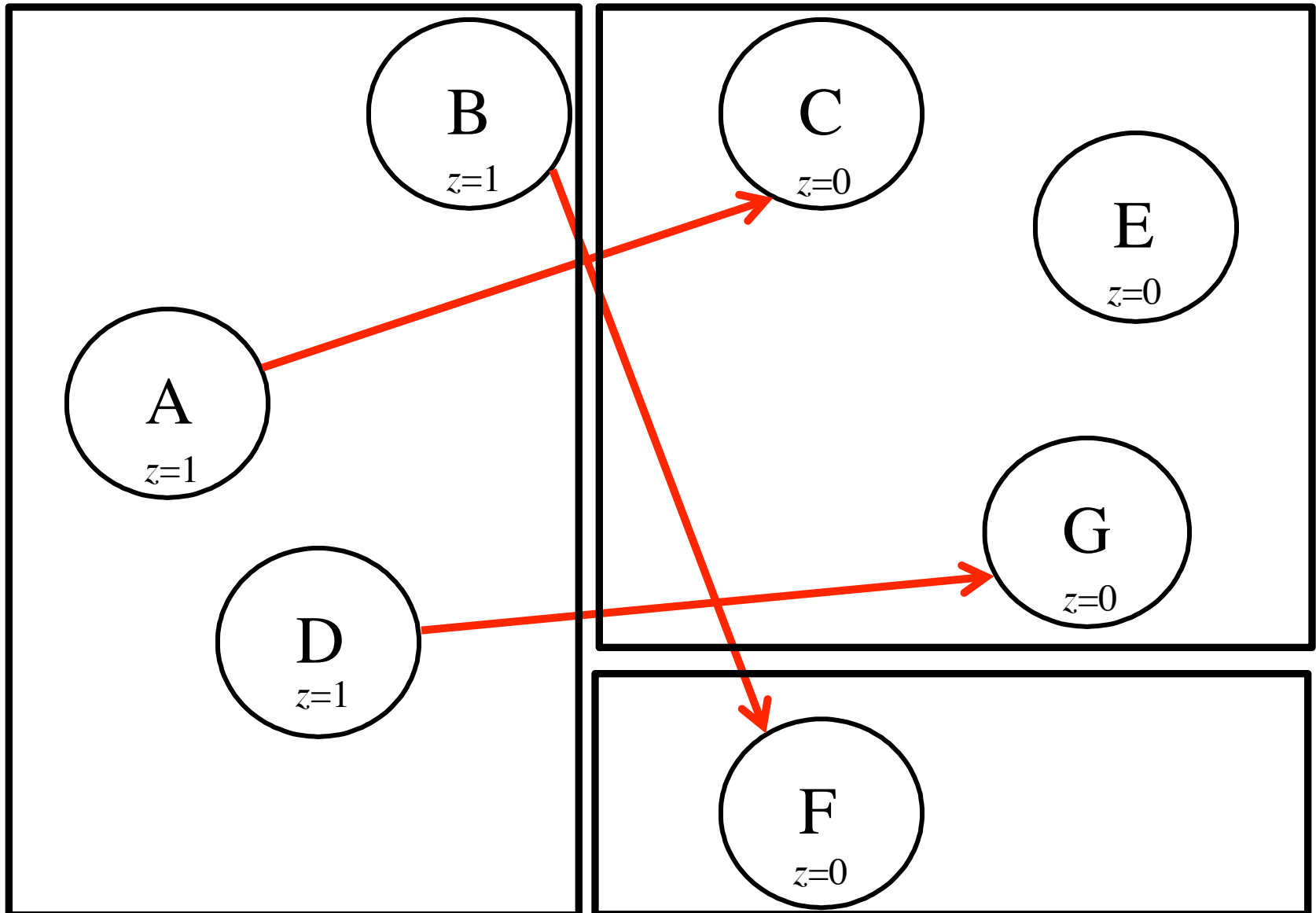
Partition #4.1

(Examine the 0-successors of ABD)



Partition #4.1

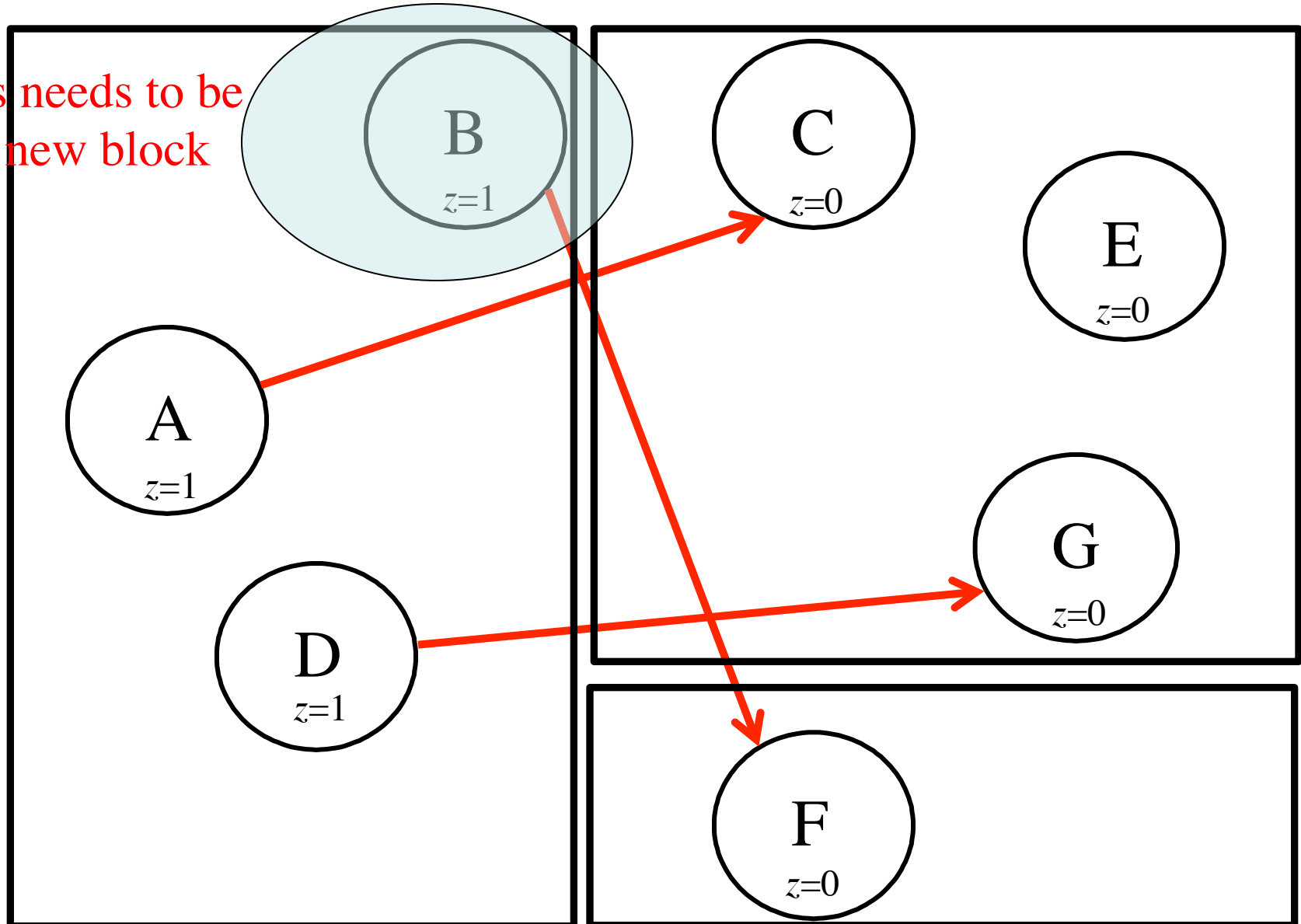
(Examine the 1-successors of ABD)



Partition #4.1

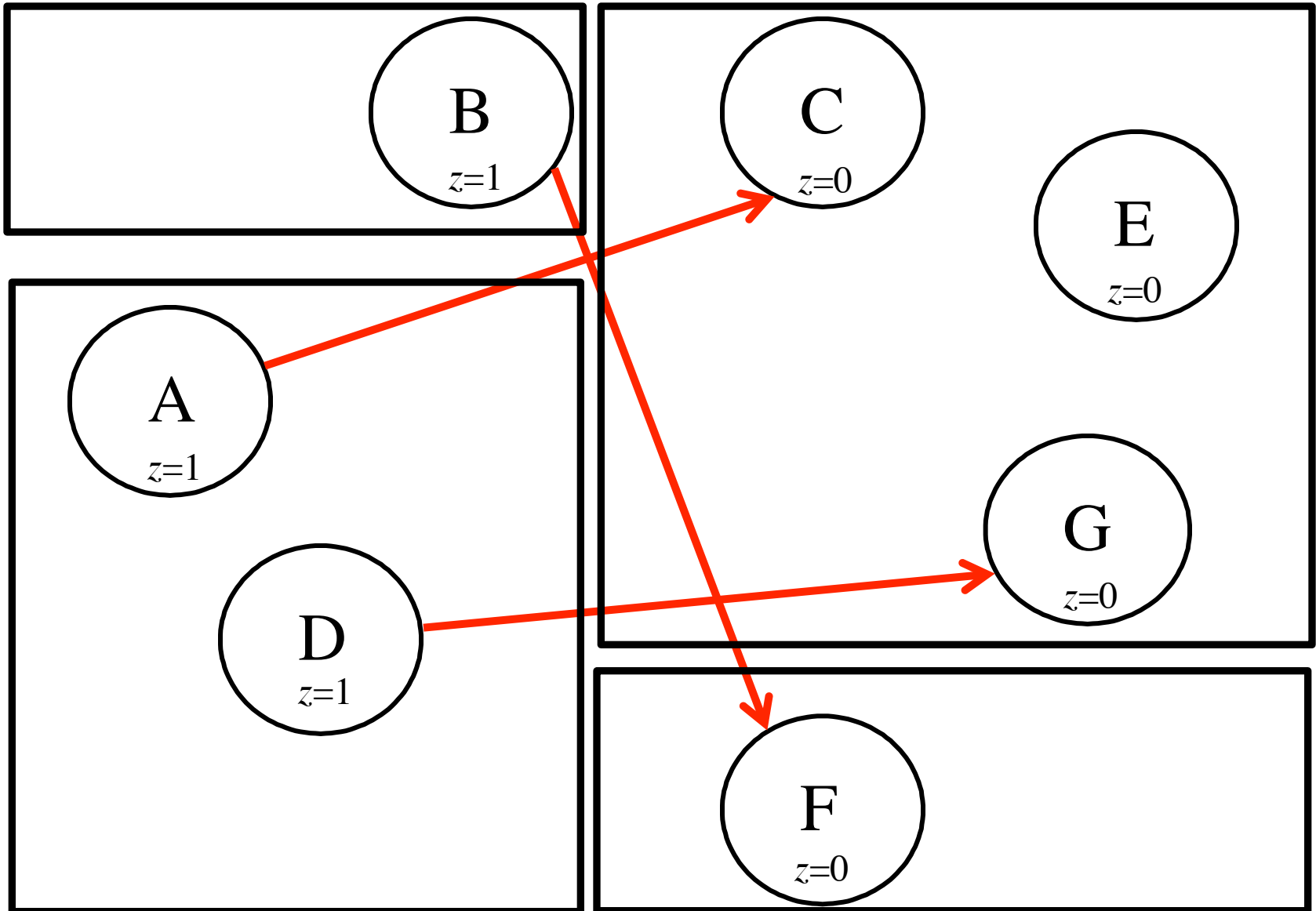
(Examine the 1-successors of ABD)

This needs to be
in a new block



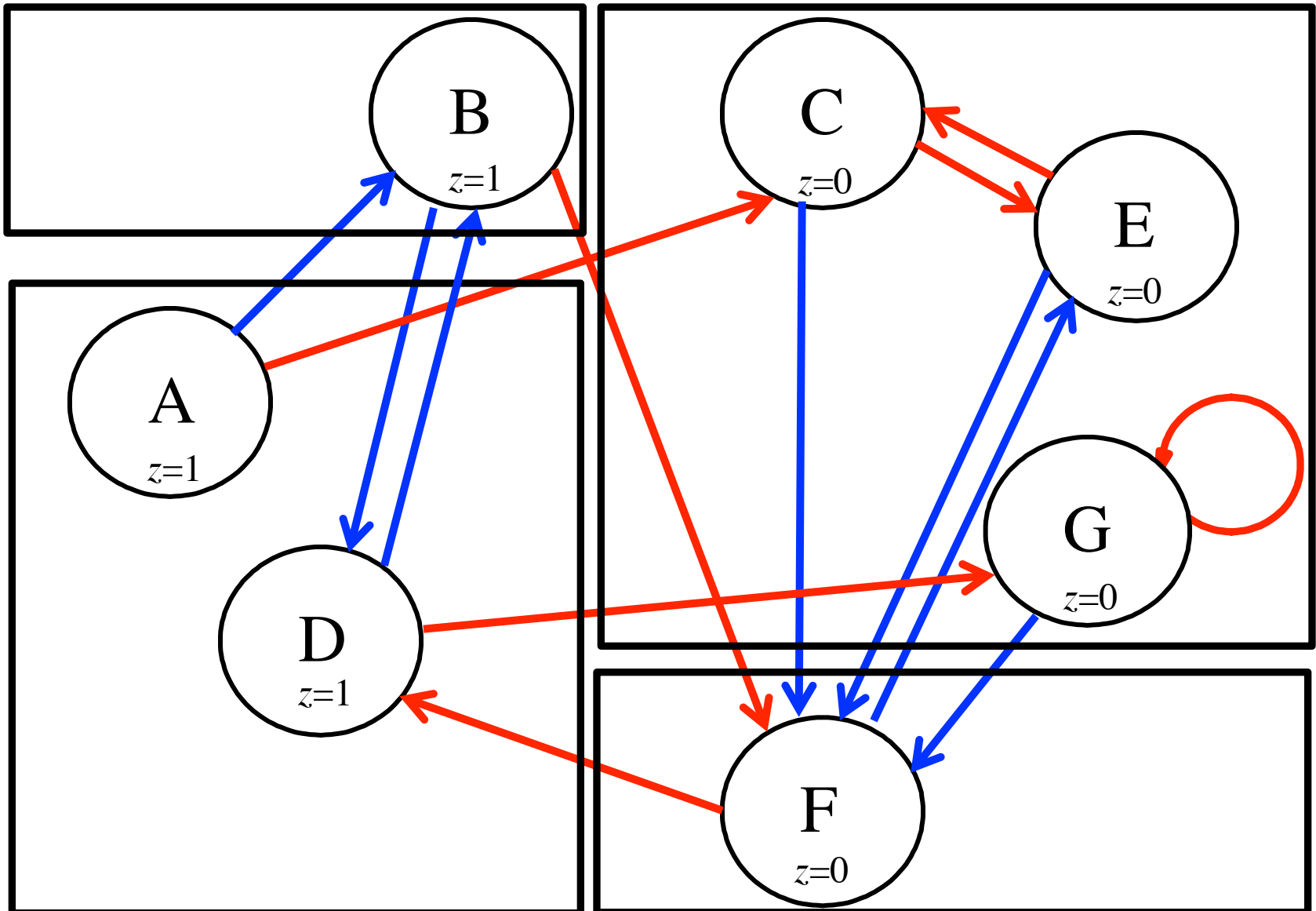
Partition #4

(AD)(B)(CEG)(F)



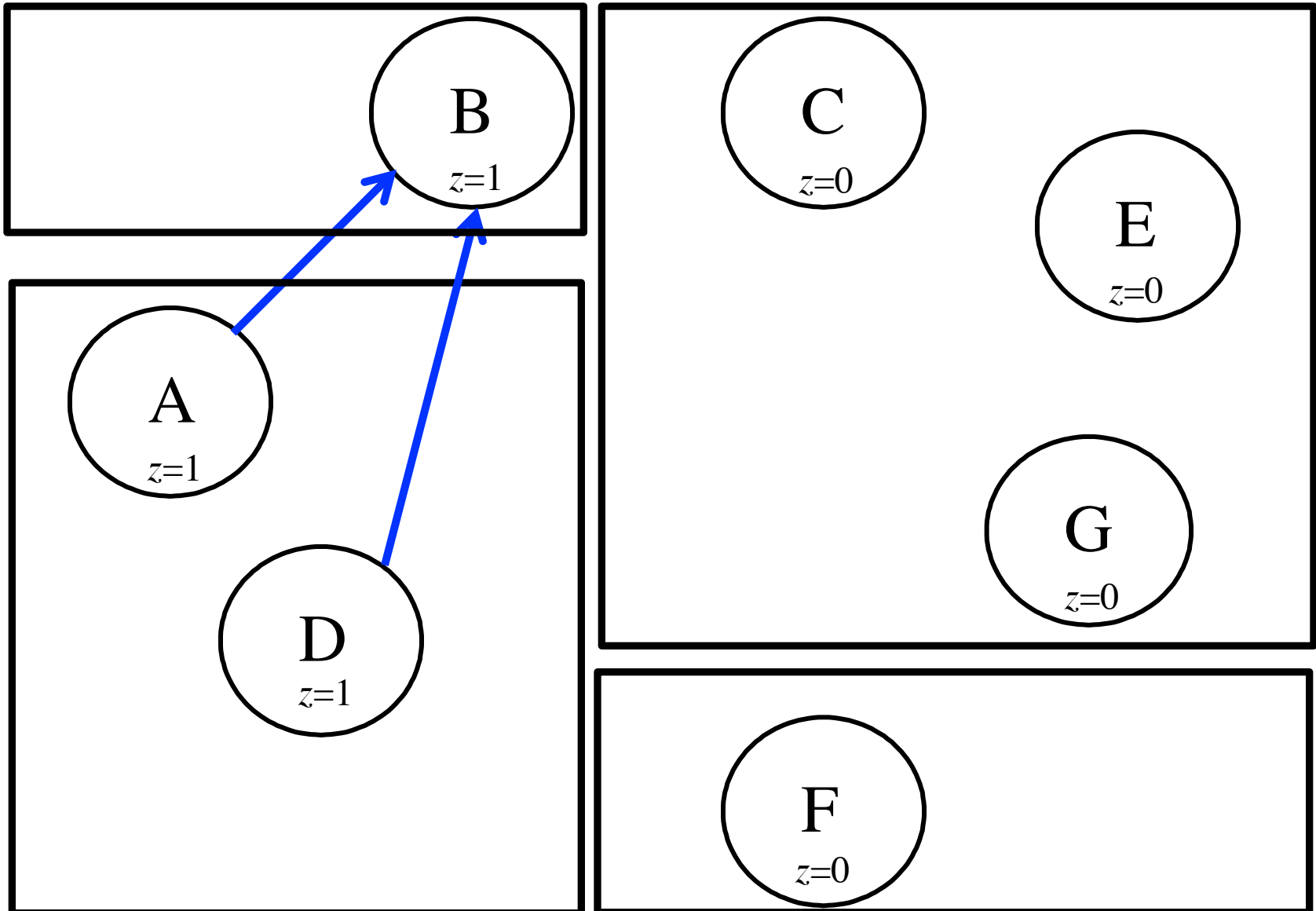
Partition #4

(AD)(B)(CEG)(F)



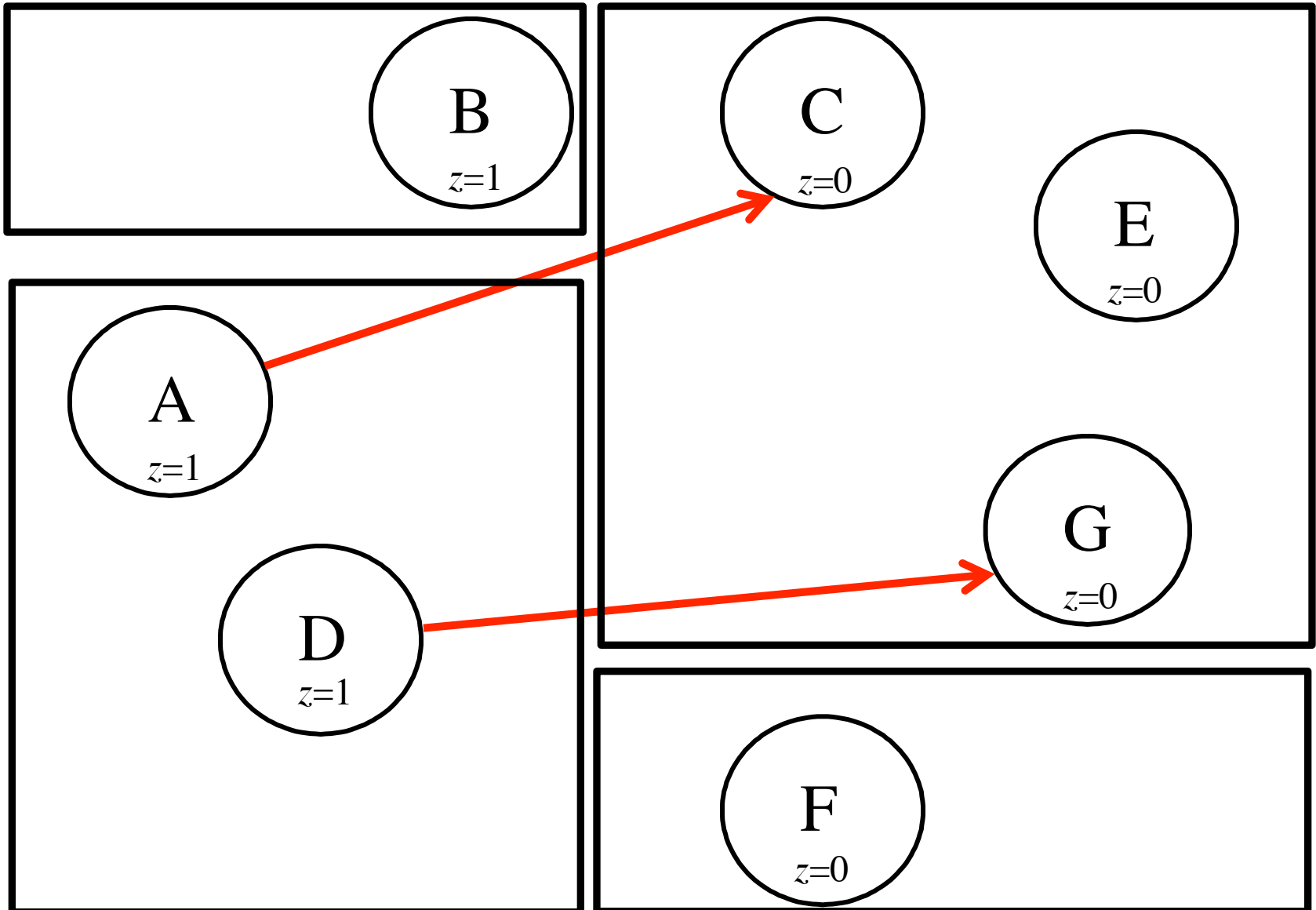
Partition #5.1

(Examine the 0-successors of AD)



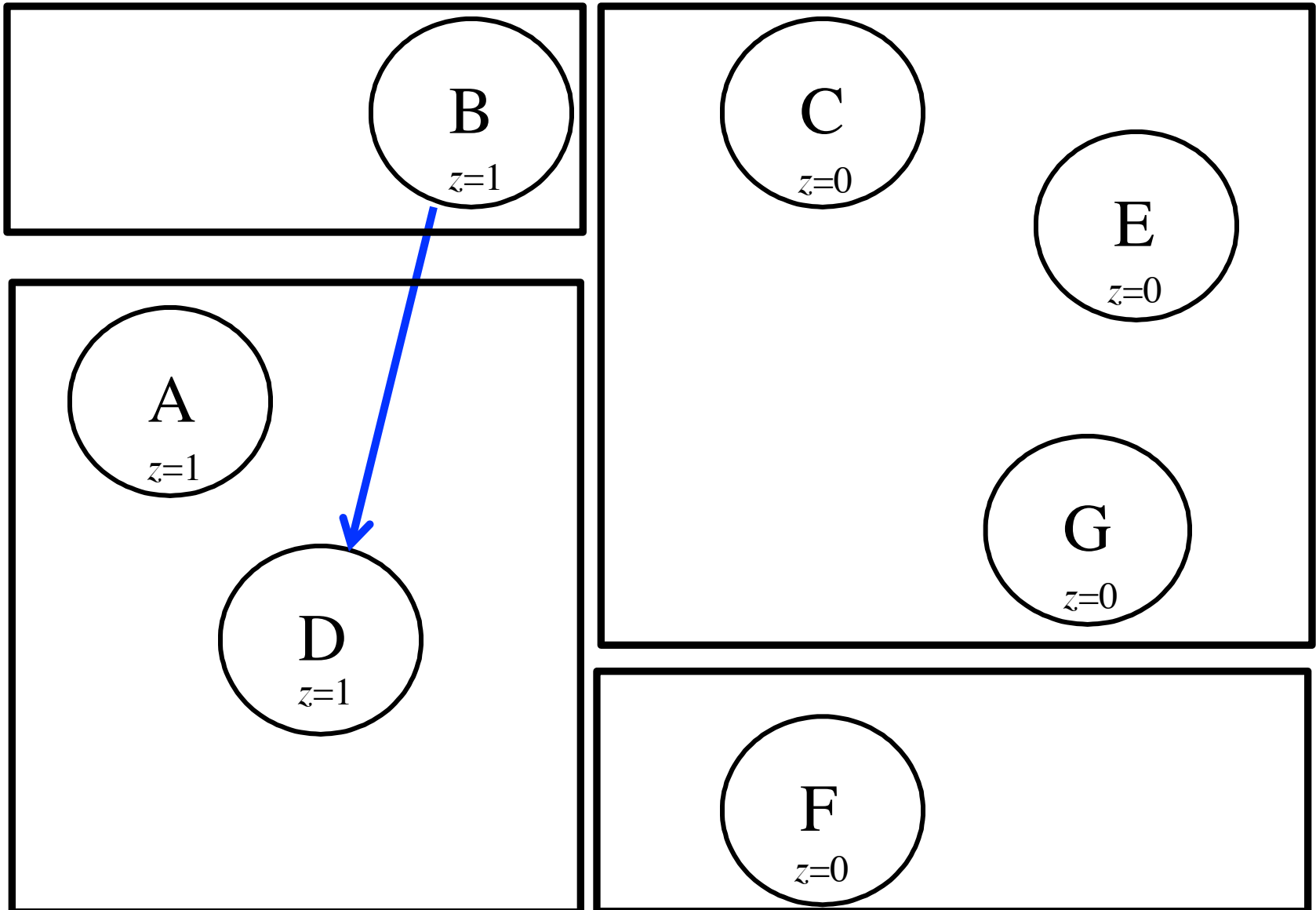
Partition #5.1

(Examine the 1-successors of AD)



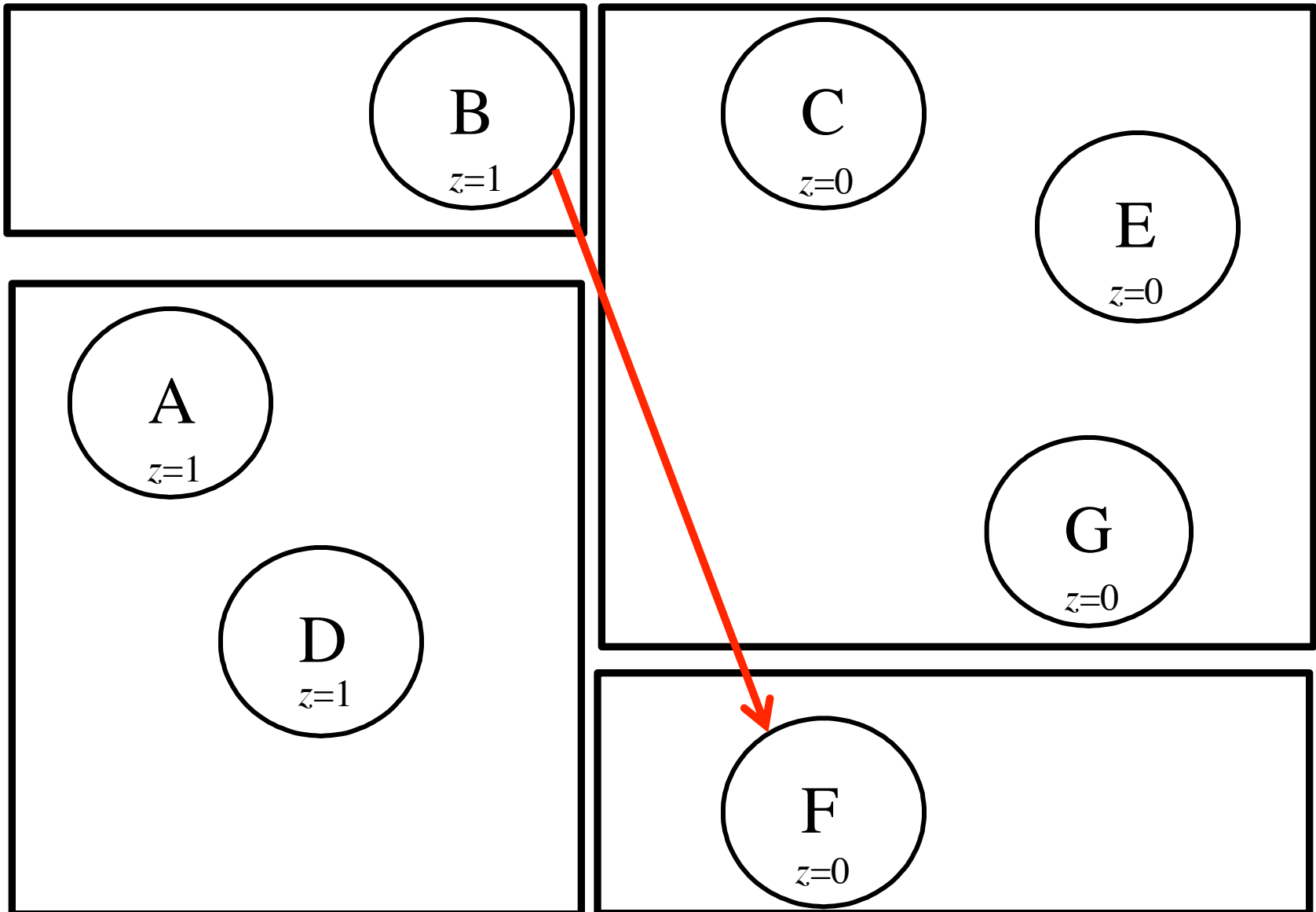
Partition #5.2

(Examine the 0-successors of B)



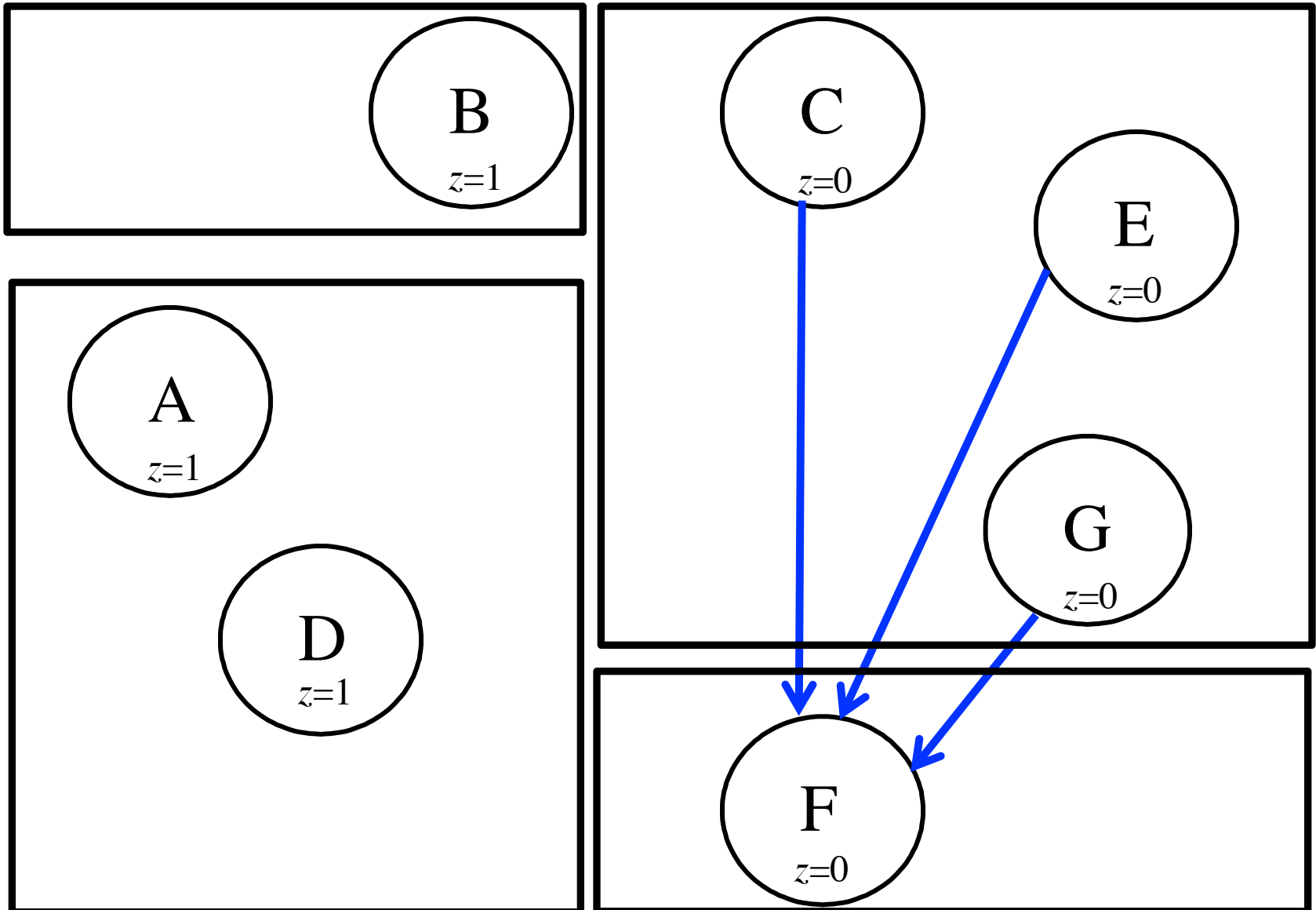
Partition #5.2

(Examine the 1-successors of B)



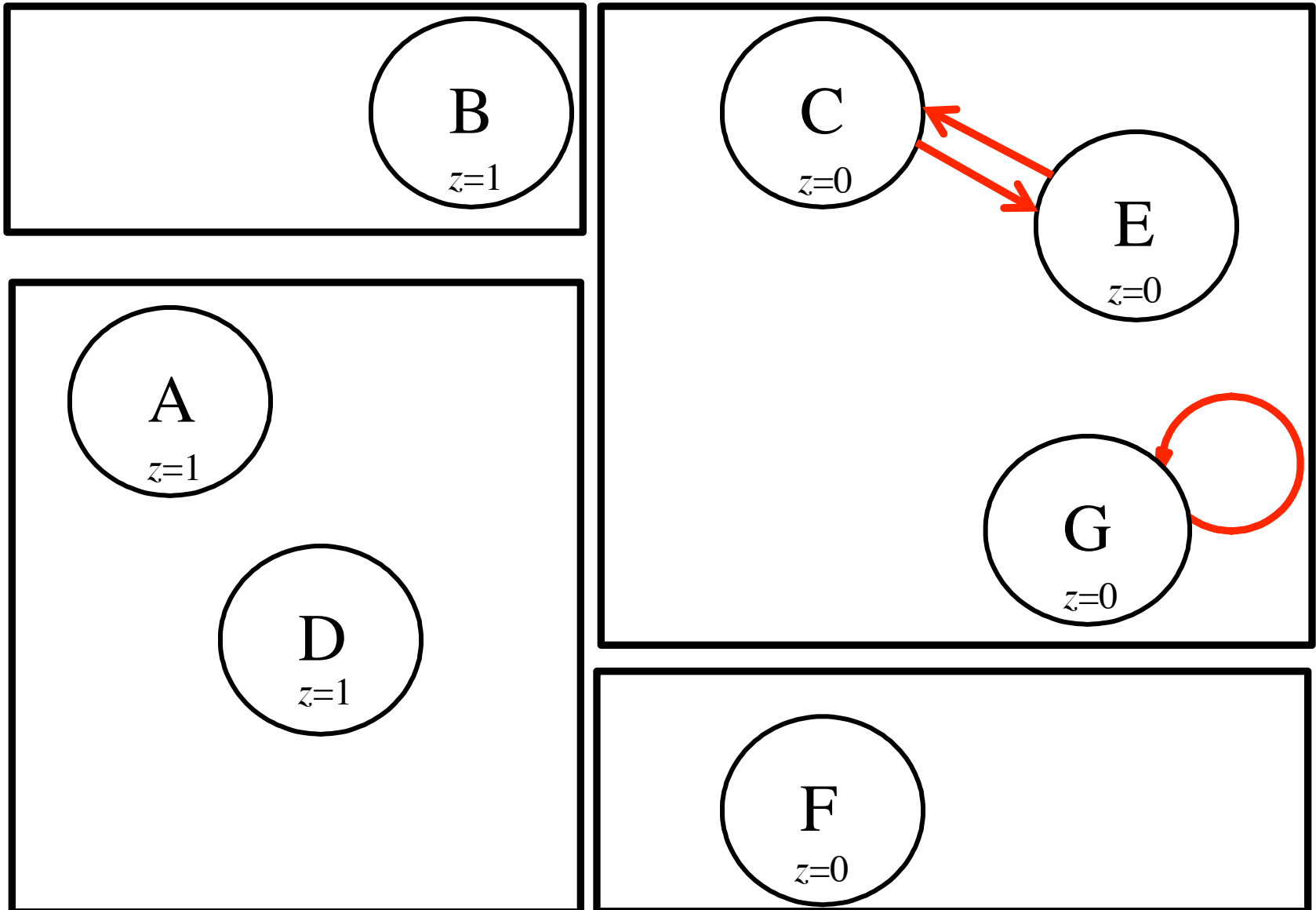
Partition #5.3

(Examine the 0-successors of CEG)



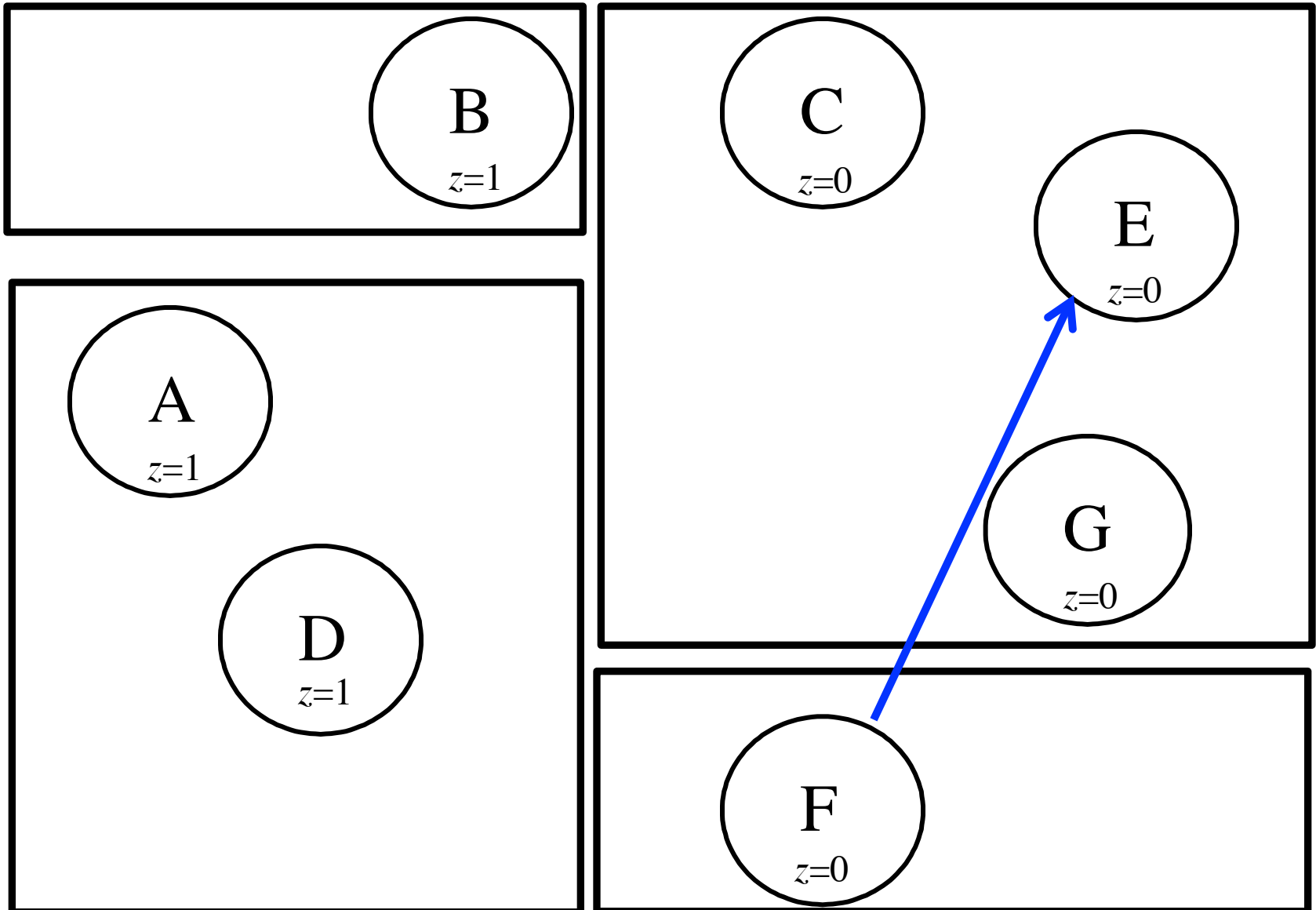
Partition #5.3

(Examine the 1-successors of CEG)



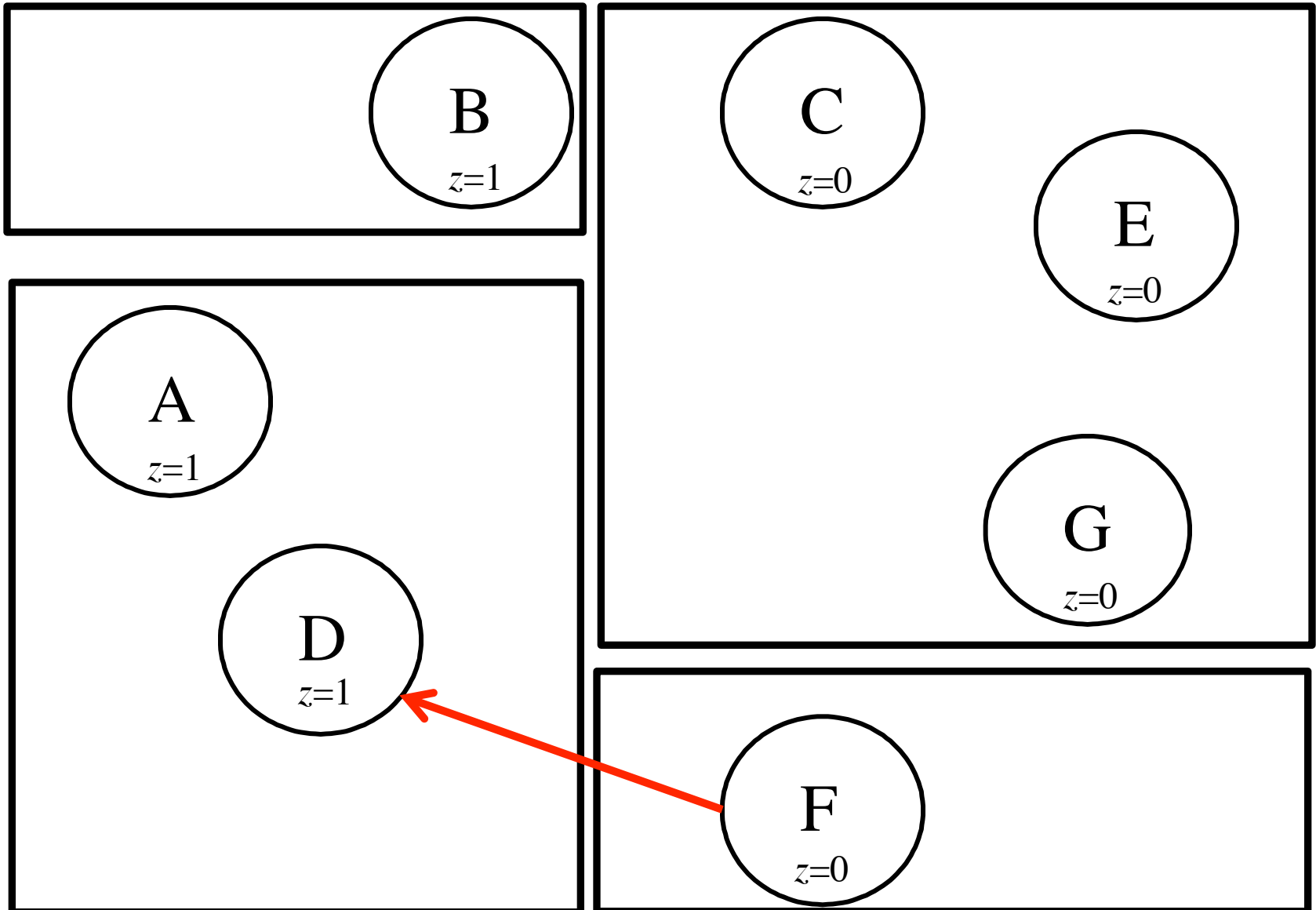
Partition #5.4

(Examine the 0-successors of F)



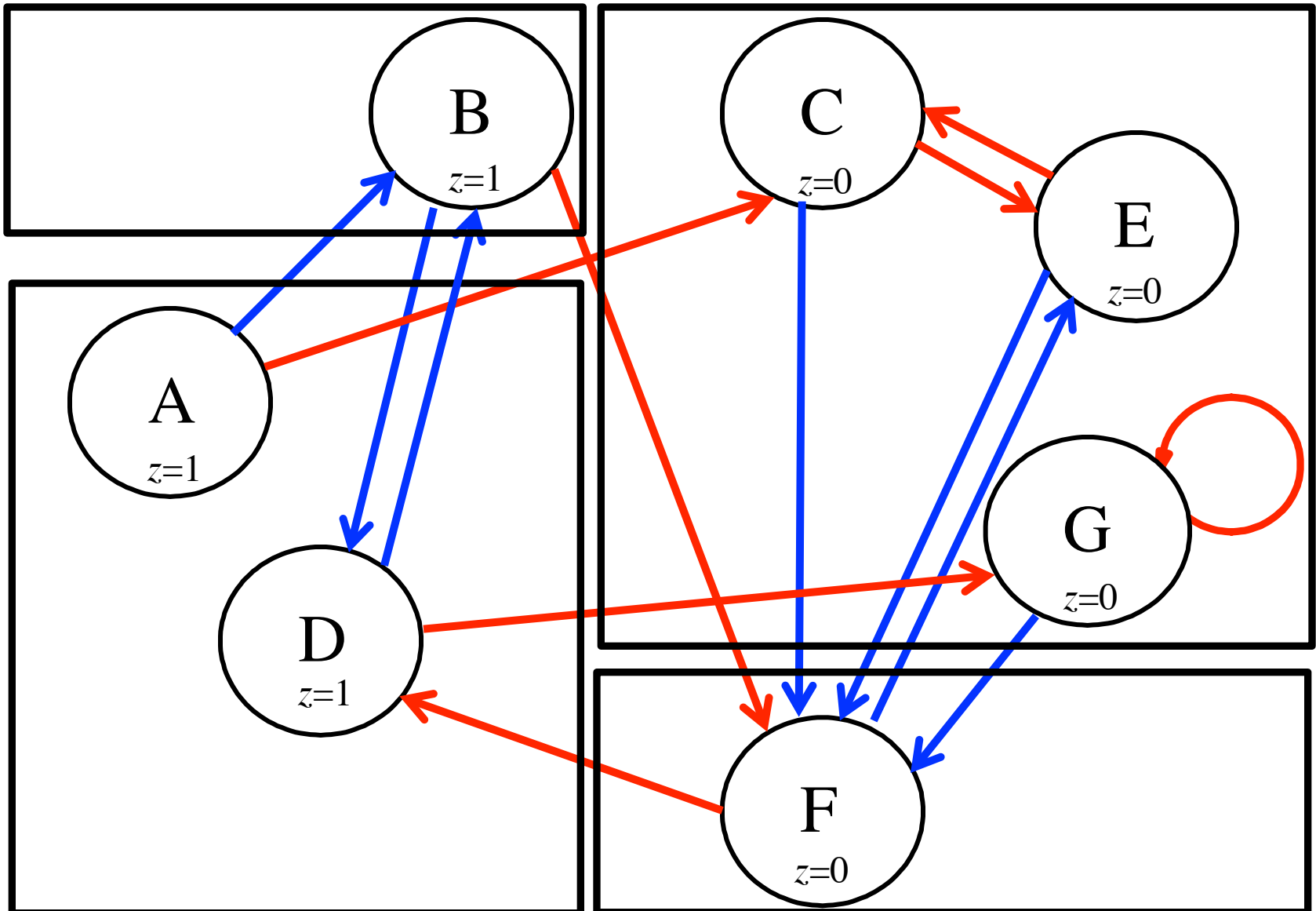
Partition #5.4

(Examine the 1-successors of F)



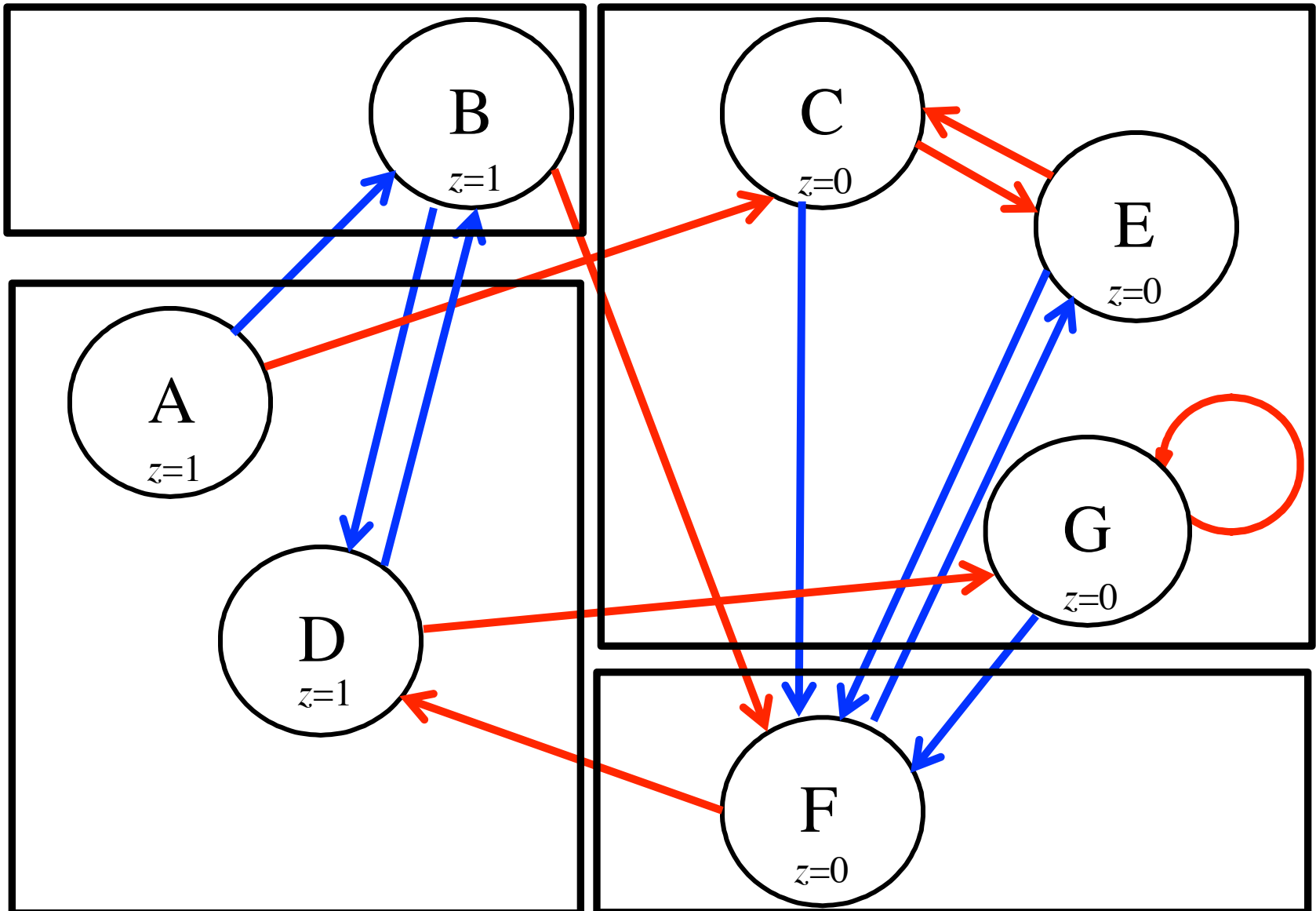
Partition #5

(AD)(B)(CEG)(F)



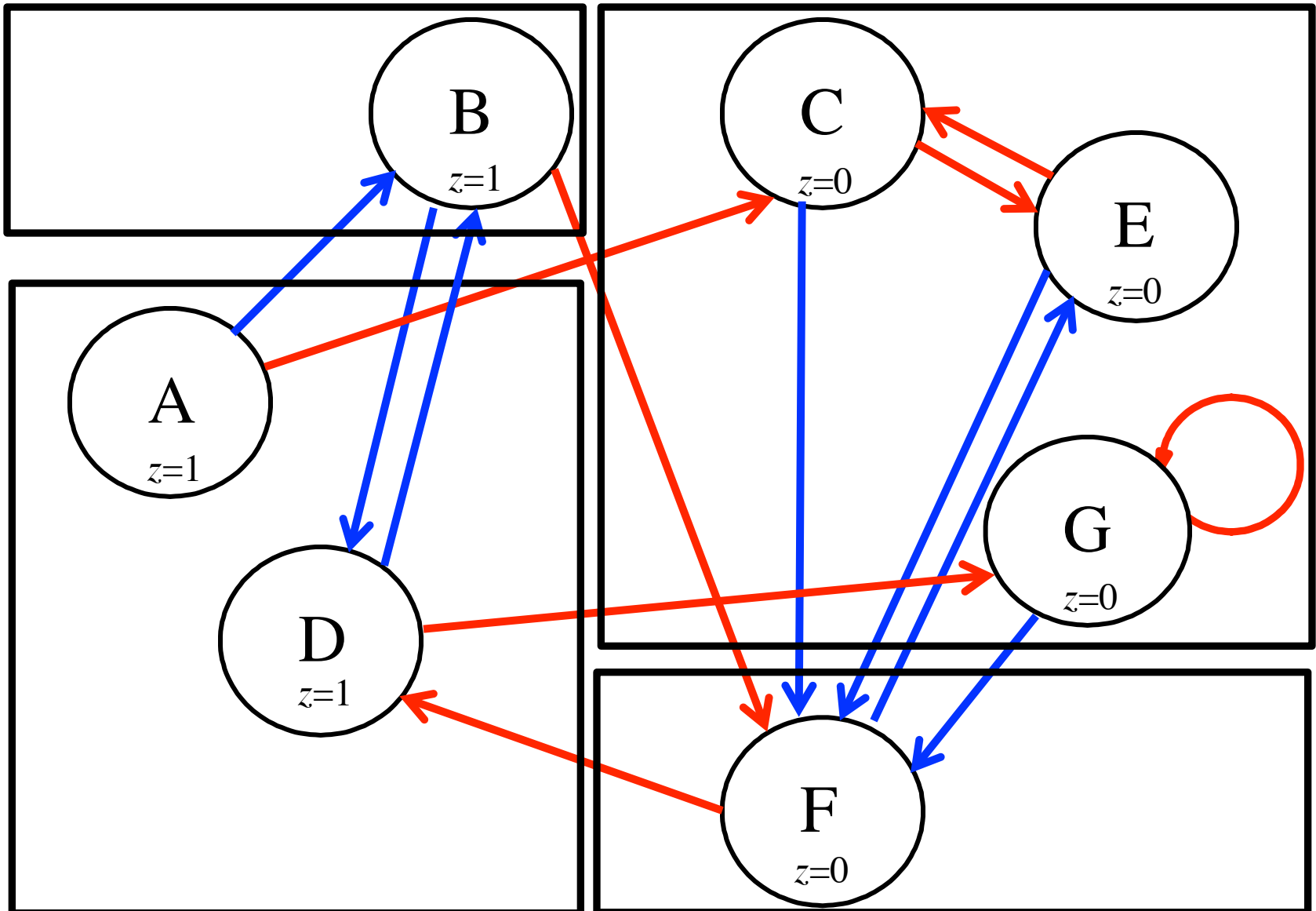
Partition #4

(AD)(B)(CEG)(F)



Partition #5

(This is the same as #4 so we can stop here)



Minimized state table

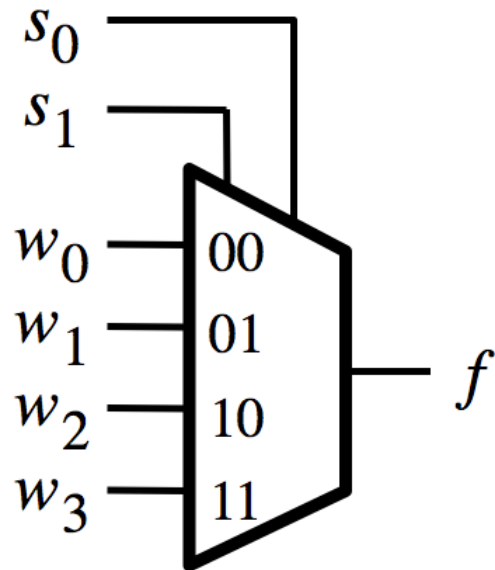
Present state	Nextstate		Output z
	w = 0	w = 1	
A	B	C	1
B	A	F	1
C	F	C	0
F	C	A	0

Multiplexers

4-1 Multiplexer (Definition)

- Has four inputs: w_0 , w_1 , w_2 , w_3
- Also has two select lines: s_1 and s_0
- If $s_1=0$ and $s_0=0$, then the output f is equal to w_0
- If $s_1=0$ and $s_0=1$, then the output f is equal to w_1
- If $s_1=1$ and $s_0=0$, then the output f is equal to w_2
- If $s_1=1$ and $s_0=1$, then the output f is equal to w_3

Graphical Symbol and Truth Table

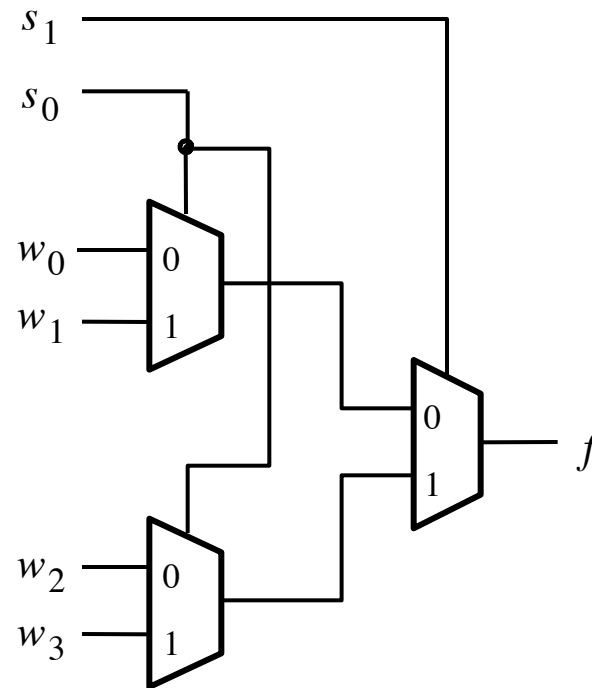


(a) Graphic symbol

s_1	s_0	f
0	0	w_0
0	1	w_1
1	0	w_2
1	1	w_3

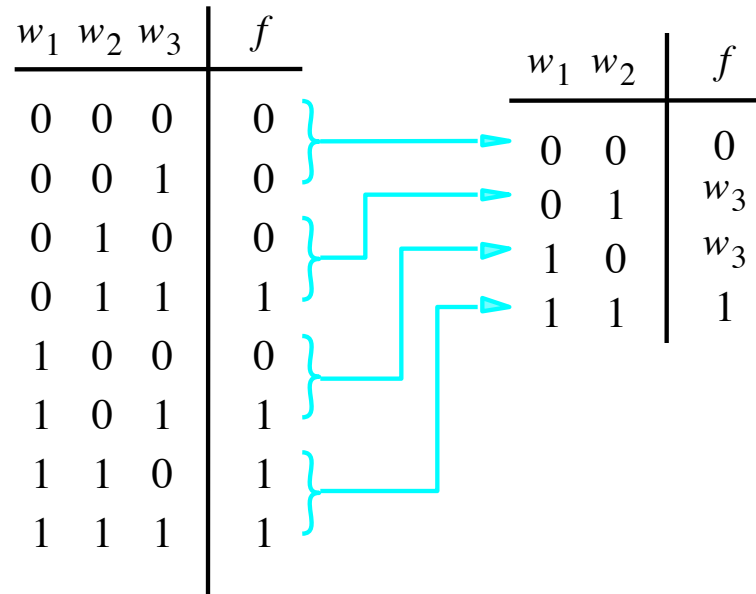
(b) Truth table

Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer

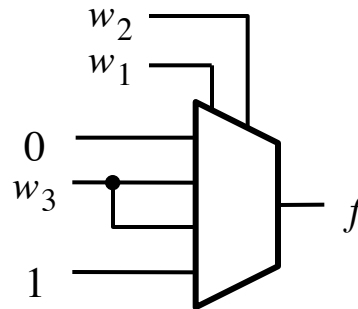


[Figure 4.3 from the textbook]

Implementation of a logic function



(a) Modified truth table



(b) Circuit

[Figure 4.7 from the textbook]

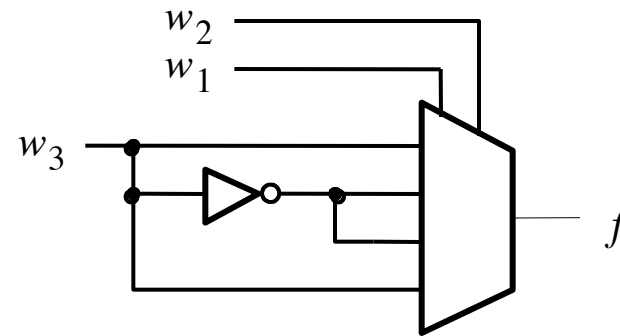
Implementation of 3-input XOR with a 4-to-1 Multiplexer

w_1	w_2	w_3	f	
0	0	0	0	} w_3
0	0	1	1	
0	1	0	1	} \bar{w}_3
0	1	1	0	
1	0	0	1	} \bar{w}_3
1	0	1	0	
1	1	0	0	} w_3
1	1	1	1	

Implementation of 3-input XOR with a 4-to-1 Multiplexer

w_1	w_2	w_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

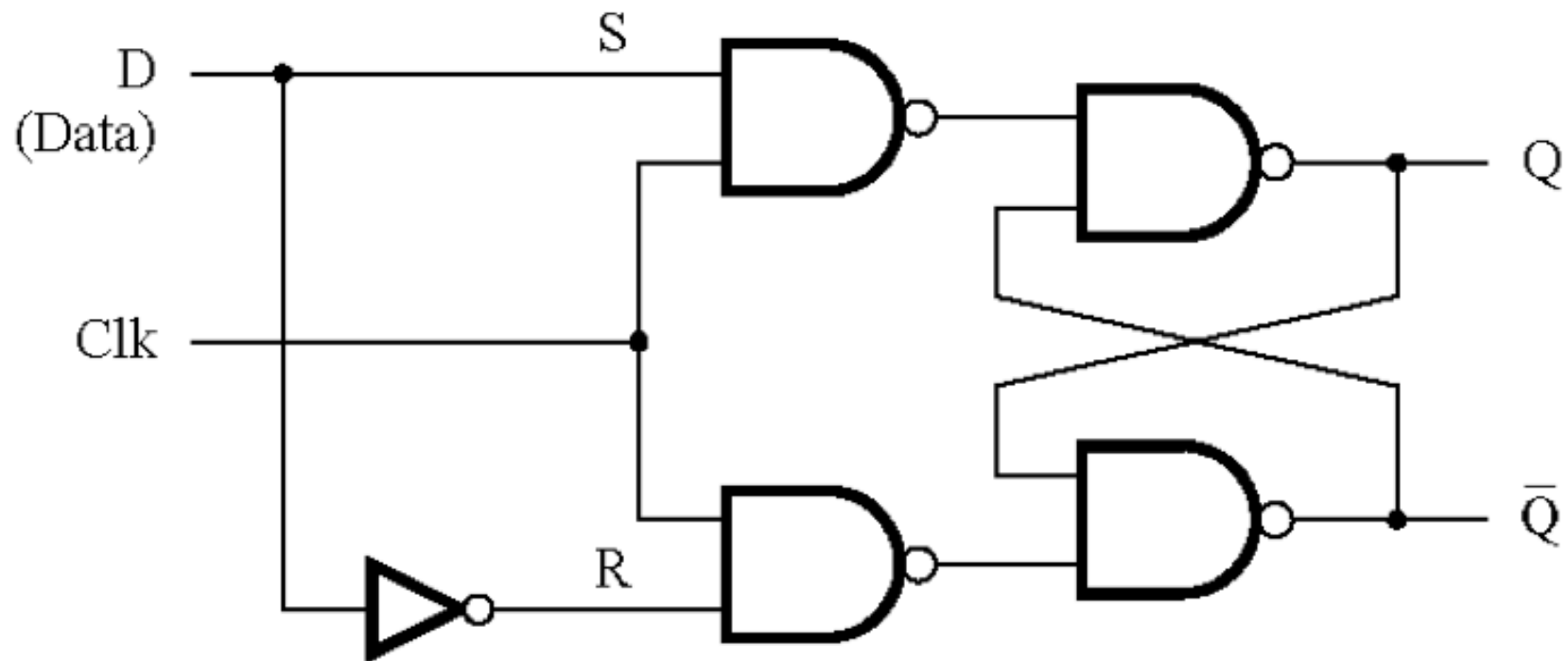
(a) Truth table



(b) Circuit

Gated D Latch

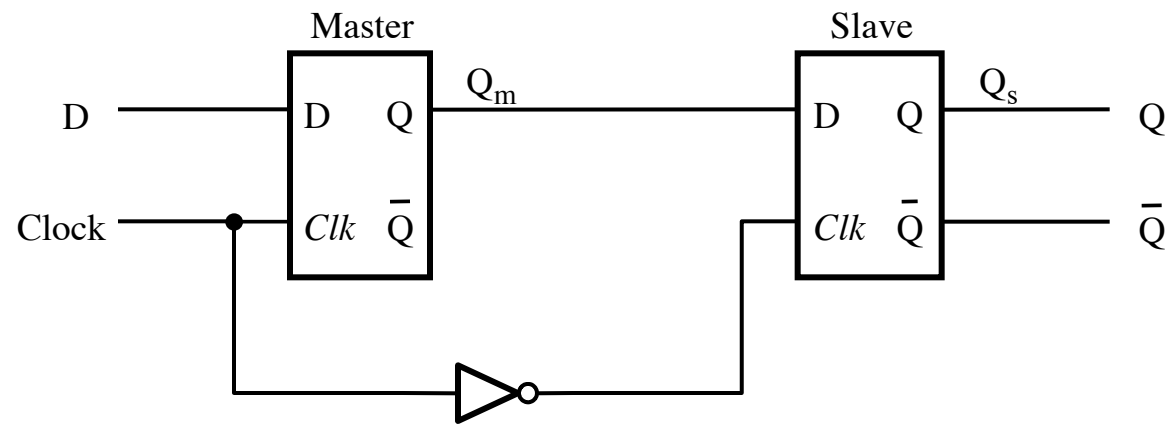
Circuit Diagram for the Gated D Latch



[Figure 5.7a from the textbook]

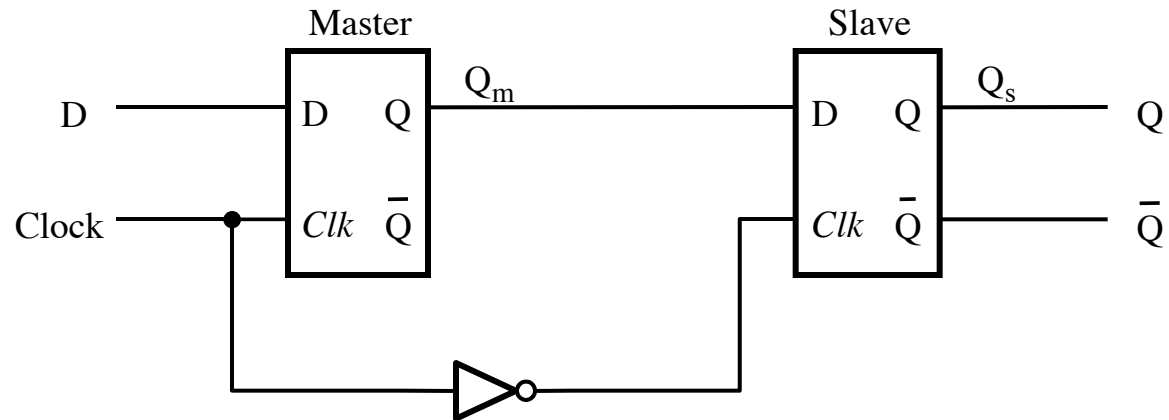
Edge-Triggered D Flip-Flops

Master-Slave D Flip-Flop

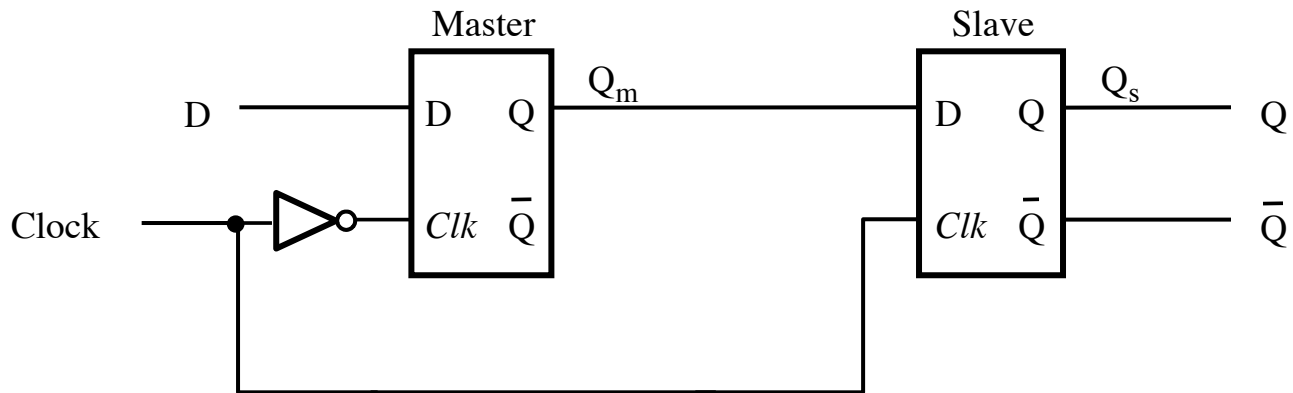


(a) Circuit

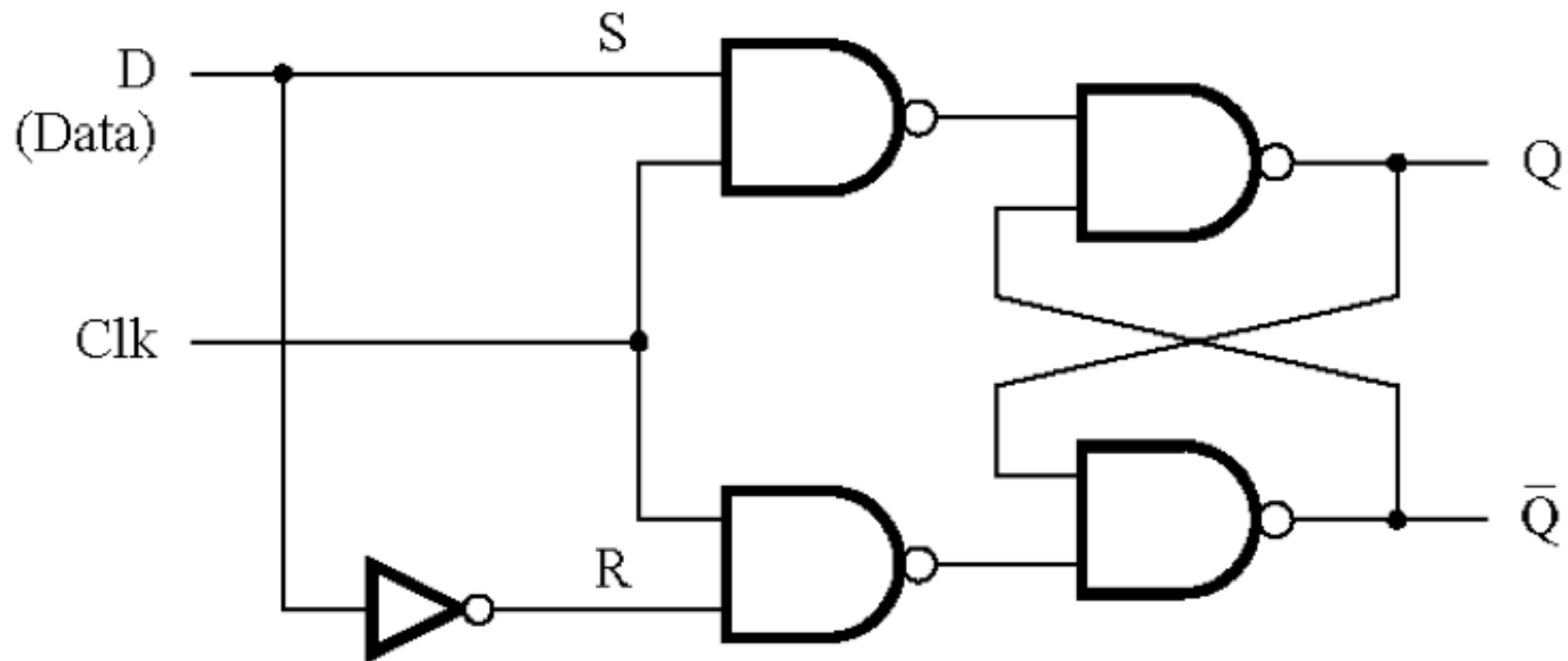
Negative-Edge-Triggered Master-Slave D Flip-Flop



Positive-Edge-Triggered Master-Slave D Flip-Flop

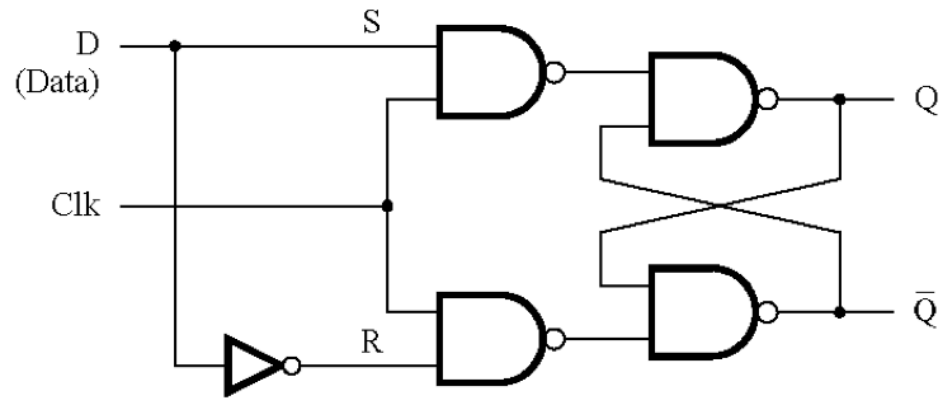
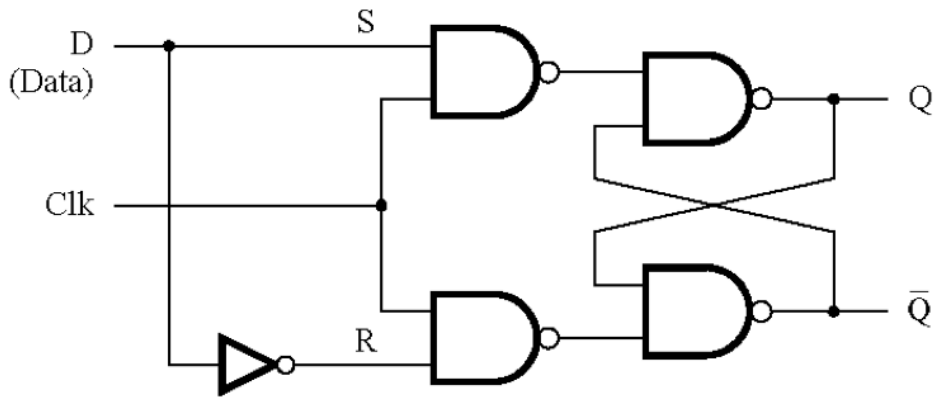


Circuit Diagram for the Gated D Latch

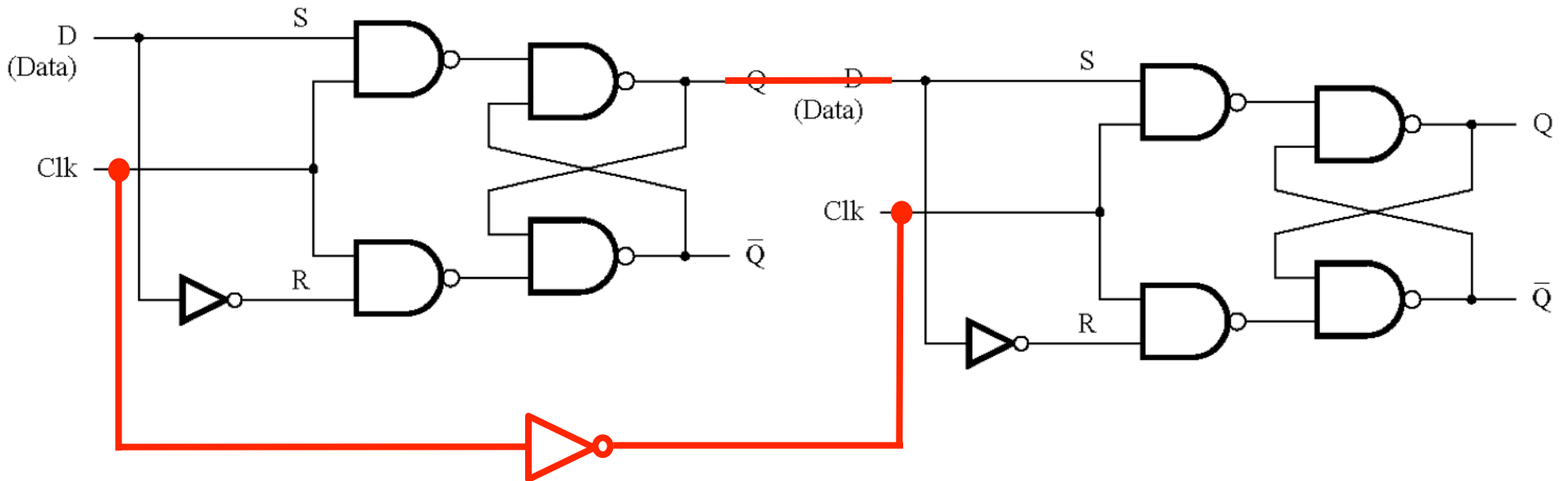


[Figure 5.7a from the textbook]

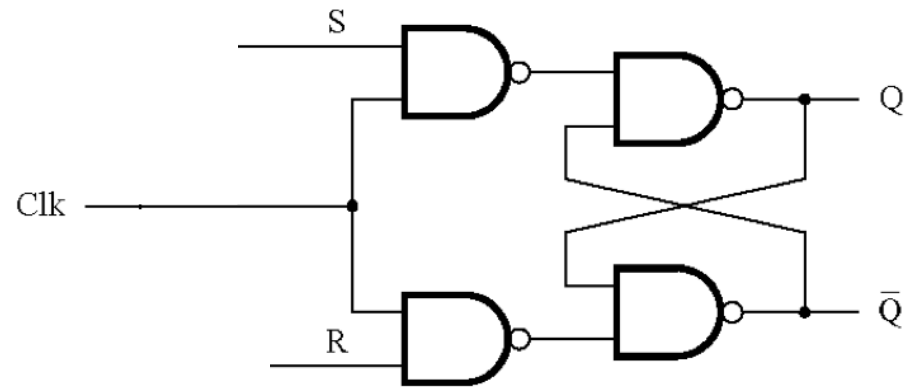
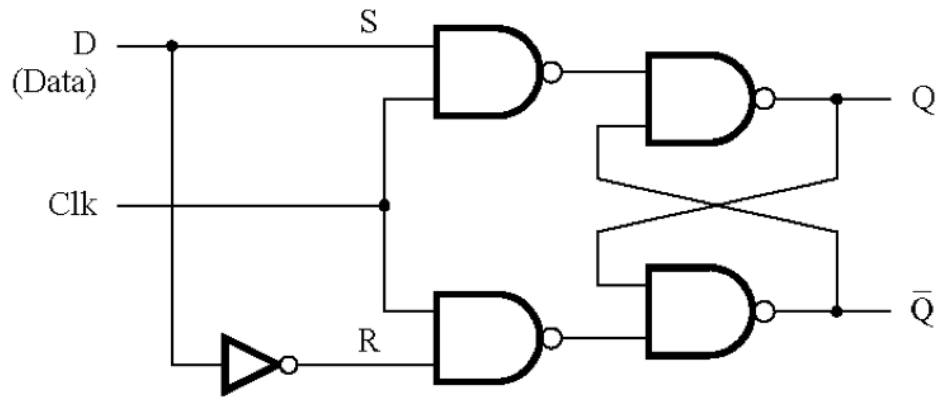
Constructing a D Flip-Flop



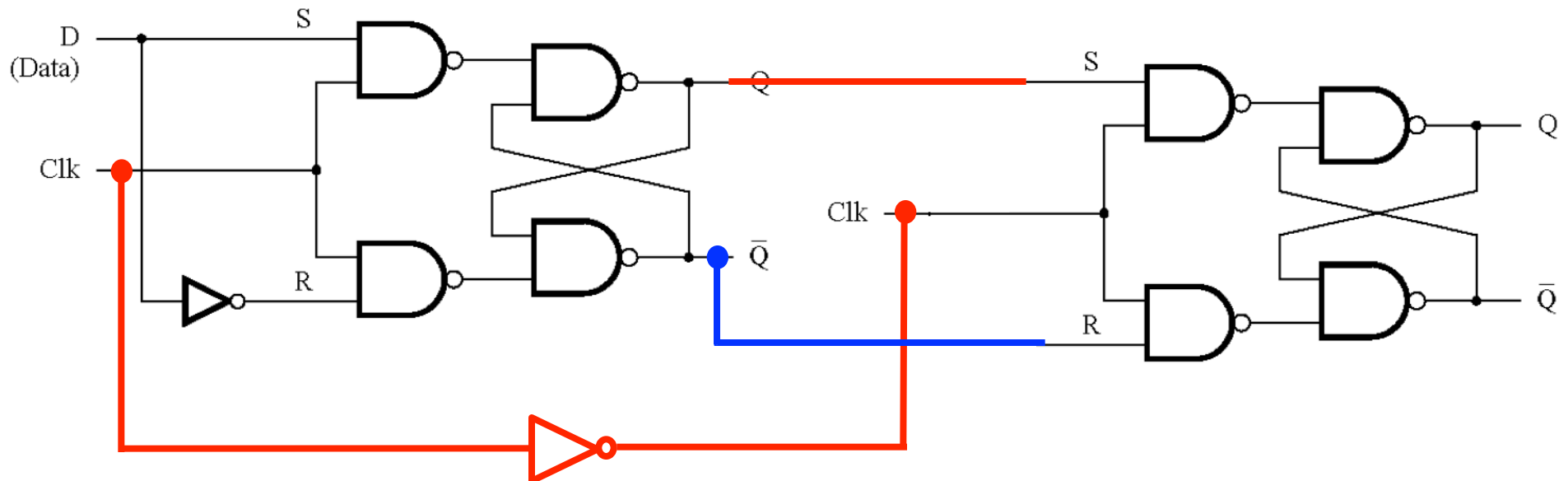
Constructing a D Flip-Flop



Constructing a D Flip-Flop (with one less NOT gate)

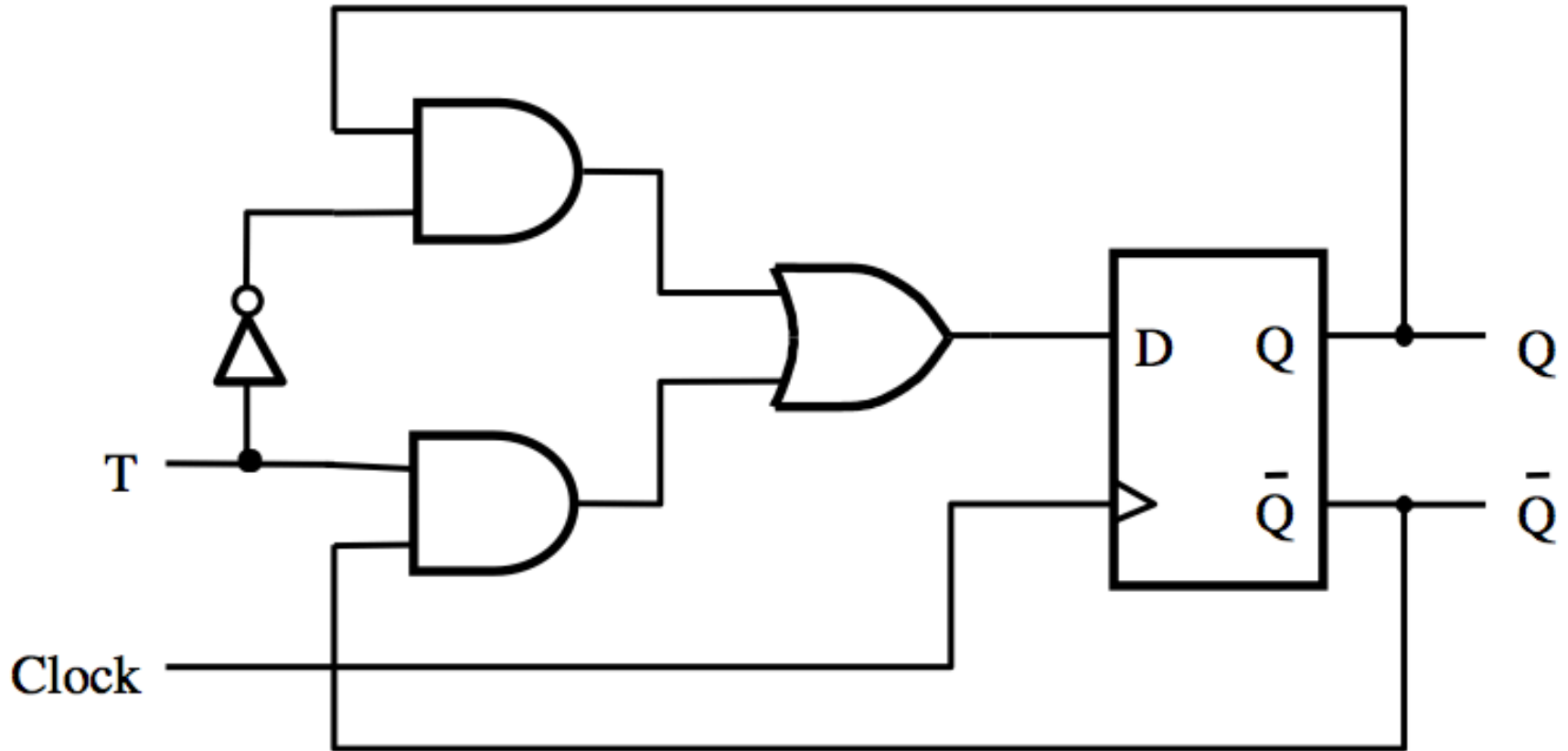


Constructing a D Flip-Flop (with one less NOT gate)



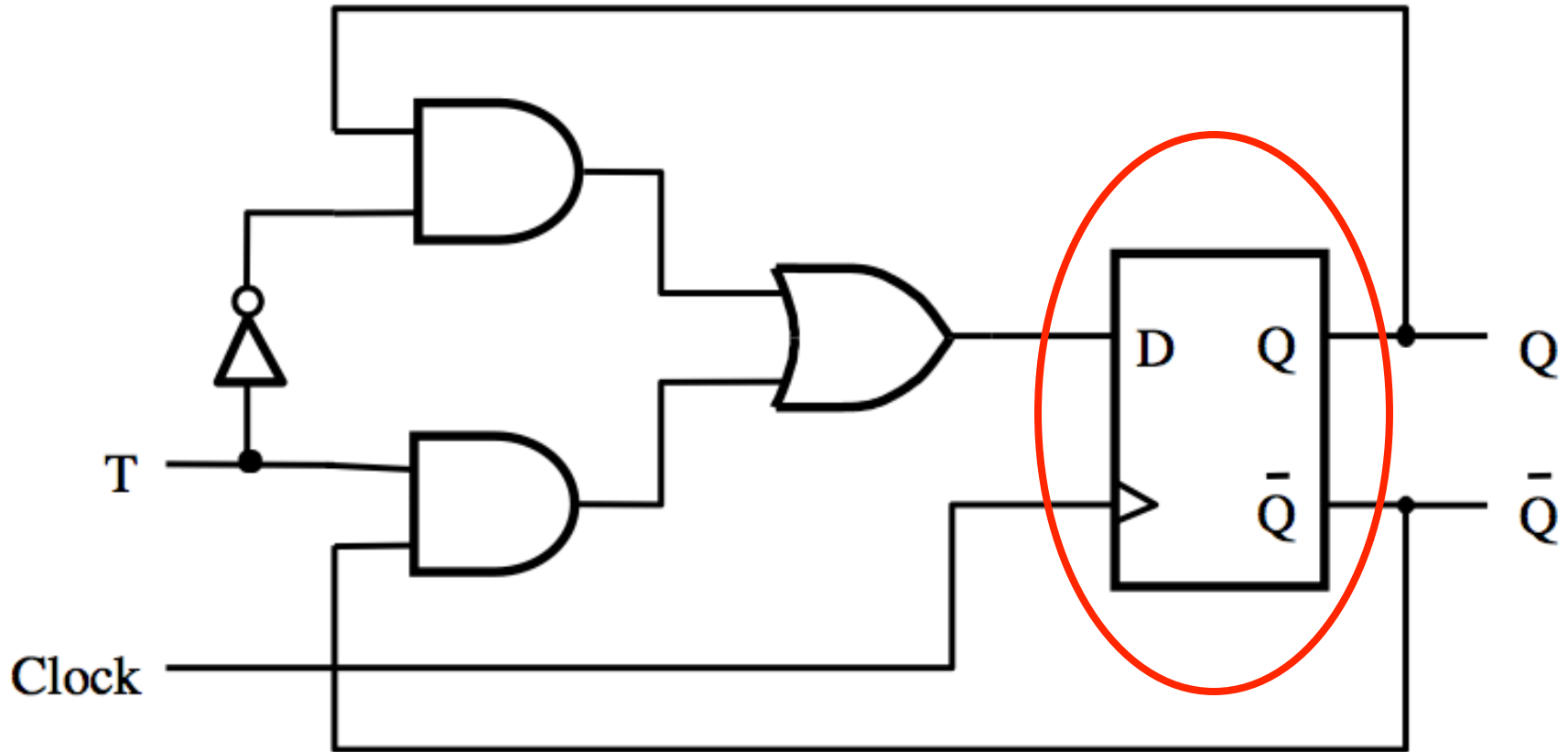
T Flip-Flop

T Flip-Flop



[Figure 5.15a from the textbook]

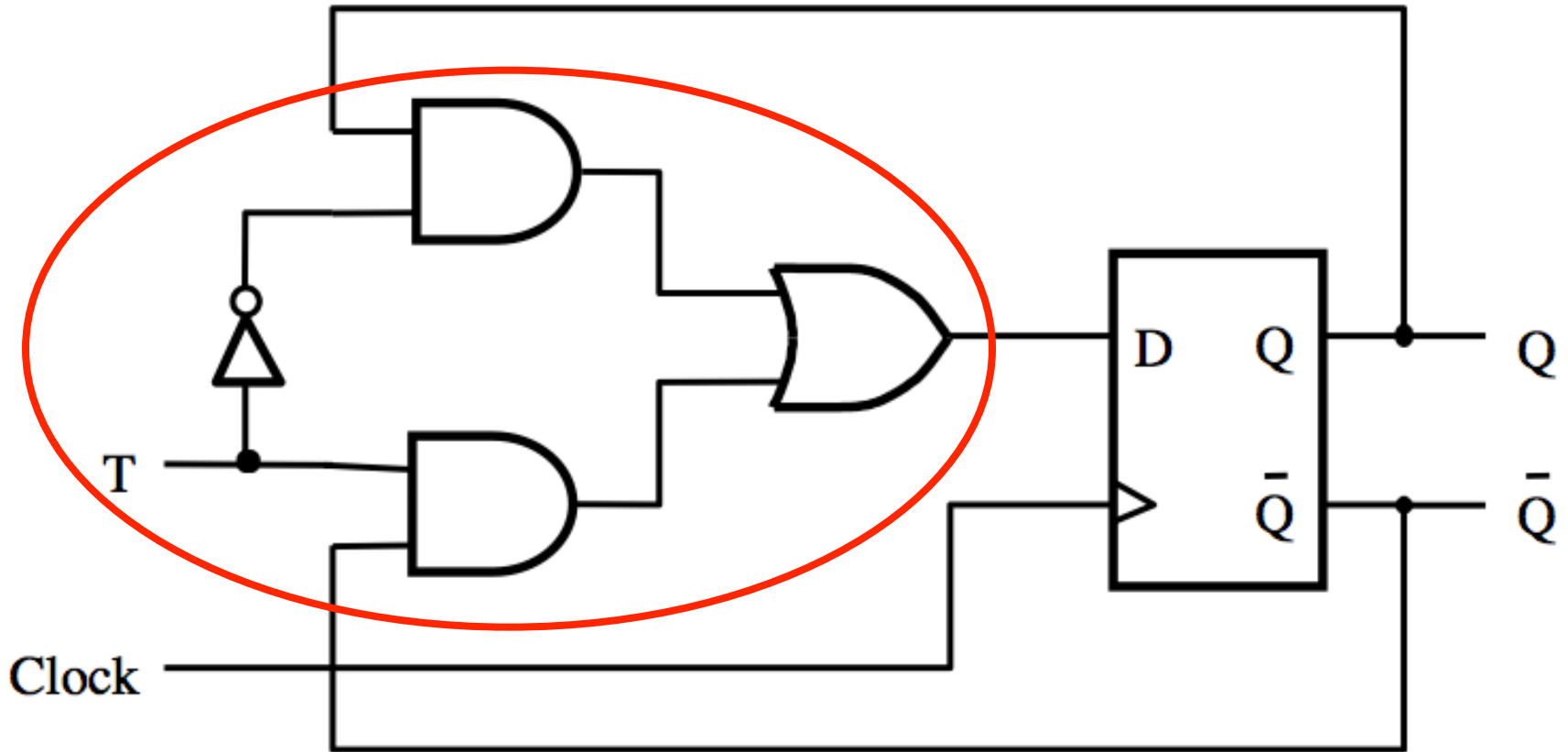
T Flip-Flop



Positive-edge-triggered
D Flip-Flop

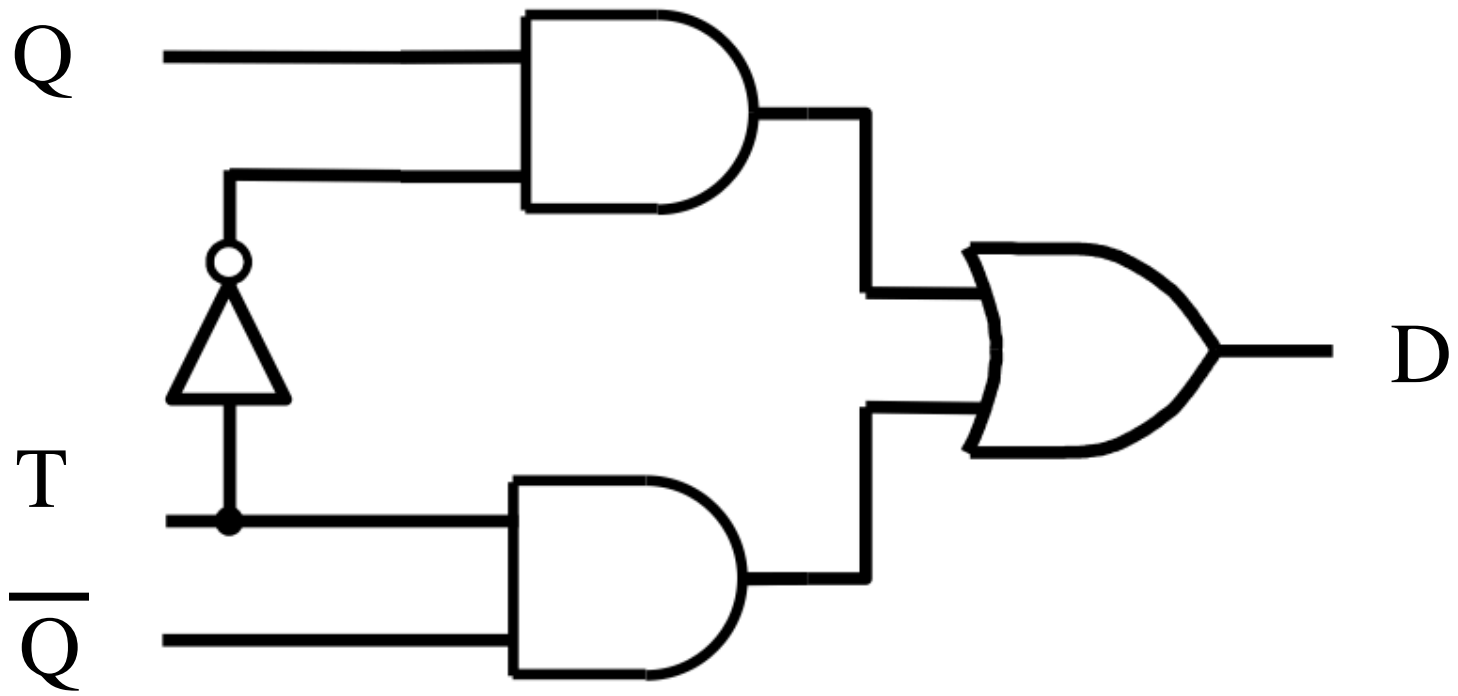
[Figure 5.15a from the textbook]

T Flip-Flop

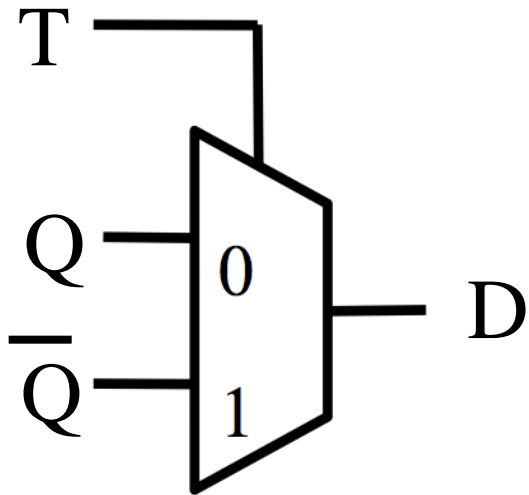


What is this?

What is this?

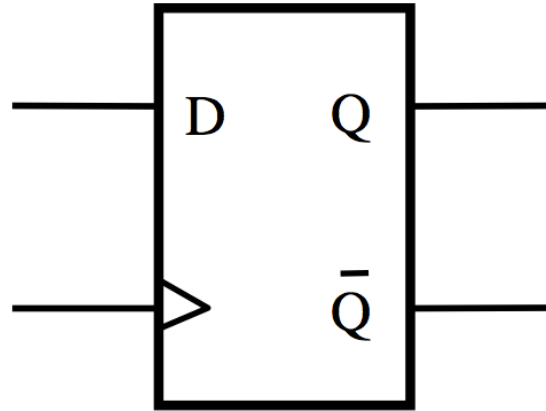


What is this?



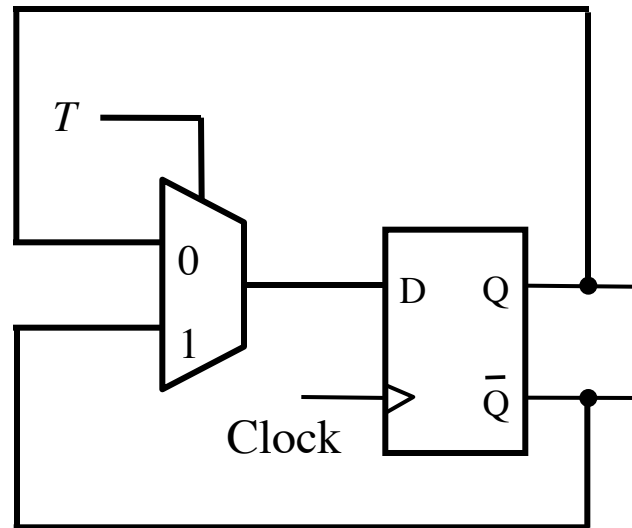
+

Clock

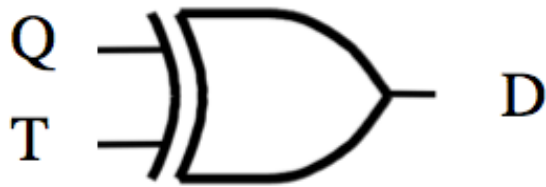


= ?

T Flip-Flop

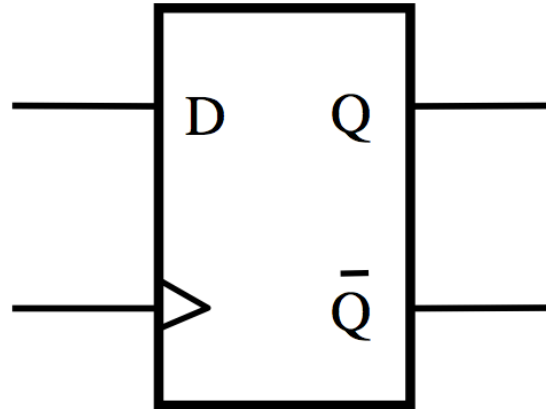


What is this?



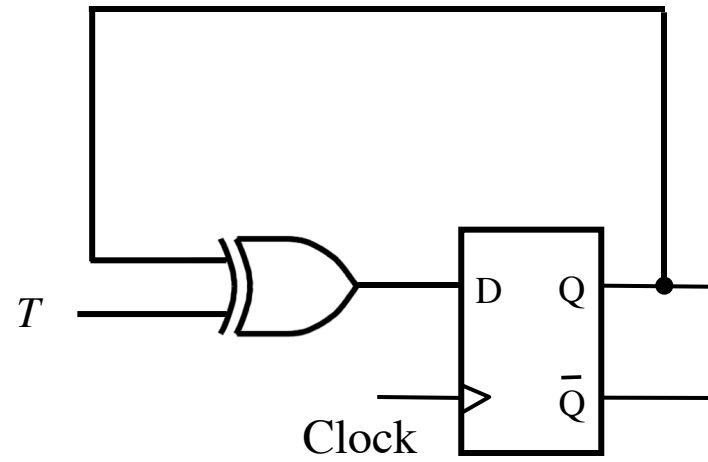
+

Clock



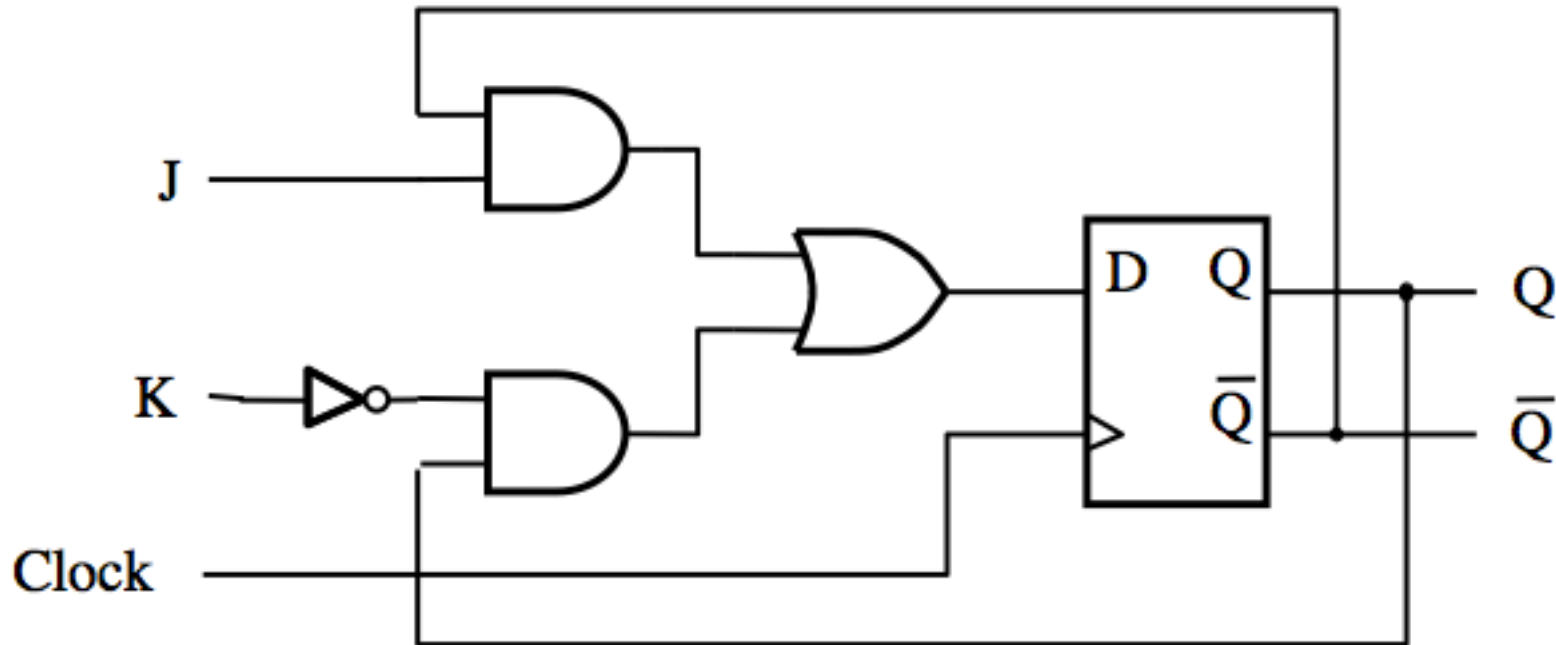
= ?

T Flip-Flop



JK Flip-Flop

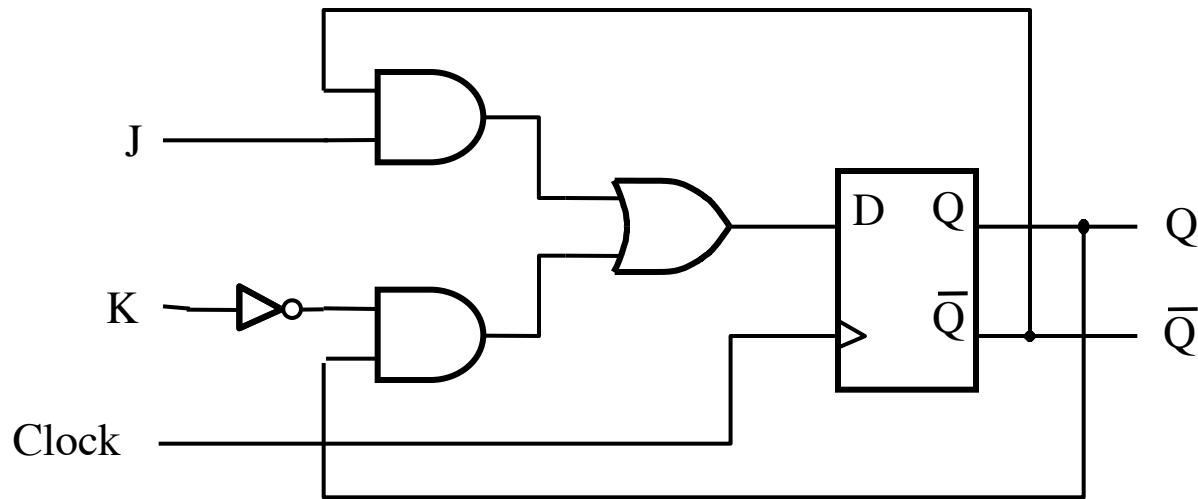
JK Flip-Flop



$$D = \overline{JQ} + \overline{KQ}$$

[Figure 5.16a from the textbook]

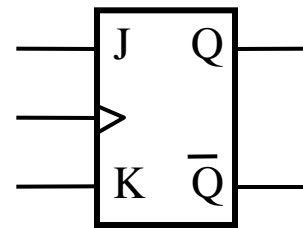
JK Flip-Flop



(a) Circuit

J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$\bar{Q}(t)$

(b) Truth table



(c) Graphical symbol

JK Flip-Flop (How it Works)

A versatile circuit that can be used both as a SR flip-flop and as a T flip flop

If $J=0$ and $S=0$ it stays in the same state

Just like SR It can be set and reset

$J=S$ and $K=R$

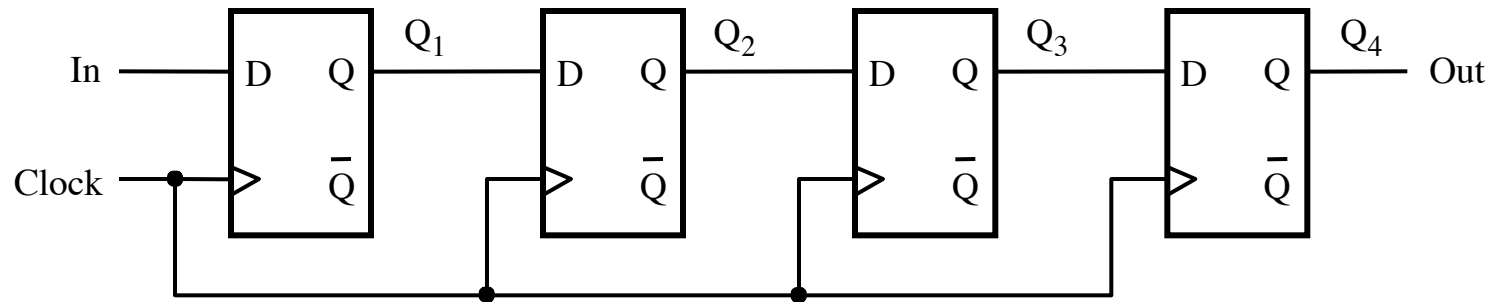
If $J=K=1$ then it behaves as a T flip-flop

Registers

Register (Definition)

An n-bit structure consisting of flip-flops

A simple shift register



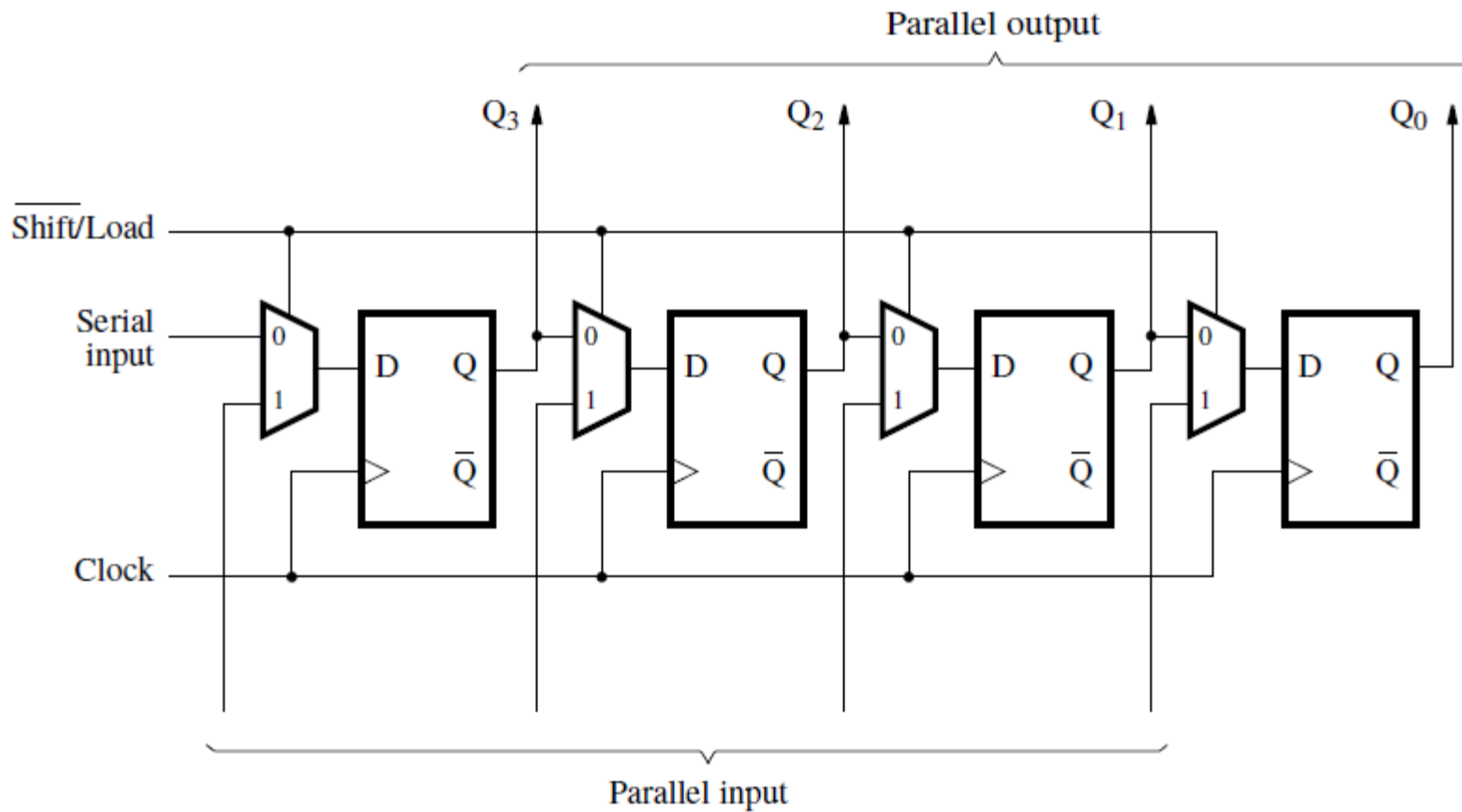
(a) Circuit

	In	Q ₁	Q ₂	Q ₃	Q ₄ = Out
t_0	1	0	0	0	0
t_1	0	1	0	0	0
t_2	1	0	1	0	0
t_3	1	1	0	1	0
t_4	1	1	1	0	1
t_5	0	1	1	1	0
t_6	0	0	1	1	1
t_7	0	0	0	1	1

(b) A sample sequence

[Figure 5.17 from the textbook]

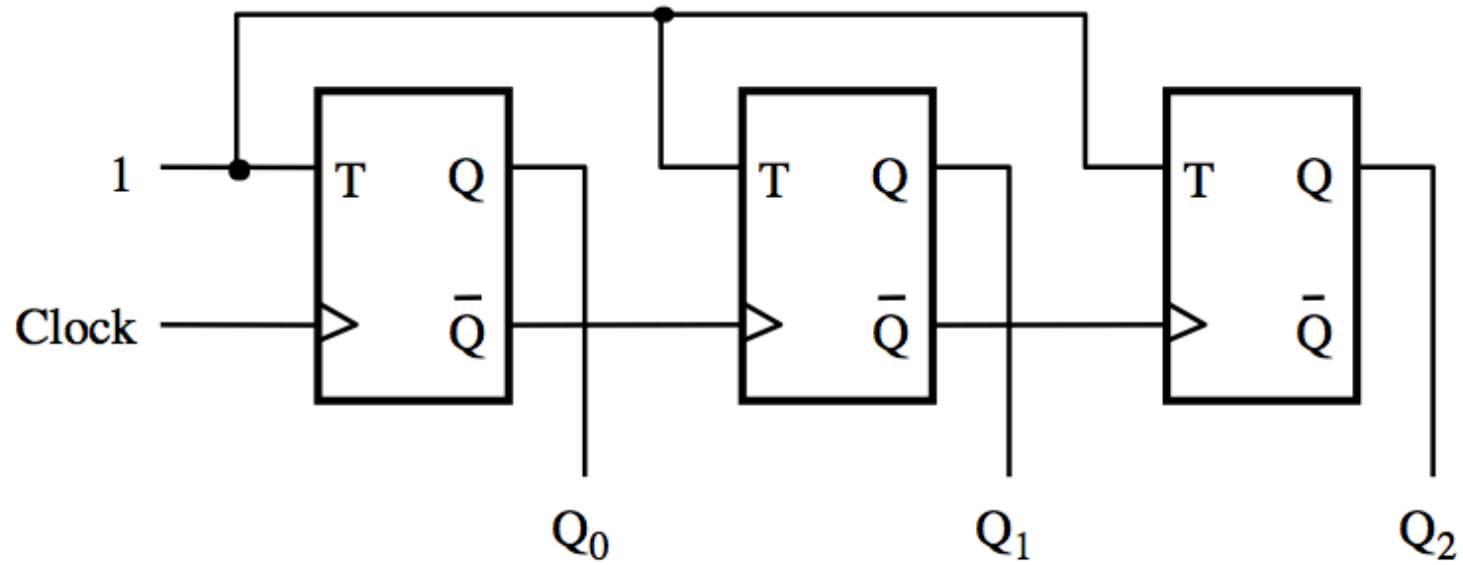
Parallel-access shift register



[Figure 5.18 from the textbook]

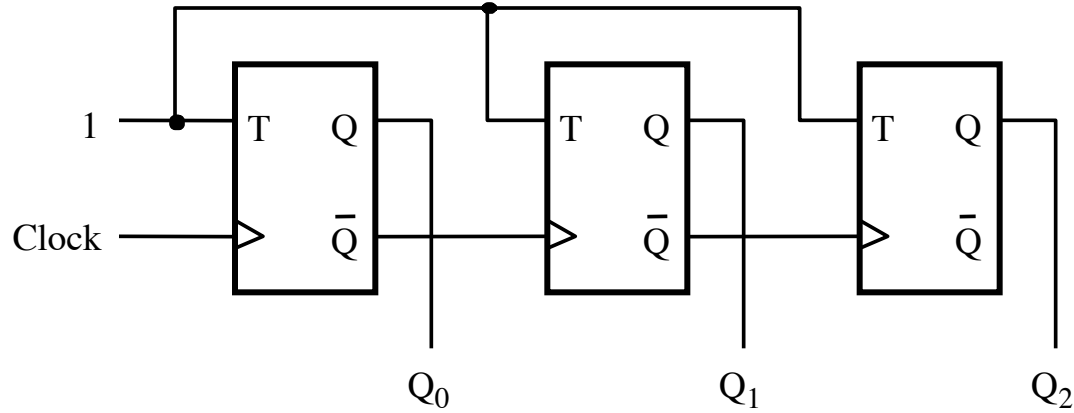
Counters

A three-bit up-counter

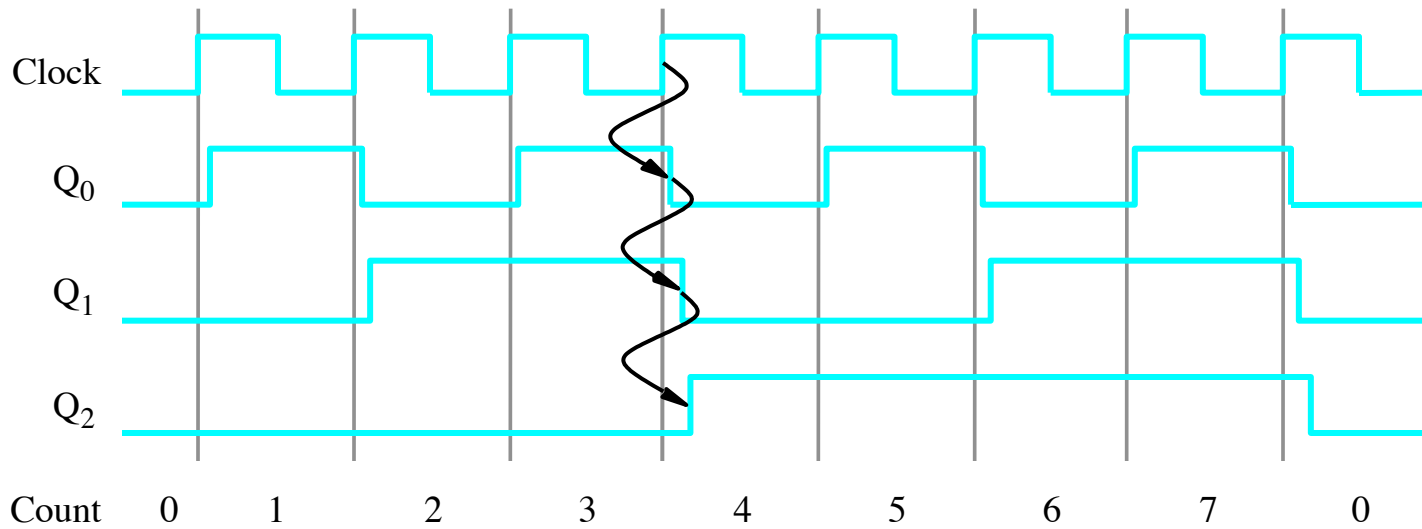


[Figure 5.19 from the textbook]

A three-bit up-counter

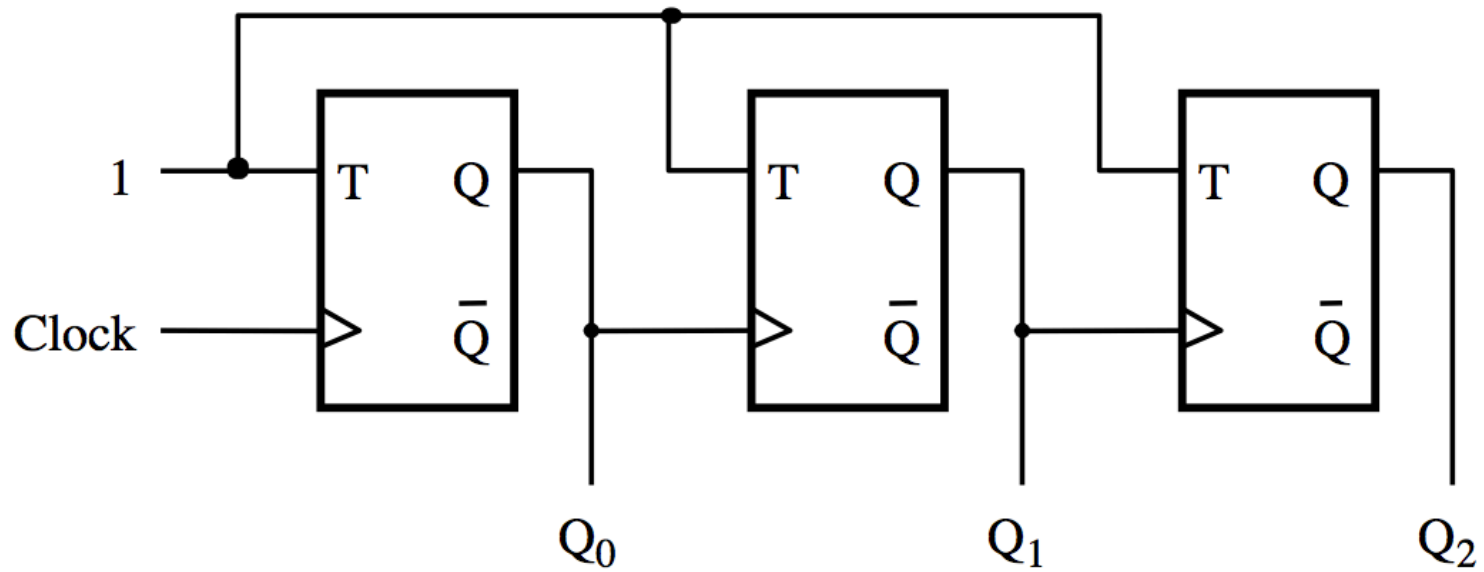


(a) Circuit



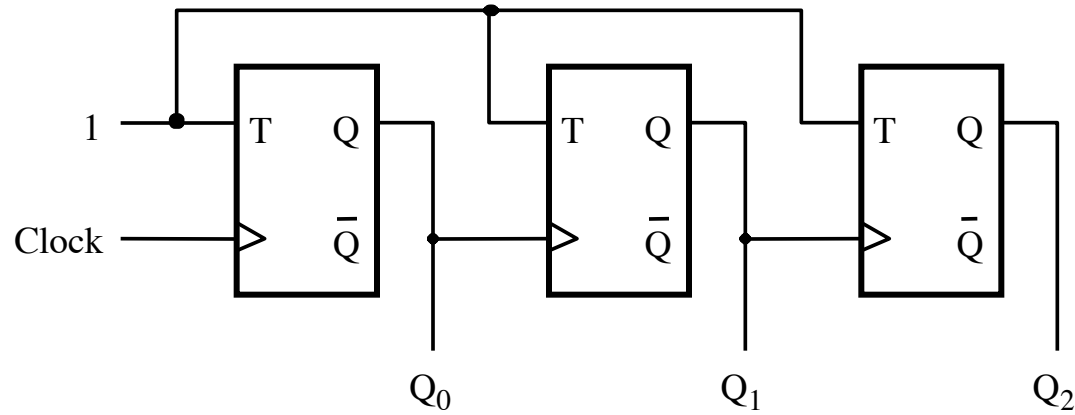
(b) Timing diagram

A three-bit down-counter

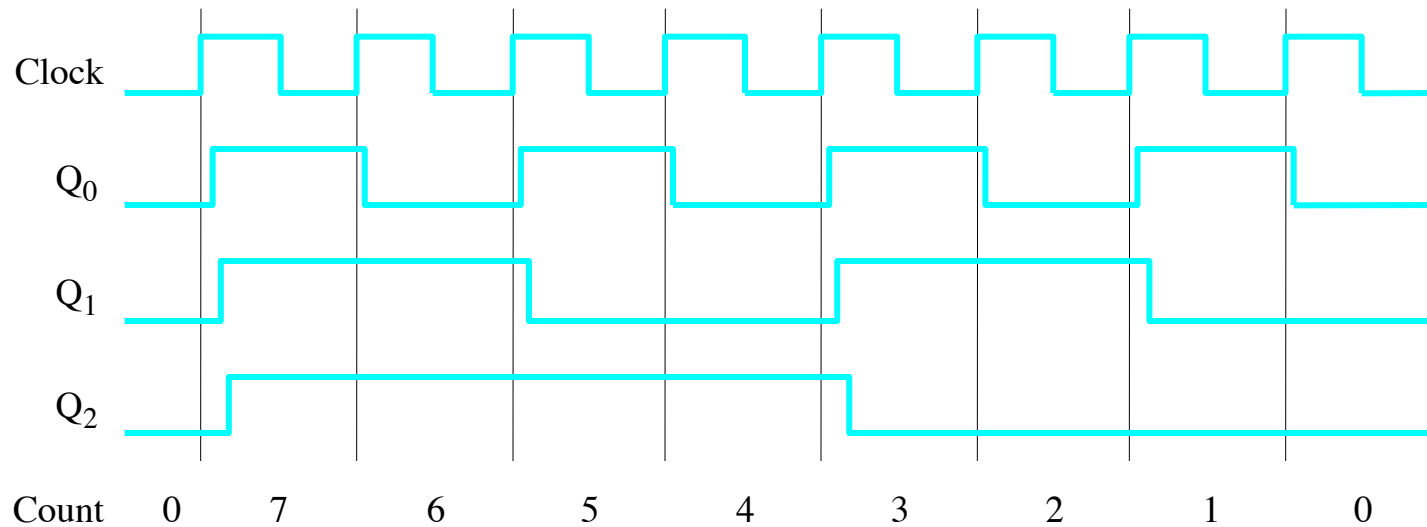


[Figure 5.20 from the textbook]

A three-bit down-counter



(a) Circuit

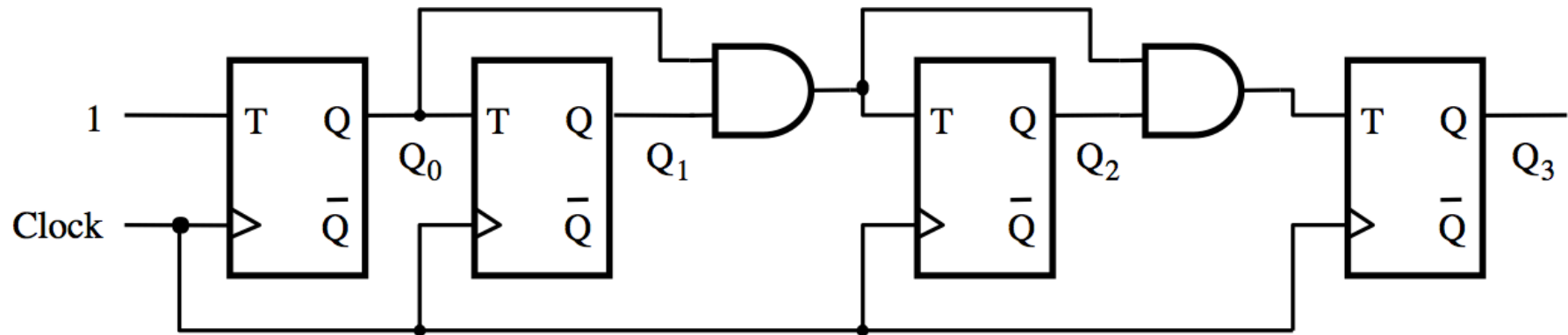


(b) Timing diagram

[Figure 5.20 from the textbook]

Synchronous Counters

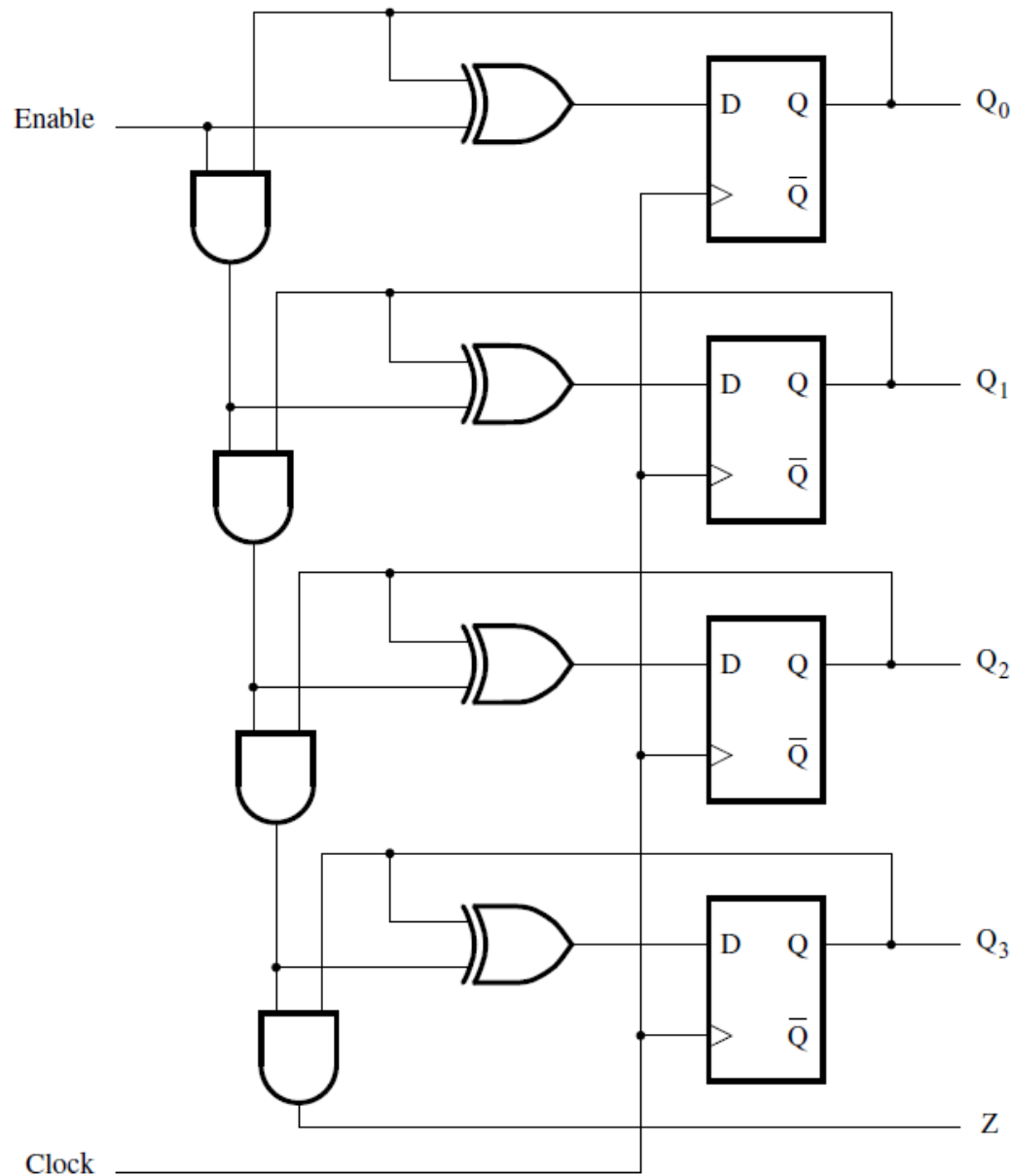
A four-bit synchronous up-counter



[Figure 5.21 from the textbook]

Synchronous Counter with D Flip-Flops

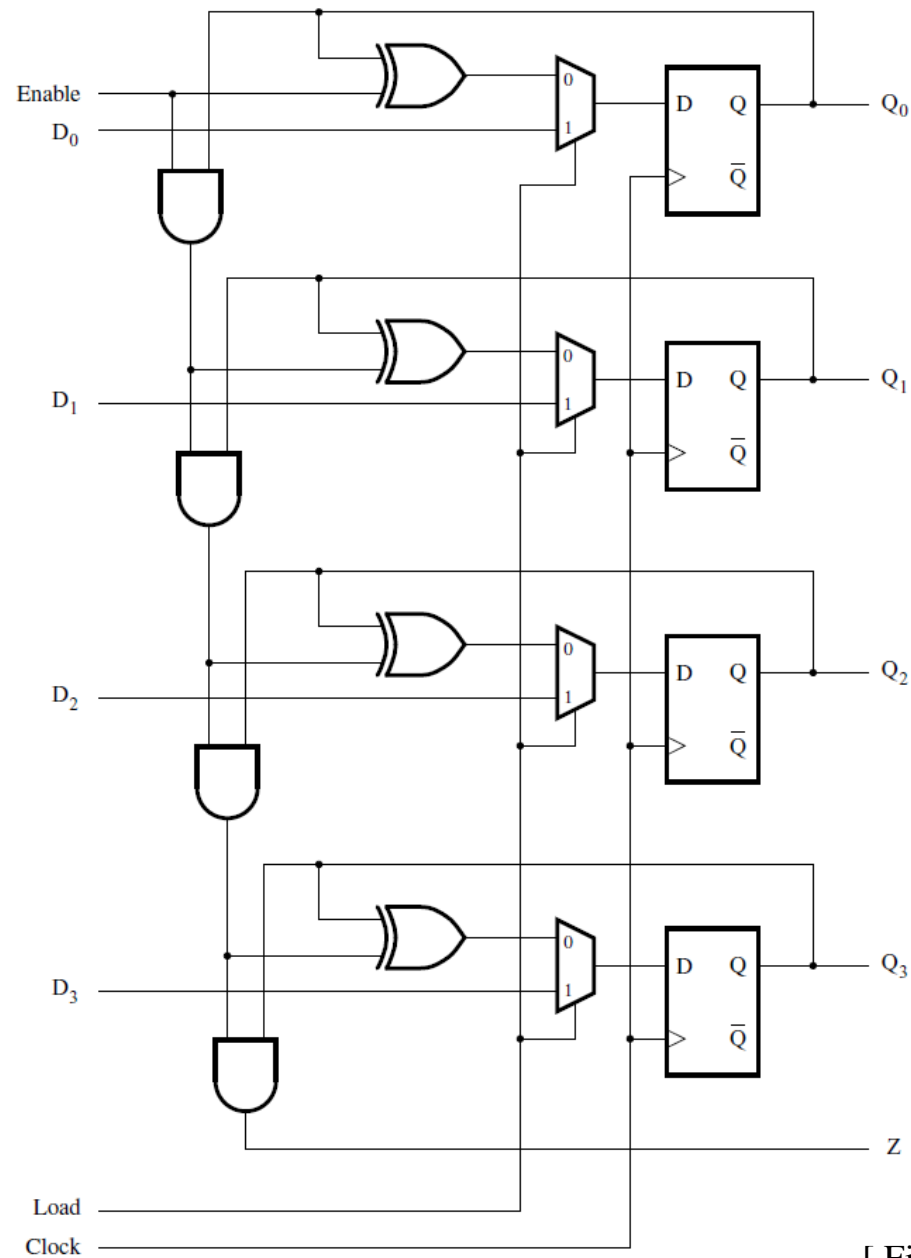
A four-bit counter with D flip-flops



[Figure 5.23 from the textbook]

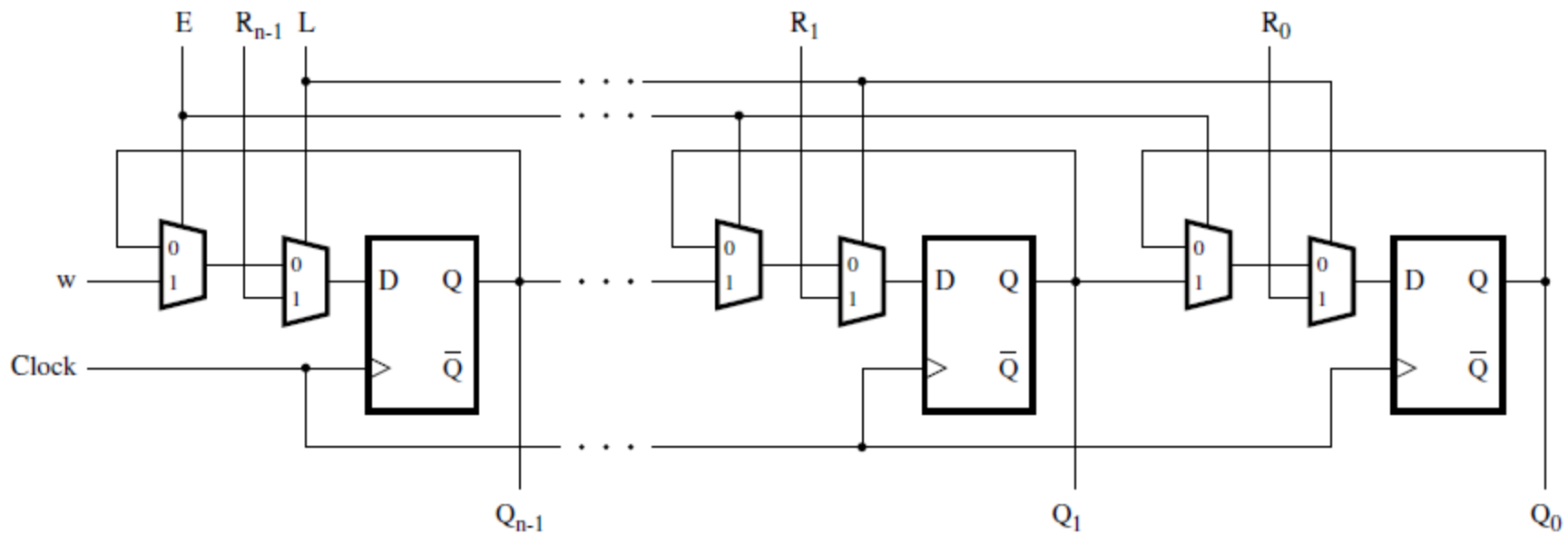
Counters with Parallel Load

A counter with parallel-load capability



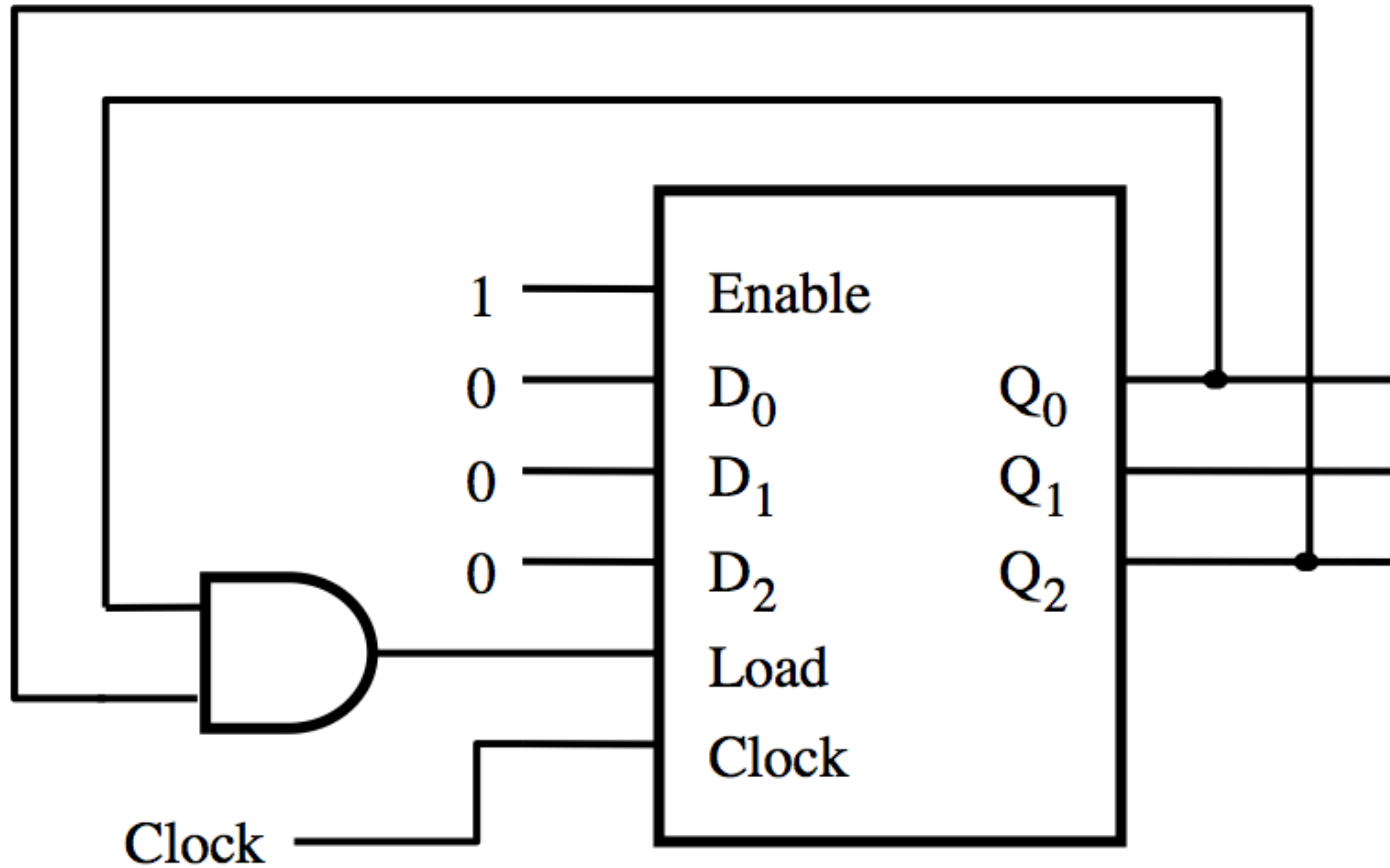
[Figure 5.24 from the textbook]

A shift register with parallel load and enable control inputs



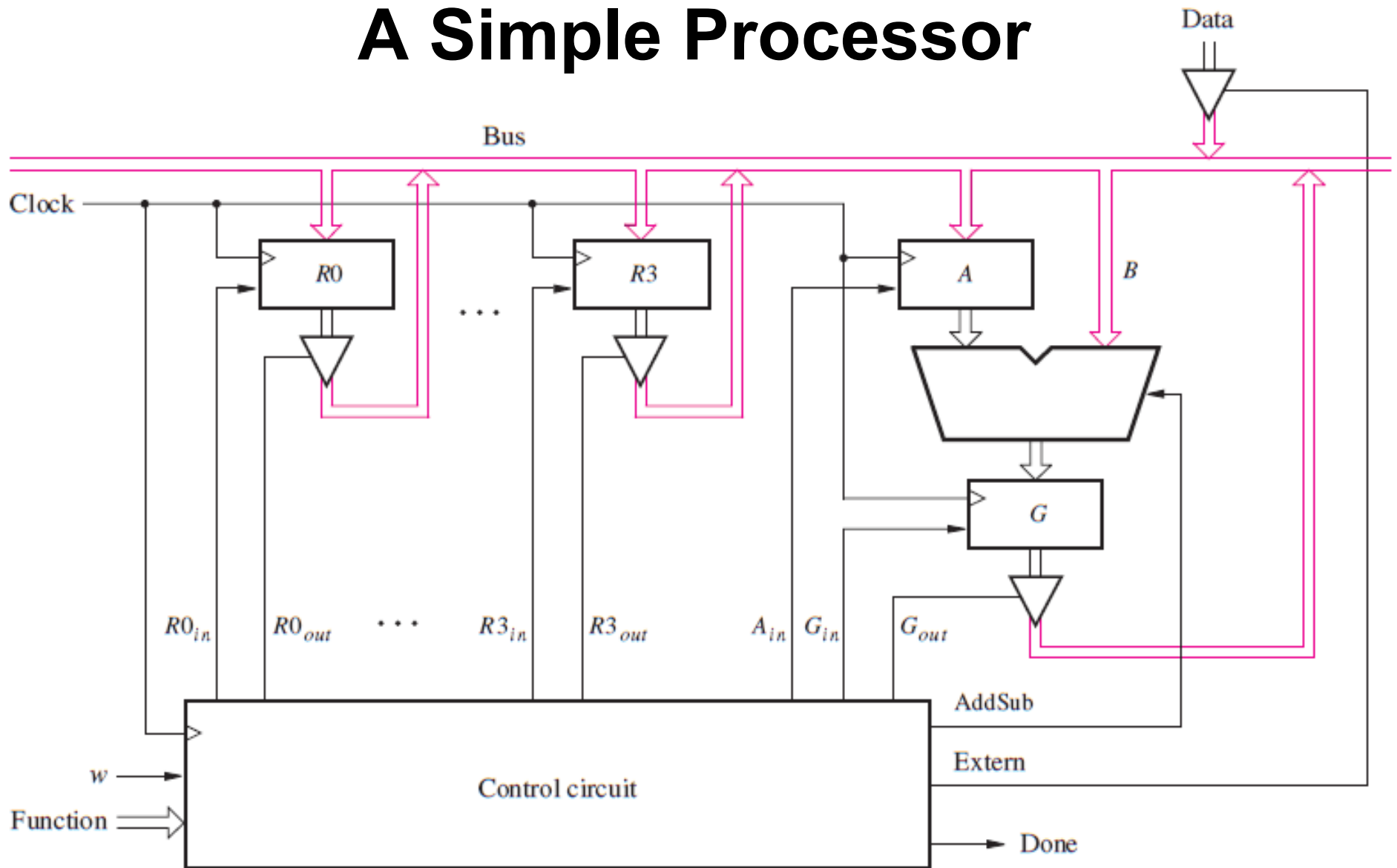
[Figure 5.59 from the textbook]

What does this circuit do?



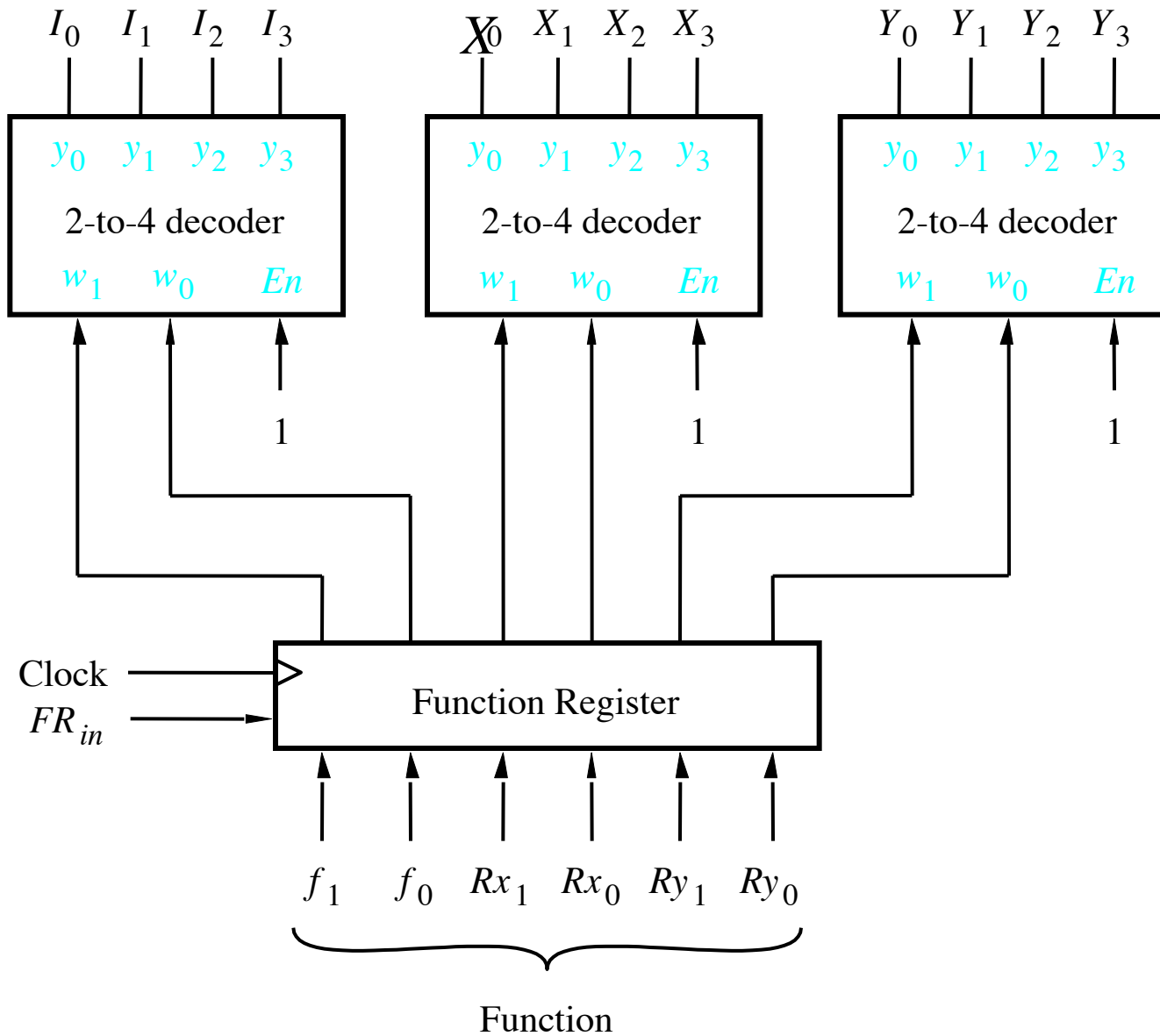
Designing The Control Circuit

A Simple Processor



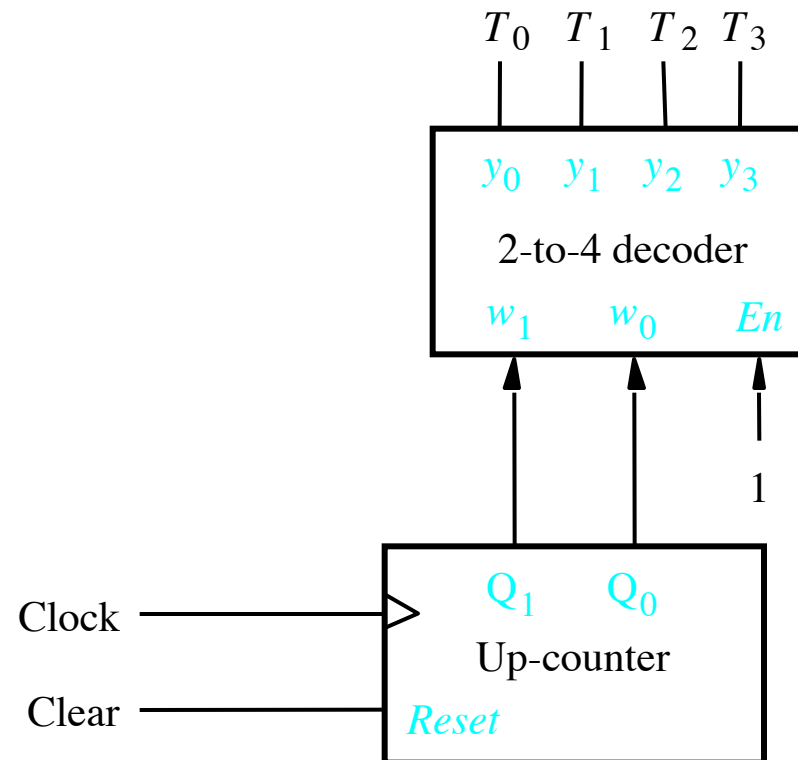
[Figure 7.9 from the textbook]

The function register and decoders



[Figure 7.11 from the textbook]

A part of the control circuit for the processor



Control signals asserted in each time step

	T_1	T_2	T_3
(Load): I_0	Extern $R_{in} = X$ Done		
(Move): I_1	$R_{in} = X$ $R_{out} = Y$ Done		
(Add): I_2	$R_{out} = X$ A_{in}	$R_{out} = Y$ G_{in} AddSub = 0	G_{out} $R_{in} = X$ Done
(Sub): I_3	$R_{out} = X$ A_{in}	$R_{out} = Y$ G_{in} AddSub = 1	G_{out} $R_{in} = X$ Done

Operations performed by this processor

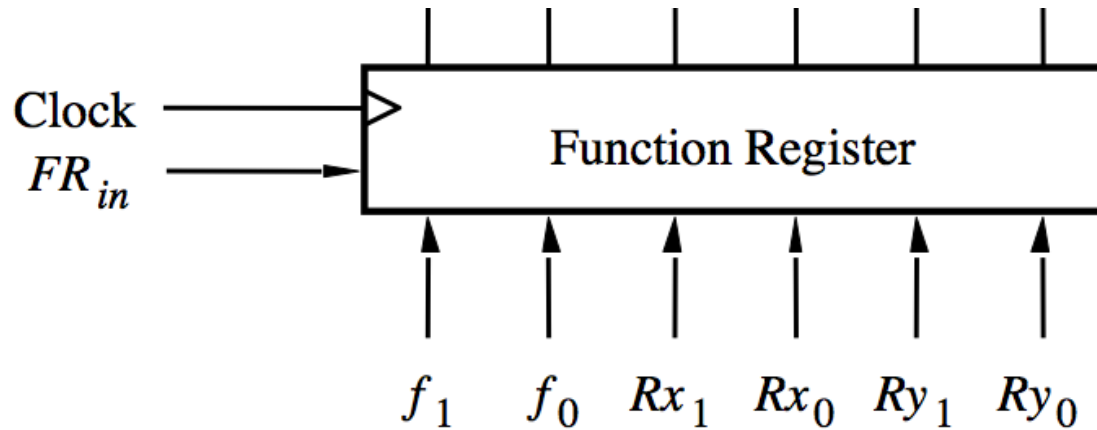
Operation	Function Performed
Load Rx, Data	Rx ← Data
Move Rx, Ry	Rx ← [Ry]
Add Rx, Ry	Rx ← [Rx] + [Ry]
Sub Rx, Ry	Rx ← [Rx] - [Ry]

Operations performed by this processor

Operation	Function Performed
Load Rx, Data	Rx ← Data
Move Rx, Ry	Rx ← [Ry]
Add Rx, Ry	Rx ← [Rx] + [Ry]
Sub Rx, Ry	Rx ← [Rx] - [Ry]

Where Rx and Ry can be one of four possible options: R0, R1, R2, and R3

Operations performed by this processor

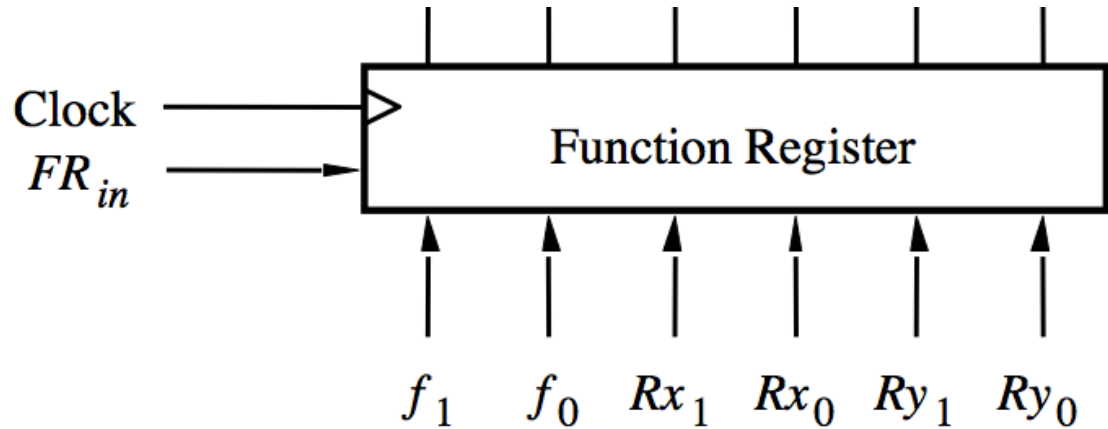


f_1	f_0	Function
0	0	Load
0	1	Move
1	0	Add
1	1	Sub

Rx_1	Rx_0	Register
0	0	R0
0	1	R1
1	0	R2
1	1	R3

Ry_1	Ry_0	Register
0	0	R0
0	1	R1
1	0	R2
1	1	R3

Operations performed by this processor



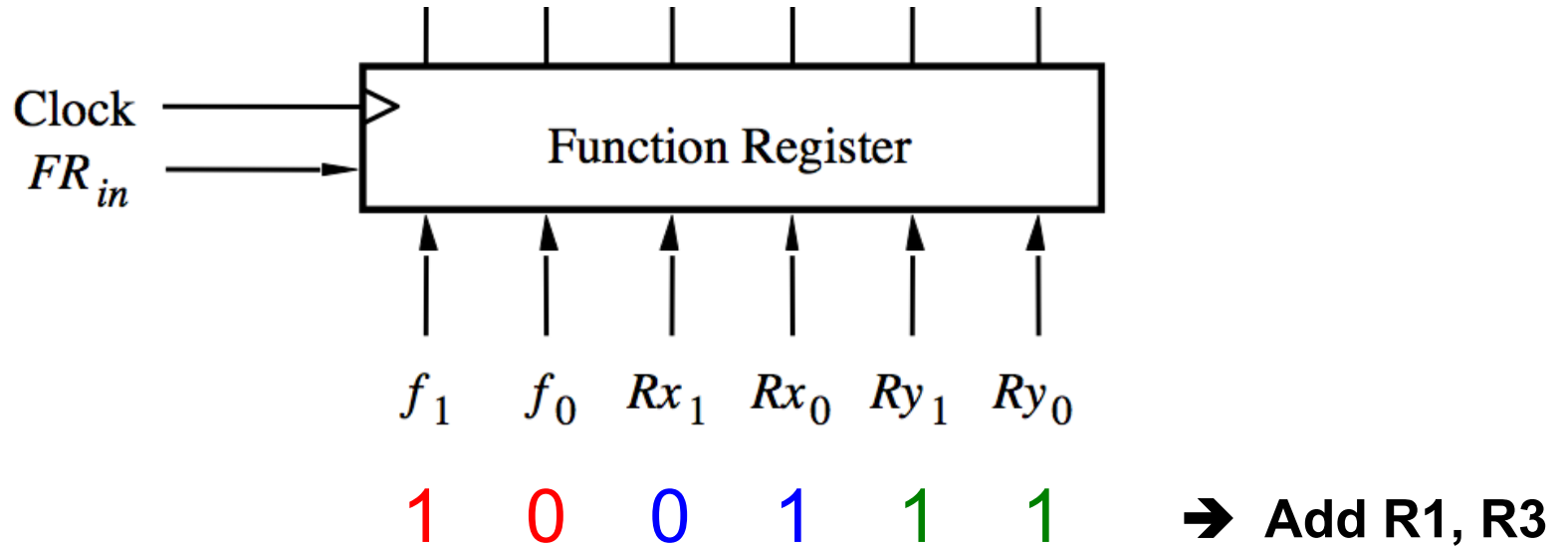
0 1 1 1 0 0 → Move R3, R0

f_1	f_0	Function
0	0	Load
0	1	Move
1	0	Add
1	1	Sub

Rx_1	Rx_0	Register
0	0	R0
0	1	R1
1	0	R2
1	1	R3

Ry_1	Ry_0	Register
0	0	R0
0	1	R1
1	0	R2
1	1	R3

Operations performed by this processor

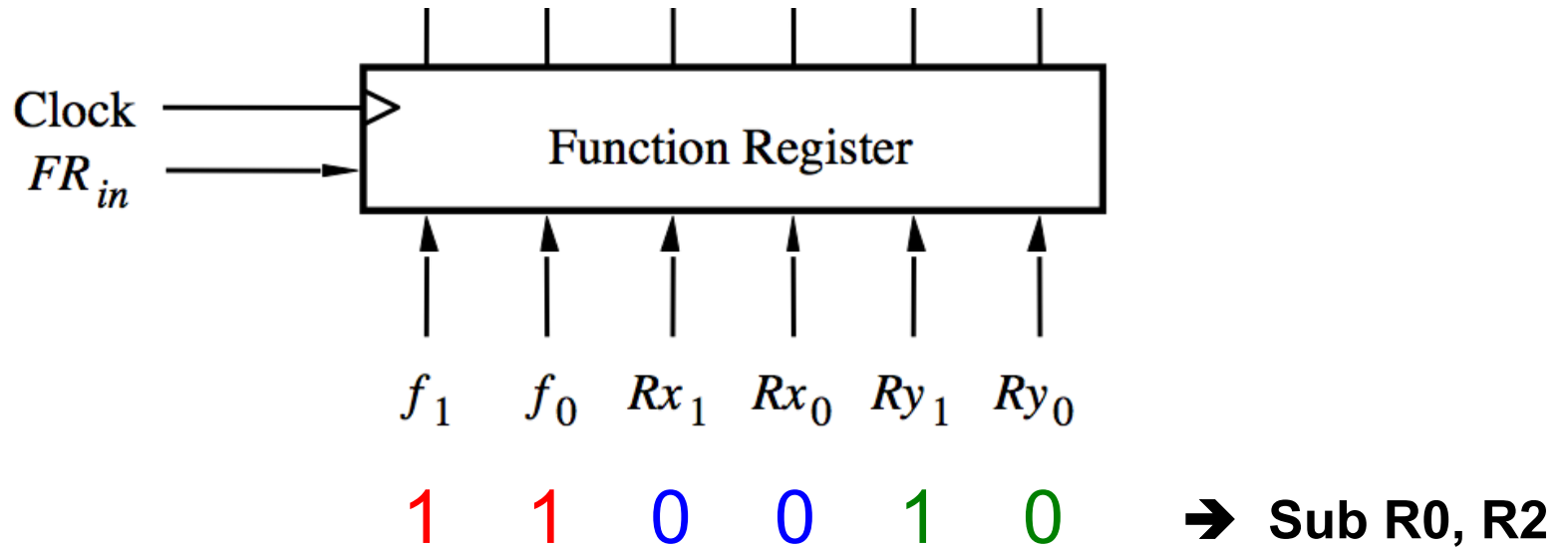


f_1	f_0	Function
0	0	Load
0	1	Move
1	0	Add
1	1	Sub

Rx_1	Rx_0	Register
0	0	R0
0	1	R1
1	0	R2
1	1	R3

Ry_1	Ry_0	Register
0	0	R0
0	1	R1
1	0	R2
1	1	R3

Operations performed by this processor

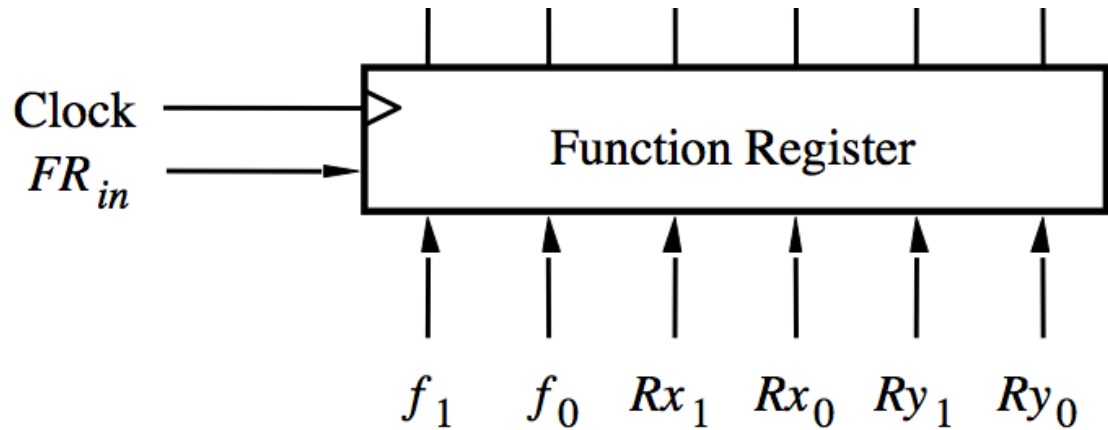


f_1	f_0	Function
0	0	Load
0	1	Move
1	0	Add
1	1	Sub

Rx_1	Rx_0	Register
0	0	R0
0	1	R1
1	0	R2
1	1	R3

Ry_1	Ry_0	Register
0	0	R0
0	1	R1
1	0	R2
1	1	R3

Operations performed by this processor



0 0 1 0 X X → Load R2, Data

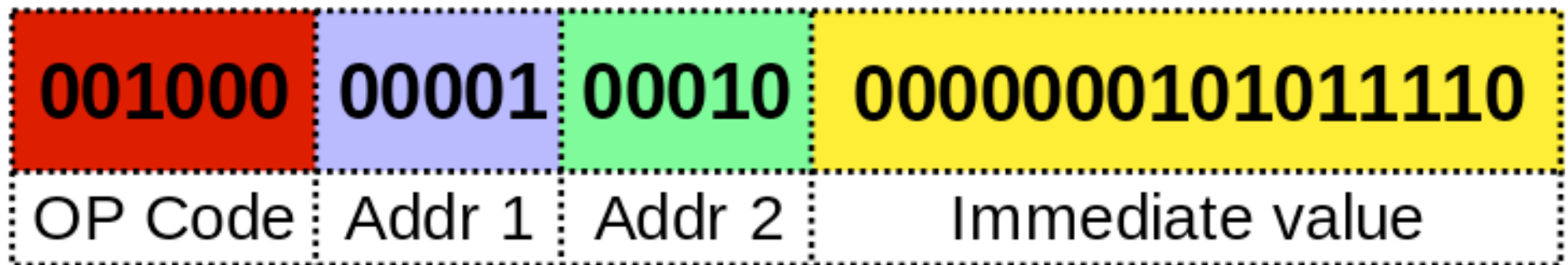
f_1	f_0	Function
0	0	Load
0	1	Move
1	0	Add
1	1	Sub

Rx_1	Rx_0	Register
0	0	R0
0	1	R1
1	0	R2
1	1	R3

Ry_1	Ry_0	Register
0	0	R0
0	1	R1
1	0	R2
1	1	R3

Similar Encoding is Used by Modern Chips

MIPS32 Add Immediate Instruction



Equivalent mnemonic:

addi \$r1, \$r2, 350

Sample Assembly Language Program For This Processor

```
Move   R3 , R0  
Add    R1 , R3  
Sub    R0 , R2  
Load  R2 , Data
```

Machine Language vs Assembly Language

Machine Language	Assembly Language	Meaning / Interpretation
011100	Move R3, R0	R3 ← [R0]
100111	Add R1, R3	R1 ← [R1] + [R3]
110010	Sub R0, R2	R0 ← [R0] - [R2]
001000	Load R2, Data	R2 ← Data

Machine Language vs Assembly Language

Machine Language	Assembly Language	Meaning / Interpretation
011100	Move R3, R0	R3 ← [R0]
100111	Add R1, R3	R1 ← [R1] + [R3]
110010	Sub R0, R2	R0 ← [R0] - [R2]
001000	Load R2, Data	R2 ← Data

Machine Language vs Assembly Language

Machine Language	Assembly Language	Meaning / Interpretation
011100	Move R3, R0	R3 ← [R0]
100111	Add R1, R3	R1 ← [R1] + [R3]
110010	Sub R0, R2	R0 ← [R0] - [R2]
001000	Load R2, Data	R2 ← Data

For short, each line
can be expressed as a
hexadecimal number

Machine Language vs Assembly Language

Machine Language	Assembly Language	Meaning / Interpretation
1C	Move R3, R0	R3 ← [R0]
27	Add R1, R3	R1 ← [R1] + [R3]
32	Sub R0, R2	R0 ← [R0] - [R2]
08	Load R2, Data	R2 ← Data

Questions?

THE END