

# **CprE 281: Digital Logic**

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**<http://www.ece.iastate.edu/~alexs/classes/>**

# Examples of Solved Problems

*CprE 281: Digital Logic*  
*Iowa State University, Ames, IA*  
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# **Administrative Stuff**

- **No homework is due next week.**

# **Administrative Stuff**

- **Midterm Exam #1**
- **When: Friday Sep 23.**
- **Where: This classroom**
- **What: Chapter 1 and Chapter 2 plus number systems**
- **The exam will be open book and open notes (you can bring up to 3 pages of handwritten notes).**

# Topics for the Midterm Exam

- **Binary Numbers**
- **Octal Numbers**
- **Hexadecimal Numbers**
- **Conversion between the different number systems**
- **Truth Tables**
- **Boolean Algebra**
- **Logic Gates**
- **Circuit Synthesis with AND, OR, NOT**
- **Circuit Synthesis with NAND, NOR**
- **Converting an AND/OR/NOT circuit to NAND circuit**
- **Converting an AND/OR/NOT circuit to NOR circuit**
- **SOP and POS expressions**

# Topics for the Midterm Exam

- **Mapping a Circuit to Verilog code**
- **Mapping Verilog code to a circuit**
  
- **Multiplexers**
- **Venn Diagrams**
- **K-maps for 2, 3, and 4 variables**
  
- **Minimization of Boolean expressions using theorems**
- **Minimization of Boolean expressions with K-maps**
  
- **Incompletely specified functions (with don't cares)**
- **Functions with multiple outputs**

# Example 1

Determine if the following equation is valid

$$\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2 = \bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3$$

$$\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2 \stackrel{?}{=} \bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3$$



$$\underbrace{\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2}_{\text{LHS}} \stackrel{?}{=} \underbrace{\bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3}_{\text{RHS}}$$

LHS

RHS

# Left-Hand Side (LHS)

Row number	$x_1$	$x_2$	$x_3$	$\overline{x_1 x_3}$	$x_2 x_3$	$x_1 \overline{x_2}$	$f$
0	0	0	0				
1	0	0	1				
2	0	1	0				
3	0	1	1				
4	1	0	0				
5	1	0	1				
6	1	1	0				
7	1	1	1				

# Left-Hand Side (LHS)

Row number	$x_1$	$x_2$	$x_3$	$\overline{x_1 x_3}$	$x_2 x_3$	$\overline{x_1 x_2}$	$f$
0	0	0	0	1	0	0	
1	0	0	1	0	0	0	
2	0	1	0	1	0	0	
3	0	1	1	0	1	0	
4	1	0	0	0	0	1	
5	1	0	1	0	0	1	
6	1	1	0	0	0	0	
7	1	1	1	0	1	0	

# Left-Hand Side (LHS)

Row number	$x_1$	$x_2$	$x_3$	$\overline{x_1 x_3}$	$x_2 x_3$	$\overline{x_1 x_2}$	$f$
0	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
2	0	1	0	1	0	0	1
3	0	1	1	0	1	0	1
4	1	0	0	0	0	1	1
5	1	0	1	0	0	1	1
6	1	1	0	0	0	0	0
7	1	1	1	0	1	0	1

# Right-Hand Side (RHS)

Row number	$x_1$	$x_2$	$x_3$	$\overline{x_1 x_2}$	$x_1 x_3$	$\overline{x_2} \overline{x_3}$	$f$
0	0	0	0				
1	0	0	1				
2	0	1	0				
3	0	1	1				
4	1	0	0				
5	1	0	1				
6	1	1	0				
7	1	1	1				

# Right-Hand Side (RHS)

Row number	$x_1$	$x_2$	$x_3$	$\overline{x_1 x_2}$	$x_1 x_3$	$\overline{x_2} \overline{x_3}$	$f$
0	0	0	0	0	0	1	
1	0	0	1	0	0	0	
2	0	1	0	1	0	0	
3	0	1	1	1	0	0	
4	1	0	0	0	0	1	
5	1	0	1	0	1	0	
6	1	1	0	0	0	0	
7	1	1	1	0	1	0	

# Right-Hand Side (RHS)

Row number	$x_1$	$x_2$	$x_3$	$\overline{x_1 x_2}$	$x_1 x_3$	$\overline{x_2} \overline{x_3}$	$f$
0	0	0	0	0	0	1	1
1	0	0	1	0	0	0	0
2	0	1	0	1	0	0	1
3	0	1	1	1	0	0	1
4	1	0	0	0	0	1	1
5	1	0	1	0	1	0	1
6	1	1	0	0	0	0	0
7	1	1	1	0	1	0	1

$$\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2 \stackrel{?}{=} \bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3$$

LHS

RHS

$f$

---

1

0

1

1

1

1

0

1

$f$

---

1

0

1

1

1

1

0

1



## Example 2

**Design the minimum-cost product-of-sums expression for the function**

$$f(x_1, x_2, x_3) = \sum m(0, 2, 4, 5, 6, 7)$$

# Minterms and Maxterms (with three variables)

Row number	$x_1$	$x_2$	$x_3$	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

# Minterms and Maxterms (with three variables)

Row number	$x_1$	$x_2$	$x_3$	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

**The function is  
1 for these rows**

# Minterms and Maxterms (with three variables)

Row number	$x_1$	$x_2$	$x_3$	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

**The function is  
1 for these rows**

**The function is  
0 for these rows**

# Two different ways to specify the same function $f$ of three variables

$$f(x_1, x_2, x_3) = \sum m(0, 2, 4, 5, 6, 7)$$

$$f(x_1, x_2, x_3) = \prod M(1, 3)$$

# The POS Expression

$$M_1 = x_1 + x_2 + \bar{x}_3$$

$$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$$

$$f(x_1, x_2, x_3) = \Pi M(1, 3)$$

$$= M_1 \cdot M_3$$

$$= (x_1 + x_2 + \bar{x}_3) \cdot (x_1 + \bar{x}_2 + \bar{x}_3)$$

# The Minimum POS Expression

$$\begin{aligned}f(x_1, x_2, x_3) &= (x_1 + x_2 + \bar{x}_3) \cdot (x_1 + \bar{x}_2 + \bar{x}_3) \\ &= (x_1 + \bar{x}_3 + x_2) \cdot (x_1 + \bar{x}_3 + \bar{x}_2) \\ &= (x_1 + \bar{x}_3)\end{aligned}$$

**Hint: Use the following Boolean Algebra theorem**

$$**14b. \quad (x + y) \cdot (x + \bar{y}) = x**$$

# Alternative Solution Using K-Maps

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

		$x_1x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map



# Alternative Solution Using K-Maps

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

		$x_1x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map

# Alternative Solution Using K-Maps

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

		$x_1x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map

# Alternative Solution Using K-Maps

		$x_1x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

# Alternative Solution Using K-Maps

		$x_1x_2$			
		00	01	11	10
$x_3$	0	1	1	1	1
	1	0	0	1	1

# Alternative Solution Using K-Maps

		$x_1x_2$			
		00	01	11	10
$x_3$	0	1	1	1	1
	1	0	0	1	1

$(x_1 + \bar{x}_3)$

## Example 3

**Problem:** A circuit that controls a given digital system has three inputs:  $x_1$ ,  $x_2$ , and  $x_3$ . It has to recognize three different conditions:

- Condition  $A$  is true if  $x_3$  is true and either  $x_1$  is true or  $x_2$  is false
- Condition  $B$  is true if  $x_1$  is true and either  $x_2$  or  $x_3$  is false
- Condition  $C$  is true if  $x_2$  is true and either  $x_1$  is true or  $x_3$  is false

The control circuit must produce an output of 1 if at least two of the conditions  $A$ ,  $B$ , and  $C$  are true. Design the simplest circuit that can be used for this purpose.

# Condition A

Condition A is true if  $x_3$  is true and either  $x_1$  is true or  $x_2$  is false

# Condition A

Condition A is true if  $x_3$  is true and either  $x_1$  is true or  $x_2$  is false

$$A = x_3(x_1 + \bar{x}_2) = x_3x_1 + x_3\bar{x}_2$$



# Condition B

Condition  $B$  is true if  $x_1$  is true and either  $x_2$  or  $x_3$  is false

# Condition B

Condition  $B$  is true if  $x_1$  is true and either  $x_2$  or  $x_3$  is false

$$B = x_1(\bar{x}_2 + \bar{x}_3) = x_1\bar{x}_2 + x_1\bar{x}_3$$

# Condition C

Condition  $C$  is true if  $x_2$  is true and either  $x_1$  is true or  $x_3$  is false

# Condition C

Condition  $C$  is true if  $x_2$  is true and either  $x_1$  is true or  $x_3$  is false

$$C = x_2(x_1 + \bar{x}_3) = x_2x_1 + x_2\bar{x}_3$$

The output of the circuit can be expressed as

$$f = \mathbf{AB} + AC + BC$$

$$AB = (x_3x_1 + x_3\bar{x}_2)(x_1\bar{x}_2 + x_1\bar{x}_3)$$

$$= x_3x_1x_1\bar{x}_2 + x_3x_1x_1\bar{x}_3 + x_3\bar{x}_2x_1\bar{x}_2 + x_3\bar{x}_2x_1\bar{x}_3$$

$$= x_3x_1\bar{x}_2 + 0 + x_3\bar{x}_2x_1 + 0$$

$$= x_1\bar{x}_2x_3$$

The output of the circuit can be expressed as

$$f = AB + AC + BC$$

$$AC = (x_3x_1 + x_3\bar{x}_2)(x_2x_1 + x_2\bar{x}_3)$$

$$= x_3x_1x_2x_1 + x_3x_1x_2\bar{x}_3 + x_3\bar{x}_2x_2x_1 + x_3\bar{x}_2x_2\bar{x}_3$$

$$= x_3x_1x_2 + 0 + 0 + 0$$

$$= x_1x_2x_3$$

The output of the circuit can be expressed as

$$f = AB + AC + BC$$

$$BC = (x_1\bar{x}_2 + x_1\bar{x}_3)(x_2x_1 + x_2\bar{x}_3)$$

$$= x_1\bar{x}_2x_2x_1 + x_1\bar{x}_2x_2\bar{x}_3 + x_1\bar{x}_3x_2x_1 + x_1\bar{x}_3x_2\bar{x}_3$$

$$= 0 + 0 + x_1\bar{x}_3x_2 + x_1\bar{x}_3x_2$$

$$= x_1x_2\bar{x}_3$$

**Finally, we get**

$$\begin{aligned} f &= x_1 \bar{x}_2 x_3 + x_1 x_2 x_3 + x_1 x_2 \bar{x}_3 \\ &= x_1 (\bar{x}_2 + x_2) x_3 + x_1 x_2 (x_3 + \bar{x}_3) \\ &= x_1 x_3 + x_1 x_2 \\ &= x_1 (x_3 + x_2) \end{aligned}$$

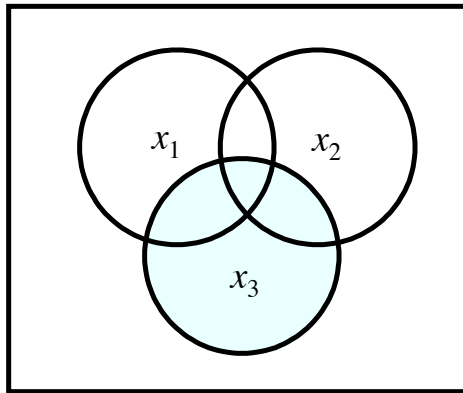


## **Example 4**

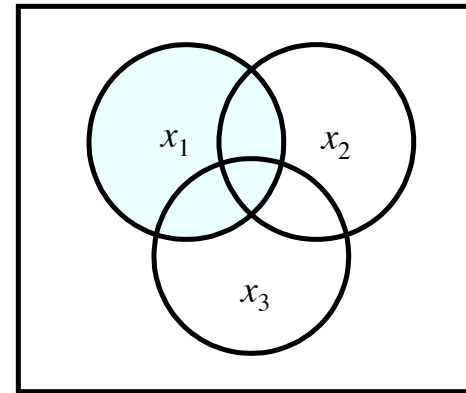
**Solve the previous problem using Venn diagrams.**

# Venn Diagrams

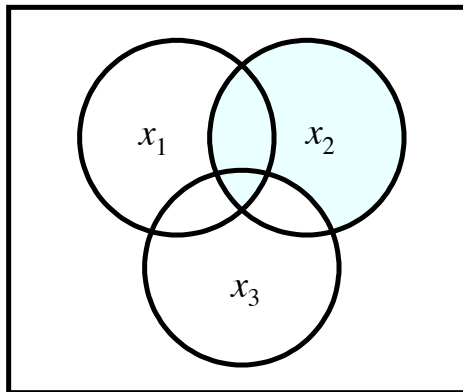
(find the areas that are shaded at least two times)



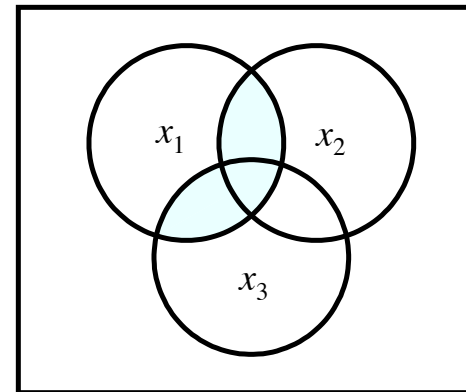
(a) Function  $A$



(b) Function  $B$



(c) Function  $C$



(d) Function  $f$

# Example 5

**Design the minimum-cost SOP and POS  
expression for the function**

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

# Let's Use a K-Map

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

$x_3x_4$ \ $x_1x_2$		$x_1$			
		00	01	11	10
$x_3$	00	$m_0$	$m_4$	$m_{12}$	$m_8$
	01	$m_1$	$m_5$	$m_{13}$	$m_9$
	11	$m_3$	$m_7$	$m_{15}$	$m_{11}$
	10	$m_2$	$m_6$	$m_{14}$	$m_{10}$

$x_2$

$x_4$

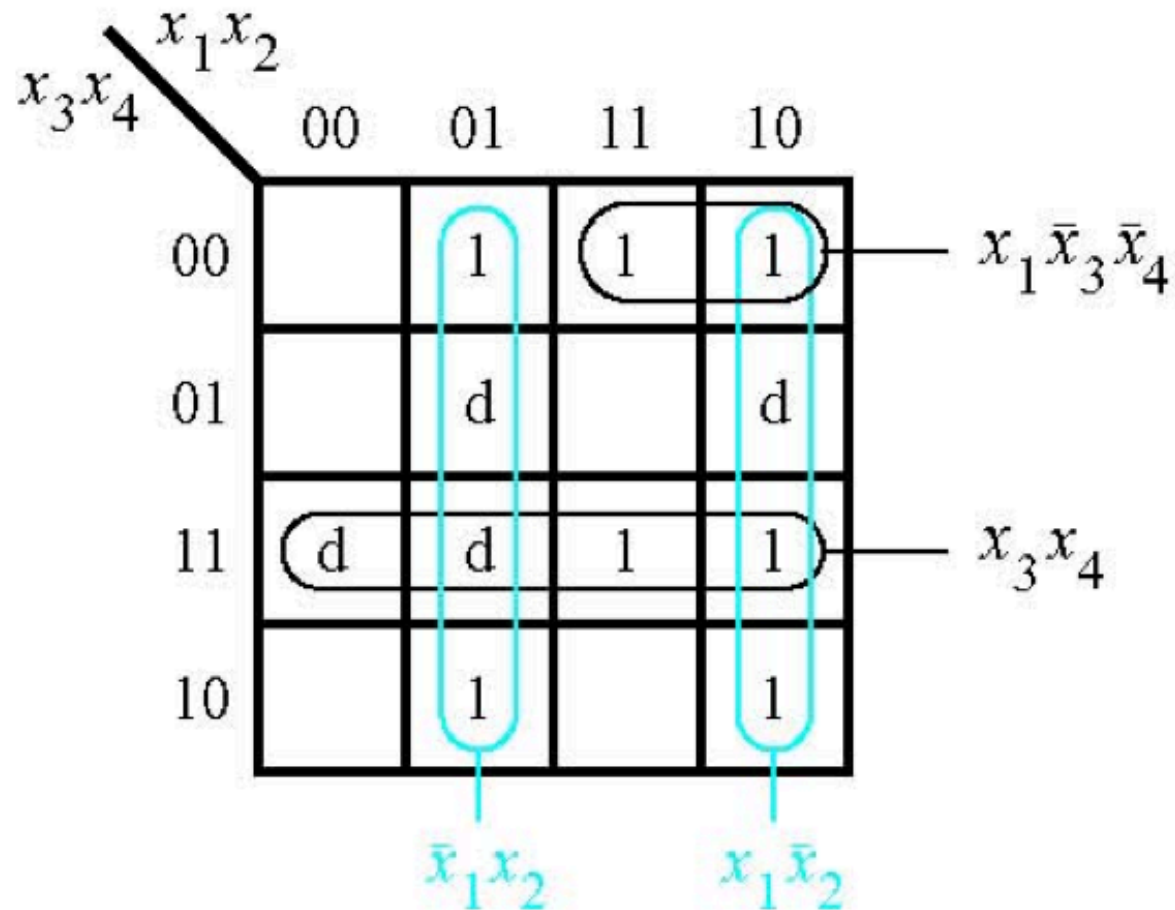
# Let's Use a K-Map

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	1	1	1
01	0	d	0	d
11	d	d	1	1
10	0	1	0	1

Brackets in the diagram indicate groupings for variables  $x_1$ ,  $x_2$ , and  $x_3$ .

# The SOP Expression



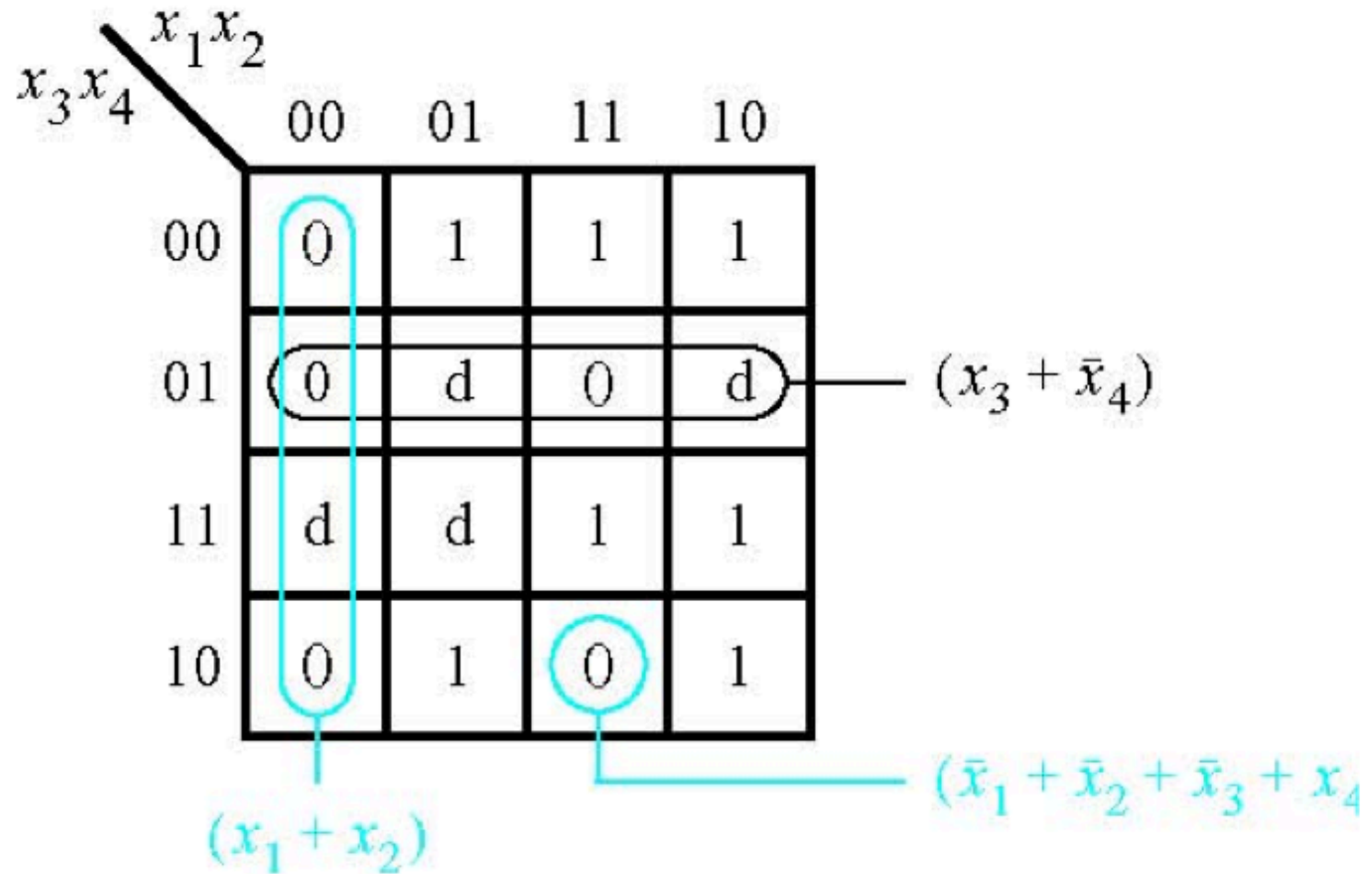
[ Figure 2.67a from the textbook ]

# What about the POS Expression?

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	1	1	1
01	0	d	0	d
11	d	d	1	1
10	0	1	0	1

# The POS Expression



[ Figure 2.67b from the textbook ]



## Example 6

**Use K-maps to find the minimum-cost SOP and POS expression for the function**

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

assuming that there are also don't-cares defined as  $D = \sum(9, 12, 14)$ .

# Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

$$D = \sum(9, 12, 14).$$

		$x_1x_2$		$x_1$	
		00	01	11	10
$x_3x_4$	00	$m_0$	$m_4$	$m_{12}$	$m_8$
	01	$m_1$	$m_5$	$m_{13}$	$m_9$
$x_3$	11	$m_3$	$m_7$	$m_{15}$	$m_{11}$
	10	$m_2$	$m_6$	$m_{14}$	$m_{10}$
		$x_2$		$x_4$	

# Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

$$D = \sum(9, 12, 14).$$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	$m_0$	$m_4$	d	$m_8$
01	$m_1$	$m_5$	$m_{13}$	d
11	$m_3$	$m_7$	$m_{15}$	$m_{11}$
10	$m_2$	$m_6$	d	$m_{10}$

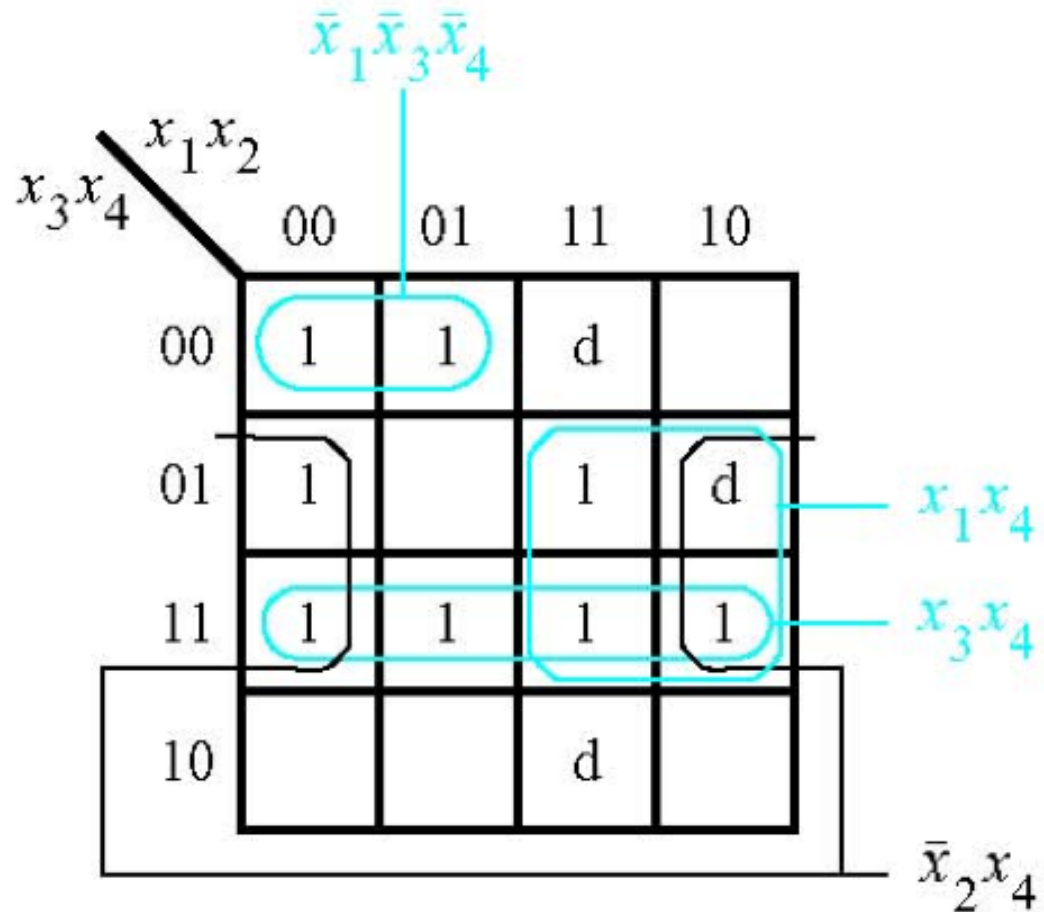
# Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	$m_0$	$m_4$	d	$m_8$
01	$m_1$	$m_5$	$m_{13}$	d
11	$m_3$	$m_7$	$m_{15}$	$m_{11}$
10	$m_2$	$m_6$	d	$m_{10}$

# The SOP Expression

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$



$$f = x_3x_4 + \bar{x}_1\bar{x}_3\bar{x}_4 + \bar{x}_2x_4 + x_1x_4$$

[ Figure 2.68a from the textbook ]

# What about the POS Expression?

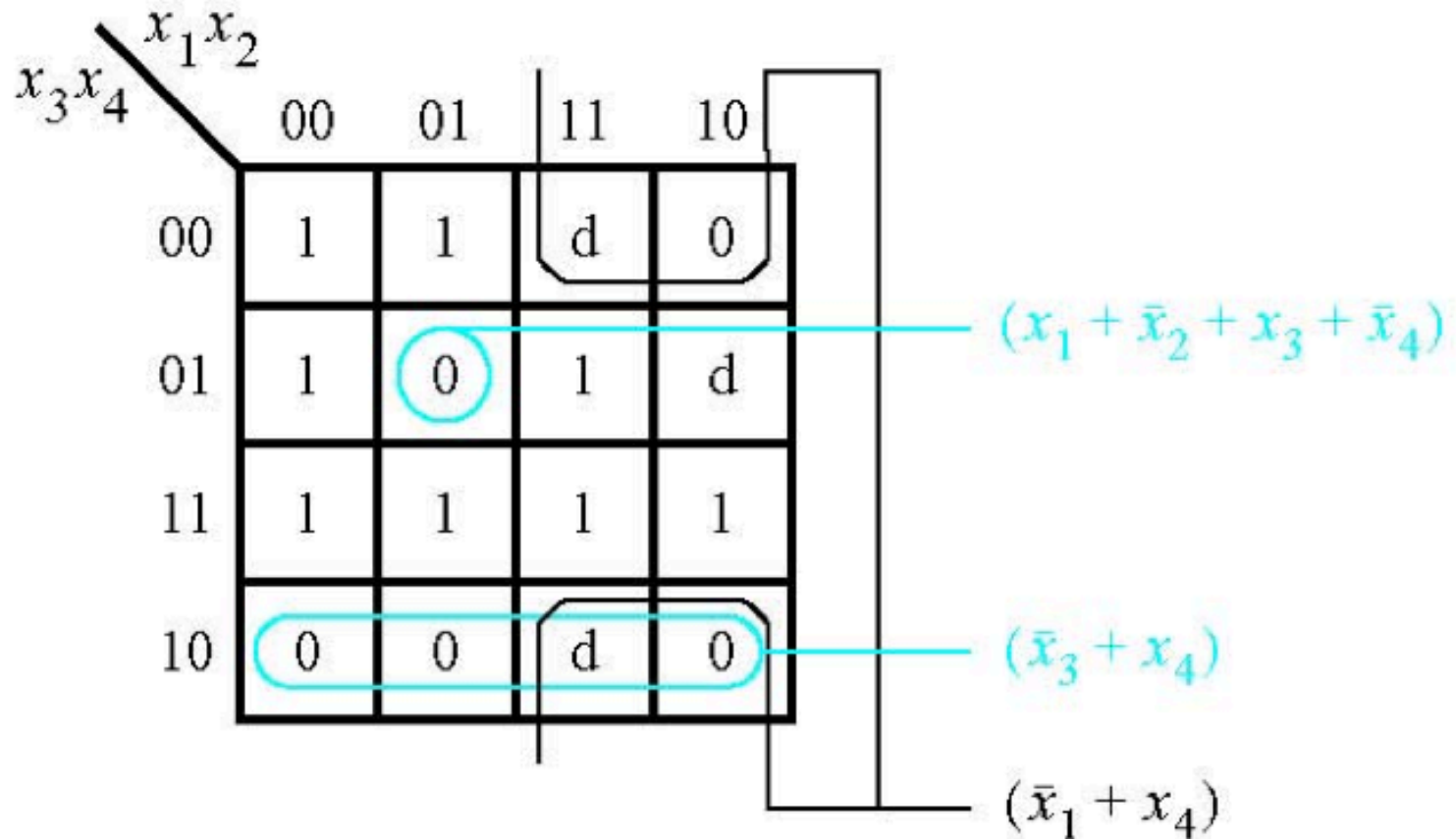
A Karnaugh map for a 4-variable function with variables  $x_1, x_2, x_3, x_4$ . The map is a 4x4 grid with the following values:

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	1	1	d	0
01	1	0	1	d
11	1	1	1	1
10	0	0	d	0

The map is annotated with cyan brackets indicating the variables  $x_1, x_2, x_3,$  and  $x_4$ :

- A horizontal bracket above the columns labeled 11 and 10 is labeled  $x_1$ .
- A horizontal bracket below the columns labeled 00 and 01 is labeled  $x_2$ .
- A vertical bracket to the left of the rows labeled 11 and 10 is labeled  $x_3$ .
- A vertical bracket to the right of the rows labeled 01 and 11 is labeled  $x_4$ .

# The POS Expression



$$f = (\bar{x}_3 + x_4)(\bar{x}_1 + x_4)(x_1 + \bar{x}_2 + x_3 + \bar{x}_4)$$

## Example 7

Derive the minimum-cost SOP expression for

$$f = s_3(\bar{s}_1 + \bar{s}_2) + s_1s_2$$



**First, expand the expression  
using property 12a**

$$f = s_3(\bar{s}_1 + \bar{s}_2) + s_1s_2$$

$$f = \bar{s}_1s_3 + \bar{s}_2s_3 + s_1s_2$$

# Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

$s_1$	$s_2$	$s_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

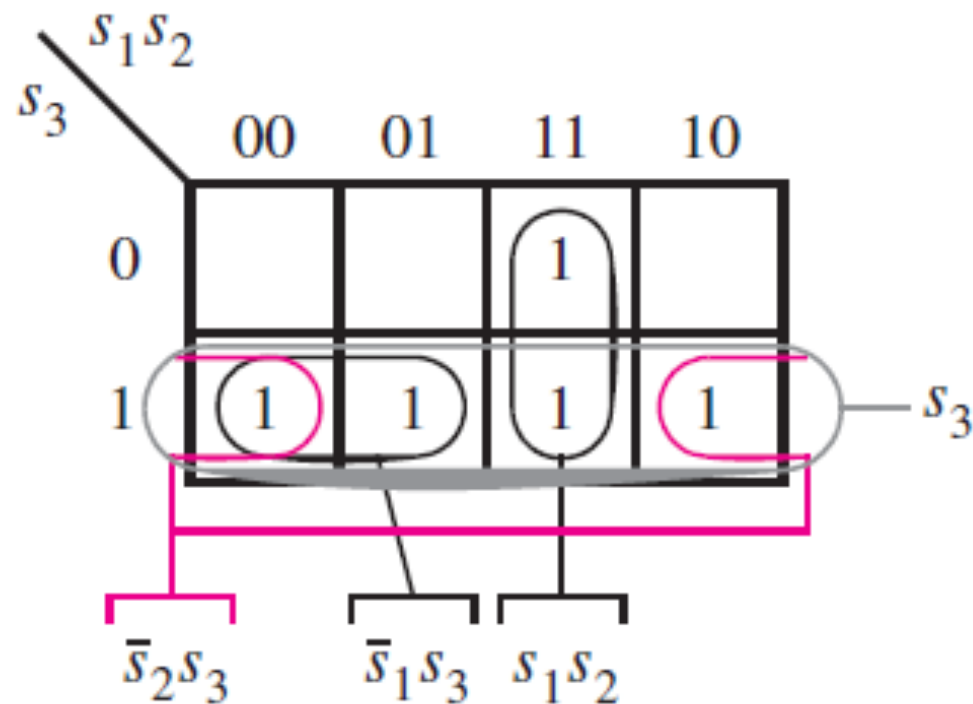
(a) Truth table

		$s_1 s_2$			
		00	01	11	10
$s_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map

# Construct the K-Map for this expression

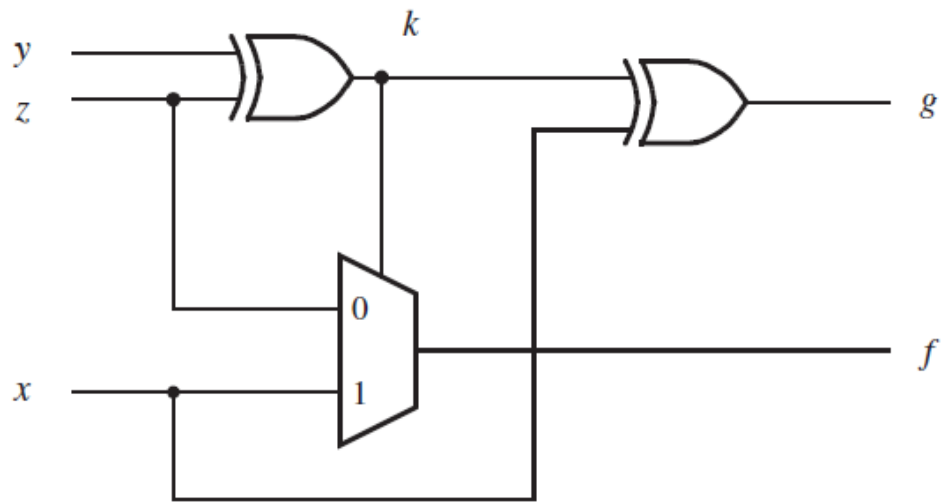
$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$



# Example 8

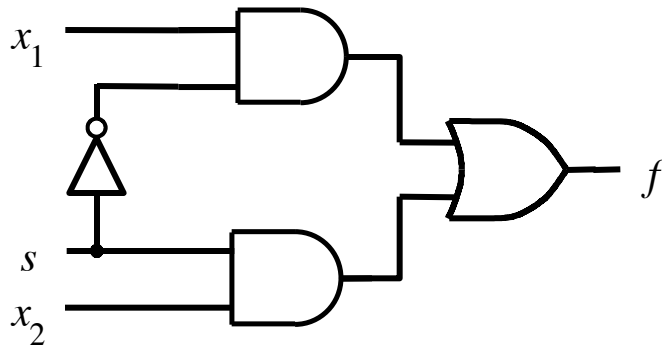
**Write the Verilog code for the following circuit ...**

# Logic Circuit

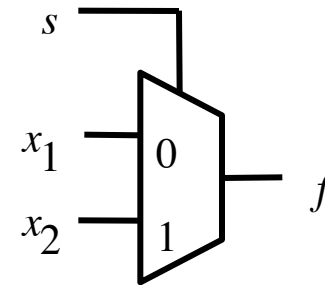


[ Figure 2.70 from the textbook ]

# Circuit for 2-1 Multiplexer



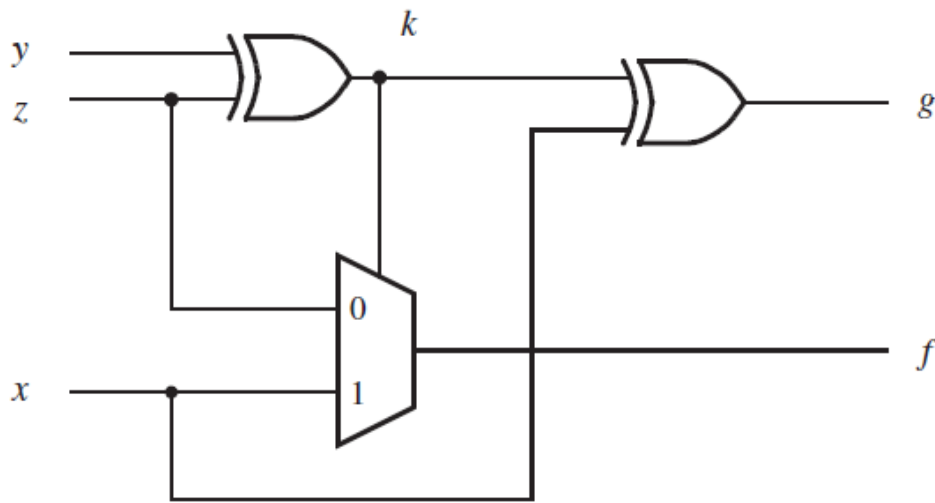
(b) Circuit



(c) Graphical symbol

$$f(s, x_1, x_2) = \bar{s} x_1 + s x_2$$

# Logic Circuit vs Verilog Code



[ Figure 2.70 from the textbook ]

```
module f_g (x, y, z, f, g);  
  input x, y, z;  
  output f, g;  
  wire k;  
  
  assign k = y ^ z;  
  assign g = k ^ x;  
  assign f = (~k & z) | (k & x);  
  
endmodule
```

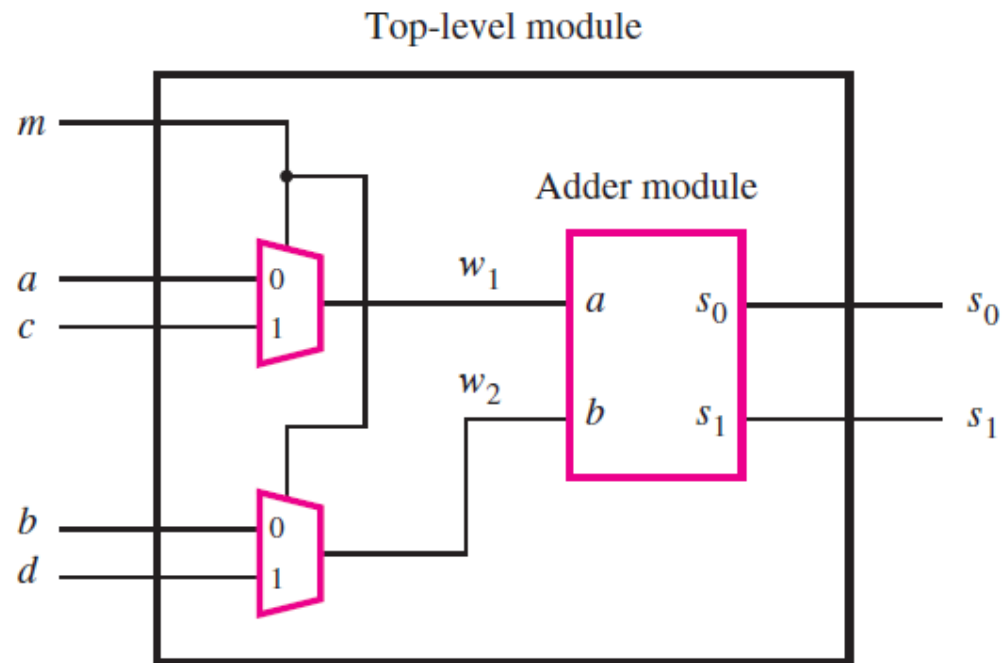
[ Figure 2.71 from the textbook ]

# Example 9

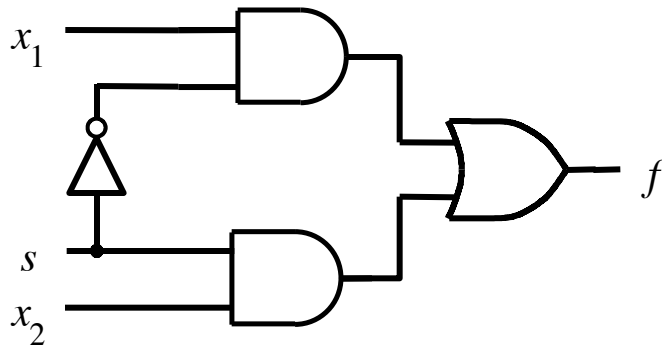
**Write the Verilog code for the following circuit ...**



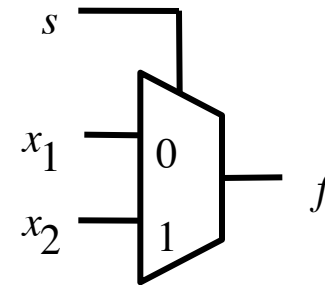
# The Logic Circuit for this Example



# Circuit for 2-1 Multiplexer



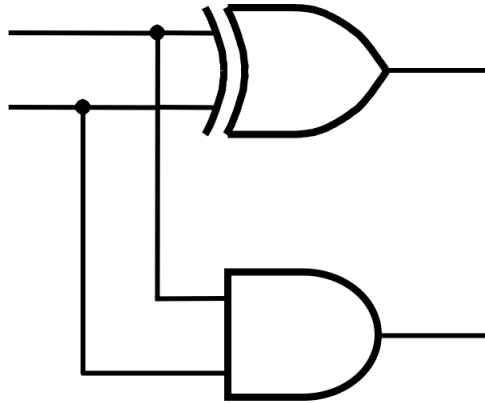
(b) Circuit



(c) Graphical symbol

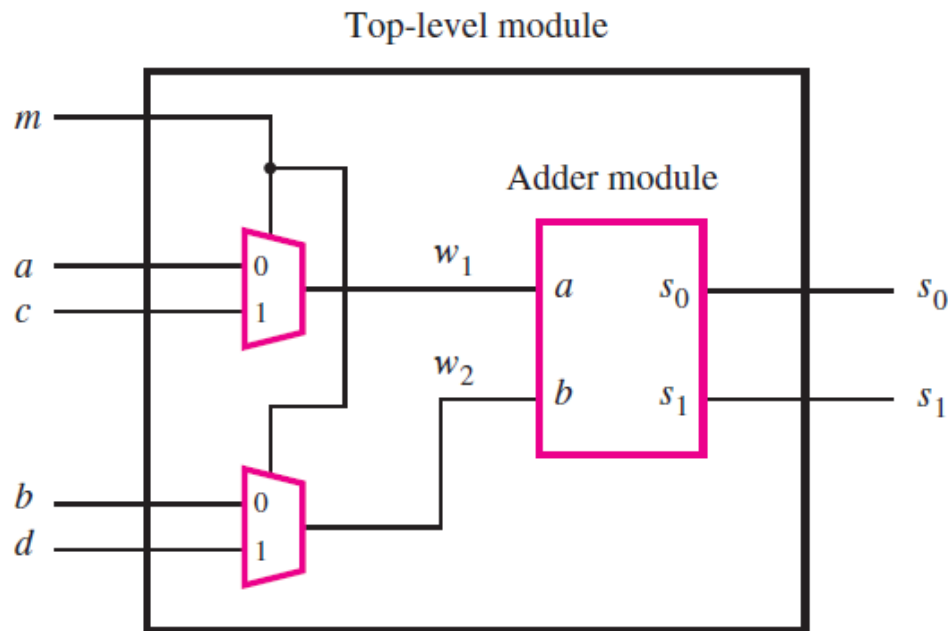
$$f(s, x_1, x_2) = \bar{s} x_1 + s x_2$$

# Addition of Binary Numbers



$a$	$b$	$s_1$	$s_0$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

# Logic Circuit vs Verilog Code



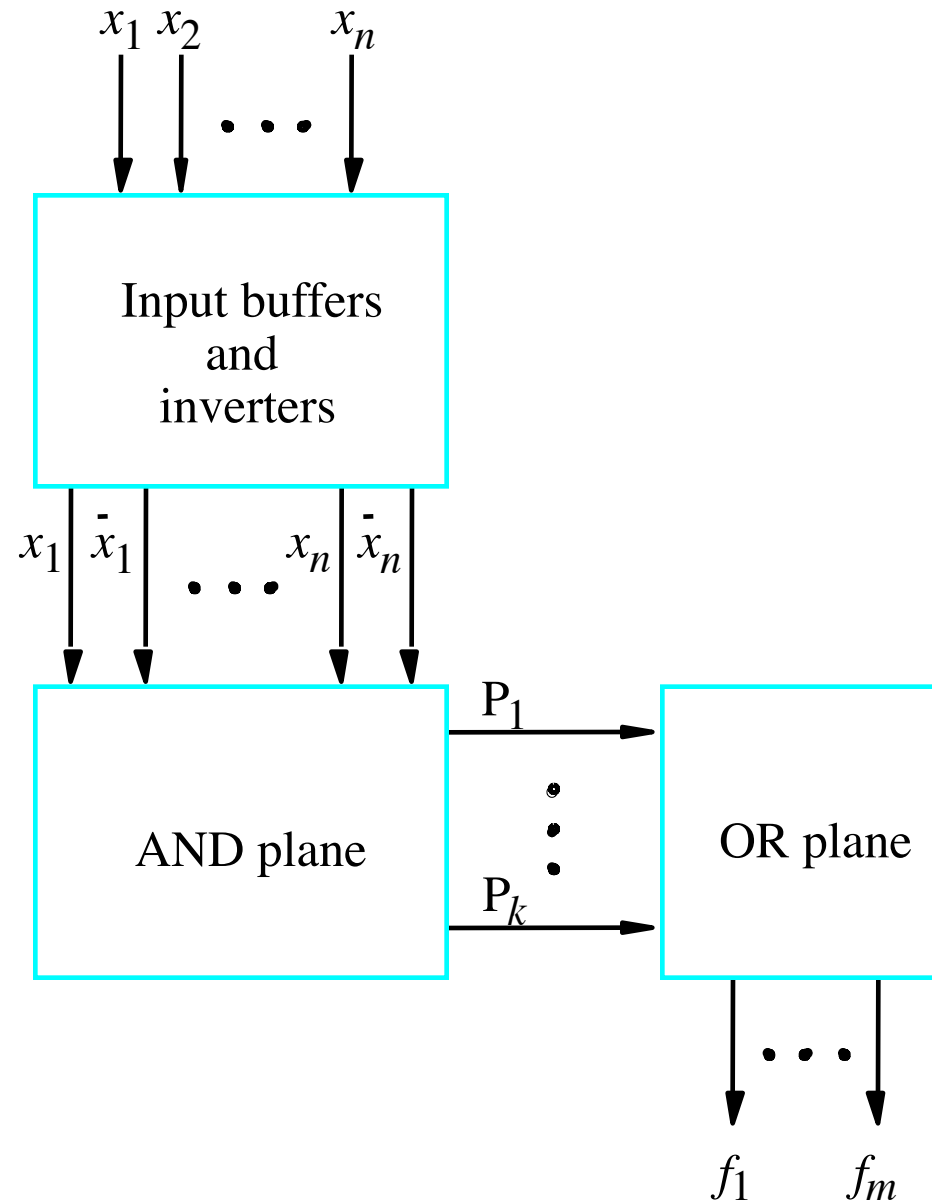
```
module shared (a, b, c, d, m, s1, s0);  
    input a, b, c, d, m;  
    output s1, s0;  
    wire w1, w2;  
    mux2to1 U1 (a, c, m, w1);  
    mux2to1 U2 (b, d, m, w2);  
    adder U3 (w1, w2, s1, s0);  
endmodule
```

```
module mux2to1 (x1, x2, s, f);  
    input x1, x2, s;  
    output f;  
    assign f = (~s & x1) | (s & x2);  
endmodule
```

```
module adder (a, b, s1, s0);  
    input a, b;  
    output s1, s0;  
    assign s1 = a & b;  
    assign s0 = a ^ b;  
endmodule
```

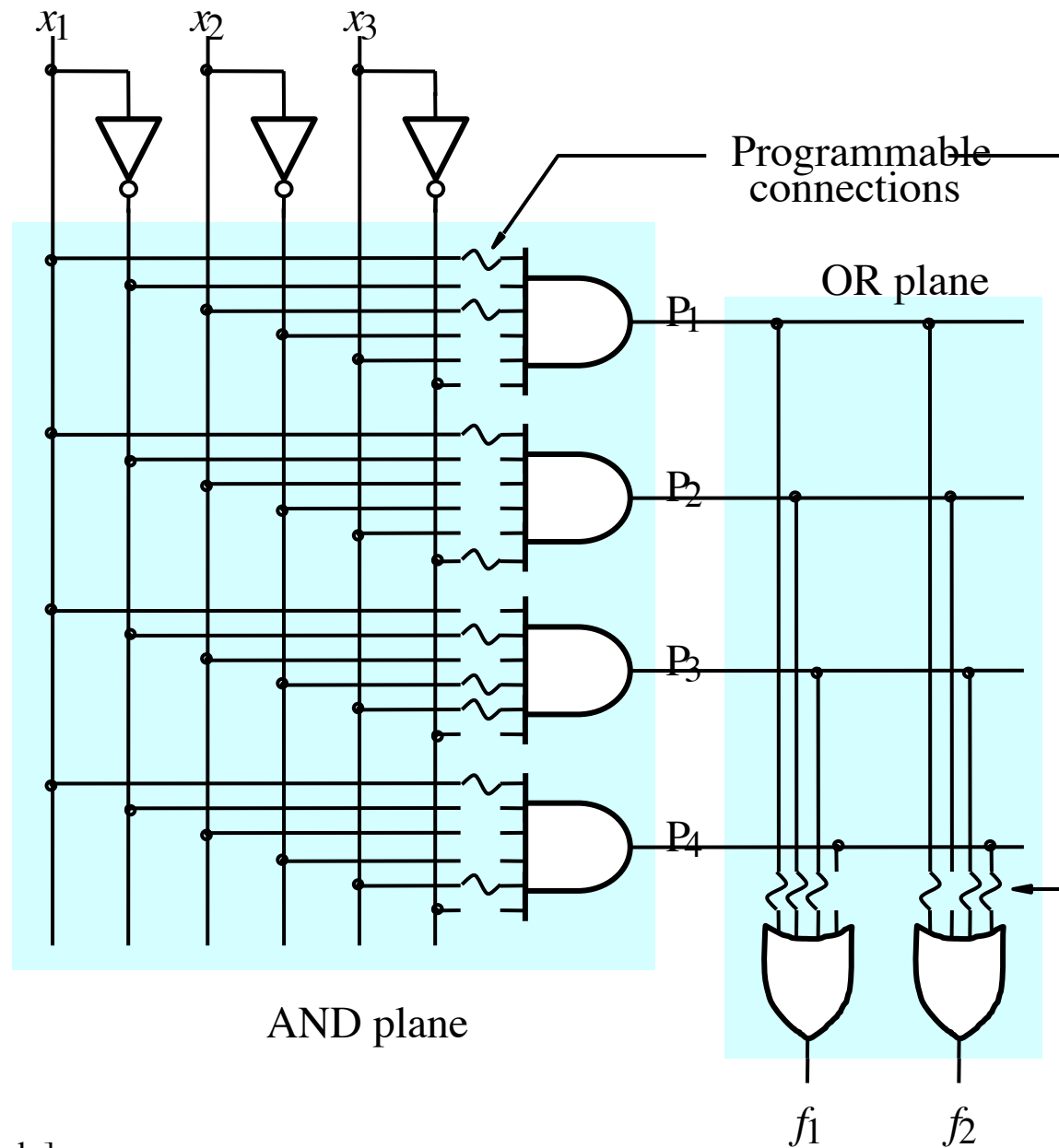
**Some material from Appendix B**

# Programmable Logic Array (PLA)



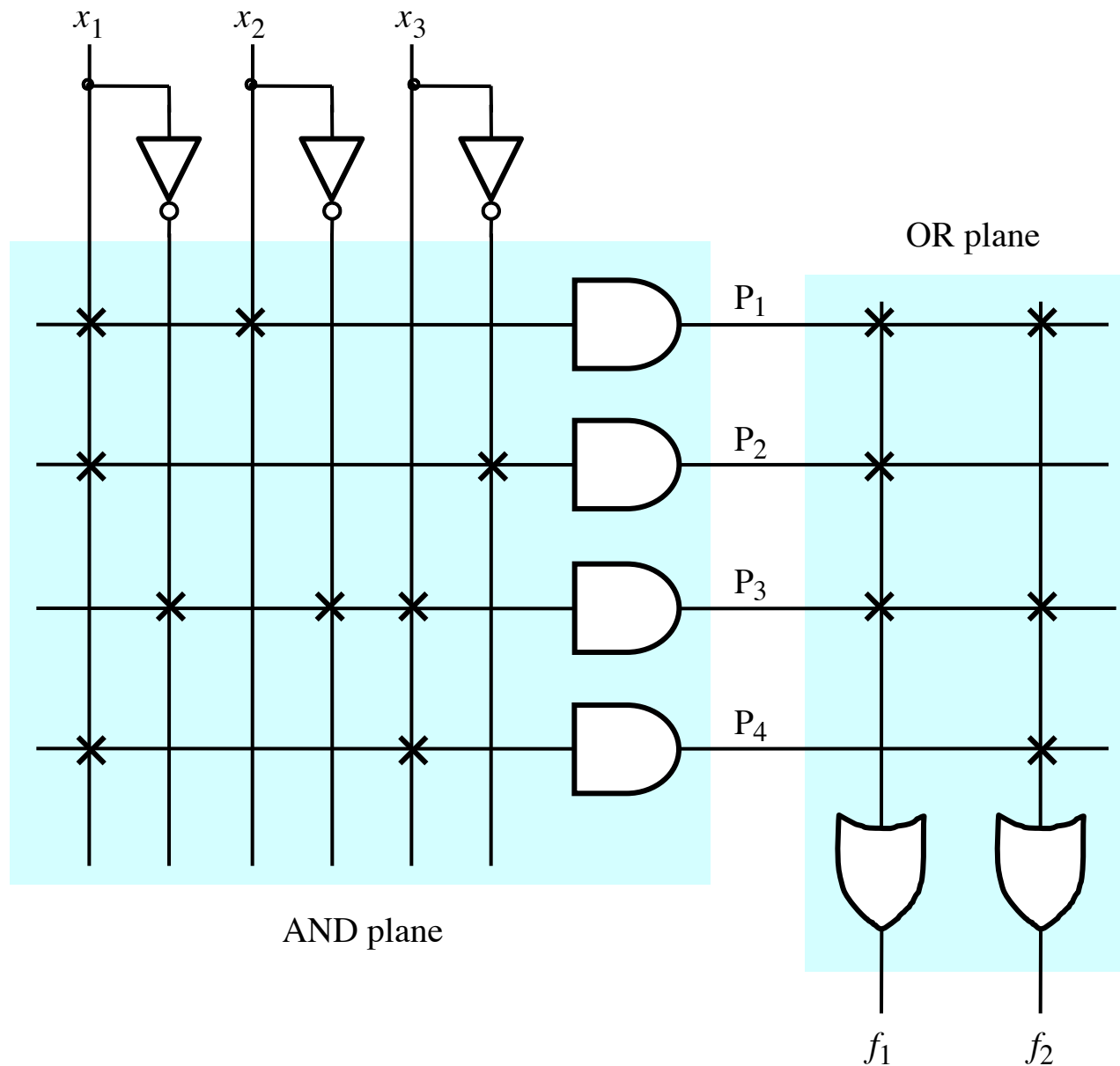
[ Figure B.25 from textbook ]

# Gate-Level Diagram of a PLA



[ Figure B.26 from textbook ]

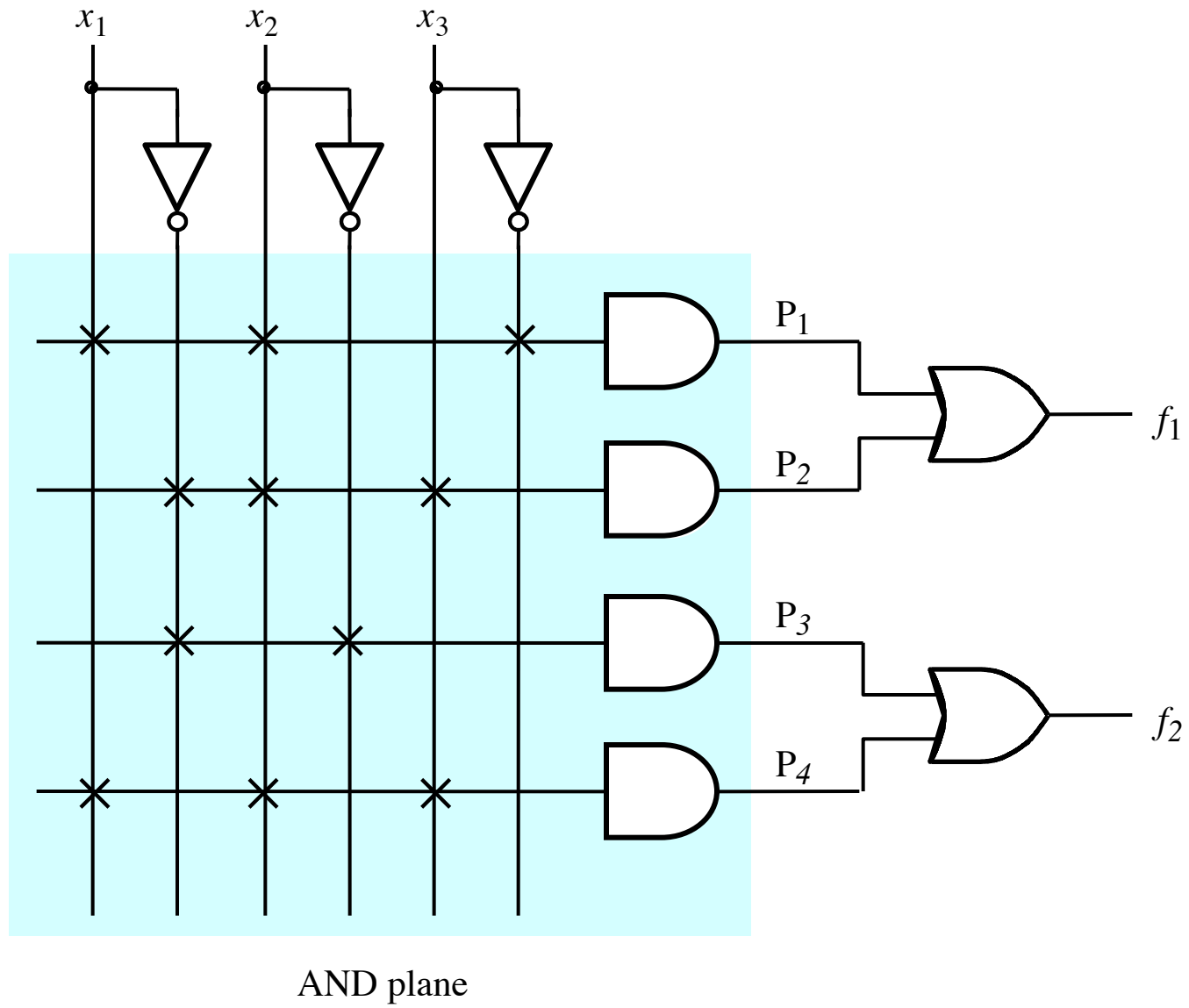
# Customary Schematic for PLA



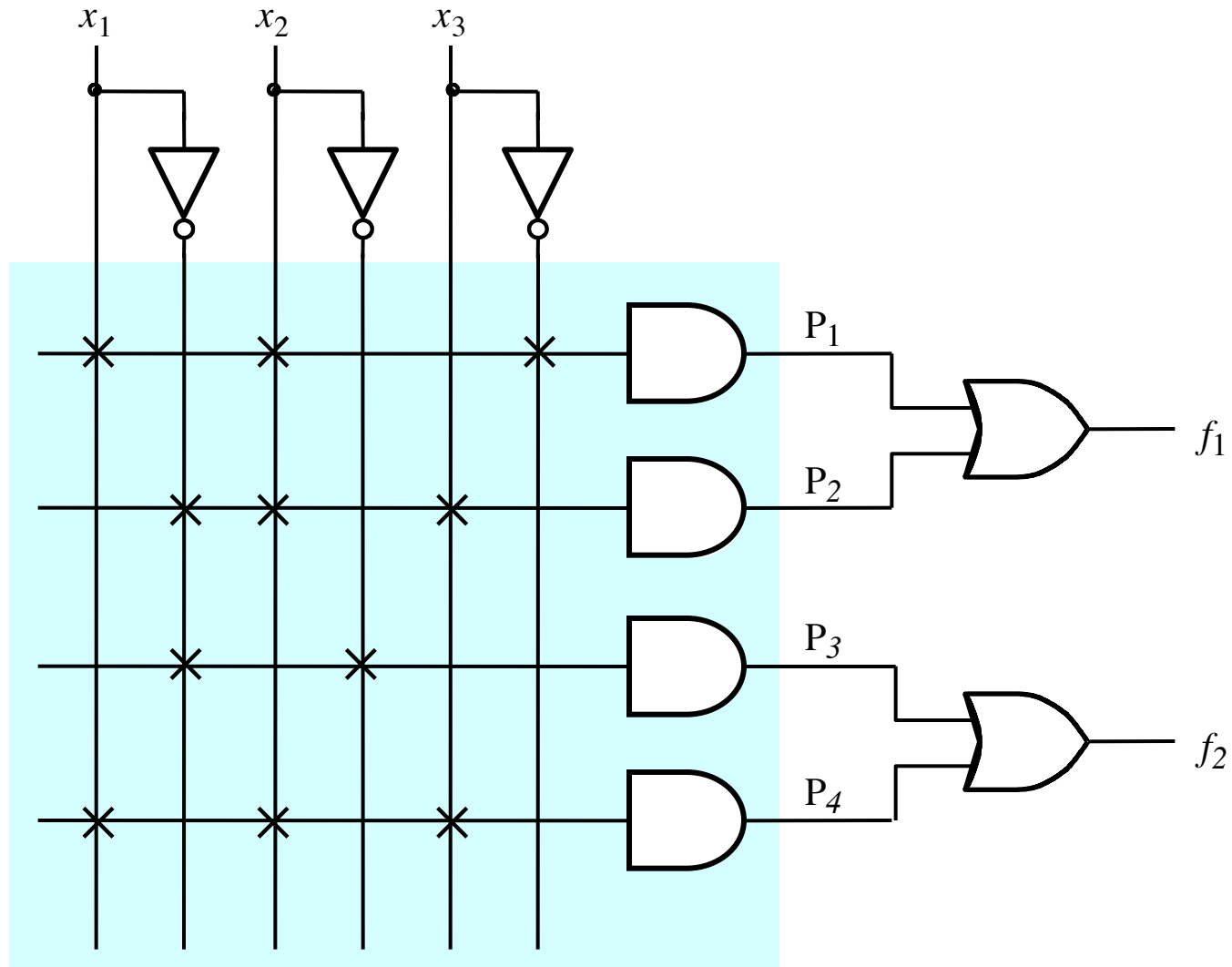
[ Figure B.27 from textbook ]



# Programmable Array Logic (PAL)



# Programmable Array Logic (PAL)



AND plane

Only the AND plane is programmable.  
The OR plane is fixed.

**Questions?**

**THE END**