

Arithmetic Circuits and  
Combinational-Circuit Building Blocks  
Assigned Date: Eighth Week  
Due Date: Mar. 10, 2014

P1. (10 points) Show that the circuit in Figure 3.4 implements the full-adder specified in Figure 3.3(a).

P2. (15 points) Consider the addition of the two  $n$ -bit 2's complement numbers  $X = x_{n-1} x_{n-2} \dots x_1 x_0$  and  $Y = y_{n-1} y_{n-2} \dots y_1 y_0$ . Suppose the sum is  $s_{n-1} s_{n-2} \dots s_1 s_0$  and the carry is  $c_n c_{n-1} \dots c_2 c_1$ .

(a) If  $X$  is positive,  $Y$  is negative, and  $c_{n-1} = 0$ , what should be the values of  $c_n$  and  $s_{n-1}$ ? Will overflow occur?

(b) If  $X$  is negative,  $Y$  is negative, and  $c_{n-1} = 0$ , what should be the values of  $c_n$  and  $s_{n-1}$ ? Will overflow occur?

(c) Following the idea in part (a) and (b), please construct a truth table list the values of  $c_n$  and  $s_{n-1}$  for all combinations of the sign of  $X$ , the sign of  $Y$ , and the value of  $c_{n-1}$ . For each combination, please also state if overflow occurs or not.

(d) Based on the truth table in part (c), prove that  $\text{Overflow} = c_n \oplus c_{n-1}$ .

P3. (10 points) Design a circuit to add 1 to a given  $n$ -bit number (i.e., design an increment-by-1 circuit) using  $n$  half-adders.

P4. (10 points) Represent the decimal number  $-7.875$  in IEEE 754 single-precision floating-point format.

P5. (10 points) What is the decimal value of the following IEEE 754 single-precision floating-point number?

00111111 01010000 00000000 00000000

P6. (10 points) Design a  $8 \times 1$  multiplexer using seven  $2 \times 1$  multiplexers. Please label all signals clearly.

P7. (10 points) The question considers the design of a  $8 \times 1$  multiplexer using gates. Assume the data inputs are  $I_0, \dots, I_7$  and the select inputs are  $S_2, S_1$  and  $S_0$ .

(a) Write a sum-of-products expression for the  $8 \times 1$  multiplexer.

(b) Implement the expression in part (a) using NOT and NAND gates with any number of inputs. Please use as few gates as possible.

P8. (10 points) Consider a function  $F$  with 4 bits of input  $A_3, A_2, A_1, A_0$  such that the output of  $F$  is 1 if the unsigned binary number represented by  $A_3 A_2 A_1 A_0$  is a prime (i.e., 2, 3, 5, 7, 11 or 13). Otherwise, the output of  $F$  is 0.

(a) Write the truth table for  $F$ .

(b) Implement  $F$  using a  $16 \times 1$  MUX and nothing else.

(c) Implement  $F$  using a  $8 \times 1$  MUX, some AND gates, some OR gates, and some NOT gates.

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P9. (5 points) Write the truth table for a 1-to-2 decoder. Draw a circuit which implements a 1-to-2 decoder using AND gates, OR gates and NOT gates only.

P10. (10 points) Given a supply of 2-to-4 decoders, show how to get a 4-to-16 decoder circuit. Assume each of the 2-to-4 decoders has an ENABLE input (ENABLE = 1 enables the decoder), but you need not include an enable capability on the 4-to-16 decoder circuit.