APPENDIX TO "THE WIENER-KHINCHIN THEOREM FOR NON-WIDE SENSE STATIONARY RANDOM PROCESSES"

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1. OUR CONDITIONS AND CONCLUSIONS

In our paper, we proved Wiener-Khinchin theorem for non-WSS processes under the condition that:

$$\int_{-\infty}^{\infty} R_x^m(\tau)d\tau < \infty, \quad \text{where } R_x^m(\tau) = \sup_t |R_x(t, t-\tau)| \quad (1)$$

We need this condition to move the limit inside the integral in equation (8) and again in (11) of [1]. This also ensures the Fourier transform of $\overline{R}_x(\tau)$ exists. We give detailed and rigorous proof and its applications in [1].

2. COOPER'S BOOK

In Cooper's book[2], they do not give the conditions on which the theorem holds while in our paper, we definitely provide the conditions. Meanwhile, their proof is also incorrect from some step.

First, he can not ensure if the Fourier transform of $\overline{R}_x(\tau)$ exists, while our condition ensures the existence of Fourier transform of $\overline{R}_x(\tau)$. Second, no conditions are given to move the limit inside the integral in equation (7.38).

3. GARDNER'S BOOK

In Gardner's book[3, Page 232], he says $S_x(f)$ is equal to the average of instantaneous PSD $S_x(t, f)$ which is Fourier transform of instantaneous autocorrelation function.

$$S_x(t, f) = \int_{-\infty}^{\infty} R_x(t + \tau/2, t - \tau/2)e^{-i2\pi f\tau}d\tau \quad (2)$$

It is a different result from our Wiener-Khinchin theorem for non-WSS processes. Again, no conditions for the existence of Fourier transform of $R_x(t, \tau)$ over τ have been given.

4. PROAKIS' BOOK

In Proakis' book[4, Page 179], the condition for the theorem to hold is not sufficient. Their condition is given as:

For any finite τ and any interval $\mathcal{A}$ with length $|\tau|$, $|\int_{\mathcal{A}} R_x(t + \tau, t)dt| < \infty$.

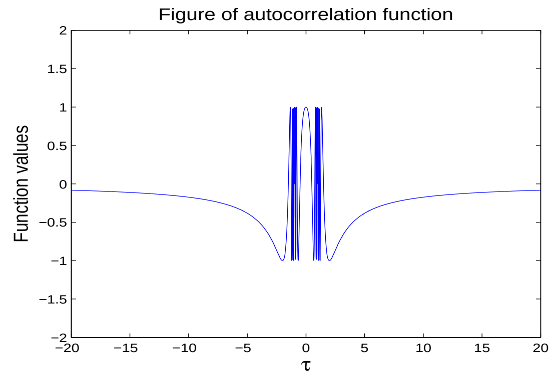

Fig. 1. Autocorrelation function $R_x(t, t-\tau) = \sin(\frac{\pi}{2(1-|\tau|)})$

His condition also does not ensure the Fourier transform of the averaged autocorrelation function exists. A counter example is the WSS process which has the autocorrelation function $R_x(t, t-\tau) = \sin(\frac{\pi}{2(1-|\tau|)})$. We know this autocorrelation function satisfies his condition since for any finite τ

$$|\int_{\mathcal{A}} R_x(t+\tau, t)dt| = |\int_{\mathcal{A}} \sin(\frac{\pi}{2(1-|\tau|)})dt| = |\tau \sin(\frac{\pi}{2(1-|\tau|)})| \leq |\tau| < \infty \quad (3)$$

However, we know Fourier transform of $\sin(\frac{\pi}{2(1-|\tau|)})$ does not exist like the function $\sin(\frac{1}{t})$ in [6]. Fig. 1 gives an intuition of the nonexistence of Fourier transform of $\sin(\frac{\pi}{2(1-|\tau|)})$. From the figure, we can see that this function will approach to 1 when $\tau \to \infty$ and infinite number of maxima and minima exist in any finite intervals close to $\tau = 1$ or containing it. Even if this autocorrelation function satisfies Proakis' condition, we are still not able to use our result.

Check if it satisfies our condition:

$$\int_{-\infty}^{\infty} R_x^m(\tau)d\tau = \int_{-\infty}^{\infty} |\sin(\frac{\pi}{2(1-|\tau|)})|d\tau = \infty \quad (4)$$

Hence, this autocorrelation function does not satisfy our condition. Consequently, Wiener-Khinchin Theorem can not be applied and our given condition is sufficient.

5. PEEBELS' BOOK

Except for Peebels' results[5, Page 227], all the results given in other books are incomplete. Peebels gives a different sufficient condition from ours. We assume that the maximum absolute autocorrelation function is integrable, while he assumes power spectral density is absolutely integrable.

6. REFERENCES

[1] W. Lu and N. Vaswani, *The Wiener-Khinchin Theorem for Non-wide Sense Stationary Random Processes*, submitted to ICASSP 2009.

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[4] J. G. Proakis and M. Salehi, *Communication Systems Engineering (2nd Edition)*, Prentice Hall, 2001.

[5] P. Z. Peebels, *Probability,Random Variables and Random Signal Principles (4th Edition)*, McGraw-Hill Publishing Company, 2001

[6] A.V. Oppenheim, A.S. Willsky, S. Hamid, *Signals and Systems(2nd Edition)*,Prentice Hall, 1996