Design and Provisioning of WDM Networks with Many-to-Many Traffic Grooming

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Abstract—A large number of network applications today allow several users to interact together using the many-to-many service mode. In many-to-many communication, also referred to as group communication, a session consists of a group of users (we refer to them as *members*), where each member transmits its traffic to all other members in the same group. In this paper, we address the problem of designing and provisioning of WDM networks to support many-to-many traffic grooming. Our objective is to minimize the overall network cost which is dominated by the cost of higher layer electronic ports (i.e., transceivers) and the number of wavelengths used. Based on different WDM node architectures, we propose four different WDM networks for many-to-many traffic grooming. For each network, we analyze the many-tomany traffic grooming problem and provide an optimal as well as a heuristic solution. A comprehensive comparison between the four networks reveals that each of the networks is the most cost-effective choice for a certain range of traffic granularities.

I. INTRODUCTION

In wavelength routing networks, using wavelength division multiplexing (WDM), it is feasible to have hundreds of wavelengths, each operating at 10 to 40 Gbps, per fiber. Bandwidth requirements of user sessions, however, are usually of subwavelength granularities. For example, an MPEG compressed HDTV channel requires less than 20 Mbps of bandwidth. In order to reduce this huge bandwidth gap, *traffic grooming* was introduced to allow a number of sessions with sub-wavelength granularities to share the bandwidth of a wavelength channel.

Early Internet applications such as TELNET and FTP are characterized as unicast or "one-to-one". A large portion of network applications today, however, are of the multipoint type. For example, video distribution and file distribution are examples of multicast or "one-to-many" applications, while resource discovery and data collection are examples of many-toone or "inverse multicasting" applications. Recently, another set of network applications has emerged such as multimedia conferencing, e-science applications, distance learning, distributed simulations, and collaborative processing [1]. In these applications, each of the participating entities both contributes and receives information to and from the other entities in the same communication session, and therefore are characterized as "many-to-many". In many-to-many communication, also referred to as group communication [2], a session consists of a group of users (we refer to them as *members*), where each



Fig. 1. A many-to-many session with members $\{A, B, C, D\}$ each with traffic denoted as a, b, c and d, respectively.

member transmits its traffic to all other members in the same group (see Fig. 1).

In this paper, we address the problem of many-to-many traffic grooming in WDM mesh networks. This problem can be formulated as follows. Given an arbitrary WDM network topology and a set of subwavelength many-to-many traffic demands, determine: 1) The set of optical channels (lightpaths and light-trees) to establish, 2) How to route and groom each of the subwavelength many-to-many traffic demands on these optical channels, and 3) The route and the wavelength to assign to each of the optical channels on the WDM network. The first two parts of the problem are referred to as the *Virtual Topology and Traffic Routing* (VTTR) problem, while the third part of the problem is referred to as the *Routing and Wavelength Assignment* (RWA) problem.

The cost of an optical network is dominated by the cost of higher layer electronic ports such as IP router ports, MPLS Label Switching Router (LSR) ports and SONET ADM ports (we will refer to these ports as *transceivers*). A transceiver is needed for each initiation or termination of an optical channel. For example, a lightpath requires two transceivers while a light-tree with N endpoints requires N transceivers. The number of wavelengths used also adds to the overall network cost. Therefore, our objective in the many-to-many traffic grooming problem is to minimize the number of transceivers used (R) and the number of wavelengths used (W).

A. Related Work:

Traffic grooming has been extensively studied for unicast traffic [3]-[13]. In [5], the authors proposed optimal and nearoptimal algorithms for traffic grooming in SONET WDM rings with the objective of minimizing the number of wavelengths and SONET ADMs. In [6], the authors proposed an auxiliary graph model for traffic grooming in heterogeneous WDM

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mesh networks and developed an integrated traffic grooming algorithm that jointly solves the traffic grooming subproblems. In [7], approximation algorithms for minimizing the total equipment cost and for minimizing the lightpath count were introduced. In [8], the authors provided a hierarchical framework for traffic grooming in a WDM network with an arbitrary topology. For an account of recent advances in unicast traffic grooming, the reader is referred to [14].

Traffic grooming has also been considered for multicast traffic [9], [15]-[20]. In [15], the authors addressed the multicast traffic grooming problem in metropolitan WDM ring networks with the objective of minimizing electronic copying. In [16], the authors introduced a graph based heuristic for the multicast traffic grooming problem in unidirectional SONET/WDM rings and compared it to the multicast extension of the best known unicast traffic grooming heuristic in [5]. In [17], the authors addressed the multicast traffic grooming problem in WDM mesh networks. They provided MILP formulations and also developed heuristic solutions. In [9], the authors considered the multicast traffic grooming problem in WDM mesh networks with sparse nodal light splitting capability. In [18], a non-linear programming formulation followed by a number of heuristic solutions were introduced for the multicast traffic grooming problem in WDM mesh networks with nodal light splitting capability. In [19], the authors addressed the problem of many-to-one traffic grooming in WDM mesh networks with the objective of minimizing the number of wavelengths and SONET ADMs. For an account of recent advances in multicast traffic grooming, the reader is referred to [20]-[21].

To the best of our knowledge, many-to-many traffic grooming is a new field of research that has been only considered in [10], [22]-[24]. In [22], the authors addressed the manyto-many traffic grooming problem in WDM ring networks with the objective of reducing the overall network cost. In our previous works [23]-[24], MILP formulations were introduced for the many-to-many traffic grooming problem in WDM mesh networks. The work in this paper is different from [23]-[24] in four important aspects: 1) In this work, we provide an efficient and a practical approach to solve the manyto-many traffic grooming problem by dividing it into two smaller problems and solving each independently, while in [23], the two subproblems were jointly considered resulting in an extremely hard problem that can only be solved for small networks, 2) Deriving properties of the optimal virtual topology for single and multiple many-to-many sessions in special cases which was the key in designing efficient nearoptimal heuristics for the general case, 3) This work proposes four different WDM network architectures for many-to-many traffic grooming and provides a comprehensive cost comparison between them, while in [23]-[24], only two network architectures were considered, 4) The cost of a WDM network in this work includes both the number of transceivers (R) and the number of wavelengths (W), while in [23]-[24], only the number of transceivers (R) was included in the cost.

B. Contributions:

The objective of this paper is to study the many-to-many traffic grooming problem in four different WDM network architectures. For two of the network architectures, our main contribution is the introduction of *lightpath cycles*. A lightpath cycle for a many-to-many session is a cycle of lighpaths that visits all members in the session (a formal definition will be given later). We will show that this cycle structure is the optimal virtual topology for single and multiple many-to-many sessions in certain special cases. Based on lightpath cycles, efficient near-optimal heuristics are developed for the general case of many-to-many traffic grooming. In another netwok architecture, we introduce a novel approach that combines optical splitting and network coding [25] to provision many-tomany sessions and we derive an optimal as well as a heuristic solution. Another contribution of this paper is a comprehensive comparison between the four networks that reveals that each of the networks is the most cost-effective choice for a certain range of traffic granularities.

C. Solution Approach:

It was shown in [3] that the unicast VTTR problem without the RWA problem is NP-hard. Since the RWA problem is also an NP-hard problem, then the overall traffic grooming problem is considered extremely hard. To obtain efficient and practical solutions to the traffic grooming problem, many researchers have adopted a decomposition approach that divides the traffic grooming problem into its subproblems and then solve each independently [3]-[10]. In [4], it was shown that this decomposition approach is efficient, practical and gives near-optimal solutions. In this work, we follow this approach and solve each of the VTTR and the RWA problems separately. More specifically, given the subwavelength many-to-many traffic demands, we first solve the VTTR problem by determining the virtual topology and the corresponding routing and grooming of each of the traffic demands with the objective of minimizing the cost R. Afterwards, we map the virtual topology on the physical WDM network topology by solving the RWA problem with the objective of minimizing the cost W. This decomposition approach also simplifies our analysis and allows us to derive useful properties of the optimal solution which will guide us to design efficient near-optimal algorithms for the many-to-many traffic grooming problem.

D. Paper Organization:

The rest of the paper is organized as follows. In Section II, we introduce different node and network architectures for many-to-many traffic grooming. In Section III, we address the VTTR problem in each of the network architectures proposed. In Section IV, we address the RWA problem. In Section V, we present experimental results and provide a comprehensive cost comparison between the different network architectures. In Section VI, we conclude the paper.

II. NODE AND NETWORK ARCHITECTURES

Designing optical WDM networks is greatly influenced by the architecture of the optical node. The following are the node architectures that we consider:

1) Opaque Node Architecture: All incoming traffic must undergo optical-to-electronic (*O/E*) conversion even if the traffic is not intended for the node. Transit traffic is switched in the electronic domain and then converted back to the optical domain for the next transmission.

2) Transparent without Optical Splitting Node Architecture: Incoming traffic not intended for the node may be switched in the optical domain without any (O/E) conversion. If the incoming traffic, however, is intended for multiple recipients or it needs to be groomed with other traffic, then (O/E) conversion is needed since traffic duplication and traffic grooming can only take place in the electronic domain.

3) *Transparent with Optical Splitting Node Architecture:* Same as transparent without optical splitting, except that multiple copies of the incoming traffic can be generated in the optical domain (using *optical splitters*) without any (*O/E*) conversion.

Based on these node architectures, we propose the following WDM networks for many-to-many traffic grooming.

Non-Splitting Opaque WDM (NSOWDM) Network: In this network, all the nodes are opaque and therefore it supports lightpaths that can only span a single physical link. A lightpath may groom traffic from different sessions and traffic from different members within the same session. This network is efficient in traffic grooming and wavelength utilization; however, it has a relatively high transceiver cost. It will be shown that this network is suitable and cost-effective for traffic granularities that are relatively low (e.g., less than one-quarter of the capacity of a wavelength).

Non-Splitting Transparent WDM (NSTWDM) Network: In this network, all the nodes are transparent without optical splitting and therefore it supports lightpaths that may span multiple physical links. A lightpath may groom traffic from different sessions and traffic from different members within the same session. Note that the NSOWDM network is a special case of the NSTWDM network and therefore it always requires at least the same number of lightpaths as the NSTWDM network. However, due to the wavelength continuity constraint, NSTWDM networks generally consume more wavelengths than NSOWDM networks. It will be shown that NSTWDM networks are also suitable and cost-effective for low traffic granularities.

Splitting Hubbed WDM (SHWDM) Network: In this network, all the nodes are transparent with optical splitting and therefore it supports lightpaths and light-trees that may span multiple physical links. Each many-to-many session has a designated hub node chosen from its set of members. All the members besides the hub transmit their traffic to the hub through direct lightpaths (upstream traffic). Using the new technique of network coding, the hub then linearly combines the traffic units received together with its own traffic units to generate a set of linear combinations. These combinations

are then groomed and sent back to the members using direct light-tree(s) (downstream traffic), see Fig. 4.(a) page 9. Each of the members will be able to recover the original traffic units transmitted by the other members in the same session by linearly combining its own traffic units with the received combinations. It will be shown that this network is suitable and cost-effective for traffic granularities that are around the half of the capacity of a wavelength.

Splitting All-Optical WDM (SAOWDM) Network: In this network, all the nodes are transparent with optical splitting. Each member in a many-to-many session transmits it traffic directly to all other members in the same session using a light-tree. Note that no traffic grooming is performed in this network, and therefore it is suitable and cost-effective for traffic granularities that are close to the full capacity of a wavelength.

III. VIRTUAL TOPOLOGY AND TRAFFIC ROUTING PROBLEM

In the VTTR problem, we need to determine what optical channels (lightpaths and light-trees) to establish and how to route and groom each of the subwavelength many-to-many traffic demands on these optical channels. The objective is to minimize the total number of transceivers used (R). As indicated by the following theorem, the many-to-many VTTR problem is an NP-hard problem.

Theorem 1. The many-to-many VTTR Problem is NP-hard.

Proof: It was shown in [3] that the unicast or one-toone VTTR problem is NP-hard. Since the one-to-one VTTR problem is a special case of the many-to-many VTTR problem (each many-to-many session has only two members), then by generalization, the many-to-many VTTR problem is NP-hard.

In this section, we analyze the VTTR problem in each of the four WDM networks proposed above. The following are the assumptions and notations used in the paper:

- The optical WDM network has an arbitrary topology represented by an undirected graph G(V, E), with a set of nodes V and a set of physical links E, where each physical link $e \in E$ is composed of two unidirectional fibers \vec{e} in opposite directions.
- The number of wavelengths per fiber is the same among all fibers and is denoted by W_{max} , while the capacity of a wavelength channel is g units of traffic (the unit of traffic may be, e.g., an OC-3 circuit).
- There is a total of K many-to-many session requests, where each session s_k $(1 \le k \le K)$ has a set of members $m_{s_k} \subseteq V$ with cardinality $N_{s_k} = |m_{s_k}|$. We assume uniformity of traffic within the same session, that is, each member in m_{s_k} has the same traffic demand t_{s_k} , where $1 \le t_{s_k} \le g$.
- We define $H_{s_k} = \lceil (N_{s_k} 1)t_{s_k}/g \rceil$ to be a lower bound on the number of incoming optical channels to a member in a session s_k in order to receive the traffic from the other $N_{s_k} - 1$ members in the same session.



Fig. 2. (a): PPLC (which is also a MIN-PPLC) for a many-to-many session s_k with a set of members $m_{s_k} = \{A, B, C, D\}$ each with one traffic unit denoted as a, b, c and d, respectively $(g = 3, H_{s_k} = 1)$. (b): Optimal provisioning of many-to-many sessions s_1 , s_2 and s_3 where $m_{s_1} = \{A, B, D\}$ each with one traffic unit denoted as a1, b1, d1, and $m_{s_2} = \{B, D, E\}$ each with one traffic unit denoted as b2, d2, e2, and $m_{s_3} = \{A, C, E\}$ each with one traffic unit denoted as a3, c3, e3 (g = 4).

• We require that the t_{s_k} traffic units originating from a member and destined to another member in a session s_k must not be bifurcated into a set of lower speed streams each taking a different route on the virtual topology.

Next, we consider the VTTR problem in each of the four WDM networks proposed above.

A. Non-Splitting Opaque WDM Network

In a NSOWDM network, a lightpath can only span a single physical link and it may groom traffic from different sessions, and traffic from different members within the same session.

Definition 1. Given a many-to-many session s_k :

- 1) A point-to-point lightpath-cycle (PPLC) for s_k is a (possibly non-simple) cycle of lightpaths that visits each member in m_{s_k} at least once given that a lightpath can only span a single physical link.
- A minimum point-to-point lightpath-cycle (MIN-PPLC) for sk is a PPLC for sk with the minimum number of lightpaths traversed.

An example of a PPLC (which is also a MIN-PPLC) for a many-to-many session s_k with a set of members $m_{s_k} = \{A, B, C, D\}$ is shown in Fig. 2.(a). Note that, depending on the physical topology, it may not always be possible to find a simple cycle of lightpaths that visits each member in m_{s_k} . Therefore, a PPLC for s_k may be a non-simple cycle of lightpaths that visits a node more than once. A MIN-PPLC for a many-to-many session serves as an optimal virtual topology in a special case, as indicated by the following theorem:

Theorem 2. An optimal virtual topology that minimizes the total number of transceivers required to provision a single many-to-many session s_k in a NSOWDM network when $H_{s_k} = 1$ consists of a MIN-PPLC for s_k .

Proof: First, we prove that any feasible virtual topology to provision s_k must contain a PPLC for s_k . Then, we prove that a PPLC for s_k by itself is feasible to provision s_k when $H_{s_k} = 1$. Then, it follows that a MIN-PPLC for s_k is an optimal virtual topology when $H_{s_k} = 1$ since it is a PPLC for s_k with the minimum number of lightpaths or transceivers.

Any feasible virtual topology to provision s_k must include a path from any member to any other member in m_{s_k} . This follows from the definition of the many-to-many traffic type where each member should transmit(receive) to(from) all the other members in the same session. Therefore, any order of the members in this virtual topology must form a PPLC for s_k that may visit a member multiple times.

To prove that a PPLC for s_k is feasible to provision s_k when $H_{s_k} = 1$, we must guarantee that in a PPLC for s_k each member in m_{s_k} receives the traffic from all the other $N_{s_k} - 1$ members in the same session and that the capacity of a lightpath is not exceeded. Now, by letting each member in m_{s_k} to transmit its traffic in the PPLC until it reaches the member just before it in the cycle (see Fig. 2.(a)), we guarantee two things. First, exactly $(N_{s_k} - 1)t_{s_k}$ traffic units are groomed between each pair of consecutive members in the PPLC and since $H_{s_k} = 1$, then a single lightpath is sufficient to groom this traffic. Second, each member in m_{s_k} receives the traffic from all the other $N_{s_k} - 1$ members in the same session. Therefore, a PPLC for s_k is a feasible virtual topology.

Note that a MIN-PPLC for s_k is the only optimal virtual topology to provision s_k when $H_{s_k} = 1$ since, as we proved, any feasible virtual topology to provision s_k must include a PPLC for s_k and a MIN-PPLC for s_k is a PPLC with the minimum number of transceivers. Unfortunately, finding a MIN-PPLC for a many-to-many session s_k is a hard problem, as indicated by the following theorem:

Theorem 3. Finding a MIN-PPLC for a many-to-many session s_k is NP-hard.

Proof: We define the decision version of the PPLC problem as follows. Given a network represented by an undirected graph G(V, E), a many-to-many session s_k with a set of members $m_{s_k} \subseteq V$ and an integer c, the problem asks whether or not there is a PPLC for s_k in G that has at most c lightpaths. Now, consider any instance G'(V', E') of the undirected Hamiltonian cycle problem. We construct an instance of the decision version of the PPLC problem by setting G = G', $m_{s_k} = V'$ and c = |V'|. If the answer is "yes" to the decision version of the PPLC problem, then

TABLE I decision variables used in the ILP for the VTTR problem in a NSOWDM network which are only defined when $P_{ij} = 1$

L_{ii} :	number of lightpaths from node <i>i</i> to node <i>j</i>						
• 5	$(i \neq j)$						
$Z_{ii}^{s_k, p, q}:$	binary number to indicate whether or not the						
-)	traffic stream originating from member $p \in$						
	m_{s_k} and destined to member $q \in m_{s_k}$ $(q \neq p)$						
	is routed on a lightpath from i to j .						
$Y_{ij}^{s_k,p}$:	binary number to indicate whether or not a traf-						
	fic stream originating from member $p \in m_{s_k}$						
	and destined to at least one other member in						
	m_{s_k} is routed on a lightpath from <i>i</i> to <i>j</i> .						

this PPLC must have exactly |V'| lightpaths since it needs to visit each member in $m_{s_k} = V'$ at least once. This means that this PPLC must visit each node in V' exactly once, and therefore it will be a Hamiltonian cycle (hence, the answer is "yes" to the Hamiltonian cycle problem). On the other hand, if the answer is "yes" to the Hamiltonian cycle problem, then this Hamiltonian cycle is a PPLC of size |V'|, and hence the answer is "yes" to the decision version of the PPLC problem. This proves that the decision version of the PPLC problem is NP-complete, and hence the optimization version (MIN-PPLC) is NP-hard.

This proves the hardness of the VTTR problem in a NSOWDM network for the simplest case of a single manyto-many session and $H_{s_k} = 1$. In the case where $H_{s_k} \ge 2$, the optimal virtual topology for a session s_k becomes harder to characterize and in the case of multiple many-to-many sessions, the problem becomes even harder due to the correlation between the sessions and the possibility of grooming traffic from different sessions on the same lightpath. Next, we formulate the VTTR problem in a NSOWDM network as an Integer Linear Program (ILP).

1) ILP Formulation: We first define P_{ij} as an input binary number to indicate whether or not there is a physical link between nodes *i* and *j* ($P_{ij} = P_{ji}$). The decision variables used in the ILP which are only defined when $P_{ij} = 1$ (since it is a NSOWDM network) are shown in Table I.

The objective of the ILP is to minimize the total number of lightpaths or transceivers:

$$\textit{Minimize } \sum_{i \in V} \sum_{j: P_{ij} = 1} L_{ij}.$$

Subject to the following constraints:

$$\sum_{i:P_{ix}=1} Z_{ix}^{s_{k},p,q} - \sum_{j:P_{xj}=1} Z_{xj}^{s_{k},p,q} = \begin{cases} 1, & \text{if } x = q \\ -1, & \text{if } x = p \\ 0, & \text{otherwise} \end{cases}$$
(1)
$$\forall s_{k}, \ p, q \in m_{s_{k}}, \ x \in V \end{cases}$$

$$Y_{ij}^{s_k,p} \ge \sum_{q \in m_{s_k}} Z_{ij}^{s_k,p,q} / N_{s_k} \quad \forall s_k, \ p \in m_{s_k}, \ i,j: P_{ij} = 1 \quad (2)$$

$$Y_{ij}^{s_k,p} \le \sum_{q \in m_{s_k}} Z_{ij}^{s_k,p,q} \quad \forall s_k, \ p \in m_{s_k}, \ i,j: P_{ij} = 1$$
(3)

$$L_{ij} \ge (\sum_{s_k} t_{s_k} \sum_{p \in m_{s_k}} Y_{ij}^{s_k, p})/g \quad \forall i, j : P_{ij} = 1$$
(4)

Constraint (1) is the flow routing constraint between each pair of members (in both directions) in a many-to-many session. Constraints (2) and (3) together set the variable $Y_{ij}^{s_k,p}$ as the logical disjunction of all the variables $Z_{ij}^{s_k,p,q}$ for all values of $q \in m_{s_k}, q \neq p$. In other words, $Y_{ij}^{s_k,p}$ will be set to 1 if at least one of the traffic streams that originate at member puses a lightpath from i to j; otherwise it is set to zero. Finally, constraint (4) computes the total number of lightpaths needed on each physical link in the network.

2) Heuristic Solution: Since the ILP has an exponential time complexity, we now introduce an efficient heuristic approach to obtain near-optimal solutions for large sized instances of the problem. As a first step, we need to find an efficient way of finding a PPLC for a session s_k with a number of lightpaths close to that of a MIN-PPLC for that session. Finding a PPLC for a session s_k in G requires us to determine two things. First, the order of the members in the PPLC, and then the path to take in G between each pair of consecutive members in the PPLC. Since we are minimizing the number of lightpaths (or links, since a lightpath can only span a single physical link), then the shortest path would be the obvious choice for the second part of the problem. The first part, however, (ordering the members) is what makes the problem hard. A very similar problem that requires this kind of hard ordering is the well-known traveling salesman problem (TSP). We map our problem to the TSP as follows. Each member in m_{s_k} corresponds to a city in the TSP instance, and the cost of traveling between two cities is the number of links on the shortest path between the corresponding members in G. Finding a least cost tour in the TSP instance becomes equivalent to finding a MIN-PPLC for s_k in G.

One of the simplest and yet powerful heuristics for the TSP is the Nearest Neighbor (NN) Algorithm, where a random member is first selected and the next member is the one with the shortest distance from the current one in G. This process is repeated until we cover all the members and determine a PPLC for that session.

After careful examination of the ILP results for small sized instances of the problem and for multiple sessions, we have noticed that many-to-many sessions tend to be provisioned through PPLCs where, for each session s_k , $(N_{s_k} - 1)t_{s_k}$ traffic units are groomed between each pair of consecutive members in the PPLCs. Since a lightpath may groom traffic from different sessions and not just traffic from different members within the same session, PPLCs of different sessions are correlated and may share lightpaths. Fig. 2.(b) clarifies this point by illustrating the optimal provisioning of three manyto-many sessions s_1 , s_2 and s_3 each with a set of members $m_{s_1} = \{A, B, D\}, \ m_{s_2} = \{B, D, E\} \text{ and } m_{s_3} = \{A, C, E\},$ respectively (g = 4). Note that the PPLC for $s_1 (A - B - D - A)$ and the PPLC for s_3 (A - B - C - E - D - A) share lightpaths $A \rightarrow B$ and $D \rightarrow A$, while the PPLC for s_2 (B - C - E - D - B) and the PPLC for s_3 share lightpaths

	Algorithm 1. VTTR Heuristic: NSOWDM Network				
	input : $G(V, E)$, K many-to-many session requests.				
	output: Virtual Topology (VT) , Routing of the K sessions on VT .				
1	sort sessions in a list S in a descending order in terms of				
	$((N_{s_k} - 1)t_{s_k})\% g.$				
2	for each session s_k in the sorted list S do				
3	order members in m_{s_k} according to the (NN) Algorithm where				
	the nearest member from the current member is the one who has				
	the shortest distance in G from the current member. The first				
	member is selected randomly.				
4	for $i = 0, 1, m_{s_k} - 1$ do				
5	provision as much traffic as possible out of the $(N_{s_k} - 1)t_{s_k}$				
	traffic units between members $m_{s_k}[i]$ and $m_{s_k}[i+1]$ using				
	the current virtual topology (VT) .				
6	for the remaining unprovisioned traffic t' (if any), establish				
	$\lceil max(0,t'-c_{\overrightarrow{e}})/g \rceil$ lightpaths on each link \overrightarrow{e} on the				
	shortest path between members $m_{s_k}[i]$ and $m_{s_k}[i+1]$ in G,				
	where the cost of a link \overrightarrow{e} in G is $\lceil max(0, t' - c_{\overrightarrow{e}})/g \rceil$.				
7	end				
8	end				

 $B \to C, \ C \to E \text{ and } E \to D.$

The heuristic we propose is based on the observation that many-to-many sessions tend to be provisioned through PPLCs and that PPLCs of different sessions may share of lightpaths. Given K many-to-many session requests, the heuristic tries to build a virtual topology (which is initially empty) to accommodate the K sessions with the minimum number of lightpaths or transceivers. The current virtual topology is represented in the heuristic as a directed graph VT with a set of nodes that includes every node in G that at least has one lightpath incoming or outgoing. A directed edge \vec{e} in VT exists only if there is at least one lightpath on link \vec{e} in G. Each directed edge \vec{e} in VT has a capacity $c_{\vec{e}}$ representing the remaining capacity on lightpaths on link \vec{e} in G.

The heuristic (shown in Algorithm 1) has three main steps. First, it sorts sessions in a list S in a descending order in terms of $((N_{s_k} - 1)t_{s_k})\% g$ (line 1). Second, for each session s_k in the sorted list S, it orders members in m_{s_k} according to the NN Algorithm (lines 2-3). Note that this is the order of the members in the sessions' PPLCs. Finally, for each session s_k , it provisions the $(N_{s_k}-1)t_{s_k}$ traffic units between each pair of consecutive members in the ordered m_{s_k} (lines 4-7). The heuristic attempts to provision as much traffic as possible out of the $(N_{s_k} - 1)t_{s_k}$ traffic units using the existing current virtual topology VT (line 5). This is done by running a maxflow algorithm (Push-relabel with FIFO vertex selection rule [26]) between the two members in VT (with edge capacities $|c_{\vec{e}}/t_{s_k}|$). Note that by setting the edge capacities in the maxflow instance to $\lfloor c_{\overrightarrow{e}}/t_{s_k} \rfloor$, we guarantee that the t_{s_k} traffic units originating from a member will not bifurcate among different routes on VT. For the remaining unprovisioned traffic t' (if any), the heuristic establishes $\lfloor max(0, t' - c_{\overrightarrow{e}})/q \rfloor$ lightpaths on each link \overrightarrow{e} on the shortest path between the two members in G (line 6). Note that the shortest path here corresponds to the path that requires the fewest number of lightpaths to provision t'.

Example: consider the 6-node network shown in Fig. 2 with three many-to-many sessions s_1 , s_2 and s_3 each with a set

of members $m_{s_1} = \{A, B, E, F\}, m_{s_2} = \{B, C, D\}$ and $m_{s_3} = \{A, B\}$, respectively. For the sake of this example, lets assume that $t_{s_1} = 1$, $t_{s_2} = 2$, $t_{s_3} = 3$ and g = 8. The heuristic first sorts sessions as follows $S = \{s_2, s_1, s_3\}$. Then, it orders members in session s_2 as follows $m_{s_2} = \{B, C, D\}$ and establishes lightpaths $B \rightarrow C, C \rightarrow E, E \rightarrow D$ and $D \rightarrow B$ each carrying four units of traffic (PPLC for $s_2 =$ $\{B-C-E-D-B\}$). The heuristic then orders members in session s_1 as follows $m_{s_1} = \{A, B, F, E\}$. It then establishes lightpaths $A \rightarrow B, C \rightarrow F, F \rightarrow E$ and $D \rightarrow A$ each carrying three units of traffic and provisions three units of traffic on lightpaths $B \to C$ and $E \to D$ which will now carry seven units of traffic (PPLC for $s_1 = \{A - B - C - F - F \}$ E - D - A). Finally, the heuristic orders members in session s_3 as follows $m_{s_3} = \{A, B\}$. It then establishes lightpath $B \rightarrow A$ carrying three units of traffic and provisions three units of traffic on lightpath $A \rightarrow B$ which will now carry six units of traffic (PPLC for $s_3 = \{A - B - A\}$). This results in 9 lightpaths (18 transceivers).

B. Non-Splitting Transparent WDM Network

In a NSTWDM network, a direct lightpath (that may span multiple physical links) can be established between any two nodes in the network. A lightpath may groom traffic from different sessions and traffic from different members within the same session.

Definition 2. A transparent lightpath cycle (TLC) for a manyto-many session s_k is a simple cycle of N_{s_k} lightpaths that visits each member in m_{s_k} exactly once given that a lightpath may span multiple physical links.

An example of a TLC for a many-to-many session s_k with a set of members $m_{s_k} = \{A, B, C, D\}$ is shown in Fig. 3.(a). Note that there is always N_{s_k} lightpaths in the TLC for s_k regardless of the order of the members and regardless of the underlying physical topology (A TLC only describes a virtual topology). TLCs serve as an optimal virtual topology, as indicated by the following theorem:

Theorem 4. An optimal virtual topology that minimizes the total number of transceivers required to provision a single many-to-many session s_k in a NSTWDM network consists of H_{s_k} . TLCs for s_k , all with the same order of members.

Proof: Any feasible virtual topology to provision s_k must at least have a total of $N_{s_k}H_{s_k}$ lightpaths. This is due to the fact that each member in m_{s_k} must at least have H_{s_k} lightpaths incoming to receive its traffic. Note that H_{s_k} TLCs for s_k have exactly $N_{s_k}H_{s_k}$ lightpaths. Therefore, if we prove it is a feasible virtual topology then it will also be an optimal one. Now, by letting each member to transmit its traffic in the H_{s_k} identically ordered TLCs until it reaches the member just before it in the TLCs (see Figure 3.(a)), we guarantee two things. First, exactly $(N_{s_k} - 1)t_{s_k}$ traffic units are groomed between each pair of consecutive members in the TLCs and therefore H_{s_k} lightpaths are sufficient to groom this traffic. Second, each member in m_{s_k} receives the traffic from the



Fig. 3. (a): TLC for a many-to-many session s_k where $m_{s_k} = \{A, B, C, D\}$ each with one traffic unit denoted as a, b, c and d, respectively $(g = 3, H_{s_k} = 1)$. denoted as b3, c3, e3 (g = 4).

other $N_{s_k} - 1$ members in the same session. Therefore, H_{s_k} TLCs all with the same order of members is a feasible and an optimal virtual topology.

Hence, for a single many-to-many session s_k , the total number of transceivers required is:

$$R = 2H_{s_k}N_{s_k}$$

In the case of multiple many-to-many sessions, the VTTR problem is still hard due to the correlation between the sessions and the possibility of grooming traffic from different sessions on the same lightpath. However, in the following two special cases, the optimal virtual topology for multiple many-to-many sessions can be efficiently found. The first special case, which follows directly from Theorem 4, is when the member sets of the many-to-many sessions are pairwise disjoint. In this case, we have the following theorem:

Theorem 5. An optimal virtual topology that minimizes the total number of transceivers required to provision a set of many-to-many sessions $s_1, s_2, ..., s_K$ in a NSTWDM network when $m_{s_k} \cap m_{s_l} = \phi$ for all $1 \leq k \leq K$ and $1 \leq l \leq K$ consists of H_{s_m} TLCs for s_m (all with the same order of *members) for all* $1 \le m \le K$.

Proof: Since the member sets of the sessions are pairwise disjoint, then the argument made in theorem 4 can be applied to each of the sessions independently.

Hence, for this special case of multiple many-to-many sessions, the total number of transceivers required is:

$$R = 2\sum_{s_k} H_{s_k} N_{s_k}$$

The second special case is when $\left\lceil \frac{\sum_{i=1}^{K} (N_{s_i}-1)t_{s_i}}{g} \right\rceil = 1$, but first we make the following definition.

Definition 3. A transparent lightpath cycle (TLC) for a set of many-to-many sessions $s_1, s_2, ..., s_K$ is a simple cycle of $|\bigcup_{i=1}^{K} m_{s_i}|$ lightpaths that visits each member in the union set $\bigcup_{i=1}^{K} m_{s_i}$ exactly once given that a lightpath may span multiple physical links.

An example of a TLC for sessions s_1 and s_2 each with a set of members $m_{s_1} = \{A, B, C\}$ and $m_{s_2} = \{C, D, E\}$, respectively is shown in Fig. 3.(b). Note that there is always $|\bigcup_{i=1}^{K} m_{s_i}|$ lightpaths in the TLC for a set of sessions s_1, s_2, \dots, s_K regardless of the order of the members and regardless of the underlying physical topology (A TLC for a set of sessions only describes a virtual topology). A TLC for a set of sessions serves as an optimal virtual topology in a special case, as indicated by the following theorem:

Theorem 6. An optimal virtual topology that minimizes the total number of transceivers required to provision a set of many-to-many sessions $s_1, s_2, ..., s_K$ in a NSTWDM network when $\lceil (\sum_{i=1}^{n} (N_{s_i} - 1)t_{s_i})/g \rceil = 1$ consists of a TLC for $s_1, s_2, ..., s_K$.

$$s_1, s_2, \ldots, i=$$

Proof: Any feasible virtual topology to provision the set of sessions s_1, s_2, \dots, s_K must at least have a total of $\bigcup_{i=1}^{K} m_{s_i}$ lightpaths. This is due to the fact that each member in $\bigcup_{i=1}^{n} m_{s_i}$ must at least have one lightpath incoming to receive its traffic. Note that a TLC for s_1, s_2, \dots, s_K has exactly $|\bigcup_{i=1}^{K} m_{s_i}|$ lightpaths. Therefore, if we prove it is a feasible virtual topology then it will also be an optimal one. Now, by letting each member in $\bigcup_{i=1}^{K} m_{s_i}$ to transmit its traffic in the TLC until it reaches the last member interested in receiving this traffic (see Figure 3.(b)), we guarantee two things. First, exactly $\sum_{i=1}^{K} (N_{s_i} - 1)t_{s_i}$ traffic units are groomed between each pair of consecutive members in the TLC and since $\lceil (\sum_{i=1}^{K} (N_{s_i} - 1)t_{s_i})/g \rceil = 1$, then a single lightpath is sufficient to groom this traffic. Second, each member in $\bigcup_{i=1}^{K} m_{s_i}$ receives the traffic from all the other $N_{s_k}-1$ members in all sessions s_k where this member appears. Therefore, a TLC for $s_1, s_2, ..., s_K$ is a feasible and an optimal virtual topology.

Hence, for this special case of multiple many-to-many

sessions, the total number of transceivers required is:

$$R = 2|\bigcup_{i=1}^{K} m_{s_i}|$$

The general case of the VTTR problem, however, remains a hard problem due to the correlation between the sessions and the possibility of grooming traffic from different sessions on the same lightpath. Next, we formulate the VTTR problem in a NSTWDM network as an ILP.

1) ILP Formulation: In a NSTWDM network, a direct lightpath (that may span multiple physical links) can be established between any two nodes in the network. Therefore, the ILP formulation for the VTTR problem in a NSTWDM network will be exactly the same as the ILP formulation introduced earlier for the NSOWDM network except that the decision variables and the constraints are now defined for all values of $i, j \in V$ ($i \neq j$) and not just when $P_{ij} = 1$.

2) Heuristic Solution: After careful examination of the ILP results for small sized instances of the problem and for multiple sessions, we have noticed that many-to-many sessions tend to be provisioned through lightpath cycles, where for each session s_k , $(N_{s_k} - 1)t_{s_k}$ traffic units are groomed between each pair of consecutive members in the lightpath cycles. Since a lightpath may groom traffic from different sessions and not just traffic from different members within the same session, lightpath cycles of different sessions are correlated and may share lightpaths. Also, a lightpath cycle for a session s_k may not be transparent (i.e., number of lightpaths in the lightpath cycle for s_k may be $> N_{s_k}$). Fig. 3.(c) clarifies these points by illustrating the optimal provisioning of three manyto-many sessions s_1 , s_2 and s_3 each with a set of members $m_{s_1} = \{A, B, C\}, \ m_{s_2} = \{B, C, D\} \text{ and } m_{s_3} = \{B, C, E\},$ respectively. Note that the TLC for $s_1 (A - B - C - A)$ and the TLC for s_2 (B-C-D-B) share lightpath $B \to C$, while the TLC for s_1 and the lightpath cycle for s_3 (B - E - C - A - B)which is not transparent) share lightpaths $C \to A$ and $A \to B$.

The heuristic we propose for the VTTR problem in NST-WDM networks is based on the observation that many-tomany sessions tend to be provisioned through lightpath cycles (which may not be transparent) and that lightpath cycles of different sessions may share lightpaths. Given K many-to-many session requests, the heuristic tries to build a virtual topology (which is initially empty) to accommodate the K sessions with the minimum number of lightpaths or transceivers. The current virtual topology is represented in the heuristic as a directed graph VT with a set of nodes that includes every node in Gthat at least has one lightpath incoming or outgoing. A directed edge from node i to node j exists in VT only if there exists at least one lightpath from node i to node j in G. Each edge (i, j)in VT has a capacity c_{ij} representing the remaining capacity on lightpaths from node i to node j in G.

The heuristic (shown in Algorithm 2) has three main steps. First, it sorts sessions in a list S in a descending order in terms of $((N_{s_k} - 1)t_{s_k})\% g$ (line 1). Second, for each session 8

Algorithm 2. VTTR Heuristic: NSTWDM Network **input** : K many-to-many session requests output: Virtual Topology VT, Routing of the K sessions on VTsort sessions in a list S in a descending order in terms of 1 $((N_{s_k} - 1)t_{s_k})\% g.$ **2** for each session s_k in the sorted list S do 3 Separate members in m_{s_k} into two disjoint sets, one set $\mathcal O$ that includes members that already exist in VT and another set Nthat includes the remaining members that do not exist in VT. 4 $order(\mathcal{O})$ 5 $order(\mathcal{N})$ for $(i = 0, 1..., |\mathcal{O}| - 2)$ do 6 7 provision as much traffic as possible out of the $(N_{s_k}-1)t_{s_k}$ traffic units between members $\mathcal{O}[i]$ and $\mathcal{O}[i+1]$ using the current virtual topology (VT). for the remaining unprovisioned traffic t' (if any), establish 8 $\lceil t'/g \rceil$ lightpaths between members $\mathcal{O}[i]$ and $\mathcal{O}[i+1]$. 9 end for $(i = 0, 1..., |\mathcal{N}| - 2)$ do 10 11 establish H_{s_k} lightpaths between members $\mathcal{N}[i]$ and $\mathcal{N}[i+1]$ 12 end 13 if $(|\mathcal{O}| = 0)$ then establish H_{s_k} lightpaths between members $\mathcal{N}[|\mathcal{N}|-1]$ and 14 $\mathcal{N}[0].$ 15 end else 16 17 if $(|\mathcal{N}| = 0)$ then establish H_{s_k} lightpaths between members $\mathcal{O}[|\mathcal{O}| - 1]$ 18 and $\mathcal{O}[0]$. 19 end 20 else 21 establish H_{s_k} lightpaths between members $\mathcal{O}[|\mathcal{O}| - 1]$ and $\mathcal{N}[0]$ and H_{s_k} lightpaths between members $\mathcal{N}[|\mathcal{N}|-1]$ and $\mathcal{O}[0]$. end 22 23 end 24 end

Procedure 1. $order(\mathcal{X})$ select a member in \mathcal{X} randomly as the *current member*; while there is at least one unselected member in \mathcal{X} do Case 1: $\mathcal{X} = \mathcal{O}$ select the *next member* (from the remaining unselected members) as the member who has the shortest logical distance in VT from the *current member*; Case 2: $\mathcal{X} = \mathcal{N}$ select the *next member* (from the remaining unselected members) as the member who has the shortest physical distance in G from the *current member*; *current member=next member*; end

 s_k , it orders members in m_{s_k} (lines 3-5). Note that this is the order of the members in the sessions' lightpath cycles. The way the heuristic orders members in a session s_k is by first separating members in m_{s_k} into two disjoint sets \mathcal{O} and \mathcal{N} (see Algorithm 2 line 3 for their definitions). Afterwards, it orders members in the \mathcal{O} set according to the NN Algorithm by minimizing the logical hop distance between each pair of consecutive members, while it orders members in the \mathcal{N} set according to the NN Algorithm by minimizing the physical hop distance between each pair of consecutive members (see Procedure 1). The third and last step of the heuristic is the provisioning of the $(N_{s_k} - 1)t_{s_k}$ traffic units between each pair of consecutive members in the ordered m_{s_k} (lines 6-23). Between each pair of consecutive members in the \mathcal{O} set, the heuristic attempts to provision as much traffic as possible out of the $(N_{s_k} - 1)t_{s_k}$ traffic units using the current virtual topology VT (line 7). This is done by running the max-flow algorithm [26] between the two members in the current VT (with edge capacities $\lfloor c_{ij}/t_{s_k} \rfloor$). For the remaining unprovisioned traffic t' (if any), the heuristic establishes $\lceil t'/g \rceil$ lightpaths between the two members (line 8). Between each pair of consecutive members in the \mathcal{N} set, the heuristic establishes H_{s_k} lightpaths to provision the $(N_{s_k} - 1)t_{s_k}$ traffic units (lines 10-11). Finally, the heuristic completes the cycle for each session s_k by connecting the \mathcal{O} set and the \mathcal{N} set by H_{s_k} lightpaths at both ends (line 13-23).

Example: We consider the same example in Section III.A, except that the 6-node network is now a NSTWDM network. The heuristic first sorts sessions as follows $S = \{s_2, s_1, s_3\}$. Afterwards, it orders members in session s_2 as follows $m_{s_2} =$ $\{B, C, D\}$ where all members belong to the \mathcal{N} set. The heuristic then establishes lightpaths $B \rightarrow C, \ C \rightarrow D$ and $D \rightarrow B$ each carrying four units of traffic (TLC for $s_2 =$ $\{B - C - D - B\}$). The heursitic then orders members in session s_1 as follows $m_{s_1} = \{B, A, E, F\}$, where member B belongs to the \mathcal{O} set and members A, E and F belong to the \mathcal{N} set. It then establishes lightpaths $B \to A, \ A \to E, \ E \to F$ and $F \rightarrow B$ each carrying three units of traffic (TLC for $s_1 =$ $\{B - A - E - F - B\}$). Finally, the heuristic orders members in session s_3 as follows $m_{s_3} = \{A, B\}$ where members A and B belong to the \mathcal{O} set. It then provisions three units of traffic on lightpaths $A \to E, E \to F, F \to B$ and $B \to A$ which will now carry six units of traffic (lightpath cycle for $s_3 = \{A - E - F - B - A\}$). Note that the lightpath cycle for session s_3 is not transparent since it consists of more than two lightpaths. This results in 7 lightpaths (14 transceivers).

C. Splitting Hubbed WDM Network

In a SHWDM network, each many-to-many session has a designated hub node chosen from its set of members. All the $N_{s_k} - 1$ members besides the hub transmit their t_{s_k} traffic units to the hub through direct lightpaths (upstream traffic). Using the new technique of *network coding* [25], the hub then linearly combines the traffic units received together with its own t_{s_k} traffic units to generate $N_{s_k} - 1$ linearly independent combinations. These combinations must also be linearly independent from the original t_{s_k} traffic units received from the members. Afterwards, the $N_{s_k} - 1$ combinations are groomed and delivered back to the members using direct light-tree(s) (downstream traffic), see Figure 4.(a).

In a SHWDM network, each member is guaranteed to recover the original traffic units transmitted by all other members in the same session, as indicated by the following theorem:

Theorem 7. In a SHWDM network, each member in a manyto-many session s_k will be able to recover the original t_{s_k} traffic units transmitted by all other members in the same session.

Proof: In a SHWDM network, each member in m_{s_k}



(b)

Fig. 4. Provisioning of a many-to-many session s_k with a set of members $m_{s_k} = \{A, B, C\}$ each with one traffic unit denoted as a, b and c, respectively $(g = 2, H_{s_k} = 1)$ in (a): a SHWDM network where $Hub(s_k) = A$. (b): a SAOWDM network.

a,b+

receives $N_{s_k} - 1$ linearly independent combinations of the original t_{s_k} traffic units transmitted by all members (except the hub which receives the original t_{s_k} traffic units directly from the other members). In addition to these combinations, each member has its own t_{s_k} traffic units which is also linearly independent from the received combinations. Therefore, each member acquires N_{s_k} linearly independent combinations which can be used to solve for the original t_{s_k} traffic units transmitted by the other $N_{s_k} - 1$ members.

Note that upstream and downstream traffic stay in the optical domain and optical-electronic-optical (O/E/O) conversion is only performed at the hub. To perform network coding at the hub, we may need to buffer traffic units that arrive early until all traffic units from the $N_{s_k} - 1$ members arrive. Using Next Generation SONET, multiservice provisioning platform (MSPP) equipments allow up to 128ms differential delay between different traffic streams.

For a single many-to-many session s_k , there will be $N_{s_k} - 1$ upstream lightpaths and H_{s_k} downstream light-tree(s). Therefore, the total number of transceivers required is:

$$R = 2(N_{s_k} - 1) + N_{s_k}H_{s_k}$$

In the case of multiple many-to-many sessions, each session s_k still requires H_{s_k} downstream light-trees (light-trees only groom the linear combinations for the corresponding session and they do not groom traffic from different sessions). However, the number of upstream lightpaths depends on the hub selection since a lightpath may groom traffic from different sessions. For example, consider two many-to-many sessions s_1 and s_2 where $m_{s_1} = \{A, B, C\}, m_{s_2} = \{A, B, D\}, t_{s_1} =$ $t_{s_2} = 1$ and g = 2. If we select $Hub(s_1) = Hub(s_2) = A$, then there will be a total of three upstream lightpaths, $B \rightarrow A$ carrying two units of traffic (one unit of traffic from each session), $C \to A$ and $D \to A$ each carrying one unit of traffic. However, if we select $Hub(s_1) = A$ and $Hub(s_2) = B$ then there will be 4 upstream lightpaths, $B \to A, C \to A, A \to B$ and $D \rightarrow B$ each carrying 1 unit of traffic. Note that, in either case, each session requires only one downstream light-tree.

Selecting the hub for each session determines the virtual topology and the corresponding routing and grooming of the traffic, and therefore it solves the VTTR problem. Next, we formulate the VTTR problem (hub selection) in a SHWDM network as an Integer Linear Program (ILP).

TABLE II decision variables used in the ILP for the VTTR problem in a SHWDM network

$I_h^{s_k}$:	binary number to indicate whether or not $h \in m_{s_k}$ is the hub node for session s_k .
L_{ij} :	number of lightpaths from node <i>i</i> to node <i>j</i> ($i \neq j$).

Algorithm 3. VTTR Heuristic: SHWDM Network input : K many-to-many session requests output: The hub for each session
1 for (each member l ∈ U_{sk} m_{sk}) do
2 | count the number of appearances of l in all the K sessions.
3 end
4 for each session sk do
5 | select the hub for sk as the element in U_{sk} m_{sk} that is a member in m_{sk} and has the largest number of appearances in all the K

sessions.

1) ILP Formulation: We first define $B_p^{s_k}$ as a binary input number to indicate whether or not $p \in m_{s_k}$. The decision variables used in ILP are shown in Table II.

The objective is to minimize the total number of upstream lightpaths (or transceivers):

 $\textit{Minimize} \quad \sum_{i \in V} \sum_{j \in V, j \neq i} L_{ij}.$

Subject to the following constraints:

$$\sum_{h \in m_{s_k}} I_h^{s_k} = 1 \quad \forall s_k \tag{5}$$

$$L_{ij} \ge \left(\sum_{s_k} t_{s_k} I_j^{s_k} B_i^{s_k}\right)/g \quad \forall i, j \in V, i \neq j \tag{6}$$

Constraint (5) ensures that there is exactly one hub node for each session chosen from its set of members, while constraint (6) computes the total number of lightpaths needed between each pair of nodes in the network. Constraint (6) calculates the total traffic between nodes *i* and *j* as the aggregate traffic from all sessions where *i* is a member and *j* is the hub. Note that there is no need to include the downstream direction in the ILP since the number of downstream light-trees is fixed $(\sum_{s_k} H_{s_k})$ and doesn't depend on the hub selection.

2) Heuristic Solution: We introduce a heuristic approach that is based on the idea of selecting the same hub node for as many sessions as possible (see Algorithm 3). The heuristic starts by counting the total number of appearances of each member in $\bigcup_{s_k} m_{s_k}$ in all the K sessions (lines 1-3). Then it selects the hub for each session s_k as the element in $\bigcup_{s_k} m_{s_k}$ that is a member in m_{s_k} and has the largest number of appearances in all the K sessions (lines 4-6). Selecting the same hub for as many sessions as possible increases the likelihood of intersession grooming on the upstream direction, which has a direct impact on reducing the number of lightpaths needed.

Example: We consider the same example in Section III.A, except that the 6-node network is now a SHWDM network. The heuristic first counts the total number of appearances of

each member in $\bigcup_{s_k} m_{s_k} = \{A, B, C, D, E, F\}$ in the three sessions as follows $\{A=2,B=3,C=1,D=1,E=1,F=1\}$. Afterwards, the heuristic selects the hub for sessions s_1 , s_2 and s_3 as follows hub $(s_1) = B$, hub $(s_2) = B$ and hub $(s_3) = B$. Based on this hub selection, there will be three upstream lightpaths for s_1 ($A \rightarrow B, E \rightarrow B$ and $F \rightarrow B$) each carrying one unit of traffic and one light-tree ($B \rightarrow \{A, E, F\}$) carrying three units of traffic. For session s_2 , there will be two upstream lightpaths ($C \rightarrow B$ and $D \rightarrow B$) each carrying two units of traffic and one light-tree ($B \rightarrow \{C, D\}$) carrying four units of traffic. Finally, for session s_3 , three units of traffic are provisioned on the lightpath $A \rightarrow B$ which will now carry four units of traffic and a light-tree ($B \rightarrow \{A\}$ which is simply a lightpath) is established carrying three units of traffic. This results in six lightpaths and two light-trees (19 transceivers).

The advantage of network coding in a SHWDM network is the reduction of downstream traffic for each session s_k from $N_{s_k}t_{s_k}$ to $(N_{s_k} - 1)t_{s_k}$ traffic units. Therefore, the total number of transceivers saved (R_{saved}) due to the use of network coding is equal to the total number of light-trees saved for each session s_k $(\lceil N_{s_k}t_{s_k}/g\rceil - \lceil (N_{s_k} - 1)t_{s_k}/g\rceil)$ times the number of transceivers per light-tree for that session (N_{s_k}) , which is indicated by the following formula:

$$R_{saved} = \sum_{s_k} N_{s_k} \left(\left\lceil N_{s_k} t_{s_k} / g \right\rceil - \left\lceil (N_{s_k} - 1) t_{s_k} / g \right\rceil \right)$$

D. Splitting All-Optical WDM Network

In a SAOWDM network, each member in a many-to-many session transmits it traffic directly to all other members in the same session using a light-tree, see Fig. 4.(b). Note that no traffic grooming is performed in this network and the virtual topology does not depend on t_{s_k} . Each session s_k requires N_{s_k} light-trees while each light-tree requires N_{s_k} transceivers. Therefore, the total number of transceivers needed is:

$$R = \sum_{s_k} N_{s_k}^2$$

For the same example in Section III.A, this network requires 16+9+4=29 transceivers.

E. A Comparative Example

In this subsection, we provide an example to compare the performance of the four networks with respect to the number of transceivers used R. We consider the 6-node network shown in Fig. 2 with a single many-to-many session s_1 with a set of members $m_{s_1} = \{A, B, C, D\}$. We obtain the session provisioning on each of the four networks for each value of $t_{s_1}=1,3,5,8$ (g=8) using the heuristics above. Table III shows the optical channels established and the corresponding value of R for each network and for each value of t_{s_k} .

We can see from Table III that NSTWDM networks are the most cost-effective for low traffic granularities ($t_{s_1} = 1$), SHWDM networks, through the novel use of network coding, are the most cost-effective for traffic granularities that lie in the middle ($t_{s_1} = 3, 5$), and SAOWDM networks are the most cost-effective for high traffic granularities ($t_{s_1} = 8$). In

 TABLE III

 Optical channels established and Values of R for each of the four networks for t_{s_1} =1,3,5,8 (g=8)

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,D}, ,C} D},
,C} D},
D},
,D},
,C}
,D},
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Section V, we will verify these results by conducting extensive experiments to compare the performance of the four networks on the costs R and W.

F. Complexity Analysis

The complexity of the ILPs for NSOWDM, NSTWDM, and SHWDM networks in terms of the number of integer variables is $O(K|E||V|^2)$, $O(K|V|^4)$, and $O(K|V| + |V|^2)$, respectively and in terms of the number of constraints is $O(K|V|^3)$, $O(K|V|^3)$, and $O(K + |V|^2)$, respectively. The time complexity of Algorithms 1 and 2 is dominated by the step of finding the max-flow using the Push-relabel Algorithm with FIFO vertex selection rule that has a time complexity of $O(|V|^3)$. This step is repeated for each member for each session, which drives the time complexity of Algorithms 1 and 2 to $O(K|V|^4)$. Finally, the time complexity of Algorithm 3 is O(K|V|).

IV. ROUTING AND WAVELENGTH ASSIGNMENT PROBLEM

Once we solve the VTTR problem and determine the virtual topology, we can then consider the RWA problem. In this problem, we need to provision each of the optical channels determined by the VTTR problem on the physical WDM network by determining: 1) the route of each optical channel on the network, and 2) the wavelength to assign to each optical channel, while taking the wavelength continuity constraint into account (assuming no wavelength conversion). The objective of the RWA is to minimize the total number of wavelengths used ($W \le W_{max}$).

The RWA problem has been extensively studied in the literature and it has been proven to be NP-complete. Many heuristics have been proposed for both the routing and the wavelength assignment problems. For example, fixed routing, fixed-alternate routing, and adaptive routing are some of the well-known heuristics for routing, while first fit, least used, and most used are some of the well-known heuristics for wavelength assignment. For a review on routing and wavelength assignment approaches, the reader is referred to [27].

Since the RWA problem has been extensively studied, we are only interested in comparing the proposed WDM networks in terms of their consumption of wavelengths. To make the comparison fair and to base it on the merit of the networks





Fig. 5. networks used in the results

only, we use very simple approaches for routing and wavelength assignment. We use fixed shortest path routing and first fit wavelength assignment for lightpaths, while we use fixed shortest path tree routing and first fit wavelength assignment for light-trees. The detailed description of the heuristic is shown in Algorithm 4.

V. RESULTS AND COMPARISONS

In this section, we verify the accuracy of our proposed heuristics and also compare the four WDM networks in terms of the costs R and W. For all the experiments we conduct, we set W_{max} large enough to guarantee feasible solutions.

A. Verification of the Heuristics

To verify the accuracy of our proposed heuristics for NSOWDM, NSTWDM and SHWDM networks, we conduct a number of experiments on small and medium sized networks. Ten experiments are conducted on the 6-node network



Fig. 6. Values of R/R_{opt} for the 20 experiments conducted on the 6-node network (exps 1-10) and on the Abilene research network (exps 11-20).

shown in Fig. 2, while another ten are conducted on the Abilene research network shown in Fig. 5.(a). Each of the 20 experiments has 10 many-to-many session requests, where the size of a session is randomly selected between [2,5]. For the 6-node experiments, members in a session are randomly selected between [0,5], while for the Abilene research network experiments they are randomly selected between [0,9]. Traffic demand of members in a session, in all the 20 experiments, is randomly selected between [1,16] (g = 16).

The optimal solution for each experiment is obtained on each of the NSOWDM, NSTWDM and SHWDM networks by solving the corresponding ILP using the CPLEX solver [28]. We have also obtained solutions for each experiment on each of the three networks by solving the corresponding heuristic. We define the normalized number of transceivers (R/R_{opt}) as the ratio of the number of transceivers obtained by a heuristic (R) over the optimal number of transceivers obtained by its corresponding ILP (R_{opt}) . Fig. 6 shows the values of R/R_{opt} for the 20 experiments conducted on the 6node network and on the Abilene research network for each of the three networks. We can see from the figure that solutions obtained from the heuristics either match or are very close to their corresponding optimal solutions (at most 29% above the optimal). Also, this closeness between the optimal and the heuristic has been consistent across all the 20 experiments on both the 6-node network and the Abilene research network.

B. Comparisons

In this subsection, the four WDM networks will be compared in terms of the costs R and W. Since the grooming capabilities of the four networks are greatly varied, their performance will be dependent on traffic granularities of sessions in the network. Therefore, we should compare them for different traffic granularities. To make this comparison, we assume a static uniform traffic with all sessions in an experiment having the same traffic demand t (e.g., $t_{s_1} = t_{s_2} = \dots = t_{s_K} = t$), where $1 \le t \le g$. Since optimal values of R in NSOWDM, NSTWDM and SHWDM networks are not possible to obtain for large sized instances of the problem, we will conduct three sets of experiments. One set of small experiments are conducted on the 6-node network shown in Fig. 2 in which optimal values of R are obtained by solving the corresponding ILPs using the CPLEX solver. Another two sets of medium and large experiments are conducted on the USNET network (shown in Figure 5.(b)) and the 47-node, 96-link network (which appeared in [29]), respectively, in which values of R are obtained by solving the corresponding heuristic.

1) Small Network Example: In this example, 8 randomly generated experiments are conducted on the 6-node network shown in Fig. 2. The number of sessions in each experiment is randomly selected between [4,6]. The size of a session is randomly selected between [2,5], while a member in a session is randomly selected between [0,5]. Assuming the static uniform traffic, each experiment is conducted for each value of t = 1, 2, ..., g (g = 16) on all four networks. We define \overline{R} to be the average value of all R values obtained from the 8 experiments at a particular value of t on a certain network. The resulting values of \overline{R} are shown in Fig. 7.(a).

After determining the optical channels for each experiment at each value of t on each network, these channels are routed and assigned a wavelength according to Algorithm 4. We define \overline{W} to be the average value of all W values obtained from the 8 experiments at a particular value of t on a certain network. The resulting values of \overline{W} are shown in Fig. 7.(b).

In relatively small networks, where optimal values of R on the NSOWDM, NSTWDM and SHWDM networks can be obtained by solving the corresponding ILP, we draw the following conclusions from Figs. 7.(a)-(b):

- In terms of the cost R: NSTWDM networks are the most cost-effective choice for low traffic granularities $(1 \le t \le 3g/8)$, while SHWDM networks are the most cost-effective choice when traffic granularities lie in the middle $(3g/8 < t \le 5g/8)$. Finally, for high traffic granularities (t > 5g/8), SAOWDM networks are the most cost-effective choice.
- In terms of the cost W: NSOWDM networks are the most cost-effective choice for all traffic granularities (1 ≤ t ≤ g). SAOWDM networks are also a cost-effective choice for high traffic granularities (t > 3g/4).

2) Medium Network Example: In this example, 100 randomly generated experiments, each with 80 many-to-many session requests, are conducted on the USNET network shown in Fig. 5.(b). The size of a session is randomly selected between [2,24], while a member in a session is randomly selected between [0,23]. Assuming the static uniform traffic, each of the 100 experiments is conducted for each value of $t = \{1,3,9,12,18,24,36,48,96,192\}$ (g = 192) on all four networks. The first eight values of t represent the recommended rates for OC streams. The resulting values of \overline{R} and \overline{W} , which are defined as before, are shown in Figs. 7.(c) and 7.(d), respectively.

3) Large Network Example: In this example, 150 randomly generated experiments, each with 100 many-to-many session requests, are conducted on the 47-node, 96-link network which appeared in [29]. The size of a session is randomly selected between [2,47], while a member in a session is randomly selected between [0,46]. Assuming the static uniform traffic, each of the 150 experiments is conducted for each value of $t = \{1, 3, 9, 12, 18, 24, 36, 48, 96, 192\}$ (g = 192) on all four



Fig. 7. (a),(c),(e): Average number of transceivers \overline{R} versus the uniform traffic t on the 6-node, USNET, and the 47-node mesh topologies, respectively. (b),(d),(f): Average number of wavelengths \overline{W} versus the uniform traffic t on the 6-node, USNET, and the 47-node mesh topologies, respectively.

networks. The resulting values of \overline{R} and \overline{W} , which are defined as before, are shown in Figs. 7.(e) and 7.(f), respectively.

In relatively medium and large networks, where values of R on the NSOWDM, NSTWDM and SHWDM are obtained using the corresponding heuristic, we draw the following conclusions from Figs. 7.(c)-(f):

- In terms of the cost R: NSTWDM networks are the most cost-effective choice for very low traffic granularities (1 ≤ t < g/16), while SAOWDM networks are the most cost-effective for very high traffic granularities (t > 15g/16). SHWDM networks, on the other hand, are the most cost-effective choice for a large portion of the traffic granularities spectrum (g/16 ≤ t ≤ 15g/16).
- In terms of the cost W: NSOWDM networks are the most cost-effective choice for the whole traffic granularities spectrum (1 ≤ t ≤ g).

Although NSTWDM networks are the most cost-effective choice only for $(1 \le t < g/16)$, this part of the traffic granularities spectrum is of practical interest in traffic grooming especially when g is relatively high. For example, many applications request only OC-1 and OC-3 circuits, while the capacity of a wavelength channel is OC-192. On the other extreme of the traffic granularities spectrum (t > 15g/16), SAOWDM are the most cost-effective choice. This part of the spectrum is also of practical interest for many applications whose bandwidth demands almost fill the capacity of a wave-

TABLE IV 95% confidence intervals for \overline{R} on the 47-node topology for $t{=}1{,}3{,}36{,}192$

t	1	3	36	192
NSOWDM	2401 ± 23	4923 ± 45	41154 ± 446	213460 ± 2342
NSTWDM	2284 ± 21	4098 ± 40	29565 ± 360	150980 ± 1900
SHWDM	2906± 18	2906± 18	16501±193	80175 ± 988
SAOWDM	77948± 968	77948 ± 968	77948± 968	77948± 968

 $\frac{\text{TABLE V}}{\text{For } R_{saved} \text{ and } \overline{R_{saved}/R} \text{ from the USNET experiments}} \\ \text{For } t = 1, 9, 18, 24, 48, 96, 192$

t	1	9	18	24	48	96	192
Rsaved	0	73.9	114.6	92	230.2	505.6	1036.5
$\overline{R_{saved}}/\overline{R}$	0%	5.1%	5%	3.3%	4.7%	5.5%	5.8%

length. Finally, SHWDM networks through the novel use of network coding, are the most cost-effective for a large portion of the traffic granularities spectrum $(g/16 \le t \le 15g/16)$.

It is to be noted that the number of experiments conducted in each of the examples above was sufficient to draw the above conclusions. Table IV shows the 95% confidence intervals for \overline{R} on the 47-node topology experiments for t=1,3,36,192. Due to space limitations, we do not show the confidence intervals for the other examples and scenarios.

Finally, Table V illustrates the advantage of network coding in reducing the number of transceivers in the SHWDM network by showing the values of $\overline{R_{saved}}$ (the average value of all R_{saved} values obtained from the 100 USNET experiments at a particular value of t on the SHWDM network) for t = 1, 9, 18, 24, 48, 96, 192. The table also shows the percentage savings due to the use of network coding $(\overline{R_{saved}}/\overline{R})$.

VI. CONCLUSION AND FUTURE WORK

We have considered and analyzed four different WDM network architectures for many-to-many traffic grooming. For NSOWDM and NSTWDM networks, we have introduced *lightpath cycles* as the optimal virtual topology for single and multiple many-to-many sessions in certain special cases. Based on lightpath cycles, efficient near-optimal heuristics were developed for the general case. For the SHWDM network, we have introduced a novel approach that combines optical splitting and network coding to provision many-tomany sessions and we derived an optimal as well as a heuristic solution. We have concluded that each of the four networks proposed is the most cost-effective choice for a certain range of traffic granularities.

In our future work, we intend to address the asymmetric many-to-many traffic grooming problem where members within the same session may have different traffic demands. This problem is more challenging and it makes the analysis more difficult. Also, it introduces new challenges to the application of network coding in the SHWDM network since the traffic combined at the hub from different members within the same session may not have the same granularity.

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