

CprE 281: Digital Logic

Instructor: Alexander Stoytchev

<http://www.ece.iastate.edu/~alexs/classes/>

Synthesis

Using AND, OR, and NOT Gates

CprE 281: Digital Logic
Iowa State University, Ames, IA
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Administrative Stuff

- **HW2 is due on Wednesday Sep 7 @ 4pm**
- **Please write clearly on the first page (in block capital letters) the following three things:**
 - **Your First and Last Name**
 - **Your Student ID Number**
 - **Your Lab Section Letter**
 - **Staple all of your pages**
- **If any of these are missing, then you will lose 10% of your grade for that homework.**

Administrative Stuff

- **Next week we will start with Lab2**
- **It will be graded!**
- **Print the answer sheet for that lab and do the prelab at home. Otherwise you'll lose 20% of your grade for that lab.**

Labs Next Week

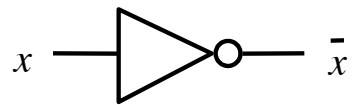
- **If your lab is on Mondays, i.e.,**
- **Section N: Mondays, 9:00 - 11:50 am (Coover Hall, room 1318)**
- **Section P: Mondays, 12:10 - 3:00 pm (Coover Hall, room 1318)**
- **Section R: Mondays, 5:10 - 8:00 pm (Coover Hall, room 1318)**
- **You will have 2 labs in one on September 12.**
- **That is, Lab #2 and Lab #3.**

Labs Next Week

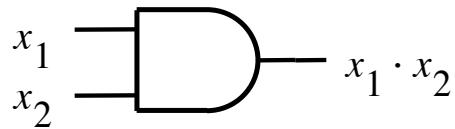
- **If your recitation is on Mondays, please go to one of the other 9 recitations next week:**
- **Section U: Tuesday 11:00 AM - 1:50 PM (Coover Hall, room 2050)**
- **Section M: Tuesday 2:10 PM - 5:00 PM (Coover Hall, room 2050)**
- **Section J: Wednesday 8:00 AM - 10:50 AM (Coover Hall, room 2050)**
- **Section T: Wednesday 6:10 PM - 9:00 PM (Coover Hall, room 1318)**
- **Section Q: Thursday 11:00 AM - 1:50 PM (Coover Hall, room 2050)**
- **Section V: Thursday 11:00 AM - 1:50 PM (Coover Hall, room 1318)**
- **Section L: Thursday 2:10 PM - 5:00 PM (Coover Hall, room 2050)**
- **Section K: Thursday 5:10 PM - 8:00 PM (Coover Hall, room 2050)**
- **Section G: Friday 11:00 AM - 1:50 PM (Coover Hall, room 2050)**
- **This is only for next week. And only for the recitation (first hour). You won't be able to stay for the lab as the sections are full.**

Quick Review

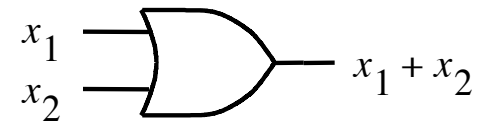
The Three Basic Logic Gates



NOT gate

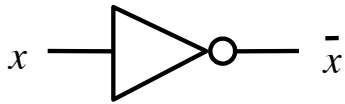


AND gate



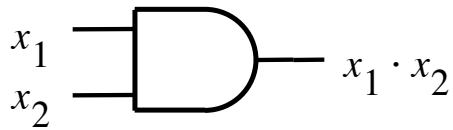
OR gate

Truth Table for NOT



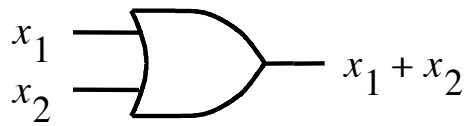
x	\bar{x}
0	1
1	0

Truth Table for AND



x_1	x_2	$x_1 \cdot x_2$
0	0	0
0	1	0
1	0	0
1	1	1

Truth Table for OR



x_1	x_2	$x_1 + x_2$
0	0	0
0	1	1
1	0	1
1	1	1

Truth Tables for AND and OR

x_1	x_2	$x_1 \cdot x_2$	$x_1 + x_2$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

AND

OR

Operator Precedence

- **In regular arithmetic and algebra multiplication takes precedence over addition**
- **This is also true in Boolean algebra**

Operator Precedence

(three different ways to write the same)

$$x_1 \cdot x_2 + \bar{x}_1 \cdot \bar{x}_2$$

$$(x_1 \cdot x_2) + ((\bar{x}_1) \cdot (\bar{x}_2))$$

$$x_1x_2 + \bar{x}_1\bar{x}_2$$

DeMorgan's Theorem

$$15a. \quad \overline{\mathbf{x} \cdot \mathbf{y}} = \overline{\mathbf{x}} + \overline{\mathbf{y}}$$

$$15b. \quad \overline{\mathbf{x} + \mathbf{y}} = \overline{\mathbf{x}} \cdot \overline{\mathbf{y}}$$

Function Synthesis

Synthesize the Following Function

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

1) Split the function into 4 functions

x_1	x_2	$f_{00}(x_1, x_2)$	$f_{01}(x_1, x_2)$	$f_{10}(x_1, x_2)$	$f_{11}(x_1, x_2)$
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	0	0
1	1	0	0	0	1

1) Split the function into 4 functions

x_1	x_2	$f_{00}(x_1, x_2)$	$f_{01}(x_1, x_2)$	$f_{10}(x_1, x_2)$	$f_{11}(x_1, x_2)$
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	0	0
1	1	0	0	0	1

2) Write Expressions for all four

x_1	x_2	$f_{00}(x_1, x_2)$	$f_{01}(x_1, x_2)$	$f_{10}(x_1, x_2)$	$f_{11}(x_1, x_2)$
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	0	0
1	1	0	0	0	1

$$x_1 x_2$$

$$\bar{x}_1 x_2$$

$$0$$

$$\bar{x}_1 \bar{x}_2$$

3) Then just add them together

x_1	x_2	$f_{00}(x_1, x_2)$	$f_{01}(x_1, x_2)$	$f_{10}(x_1, x_2)$	$f_{11}(x_1, x_2)$
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	0	0
1	1	0	0	0	1

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1x_2 + 0 + \bar{x}_1\bar{x}_2$$

A function to be synthesized

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

[Figure 2.19 from the textbook]

**Let's look at it row by row.
How can we express the last row?**

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1


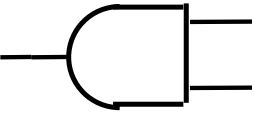
**Let's look at it row by row.
How can we express the last row?**

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

x_1x_2

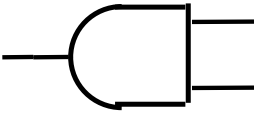
**Let's look at it row by row.
How can we express the last row?**

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

What about this row?

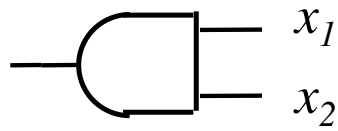
x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

 x_1
 x_2

What about this row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

$\bar{x}_1 x_2$



What about this row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

The output value 1 in the second row of the truth table is highlighted in green. To the right of the truth table, two logic diagrams are shown. The top diagram is an AND gate with inputs x_1 and x_2 , followed by a NOT gate. The bottom diagram is a simple AND gate with inputs x_1 and x_2 .

What about the first row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

Logic circuit diagrams for the function $f(x_1, x_2)$:

- The first row (0, 0, 1) is highlighted in green in the original image. It is implemented by an AND gate with inputs x_1 and x_2 , followed by a NOT gate.
- The fourth row (1, 1, 1) is implemented by an AND gate with inputs x_1 and x_2 .

What about the first row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

$\bar{x}_1\bar{x}_2$

x_1
 x_2

x_1
 x_2

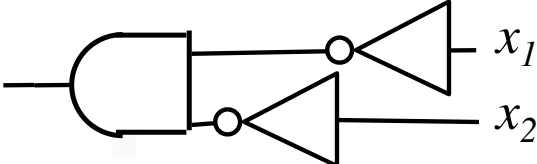
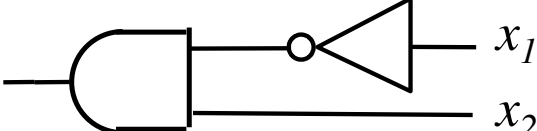
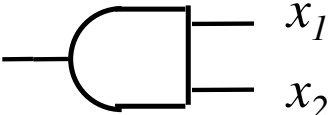
What about the first row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

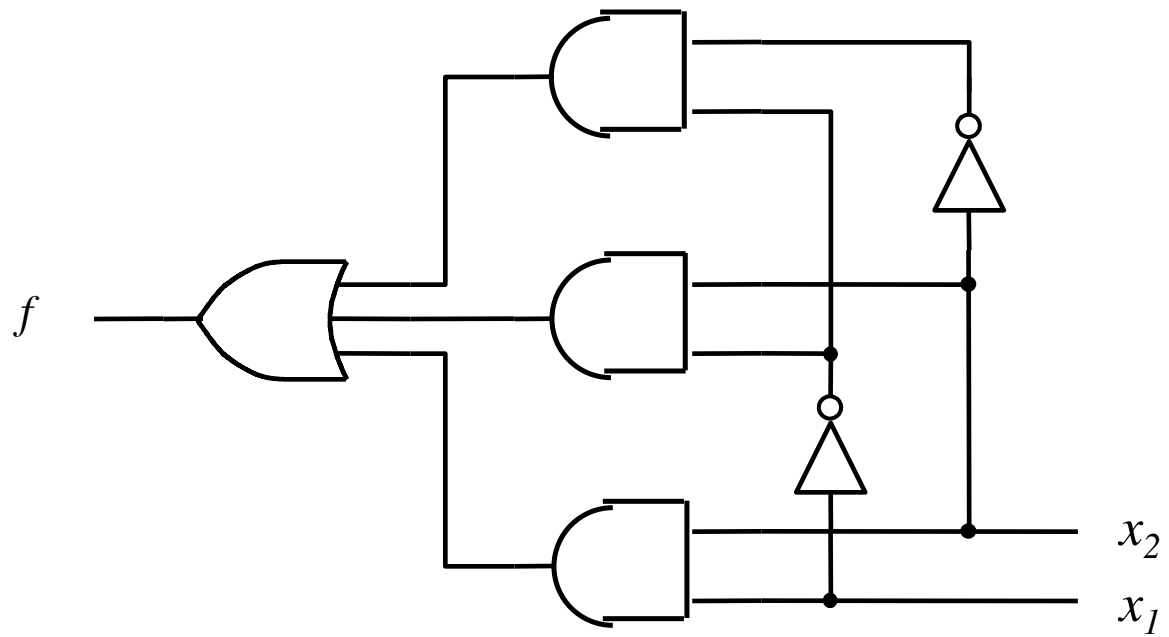
The first row of the truth table (0, 0) is highlighted with a green background. To the right of the truth table, three logic circuit diagrams are shown, each corresponding to a row of the truth table:

- The first diagram (corresponding to the first row) shows an AND gate with inputs x_1 and x_2 . The output of the AND gate is connected to an inverter, and the output of the inverter is connected to an OR gate. The output of the OR gate is labeled x_1 . This circuit implements the function $f(x_1, x_2) = \neg(x_1 \wedge x_2)$.
- The second diagram (corresponding to the second row) shows an AND gate with inputs x_1 and x_2 . The output of the AND gate is connected to an inverter, and the output of the inverter is connected to an OR gate. The output of the OR gate is labeled x_1 . This circuit implements the function $f(x_1, x_2) = \neg(x_1 \wedge x_2) \vee x_2$.
- The third diagram (corresponding to the fourth row) shows an AND gate with inputs x_1 and x_2 . The output of the AND gate is labeled x_1 . This circuit implements the function $f(x_1, x_2) = x_1 \wedge x_2$.

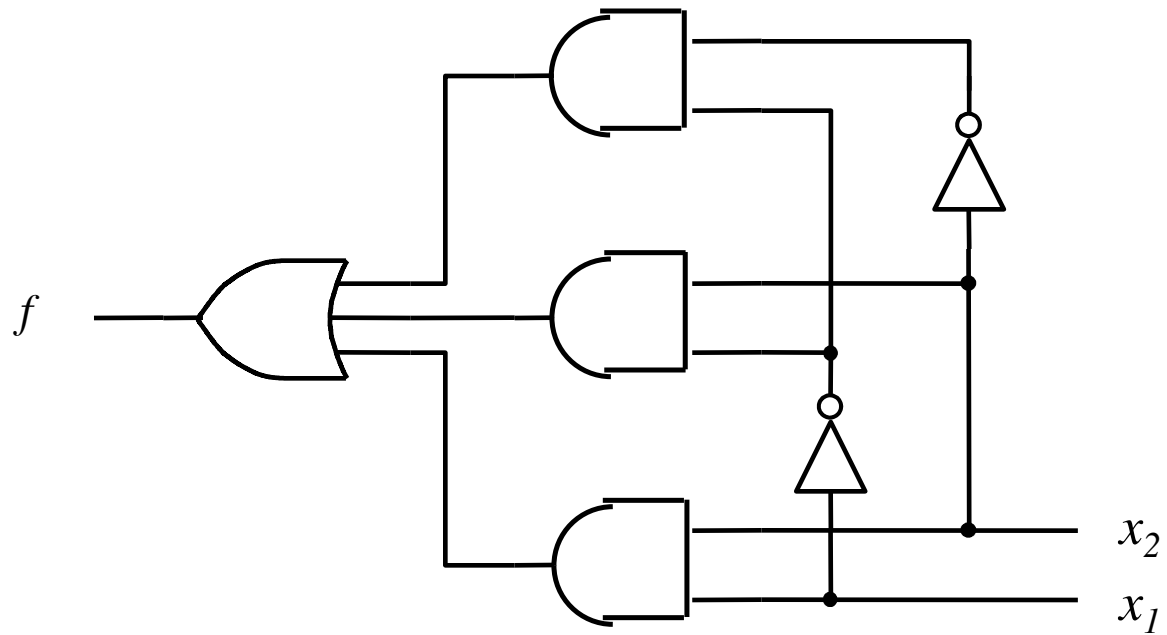
Finally, what about the zero?

x_1	x_2	$f(x_1, x_2)$	
0	0	1	
0	1	1	
1	0	0	
1	1	1	

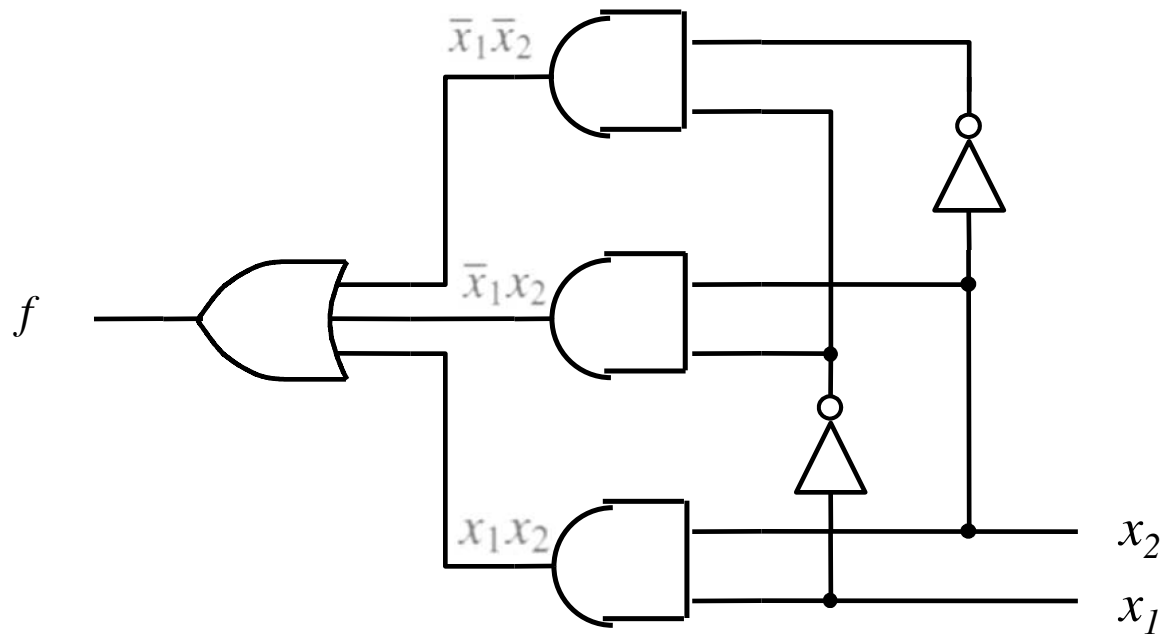
Putting it all together



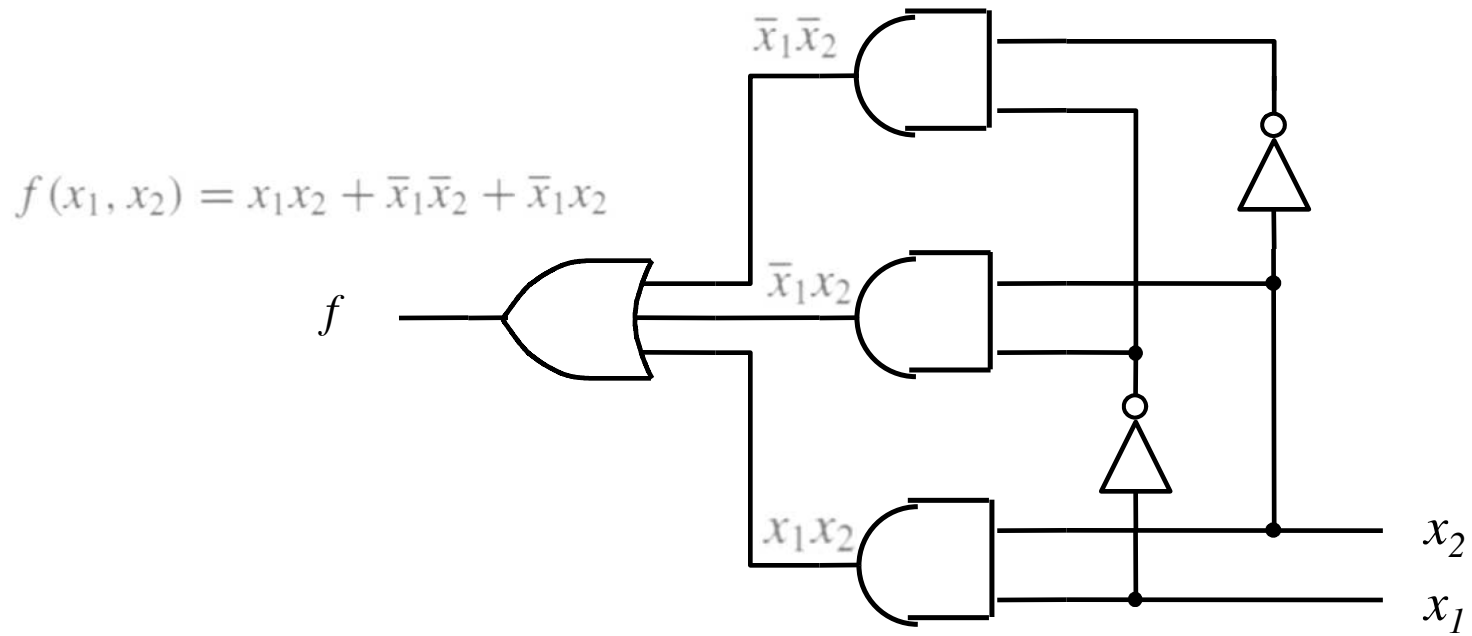
Let's verify that this circuit implements correctly the target truth table



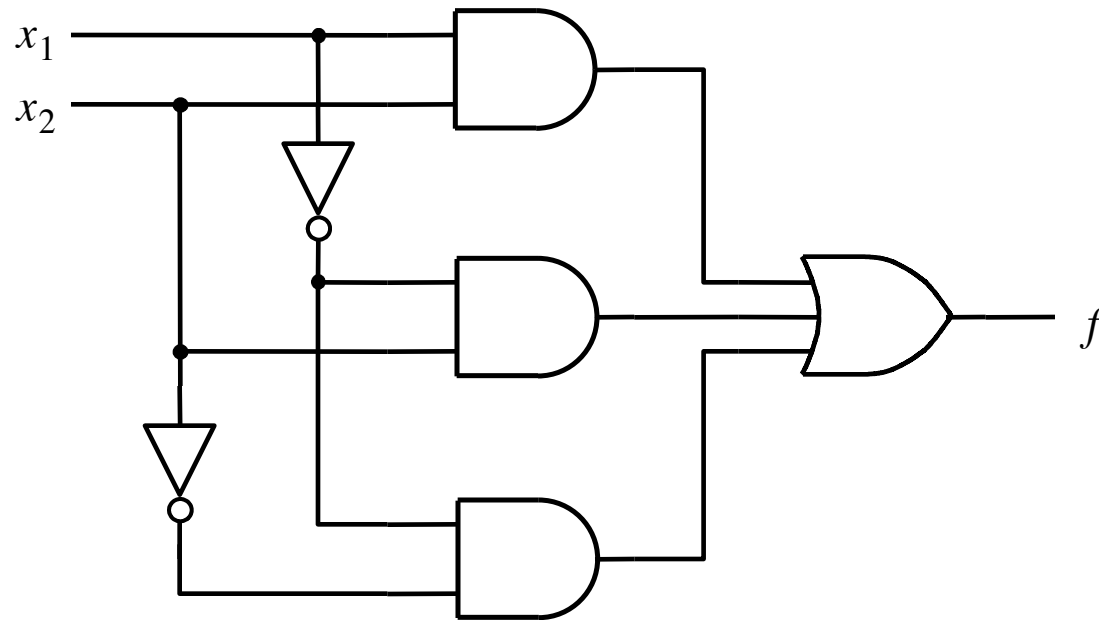
Putting it all together



Putting it all together



Canonical Sum-Of-Products (SOP)

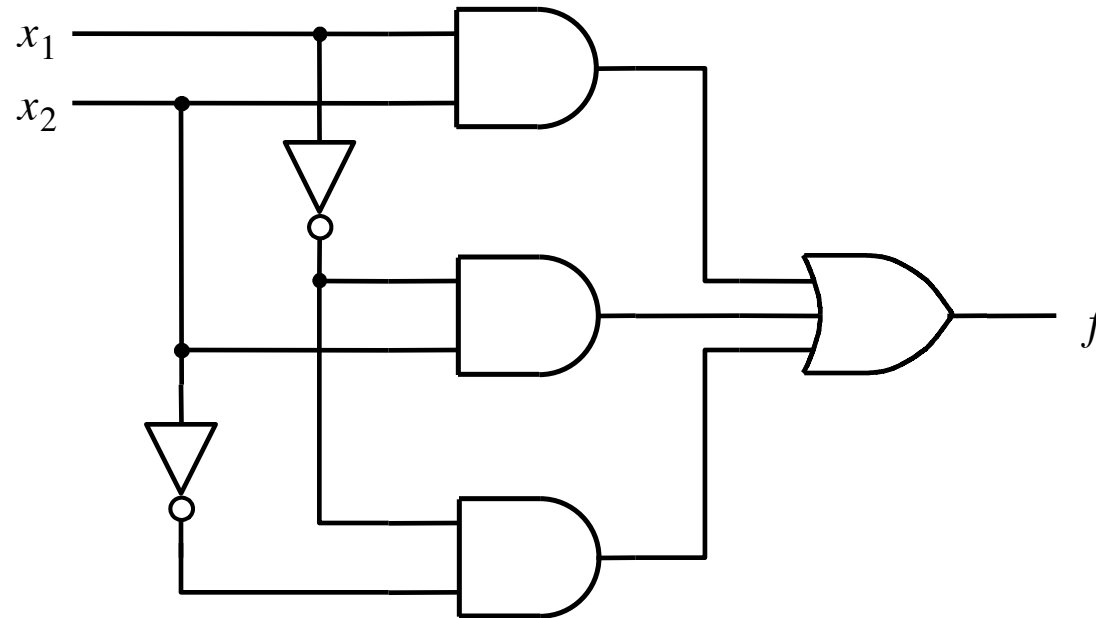


$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

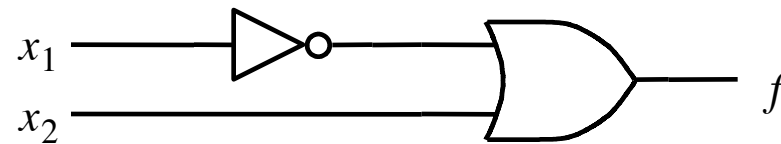
Summary of This Procedure

- **Get the truth table of the function**
- **Form a product term (AND gate) for each row of the table for which the function is 1**
- **Each product term contains all input variables**
- **In each row, if $x_i=1$ enter it as x_i , otherwise use $\overline{x_i}$**
- **Sum all of these products (OR gate) to get the function**

Two implementations for the same function



(a) Canonical sum-of-products



(b) Minimal-cost realization

[Figure 2.20 from the textbook]

Simplification Steps

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

Simplification Steps

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

replicate this term

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2 + \bar{x}_1x_2$$

Simplification Steps

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

group
these terms

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2 + \bar{x}_1x_2$$

$$f(x_1, x_2) = (x_1 + \bar{x}_1)x_2 + \bar{x}_1(\bar{x}_2 + x_2)$$

Simplification Steps

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2 + \bar{x}_1x_2$$

These two terms are trivially equal to 1

$$f(x_1, x_2) = (x_1 + \bar{x}_1)x_2 + \bar{x}_1(\bar{x}_2 + x_2)$$

$$f(x_1, x_2) = 1 \cdot x_2 + \bar{x}_1 \cdot 1$$

Simplification Steps

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2 + \bar{x}_1x_2$$

$$f(x_1, x_2) = (x_1 + \bar{x}_1)x_2 + \bar{x}_1(\bar{x}_2 + x_2)$$

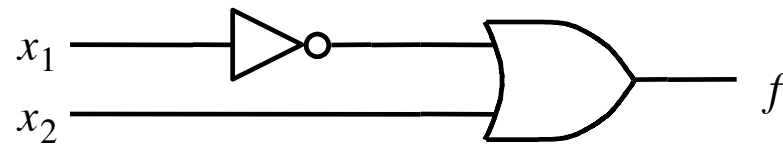
$$f(x_1, x_2) = \boxed{1} \cdot x_2 + \bar{x}_1 \cdot \boxed{1}$$

Drop the 1's

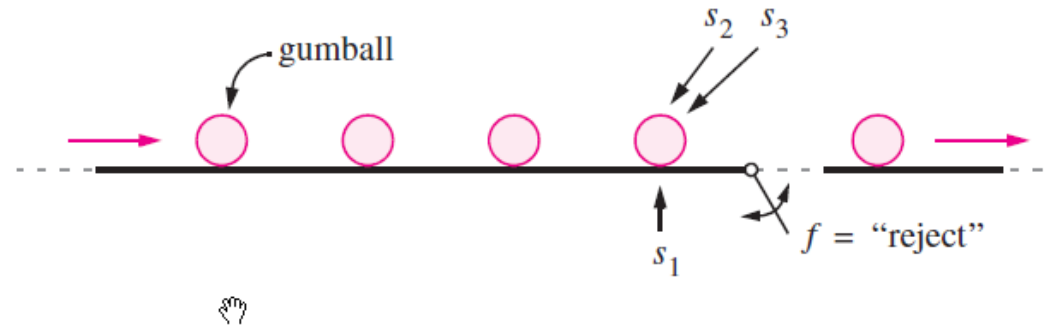
$$f(x_1, x_2) = x_2 + \bar{x}_1$$

Minimal-cost realization

$$f(x_1, x_2) = x_2 + \bar{x}_1$$



Let's look at another problem



(a) Conveyor and sensors

s_1	s_2	s_3	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(b) Truth table

Let's look at another problem

s_1	s_2	s_3	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Let's look at another problem

s_1	s_2	s_3	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Let's look at another problem

s_1	s_2	s_3	f	
0	0	0	0	
0	0	1	1	$\bar{s}_1 \bar{s}_2 s_3$
0	1	0	0	
0	1	1	1	$\bar{s}_1 s_2 s_3$
1	0	0	0	
1	0	1	1	$s_1 \bar{s}_2 s_3$
1	1	0	1	$s_1 s_2 \bar{s}_3$
1	1	1	1	$s_1 s_2 s_3$

Let's look at another problem

s_1	s_2	s_3	f	
0	0	0	0	
0	0	1	1	$\bar{s}_1\bar{s}_2s_3$
0	1	0	0	
0	1	1	1	$\bar{s}_1s_2s_3$
1	0	0	0	
1	0	1	1	$s_1\bar{s}_2s_3$
1	1	0	1	$s_1s_2\bar{s}_3$
1	1	1	1	$s_1s_2s_3$

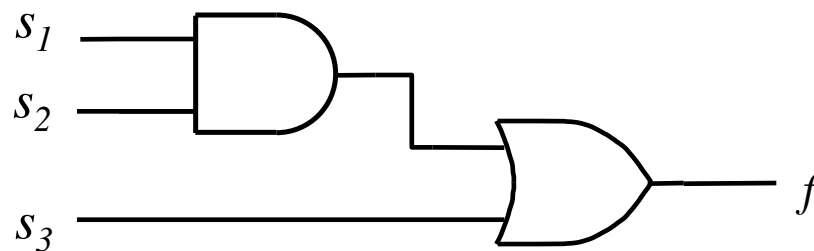
$$f = \bar{s}_1\bar{s}_2s_3 + \bar{s}_1s_2s_3 + s_1\bar{s}_2s_3 + s_1s_2\bar{s}_3 + s_1s_2s_3$$

Let's look at another problem (minimization)

$$\begin{aligned}f &= \bar{s}_1\bar{s}_2s_3 + \bar{s}_1s_2s_3 + s_1\bar{s}_2s_3 + s_1s_2s_3 + s_1s_2\bar{s}_3 + s_1s_2s_3 \\ &= \bar{s}_1s_3(\bar{s}_2 + s_2) + s_1s_3(\bar{s}_2 + s_2) + s_1s_2(\bar{s}_3 + s_3) \\ &= \bar{s}_1s_3 + s_1s_3 + s_1s_2 \\ &= s_3 + s_1s_2\end{aligned}$$

Let's look at another problem (minimization)

$$\begin{aligned} f &= \bar{s}_1\bar{s}_2s_3 + \bar{s}_1s_2s_3 + s_1\bar{s}_2s_3 + s_1s_2s_3 + s_1s_2\bar{s}_3 + s_1s_2s_3 \\ &= \bar{s}_1s_3(\bar{s}_2 + s_2) + s_1s_3(\bar{s}_2 + s_2) + s_1s_2(\bar{s}_3 + s_3) \\ &= \bar{s}_1s_3 + s_1s_3 + s_1s_2 \\ &= s_3 + s_1s_2 \end{aligned}$$



Minterms and Maxterms

Row number	x_1	x_2	Minterm	Maxterm
0	0	0	$m_0 = \bar{x}_1\bar{x}_2$	$M_0 = x_1 + x_2$
1	0	1	$m_1 = \bar{x}_1x_2$	$M_1 = x_1 + \bar{x}_2$
2	1	0	$m_2 = x_1\bar{x}_2$	$M_2 = \bar{x}_1 + x_2$
3	1	1	$m_3 = x_1x_2$	$M_3 = \bar{x}_1 + \bar{x}_2$

Sum-of-Products Form

Row number	x_1	x_2	Minterm	$f(x_1, x_2)$
0	0	0	$m_0 = \bar{x}_1\bar{x}_2$	1
1	0	1	$m_1 = \bar{x}_1x_2$	1
2	1	0	$m_2 = x_1\bar{x}_2$	0
3	1	1	$m_3 = x_1x_2$	1

Sum-of-Products Form

Row number	x_1	x_2	Minterm	$f(x_1, x_2)$
0	0	0	$m_0 = \bar{x}_1\bar{x}_2$	1
1	0	1	$m_1 = \bar{x}_1x_2$	1
2	1	0	$m_2 = x_1\bar{x}_2$	0
3	1	1	$m_3 = x_1x_2$	1

$$\begin{aligned}f &= m_0 \cdot 1 + m_1 \cdot 1 + m_2 \cdot 0 + m_3 \cdot 1 \\&= m_0 + m_1 + m_3 \\&= \bar{x}_1\bar{x}_2 + \bar{x}_1x_2 + x_1x_2\end{aligned}$$

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1
1	0	1	$M_1 = x_1 + \bar{x}_2$	1
2	1	0	$M_2 = \bar{x}_1 + x_2$	0
3	1	1	$M_3 = \bar{x}_1 + \bar{x}_2$	1

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1
1	0	1	$M_1 = x_1 + \bar{x}_2$	1
2	1	0	$M_2 = \bar{x}_1 + x_2$	0
3	1	1	$M_3 = \bar{x}_1 + \bar{x}_2$	1

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1
1	0	1	$M_1 = x_1 + \bar{x}_2$	1
2	1	0	$M_2 = \bar{x}_1 + x_2$	0
3	1	1	$M_3 = \bar{x}_1 + \bar{x}_2$	1

$$\begin{aligned}\bar{f}(x_1, x_2) &= m_2 \\ &= x_1 \bar{x}_2\end{aligned}$$

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1
1	0	1	$M_1 = x_1 + \bar{x}_2$	1
2	1	0	$M_2 = \bar{x}_1 + x_2$	0
3	1	1	$M_3 = \bar{x}_1 + \bar{x}_2$	1

$$\begin{aligned}\bar{f}(x_1, x_2) &= m_2 \\ &= x_1 \bar{x}_2\end{aligned}$$

$$\begin{aligned}\bar{\bar{f}} &= f = \overline{x_1 \bar{x}_2} \\ &= \bar{x}_1 + x_2\end{aligned}$$

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1
1	0	1	$M_1 = x_1 + \bar{x}_2$	1
2	1	0	$M_2 = \bar{x}_1 + x_2$	0
3	1	1	$M_3 = \bar{x}_1 + \bar{x}_2$	1

$$\begin{aligned}\bar{f}(x_1, x_2) &= m_2 \\ &= x_1 \bar{x}_2\end{aligned}$$

$$\begin{aligned}\bar{\bar{f}} &= f = \overline{x_1 \bar{x}_2} \\ &= \bar{x}_1 + x_2\end{aligned}$$

$$f = \bar{m}_2 = M_2$$

Minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

A three-variable function

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

Sum-of-Products Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

Sum-of-Products Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$f(x_1, x_2, x_3) = \bar{x}_1\bar{x}_2x_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2x_3 + x_1x_2\bar{x}_3$$

Sum-of-Products Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$f(x_1, x_2, x_3) = \bar{x}_1\bar{x}_2x_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2x_3 + x_1x_2\bar{x}_3$$

$$\begin{aligned} f &= (\bar{x}_1 + x_1)\bar{x}_2x_3 + x_1(\bar{x}_2 + x_2)\bar{x}_3 \\ &= 1 \cdot \bar{x}_2x_3 + x_1 \cdot 1 \cdot \bar{x}_3 \\ &= \bar{x}_2x_3 + x_1\bar{x}_3 \end{aligned}$$

A three-variable function

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

Product-of-Sums Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

Product-of-Sums Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$f = \overline{m_0 + m_2 + m_3 + m_7}$$

$$= \overline{m_0} \cdot \overline{m_2} \cdot \overline{m_3} \cdot \overline{m_7}$$

$$= M_0 \cdot M_2 \cdot M_3 \cdot M_7$$

$$= (x_1 + x_2 + x_3)(x_1 + \bar{x}_2 + x_3)(x_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + \bar{x}_2 + \bar{x}_3)$$

Product-of-Sums Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$f = ((x_1 + x_3) + x_2)((x_1 + x_3) + \bar{x}_2)(x_1 + (\bar{x}_2 + \bar{x}_3))(\bar{x}_1 + (\bar{x}_2 + \bar{x}_3))$$

$$f = (x_1 + x_3)(\bar{x}_2 + \bar{x}_3)$$

Shorthand Notation

- **Sum-of-Products**

$$f(x_1, x_2, x_3) = \sum (m_1, m_4, m_5, m_6)$$

or

$$f(x_1, x_2, x_3) = \sum m(1, 4, 5, 6)$$

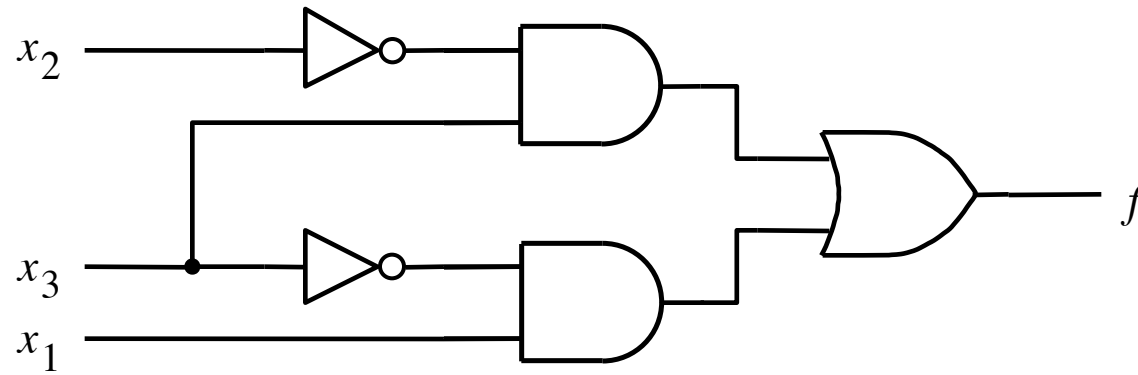
- **Product-of-sums**

$$f(x_1, x_2, x_3) = \Pi(M_0, M_2, M_3, M_7)$$

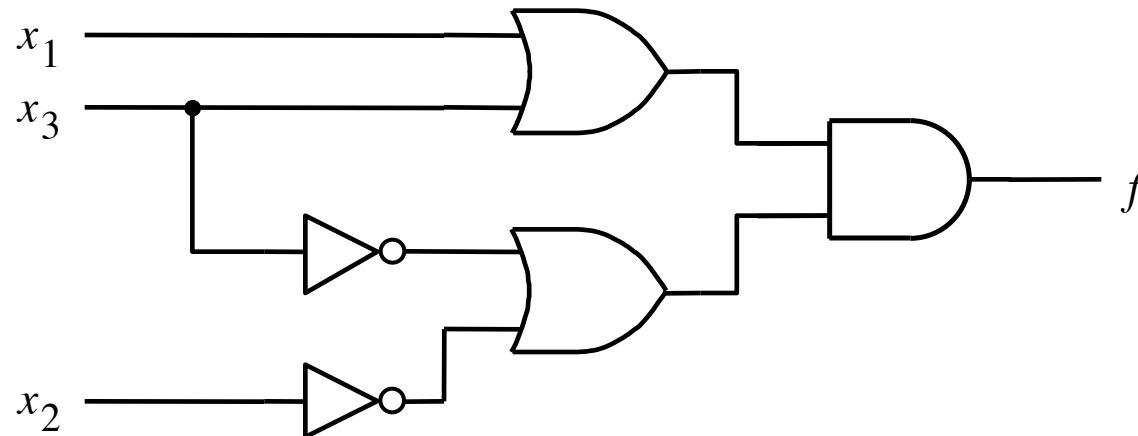
or

$$f(x_1, x_2, x_3) = \Pi M(0, 2, 3, 7)$$

Two realizations of that function



(a) A minimal sum-of-products realization



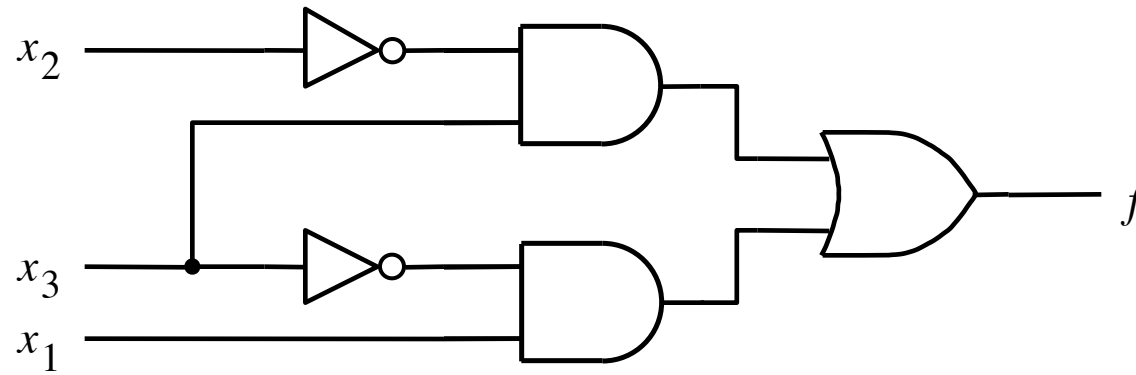
(b) A minimal product-of-sums realization

[Figure 2.24 from the textbook]

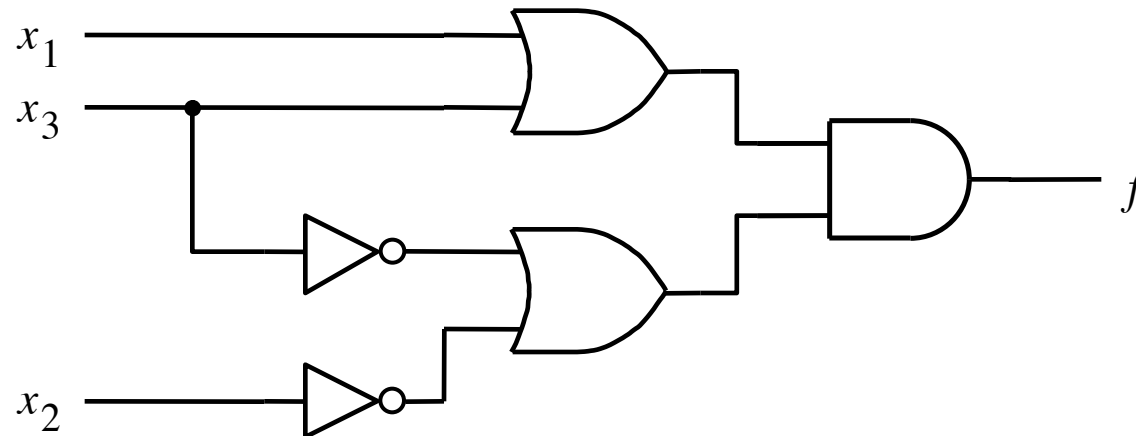
The Cost of a Circuit

- **Count all gates**
- **Count all inputs/wires to the gates**

What is the cost of each circuit?



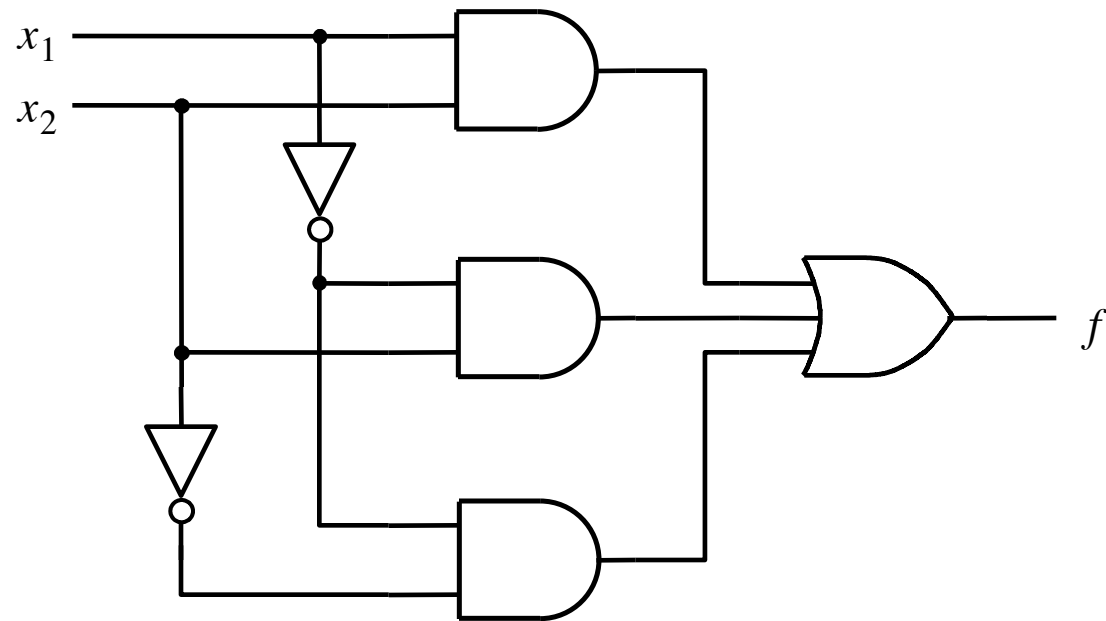
(a) A minimal sum-of-products realization



(b) A minimal product-of-sums realization

[Figure 2.24 from the textbook]

What is the cost of this circuit?



Questions?

THE END