

## Make-up Midterm, Fall 06 (Out of 30)

5 questions, 6 points for each question

1.  $X$  is a geometric random variable with parameter  $p$ . Let  $Y = \min(a, X)$  and let  $Z = a - Y$ .

**2+2+2 points**

- (a) Compute PMF of  $Y$
- (b) Compute PMF of  $Z$ .
- (c) Compute  $E[Z]$  either directly or using the PMF.

2. You drive  $a = 250$  days in a year. Every day, there is a probability  $p = 0.02$  that you will get a traffic ticket, independently of other days.

**1+1+2+2 points**

- (a) Compute the probability that you will get  $b$  tickets in the whole year.
- (b) What is the probability that you get more than *twice* the expected number of tickets in the whole year.
- (c) Any of the tickets is 50, 100 or 200 dollars with respective probabilities 0.5, 0.3 and 0.2. Find the mean of the amount of money you will pay in traffic tickets during the year.
- (d) Find its variance.

3. The median of a continuous random variable  $X$  is the value  $m$  such that  $P(X \leq m) = 0.5$ . The lifetime (time until it breaks down) of an air-conditioner is an exponentially distributed random variable with a **median** value of 20 years.
- (a) What is the probability that it breaks down in less than a day?
  - (b) What is the probability that it lasts **at least** 30 years?

4.  $X$  is a random variable with PDF  $f_X(x)$ .  $Y$  is a random variable dependent on  $X$  with conditional PDF  $f_{Y|X}(y|x)$  and conditional CDF  $F_{Y|X}(y|x)$ .

**4+2 points**

- (a) Let  $Z = XY$ . Find the PDF of  $Z$ .
- (b) Let  $Z = X^2 + Y$ . Find the PDF of  $Z$ .

5. Let  $R$  be uniformly distributed between 0 and 1. Given  $R = r$ ,  $X$  is uniformly distributed between 0 and  $r$ . Find the conditional PDF of  $R$  given  $X$ .