EE 501 Lab 4 Design of two stage op amp with miller compensation

**Objectives:**

1. Design a two stage op amp
2. Investigate how to miller compensate a two-stage operational amplifier.

**Tasks:**

1. Build a two-stage operational amplifier as given in Fig. 1 to meet the following requirement without compensation <load cap =2 pF>
   a. DC Gain >= 55 dB
   b. GB > = 50 MHz
2. Simulate the frequency response of this opamp without compensation. Record the value of DC gain, f_{3db}, GBW, Unity Gain Frequency, Phase Margin.
3. Apply Miller compensation by connecting a capacitor C_c from output to the input of second transconductance stage. Parametrically sweep the capacitance from 0 to 2C_c to achieve the best UGF while achieving >= 50 ° PM. Then repeat task 2. In addition, report the compensation capacitance when you get 50 PM or above. (To achieve the best UGF, you may tweak the first stage current and/or input pair size. When you change the tail current, you may change the W/L of the tail transistor and the current mirror load transistors by the same percentage.)
4. Apply lead compensation by inserting a resistor R_c in series with C_c. Start with ½ the C_c from step3, sweep R_c parametrically to maximize UGF with >= 50 ° PM. Try different values of C_c and repeat. After both C_c and R_c are optimized, repeat task 2. (Same comments as in last paragraph apply here.)
5. Replace the resistor R_c with a triode transistor with the same type as the second stage input transistor. Maximized UGF by sizing and biasing this transistor. A scheme for generating this bias voltage is provided as in figure 4.3. Then repeat task 2.
6. Comment on the comparisons you made. Draw some conclusions regarding the maximum UGF that can be achieved with various compensation methods.

Fig 1 Two Stage amplifier

Fig 4.2 Two Stage amplifier with Miller compensation
Fig 4.3 Resistor replacement
Derivation Hints:

The symbol for an operational amplifier is shown in Figure 4-4. The basic device has two inputs and a single output. A fully-differential version of the opamp is often used in high performance integrated circuit designs.

![Operational amplifier symbol](image)

**Figure 4.4: Operational amplifier symbol**

The operational amplifier functions as a voltage amplifier. The relationship between the input and output voltage is given by:

$$V_o = A_{v0} \cdot (V_{i+} - V_{i-})$$

The amplifier has a high voltage gain ($A_{v0} > 60\text{dB}$ for CMOS opamps). Due to the high gain, the linear region is very narrow. Figure 4-5 illustrates the typical input-output characteristic for an operational amplifier used with and without feedback. The open-loop plot shows the linear region is only a few millivolts wide.

![Input-output characteristic for an opamp](image)

**Fig4.5: Input–output characteristic for an opamp**

Due to the high gain and limited linear region, the opamp is commonly used in a negative feedback loop. Figure 4.5 shows also the input-output characteristic when the amplifier is used with feedback. Notice the closed-loop linear region consists of almost the entire input voltage range. The application of feedback reduces the non-linearity, but also reduces the voltage gain.

The two-stage amplifier shown in Figure 4.2 is referred to as a Miller compensation opamp. The two stages create two significant poles which affect the stability of the system. Usually some form of compensation is required to assure the amplifier is unity-gain stable. Additional gain stages can be employed to increase the gain, but this increases the stability problems and requires complex compensation techniques.

The frequency response of an operational amplifier will be analyzed with help from a small-signal model of the structure. Figure 4.6 shows a small-signal model of the opamp.
The capacitor $C_{in}$ models the input capacitance of the opamp, which is due mostly from $V_{gs}$. The subcircuit consisting of $G_{mA}$, $R_A$, and $C_A$ model the gain and frequency response of the input stage. The capacitance $C_A$ includes the input capacitance of the second stage and the output capacitance of the first stage. The components $G_{mB}$, $R_B$, and $C_B$ model the second stage. The load capacitor and resistor are also included in $R_B$ and $C_B$.

The transfer function of the small-signal model is given by:

$$H(s) = \frac{AV_0'p1\cdot p2}{(s+p1)(s+p2)} = \frac{\frac{G_{mA}R_{mB}}{C_A}}{\frac{1}{R_A C_A} + \frac{1}{R_B C_B}}$$

This transfer function assumes the source resistance is zero ohms. Notice the two poles are approximately equal. The capacitors $C_A$ and $C_B$ are dominated by $V_{gs}$, and $R_A$ and $R_B$ are the parallel connected small-signal $r_{ds}$ resistance. The pole-zero plot of this transfer function is illustrated in Figure 4.7.

Due to the two poles being located close together, the system is unlikely to be unity-gain stable due to the large DC gain of an operational amplifier. Clearly, some form of compensation is required. The modified small-signal shown in 4.8 uses capacitor $C_C$ to compensate the frequency response of the opamp by splitting the distance between the two poles.
Figure 4.8 Operational amplifier small-signal model with compensation capacitor $C_c$

The transfer function for the operational amplifier with the compensation capacitor is:

$$H(s) = \frac{V_{o}p_{2}}{Z(s+p1)(s+p2)} = \frac{g_{mA}g_{mB}R_{A}R_{B}}{(s+1)g_{mB}R_{A}R_{B}C_{C}}$$

These simplifying assumptions hold because capacitance $C_B$ will include the capacitance of the load, and the compensation capacitance $C_C$ can be chosen to be the size of the load capacitor. Also, for the two-stage Opamp, capacitance $C_B$ will include the load capacitance $C_L$.

With the transfer function in factored form, we can find the open-loop DC gain, poles, and zero of the compensated opamp are given by:

$$A_{V0} = g_{mA}g_{mB}R_{A}R_{B}$$

$$p_1 = \frac{1}{g_{mB}R_{A}R_{B}C_{C}} = \frac{-1}{|A_{V2}|R_{A}C_{C}}$$

$$p_2 = \frac{-g_{mB}}{C_{A}+C_{B}}$$

$$z = \frac{g_{mB}}{C_{C}}$$

$$GBW = \frac{g_{mA}}{C_{C}}$$

Notice the addition of the compensation capacitor $C_C$ caused the poles to separate. One pole moved closer to the origin by a factor of $A_{V2} = g_{mB} \cdot R_B$, while the other pole moved away from the origin by a factor of $A_{V2}$. This compensation technique is called “pole splitting”. The pole-zero plot of this transfer function is illustrated in Figure 4.9.

Also, notice the creation of a zero as a result of the transition path created by the capacitor.

![Figure 4.9: Pole-zero plot for a compensated opamp](image)

Using the compensated opamp in a feedback loop produces the following transfer function:

$$A_c(s) = \frac{H(s)}{1+H(s) \cdot \beta} = \frac{A_0(s-z)}{(s+p1)(s+p2)+A_0 \beta(s-z)}$$

$$= \frac{A_0(s-z)}{s^2+\left(\frac{1}{g_{mB}R_{A}R_{B}C_{C}}+\frac{g_{mB}g_{mB}R_{A}R_{B}C_{C}}{C_{B}}\right)s+\left(\frac{1}{R_{A}C_{B}R_{B}C_{C}}+\frac{g_{mA}g_{mB}R_{A}R_{B}C_{C}}{C_{B}C_{C}}\right)}$$

The closed-loop transfer function using the compensated amplifier can be approximated by:

$$A_c(s) = \frac{\frac{g_{mA}g_{mB}}{C_{B}}}{s^2+\left(\frac{g_{mB}g_{mB}R_{A}R_{B}C_{C}}{C_{B}}\right)s+\left(\frac{g_{mA}g_{mB}R_{A}R_{B}C_{C}}{C_{B}C_{C}}\right)}$$
The effect of the above simplification on the system is to assume the dominant pole is at the origin. Notice that when the system is in open-loop ($\beta = 0$), the transfer function reduces to

$$A_c(s) = \frac{\frac{g_m A(s - \frac{g_m B}{C_c})}{R_b}}{s(s + \frac{g_m B}{C_b})}$$

To assure the feedback system is unity-gain stable ($\beta = 1$), the phase margin must be examined. The phase margin is the amount of phase before phase inversion (180°) at the unity gain frequency. The expression for the phase margin is given by:

$$PM = 180^\circ - \tan^{-1}\left(\frac{\omega_{\text{dB}}}{p_1}\right) - \tan^{-1}\left(\frac{\omega_{\text{dB}}}{p_2}\right) - \tan^{-1}\left(\frac{\omega_{\text{dB}}}{z}\right)$$

$$= 180^\circ - \tan^{-1}\left(\frac{\text{GBW}}{p_1}\right) - \tan^{-1}\left(\frac{\text{GBW}}{p_2}\right) - \tan^{-1}\left(\frac{\text{GBW}}{z}\right)$$

$$= 180^\circ - \tan^{-1}\left(\frac{\text{GBW}}{p_2}\right) - \tan^{-1}\left(\frac{\text{GBW}}{z}\right)$$

$$\approx 90^\circ - \tan^{-1}\left(\frac{\text{GBW}}{p_2}\right) - \tan^{-1}\left(\frac{\text{GBW}}{z}\right)$$

The phase margin is improved by moving the non-dominant pole and zero to higher frequencies away from the unity-gain frequency. The phase margin can also be improved by using compensation techniques which place the zero in the left-half plane.

The slew rate is determined by the compensation capacitance and the tail current:

$$\text{SR} = \frac{I_{\text{tail}}}{C_c}$$

The performance characteristics of the two-stage amplifier are summarized below:

$$A_{V0} = g_{m1}(r_{ds2}||r_{ds4})g_{m6}(r_{ds6}||r_{ds7})$$

$$p_1 = \frac{g_{m6}(r_{ds6}||r_{ds7})C_c(r_{ds2}||r_{ds4})}{C_L}$$

$$p_2 = \frac{g_{m6}}{C_c}$$

$$z = \frac{g_{m6}}{C_c}$$

$$\text{GBW} = \frac{g_{m1}}{C_c}$$

$$\text{SR} = \frac{I_{\text{tail}}}{C_c}$$

$$PM = 90^\circ - \tan^{-1}\left(\frac{\text{GBW}}{p_2}\right) - \tan^{-1}\left(\frac{\text{GBW}}{z}\right)$$