Cpr E 281 HW02

ELECTRICAL AND COMPUTER ENGINEERING
IOWA STATE UNIVERSITY

Initial Stuff and Basics
Assigned Date: Second Week
Due Date: Sep. 9, 2015

P1. (10 points) Use Venn diagrams to prove the following two equations:

$$(x_1 + x_2 + x_3) \cdot (x_1 + x_2 + \overline{x}_3) = x_1 + x_2$$

$$\overline{x + y} = \overline{x} \cdot \overline{y}$$

P2. (10 points) Use Boolean algebra to prove the first equation in P1.

P3. (10 points) Using a truth table to check the validity of the following:

$$(x_1 + x_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3)(\overline{x}_1 + x_2) = (x_1 + x_2)(x_2 + x_3)(\overline{x}_1 + \overline{x}_3)$$

P4. (10 points) Draw logic circuits for the following three expressions:

$$\overline{x}_1 x_3 + x_1 x_2 \overline{x}_3 + \overline{x}_1 x_2 + x_1 \overline{x}_2$$

$$x_1 \overline{x}_3 + x_2 x_3 + \overline{x}_2 \overline{x}_3$$

$$(x_1 + x_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3)(\overline{x}_1 + x_2)$$

P5. (20 points) Consider the logic function:

f(a,b,c) = a'b'c' + ab'c + a'bc + a'b'c + ab'c',

where a' is an alternative notation for $\overline{a}$, b' for $\overline{b}$, and c' for $\overline{c}$.

(a) (4 points) Draw the logic circuit for the function f given above.

(b) (4 points) Let the cost of a logic circuit be the total number of gates plus the total number of inputs to all gates in the circuit. (See pages 49-50 in the book for examples.) What is the cost of the circuit in (a)?

(c) (4 points) Simplify f using Boolean algebra as much as possible.

(d) (4 points) Draw the logic circuit for the simplified version of f in (c).

(e) (4 points) What is the cost of the circuit in (d)?

P6. (20 points) A function f has four inputs x, y, z, and w and one output such that the output is a 1 if and only if the number of 1s in the inputs is either one or three.

(a) (4 points) Derive the truth table for f.
(b) (4 points) Write the canonical sum-of-products expression for f. Do not use the shorthand notation.
(c) (2 points) Write the canonical sum-of-products expression for f in shorthand notation.
(d) (2 points) Write the canonical sum-of-products expression for $\bar{f}$ in shorthand notation.
(e) (4 points) Write the canonical product-of-sums expression for f. Do not use the shorthand notation.
(f) (2 points) Write the canonical product-of-sums expression for f in shorthand notation.
(g) (2 points) Write the canonical product-of-sums expression for $\bar{f}$ in shorthand notation.

P7. (10 points) Use algebraic manipulation to show that for three input variables x_1, x_2, and x_3 the following is true:

$$\Pi\, \mathrm{M}(0, 1, 2, 3, 4, 5, 6) = x_1 x_2 x_3$$

Show the steps in your derivation.

P8. (10 points) Use algebraic manipulation to find the minimum sum-of-products expression for the following function:

$$f = x_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 x_4 + x_1 \bar{x}_2 x_3 \bar{x}_4$$