Particle Filtered Modified-CS (PaFiMoCS) for tracking signal sequences

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Our Goal: recursive causal sparse reconstruction

- Causally & recursively recons. a time seq. of sparse signals
- with slowly changing sparsity patterns
- from as few linear measurements at each time as possible
  - "recursive": use current measurements & previous reconstruction to get current reconstruction

- Potential applications
  - real-time dynamic MRI, e.g. for interventional radiology apps
  - single-pixel video imaging with a real-time video display, ...
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- Most existing work:
  - is either for static sparse reconstruction or is offline & batch, e.g. [Wakin et al (video)], [Gamper et al,Jan'08 (MRI)], [Jung et al'09 (MRI)]

Notation

- support of x: the set $\{i \in [1, 2, \ldots m] : |(x)_i| > 0\}$
- $|T|$: cardinality of set T
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- A': denotes the transpose of matrix A
- A_T: sub-matrix containing columns of A with indices in set T
- β_T: sub-vector containing elements of β with indices in set T
- $\|\beta\|_k$: ℓ_k norm

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  - unique solution if $\delta_{2|N|} < 1$
  - exponential complexity
- Practical approaches (polynomial complexity in m)
  - greedy methods, e.g. MP, OMP,..., Subspace Pursuit, CoSaMP [Mallat,Zhang'93], [Pati et al'93],...[Dai,Milenkovic'08],[Needell,Tropp'08]
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  - convex relaxation approaches, e.g. BP, BPDN,..., DS, [Chen,Donoho'95], ..., [Candes,Tao'06],...
- Compressed Sensing (CS) literature [Candes,Romberg,Tao'05], [Donoho'05]
  - provides exact reconstruction conditions and error bounds for the practical approaches

Sparse recon. w/ partly known support [Vaswani,Lu, ISIT'09, IEEE Trans. SP'10]

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  - $\Delta := N \setminus T$: misses in T (unknown)
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  - exact recon. conditions for modCS much weaker than for CS
    - **when** $|\Delta| \ll |N|$ **and** $|\Delta_e| \ll |N|$

Problem definition

- Measure $y_t = \Phi x_t + w_t$
  - Φ: measurement matrix times sparsity basis matrix
    - e.g. for MR imaging of wavelet sparse images, $\Phi = F_p W$, F_p is a partial Fourier matrix and W is the inverse DWT matrix
  - y_t: measurements' vector $(n \times 1)$
  - x_t: sparsity basis coefficients' vector $(m \times 1)$, $m > n$
  - N_t: support of x_t (set of indices of nonzero elements of x_t)

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  - i.e. use only $\hat{x}_{t-1}$ and y_t for reconstructing x_t
- Assumptions:
  - *support set of x_t, N_t, changes slowly over time:*
    - empirically verified for dynamic MRI sequences [Lu,Vaswani,ICIP'09]
  - *nonzero values of x_t, i.e. $(x_t)_{N_t}$, also change slowly*
    - commonly used tracking assumption

Related work

- Kalman filtered CS-residual (KF-CS) and Least Squares CS-residual (LS-CS) [Vaswani,ICIP'08,Trans.SP,Aug'10]
- Modified-CS and Regularized Mod-CS [Vaswani,Lu,ISIT'09,Trans.SP,Sept'10, Lu,Vaswani,ICASSP'10,Asilomar'10]
- Sequential Compressed Sensing in Sparse Dynamical Systems, [Sejdjnovic et al,Allerton 2010]: uses particle filters

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- RLS Lasso, Dynamic Lasso [Angelosante et al,ICASSP09, DSP09]: assume support set does not change with time; causal but batch methods

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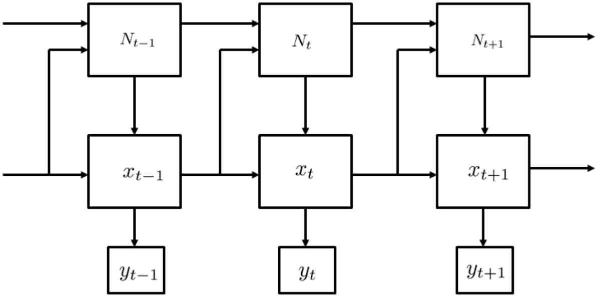
- Static approaches
  - weighted ℓ_1 [Khajenejad et al, ISIT'09]
  - approach similar to modified-cs [vonBorries et al,CAMSAP'07]
- Static Bayesian approaches [Baron et al, Trans.SP'09], [Ji et al,Trans.SP'08], [Schniter et al,ITA'08], [Babacan et al,Trans.IP'10], [Blackhall et al,IFAC'08]

Key Question

- Given a Markov model on slow support change, and on slow nonzero signal value change, what is the "best" way to use it?
  - what does "best" even mean?



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- Find the causal MMSE or MAP estimate
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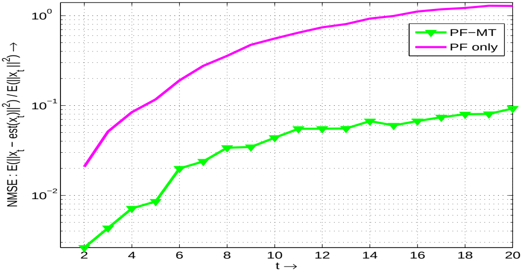
- Find the causal MMSE or MAP estimate
  - approximate using a K-particle particle filter (PF)
  - limitation: if K not large enough, none of the particles at t may contain the correct new support $\Rightarrow$ support error at $t+1$ larger $\Rightarrow$ error propagation over time
    - not explicitly using slow sparsity change

PF, PF-MT error is unstable



PF-MT: PF designed for large dimensional multimodal problems

- importance sample on dominant part of state space (here N_t)
- replace importance sampling by posterior mode tracking (MT) for the rest of the states (here $(x_t)_{N_t}$) [Vaswani,Trans.SP'08]
  - here we used reg-mod-CS in the MT step with $T = N_t{}^i$

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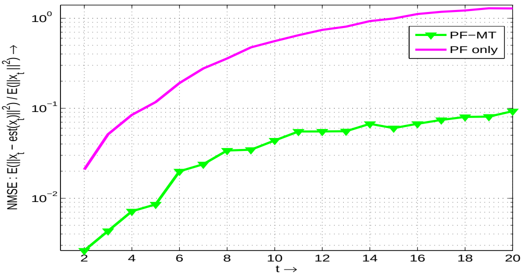

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PF-MT uses slow sparsity change to estimate x_t but not for N_t

Possible solution strategies – 2

- Find the sparsest change of support
  - modified-CS: $\min_x \gamma \|x_{T^c}\|_1 + \|y_t - \Phi x\|_2^2$ with $T := \hat{N}_{t-1}$
    - no constraint on x_T: it can become too large in noise

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  - weighted ℓ_1: $\min_x \gamma \|x_{T^c}\|_1 + \gamma' \|x_T\|_1 + \|y_t - \Phi x\|_2^2$, $\gamma' < \gamma$
    - improves upon mod-cs only for carefully chosen γ'/γ
  - both do not utilize slow signal value change

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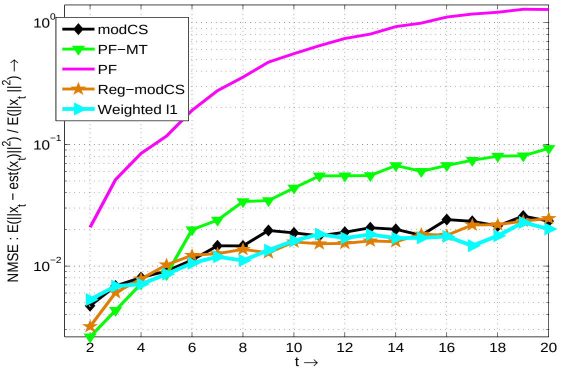
  - reg mod-CS: $\min \gamma \|x_{T^c}\|_1 + \|(x - \hat{x}_{t-1})_T\|_2^2 + \|y_t - \Phi x\|_2^2$
    - does not utilize the "model" on support change

Mod-CS, Reg-Mod-CS, Weighted ℓ_1 are better



but still unstable (when using only 25% measurements for a 10%-sparse signal sequence with min-SNR 26, & support change 1%)

PFed Modified CS (PaFiMoCS): key idea

- PF and PF-MT: missed support changes accumulate over time
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  - weight appropriately and resample

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- Support change model:

$$\begin{aligned} A_t &\sim \mathrm{Unif}_p({N_{t-1}}^c) \\ R_t &\sim \mathrm{Unif}_p(N_{t-1,\mathrm{small}}) \\ N_t &= (N_{t-1} \cup A_t) \setminus R_t \end{aligned}$$

where $N_{t-1,\mathrm{small}} := \{j \in N_{t-1} : |(x_{t-1})_j| < b\}$

- Signal amplitude change model:

$$\begin{aligned} (x_t)_{N_t} &= (x_{t-1})_{N_t} + \boldsymbol{\nu}, \quad \boldsymbol{\nu} \sim \mathcal{N}(\mathbf{0}, \Sigma_\nu) \\ (x_t)_{N_t^c} &= \mathbf{0} \end{aligned}$$

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Observation Model:

$$y_t = \Phi x_t + w_t, \;\; w_t \sim \mathcal{N}(0, \Sigma_o)$$

Basic PF [Gordon, Salmon, Smith'93]

At each time $t > 0$, for all particles $i = 1, 2, ...$

1. Importance sample on support change

$$\begin{aligned} A_t^i &\sim \text{Unif}_p(\ ({N_{t-1}}^i)^c\ ) \\ R_t^i &\sim \text{Unif}_p(\ {N_{t-1}}^i\ ) \\ N_t^i &= N_{t-1}^i \cup A_t^i \setminus R_t^i \end{aligned}$$

2. Importance sample on non-zero signal values' change

$$\begin{aligned} (x_t^i)_{N_t^i} &\sim \mathcal{N}((x_{t-1}^i)_{N_t^i}, \Sigma_\nu) \\ (x_t^i)_{(N_t^i)^c} &= \mathbf{0} \end{aligned}$$

3. Weight appropriately and resample

$$w_t^i \propto p(y_t | x_t^i)$$

4. MAP estimate : output maximum weight particle

PFed posterior Mode Tracker (PF-MT) [Vaswani, Trans.SP'08]

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2. Mode Track on non-zero signal values' change

$$x_t^i = \arg\min_x \underbrace{\gamma \|x_{T^c}\|_1 + \lambda \|x - x_{t-1}^i\|_2^2 + \|y_t - \Phi x\|_2^2}_{\text{reg-mod-cs}(x)} \text{ with } T = N_t^i$$

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3. Weight appropriately and resample

$$w_t^i \propto \frac{p(y_t|x_t^i) p(x_t^i|N_t^i, x_{t-1}^i)}{e^{-\text{reg-mod-cs}(x_t^i)}}$$

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Particle filtered Modified-CS (PaFiMoCS)

PF-MT with support estimation after reg-mod-cs

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3. **Update the support particle N_t^i by thresholding x_t^i**

$$N_t^i = \{j : |(x_t^{(i)})_j| > \alpha\}$$

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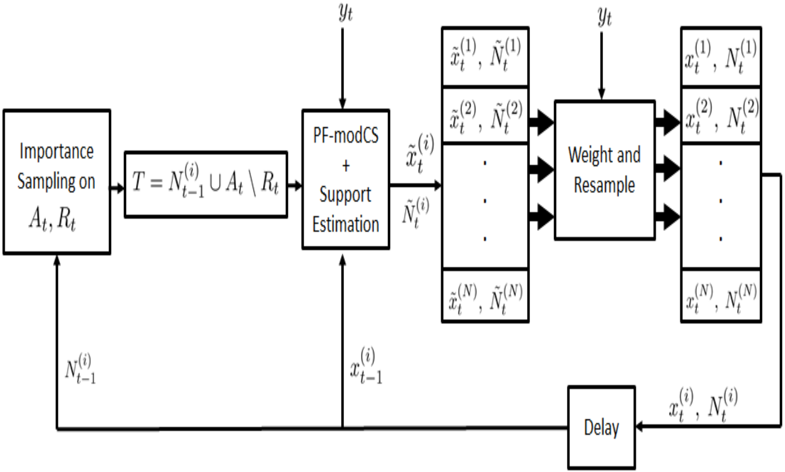
PaFiMoCS algorithm

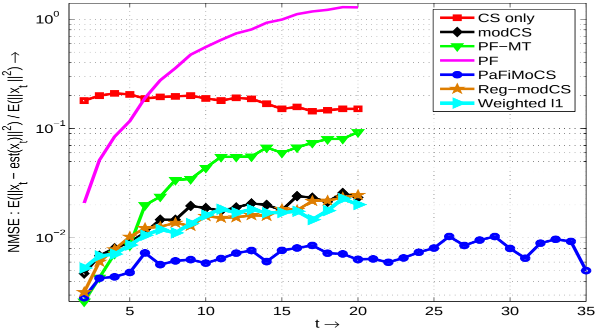


Simulation Experiments

- Generated a sparse signal sequence with $m = 200$, $|N_t| = S_0 = 20$, $p = 2$
  - Initialization: $N_0 = \{10, 15, 20, \ldots, 105\}$, $(x_0)_i \sim \mathcal{N}(5, 3)$ for all $i \in N_0$
  - Signal value change covariance matrix $\Sigma_\nu = \mathbf{I}_{20}$
- Number of observations, $n = 50$, used a random Gaussian matrix Φ
- Observation noise covariance matrix, $\Sigma_o = \mathbf{I}_n$
- Algorithm parameters
  - We used 100 particle for PaFiMoCS and other PF methods
  - $\lambda = 0.5$, $\gamma = 0.1$

Performance Comparison : Normalized MSE

- Compared among : PF, PF-MT, CS, modCS, reg-modCS, weighted l-1, PaFiMoCS (our algorithm)
- $NMSE = E(\|x_t - \hat{x}_t\|^2)/E(\|x_t\|^2)$ (over 50 MC runs)



Discussion

- PaFiMoCS is a PF-MT like algorithm but it is not a PF-MT for the assumed state space model
  - a heuristic that attempts to improve upon reg-mod-cs using importance sampling

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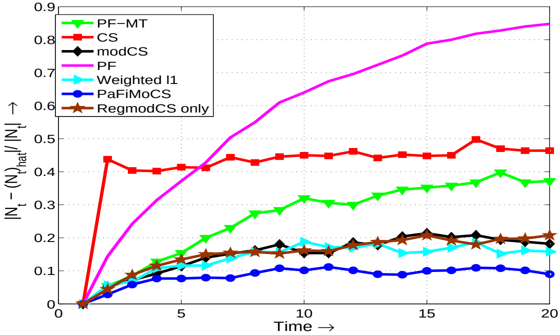
- PaFiMoCS is a PF-MT like algorithm but it is not a PF-MT for the assumed state space model
  - a heuristic that attempts to improve upon reg-mod-cs using importance sampling
- Open question: can we interpret it as a PF-MT under some prior model?
  - if we assume $N_t = N_{t-1} \cup A_t \cup N_{noise,a} \setminus (R_t \cup N_{noise,r})$
  - and let $\mathcal{X}_{\text{effective}} = \{A_t, R_t\}$ and $\mathcal{X}_{\text{rest}} = \{N_t, x_t\}$
  - then MT step needs to estimate both x_t and N_t
  - is this a PF-MT?

Performance Comparison : Support Error

- Comparison for $\frac{|N_t \setminus \hat{N}_t|}{|N_t|}$



Performance Comparison : Support Error

- Comparison for $\frac{|\hat{N}_t \setminus N_t|}{|N_t|}$