

For this homework, the cost of a circuit is considered to be the total number of gates plus the total number of inputs.

P1. (10 points) Find the simplest realization of the following function:

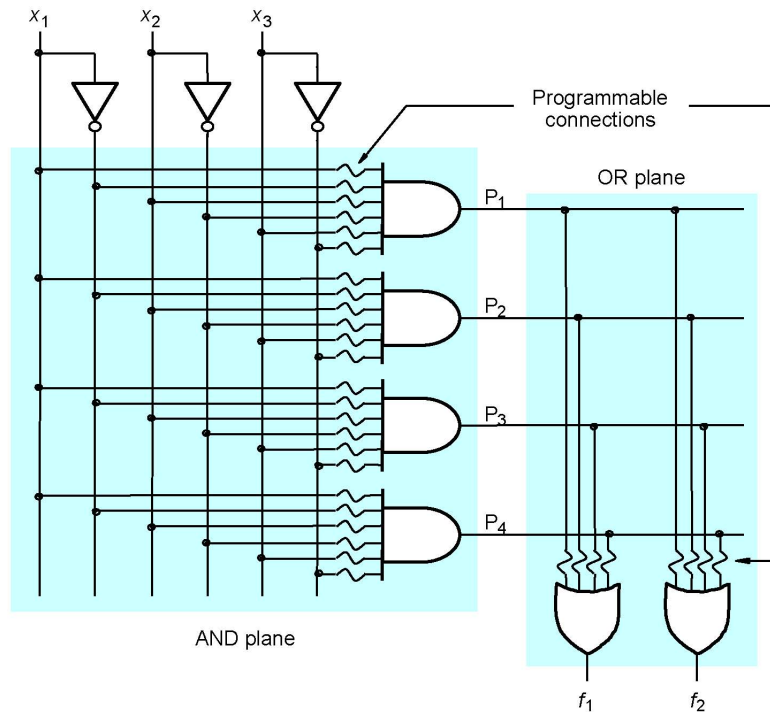
$$f(x_1, \dots, x_4) = \sum m(0, 1, 6, 7, 10, 11, 12, 13)$$

Assume that you can only use NOT gates, 2-input AND gates and 2-input OR gates. Note that it is **not** necessary to restrict your circuit to SOP or POS form. Hint: Look at the section on Functional Decomposition in the book.

P2. (10 points) Implement the following two functions at the same time into the following PLA (with 3 inputs, four AND gates in AND plane, and two OR gates in OR plane).

$$f_1 = x_1' x_2' x_3' + x_1 x_2' x_3 + x_1 x_2 x_3' + x_1 x_2 x_3$$

$$f_2 = x_1' x_2' x_3' + x_1 x_2' x_3 + x_1' x_2 x_3 + x_1 x_2 x_3$$



You may need to simplify the functions in order to incorporate all the product terms into the PLA. To simplify your figure, you may draw the PLA in customary schematic as shown in Figure B.27 in textbook.

P3. (10 points) A circuit with two outputs must implement the two functions:

$$f(x_1, \dots, x_4) = \sum m(1, 4, 5, 6, 8, 9, 11, 14) + D(7, 13, 15)$$

$$g(x_1, \dots, x_4) = \sum m(1, 3, 4, 11, 12, 14, 15) + D(5, 7, 13)$$

Design the minimum-cost circuit and compare its cost with combined costs of two circuits that implement  $f$  and  $g$  separately. Assume that the input variables are available in both uncomplemented and complemented forms.

P4. (10 points) Prove or disprove that  $f = g$ :

$$f(x_1, \dots, x_4) = x_1' x_2' x_3' x_4' + x_1 x_2 x_3' + x_2 x_4 + x_1' x_2' x_3 x_4 + x_1 x_2' x_3 x_4'$$

$$g(x_1, \dots, x_4) = (x_1 + x_2' + x_3 + x_4) \cdot (x_2 + x_3 + x_4') \cdot (x_1 + x_3' + x_4) \cdot (x_2' + x_3' + x_4)$$

$$(x_2' + x_3' + x_4) \cdot (x_1' + x_2 + x_3) \cdot (x_1' + x_2 + x_3' + x_4')$$

Solve this problem by drawing K-maps for both functions and then compare them.

P5. (10 points) A circuit with two outputs is defined by the logic functions

$$f(x_1, \dots, x_4) = x_2' x_4' + x_1' x_2 x_4' + x_1' x_3 x_4' + x_1 x_2 x_3' x_4$$

$$g(x_1, \dots, x_4) = x_1' x_2 x_3' + x_1 x_2 x_4 + x_1 x_2' x_4' + x_1 x_3 x_4'$$

Derive a minimum-cost implementation of this circuit and find its cost.

P6. (10 points) Repeat problem 5 for

$$f(x_1, \dots, x_4) = (x_2 + x_3 + x_4) \cdot (x_1' + x_3 + x_4) \cdot (x_1 + x_3' + x_4') \cdot (x_2' + x_3' + x_4')$$

$$(x_1 + x_2' + x_3' + x_4)$$

$$g(x_1, \dots, x_4) = (x_1' + x_3 + x_4)(x_2' + x_3 + x_4')(x_1' + x_2' + x_4')(x_1 + x_2 + x_3' + x_4')$$

P7. (10 points) Find the minimum-cost circuit consisting only of two-input NAND gates for the function  $f(x_1, \dots, x_4) = \sum m(0, 1, 2, 3, 4, 6, 8, 9, 12)$ . Assume that the input variables are available in uncomplemented and complemented forms. How many NAND gates do you use?

P8. (10 points) Find the minimum-cost circuit consisting only of two-input NOR gates for the function  $f(x_1, \dots, x_4) = \sum m(6, 7, 8, 10, 12, 14, 15)$ . Assume that the input variables are available in uncomplemented and complemented forms. How many NOR gates do you use?

P9. (10 points) Problem 2.77 in the textbook.

P10. (10 points) Jointly minimize the functions for segments F and G to display digits from 0 to 9 in a 7-segment display. Use W, X, Y, and Z as your variables.