Homework 7, EE 527, Spring 2008

Good if you do by April 3. Due April 7 (for both EDE and on-campus students)

The questions are based on what I taught in class. The material was taken from Vincent Poor's book Chapter IV-B and Chapter V. Some answers may be available in the book itself.
I will grade any two of the first 5 problems.

1. Prove the conditional variance inequality, i.e. show that

$$Cov[X] = E[Cov[X|Y]] + Cov[E[X|Y]] \tag{1}$$

2. Prove that the minimum mean absolute error (MMAE) estimate is the conditional median (median of the posterior $p_{X|Y}(x|y)$).

3. Suppose that N_1 and N_2 are two jointly Gaussian random variables with zero means, unit variances and correlation coefficient ρ. Suppose further that we observe $Y_k = N_k/\sqrt{\Theta}, \quad k = 1, 2$ where Θ is a random parameter with a uniform prior, $w(\theta) = 1/\alpha$ for $\theta \in [0, \alpha]$ and $w(\theta) = 0$ otherwise. $\alpha > 0$ is known.
   Find the MMSE, MMAE, MAP estimates of θ.

4. Write out the derivation for the Kalman filter measurement update step. Convince yourself that the prediction step derivation makes sense.

5. Consider the model

$$\begin{aligned} Y_t &= \Theta s_t + \nu_t, \quad \nu_t \sim \mathcal{N}(0, \sigma^2) \\ \Theta &\sim \mathcal{N}(\mu_0, \sigma_0^2) \end{aligned} \tag{2}$$

   where $\{\nu_t\}$ are i.i.d.. Let θ_t denote the MMSE estimate of Θ given $Y_1, \ldots Y_t$. Find the recursion for θ_t by recasting this as a Kalman filtering problem.

6. *Extra credit: submit them anytime. I do not help in these questions.*

   (a) Derive the recursion for a Kalman smoother, i.e. derive a recursion for $\hat{X}_{j|t} \triangleq E[X_j|Y_{0:t}]$ in terms of $\hat{X}_{j|t-1}$ where $j < t$.

   (b) In the usual Kalman model, if the measurement noise at t and the system noise at t are correlated, i.e. $E[U_t \nu_t] = C_t$, and everything else is the same, how does the Kalman recursion for $\hat{X}_{t|t}$ change.