Homework 4 (due on Oct. 29th)

Consider a wireless downlink network with a single base-station and two mobile users. Let $C_i[t]$ denote the state of the channel to user i at time t. We assume that

$$C_1[t] = \begin{cases} 2, & \text{with probability } 0.5; \\ 1, & \text{with probability } 0.5. \end{cases},$$

$$C_2[t] = \begin{cases} 4, & \text{with probability } 0.7; \\ 1, & \text{with probability } 0.3. \end{cases},$$

and $C_1[t]$ and $C_2[t]$ are independent across users and time. Let $Z_i[t] = 1$ if user i is selected for transmission at time t, and $Z_i[t] = 0$ otherwise. Only one user can be selected at a time.

Consider the following algorithm:

(1) At time t, the base station selects a user i^* such that

$$i^* \in \arg\max_{i=1,2} \left(KC_i[t] + P_i[t]\right),$$

where $K > 0$ is a constant. Then the base station will serve user i^* so that $Z^*_{i^*}[t] = 1$ and $Z^*_j[t] = 0$ for $j \neq i^*$.

(2) The virtual queues are updated as follows:

$$P_i[t] = \left(P_i[t] + \gamma_i - Z^*_i[t]\right)^+,$$

where $\gamma_1 = \gamma_2 = 0.3$.

Define the following optimization problem

$$\text{Opt} = \max_{\mathbf{Z}} E\left[\textstyle\sum_i C_i[t]Z_i[t]\right]$$

$$\text{subject to:} \qquad \Pr(Z_i[t]) \geq 0.3, i = 1, 2.$$

Problem 1: Prove that

$$\lim_{K\to\infty} \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} E\left[\sum_i C_i[t]Z^*_i[t]\right] = \text{Opt}.$$

Problem 2: Simulate the algorithm with $K = 1, 10, 100$. For each K, plot the figure of t versus $\frac{1}{t}\sum_{s=1}^{t}\sum_i C_i[s]Z^*_i[s]$.