CprE 528, Spring 2012 Probabilistic Methods in Computer Engineering

Homework 2

Due: Thu, 9 Feb

Problem 2.1

Coupon Collector Variant. Exercise 2.13, page 40.

Problem 2.2

Random Permutation. Exercise 2.21, Page 41.

Problem 2.3

Hiring Problem: Exercise 2.32, page 43.

Problem 2.4

Reservoir Sampling: Exercise 2.18, page 40, and 2.19, page 41.

Problem 2.5

a. The covariance of two random variables X and Y, denoted by $cov(X, Y)$, is defined as $E[XY] - E[X]E[Y]$. Prove the following:

$$Var[X + Y] = Var[X] + Var[Y] + 2cov(X, Y)$$

b. Using the above, show that if X and Y are independent, then

$$Var[X + Y] = Var[X] + Var[Y]$$

c. Using induction, prove that for any set of n random variables $X_i, i = 1 \dots n$, if the X_is are mutually independent, then

$$Var\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} Var[X_i]$$