Many-to-Many Traffic Grooming in WDM Networks

Mohammad A. Saleh Ahmed E. Kamal

***Abstract*—A large number of network applications today allow several users to interact together using the *many-to-many* service mode. In many-to-many communication, also referred to as group communication, a session consists of a group of users (we refer to them as *members*), where each member transmits its traffic to all other members in the same group. In this paper, we address the problem of many-to-many traffic grooming in WDM mesh networks. In this problem, a set of many-to-many session requests, each with an arbitrary sub-wavelength traffic demand, are given and the objective is to provision the sessions on the WDM network with the minimum network cost. The cost of a WDM network is dominated by the cost of higher layer electronic ports (we refer to them as *transceivers*). Therefore, our objective is to minimize the total number of transceivers used.**

We address the problem in both *non-splitting networks* where the nodes do not have optical splitting capabilities and in *splitting networks* where the nodes do have optical splitting capabilities. First, we formulate the problem in each of the two networks as a Mixed Integer Linear Programs (MILP). Afterwards, based on observations from optimal solutions, we develop a heuristic approach for each network by relaxing and simplifying its corresponding MILP. Through extensive experiments, we verify the accuracy of our proposed heuristics and also show when each of the two networks is a more cost-effective choice for many-to-many traffic grooming.

I. Introduction

In wavelength routing networks, using wavelength division multiplexing (WDM), the bandwidth of a fiber is divided into multiple disjoint optical channels (wavelengths). Currently, it is feasible to have hundreds of wavelengths, each operating at 10 to 40 Gbps, per fiber. Bandwidth requirements of user sessions, however, are usually of sub-wavelength granularities. For example, an MPEG compressed HDTV channel requires less than 20 Mbps of bandwidth. In order to reduce this huge bandwidth gap, *traffic grooming* was introduced to allow a number of sessions with sub-wavelength granularities to share the bandwidth of a wavelength channel. In addition to determining the virtual topology and the routing and wavelength assignment of each of the wavelength channels, the traffic grooming problem deals with the intelligent assignment of sub-wavelength traffic demands onto the existing wavelength channels.

The cost of a WDM network is dominated by the cost of higher layer electronic ports such as IP router ports, MPLS Label Switching Router (LSR) ports and SONET ADM ports (we refer to these ports as *transceivers*). A transceiver is needed for each initiation or termination of an optical channel. For example, a lightpath requires two transceivers, while a *light-tree* [1] with N endpoints requires N transceivers. Therefore, most of the studies on traffic grooming focus on minimizing the total number of transceivers used. Note that associated with each electronic port that terminates or originates an optical channel are optical transceivers for transmitting and receiving the optical signal. Therefore, the cost of a transceiver includes both the cost of the electronic port and the cost of the associated optical transceivers.

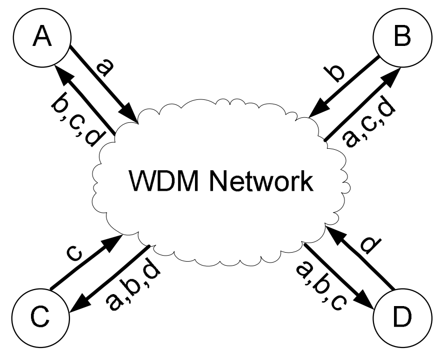

Fig. 1. A many-to-many session with members $\{A, B, C, D\}$ each with traffic denoted as a, b, c and d, respectively.

The traffic grooming problem even with unicast traffic and on simple topologies has been shown to be NP-complete [2]. Most of the work in traffic grooming has dealt with unicast or "one-to-one" traffic. A large portion of the traffic in high performance networks, however, is becoming of the multipoint type. This traffic type includes multicast, many-to-one, and many-to-many. In multicast or "one-to-many", a single source communicates with a set of destinations, while in many-to-one or "inverse multicasting" a set of sources communicate with a single destination. In many-to-many or group communication [3], a session consists of a group of users (we refer to them as *members*), where each member transmits its traffic to all other members in the same group (see Fig. 1). On-demand video distribution and file distribution are examples of multicast applications, while resource discovery and data collection are examples of many-to-one applications. In the case of many-to-many, where several users interact together, multimedia conferencing, distance learning, e-science applications, distributed simulations, and collaborative processing are some of the applications [4]. Bandwidth requirements of these user applications are usually of sub-wavelength granularities. Therefore, finding efficient ways of grooming them at the optical layer has become prominent.

In order to effectively support many-to-many communication, nodes in a WDM network must be able to duplicate

The authors are with the Department of Electrical and Computer Engineering, Iowa State University, Ames, Iowa 50011, USA (e-mail: msaleh@iastate.edu; kamal@iastate.edu).

incoming traffic into multiple copies, each going to a different output port. Two main node architectures were proposed in the literature to implement this functionality. In the first one, nodes can only duplicate an incoming optical signal by applying optical-electronic-optical (*O/E/O*) conversion and duplication takes place in the electronic domain; we refer to networks with these nodes as *non-splitting networks*. In the second one, nodes are capable of splitting the incoming optical signal (using *optical splitters*) into multiple copies without any *O/E/O* conversion. Therefore, in this node architecture, traffic duplication can take place in both the electronic and the optical domains; we refer to networks with these nodes as *splitting networks*. Note that non-splitting networks support only lightpaths, while splitting networks support lightpaths and light-trees.

In this paper, we address the many-to-many traffic grooming problem on arbitrary mesh topologies in both non-splitting and splitting networks with the objective of minimizing the total number of transceivers used. Since the total number of wavelengths used only marginally adds to the overall network cost, we are only interested in a feasible routing and wavelength assignment. A feasible routing and wavelength assignment must ensure that each optical channel uses the same wavelength on all the fiber links it traverses (assuming no wavelength conversion), and that no two optical channels use the same wavelength on the same fiber link.

The rest of the paper is organized as follows. In Section II, we review related work, while in Section III, we describe and compare the many-to-many traffic grooming problem in both non-splitting and splitting networks. In Section IV, we present the network model, while in Section V, we formulate Mixed Integer Linear Programs (MILPs) for the many-to-many traffic grooming problem in both non-splitting and splitting networks, followed in Section VI by a detailed numerical example from MILPs solutions. In Section VII, we propose heuristic solutions for the many-to-many traffic grooming problem in both non-splitting and splitting networks. In Section VIII, numerical results from both networks are presented and compared, while the paper is concluded in Section IX.

II. Related Work

Traffic grooming has been extensively studied for unicast traffic [5]-[14]. In [5], the authors addressed the traffic grooming problem on a number of WDM ring architectures with the objective of minimizing the overall network cost. In [6], the authors proposed optimal and near-optimal algorithms for traffic grooming in SONET WDM rings with the objective of minimizing the number of wavelengths and SONET ADMs. In [7], the authors considered the traffic grooming problem in a WDM mesh network. They introduced an ILP formulation and then developed heuristic solutions. In [8], the authors provided a decomposition method that divides the traffic grooming problem into two smaller problems and then solved each problem independently. In [9], approximation algorithms for minimizing the total equipment cost and for minimizing the lightpath count were introduced. In [10], the authors provided a hierarchical framework for traffic grooming in a WDM mesh network. For a survey of advances in unicast traffic grooming, the reader is referred to [11].

Traffic grooming has also been considered for multicast traffic [15]-[19]. In [15], the authors addressed the multicast traffic grooming problem in metropolitan WDM ring networks with the objective of minimizing electronic copying. They presented an ILP formulation and then developed a heuristic approach that consists of three phases: routing, circle construction, and grouping of circles. In [16], the authors introduced a graph based heuristic for the multicast traffic grooming problem in unidirectional SONET/WDM rings and compared it to the multicast extension of the best known unicast traffic grooming heuristic in [6]. In [17], the authors addressed the multicast traffic grooming problem in WDM mesh networks. They provided MILP formulations and also developed heuristic solutions. In [18], the authors considered the multicast traffic grooming problem in WDM mesh networks with sparse nodal light splitting capability. In [19], a non-linear programming formulation was introduced as an analytical model for the multicast traffic grooming problem in splitting networks followed by a number of heuristic solutions. In [20], the authors addressed the problem of many-to-one traffic grooming in WDM mesh networks with the objective of minimizing the number of wavelengths and SONET ADMs. They introduced an MILP formulation and a dynamic programming style approach that builds the solution progressively as a heuristic solution. For a survey of advances in multicast traffic grooming, the reader is referred to [21].

To the best of our knowledge, many-to-many traffic grooming is a new research problem that has been only considered in the authors' work [22]. In this paper, we address the many-to-many traffic grooming problem in both non-splitting and splitting networks. In non-splitting networks, we introduce an MILP formulation and then a heuristic solution based on observations from optimal solutions. In splitting networks, we introduce a novel hub-based approach that combines optical splitting and network coding to provision many-to-many sessions. We also introduce an MILP formulation and a heuristic solution for the hub-based approach. A comprehensive comparison between non-splitting and splitting networks reveals that each of the two networks is a cost-effective choice for a certain range of traffic granularities.

III. Problem Description and Motivation

We formally define the many-to-many traffic grooming problem as follows. Given the physical WDM network topology, number of wavelengths per fiber, grooming factor, and a set of many-to-many session requests each with an arbitrary sub-wavelength traffic demand, determine:

1) What optical channels (lightpaths and light-trees) to establish and how to route and groom each of the sub-wavelength many-to-many traffic demands on these optical channels; the *virtual topology and traffic routing* problem.

2) How to route and assign a wavelength to each of the optical channels on the WDM network; the *routing and wavelength assignment* problem.

The objective is to minimize the total number of transceivers used. As indicated earlier, the traffic grooming problem even

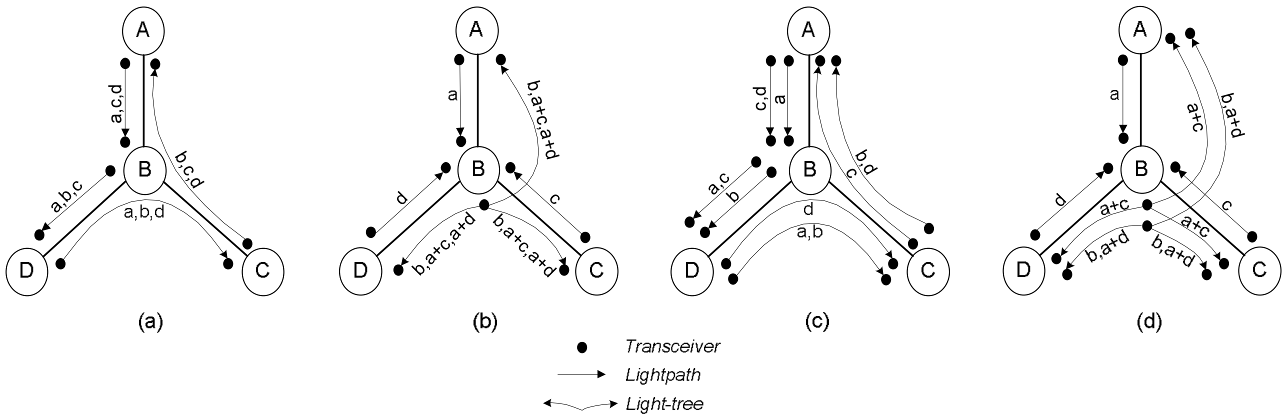

Fig. 2. Provisioning of a many-to-many session with a set of members $\{A, B, C, D\}$ each with traffic denoted as a, b, c and d, respectively (grooming factor=4) in: (a) a non-splitting network case where a, b, c and d are one unit of traffic each. (b) a splitting network case where a, b, c and d are one unit of traffic each ($hub = B$). (c) a non-splitting network case where a, b, c and d are two units of traffic each. (d) a splitting network case where a, b, c and d are two units of traffic each ($hub = B$).

with unicast traffic and on simple topologies has been proven to be NP-Complete. Furthermore, each of the two subproblems above is considered hard on general topologies. Although solving each subproblem independently is a more tractable approach than solving them combined, it will not guarantee an optimal solution. To guarantee an optimal solution, the two subproblems must be jointly considered and this is the approach we follow in this paper.

A. *Many-to-Many Traffic Grooming in non-Splitting Networks*

In non-splitting networks, lightpaths are the only optical communication channels available to provision many-to-many sessions. The traffic originating from a member in a many-to-many session may traverse multiple lightpaths to reach any other member in the same session, while a lightpath may traverse multiple fiber links. A lightpath may groom traffic from different sessions and traffic from different members within the same session. Therefore, the many-to-many traffic grooming problem in non-splitting networks is to determine: 1) how many lightpaths to establish between each pair of nodes in the network and how to route and groom each of the sub-wavelength many-to-many traffic demands on these lightpaths; 2) how to route and assign a wavelength to each of the lightpaths on the physical WDM network. The objective is to minimize the total number of transceivers used.

B. *Many-to-Many Traffic Grooming in Splitting Networks*

In splitting networks, light-trees in addition to lightpaths can be used to provision many-to-many sessions. We introduce a novel *hub-based* approach for many-to-many traffic grooming in splitting networks. In this approach, each many-to-many session has a designated hub node chosen from the set of nodes in the network including the members themselves. All the members, in a session with N members, transmit their traffic units to the hub through lightpaths (upstream traffic). Using the new technique of *network coding* [23], the hub then linearly combines the traffic units received together with its own traffic units (if it is a member) to generate $N-1$ linearly independent combinations. These combinations must also be linearly independent from the original traffic units received from the members. Afterwards, the $N-1$ combinations are groomed and delivered back to the members using light-tree(s) (downstream traffic), see Fig. 2.(b).

According to the hub-based approach, each member in many-to-many session will be able to recover the original traffic units transmitted by the other $N-1$ members in the same session by linearly combining its own traffic units with the received combinations (i.e., solving N linearly independent combinations). For simplicity, we assume that all members in a many-to-many session have the same traffic demand. This assumption is needed to facilitate network coding at the hub node by linearly combining equal sized data units. We also assume that the linear combinations are performed using coefficients taken from a field of size two (i.e., addition modulo two or bitwise XOR). Note that traffic streams originating from the members are first terminated at the hub node and then the bitwise XOR operation is performed in the electronic domain.

To perform network coding at the hub node, we may need to buffer traffic units that arrive early until all the traffic units arrive from the members. Using Next Generation SONET, multiservice provisioning platform (MSPP) equipment allows up to 128ms differential delay between different traffic streams. The use of network coding with the hub-based approach reduces the downstream traffic for each session (with N members) from N to $N-1$ traffic streams. This has a direct impact on reducing the number of required light-trees, and hence the number of transceivers.

C. *A Motivating Example*

Consider the example shown in Fig. 2, where nodes A, B, C and D are members of a many-to-many session. Each of the members needs to transmit one unit of traffic denoted as a, b, c and d, respectively, to the other three members. For the sake of this example, we assume that the capacity of a wavelength channel (grooming factor) is four units of traffic. In the non-splitting network case, Figure 2.(a) illustrates the optimal provisioning of the session, which requires four lightpaths (eight transceivers). In the splitting network case, Figure 2.(b) illustrates the optimal provisioning of the session by the hub-based approach ($hub = B$), which requires three lightpaths and one light-tree (ten transceivers). Note that each of the members A, C and D will be able to recover the original traffic units by performing bitwise XOR operations between $a + c, a + d$ and their own traffic unit. For example, node C will perform XOR between $a + c$ and c to recover a and then perform XOR between $a + d$ and a to recover d. Note that the hub B did not combine its own traffic unit b with other traffic, however it could, for example, combine b with c and send $b + c$ instead of b. In either case, the solution requires a total of ten transceivers, which costs two more transceivers than the non-splitting network case. On the other hand, if a, b, c and d are two units of traffic instead of one, then the optimal provisioning in the non-splitting network case is shown in Figure 2.(c), which requires eight lightpaths (16 transceivers). However, in the splitting network case, the hub-based approach ($hub = B$), as shown in Figure 2.(d), requires three lightpaths and two light-trees (14 transceivers), which saves two transceivers compared to the non-splitting network case.[1]

IV. The Network Model

In this section, we introduce the network model that we consider for many-to-many traffic grooming in both non-splitting and splitting networks (the hub-based approach). First, we introduce the assumptions used in the paper.

- The WDM network is represented by an undirected graph $G(V, E)$, where V is the set of nodes and E is the set of physical links. Each physical link corresponds to two unidirectional fibers in opposite directions. Each unidirectional fiber has the same number of wavelength channels W, and each wavelength channel has the same grooming factor g.
- There is a total of K many-to-many session requests, where each session s_k ($1 \leq k \leq K$) has a set of members $m_{s_k} \subseteq V$ with cardinality $N_{s_k} = |m_{s_k}|$. Each member in m_{s_k} has the same traffic demand t_{s_k}, where $1 \leq t_{s_k} \leq g$.
- We define $H_{s_k} = \lceil (N_{s_k} - 1)t_{s_k}/g \rceil$ to be a lower bound on the number of incoming channels to a member in a session s_k in order to receive the traffic from the other $N_{s_k} - 1$ members in the same session.
- The traffic stream originating from a member and destined to any other member in a many-to-many session must not be bifurcated into a set of lower speed streams each taking a different route on the virtual topology.

We view the network at three different layers:

1) The Physical Layer: This layer includes the fiber network consisting of optical nodes and fiber links. We assume that any two nodes in the network are connected by at most one physical link (two unidirectional fibers). In our problem, we assume that the physical WDM network is given. Therefore, if the distance between any two connected nodes is relatively long and requires optical amplification and/or *O/E/O* regeneration, then we assume that optical amplifiers and *O/E/O* regenerators are already deployed.

2) The Optical Layer: This layer includes the optical channels (lightpaths and light-trees) that are established on the fiber network. A lightpath may traverse multiple fiber links and it may be established between any two nodes in the network. We assume that intermediate nodes have no wavelength conversion capability. This constrains a lightpath to use the same wavelength on all the fiber links it traverses. A light-tree, in our model, is always rooted at the hub of a session and its leaves are the members (or the remaining members if the hub is a member) of that session. Accordingly, a light-tree is associated with a particular session, e.g, when we say "light-tree for session s_k" we mean a light-tree that is rooted at the hub of s_k and its leaves are the members (or the remaining members) of s_k. Consider, for example, a many-to-many session s_1 with a set of members $m_{s_1} = \{A, B, C, D, E\}$, and let us assume that $hub(s_1) = A$. A "light-tree for s_1" is a light-tree that is rooted at A and its leaves are $\{B, C, D, E\}$. Similar to a lightpath, a light-tree must use the same wavelength on all the fiber links it traverses.

3) The Session Layer: This layer includes the routing and the grooming of the many-to-many traffic demands on the optical channels. Lightpaths and light-trees may groom traffic from different sessions and traffic from different members within the same session. In non-splitting networks, the traffic originating from a member may traverse multiple lightpaths to reach any other member in the same session. In splitting networks, to deliver the traffic according to the hub-based approach, we must determine the following:

- *hub selection:* selecting the hub node for each session from the set of nodes in the network.
- *members-to-hub journey:* determining how to route the traffic from each of the members to the hub. The traffic originating from a member may traverse multiple lightpaths to reach the hub.
- *hub-to-members journey:* determining how to route the linear combinations of the original traffic units from the hub node back to the members. This traffic is either delivered through light-tree(s) for the corresponding session or through light-tree(s) for other sessions. For example, consider the 4-node network shown in Fig. 2 with three many-to-many session requests s_1, s_2 and s_3 each with a set of members $m_{s_1} = \{A, B, C, D\}$, $m_{s_2} = \{A, B, C\}$ and $m_{s_3} = \{A, B, D\}$, respectively. Let us assume that $hub(s_1) = hub(s_2) = hub(s_3) = B$, then one possible routing of the hub-to-members journey of the three sessions is to establish a light-tree for s_2 ($B \rightarrow \{A, C\}$) and a light-tree for s_3 ($B \rightarrow \{A, D\}$) and to route the hub-to-members journey of s_1 on the two established light-trees (assuming that each of the light-trees has enough capacity to accommodate session s_1 traffic units). This example also shows the significance of the hub selection since the hub-to-members journey of a session cannot be routed on a light-tree for another session unless the two sessions share the same hub node.

[1] There is also a saving of three wavelength links and an addition of two optical splitters, but the effect of these components on the cost is neglected.

To illustrate the three layers of the network model, we consider the two examples shown in Fig. 2.(a) and Fig. 2.(b). The physical layer in both examples is the fiber network consisting of four nodes and three physical links (a total of 6 fibers). The optical layer in the non-splitting example consists of four lightpaths, $A \rightarrow B$, $B \rightarrow D$, $D \rightarrow C$ and $C \rightarrow A$, while in the splitting example, it consists of three lightpaths ($A \rightarrow B$, $D \rightarrow B$ and $C \rightarrow B$) and a light-tree ($B \rightarrow \{A, C, D\}$). The session layer in the non-splitting example consists of routing the traffic between members on lightpaths. For example, the traffic from member A to member C is routed on lightpaths $A \rightarrow B$, $B \rightarrow D$ and $D \rightarrow C$. In the splitting example, the hub is selected as member B and the members-to-hub journey consists of routing the traffic from members A, C and D to the hub B through lightpaths $A \rightarrow B$, $C \rightarrow B$ and $D \rightarrow B$, respectively. After linearly combining the traffic units received from A, C and D, the hub B delivers the linear combinations back to the members through the light-tree $B \rightarrow \{A, C, D\}$ (hub-to-members journey).

Regarding notation, we use p and q to refer to any two members in a many-to-many session, while we use h to refer to the hub of a session. Also, we use i and j to refer to the source and destination nodes of a lightpath, while we use m and n to refer to the end nodes of a fiber link.

V. Problem Formulation

In this section, we formulate Mixed Integer Linear Programs (MILPs) for the many-to-many traffic grooming problem in both non-splitting and splitting networks (the hub-based approach). First, we introduce the input parameters used in the MILPs.

Input Parameters:
P_{mn} : binary number equals to 1 if there is a fiber link from node m to node n; otherwise it is set to 0 ($P_{mn} = P_{nm}$).
W : number of wavelengths per fiber, which we set large enough to guarantee a feasible solution.
g : grooming factor.
K : number of many-to-many sessions.
m_{s_k} : set of members in session s_k, where $1 \leq k \leq K$.
$B_l^{s_k}$: binary number equals to 1 if $l \in m_{s_k}$; otherwise it is set to 0.
N_{s_k} : number of members in session s_k; $N_{s_k} = |m_{s_k}|$.
t_{s_k} : number of basic units of traffic demanded by each member in session s_k, where $1 \leq t_{s_k} \leq g$.
Q : a large integer ($Q \geq K \cdot |V|$).
Next, we introduce the common decision variables that are used in both MILPs.

Common Decision Variables:
TR_n : number of transceivers at node n.
L_{ij}^w : number of lightpaths from node i to node j on wavelength w.
L_{ij} : number of lightpaths from node i to node j on all wavelengths; $L_{ij} = \sum_w L_{ij}^w$.
$F_{mn}^{ij,w}$: binary number equals to 1 if there is a lightpath from node i to node j that uses fiber link mn on wavelength w; otherwise it is set to 0.
$X_{ij}^{s_k}$: real number equals to the amount of traffic carried on lightpaths from node i to node j due to all members in m_{s_k}.

A. MILP Formulation for non-Splitting Networks

In this subsection, we introduce an MILP formulation for the many-to-many traffic grooming problem in non-splitting networks. First, we introduce the decision variables that are only used in this MILP formulation.

Decision Variables for non-Splitting Networks:
$Z_{ij}^{s_k,p,q}$: binary number equals to 1 if the traffic stream originating from member $p \in m_{s_k}$ and destined to member $q \in m_{s_k}$ is routed on a lightpath from node i to node j; otherwise it is set to 0 ($p \neq q$).
$Y_{ij}^{s_k,p}$: binary number equals to 1 if a traffic stream originating from member $p \in m_{s_k}$ and destined to at least one other member in m_{s_k} is routed on a lightpath from node i to node j; otherwise it is set to 0.

Objective Function:

$$\textit{Minimize: } \sum_n TR_n$$

Subject to:
Number of Transceivers Constraints:
The following constraint ensures that at the source and at the destination of each lightpath there is a transceiver present.

$$TR_i \geq \sum_{j:j \neq i} (L_{ij} + L_{ji}) \quad \forall i \tag{1}$$

Lightpath Level Constraints:
The following constraint ensures that for each lightpath from i to j there is a corresponding physical path from i to j that uses the same wavelength on all the fiber links it traverses.

$$\sum_{m:P_{mx}=1} F_{mx}^{ij,w} - \sum_{n:P_{xn}=1} F_{xn}^{ij,w} = \begin{cases} L_{ij}^w, & \text{if } x = j \\ -L_{ij}^w, & \text{if } x = i \\ 0, & \text{otherwise} \end{cases} \tag{2}$$
$$\forall i, j, w, x \in V$$

The following constraint ensures that for any wavelength w on any fiber link mn no more than one lightpath can be present.

$$\sum_i \sum_j F_{mn}^{ij,w} \leq 1 \quad \forall w, m, n : P_{mn} = 1 \tag{3}$$

Session Level Constraints:
The following is the traffic routing constraint between each pair of members in a many-to-many session. It ensures that the traffic originating from a member and destined to any other member in the same session may traverse multiple lightpaths.

$$\sum_i Z_{ix}^{s_k,p,q} - \sum_j Z_{xj}^{s_k,p,q} = \begin{cases} 1, & \text{if } x = q \\ -1, & \text{if } x = p \\ 0, & \text{otherwise} \end{cases} \tag{4}$$
$$\forall s_k, (p,q) \in m_{s_k}, x \in V$$

Constraints (5) and (6) together set the variable $Y_{ij}^{s_k,p}$ as the logical disjunction of all the variables $Z_{ij}^{s_k,p,q}$ for all values of $q \in m_{s_k},\ q \neq p$.

$$Y_{ij}^{s_k,p} \geq \sum_{q \neq p} Z_{ij}^{s_k,p,q}/Q \quad \forall s_k, p \in m_{s_k}, i, j \tag{5}$$

$$Y_{ij}^{s_k,p} \leq \sum_{q \neq p} Z_{ij}^{s_k,p,q} \quad \forall s_k, p \in m_{s_k}, i, j \tag{6}$$

$Y_{ij}^{s_k,p}$ will be set to 1 if at least one of the traffic streams that originate from member p uses a lightpath from i to j. Note that when $Y_{ij}^{s_k,p} = 1$, then lightpaths from i to j carry the t_{s_k} traffic units that originate from member p. The following constraint determines the amount of traffic carried on lightpaths from i to j due to all members in session s_k.

$$X_{ij}^{s_k} = t_{s_k} \sum_{p \in m_{s_k}} Y_{ij}^{s_k,p} \quad \forall s_k, i, j \tag{7}$$

The following constraint computes the total number of lightpaths needed between each pair of nodes in the network.

$$L_{ij} \geq (\sum_{s_k} X_{ij}^{s_k})/g \quad \forall i, j \tag{8}$$

B. MILP Formulation for Splitting Networks

In this subsection, we introduce an MILP formulation for the many-to-many traffic grooming problem in splitting networks according to the hub-based approach. First, we introduce the decision variables that are only used in this MILP formulation.

Decision Variables for Splitting Networks:
$I_h^{s_k}$: binary number equals to 1 if node h is the hub node for session s_k; otherwise it is set to 0.
$E_{s_l}^{s_k,h}$: binary number equals to 1 if sessions s_k and s_l share node h as their hub node; otherwise it is set to 0.
$E_{s_l}^{s_k}$: binary number equals to 1 if sessions s_k and s_l share the same hub node; otherwise it is set to 0.
$D_{ij}^{s_k,p}$: binary number equals to 1 if the traffic stream originating from member $p \in m_{s_k}$ and destined to the hub of session s_k is routed on a lightpath from i to j; otherwise it is set to 0.
$R_{mn}^{s_k,p,w}$: binary number equals to 1 if there is a light-tree for session s_k with root (hub of s_k) to leaf (member $p \in m_{s_k}$) path that uses fiber link mn on wavelength w; otherwise it is set to 0.
$R_{mn}^{s_k,w}$: binary number equals to 1 if at least one of the root (hub of s_k) to leaf (member in m_{s_k}) paths of a light-tree for session s_k uses fiber link mn on wavelength w; otherwise it is set to 0.
$U_{s_l}^{s_k}$: binary number equals to 1 if session s_k is routed on a light-tree for session s_l; otherwise it is set to 0.
$T_{s_l}^{s_k}$: real number equals to the amount of traffic carried on light-trees for session s_l due to members in session s_k.
$LT_{s_k}^w$: number of light-trees for session s_k on wavelength w.
LT_{s_k} : number of light-trees for session s_k on all wavelengths ($LT_{s_k} = \sum_w LT_{s_k}^w$).
$A_h^{s_k}$: real number equals to the product of LT_{s_k} and $I_h^{s_k}$.

Objective Function:

$$\textit{Minimize: } \sum_n TR_n$$

Subject to:
Number of Transceivers Constraints:
The following constraint ensures that at the source and at the destination of each lightpath there is a transceiver present. Also, it ensures that at the root and at the leaves of each light-tree there is a transceiver present.

$$TR_i \geq \sum_{j:j \neq i} (L_{ij} + L_{ji}) + \sum_{s_k} LT_{s_k} B_i^{s_k} + \sum_{s_k: i \notin m_{s_k}} A_i^{s_k} \quad \forall i \tag{9}$$

The first term counts all the lightpaths originating and terminating at node i. The second term counts all light-trees for sessions where node i is a member, while the third term counts all light-trees for sessions where node i is a hub but not a member. The nonlinear term $A_i^{s_k}$ can be computed using the following set of linear constraints (together with the minimization in the objective function).

$$A_i^{s_k} \geq Q I_i^{s_k} - Q + LT_{s_k} \quad \forall s_k, i \tag{10}$$

$$A_i^{s_k} \leq LT_{s_k} \quad \forall s_k, i \tag{11}$$

Note that constraint (11) (the upper bound for $A_i^{s_k}$) is not needed due to the minimization in the objective function; however, keeping it limits the search space for the MILP.
Lightpath Level Constraint:
This will be exactly the same as the lightpath level constraint (2) in the non-splitting networks MILP.
Light-tree Level Constraints:
In this set of constraints, we visualize a light-tree for session s_k as a set of paths, each originating from the root of the light-tree (hub of s_k) and terminating at one of its leaves (one of the members of s_k). We refer to these paths as *root-to-leaf paths*. Note that the root of a light-tree (the hub for the corresponding session) is a decision variable and it is not known in advance. The following constraints ensure that for each leaf of a light-tree there should be a root-to-leaf path originating from the root.

$$\forall s_k, p \in m_{s_k}, h \neq p, w:$$

$$\sum_{n:P_{hn}=1} R_{hn}^{s_k,p,w} \geq LT_{s_k}^w - (1 - I_h^{s_k})Q \tag{12}$$

$$\sum_{n:P_{hn}=1} R_{hn}^{s_k,p,w} \leq LT_{s_k}^w + (1 - I_h^{s_k})Q \tag{13}$$

Note that when h is the hub node for session s_k ($I_h^{s_k} = 1$), then $\sum_{n:P_{hn}=1} R_{hn}^{s_k,p,w} = LT_{s_k}^w$; otherwise there will be no constraint ($-Q \leq \sum_{n:P_{hn}=1} R_{hn}^{s_k,p,w} \leq Q$).

The following constraints ensure that for each leaf of a light-tree there should be a root-to-leaf path terminating at the leaf.

$$\sum_{m:P_{mp}=1} R_{mp}^{s_k,p,w} \geq LT_{s_k}^w - Q I_p^{s_k} \quad \forall s_k, p \in m_{s_k}, w \tag{14}$$

$$\sum_{m:P_{mp}=1} R_{mp}^{s_k,p,w} \leq LT_{s_k}^w + Q I_p^{s_k} \quad \forall s_k, p \in m_{s_k}, w \tag{15}$$

Note that when member p is not the hub node for session s_k ($I_p^{s_k} = 0$), then $\sum_{m:P_{mp}=1} R_{mp}^{s_k,p,w} = LT_{s_k}^{w}$; otherwise there will be no constraint ($-Q \leq \sum_{m:P_{mp}=1} R_{mp}^{s_k,p,w} \leq Q$).

The following constraints ensure flow conservation at all intermediate nodes of a root-to-leaf path. They also guarantee that the same wavelength is used on all the fiber links traversed by the root-to-leaf path.

$$\forall s_k, p \in m_{s_k}, w,\ x \in V (x \neq p) :$$

$$\sum_{m:P_{mx}=1} R_{mx}^{s_k,p,w} \leq \sum_{n:P_{xn}=1} R_{xn}^{s_k,p,w} \quad (16)$$

$$\sum_{m:P_{mx}=1} R_{mx}^{s_k,p,w} \geq \sum_{n:P_{xn}=1} R_{xn}^{s_k,p,w} - Q I_x^{s_k} \quad (17)$$

Note that when x is the not the hub node for session s_k ($I_x^{s_k} = 0$), then flow conservation is maintained at x (i.e., $\sum_{m:P_{mx}=1} R_{mx}^{s_k,p,w} = \sum_{n:P_{xn}=1} R_{xn}^{s_k,p,w}$).

Since a light-tree must use the same wavelength on all the fiber links it traverses, then the same wavelength must be used on all the root-to-leaf paths that belong to the same light-tree, which is guaranteed by the following constraints.

$$\forall s_k, (p, q) \in m_{s_k}, w :$$

$$\sum_{m:P_{mp}=1} R_{mp}^{s_k,p,w} \geq \sum_{m:P_{mq}=1} R_{mq}^{s_k,q,w} - (I_p^{s_k} + I_q^{s_k})Q \quad (18)$$

$$\sum_{m:P_{mp}=1} R_{mp}^{s_k,p,w} \leq \sum_{m:P_{mq}=1} R_{mq}^{s_k,q,w} + (I_p^{s_k} + I_q^{s_k})Q \quad (19)$$

Constraints (12)-(19) ensure that for each light-tree for a session s_k there is a corresponding physical tree from the root (hub of s_k) to the leaves (members or remaining members of s_k), that uses the same wavelength all the fiber links it traverses.

$$R_{mn}^{s_k,w} \geq \sum_{p \in m_{s_k}} R_{mn}^{s_k,p,w}/Q \quad \forall s_k, w, m, n : P_{mn} = 1 \quad (20)$$

$$R_{mn}^{s_k,w} \leq \sum_{p \in m_{s_k}} R_{mn}^{s_k,p,w} \quad \forall s_k, w, m, n : P_{mn} = 1 \quad (21)$$

$R_{mn}^{s_k,w}$ is set to 1 if at least one of the $R_{mn}^{s_k,p,w}$ variables is set to 1 for any leaf p; otherwise it is set to 0. The following constraint ensures that for any wavelength w on any fiber link mn no more than one lightpath or light-tree can be present.

$$\sum_{s_k} R_{mn}^{s_k,w} + \sum_{i} \sum_{j} F_{mn}^{ij,w} \leq 1 \quad \forall w, m, n : P_{mn} = 1 \quad (22)$$

Note, however, that root-to-leaf paths that belong to the same light-tree can use the same wavelength on the same fiber link.

Hub Node Selection Constraints:
The following constraint ensures that there is exactly one hub node for each session s_k chosen from the set of nodes in the network.

$$\sum_{h \in V} I_h^{s_k} = 1 \quad \forall s_k \quad (23)$$

The following constraints set the variable $E_{s_l}^{s_k,h}$ as the logical conjunction of the variables $I_h^{s_k}$ and $I_h^{s_l}$.

$$E_{s_l}^{s_k,h} \leq (I_h^{s_k} + I_h^{s_l})/2 \quad \forall s_k, s_l, h \quad (24)$$

$$E_{s_l}^{s_k,h} \geq I_h^{s_k} + I_h^{s_l} - 1 \quad \forall s_k, s_l, h \quad (25)$$

The following constraints set the variable $E_{s_l}^{s_k}$ as the logical disjunction of $E_{s_l}^{s_k,h}$ variables for all values of h.

$$E_{s_l}^{s_k} \geq \sum_{h} E_{s_l}^{s_k,h}/Q \quad \forall s_k, s_l \quad (26)$$

$$E_{s_l}^{s_k} \leq \sum_{h} E_{s_l}^{s_k,h} \quad \forall s_k, s_l \quad (27)$$

Members-to-Hub Journey Constraints:
In this set of constraints, we visualize the members-to-hub journey of a session as a set of streams, each originating from a member and terminating at the hub. Each of these streams, which we refer to as *member-to-hub streams*, may traverse multiple lightpaths from the member to the hub. It is to be noted that the destination of a member-to-hub stream is a decision variable and it is not known in advance. The following constraint ensures that for each member-to-hub stream, there is a lightpath originating from the member unless it is the hub.

$$\sum_{i:i \neq p} D_{pi}^{s_k,p} = 1 - I_p^{s_k} \quad \forall s_k, p \in m_{s_k} \quad (28)$$

The following constraint ensures that for each member-to-hub stream, there is a lightpath terminating at the hub.

$$\sum_{i:i \neq h} D_{ih}^{s_k,p} \geq I_h^{s_k} \quad \forall s_k, p \in m_{s_k}, h \neq p \quad (29)$$

The following constraint ensures the continuity of a member-to-hub stream on multiple lightpaths.

$$\forall s_k, p \in m_{s_k},\ x \in V (x \neq p) :$$

$$\sum_{i:i \neq x} D_{ix}^{s_k,p} = \sum_{j:j \neq (x,p)} D_{xj}^{s_k,p} + I_x^{s_k} \quad (30)$$

The following constraint determines the amount of traffic carried on lightpaths from i to j due to all members in session s_k.

$$X_{ij}^{s_k} = t_{s_k} \sum_{p \in m_{s_k}} D_{ij}^{s_k,p} \quad \forall s_k, i, j \quad (31)$$

Finally, the the total number of lightpaths needed between each pair of nodes in the network is computed exactly the same way as in constraint (8) in the non-splitting networks MILP.

Hub-to-Members Journey Constraints:
In this set of constraints, we determine which light-trees are used in the hub-to-members journey of a session. The following constraint ensures that the hub-to-members journey of a session cannot be routed on a light-tree for another session unless the two sessions share the same hub node.

$$U_{s_l}^{s_k} \leq E_{s_l}^{s_k} \quad \forall s_k, s_l \quad (32)$$

The following constraint ensures that each member in a session is reached by at least one of the light-trees used in the hub-to-members journey of that session.

$$\sum_{s_l : p \in m_{s_l}} U_{s_l}^{s_k} \geq 1 \quad \forall s_k, p \in m_{s_k} \quad (33)$$

The following constraint determines the amount of traffic carried on light-trees for session s_l due to members in session s_k.

$$T_{s_l}^{s_k} = U_{s_l}^{s_k} \times (N_{s_k} - 1) \times t_{s_k} \quad \forall s_k, s_l \quad (34)$$

TABLE I
SAMPLE TRAFFIC USED IN THE EXAMPLE

Session	Members	Traffic Demand
s_1	{0,8}	3
s_2	{0,3,8}	16
s_3	{0,4,7,8,9}	8
s_4	{0,1,2}	13
s_5	{1,8}	11
s_6	{1,4}	5

The $(N_{s_k} - 1)t_{s_k}$ traffic units represent the total amount of traffic after linearly combining the traffic units transmitted by members of session s_k at the hub node of session s_l. The following constraint determines the total number of light-trees needed for session s_k.

$$LT_{s_k} \geq (\sum_{s_l} T^{s_l}_{s_k})/g \quad \forall s_k \tag{35}$$

VI. ILLUSTRATIVE NUMERICAL EXAMPLE

In this section, we provide a detailed numerical example from MILPs solutions. The example is conducted on the Abilene network (shown in Fig. 4.(b)) with $W = 6$ and $g = 16$. A sample traffic consisting of six many-to-many sessions is shown in Table I. Optimal solutions for both non-splitting and splitting network cases are obtained by solving the corresponding MILP using the CPLEX solver [24].

Table II illustrates the many-to-many sessions provisioning in the non-splitting network case. The second column of the table shows all the lightpaths traversed to deliver traffic between members in the corresponding session. For example, the traffic from member 0 to member 8 in session s_1 traverses lightpaths $0 \rightarrow 2$ and $2 \rightarrow 8$, while the traffic from member 1 to member 8 in session s_5 traverses lightpaths $1 \rightarrow 4,\ 4 \rightarrow 2$ and $2 \rightarrow 8$. Traffic streams traversing the same lightpath are groomed together on that lightpath. For example, the traffic streams from members 0 and 4 in session s_3 are groomed together on lightpath $4 \rightarrow 9$, while the traffic streams from members 0 and 1 in sessions s_1 and s_5, respectively are groomed together on lightpath $2 \rightarrow 8$. Totally, 26 lightpaths were established where some node pairs had two lightpaths between them ($8 \rightarrow 0$ and $2 \rightarrow 1$), which required a total of 52 transceivers.

Table III illustrates the many-to-many sessions provisioning in the splitting network case according to the hub-based approach. It shows the hub selected, the members-to-hub journey, and the hub-to-members journey for each session. For example, in the members-to-hub journey of session s_3, the traffic from member 7 to the hub 8 traverses lightpaths $7 \rightarrow 9$ and $9 \rightarrow 8$, while the traffic from member 0 to the hub 1 in the members-to-hub journey of session s_4 traverses lightpath $0 \rightarrow 1$. The hub-to-members journey of a session either traverses light-trees for that session or 1ight-trees for other sessions. For example, the hub-to-members journey of session s_3 traverses the two light-trees for s_3, while the hub-to-members journey of session s_5 traverses the light-tree for session s_6. Note that the light-tree for session s_6 ($8 \rightarrow \{1, 4\}$) grooms the linear combinations for both sessions s_5 and s_6. Note also that the lightpath $7 \rightarrow 9$ grooms the traffic from members 7 and 9 in session s_3, while the lightpath $0 \rightarrow 8$ grooms the two traffic streams that originate from member 0 in sessions s_1 and s_3. Totally, ten lightpaths and seven light-trees were established, which required a total of 45 transceivers. For sessions s_2, s_3 and s_4, two light-trees were established, while a single light-tree was established for session s_6. Note that the light-tree for session s_1 is simply a lightpath $8 \rightarrow 0$.

TABLE II
MANY-TO-MANY SESSIONS PROVISIONING IN THE NON-SPLITTING NETWORK CASE

Session	Lightpaths Traversed
s_1	$0 \rightarrow 2$, $2 \rightarrow 8$, $8 \rightarrow 1$, $1 \rightarrow 0$
s_2	$0 \rightarrow 8$, $8 \rightarrow 3$, $3 \rightarrow 0$, $3 \rightarrow 8$, $8 \rightarrow 0$, $0 \rightarrow 3$
s_3	$0 \rightarrow 4$, $0 \rightarrow 7$, $4 \rightarrow 8$, $4 \rightarrow 9$, $7 \rightarrow 0$, $7 \rightarrow 4$, $7 \rightarrow 9$, $8 \rightarrow 0$, $9 \rightarrow 7$, $9 \rightarrow 8$
s_4	$0 \rightarrow 2$, $1 \rightarrow 0$, $1 \rightarrow 2$, $2 \rightarrow 0$, $2 \rightarrow 1$
s_5	$1 \rightarrow 4$, $2 \rightarrow 8$, $4 \rightarrow 2$, $8 \rightarrow 1$
s_6	$1 \rightarrow 4$, $2 \rightarrow 1$, $4 \rightarrow 2$

TABLE III
MANY-TO-MANY SESSIONS PROVISIONING IN THE SPLITTING NETWORK CASE ACCORDING TO THE HUB-BASED APPROACH

Session	Hub Node	Members-to-Hub Journey	Hub-to-Members Journey
s_1	8	$0 \rightarrow 8$	$8 \rightarrow \{0\}$
s_2	3	$0 \rightarrow 3$, $8 \rightarrow 3$	$3 \rightarrow \{0, 8\}$
s_3	8	$0 \rightarrow 8$, $4 \rightarrow 8$, $7 \rightarrow 9$, $9 \rightarrow 8$	$8 \rightarrow \{0, 4, 7, 9\}$
s_4	1	$0 \rightarrow 1$, $2 \rightarrow 1$	$1 \rightarrow \{0, 2\}$
s_5	8	$1 \rightarrow 8$	$8 \rightarrow \{1, 4\}$
s_6	8	$1 \rightarrow 8$, $4 \rightarrow 8$	$8 \rightarrow \{1, 4\}$

VII. HEURISTIC SOLUTIONS

In this section, we introduce heuristic solutions for the many-to-many traffic grooming problem in both non-splitting and splitting networks.

A. Heuristic Solution for non-Splitting Networks

After careful examination of the MILP results for small and medium sized instances of the problem, we have made an observation on how many-to-many sessions tend to be provisioned in non-splitting networks. Before we state our observation, we make the following definition.

Definition 1. *A lightpath cycle (LC) for a many-to-many session s_k is a simple cycle of N_{s_k} lightpaths that visits each member in m_{s_k} exactly once.*

An example of a *LC* for a many-to-many session s_k with a set of members $m_{s_k} = \{A, B, C, D\}$ is shown in Fig. 3.(a). Note that the *LC* only describes a virtual topology and therefore it always contains N_{s_k} lightpaths regardless of the order of the members and regardless of the underlying physical topology. Next, we make the following observation.

Observation 1. *Many-to-many sessions in non-splitting networks tend to be provisioned through LCs, where for each session s_k, $(N_{s_k} - 1)t_{s_k}$ traffic units are groomed between each pair of consecutive members in the LCs.*

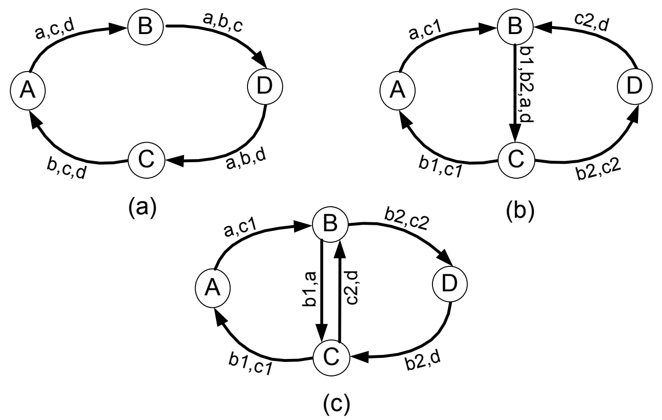

Fig. 3. (a): *LC* for a session s_k where $m_{s_k} = \{A, B, C, D\}$ each with one traffic unit denoted as a, b, c and d, respectively ($g = 3, H_{s_k} = 1$). (b): Provisioning of sessions s_1 and s_2, where $m_{s_1} = \{A, B, C\}$ each with one traffic unit denoted as a, b_1, c_1, and $m_{s_2} = \{B, C, D\}$ each with one traffic unit denoted as b_2, c_2, d ($g = 4$). The order of the members in the *LCs* for s_1 and s_2 is $A-B-C-A$ and $B-C-D-B$, respectively. (c): same as part (b) except that the order of the members in the *LCs* for s_1 and s_2 is $A-B-C-A$ and $B-D-C-B$, respectively.

Fig. 3.(a) demonstrates how session s_k traffic is routed on its *LC*. Each member transmits its traffic through the *LC* until it reaches the member just before it in the cycle. Using this routing strategy, we are always guaranteed that each member receives the traffic from the other $N_{s_k}-1$ members in the same session. In this particular example, we assumed that a single lightpath was sufficient to carry or groom the $(N_{s_k}-1)t_{s_k}$ traffic units between each pair of consecutive members (i.e., $H_{s_k} = 1$). As a general rule, we need H_{s_k} lightpaths to groom the $(N_{s_k}-1)t_{s_k}$ traffic units between each pair of consecutive members, and therefore we need H_{s_k} identically ordered *LCs* for s_k to provision session s_k traffic.

Since a lightpath may groom traffic from different sessions and not just traffic from different members within the same session, *LCs* of different sessions may share lightpaths. This introduces a correlation between *LCs* where the order of the members becomes significant and must be taken into account. Fig. 3.(b) clarifies this point by illustrating the provisioning of two many-to-many sessions s_1 and s_2 each with a set of members $m_{s_1} = \{A, B, C\}$ and $m_{s_2} = \{B, C, D\}$, respectively through *LCs*. Note that the *LC* for session s_1 ($A-B-C-A$) and the *LC* for session s_2 ($B-C-D-B$) share the lightpath $B \to C$. Precisely, the lightpath $B \to C$ grooms the two traffic units b_1, a belonging to session s_1 and the two traffic units b_2, d belonging to session s_2. Note that the order of the members in the *LCs* is significant. For example, if order of the members in the *LC* for s_2 is $B-D-C-B$ instead of $B-C-D-B$, then the two *LCs* for s_1 and s_2 will not share a lightpath and we would require six lightpaths instead of five (see Fig. 3.(c)).

The above observation is the basis for designing our heuristic for the many-to-many traffic grooming problem in non-splitting networks. In the heuristic, we assume that every many-to-many session s_k is provisioned through H_{s_k} identically ordered *LCs* for s_k. Although this assumption may not result in an optimal solution, assuming it always holds, as we shall see, will lead to near optimal solutions. Based on this assumption, we just need to determine the order of the members in the sessions' *LCs* and then route the traffic on the *LCs* as described before (see Fig. 3.(a)). Note that, between each pair of nodes i and j, the heuristic grooms the $\sum_{s_k}(N_{s_k}-1)t_{s_k}$ traffic units for all sessions s_k where $i, j \in m_{s_k}$ and member j follows member i immediately in the session's LCs.

TABLE IV
MANY-TO-MANY SESSIONS PROVISIONING IN THE NON-SPLITTING NETWORK CASE USING THE HEURISTIC MILP

Session	Lightpaths Traversed
s_1	$0 \to 8$, $8 \to 0$
s_2	$0 \to 3$, $3 \to 8$, $8 \to 0$
s_3	$0 \to 9$, $9 \to 8$, $8 \to 4$, $4 \to 7$, $7 \to 0$
s_4	$0 \to 1$, $1 \to 2$, $2 \to 0$
s_5	$1 \to 8$, $8 \to 1$
s_6	$1 \to 4$, $4 \to 1$

Applying the above observation to the MILP means a significant simplification, since we do not need to consider the sessions' traffic routing once we determine the order of the members in the sessions' *LCs*. Therefore, we no longer require the $Z_{ij}^{s_k,p,q}$ and $Y_{ij}^{s_k,p}$ variables, however, we require the following two new variables.
$C_{p,q}^{s_k}$: binary number equals to 1 if member $p \in m_{s_k}$ is followed immediately by member $q \in m_{s_k}$ in the *LCs* for session s_k; otherwise it is set to 0.
$u_p^{s_k}$: an arbitrary real number.

The lightpath level constraints remain unchanged, while the session level constraints are replaced by the following set of constraints.
Session Level Constraints:

$$\sum_{q \neq p} C_{p,q}^{s_k} = \sum_{q \neq p} C_{q,p}^{s_k} = 1 \quad \forall s_k,\ p \in m_{s_k} \tag{36}$$

$$\forall s_k,\ p \in (m_{s_k} - m_{s_k}[0]),\ q \in m_{s_k}(q \neq p):$$

$$u_p^{s_k} - u_q^{s_k} + N_{s_k} C_{p,q}^{s_k} \leq N_{s_k} - 1 \tag{37}$$

$$L_{ij} \geq \sum_{s_k} (N_{s_k} - 1) t_{s_k} C_{i,j}^{s_k} / g \quad \forall i, j \tag{38}$$

Constraint (36) determines the order of the members in each session's *LCs*, while constraint (37) ensures that a *LC* for session s_k must include all members in m_{s_k} ($m_{s_k}[0]$ represents the first member in m_{s_k}). In other words, constraint (37) eliminates all sub-*LCs* (*LCs* that visit only a subset of the members). Constraint (38) computes the total number of lightpaths needed between each pair of nodes in the network. It calculates the total traffic from node i to node j as the aggregate traffic from all sessions who have member i followed immediately by member j in their *LCs*.

Although this heuristic approach remains an MILP, it will be shown that it is a practical one that leads to near optimal solutions of large networks in a reasonable time. Solving the example in Section VI using this heuristic MILP, we obtain the many-to-many sessions provisioning shown in Table IV. Totally, 28 lightpaths were established where some node pairs had two lightpaths between them ($0 \to 1,\ 2 \to 0,\ 0 \to 3,\ 7 \to 0,\ 0 \to 9,\ 1 \to 2,\ 3 \to 8,\ 4 \to 7,\ 8 \to 4,\ 9 \to 8$) and some node pairs had three lightpaths between them ($8 \to 0$), which required a total of 56 transceivers compared to 52 transceivers in the optimal solution in Section VI. Note that sessions s_1, s_5 and s_6 are provisioned by a single *LC*, while sessions s_2, s_3 and s_4 are provisioned by two *LCs*.

The solution from the heuristic MILP was obtained in 2.5 seconds, while the optimal solution in Section VI was obtained in almost one hour and six minutes. This is a significant reduction in the running time, while still obtaining near optimal solutions (7.7% more than the optimal solution).

B. Heuristic Solution for Splitting Networks

After careful examination of the MILP results for small and medium sized instances of the problem, we have made observations on how many-to-many sessions tend to be provisioned in splitting networks according to the hub-based approach.

Observation 2. *The hub for a many-to-many session is usually selected from its set of members.*

Observation 3. *A member-to-hub stream in the members-to-hub journey of a many-to-many session usually traverses a single direct lightpath from the member to the hub.*

Observation 4. *Light-trees usually do not groom traffic from different sessions; they only groom the linear combinations for the corresponding session.*

The heuristic we propose is based on the assumption that these observations always hold. Although this assumption may not result in an optimal solution, assuming it always holds, as we shall see, will lead to near optimal solutions. Applying the above observations to the MILP means a significant simplification.

Number of Transceivers Constraints:
Since the hub for a many-to-many session s_k can only be selected from its set of members m_{s_k}, a light-tree for s_k places one transceiver at each member in m_{s_k} and does not place a transceiver at any other node. Therefore, we no longer require the non-linear variable $A_h^{s_k}$ and the number of transceivers constraints are replaced by the following constraint.

$$TR_i \geq \sum_{j:j\neq i}(L_{ij}+L_{ji}) + \sum_{s_k} LT_{s_k} B_i^{s_k} \quad \forall i \tag{39}$$

The lightpath/light-tree level constraints remain unchanged.
Hub Node Selection Constraints:
Since the hub for a many-to-many session s_k can only be selected from its set of members m_{s_k}, we no longer require the variable $I_h^{s_k}$ to be defined for all $h \in V$, rather it is defined only for $h \in m_{s_k}$. Also, since light-trees do not groom traffic form different sessions, we no longer require the $E_{s_l}^{s_k,h}$ and $E_{s_l}^{s_k}$ variables. Accordingly, the hub selection constraints are replaced by the following constraint, which ensures that there is exactly one hub node for each session chosen from its set of members.

$$\sum_{h\in m_{s_k}} I_h^{s_k} = 1 \quad \forall s_k \tag{40}$$

Members-to-Hub Journey Constraints:
Assuming that a member-to-hub stream traverses a single direct lightpath from the member to the hub, we no longer require the $D_{ij}^{s_k,p}$ variables. Accordingly, the members-to-hub journey constraints are replaced by the following constraint.

$$L_{ij} \geq (\sum_{s_k} t_{s_k} I_j^{s_k} B_i^{s_k})/g \quad \forall i,j\ (i\neq j) \tag{41}$$

The above constraint ensures that if node $i \in m_{s_k}$ and node j is the hub for s_k, then lightpaths from i to j carry the t_{s_k} traffic units that originate from member i.
Hub-to-Members Journey Constraints:
Since light-trees do not groom traffic from different sessions and they only groom the linear combinations for the corresponding session, we no longer require the $U_{s_l}^{s_k}$ and $T_{s_l}^{s_k}$ variables. Accordingly, the hub-to-members journey constraints are replaced by the following constraint.

$$LT_{s_k} = H_{s_k} \quad \forall s_k \tag{42}$$

Although this heuristic approach remains an MILP, it will be shown that it is a practical one that leads to near optimal solutions of large networks in a reasonable time. Solving the example in Section VI using this heuristic MILP, we obtain the many-to-many sessions provisioning shown in Table V. It shows the hub selected, the members-to-hub journey, and the hub-to-members journey for each session. Totally, 13 lightpaths and 6 light-trees were established, which required a total of 48 transceivers compared to 45 transceivers in the optimal solution in Section VI. For sessions s_2, s_3 and s_4, two light-trees were established, while the light-trees for sessions s_1, s_5 and s_6 were simply the lightpaths $8 \to 0$, $1 \to 8$ and $1 \to 4$, respectively.

TABLE V
MANY-TO-MANY SESSIONS PROVISIONING IN THE SPLITTING NETWORK CASE USING THE HEURISTIC MILP

Session	Hub node	members-to-hub journey	hub-to-members journey
s_1	8	$0 \to 8$	$8 \to \{0\}$
s_2	3	$0 \to 3$, $8 \to 3$	$3 \to \{0,8\}$
s_3	8	$0 \to 8$, $4 \to 8$, $7 \to 8$, $9 \to 8$	$8 \to \{0,4,7,9\}$
s_4	0	$1 \to 0$, $2 \to 0$	$0 \to \{1,2\}$
s_5	1	$8 \to 1$	$1 \to \{8\}$
s_6	1	$4 \to 1$	$1 \to \{4\}$

The solution from the heuristic MILP was obtained in 13 seconds, while the optimal solution in Section VI was obtained in almost two hours and one minute. This is a significant reduction in the running time, while still obtaining near optimal solutions (6.7% more than the optimal solution).

The advantage of using network coding in this heuristic is the reduction of downstream traffic for each session s_k from $N_{s_k} t_{s_k}$ to $(N_{s_k}-1)t_{s_k}$ traffic units. The total number of transceivers saved due to the use of network coding (R_{saved}) is equal to the total number of light-trees saved for each session s_k ($\lceil N_{s_k} t_{s_k}/g\rceil - \lceil (N_{s_k}-1)t_{s_k}/g\rceil$) times the total number of transceivers per light-tree for that session (N_{s_k}), which is indicated by the following equation:

$$TR_{saved} = \sum_{s_k} N_{s_k}(\lceil N_{s_k} t_{s_k}/g\rceil - \lceil (N_{s_k}-1)t_{s_k}/g\rceil) \tag{43}$$

Note that the value of R_{saved} in the optimal approach can only be determined by solving the optimal MILP with network coding (downstream traffic for each session s_k is $(N_{s_k}-1)t_{s_k}$) and without network coding (downstream traffic for each session s_k is $N_{s_k}t_{s_k}$) and then taking the difference.

C. Complexity Analysis

The complexity of the optimal MILP for non-splitting networks in terms of the number of integer variables is $O(K|V|^4 + W|E||V|^2)$, and in terms of the number of constraints is $O((K+W)|V|^3)$. The complexity of the optimal MILP for splitting networks in terms of the number of integer variables is $O(K|V|^3 + KW|E||V| + K^2|V| + W|E||V|^2)$, and in terms of the number of constraints is $O(W|V|^3 + KW|V|^2 + K^2|V|)$.

The complexity of the heuristic MILP for non-splitting networks in terms of the number of integer variables is $O(K|V|^2 + W|E||V|^2)$, and in terms of the number of constraints is $O(W|V|^3 + K|V|^2)$. The complexity of the heuristic MILP for splitting networks in terms of the number of integer variables is $O(KW|E||V| + W|E||V|^2)$, and in terms of the number of constraints is $O(W|V|^3 + KW|V|^2)$.

VIII. Numerical Results

A. Heuristics Performance

To verify the performance of our proposed heuristics, we conduct a number of experiments on small, medium and large sized networks. Five experiments are conducted on a small sized network (the 6-node network shown in Fig. 4.(a)). The number of sessions in each experiment is randomly selected between [2,4]. The number of members in a session is randomly selected between [2,5], while members are randomly selected between [0,5]. Another five experiments are conducted on a medium sized network (the Abilene network shown in Fig. 4.(b)). The number of sessions in each experiment is randomly selected between [4,6]. The number of members in a session is randomly selected between [2,5], while members are randomly selected between [0,9]. Another five experiments are conducted on a large sized network (the NSF network shown in Fig. 4.(c)). The number of sessions in each experiment is randomly selected between [6,8]. The number of members in a session is randomly selected between [2,5], while members are randomly selected between [0,13]. Finally, traffic demand of members in a session, in all the 15 experiments, is randomly selected between [1,16] ($g = 16$).

For both non-splitting and splitting network cases, we solve each of the 15 experiments using the optimal MILP and the heuristic MILP. Tables VI and VII present the results by showing the average running time and the average number of transceivers for the five experiments on each of the three topologies in non-splitting and splitting network cases, respectively.

We can see from tables VI and VII that solutions from the heuristics on the 6-node network are significantly close to their corresponding optimal solutions. For example, in the non-splitting network case, they are, on average, 6.2% of their corresponding optimal solutions, while in the splitting network case they are, on average, 5.5% of their corresponding optimal solutions.

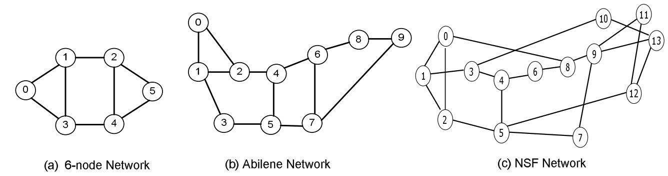

Fig. 4. networks used in the results

TABLE VI
Average running time and average number of transceivers for the 5 experiments conducted on each of the 6-node, Abilene and NSF networks in the non-splitting network case

Network	MILP	Avg. Run Time	Avg. # of TRs
6-node	Optimal MILP	15 hours	46.2
	Heuristic MILP	6 seconds	49.1
Abilene	Optimal MILP	108 Hours	62.4
	Heuristic MILP	3 Minutes	66
NSF	Optimal MILP	>150 hours	No Solution
	Heuristic MILP	1 Hour	88.8

TABLE VII
Average running time and average number of transceivers for the 5 experiments conducted on each of the 6-node, Abilene and NSF networks in the splitting network case (hub-based approach)

Network	MILP	Avg. Run Time	Avg. # of TRs
6-node	Optimal MILP	6 hours	41
	Heuristic MILP	33 seconds	43.2
Abilene	Optimal MILP	57 Hours	54.4
	Heuristic MILP	4 Minutes	57
NSF	Optimal MILP	>150 hours	No Solution
	Heuristic MILP	2 Hours	78.6

In some experiments on the Abilene network, we could not obtain the optimal solution after 150 hours of running time at which we have terminated the CPLEX program and recorded the best feasible solution. The largest gap we have encountered between the best feasible solution and the best lower bound found by CPLEX was only 3%. This means that the best feasible solutions obtained were very close to their corresponding optimal solutions. We can see from tables VI and VII that solutions from the heuristics on the Abilene network are significantly close to their corresponding optimal (or best feasible) solutions. For example, in the non-splitting network case, they are, on average, 5.8% of their corresponding optimal (or best feasible) solutions, while in the splitting network case they are, on average, 4.7% of their corresponding optimal (or best feasible) solutions.

In the NSF experiments, the CPLEX program did not return a feasible solution for any of the five experiments (using the optimal MILP) after 150 hours of running time at which we have terminated the program. On the other hand, the heuristic MILPs for both non-splitting and splitting network cases were able to return solutions in a reasonable time.

B. Comparisons

In this subsection, non-splitting and splitting networks will be compared in terms of the number of transceiver needed

(TR). We will show when each of these two networks is a more cost-effective choice (requires fewer transceivers) for many-to-many traffic grooming. Since the grooming capabilities of the two networks are varied, their performance will be dependent on traffic granularities of sessions in the network. Therefore, we should compare them for different traffic granularities.

1) *Uniform Traffic:* We assume a static uniform traffic with all sessions in an experiment having the same traffic demand t (i.e., $t_{s_1} = t_{s_2} = ... = t_{s_K} = t$), where $1 \leq t \leq g$.

Fifteen randomly generated experiments are conducted on the Abilene network. The number of sessions in each experiment is randomly selected between [2,6]. The size of each session is randomly selected between [2,5], while a member in a session is randomly selected between [0,9]. Based on the uniform traffic assumption, each of the fifteen experiments is conducted for each value of $t = 1, 2.., g$ ($g = 16$) on both non-splitting and splitting networks using the corresponding heuristic MILP. We define $\overline{TR}$ to be the average value of all TR values obtained from the fifteen experiments at a particular value of t on a certain network. The resulting values of $\overline{TR}$ are shown in Fig. 5.

From Fig. 5, we draw the following conclusions:

- non-splitting networks are more cost-effective when traffic granularities of sessions are relatively low ($t \leq g/4$). The intuition behind this is that lightpaths are more efficient than light-trees in grooming and packing low granularity traffic. This is a result of the point-to-point nature of a lightpath where it is possible to route many sessions or members with sub-wavelength granularities through it. Note that, contrary to a lightpath, it is not easy to route many sessions with sub-wavelength granularities through a point-to-multipoint channel (i.e., a light-tree).
- splitting networks are more cost-effective for almost three quarters of the traffic granularities spectrum ($t > g/4$). The intuition behind this is that when traffic granularities of sessions are relatively high, inter-session grooming is rarely performed and in that case light-trees are more cost-effective than lightpaths. For example, a light-tree from a source to a set of destinations requires fewer transceivers than a set of lightpaths each from the source to one of the destinations. Also, the use of network coding in splitting networks has a direct impact on reducing the number of needed light-trees, and hence the number of transceivers.

2) *Non-Uniform Traffic:* Although the above conclusions are drawn from the uniform traffic assumption, we will now show that they remain valid even when traffic demands of user sessions are non-uniform. In this case, however, we define $\bar{t}$ to be the average amount of traffic demanded by a member in an experiment, which is expressed by the following equation:

$$\bar{t} = \sum_{s_k} N_{s_k} t_{s_k} / \sum_{s_k} N_{s_k}$$

We claim that the above conclusions remain valid for different values of $\bar{t}$. To verify this, we randomly generate six experiments on the Abilene network with the same parameters as the fifteen experiments generated earlier, however, the traffic demand of members in a session is now randomly selected between [1,16] (non-uniform traffic). Each of the experiments is conducted in both non-splitting and splitting networks using the corresponding heuristic MILP. The resulting values of TR are shown in Table VIII.

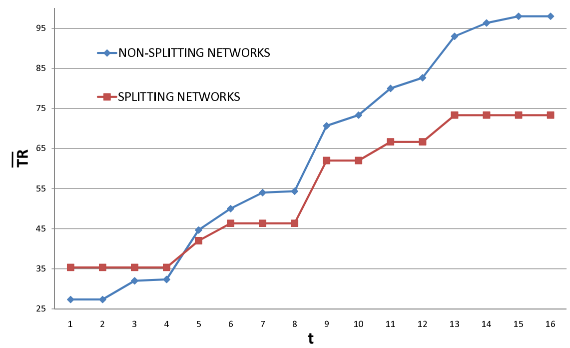

Fig. 5. Values of $\overline{TR}$ for $t = 1, 2..., g$ on the Abilene network ($g = 16$)

TABLE VIII
NUMBER OF TRANSCEIVERS (TR) COMPARISON ON THE ABILENE NETWORK WITH NON-UNIFORM TRAFFIC

Exp #	1	2	3	4	5	6
$\bar{t}$	2.1	3.6	7.7	9.5	11.1	13.8
non-splitting networks	28	38	48	56	58	70
splitting networks	35	40	45	50	50	55

We can see from the table that non-splitting networks are more cost-effective when $\bar{t} \leq g/4$ (Exps. 1 and 2) , while splitting networks are more cost-effective when $\bar{t} > g/4$ (Exps. 3, 4, 5 and 6).

C. *Advantage of Network Coding in Splitting Networks*

To illustrate the advantage of network coding in reducing the number of transceivers in splitting networks, we compute the values of TR_{saved} for each of the fifteen uniform traffic experiments at each value of $t = 1, 2, 3..., 16$ using Eq. (43). We define $\overline{TR_{saved}}$ to be the average value of all TR_{saved} values obtained from the fifteen experiments at a particular value of t. Table IX shows the corresponding values of $\overline{TR_{saved}}$ and the corresponding percentage savings due to the use of network coding ($\overline{TR_{saved}}/\overline{TR}$).

IX. CONCLUSIONS

In this paper, we have studied the many-to-many traffic grooming problem in both non-splitting and splitting WDM networks. In non-splitting networks, we have introduced an MILP formulation and then a heuristic solution based on observations from optimal solutions. In splitting networks, we have introduced a novel hub-based approach that combines optical splitting and network coding to provision many-to-many sessions. We have also introduced an MILP formulation and a heuristic solution for the hub-based approach. Through extensive experiments, we have verified the accuracy and the performance of the proposed heuristics and concluded that each of the two networks is a cost-effective choice for a certain range of traffic granularities. For example, we have shown that non-splitting networks are cost-effective for low

TABLE IX
VALUES OF $\overline{TR_{saved}}$ AND ($\overline{TR_{saved}}/\overline{TR}$) FOR $t = 1, 2..., g$ ($g = 16$) ON THE ABILENE NETWORK

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\overline{TR_{saved}}$	0	0	0	2.7	1.6	5.2	7.9	7.9	3.7	6.4	10	10	11.6	11.6	11.6	11.6
$\overline{TR_{saved}}/\overline{TR}$	0%	0%	0%	7.6%	3.8%	11.2%	17%	17%	6%	10.3%	15%	15%	15.8%	15.8%	15.8%	15.8%

traffic granularities ($t \leq g/4$), while splitting networks are cost-effective for high traffic granularities ($t > g/4$). Finally, we have illustrated the advantage of network coding in reducing the number of transceivers in splitting networks through extensive experiments.

ACKNOWLEDGMENT

This research was supported in part by grant CNS-0626741 from the National Science Foundation.

REFERENCES

[1] L. H. Sahasrabuddhe and B. Mukherjee,"Light-Trees: Optical Multicasting for Improved Performance in Wavelength Routed Networks," *IEEE Communications Magazine,* vol. 37, no. 2, pp. 67-73, Feb. 1999.

[2] A. L. Chiu and E. H. Modiano, "Traffic grooming algorithms for reducing electronic multiplexing costs in WDM ring networks" *IEEE J. Lightwave Technol.* vol. 18, no. 1, pp. 2-12, Jan. 2000.

[3] C. Diot et al."Multipoint Communication: A Survey of Protocols, Functions, and Mechanisms," *IEEE J. Select. Areas Commun.,* vol. 15, pp. 277-290, 1997.

[4] B. Quinn and K. Almeroth, "IP multicast applications: challenges and solutions," IETF Request for Comments (RFC) 3170, Sept. 2001.

[5] O. Gerstel, R. Ramaswami, and G. Sasaki, "Cost-effective Traffic Grooming in WDM Rings," *IEEE/ACM Transactions on Networking,* vol. 8, no. 5, pp. 618-630, Oct. 2000.

[6] X. Zhang and C. Qiao,"An effective and comprehensive approach for traffic grooming and wavelength assignment in SONET/WDM rings," *IEEE/ACM Trans. Networking,* vol. 8, no. 5, pp. 608-617, Oct. 2000.

[7] K. Zhu and B. Mukherjee,"Traffic Grooming in an optical WDM Mesh Network," *IEEE Journal on Selected Areas in Communications,* vol. 20, no. 1, pp. 122-133, Jan 2002.

[8] J.Q. Hu and B. Leida, "Traffic grooming, routing, and wavelength assignment in optical WDM mesh networks," in *Proc. IEEE Infocom'04* pp. 495-501.

[9] S. Antonakopoulos and L. Zhang,"Approximation Algorithms for Grooming in Optical Network Design," in *Proc. IEEE Infocom'09* pp. 1548-1556.

[10] B. Chen, G. Rouskas, and R. Dutta, "On Hierarchical Traffic Grooming in WDM Networks," *IEEE/ACM Transactions on Networking,* vol. 16, no. 5, pp. 1226-1238, Oct. 2008.

[11] R. Dutta and G. N. Rouskas,"Traffic grooming in WDM networks: past and future," *IEEE Network,* vol. 16, no. 6, pp. 46-56, Nov./Dec. 2002.

[12] P-J. Wan, G. Calinescu, L. Liu, and O. Frieder, "Grooming of Arbitrary Traffic in SONET/WDM BLSRs. *IEEE J. Select. Areas Commun.,* vol. 18, pp. 1995-2003, 2000.

[13] H. Zhu, H. Zang, K. Zhu, and B. Mukherjee,"A novel generic graph model for traffic grooming in heterogeneous WDM mesh networks," *IEEE/ACM Trans. Networking,* vol. 11, no. 2, pp. 285-299, April 2003.

[14] R. Dutta, S. Huang, and G. N. Rouskas, "Traffic grooming in path, star, and tree networks: Complexity, bounds, and algorithms," *IEEE J. Sel. Areas Commun.* vol. 24, no. 4, pp. 66-82, Apr. 2006.

[15] H. Madhyastha et al., "Grooming of multicast sessions in metropolitan WDM ring networks," *Computer Networks,* vol. 49, pp. 561-579, 2005.

[16] A. Rawat, R. La, S. Marcus, and M. Shayman, "Grooming Multicast Traffic in Unidirectional SONET/WDM Rings," *IEEE Journal on Selected Areas in Communication,* vol. 25, no. 6, pp. 70-83, August 2007.

[17] R. Ul-Mustafa and A. E. Kamal,"Design and provisioning of WDM networks with multicast traffic grooming," *IEEE J. Select. Areas Commun.,* vol. 24, no. 4, pp. 37-53, Apr. 2006.

[18] A. Billah, B. Wang, and A. Awwal, "Multicast traffic grooming in WDM optical mesh networks," in *Proc. GLOBECOM'03,* vol. 5, pp. 2755- 2760, Dec. 2003.

[19] G. Chowdhary and C. S. R. Murthy,"Grooming of multicast sessions in wdm mesh networks," in *Workshop on Traffic Grooming,* 2004.

[20] R. Ul-Mustafa and A. E. Kamal,"Many-to-one Traffic Grooming with Aggregation in WDM Networks " *IEEE Journal on Selected Areas in Communication,* vol. 24, no. 8, pp. 68-81, August 2006.

[21] A. E. Kamal "Algorithms for Multicast Traffic Grooming in WDM Mesh Networks," *IEEE Communications* pp. 96-105, Nov. 2006.

[22] M. Saleh and A. Kamal, "Many-to-Many Traffic Grooming in WDM Mesh Networks," in *proc. IEEE Globecom'08* pp. 1-5.

[23] R. Ahlswede, N. Cai, S. R. Li, and R. W. Yeung,"Network information flow" *IEEE Trans. Inf. Theory* vol. 46, no. 4, pp. 1204-1216, Jul. 2000.

[24] http://www.ilog.com/products/cplex/

Mohammad A. Saleh received his B.Sc. in Computer Engineering from Jordan University of Science and Technology (JUST), Jordan, in 2005. He is currently working towards his Ph.D. in Computer Engineering at Iowa State University, USA. His research interests include optical WDM networks, wireless networks and network survivability. He is a student member of the IEEE.

Ahmed E. Kamal (S'82-M'87-SM'91) received a B.Sc. (distinction with honors) and an M.Sc. both from Cairo University, Egypt, and an M.A.Sc. and a Ph.D. both from the University of Toronto, Canada, all in Electrical Engineering in 1978, 1980, 1982 and 1986, respectively. He is currently a professor of Electrical and Computer Engineering at Iowa State University. Earlier he held faculty positions in the Department of Computing Science at the University of Alberta, Canada, and the Department of Computer Engineering at Kuwait University, Kuwait. He was also an adjunct professor at the Telecommunications Research Labs, Edmonton, Alberta.

Kamal's research interests include high-performance networks, optical networks, wireless and sensor networks and performance evaluation. He is a senior member of the IEEE, a senior member of the Association of Computing Machinery, and a registered professional engineer. He was the co-recipient of the 1993 IEE Hartree Premium for papers published in Computers and Control in IEE Proceedings for his paper entitled Study of the Behaviour of Hubnet, and the best paper award of the IEEE Globecom 2008 Symposium on Ad Hoc and Sensors Networks Symposium. He served on the technical program committees of numerous conferences and workshops, was the organizer and co-chair of the first and second Workshops on Traffic Grooming 2004 and 2005, respectively, and was the co-chair of the Technical Program Committees of a number of conferences including the Communications Services Research (CNSR) conference 2006, the Optical Symposium of Broadnets 2006, and the Optical Networks and Systems Symposium of the IEEE Globecom 2007, the 2008 ACS/IEEE International Conference on Computer Systems and Applications (AICCSA-08), and the ACM International Conference on Information Science, Technology and Applications, 2009. He is also the Technical Program co-chair of the Optical Networks and Systems Symposium of the IEEE Globecom 2010. He is on the editorial boards of the Computer Networks journal, and the Journal of Communications.