ComS 401: Projects in Computing
Lab Assignment #1
Out: Wed. Aug 29, 2007
Due: Fri. Sep 7, 2007
For extra credit submit by Fri. Aug 31 before 4pm.
(submit it to Karen in VRAC front office, Howe Hall, room 1620.)

Note: Not all assignments are created equal :(

These problems are designed to test your background knowledge in linear algebra and statistics. Please show the intermediary steps of your calculations (not just the final answers).

1. **Calculate the following determinants.**

$$(a)\ \begin{vmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{vmatrix} \quad (b)\ \begin{vmatrix} a-3 & 5 \\ -3 & a-2 \end{vmatrix} \quad (c)\ \begin{vmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{vmatrix} \quad (d)\ \begin{vmatrix} c & -4 & 3 \\ 2 & 1 & c^2 \\ 4 & c-1 & 2 \end{vmatrix}$$

2. **Show that the value of the determinant does not depend on θ.**

$$\begin{vmatrix} sin\theta & cos\theta & 0 \\ -cos\theta & sin\theta & 0 \\ sin\theta - cos\theta & sin\theta + cos\theta & 1 \end{vmatrix}$$

3. **Calculate the inverse of the given matrix.**

$$(a)\ \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix} \quad (b)\ \begin{bmatrix} -3 & 6 \\ 4 & 5 \end{bmatrix} \quad (c)\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad (d)\ \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix}$$

4. **Solve the following systems of equations.**

$$(a)\ \begin{array}{rcrcr} x_1 & + & x_2 & = & 2 \\ 5x_1 & + & 6x_2 & = & 9 \end{array} \qquad (b)\ \begin{array}{rcrcrcr} x_1 & + & 3x_2 & + & x_3 & = & 4 \\ 2x_1 & + & 2x_2 & + & x_3 & = & -1 \\ 2x_1 & + & 3x_2 & + & x_3 & = & 3 \end{array}$$

5. **Let** $u = (3, 2, -1)$**,** $v = (0, 2, -3)$**, and** $w = (2, 6, 7)$**. Compute:**

$$(a)\ u \cdot v \qquad (b) \|u\| \qquad (b)\ v \times w \qquad (c)\ u \times (v \times w) \qquad (d)\ (u \times v) \times w$$

6. **Find the orthogonal projection of u on a (i.e., find $proj_a u = \frac{u \cdot a}{\|a\|^2} a$)**

$$(a)\ u = (6,2), a = (3,-9)$$
$$(b)\ u = (3,1,-7), a = (1,0,5)$$

7. **Find the angle θ between the two vectors u and v.**

$$(a)\ u = (2,3), v = (5,-7)$$
$$(b)\ u = (1,0,0), v = (1,1,1)$$

8. **Find an equation for the plane passing through the given points.**

$$(a)\ \ P(-4,-1,-1), Q(-2,0,1), R(-1,-2,-3) \qquad (b)\ \ P(5,4,3), Q(4,3,1), R(1,5,4)$$

9. **Find all values of λ for which the determinant of the matrix is equal to 0.**

$$(a)\ \begin{vmatrix} \lambda - 2 & 1 \\ -5 & \lambda + 4 \end{vmatrix} \quad (b)\ \begin{vmatrix} \lambda - 4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda - 1 \end{vmatrix}$$

10. **Find the eigenvalues of the following matrices.**

$$(a)\ \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \quad (b)\ \begin{bmatrix} 3 & 0 & -5 \\ \frac{1}{5} & -1 & 0 \\ 1 & 1 & -2 \end{bmatrix}$$

11. **Given the vector $v = [x, y]^T$ find a rotation matrix R which rotates the vector by 60° counter-clockwise. Give the values for the vector $v' = [x', y']^T$ in terms of x and y, where $v' = Rv$.**

12. **Write down the analytical form of the 3D rotation matrices R_x, R_y, and R_z which rotate a vector about the X, Y, and Z axis, respectively.**

13. **By example, show that the 3D rotation matrices are not commutative.**