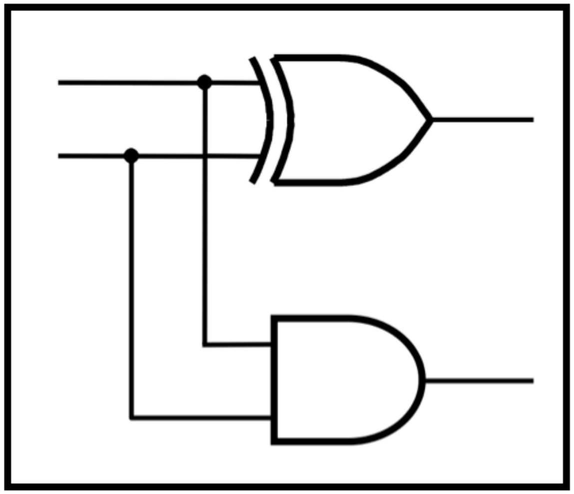


# CprE 281: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

# Code Converters

*CprE 281: Digital Logic*
*Iowa State University, Ames, IA*

# Administrative Stuff

- HW 7 is out
- It is due next Monday (Oct 19) @ 4pm

# Administrative Stuff

- The second midterm is in 2 weeks.

# Administrative Stuff

- **Midterm Exam #2**
- **When: Friday October 30 @ 4pm.**
- **Where: This classroom**
- **What: Chapters 1, 2, 3, 4 and 5.1-5.8**
- **The exam will be open book and open notes (you can bring up to 3 pages of handwritten notes).**

# Midterm 2: Format

- The exam will be out of 130 points
- You need 95 points to get an A
- It will be great if you can score more than 100 points.
  - but you can't roll over your extra points ☹

# Quick Review

# Decoders

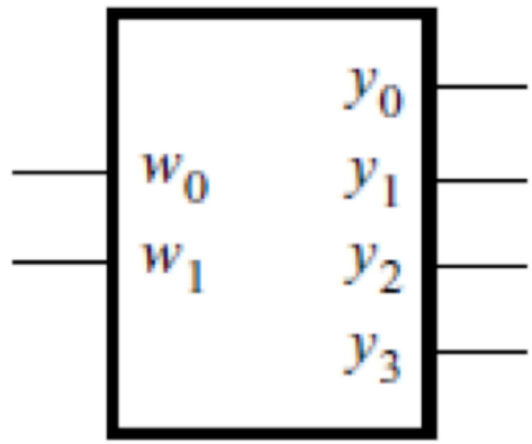
# 2-to-4 Decoder (Definition)

- **Has two inputs: $w_1$ and $w_0$**
- **Has four outputs: $y_0$ , $y_1$ , $y_2$ , and $y_3$**
- **If $w_1$=0 and $w_0$=0, then the output $y_0$ is set to 1**
- **If $w_1$=0 and $w_0$=1, then the output $y_1$ is set to 1**
- **If $w_1$=1 and $w_0$=0, then the output $y_2$ is set to 1**
- **If $w_1$=1 and $w_0$=1, then the output $y_3$ is set to 1**
- **Only one output is set to 1. All others are set to 0.**

# Truth Table and Graphical Symbol for a 2-to-4 Decoder

| $w_1$ | $w_0$ | $y_0$ | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 |

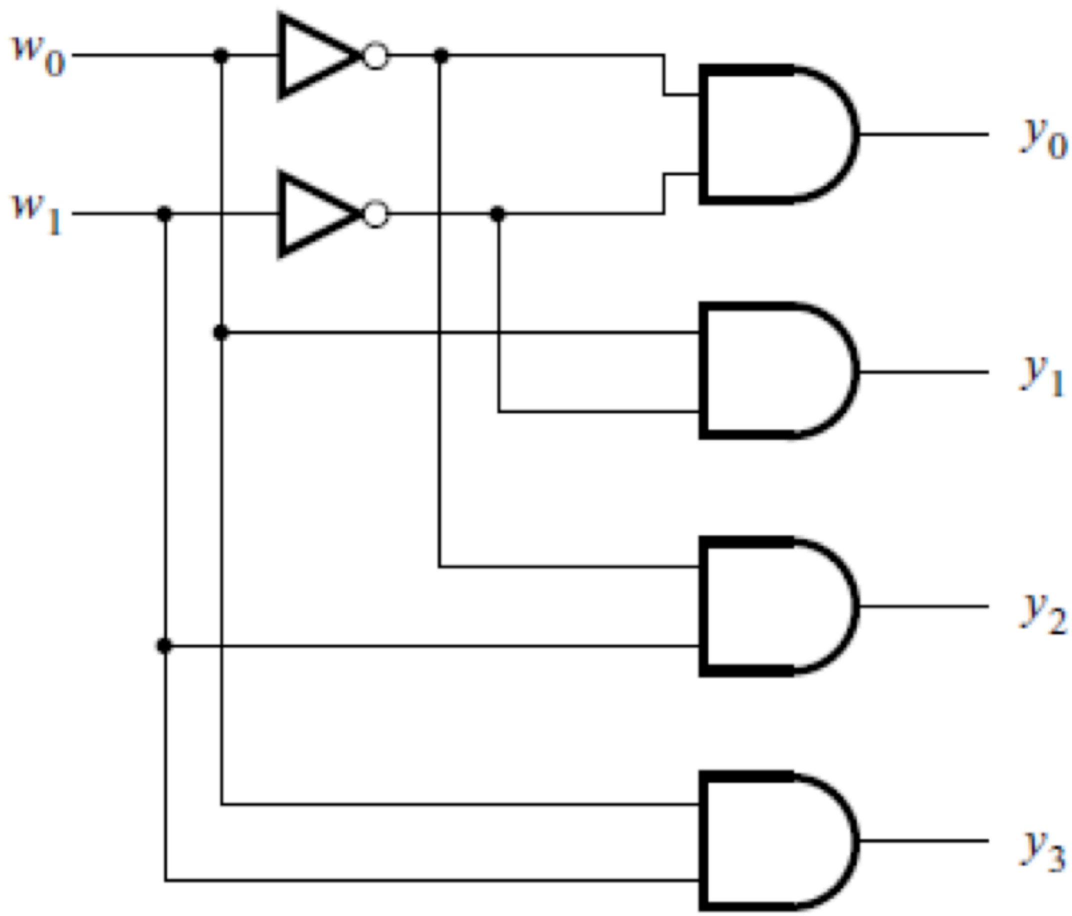
(a) Truth table




(b) Graphical symbol

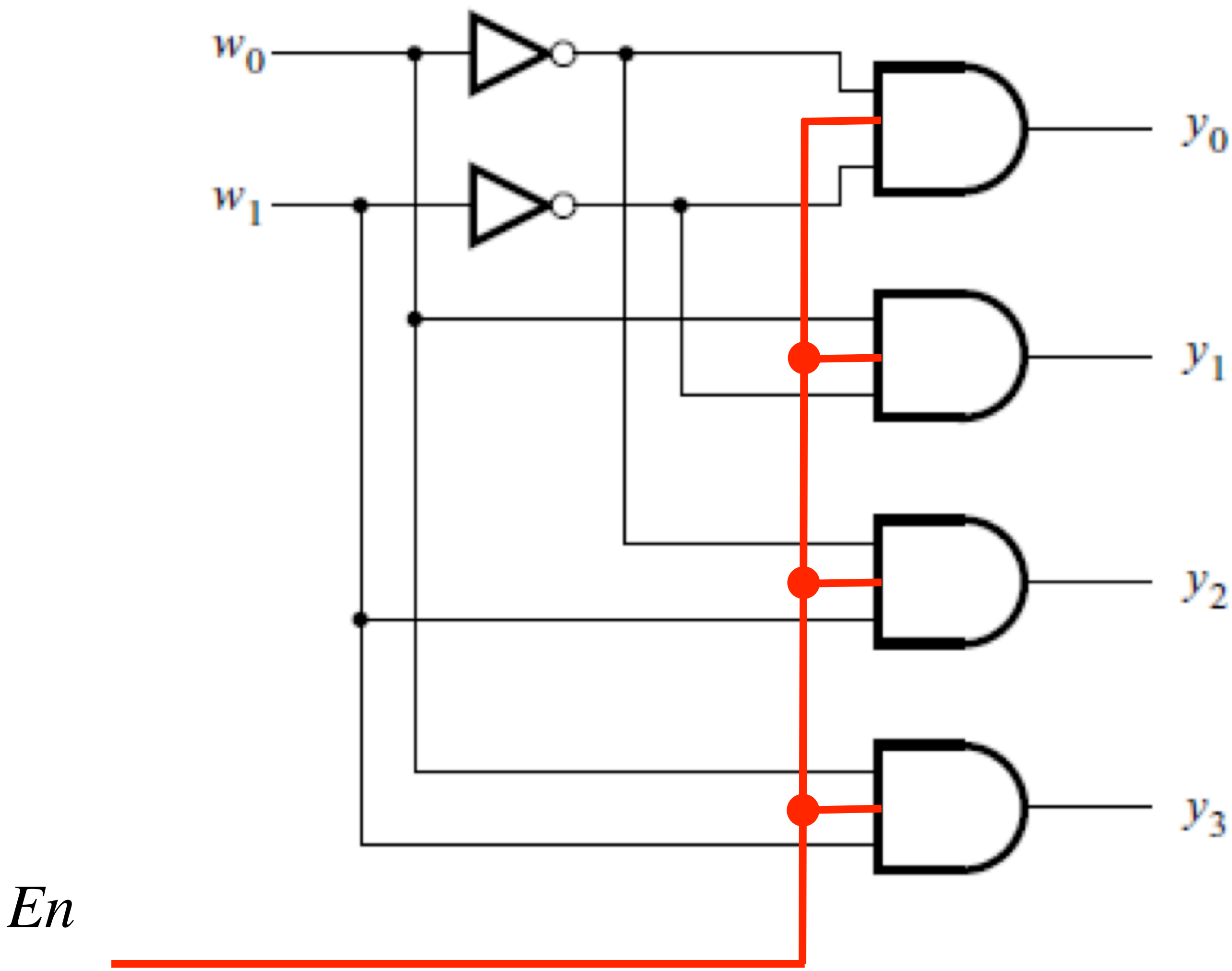
[ Figure 4.13a-b from the textbook ]

# Truth Logic Circuit for a 2-to-4 Decoder




[ Figure 4.13c from the textbook ]

# Adding an Enable Input

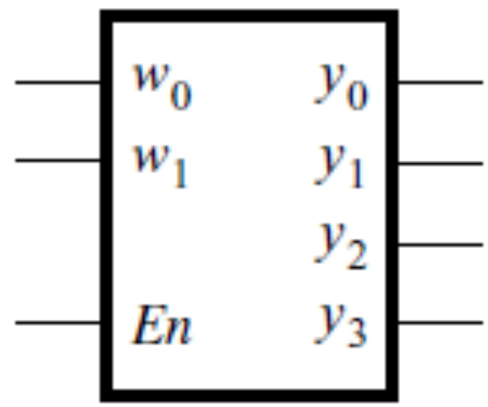



[ Figure 4.13c from the textbook ]

# Truth Table and Graphical Symbol for a 2-to-4 Decoder with an Enable Input

| $En$ | $w_1$ | $w_0$ | $y_0$ | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| 0 | x | x | 0 | 0 | 0 | 0 |

(a) Truth table




(b) Graphical symbol

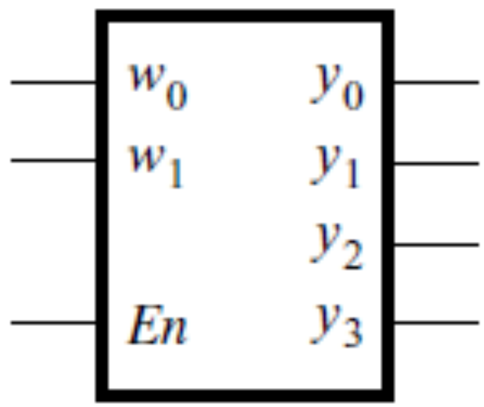
[ Figure 4.14a-b from the textbook ]

# Truth Table and Graphical Symbol for a 2-to-4 Decoder with an Enable Input

| $En$ | $w_1$ | $w_0$ | $y_0$ | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| 0 | x | x | 0 | 0 | 0 | 0 |

(a) Truth table

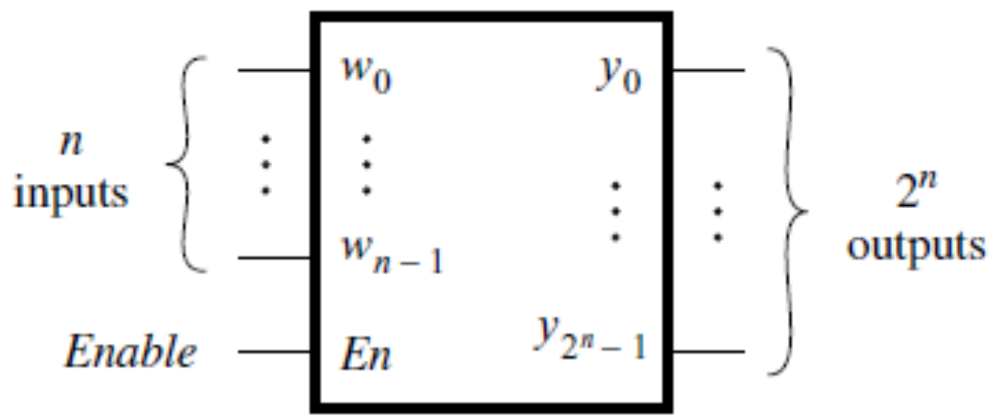
x indicates that it does not matter what the value of these variable is for this row of the truth table




(b) Graphical symbol

[ Figure 4.14a-b from the textbook ]

# Graphical Symbol for a Binary n-to-$2^n$ Decoder with an Enable Input



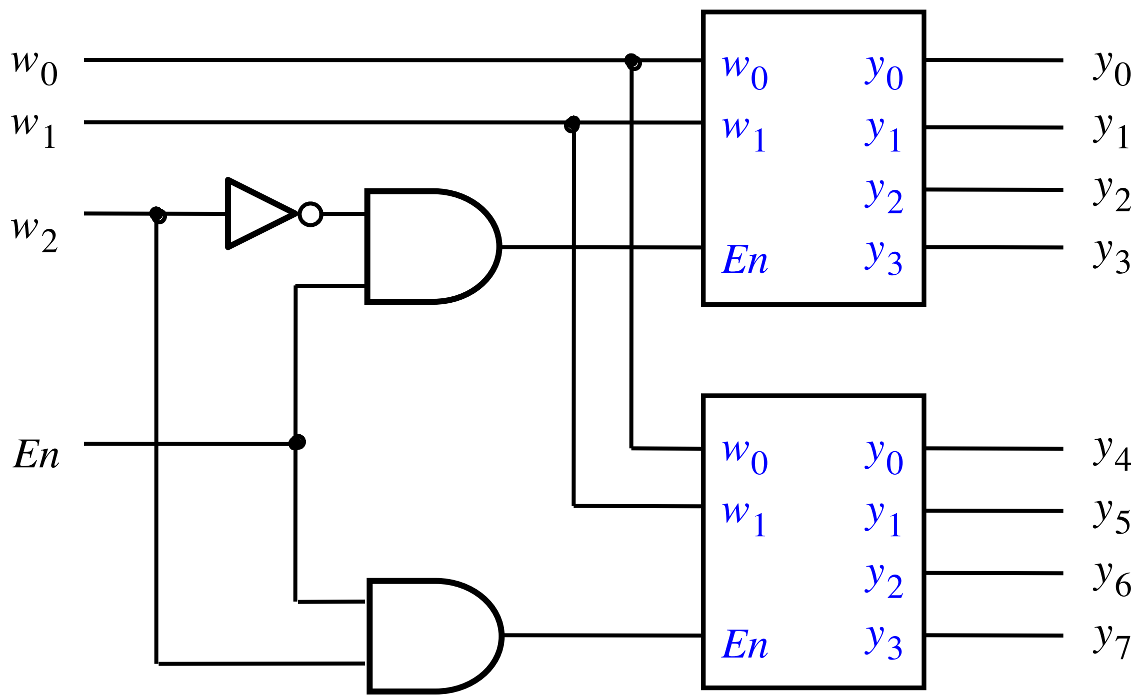

(d) An n-to-$2^n$ decoder

A binary decoder with n inputs has $2^n$ outputs

The outputs of an enabled binary decoder are "one-hot" encoded, meaning that only a single bit is set to 1, i.e., it is *hot*.

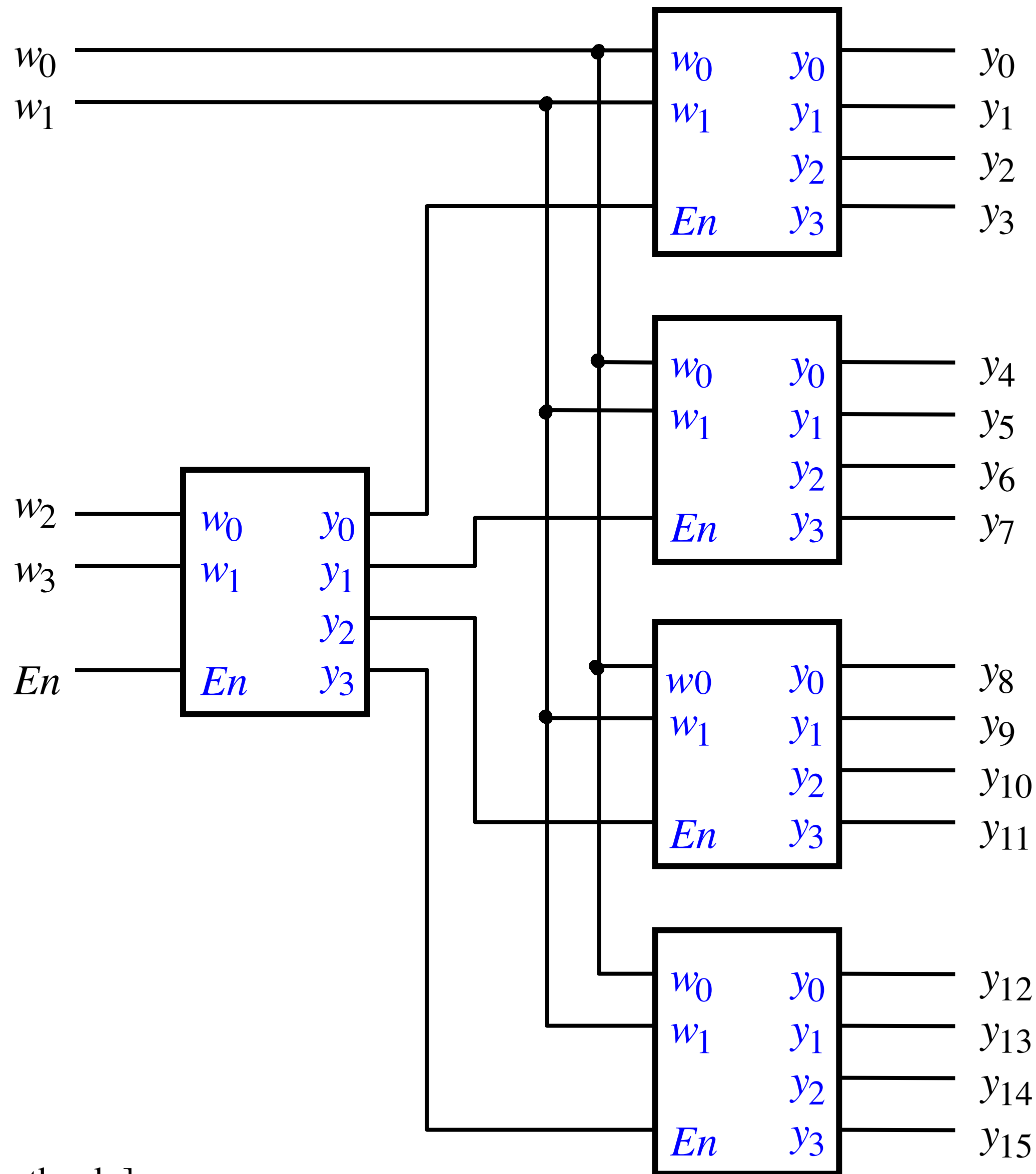
[ Figure 4.14d from the textbook ]

# A 3-to-8 decoder using two 2-to-4 decoders



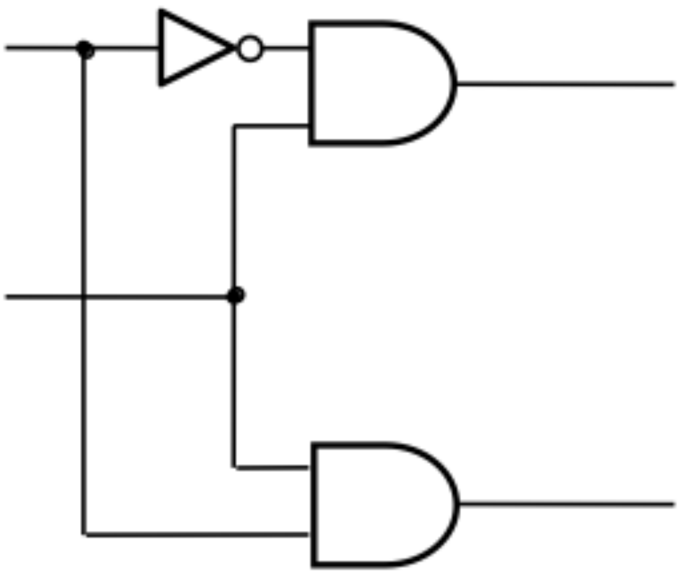

[ Figure 4.15 from the textbook ]

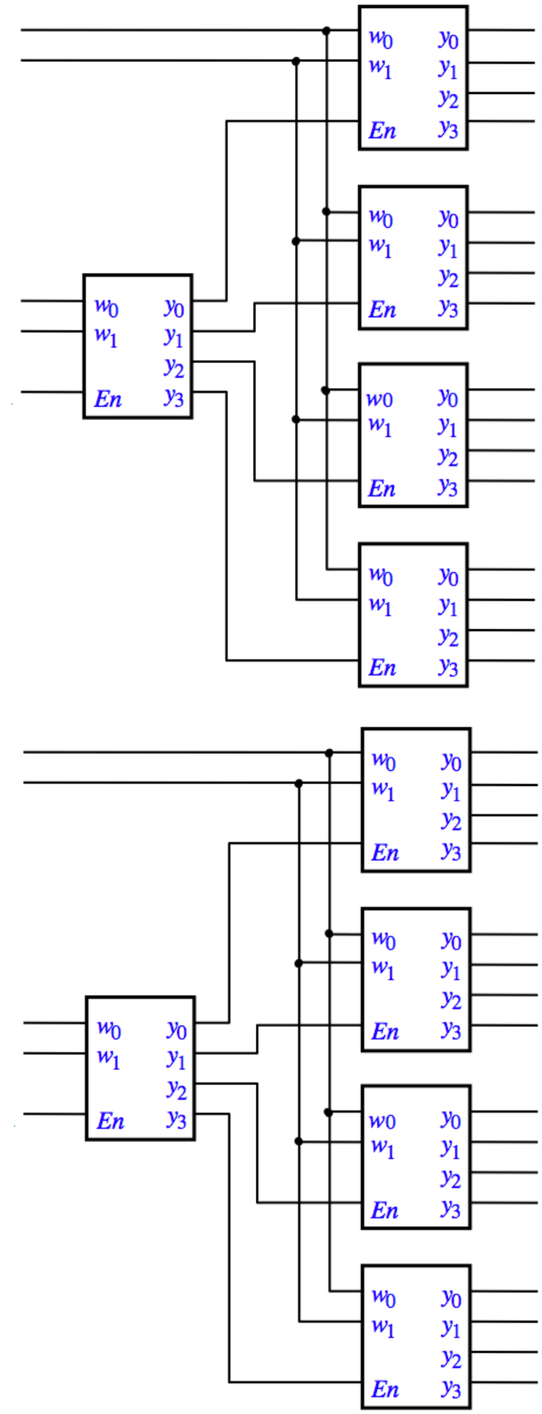
# A 4-to-16 decoder built using a decoder tree



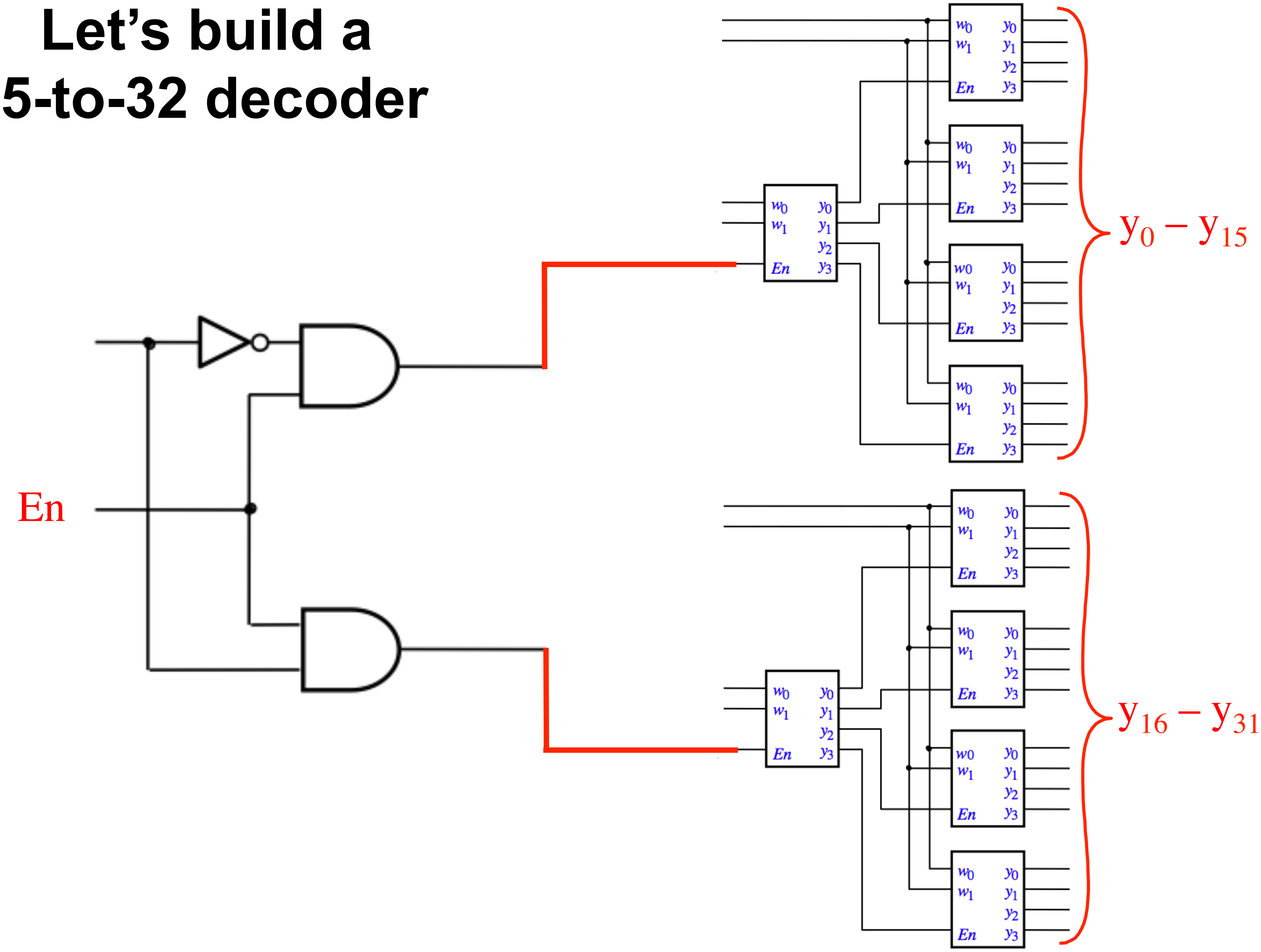

[ Figure 4.16 from the textbook ]

# Let's build a 5-to-32 decoder

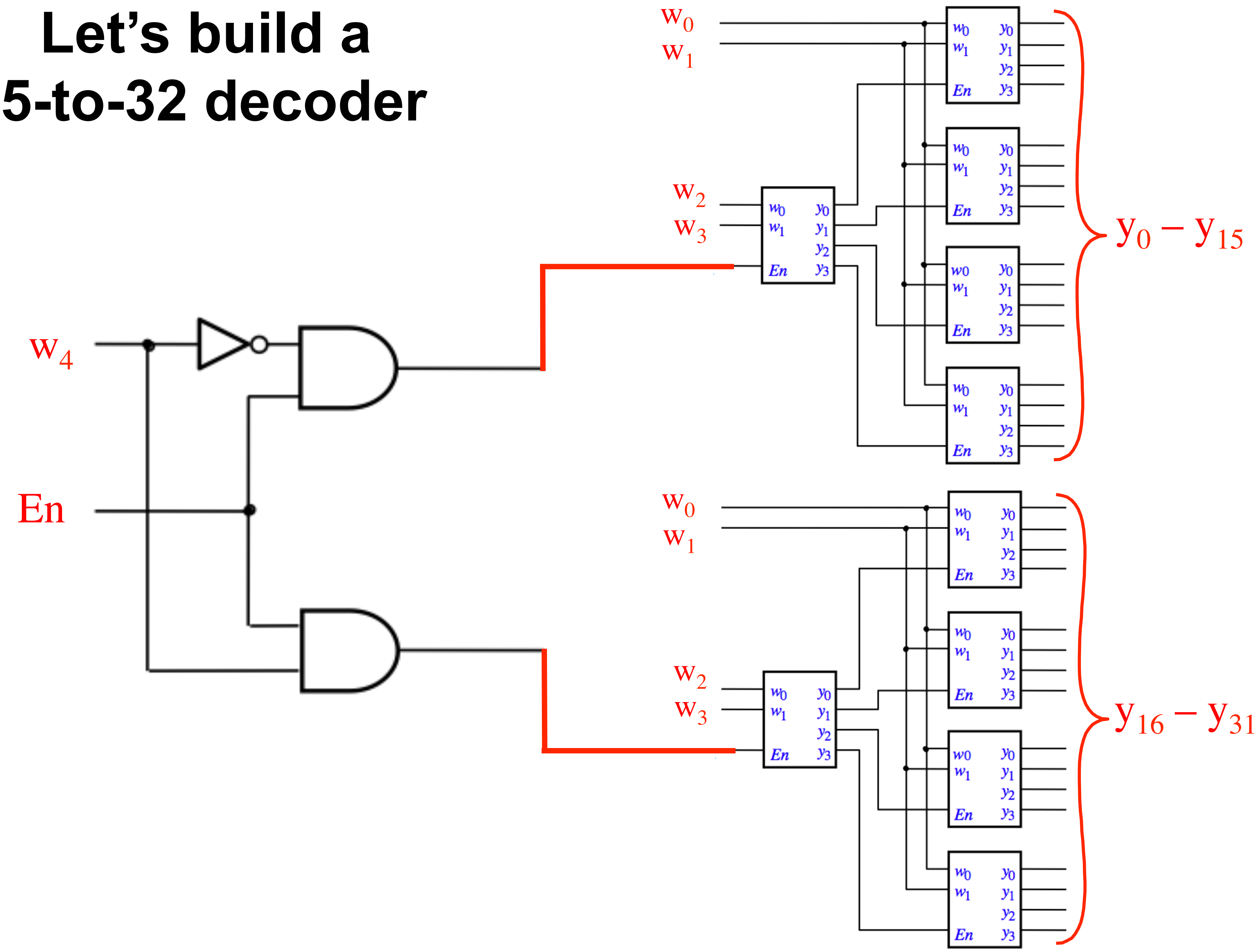




# Let's build a 5-to-32 decoder



# Let's build a 5-to-32 decoder

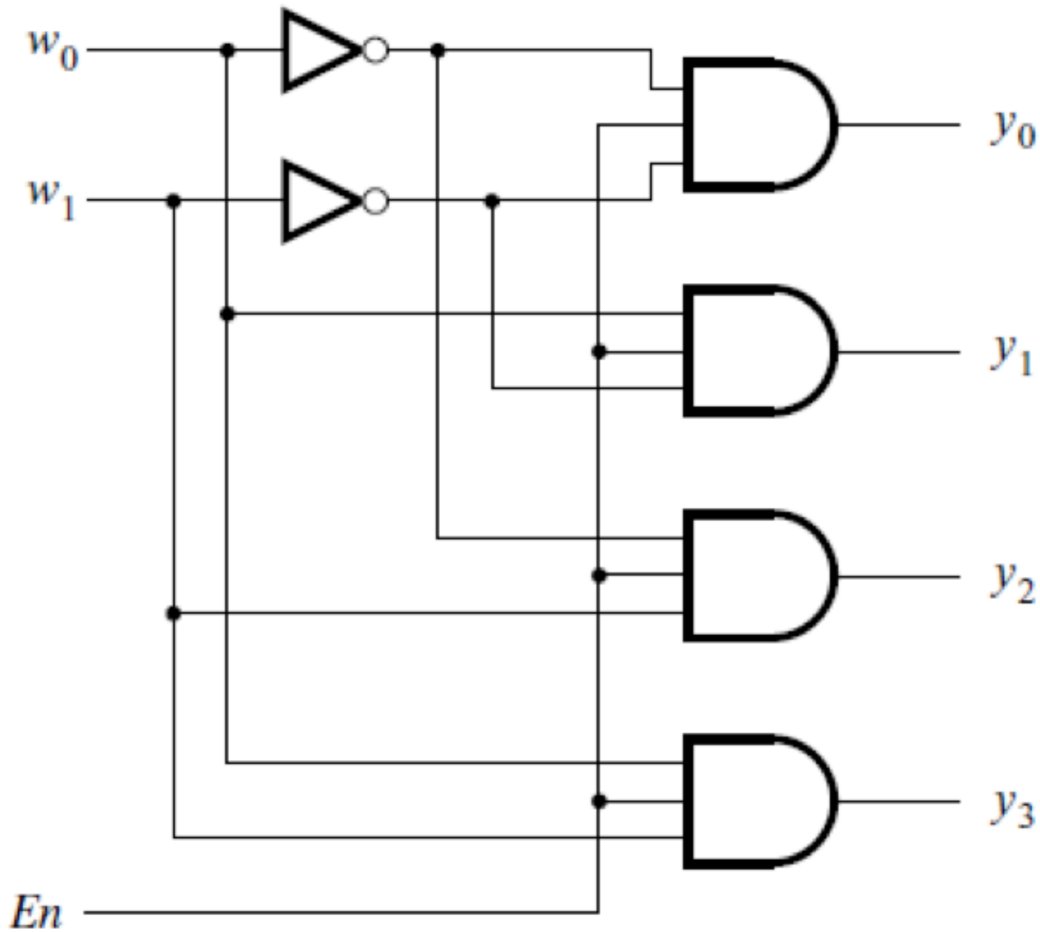


# Demultiplexers

# 1-to-4 Demultiplexer (Definition)

- Has one data input line: D
- Has two output select lines: $w_1$ and $w_0$
- Has four outputs: $y_0$ , $y_1$ , $y_2$ , and $y_3$
- If $w_1$=0 and $w_0$=0, then the output $y_0$ is set to D
- If $w_1$=0 and $w_0$=1, then the output $y_1$ is set to D
- If $w_1$=1 and $w_0$=0, then the output $y_2$ is set to D
- If $w_1$=1 and $w_0$=1, then the output $y_3$ is set to D
- Only one output is set to D. All others are set to 0.

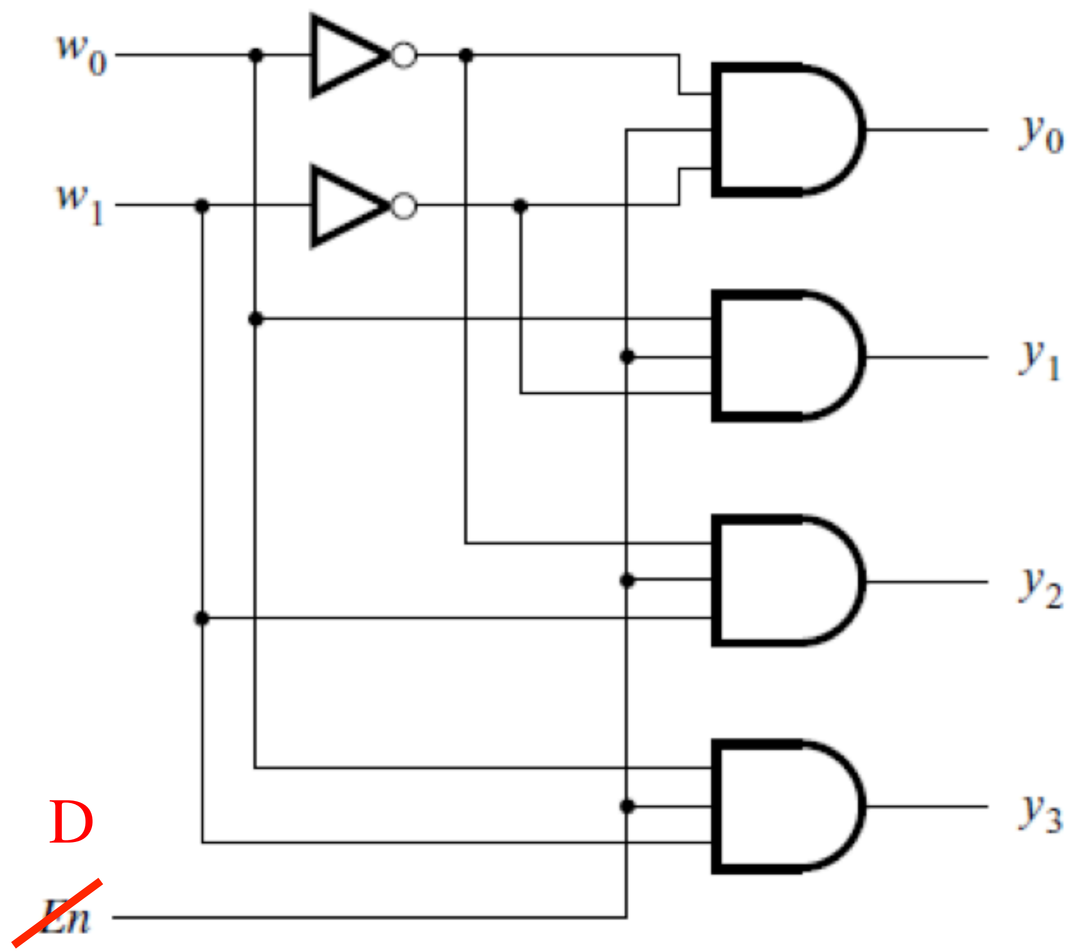
# A 1-to-4 demultiplexer built with a 2-to-4 decoder




[ Figure 4.14c from the textbook ]

# A 1-to-4 demultiplexer built with a 2-to-4 decoder

output select lines



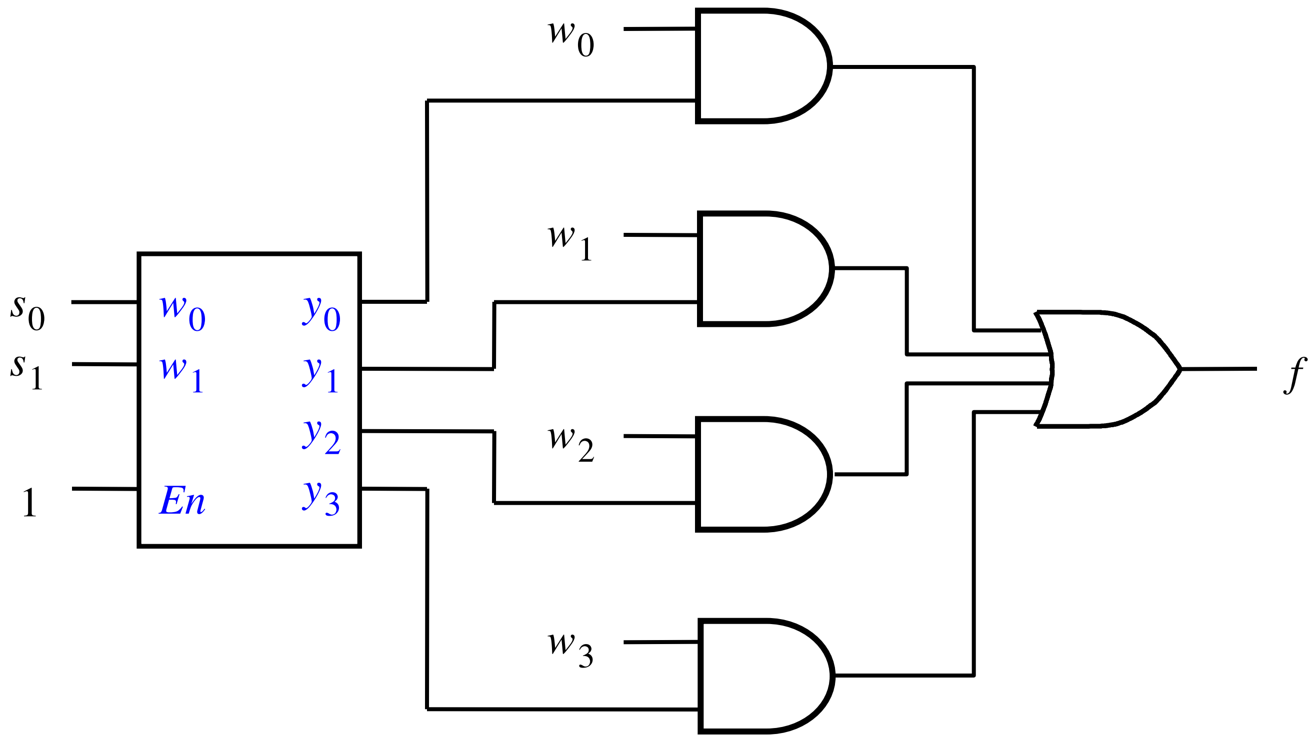

the four output lines

data input line

[ Figure 4.14c from the textbook ]

# Multiplexers
# (Implemented with Decoders)

# A 4-to-1 multiplexer built using a 2-to-4 decoder

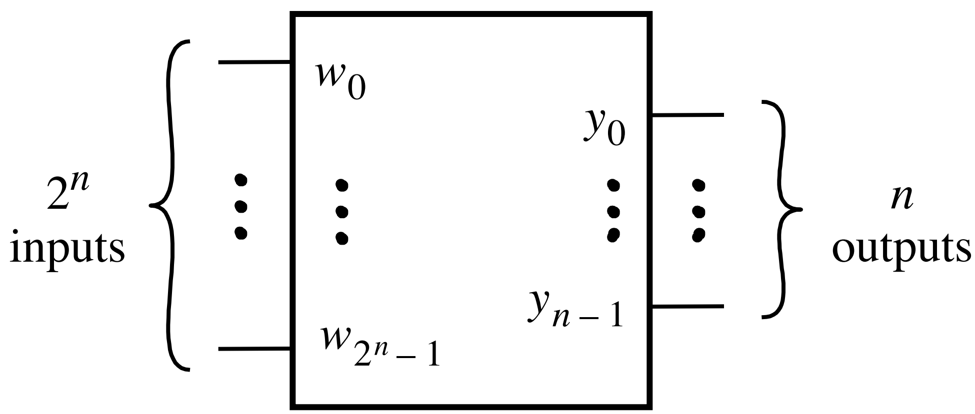



[ Figure 4.17 from the textbook ]

# Encoders

# Binary Encoders

# A $2^n$-to-n binary encoder

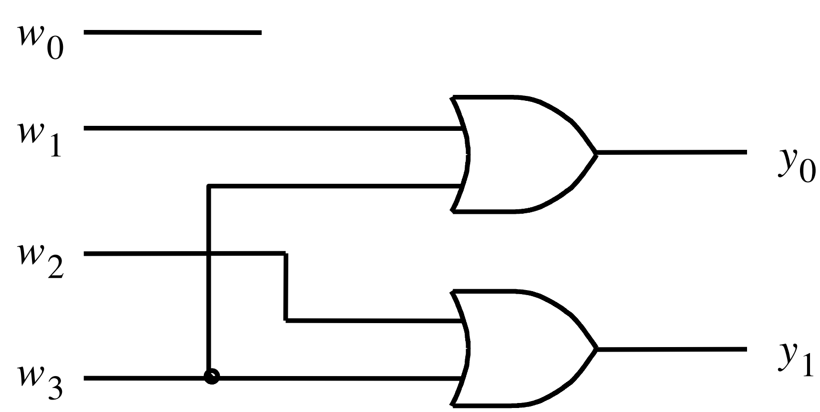



[ Figure 4.18 from the textbook ]

# A 4-to-2 binary encoder

| $w_3$ | $w_2$ | $w_1$ | $w_0$ | $y_1$ | $y_0$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 |

(a) Truth table



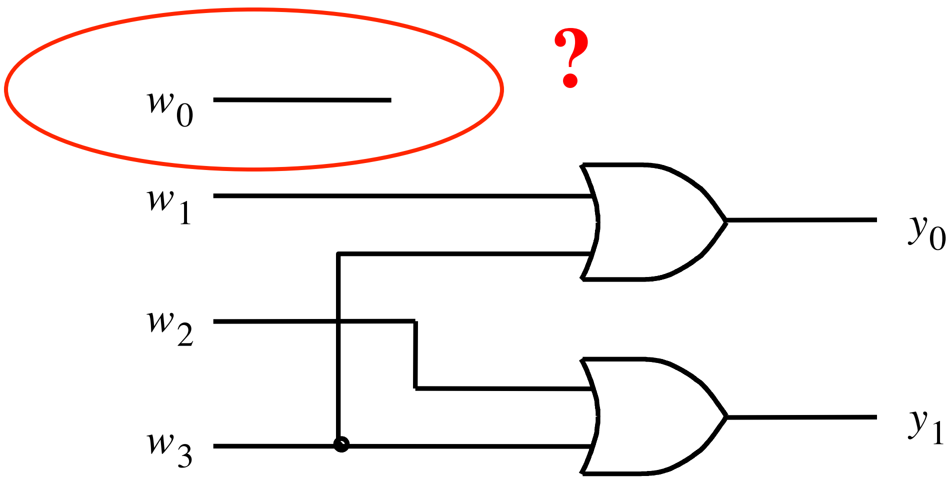

(b) Circuit

[ Figure 4.19 from the textbook ]

# A 4-to-2 binary encoder

| $w_3$ | $w_2$ | $w_1$ | $w_0$ | $y_1$ | $y_0$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 |

(a) Truth table



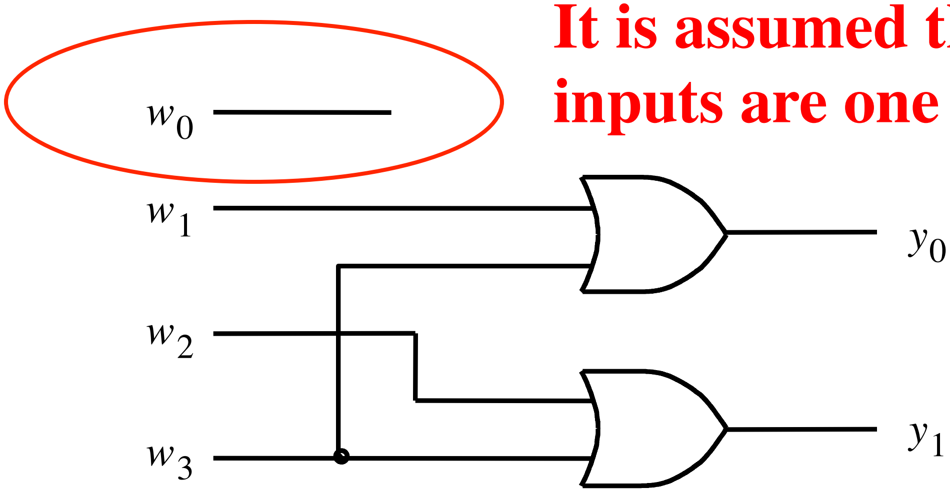

(b) Circuit

[ Figure 4.19 from the textbook ]

# A 4-to-2 binary encoder

| $w_3$ | $w_2$ | $w_1$ | $w_0$ | $y_1$ | $y_0$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 |

(a) Truth table




**It is assumed that the inputs are one hot encoded**

(b) Circuit

[ Figure 4.19 from the textbook ]

# Priority Encoders

# Truth table for a 4-to-2 priority encoder

| $w_3$ | $w_2$ | $w_1$ | $w_0$ | $y_1$ | $y_0$ | $z$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | d | d | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | x | 0 | 1 | 1 |
| 0 | 1 | x | x | 1 | 0 | 1 |
| 1 | x | x | x | 1 | 1 | 1 |

[ Figure 4.20 from the textbook ]

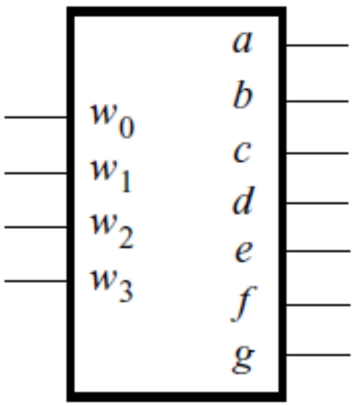
# Code Converter (Definition)

- **Converts from one type of input encoding to a different type of output encoding.**

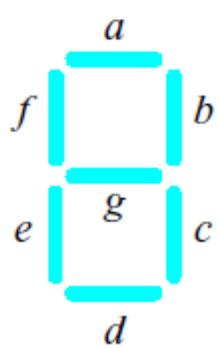
# Code Converter (Definition)

- **Converts from one type of input encoding to a different type of output encoding.**
- **A decoder does that as well, but its outputs are always one-hot encoded so the output code is really only one type of output code.**
- **A binary encoder does that as well but its inputs are always one-hot encoded so the input code is really only one type of input code.**

# A hex-to-7-segment display code converter




(a) Code converter



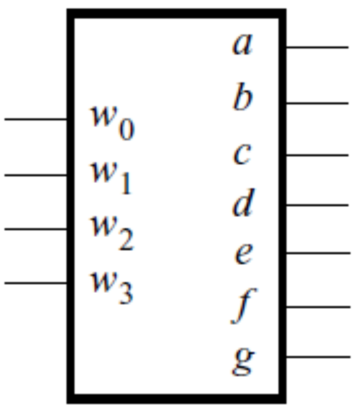

(b) 7-segment display

| $w_3$ | $w_2$ | $w_1$ | $w_0$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |

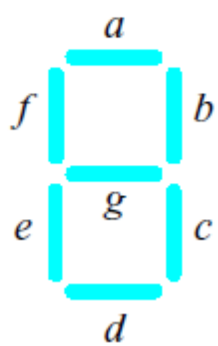
(c) Truth table

[ Figure 4.21 from the textbook ]

# A hex-to-7-segment display code converter




(a) Code converter



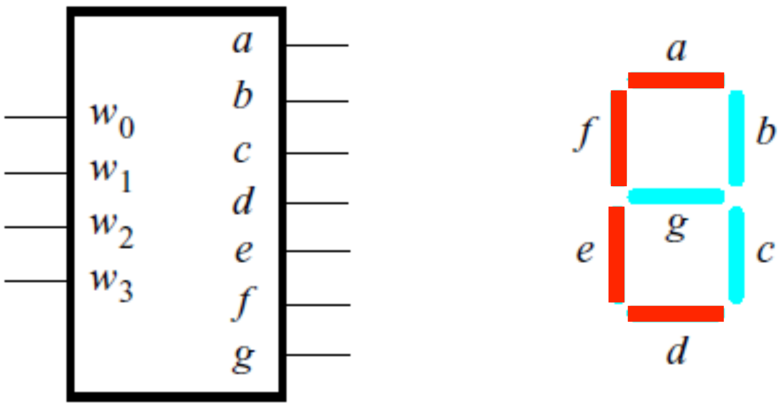

(b) 7-segment display

| $w_3$ | $w_2$ | $w_1$ | $w_0$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |

(c) Truth table

[ Figure 4.21 from the textbook ]

# A hex-to-7-segment display code converter




(a) Code converter

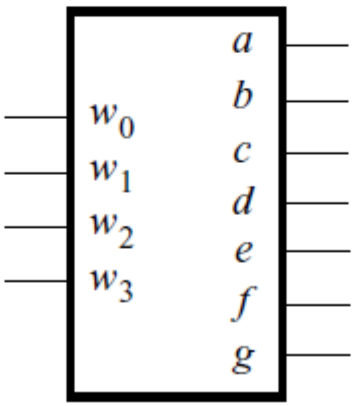
(b) 7-segment display

| $w_3$ | $w_2$ | $w_1$ | $w_0$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |

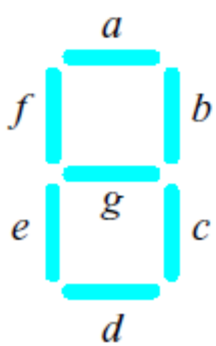
(c) Truth table

[ Figure 4.21 from the textbook ]

# A hex-to-7-segment display code converter




(a) Code converter



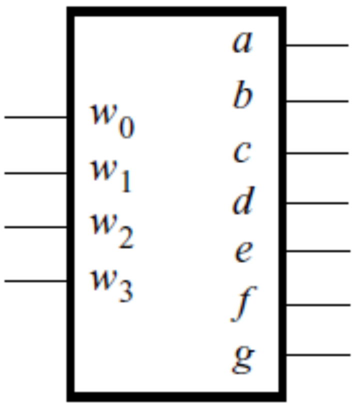

(b) 7-segment display

| $w_3$ | $w_2$ | $w_1$ | $w_0$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |

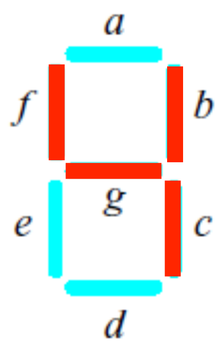
(c) Truth table

[ Figure 4.21 from the textbook ]

# A hex-to-7-segment display code converter




(a) Code converter




(b) 7-segment display

| $w_3$ | $w_2$ | $w_1$ | $w_0$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |

(c) Truth table

[ Figure 4.21 from the textbook ]

# What is the circuit for this code converter?

| $x_3$ | $x_2$ | $x_1$ | $x_0$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |

# What is the circuit for this code converter?

| $x_3$ | $x_2$ | $x_1$ | $x_0$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(0, 2, 3, 5, 6, 7, 8, 9, 10, 12, 14, 15)$$

# What is the circuit for this code converter?

| $x_3$ | $x_2$ | $x_1$ | $x_0$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |

| $x_3x_4$ \ $x_1x_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 1 | 1 |
| 01 | 0 | 1 | 0 | 1 |
| 11 | 1 | 1 | 1 | 0 |
| 10 | 1 | 1 | 1 | 1 |

$x_1$, $x_2$, $x_3$, $x_4$

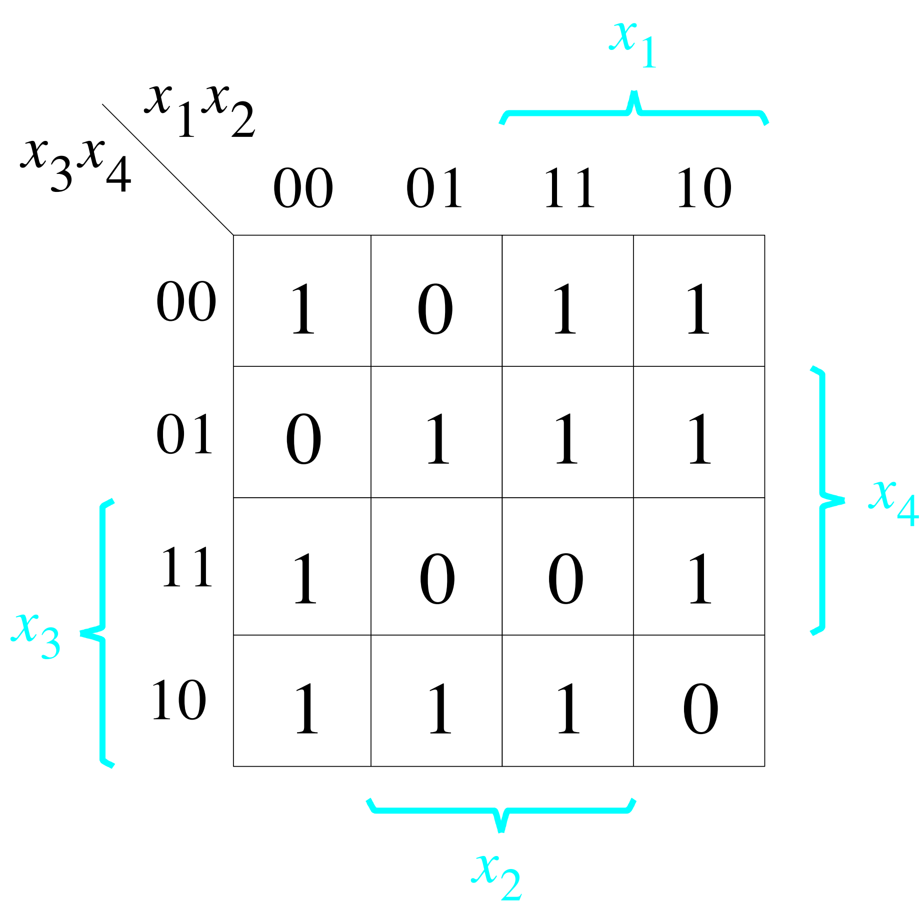
$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(0, 2, 3, 5, 6, 7, 8, 9, 10, 12, 14, 15)$$

# What is the circuit for this code converter?

| $x_3$ | $x_2$ | $x_1$ | $x_0$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |

# What is the circuit for this code converter?

| $x_3$ | $x_2$ | $x_1$ | $x_0$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |




$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(0, 2, 3, 5, 6, 8, 9, 11, 12, 13, 14)$$

# Arithmetic Comparison Circuits

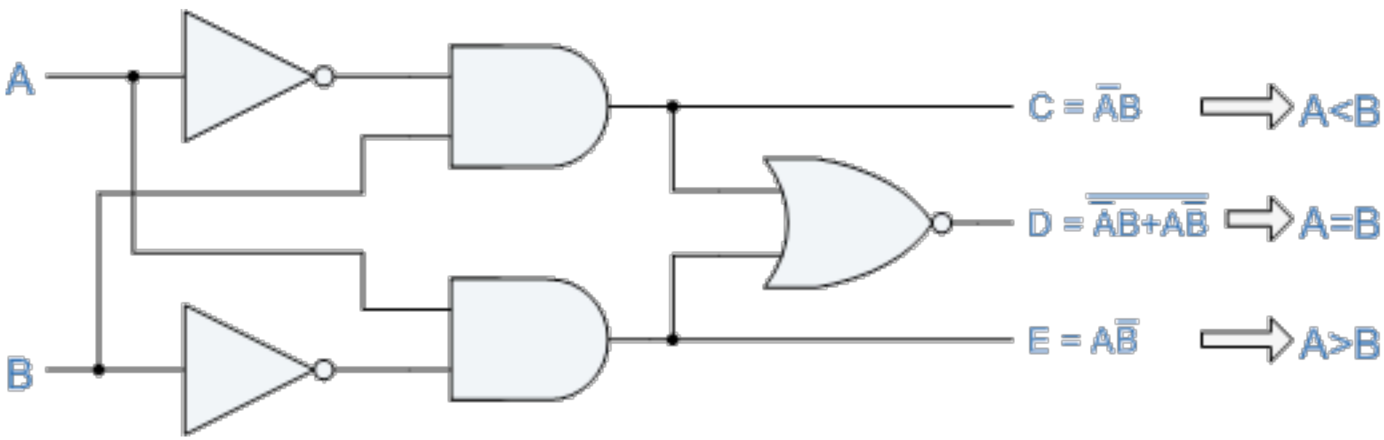
# Truth table for a one-bit digital comparator

| Inputs | | Outputs | | |
|---|---|---|---|---|
| $A$ | $B$ | $A > B$ | $A = B$ | $A < B$ |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |

[http://en.wikipedia.org/wiki/Digital_comparator]

# A one-bit digital comparator circuit

| Inputs | | Outputs | | |
|---|---|---|---|---|
| $A$ | $B$ | $A > B$ | $A = B$ | $A < B$ |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |



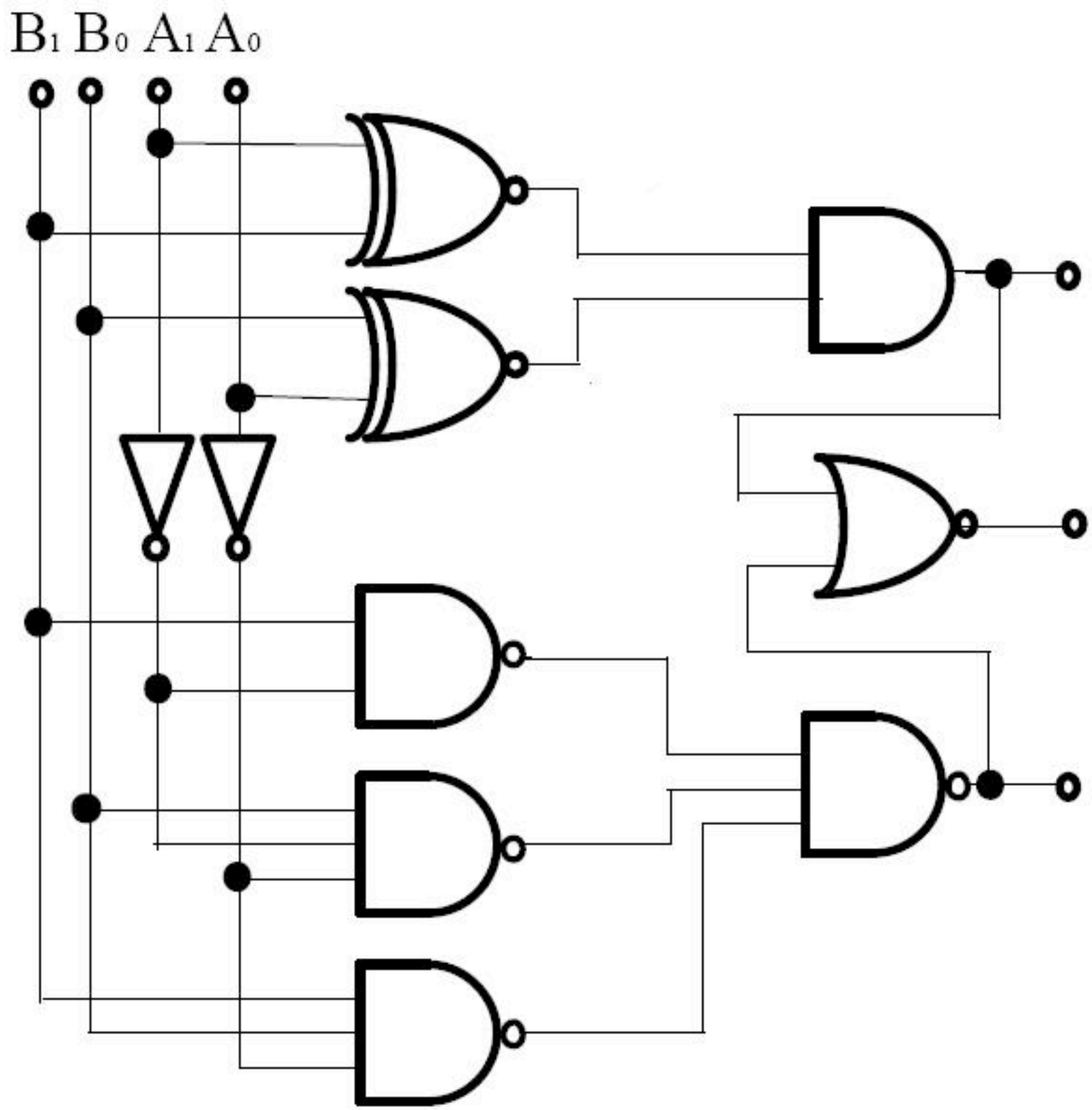

[http://www.personal.psu.edu/users///c/w/cwb5096/Old%20Site/index_files/cmpenlab4.htm]

# Truth table for a two-bit digital comparator

| Inputs | | | | Outputs | | |
|---|---|---|---|---|---|---|
| $A_1$ | $A_0$ | $B_1$ | $B_0$ | $A < B$ | $A = B$ | $A > B$ |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 | 1 | 0 |

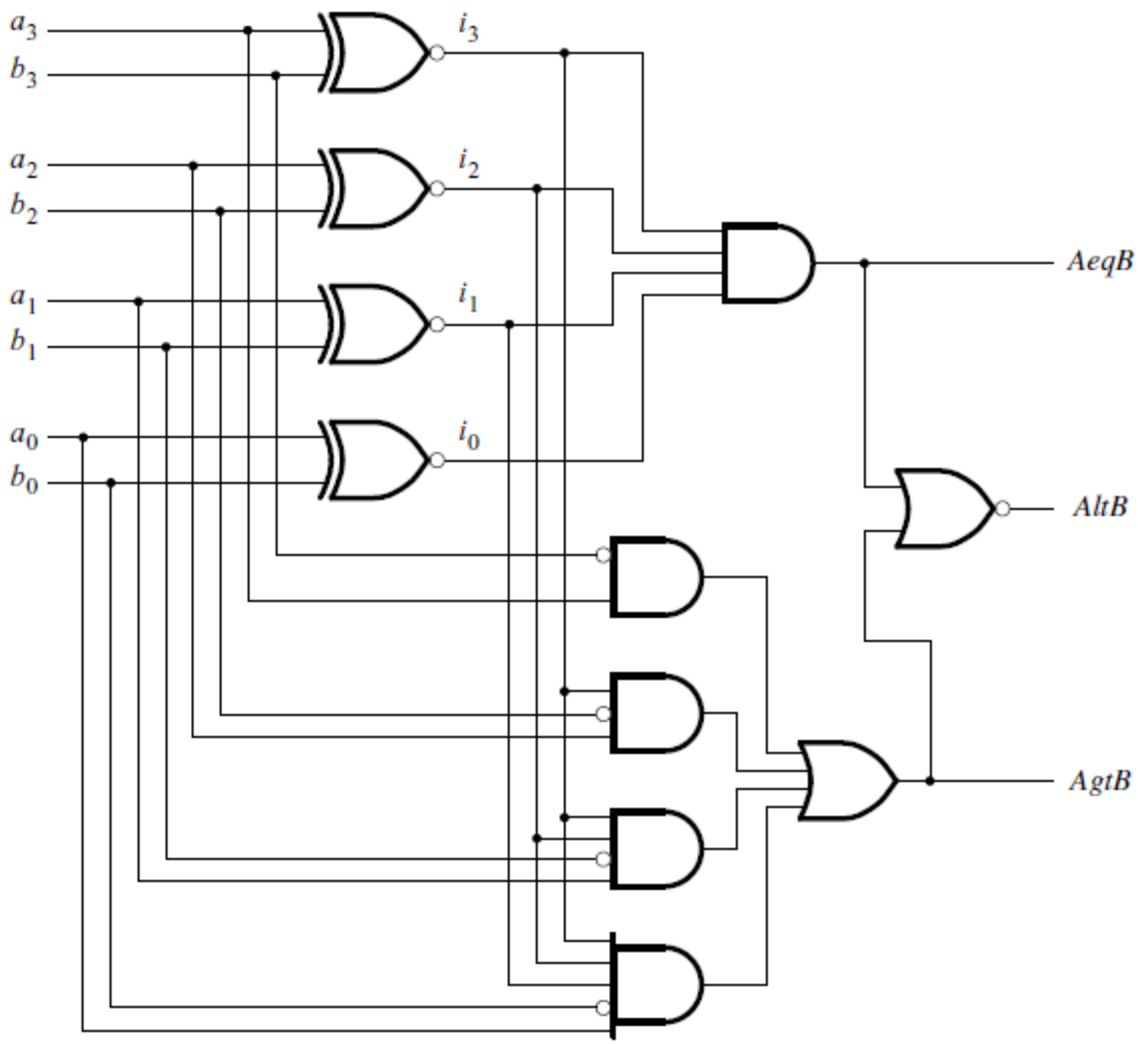
[http://en.wikipedia.org/wiki/Digital_comparator]

# A two-bit digital comparator circuit




[http://forum.allaboutcircuits.com/showthread.php?t=10561]

# A four-bit comparator circuit




[ Figure 4.22 from the textbook ]

# Example Problems from Chapter 4

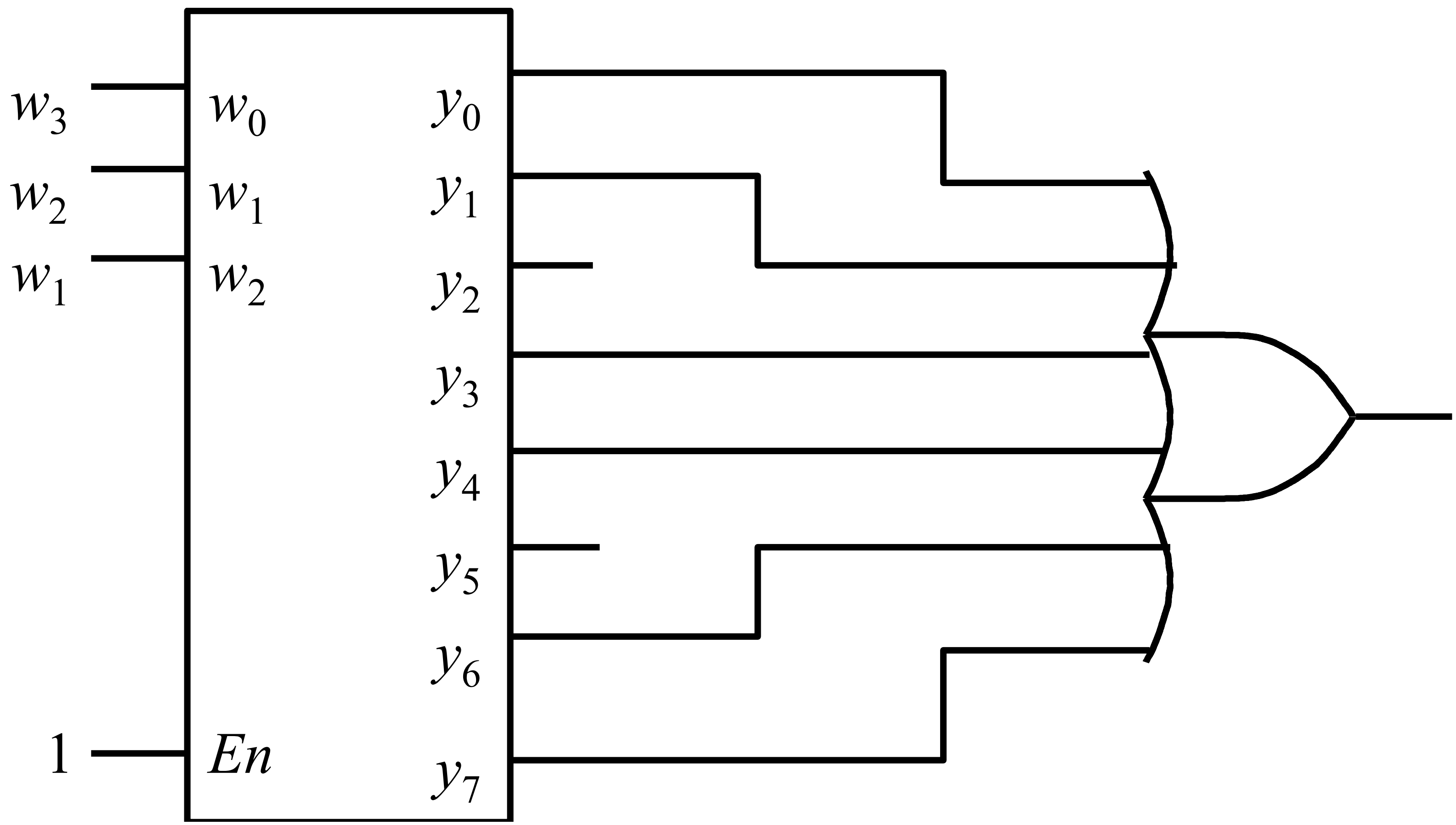
# Example 1: SOP vs Decoders

**Implement the function**

$$f(w_1, w_2, w_3) = \Sigma\, m(0, 1, 3, 4, 6, 7)$$

**by using a 3-8 binary decoder and one OR gate.**

# Solution Circuit




$$f(w_1, w_2, w_3) = \Sigma\, m(0, 1, 3, 4, 6, 7)$$

[ Figure 4.44 from the textbook ]

# Example 2: Implement an 8-to-3 binary encoder

| $w_7$ | $w_6$ | $w_5$ | $w_4$ | $w_3$ | $w_2$ | $w_1$ | $w_0$ | $y_2$ | $y_1$ | $y_0$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |

[ Figure 4.45 from the textbook ]

# Example 2: Implement an 8-to-3 binary encoder

| $w_7$ | $w_6$ | $w_5$ | $w_4$ | $w_3$ | $w_2$ | $w_1$ | $w_0$ | $y_2$ | $y_1$ | $y_0$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |

$$y_2 = w_4 + w_5 + w_6 + w_7$$
$$y_1 = w_2 + w_3 + w_6 + w_7$$
$$y_0 = w_1 + w_3 + w_5 + w_7$$

[ Figure 4.45 from the textbook ]

# Example 3:Circuit implementation using a multiplexer

**Implement the function**

$$f(w_1, w_2, w_3, w_4, w_5) = \bar{w}_1\bar{w}_2\bar{w}_4\bar{w}_5 + w_1w_2 + w_1w_3 + w_1w_4 + w_3w_4w_5$$

**using a 4-to-1 multiplexer**

# Some Boolean Algebra Leads To

$$\overline{w}_1\overline{w}_2\overline{w}_4\overline{w}_5 + w_1w_2 + w_1w_3 + w_1w_4 + w_3w_4w_5$$

$$\overline{w}_1\overline{w}_4\,(\overline{w}_5\,\overline{w}_2) + w_4\,(w_3w_5) + w_1\,(w_2 + w_3) + w_1w_4\,(1)$$

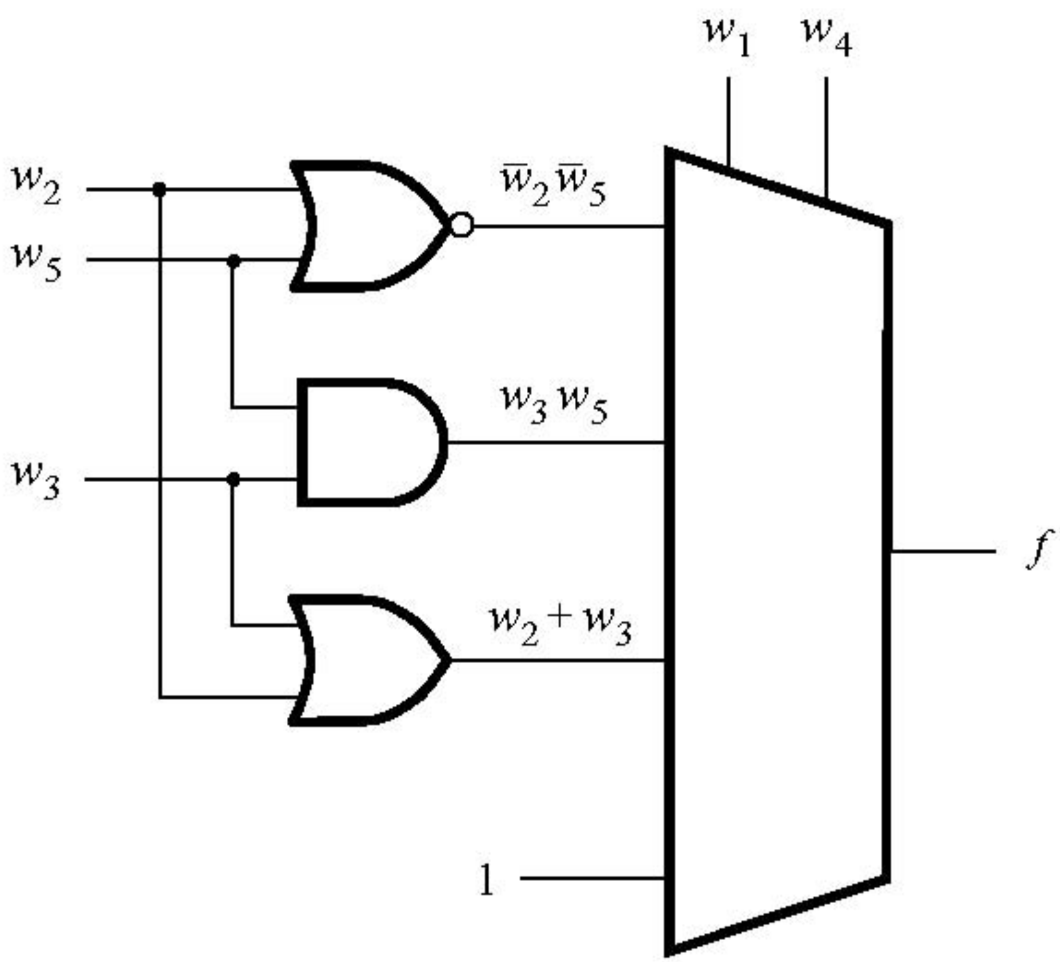
$$\overline{w}_1\overline{w}_4\,(\overline{w}_5\,\overline{w}_2) + (\,\overline{w}_1 + w_1)w_4\,(w_3w_5) + w_1(\,\overline{w}_4 + w_4)\,(w_2 + w_3) + w_1w_4\,(1)$$

$$\overline{w}_1\overline{w}_4\,(\overline{w}_5\,\overline{w}_2) + \overline{w}_1w_4\,(w_3w_5) + w_1\overline{w}_4\,(w_2 + w_3) + w_1w_4\,(\,w_3w_5 + (w_2 + w_3) + 1)$$

$$\overline{w}_1\overline{w}_4\,(\overline{w}_5\,\overline{w}_2) + \overline{w}_1w_4\,(w_3w_5) + w_1\overline{w}_4\,(w_2 + w_3) + w_1w_4\,(1)$$

**Note that the split is by $w_1$ and $w_4$, not $w_1$ and $w_2$**

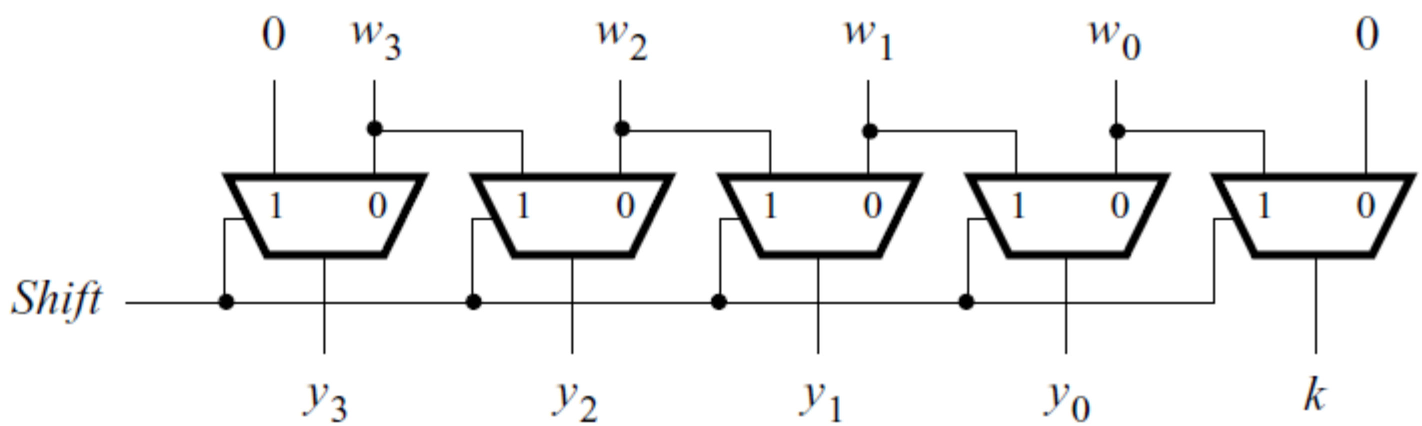
# Solution Circuit




[ Figure 4.46 from the textbook ]

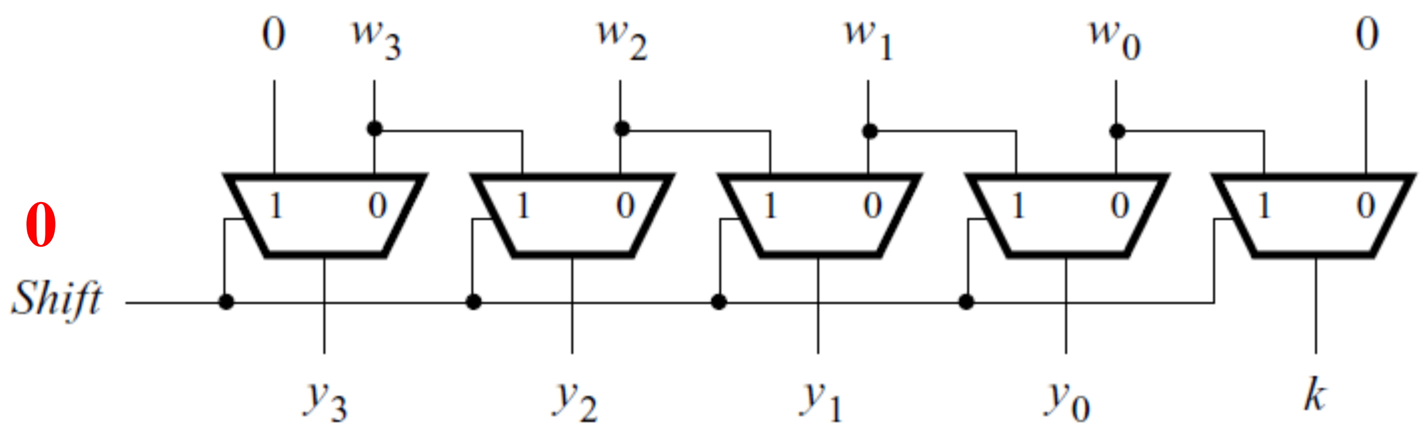
# Some Final Things from Chapter 4

# A shifter circuit

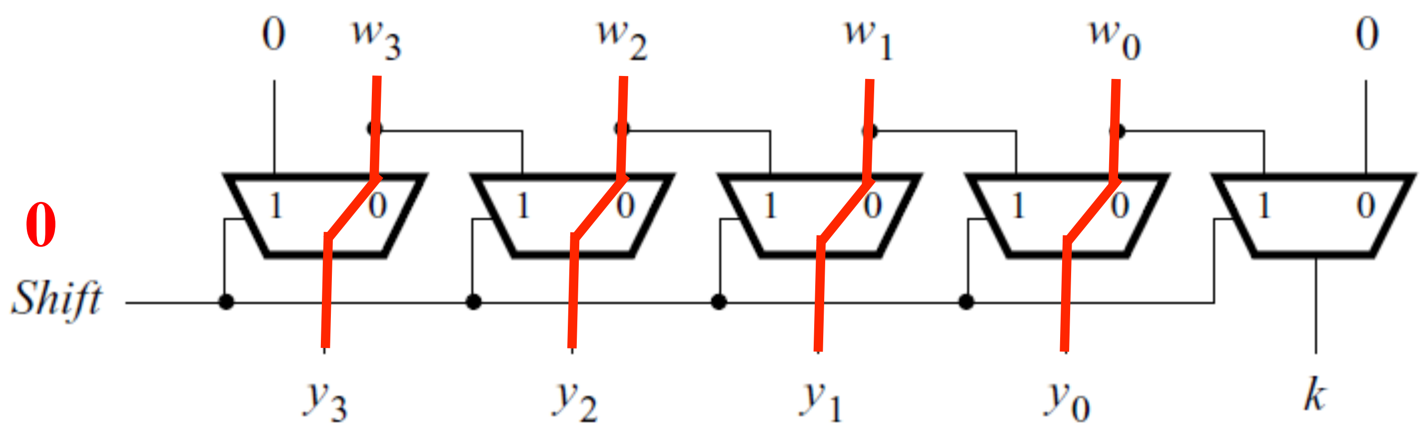



[ Figure 4.50 from the textbook ]

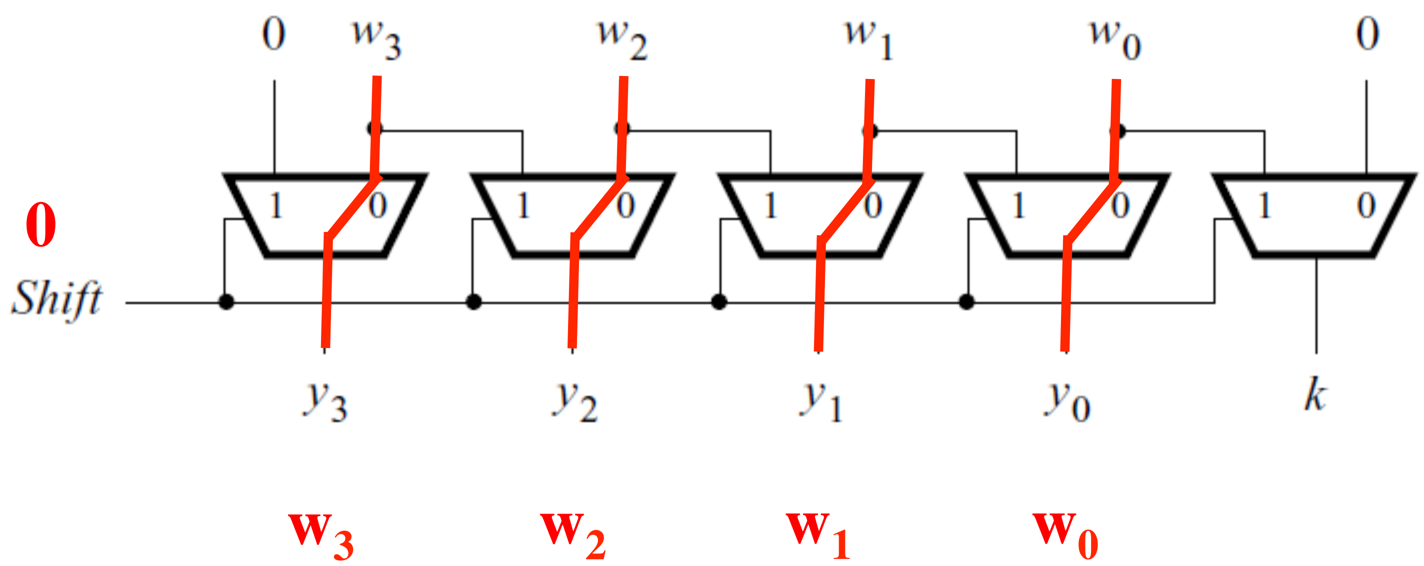
# A shifter circuit



# A shifter circuit

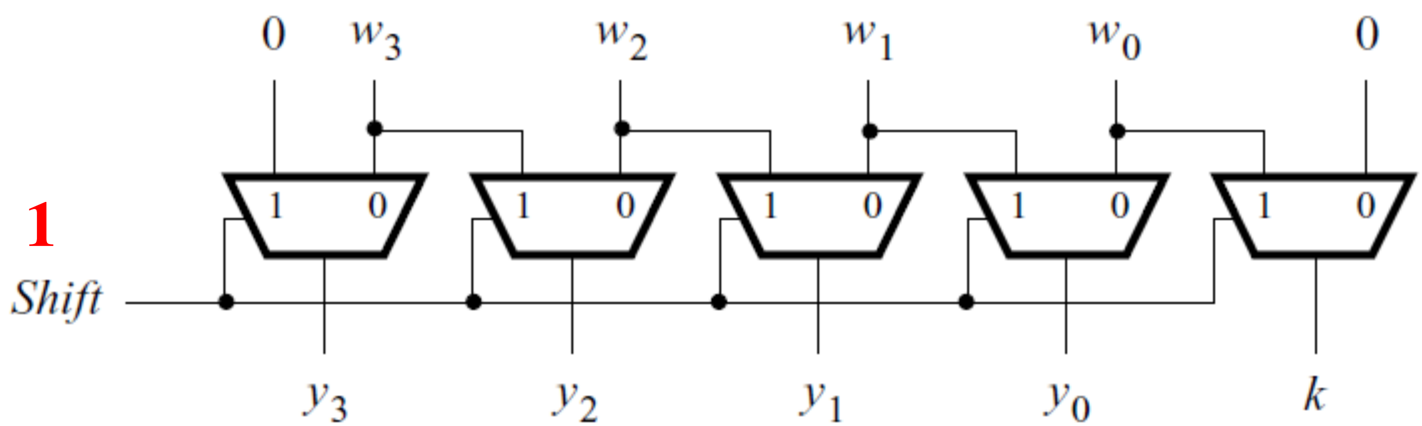


# A shifter circuit

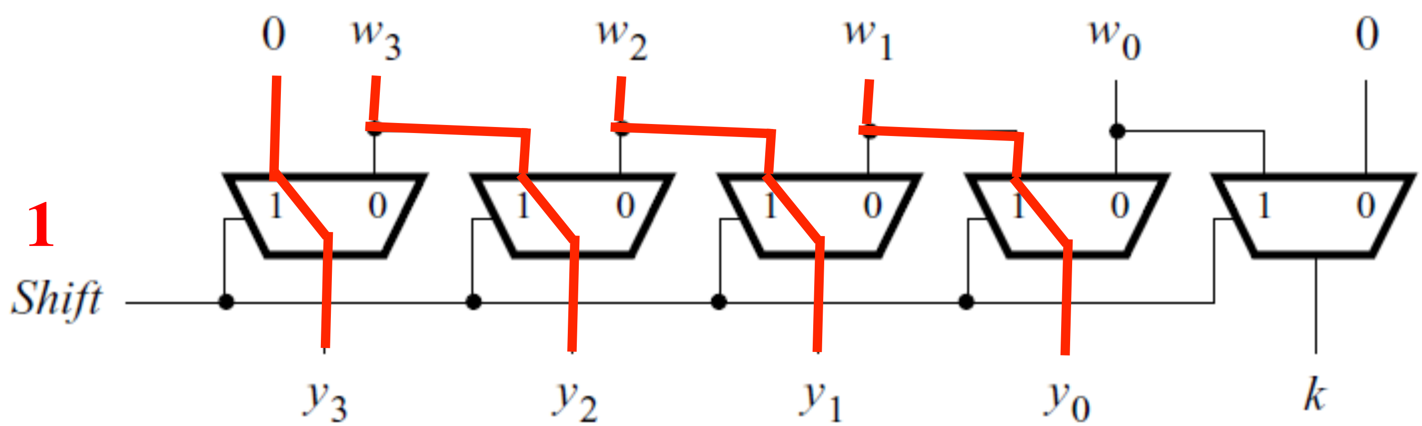



No shift in this case.

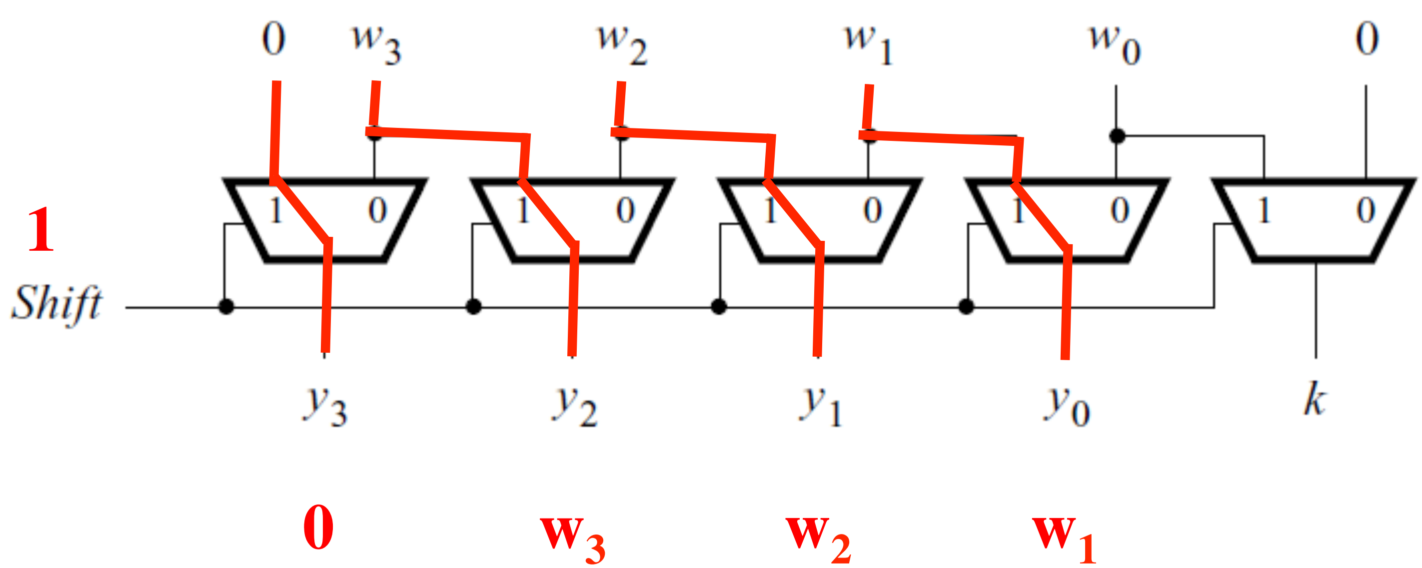
# A shifter circuit



# A shifter circuit

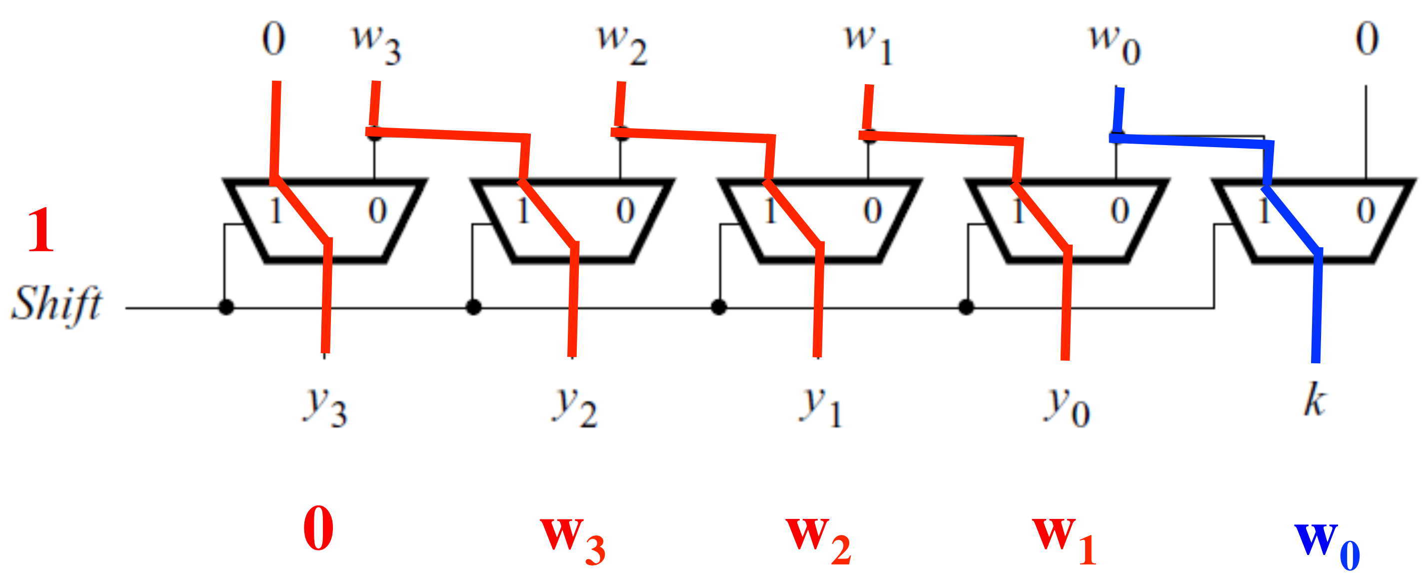


# A shifter circuit




Shift to the right by 1 bit

# A shifter circuit

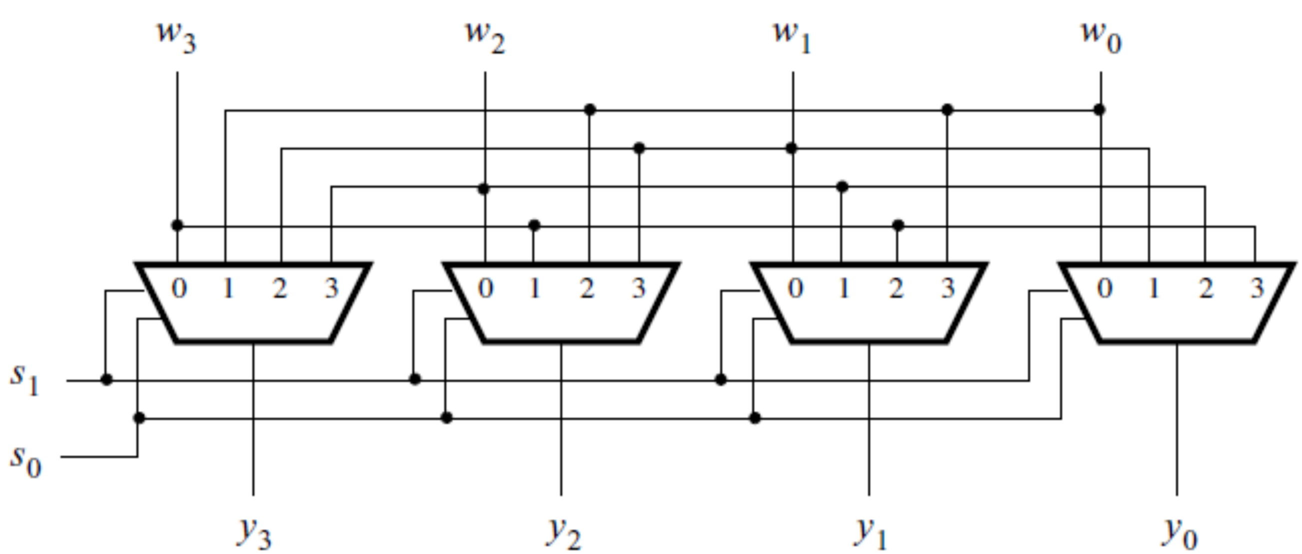



Shift to the right by 1 bit

# A barrel shifter circuit

| $s_1$ | $s_0$ | $y_3$ | $y_2$ | $y_1$ | $y_0$ |
|---|---|---|---|---|---|
| 0 | 0 | $w_3$ | $w_2$ | $w_1$ | $w_0$ |
| 0 | 1 | $w_0$ | $w_3$ | $w_2$ | $w_1$ |
| 1 | 0 | $w_1$ | $w_0$ | $w_3$ | $w_2$ |
| 1 | 1 | $w_2$ | $w_1$ | $w_0$ | $w_3$ |

(a) Truth table




(b) Circuit

[ Figure 4.51 from the textbook ]

# Questions?

# THE END