1 MMSE estimation

1. Define Bayesian MSE

2. Show that $\mathbb{E}[X|Y]$ is the MMSE estimator of $X$ from $Y$

3. Also show that variance of the error of $\mathbb{E}[X|Y]$ is smallest in any direction (i.e. it maximizes the error covariance matrix).

4. Read Chapter IV-B of Poor’s book.

5. Applications: most important one is Kalman filtering to recursively obtain a causal MMSE estimate (MMSE estimate $X_t$ from $Y_{0:t}$): see Kalman Filter details handout and Chapter V of Poor’s book.

2 Linear MMSE estimation

1. Define linear MMSE estimator

2. Orthogonality principle 1: $\hat{X}$ is the linear MMSE of $X$ from observations $Y_{0:n-1}$ if and only if

$$\mathbb{E}[(X - \hat{X})^T Z] = 0, \ \forall \ Z = \sum_{k=0}^{n-1} H_k Y_k + b$$

for any choice of $H_k$’s and $b$. $H_k$’s are matrices of appropriate size, $b$ is a vector.

3. Orthogonality principle 2: $\hat{X}$ is the linear MMSE of $X$ from observations $Y_{0:n-1}$ if and only if

$$\mathbb{E}[(X - \hat{X})] = 0 \ \text{and} \ \mathbb{E}[(X - \hat{X}) Y_l^T] = 0, \ \forall \ l = 0, 1, \ldots n - 1,$$

4. Proofs of both of them.

5. Wiener-Hopf equations

6. See Chapter V of Poor’s book for the proof for scalar case. Vector case proof is only slightly different.

7. Applications: most important one is Kalman filtering to recursively obtain a causal L-MMSE estimate (linear MMSE estimate of $X_t$ from $Y_{0:t}$): see Kalman Filter details handout and Chapter V of Poor’s book.

8. Applications: Wiener filtering (non-causal and causal), Levinson recursion, many more. See Chapter V of Poor’s book.