Efficient Spectrum Searching and Monitoring in Cognitive Radio Network

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***Abstract*—Two objectives of sensing in cognitive radio (CR) are to detect the primary user (PU) accurately and quickly, which are contradicting objectives. Therefore, many papers try to optimize this tradeoff and find the minimum sensing time which protects the PU. The trends are classified in enhancing false alarm probability (P_f) and detection probability (P_d), optimizing inter-sensing time, in-band sensing (monitoring) time optimization, and out-of-band sensing (search) time optimization. The PU model used in most of these work was a simple two states model (busy/idle renewal process). In this work, we developed a model for the PU in its idle state. The model enables the CR node to benefit from its previous measurements. It assumes that there are multi-idle states, each with specific length and known probability of staying in it. We used this model to find the best sensing time, energy detection threshold, and false alarm probability of the channel being sensed in monitoring. Also, we developed an out-of-band optimization formulation. The formulation finds the best number of channels to sense, the threshold of each channel, the sensing time of each channel, and P_f of each channel such that the PU is protected, the sensing time is minimized, and the CR will find an available channel with very high probability.**

I. Introduction

Wireless spectrum is highly underutilized, where significant parts of it are used only for some time periods in an on/off manner and with large geographical variations. Such low utilization in spectrum usage resulted from currently deployed static frequency allocation policy. Apparently, in order to increase wireless spectrum utilization, more flexible spectrum management techniques are required (like Opportunistic Spectrum Sharing (OSS)), where secondary users are allowed to operate in frequency bands without the permission of the Primary User (PU), provided that they do not introduce harmful interference with the PU. Cognitive Radios (CRs) have been proposed as a technology to implement OSS, since they are able to sense the spectrum and adapt their spectrum usage accordingly [1].

CRNs impose unique challenges due to the high fluctuation in the available spectrum in time. Different CR nodes see different available channels at different times. This introduces some challenges such as: (1) spectrum sensing which needs to be done frequently and accurately, (2) multi-hop routing and the availability of routes between nodes that see different channels, (3) spectrum decision and sharing in a distributed setting without a central coordinator, (4) coordination among the nodes with/without the availability of a common control channel, etc.

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One of the most important and challenging issues is to have fast and reliable spectrum sensing. Spectrum sensing is a key enabling technology for CRN. This is because the decisions of using and vacating channels and other decisions are based on sensing results. Also, reliable sensing can help avoiding interference with the PU. The design of a sensing algorithm involves balancing a tradeoff between the quality and the speed of sensing, through an appropriate selection of the sensing time. Increasing sensing time enhances detection probability, however, results in shorter transmission time.

Spectrum sensing can be classified into two types [2]: out-of-band sensing (searching for an idle channel), and in-band sensing (monitoring a channel periodically while using it, in order to prevent the interference with the PU). Since increasing sensing time will reduce room left for transmission, several algorithms were developed to optimize required sensing time. These work can be divided into four main axes: firstly, optimizing the detection probability and false alarm probability [3]; secondly, reducing inter-sensing time in case of monitoring [4], [5]; thirdly, reducing required monitoring time [6], [7]; and finally, reducing search time [8], [9].

In this paper, we present two optimization formulations; one for search and the other for monitoring. Optimization of inter-sensing time is not part of this work. Our work differs from previous work in: 1) In monitoring, we find the sensing time jointly with the detection threshold such that the PU is protected and the sensing time is minimized. We take into account the PU statistics, where we will show how PU statistics affect the required sensing time; 2) In search, we increase the degrees of freedom where we find sensing time of each channel, energy detection threshold of each channel, false alarm probability of each channel, and the number of channels to be sensed; 3) Moreover in search, we do joint optimization of multiple heterogeneous channels with different detection probabilities, probabilities of being idle (P($\mathcal{H}0$)), and primary SNR; 4) In both optimizations, we relaxed the false alarm probability. Usually false alarm probability is required to be small enough. But, we proved that in some cases sensing with higher false alarm probability requires less sensing time.

Another difference from existing research is the PU model. Most of the current methods use only simple partially observable Markov decision processes (POMDPs), where each radio channel is modeled with two states: Busy and Idle states [5], [8], [10], [11]. Such limited channel models do not allow the CR node to benefit from its previous measurements: in particular the sensing decisions done by the node in the previous sensing periods. In this paper, we model the PU idle state into multiple idle states instead of just one idle state. This allows the CR node to benefit from the previous sensing decisions done in the last sensing periods.

We use energy detection as the basic detection method because it is the fastest sensing method and the simplest for CRNs. In this method, the energy in the received waveform over an observation interval (sensing time) is measured [12], and compared to a threshold value (γ), if the measured value is larger than γ, the PU is assumed to be using the channel and the CR nodes should not use the channel. Otherwise, the PU is considered to be absent and the CR nodes can use the channel.

Such threshold-based decision is subject to two types of errors; false alarm and miss detection. In false alarm, the CR will detect the existence of a PU while there is no PU exists. Higher false alarm probability (P_f), reduces spectrum utilization by the CR. However, having less strict requirements on the false alarm has several advantages: (a) decreasing miss detection probability (P_m) due to the tradeoff between the detection probability ($P_d = 1 - P_m$) and P_f, (b) sensing potentially becomes less complex, and (c) less required sensing time for sensing a channel. On the other hand, in miss-detection, the CR detects that there is no PU while there is a PU. Higher miss detection probability ($P_m = 1 - P_d$), increases the interference with the PU, which should be avoided.

The contribution of this paper can be summarized in: 1) We introduce a PU multi-idle states model which allows us to benefit from the previous sensing measurements done by the node itself to optimize the monitoring time for the current sensing period; 2) we formulate two convex non-linear optimization formulations; one for search and the other for monitoring. The formulations have more degrees of freedom than previous work, where for search we jointly find: the sensing time of each channel (N_i), the energy detection threshold of each channel (γ_i), and P_f of each channel. For monitoring, we find N, γ, and P_f of the channel being used. For search, we also find the number of channels to sense; 3) We prove that by relaxing P_f, we reduce the total sensing time; 4) Search optimization formulation considers channels with different $\bar{P}_d(i)$, primary SNR, and $Pr_i(\mathcal{H}0)$.

Section 5 shows that monitoring and searching results appeared very fast. Numerical results show that in monitoring, in some cases, sensing with higher P_f is better, while in search, sensing more channels under forcing P_f to be less than 0.5 requires less sensing time than forcing it to be less than 0.1 or 0.05.

The rest of this paper is organized as follows. System model overview and problem definition are introduced in Section 2. Monitoring optimization formulation and solution is shown in section 3. Section 4 shows the search: definition, optimization formulation, solution and protocol. Results and analysis are evaluated in Section 5. Section 6 shows the related work and finally we conclude in section 7.

II. System Model and Problem Statement

The main targets of this work is to design two sensing time optimization algorithms for monitoring and search. For monitoring we find jointly the threshold of the energy detection algorithm and the required sensing time which minimizes the sensing time and protects the PU. For search, we will jointly find the number of channels to be sensed, the detection threshold of each channel, sensing time of each channel, and P_f of each channel. We also consider channels with different characteristics.

To detect a weak primary signal on specific channel, one could pose a binary hypothesis testing as follows:

$$y_i \sim \begin{cases} v(i) & \text{Under } \mathcal{H}0 \\ s(i) + v(i), i = 1, 2, ..., N & \text{Under } \mathcal{H}1 \end{cases} \tag{1}$$

where $\mathcal{H}0$ represents the absence of the primary signal, i.e., the received baseband complex signal y_i contains only additive white Gaussian noise (AWGN), $v(i) \sim \mathcal{N}(0, \sigma_v^2)$, and $\mathcal{H}1$ represents the presence of the primary signal, i.e., y_i consists of a primary signal $s(i)$ corrupted by $v(i)$. Moreover, N corresponds to the number of measured samples. Energy detection is a threshold-based hypothesis test. It means that the energy on a specific channel is measured and compared to a threshold value similar to the following hypothesis test:

$$V(\mathbf{y}) = \sum_{\mathbf{i=1}}^{\mathbf{N}} \mathbf{y_i^2} \underset{\mathcal{H}\mathbf{0}}{\overset{\mathcal{H}\mathbf{1}}{\gtrless}} \gamma \tag{2}$$

where $V(\mathbf{y})$ is the test statistics, N is the number of measured samples, y_i^2 is the energy measured on sample i, and γ is the threshold.

$V(\mathbf{y})$ is a random variable whose probability density function is $t_0(x)$ under $\mathcal{H}0$ and $t_1(x)$ under $\mathcal{H}1$. Therefore, $V(\mathbf{y})$ is a random variable with Chi square distribution with $2N$ degrees of freedom for complex valued case, and with N degrees of freedom for real-valued case. According to the central limit theorem, $V(\mathbf{y})$ is asymptotically normally distributed if N is large enough ($N \geq 20$ is practically sufficient). For large N, $V(\mathbf{y})$ can be modeled as follows:

$$V(\mathbf{y}) \sim \begin{cases} \mathcal{N}(N\sigma_v^2, 2N\sigma_v^4), & \text{Under } \mathcal{H}0 \\ \mathcal{N}(N\sigma_v^2 + Np_s, 2N\sigma_v^4 + 4N\sigma_v^2 p_s), & \text{Under } \mathcal{H}1 \end{cases} \tag{3}$$

where p_s is the average PU signal power. Therefore, when doing energy detection of a channel, k, and for large N_k, the false alarm probability, P_f^k, and the detection probability, P_d^k, can be approximated by the following two equations:

$$
\begin{aligned}
P_f^k(\gamma_k, N_k) &= Pr(\mathcal{H}1|\mathcal{H}0) \\
&= Pr(V(y) \geq \gamma|\mathcal{H}0) \\
&= \int_\gamma^\infty t_0(x).dx \\
&= Q((\frac{\gamma_k}{\sigma_v^2} - 1)\sqrt{N_k})
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
P_d^k(\gamma_k, N_k, SNR) &= Pr(\mathcal{H}1|\mathcal{H}1) \\
&= Pr(V(y) > \gamma|\mathcal{H}1) \\
&= \int_\gamma^\infty t_1(x).dx \\
&= Q\left[(\frac{\gamma_k}{\sigma_v^2} - SNR - 1)\sqrt{\frac{N_k}{2SNR+1}}\right]
\end{aligned} \tag{5}
$$

where:

$$
Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-\tau^2/2} d\tau \tag{6}
$$

is the tail probability of a zero-mean unit-variance Gaussian random variable. We can notice that both P_f^k and P_d^k are functions of the sensing time (N_k) and the threshold (γ_k). But, P_d^k is also a function of the primary SNR.

In this paper, we assume that we have statistical information about the PU through long monitoring. This information could be represented by radio environment maps (REM) [13], [14], [15] through accurate cooperative sensing [16]. Supposing that we are doing sensing every $\tau seconds$, where τ is fixed and it is PU-dependent (i.e., 2 seconds for TV PU [17]), then we derive the probability that the PU will stay idle for $i * \tau seconds$, $i \in [1, I]$, P_i^I.

Since in monitoring, the CR node do sensing every τ, then it can maintain a memory of the last sensing decisions. This memory as we will see later will affect the required sensing time. We assume that instead of only one idle state of the PU, we have multiple idle states (I). PU behavior model could be represented by Figure 1 for the duration of idle periods. In Figure 1, each circle represents how long the PU is going to stay idle in terms of sensing periods (τ). For example the circle with caption $i * \tau$ means that the PU is going to stay idle i sensing periods with probability P_i^I, and then becomes busy with probability 1.

We assume that all probabilities in Figure 1 and ($Pr(\mathcal{H}1)$) are known, and these can be obtained through long term measurements, and stored in the form of REM. The multi-idle states are useful in monitoring, where a CR node does sensing every sensing period (τ). Suppose that the CR node detected that the PU was idle in the last i sensing periods. Then, let q_i be the probability that the PU becomes busy after i sensing periods given it was idle in the previous i sensing periods. Using the simplified model in Figure 2, q_i can be calculated using the following equation:

$$
p_i^I = \prod_{j=1}^{i-1} (1 - q_j) * q_i \tag{7}
$$

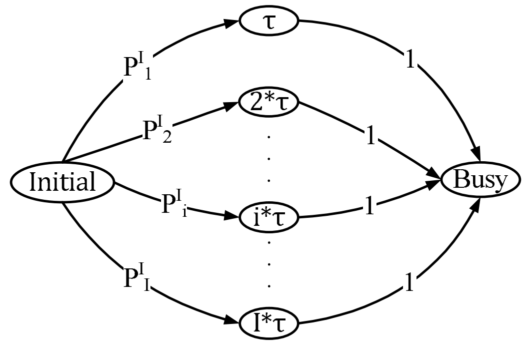


Fig. 1: Idle states

where $q_1 = p_1^I$. Then, from Equation (7) we can find the probabilities that the channel is busy or idle at each time slot given the state of the channel in the previous time slots.

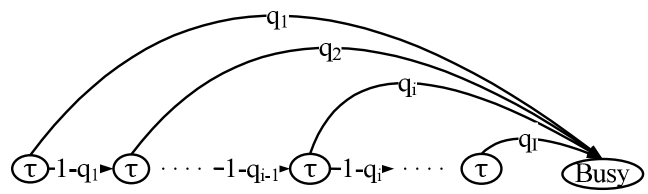


Fig. 2: Idle states

Our problem can be defined as: First, given the primary SNR, $P_i^I\ \forall\ i \in [1, \infty]$, in which state of Figure 1 the CR is in, the required detection probability of the PU ($\bar{P}_d$), and the average search time, we formulate a monitoring time convex optimization formulation that finds jointly the optimal monitoring time, detection threshold, and false alarm probability. Second, given the primary SNR_i, $Pr_i(\mathcal{H}0)$, $\bar{P}_d(i)$ $\forall\ i \in [1, M]$, and the best order of the M channels to be followed when doing search, we design a convex non-linear search time optimization formulation that finds the sensing time of each channel, γ of each channel, P_f of each channel, and the best number of channels to search such that the total search time is minimized, PUs are protected, and the CR node will find an idle channel with high probability.

III. Monitoring Optimization

The goal of in-band sensing (monitoring) is to prevent interfering with the PU. Monitoring should satisfy two conditions: 1) Detection time should be less than PU tolerable interference delay (TID). We assume a time slotted system, where monitoring should be repeated every τ. Note that τ is strictly upper bounded by the TID: no interference to PUs longer than TID is allowed. In practice, TID will be defined by the PU itself or by regulations [17], [18]. For example, for TV bands ($TID = 2s$). Assuming the CR node is going to sense the channel every τ seconds for time t_m, and transmit for $\tau - t_m$ if the channel is still idle. If the channel is found to be busy, then the CR node will search for an empty channel for T_{search} seconds on average. 2) The detection probability from doing sensing (P_d or $Pr(\mathcal{H}1|\mathcal{H}1)$) given by Equation

(5) should be greater than the given detection probability of the PU ($\bar{P}_d$).

Monitoring a channel has four candidate results:

1) $\mathcal{H}0|\mathcal{H}0$ (true positive): detects it idle while it is idle. In this case, the CR node will start sending on the channel.
2) $\mathcal{H}0|\mathcal{H}1$ (false positive or miss detection): detects it idle while it is indeed busy. In this case, the CR node will start sending, but interfering with the PU.
3) $\mathcal{H}1|\mathcal{H}1$ (true negative or detection probability): detects it busy and it is indeed busy. In this case, the CR node should vacate the channel, and search for another available channel.
4) $\mathcal{H}1|\mathcal{H}0$ (false negative or false alarm): detects it busy while it is indeed idle. In this case, the CR node should vacate the channel, and search for another available channel.

Note that false positives should be avoided since they result in collisions with PU transmission, while false negatives should also be avoided since they waste available transmission opportunities by SUs.

Increasing sensing time increases detection probability and reduces false alarm probability, but also reduces available transmission time. Moreover, the longer the PU was idle, the higher the probability he will be active in the next detection period. Algorithm 1 describes a non-linear optimization algorithm that minimizes the monitoring time.

Algorithm 1 : Monitoring given that the CR is in state i

1: Min $\quad t_m + T_{search} * (Pr^{(i)}(\mathcal{H}0) * Pr(\mathcal{H}1|\mathcal{H}0) + Pr^{(i)}(\mathcal{H}1))$
2: s.t. $\quad Pr(\mathcal{H}1|\mathcal{H}1) \geq \bar{P}_d$
3: $\quad t_m \leq \tau$

In Algorithm 1, the first line is the objective function that we are trying to minimize. It consists of three parts: 1) monitoring time (t_m) which will be executed every detection cycle 2) Search time (T_{search}) which will be executed in case the channel is idle, but detects it as busy. This will happen with probability $Pr^{(i)}(\mathcal{H}0) * Pr(\mathcal{H}1|\mathcal{H}0)$, and 3) search time ($T_{search}$) which will be executed in case the channel is busy. This will happen with probability $Pr^{(i)}(\mathcal{H}1)$.

The second line in algorithm 1 is a constraint to satisfy the second condition of monitoring mentioned above. In this constraint, we are trying to guarantee the PU detection probability condition. Therefore, on monitoring a PU, the probability of true negative ($Pr(\mathcal{H}1|\mathcal{H}1)$) of that PU must be greater than or equal to the given detection probability ($\bar{P}_d$) of the same PU. $Pr(\mathcal{H}1|\mathcal{H}1)$ is given in Equation (5). The third line is that the monitoring time should be less than the detection period. In case when the SNR is below the SNR wall [19], the monitoring algorithm will not be able to detect the primary signal.

The decision variables that we are trying to find are the sensing time (N) and the detection threshold (γ) of the channel being sensed. The enhancement in sensing time comes from the model described in Figures. 1 and 2. For example, if we know that with very low probability the PU will keep idle in the current detection cycle, then the CR node has to do sensing for longer time as section 5 shows to avoid the false alarm and doing longer search.

The parameters to this algorithm are: $Pr^{(i)}(\mathcal{H}0)$, primary SNR, and average search time (T_{search}) that will be needed in case the PU found to be active whether it is correct or not. SNR is the signal-to-noise ratio between the CR node and the PU that the CR node is monitoring. $Pr^{(i)}(\mathcal{H}0)$ could be calculated using the model in Figure 1, 2 and Equation (7). Depending on in which state the CR node is in, it calculates $Pr^{(i)}(\mathcal{H}0) = 1 - q_i$. For example, if the CR node was using the channel in the last i periods, where it sensed the channel in the last i periods and found the channel to be idle, then the CR node knows it is in state i and $Pr^{(i)}(\mathcal{H}0) = 1 - q_i$ and $Pr^{(i)}(\mathcal{H}1) = q_i$.

There is a hidden convexity in this optimization under certain conditions. First, this is a minimization problem, which means that the constraints and the objective function should be convex for the problem to be convex. The objective function composed of the monitoring time which is linear (convex), and a constant multiplied by the false alarm probability $(Pr(\mathcal{H}1|\mathcal{H}0))$. False alarm probability is given in Equation (4), and it is a Q function. Figure 3 shows the curve of the Q-function. It is clear that it will be convex for values $\leq$ 0.5, which means that the false alarm probability will be convex in case it is less than 0.5. Since the false alarm probability is required to be small, then this range is a desired range for false alarm in monitoring.

The first constraint is a greater than or equal inequality. For a non-linear constraint to be convex, it should look like: *"convex non-linear terms $\leq$ constant"*. Also, since the negative of a concave non-linear term is indeed convex, then the non-linear term should be concave for the constraint to be convex. The detection probability $(Pr(\mathcal{H}1|\mathcal{H}1))$ is given in Equation (5). Again, it is a Q function that is concave for values ≥ 0.5 which is a desired range for detection probability, where it is usually required to be greater than 0.9. Therefore, the optimization problem is a convex optimization for $\bar{P}_d \geq 0.5$ and $P_f \leq 0.5$. This convex optimization could be solved using convex optimization algorithms that have quadratic convergence, such as Newton's algorithm and Sequential Quadratic Programming (SQP).

IV. Search Optimization

In this section, we consider out-of-band sensing (search) optimization. First, we will define out-of-band sensing and its requirements. Then, we introduce a non-linear optimization formulation that minimizes search time. After that, we show a solution strategy that solves the non-linear optimization very fast. Finally, we discuss the protocol and when do we need to re-run the search process.

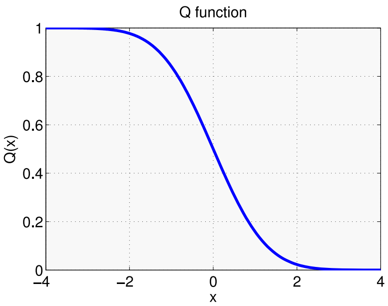


Fig. 3: Q Function

A. Search Definition

Out-of-band sensing (search) target is to find an available channel to use. Search is also required in case of spectrum hand-off (when the PU re-appears). Therefore, search process needs to be done very fast in order to enhance the quality of service (QoS) of the CR nodes, and transmission not to be cut for longer time.

During the search, the PU detection probability (P_d) condition should be satisfied. This condition is a little bit different from that in monitoring. In monitoring, the CR node is monitoring one channel. In search, the CR node is looking for an available channel. It searches multiple channels until finding an available one. Therefore, there will be multiple channels with multiple PUs and different detection probabilities to be satisfied.

B. Optimization

Usually, sensing is done such that the false alarm probability (P_f) is reduced [20], [21]. But, sometimes sensing more channels with higher P_f will be better than sensing less number of channels with lower P_f. In this section, our goal is to find the number of channels to sense, the detection threshold of each channel, the sensing time of each channel, and the false alarm probability of each channel such that the detection probabilities of PUs are satisfied, total search time is minimized, and the CR node will find an available channel with very high probability. Algorithm 2 shows channels search-time optimization.

Algorithm 2 :Search Optimization

1: Min $\sum_{i=1}^{K}(t_s(i)+F(\alpha,f_i,f_{i-1}))*Pr(sw)$

2: s.t. $Pr_i(\mathcal{H}1|\mathcal{H}1) \geq \bar{P}_d(i)$ for $i \in [1,K]$

3: $1-\prod_{i=1}^{K}(Pr_i(\mathcal{H}1)+Pr_i(\mathcal{H}0)*Pr_i(\mathcal{H}1|\mathcal{H}0)) \geq \zeta$

Algorithm 2 is a non-linear programming formulation. The intuition of this optimization is to try to minimize the search time such that the PU is protected against interference, and the CR node will find an available channel with high probability. First line is the objective function where we minimize the total search time. Line 2 says that the detection probability constraint of each PU must be satisfied. Line 3, means that the CR node will find an idle channel with a probability that is at least equals to ζ.

The objective function is the expected search time. It is composed of the sensing time of each channel ($t_s(i)$), and the switching delay between the channels ($F(\alpha, f_i, f_{i-1})$). The switching delay could be zero if multiple narrow-band detectors are used [22]. Otherwise, it is a function of three things: 1) the previous frequency that the CR will switch from (f_{i-1}), 2) the current frequency that the CR switched to (f_i), and 3) a technology factor (α). For example, a linear switching delay function can be expressed as $\alpha*(f_i-f_{i-1})+c$ where c is a constant time.

Indeed, α depends on many factors like the energy consumed, the error rate, the SNR, and the technology that is used. According to [23], the switching time required for frequency hopping is primarily determined by the design of the phase locked loop (PLL) used in the frequency synthesizer that generates the channel carrier frequencies. A decrease in switching time also comes at the expense of an increase in power dissipation. Table I shows switching times and the power consumed for switching. Each of these values is for different frequency steps, i.e., the PLL needs 120 μs for 75 MHz steps [24]. We will use the 120μs in our experiments because it consumes the minimum amount of power.

Switching time (μs)	Power(mW)
0.009	124
0.15	57.6
20	20
70	11.4
120	4.2

TABLE I: Relationship between switching time and power consumption

The objective function is also a function of the probability of switching ($Pr(sw)$). This probability is multiplied by the sum of the sensing and switching times. This results in the expected searching time. $Pr(sw)$ is given in the following equation:

$$Pr(sw)=Pr_{i-1}(\mathcal{H}1)+Pr_{i-1}(\mathcal{H}0)*Pr_{i-1}(\mathcal{H}1|\mathcal{H}0) \quad (8)$$

where $Pr_{i-1}(\mathcal{H}1)$ is the probability that channel $i-1$ is busy, $Pr_{i-1}(\mathcal{H}0)$ is the probability that channel $i-1$ is idle, and $Pr_{i-1}(\mathcal{H}1|\mathcal{H}0)$ is the probability that channel $i-1$ is idle but it is detected as busy (false alarm). This yields the probability of switching to sense channel i after concluding that the previous channel, ($i-1$), is busy, either correctly or mistakenly.

C. Solution

Algorithm 2 minimizes total sensing time when the CR node is going to sense K channels. In order to find K that achieves the minimum sensing time, we evaluate it iteratively. In each iteration, K will be incremented by 1 and given this K, Algorithm 2 will be solved for total sensing time and thresholds. We keep on incrementing K from 1 towards M until finding the minimum total sensing time, i.e., it decreases, and then starts increasing. At that point, K will be assumed the optimal value.

Some cases will be infeasible. For example, if each channel is idle with probability ($P(\mathcal{H}0) = 0.6$), then it is infeasible to find an available channel with probability ($\zeta = 0.9$) by searching only 1 or 2 channels even if the CR node conducted perfect sensing with zero false alarm probability. To exclude the infeasible cases, we do not start from $K = 1$, instead, we start it from a larger value, say J. To find J, we initialize J to 1. After that, we assume that $Pr_i(\mathcal{H}1|\mathcal{H}0) = 0$ (which means perfect sensing), $\forall\ i \in [1, M]$. Then, we start incrementing J until the following constraint $1 - \prod_{i=1}^{J}(Pr_i(\mathcal{H}1)) \geq \zeta$ is satisfied. This yields the required value of J.

Non-linear optimization is usually solved iteratively, and the accuracy of the solution improves with the number of iterations. The optimization starts by initializing the decision variables to some initial values. Then, each iteration gives a solution that is closer to the optimal solution. The process will be repeated until a stopping criteria is satisfied. An example of a stopping criteria is that the difference between the solutions of the two iterations is smaller than a given small number, ϵ.

This search formulation is indeed convex. Lines 1 and 2 are convex for the same reasons mentioned above about the convexity of lines 1 and 2 in monitoring formulation. To prove the convexity of line 3: in general the product of two convex functions is not convex. However, If f and g are convex, both non decreasing (or non increasing), and positive functions on an interval, then $f * g$ is convex. The proof of this claim follows from Jensen's inequality. Line 3 is a product of K Q-functions. The Q-function is convex for input values greater than 0.5, non-increasing and positive function. Therefore, line 3 is convex. Consequently, the search optimization is convex for $\bar{P}_d \geq 0.5$ and $P_f \leq 0.5$

As explained before, to reach the global minimum solution quickly, we use a method to find the initial values of the decision variables. Since in algorithm 2, in each iteration, we find values for γ_i and N_i, $\forall i \in [1, K]$, we use initial values for N_i, such as 2000. Then, we find the initial values for γ_i using the following equation which is obtained by inverting Equation (5):

$$\gamma_i = \left[\frac{Q^{-1}(P_d(i))}{\sqrt{\frac{N_i}{2*SNR_i+1}}} + SNR_i + 1 \right] * \sigma_v^2 \tag{9}$$

Using these initial values, and by adding the constraints ($P_f^i \leq 0.5$, $\forall i \in [1, K]$), convergence to the optimal solution is achieved quickly as we will see in section 5. We used the sequential quadratic programming (SQP) algorithm [25] for solving this optimization problem which achieves convergence very fast.

D. Protocol

Channel search is required in case the CR node wants to find an available channel to transmit on, or after doing the in-band sensing and finding that the PU became active, hence the CR must perform spectrum hand-off. The optimization will be applied when the CR node initially switch on and wants to find an available channel to start using it. As a result of the optimization, the node will determine the best number of channels (K), sensing time and threshold value of each of the K channels. The optimization will be required again in case the input parameters to Algorithm 2 change.

The parameters to Algorithm 2 are: 1) $Pr_i(\mathcal{H}0)$, and SNR_i $\forall\ i \in [1, M]$. 2) The order in which channels are sensed, since channel sensing is done sequentially. Authors in [8], [9], [26] and others introduced methods to find the optimal sequence of channels to be sensed such that sensing time is minimized. Therefore, we assume that the sequential order of channels is known. During search, the CR node will follow the order in that sequence.

One important thing to notice is that this algorithm is a probabilistic algorithm. This means that after calculating the recommended number of channels to be sensed (K), threshold values (γ_i, $\forall\ i \in [1, K]$), and sensing time (N_i, $\forall\ i \in [1, K]$), the CR node will start sensing the channels. It is expected that the node is going to find an available channel by following the sequence and the recommended values with high probability (ζ). It may find an available channel by sensing a fewer number of channels. Also, it may sense the K channels without finding an available channel. In case it did not find an available channel, the CR node can re-apply the optimization problem on the remaining channels, then continue sensing the new K channels with the sensing time and threshold values returned by solving the optimization problem with the new parameters.

Algorithm 3 shows the search protocol. In it, X[V] means an array X of V elements. The inputs to Algorithm 3 are the M channels with known P0$[M]$ ($Pr_i(\mathcal{H}0)\ \forall\ i \in [1, M]$), $\bar{P}_d[M]$, $SNR[M]$, and $\sigma_v[M]$. Also as we stated previously, the channels are sorted in a way to reduce sensing time. The subtraction of the first K values that is done on $ChSeq, SNR, P0$, and $\bar{P}_d$ is by trimming the first K values in the array. For example, if the arrays have 100 elements related to 100 channels, and K=10, then we remove the IDs of the first 10 channels from $ChSeq$ and the first 10 values that are related to the 10 deleted channels from the arrays $SNR, P0$, and $\bar{P}_d$.

The function $Optimiz$ takes the sequence of M channels sorted according to one of the sorting algorithms, their related (SNR, $Pr_i(\mathcal{H}0)$, and $\bar{P}_d$), and ζ. It finds the best number of channels to be sensed (K), N_i, γ_i, and $P_f(i)\ \forall\ i \in [1, K]$. Function $FindCh$ takes an array of K channels and their related sensing time ($N[K]$) and threshold value ($\gamma[K]$). Then, $FindCh$ senses them sequentially, it stops when it finds an idle channel. Function $FindCh$ conducts one of the physical layer energy detection algorithm [27] by calling the function $DoEnergyDetection$ with the parameters of channel id, its sensing time, and threshold value.

V. Results and Analysis

We implemented our optimization formulations using Matlab. Throughout the implementation phase, we used the values in table II, unless it is explicitly stated otherwise. We used

Algorithm 3 Search Protocol

if Node Just Started or In-band sensing returns channel is busy **then**
  while $ChannelFound$=False **do**
    $[K, \gamma[K], N[K]] \leftarrow Optimiz(ChSeq[M], SNR[M], P0[M], \bar{P}_d$[M], ζ)
    $ChannelFound \leftarrow FindCh(K, \gamma[K], N[K])$
    if $ChannelFound$ **then**
      Start using the channel;
      exit while
    else
      $ChSeq \leftarrow ChSeq$ - First K values;
      $SNR \leftarrow SNR$ - first K values;
      $P0 \leftarrow P0$ - First K values;
      $\bar{P}_d \leftarrow \bar{P}_d$ - First K values
    end if
  end while
end if

//$FindCh$ senses the K channels sequentially until finds a channel or exhausts all channels
Function $FindCh(K, \gamma[K], N[K])$
  $i \leftarrow$1;
  $ChAvailable \leftarrow$ false;
  while $i \leq K$ and $ChAvailable$=false **do**
    $ChAvailable \leftarrow DoEnergyDetection$(Ch i, γ_i, N_i)
//DoEnergyDetection function conducts energy detection on channel i
    if $ChAvailable$ = True **then**
      return $ChAvailable$;
    end if
    $i \leftarrow i + 1$
  end while
  return $ChAvailable$;
End Function

//Optimiz Applies Algorithm 2 on the list of channels
Function $Optimiz(ChSeq[M], SNR[M], P0[M], \bar{P}_d$[M], ζ)
  $N0[1:M] \leftarrow$ 5000;
  γ0[1:M] $\leftarrow$ apply Equation 9;
  $[K, \gamma[K], N[K]] \leftarrow$ Alg2($N0[M], \gamma0[M], ChSeq[M], SNR[M], P0[M]$, $\bar{P}_d$[M], ζ)
  return $(K, \gamma[K], N[K])$
End Function

sequential quadratic programming (SQP) for solving the optimization formulation because it converges very fast.

Parameter	Value
$P(\mathcal{H}0)$	0.6
required detection probability (P_d)	0.94
primary SNR	-16dB
σ_v	1
ζ	0.95
sampling frequency	6Msps
switching time	120μs = 720 samples
ϵ used in the optimization	10^{-6}
number of channels (M)	25

TABLE II: Default parameter values used for obtaining results

In order to facilitate a fair comparison to other sensing algorithms who try to minimize the false alarm probability to values less than 0.1 or less than 0.05, in our optimization formulations we expanded the acceptable values of false alarm probability. For example, we will compare our algorithm (referred to by curves with $P_f \leq$0.5) to the approaches that force the false alarm probability to be less than 0.1 and less than 0.05. We will see that relaxing the false alarm probability will reduce required sensing time.

As stated in the previous subsection, non-linear optimization is solved iteratively. It starts from initial values of N_i and γ_i $\forall i \in [1, M]$, then in each iteration, new values for N_i and γ_i are found such that the objective value is closer to the optimal value. This process will be repeated until a stopping criteria is satisfied.

A. Search optimization results

Using the initial values of: $N_i = 5000$, and γ_i according to Equation (9) $\forall i \in [1, M]$, we obtained the optimal results for search optimization on average in 25 iterations. These results show that the convergence is fast. Moreover, from experimental results, the total sensing time in the search optimization followed a convex behavior versus the number of channels as Figure 4 shows. Note that the figure starts from 4 channels; this is due to the in-feasibility of the optimization for fewer number of channels (i.e., ζ constraint will not be satisfied).

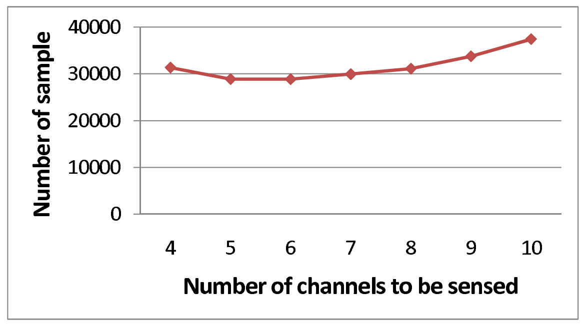


Fig. 4: Convex behavior of the optimization in algorithm 2

Figures 5.a-d show the effects of changing $\bar{P}_d$, $P(\mathcal{H}0)$, primary SNR, and ζ on the total required sensing time in terms of the total number of samples (N). To calculate the sensing time in seconds, N should be divided by the sampling frequency (6 Msps). From the figures, it is clear that using our approach ($P_f \leq 0.5$) requires shorter sensing time in all cases.

Figure 5.a shows that by increasing the value of $\bar{P}_d$, the required sensing time is increased. This is because larger $\bar{P}_d$ means that the PU tolerates less interference and the results have to be more accurate with higher detection probability (true negative). More accurate results can be achieved by sensing for longer time (Equation (5)). Figure 5.b shows that if the channels have smaller probability of being idle ($P(\mathcal{H}0)$), then we need to do sensing longer to find an available channel with probability (ζ). This is because it is less probable that the channel is idle, and hence, the CR node has to sense more channels, consequently, longer sensing time.

Figure 5.c Shows that higher SNR requires less sensing time. By looking at Equation (5), the smaller the SNR is, the smaller the detection probability will be. Therefore, to compensate for the reduction in detection probability, the CR must do longer sensing. Therefore, more sensing time is required in lower SNR. Moreover, Figure 5.d studies the effect of the required probability of finding an available channel (ζ) on the expected sensing time. Increasing ζ requires more sensing time.

Figures 6.a and 6.b are related to Figures 5.a and 5.b respectively. The figures show that sensing more channels with relaxed false alarm probability will require less sensing

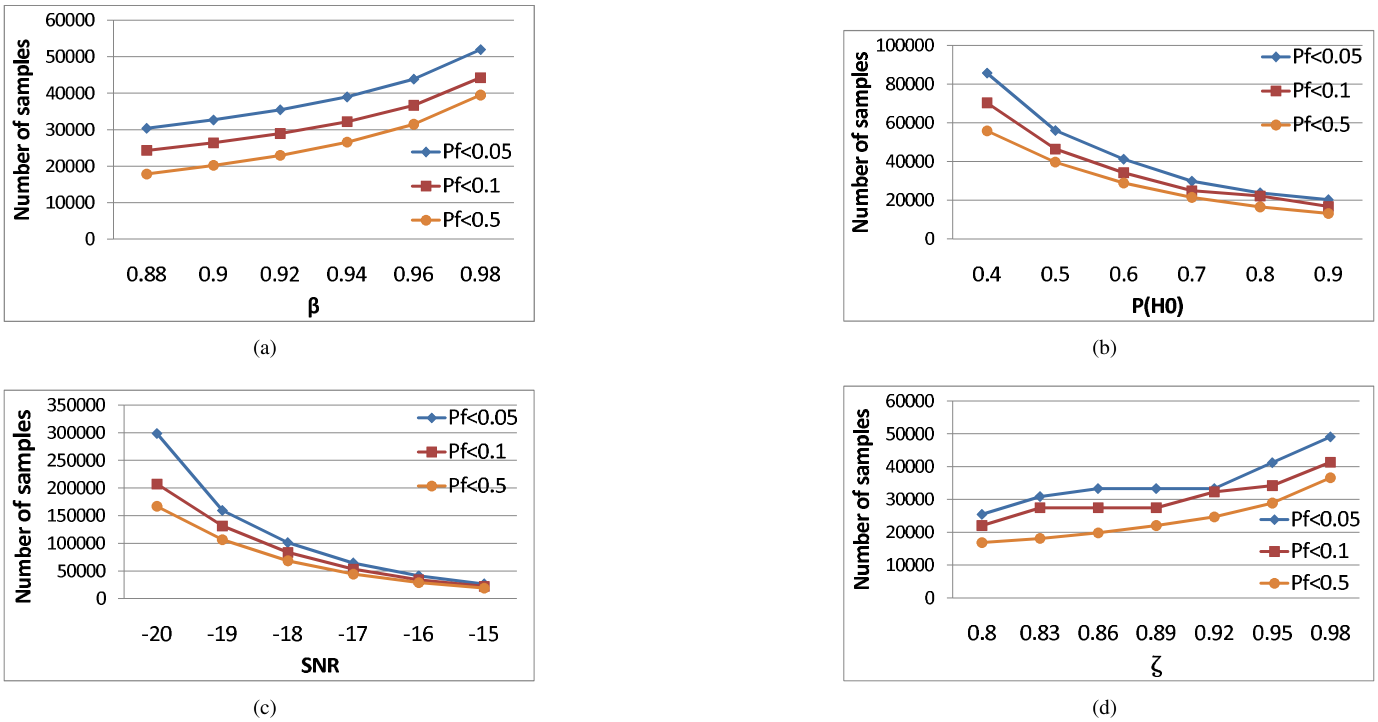


Fig. 5: Effects of: a) $\bar{P}_d$, b) $P(\mathcal{H}0)$, c) primary SNR, and d) ζ on the required sensing time

time than sensing fewer channels with more strict false alarm probability.

B. Monitoring optimization results

As we mentioned previously, Algorithm 1 is convex. From experiments, and using initial number of samples ($N = 5000$), and using initial threshold value calculated according to Equation (9), we obtained the optimal value in 13 iterations on average. In Figure 7, we have drawn the results for 4 average search times (20000-140000 samples). In this section we obtain the results of different states, in Figure 7, states 1-7 refer to the probabilities of being idle ($P(\mathcal{H}0)$) equal to 0.9-0.3 respectively, i.e., state 2 means $P(\mathcal{H}0)$=0.8, and so on. We made this assumption due to the lack of experimental data which will be part of the future work.

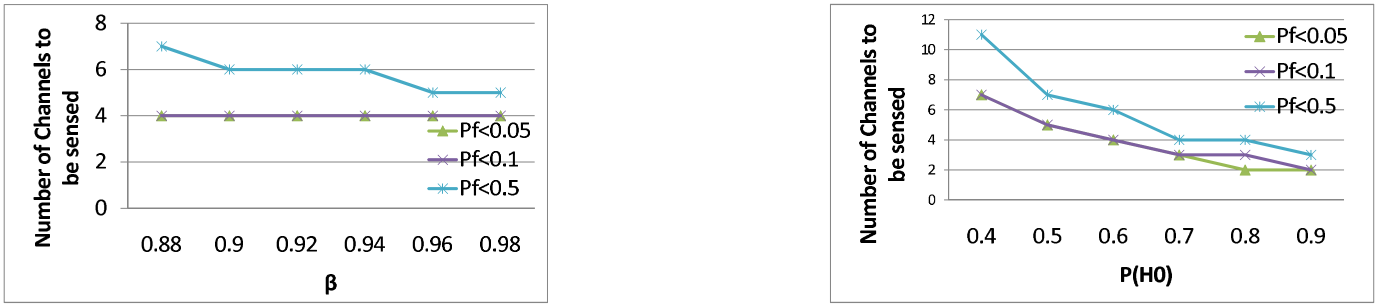
Figure 7.a shows the optimal monitoring time for different search times and for different $P(\mathcal{H}0)$values (states). By increasing the search time, the required monitoring time increases for the same $P(\mathcal{H}0)$. This is because as the average search time increases, it is better to reduce P_f as Figure 7.b shows, otherwise, the node has to do long search while the channel is free. Therefore, the node will favor monitoring the current channel longer which reduces P_f (Equation (4)). In addition to that, as $P(\mathcal{H}0)$ decreases, the required monitoring time will be increased. Figure 7.c shows the threshold values to achieve the minimum sensing time.

VI. Related Work

Sensing time optimization usually makes tradeoff between protecting the PU and enhancing the performance of the CRNs. Since increasing sensing time reduces the quality of service (QoS) for the CR node, many algorithms have been developed to minimize sensing time. These work can be categorized into four major classes: Firstly, optimizing P_d and P_f such that the performance of the CR nodes are optimized [3]. Secondly, reducing inter-sensing time while monitoring [4], [5]. Thirdly, reducing the required monitoring time [5], [6], [7]. Lastly, reducing search time [22], [8], [9].

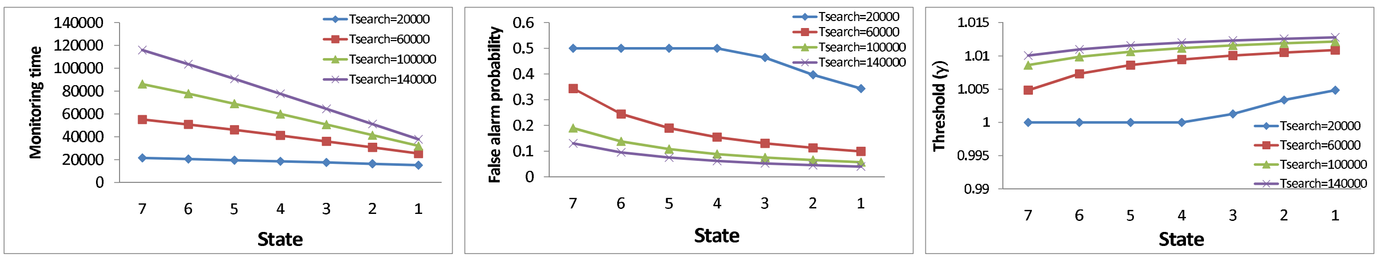
In [3], the paper presented a REM-assisted threshold adaptation technique for spectrum sensing. The approach tried to maximize the total utility of primary and secondary systems. The detection probability was defined as a function of the false alarm probability. Due to the concavity of the receiver operating characteristics (ROC) curve, there exists an optimal point that achieves optimal (P_d^*, P_f^*). The optimal threshold was found to optimize P_d and P_f. But, this approach did not minimize the sensing time. In our paper, we find the optimal sensing time as well as the threshold. Our solution can satisfy the detection probability constraint and make false alarm probability adaptive such that the total sensing time is minimized.

[5] studied how to decide monitoring time to maximize the achievable throughput of the CR nodes under the constraint that the PUs are sufficiently protected. They found the sensing time, and for that sensing time, they find the threshold. [7] discussed the issue of the optimal detection time selection that leads to the highest channel efficiency. They first gave the definition of channel efficiency and formulated it as an optimization problem. Then, they developed a numerical optimization algorithm from which numerical solutions and numerical insights to the problem obtained. For monitoring



(a) (b)

Fig. 6: Required number of channels (K) to achieve minimum sensing times shown in Figure a) 5.a and b) Figure 5.b



(a) Monitoring time (b) False alarm probability (P_f) (c) Threshold (γ)

Fig. 7: Effects of $P(\mathcal{H}0)$ on the required monitoring time

in our approach, we jointly find the sensing time and the detection threshold. From the calculated sensing time and threshold, we find the false alarm probability that achieves the minimum sensing time while protecting the PU. The PU model for the idle state is different from theirs which also affect the optimal monitoring time. Moreover, we introduce a solution for search optimization.

[6] studied both optimizing monitoring and search times. In particular, the optimal sensing times for channel-search and channel-monitoring were obtained in a way to maximize the average throughput of the CR node while protecting the PU from harmful interference. They handled only one channel, and found the optimal sensing time for a channel given the threshold. They did not adapt the threshold, the number of channels, and the false alarm probability to minimize the sensing time. Also, they used average value analysis without taking care of heterogeneous channels characteristics.

The authors in [22] introduced the multi-band joint detection framework for wide-band spectrum sensing in a single CR. They jointly optimized a bank of multiple narrow-band detectors to improve the aggregate opportunistic throughput of a CR system while limiting the interference to the PU. They formulated the design of wide-band spectrum sensing into a class of optimization problems. They exploited the hidden convexity of the seemingly non-convex problems to show that the optimization problem can be reformulated into a convex program under practical conditions. They developed search time optimization problem that finds the optimal thresholds for the sub-bands in order to collectively maximize the aggregate opportunistic throughput subject to some interference constraints for each PU. In our paper we find also the required sensing time for each channel and the optimal number of channels to be sensed. Also, we do sequential sensing instead of wide-band sensing.

Another trend to minimize the search time is by optimizing the order of the channels to be sensed. Kim and Shin [8] introduced a sensing-sequence that sorts channels in descending order of the probability of being idle. [9] finds a sensing-sequence that helps finding spectrum opportunities with minimal delay. To achieve their goal, [9] maintains two channel lists; back-up channel list (BCL) and candidate channel list (CCL), and developed a method to construct and update BCL by importing/exporting channels from/to CCL. However, such algorithms maximize the chance of finding an idle channel. They reduce the overall sensing time by optimizing the order of channels. But, they do not optimize the sensing time per channel. We can use these sorting algorithms in our work to find the order of channels to be followed during search.

Some other approaches try to minimize P_f. [20] finds the optimal sensing time and inter-sensing duration τ given P_d and P_f when an energy detector with a certain threshold γ is used. They do not relax P_f and they assume known threshold value. [21] finds the achievable minimum probability of false alarm in the network given a targeted probability of detection for the channel. They also find the achievable maximum P_d in the network given targeted P_f. In our work, we proved that relaxing P_f enhances sensing time.

In [4], a large-scale measurement-driven characterization of primary usage in cellular networks was conducted. They optimized the inter-sensing time, derived a formula for optimal inter-sensing time, and showed that large variations in inter-sensing time exist for different PU and different detection required probabilities. [5] also minimizing inter-sensing time. We can use the results of their work in monitoring since we do not consider inter-sensing time optimization.

VII. Conclusion And Future Work

In this paper we developed a PU multi-idle states model which allows the CR node to benefit from the previous sensing measurements. Based on this model, convex non-linear optimization formulations were introduced for monitoring and search. Monitoring optimization finds the sensing time, the detection threshold, and the false alarm probability of the channel being used. Search optimization formulation has more degrees of freedom than previous work, it jointly finds: the sensing time of each channel (N_i), the energy detection threshold of each channel (γ_i), the number of channels to sense, and the false alarm probability of each channel ($P_f(i)$), such that the sensing time is minimized, the PU is protected, and the CR node finds an idle channel with very high probability. The optimization considered channels with different characteristics.

We also proved that by relaxing P_f, we reduce the total sensing time; where in some cases sensing more channels with higher P_f, requires less time than sensing fewer channels with more strict P_f. Numerical results showed that in monitoring, in some cases, sensing with higher P_f is better. Moreover in search, sensing more channels under forcing P_f to be less than 0.5 requires less sensing time than forcing it to be less than 0.1 or 0.05.

As a future work, we are planning to design a sorting algorithm that also minimizes the search time. The sorting algorithm should be considered in the optimization formulation. Moreover, we are planning to conduct either simulation experiments or collect real experimental data that allow us to model the multi idle states with more realistic values.

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